APPLICATION OF ADDITIVE REGIONAL KALMAN FILTERING
TO X-RAY IMAGES IN NDE

John P. Basart, Yi Zheng, and Edward R. Doering

Center for NDE
Iowa State University
Ames, Iowa 50011

INTRODUCTION

One of the time consuming procedures in inspecting parts by x-ray film is the identification of a flaw. Low contrast films of dense objects especially cause problems. A radiologist must have considerable experience in identification in order to keep the examination time relatively small, but also keep the reliability high. Our objective in this project is to develop a computer procedure that will sufficiently enhance flaws in an image in a manner that will reduce the time it takes a human to locate and identify a flaw.

Factors limiting the quality of an X-ray image are image unsharpness, quantum fluctuation, film grain and film contrast [1,2]. The unsharpness caused by scattered radiation reduces the image contrast. The quantum fluctuation caused by random emission and absorption of X-ray quanta smears or masks the contrast. The film grain and contrast limit the recorded information capacity. A coarse-grained image conveys less detail than one of fine grain. In this paper, we discuss a method for enhancing the image by reducing the fluctuation due to disturbances, such as quantum fluctuation and granularity, etc. The main tool used is the Kalman filter. The basic idea is to estimate a pixel optimally in an image using a given pixel and its near neighbors. One advantage of a Kalman filter is that it incorporates information about every aspect of the process. It can include a model of the process that generated the desired information, a model of the noise added to this process, a model of the measurement system, and a model of the noise within the measurement system. In addition, there can be multiple models representing multiple processes at any one, or all, of these stages. Another advantage of the Kalman filter is that it can distinguish between stochastic processes that have strongly overlapping spectra. Ordinary spectral filters are of limited benefit under such conditions. When processing noisy images one often finds that the noise, system, and signal processes overlap in frequency.

Three steps are involved when implementing our method of filtering. They are 1) image segmentation, 2) image modeling, and 3) Kalman filtering. Each of these procedures will be explained. The results of filtering a low contrast flaw in an x-ray image will be discussed at the end of the article.
SEGMENTATION

Segmentation is an image classification procedure. Autoregressive modeling, which we incorporate in our method, requires stationarity. Generally, the stationarity assumption is not true for the processes in an image over the whole image and this violation will cause blurred edges and reduced contrast in a filtered image. Therefore segmentation is necessary to find regions in which the statistics, mean and variance, are stationary.

An image is segmented by partitioning it with respect to local mean and local spatial activity of the image [3,4]. Spatial activity is defined as the rate of change of spatial luminance from one pixel to another. It is related to the concept of variance. The formula used to calculate the spatial activity is called the masking function. Regions of stationary mean and stationary variance can be found by segmenting an image by local means and by the masking function, respectively. With these two segmentations in hand, they can be combined to produce new segments that are wide-sense stationary.

Local means are found by a window of running average. A \((2n+1) \times (2n+1)\) window is selected in one corner of the image. All pixels within the window are averaged. This average is assigned to the center pixel. The window is then moved and the process is repeated. The mathematical expression for the local mean is

\[
m_n(i,j) = \frac{1}{(2n+1)^2} \sum_{p=i-n}^{i+n} \sum_{q=j-n}^{j+n} z(p,q)
\]

where \(z(p,q)\) is the image intensity at pixel \(p,q\). After calculation of the mean for all windows, a file of the local means is set aside for later use.

The next step is to determine the masking function for the image. The masking function is defined by

\[
M_r(i,j) = \frac{1}{(2n+1)^2} \sum_{p=i-r}^{i+r} \sum_{q=j-r}^{j+r} e^{-\frac{1}{2} (x,y) - (p,q) - \frac{1}{7} \sum_{q=0}^{7} D_{pqn}}
\]

where \(\| (x,y) - (p,q) \|\) is the Euclidean distance between points \((x,y)\) and \((p,q)\), \((x,y)\) is center pixel of a window, \((p,q)\) is any other point in the window, and \(D\) is the difference in intensity between a pixel adjacent to \((p,q)\) and the pixel at \((p,q)\). The difference, \(D\), is summed over all pixels adjacent to \((p,q)\). The average of these differences is weighted exponentially by the distance from \((p,q)\) to \((x,y)\). After the masking function is calculated for all the windows, it is recorded in a file.

The next step is to use the local means and masking function to segment the image. A cluster seeking procedure, somewhat similar to the K-means cluster seeking algorithm [5], is used to cluster local means and masking functions. It differs from the standard K-means cluster seeking algorithm in that the thresholds of the distance between the cluster center are given for simplicity. Each local mean and masking function is assigned to a certain cluster. All combinations of local mean clusters and masking clusters form wide-sense stationary regions which we desire.
After completing the segmentation, the process in each segmented region is represented by a p-order AR process:

$$s(k) = \sum_{n=1}^{p} \phi_n s(k-n) + w(k)$$

(3)

$$z(k) = Hs(k) + v(k)$$

(4)

where $z(k)$ is the measurement of intensity at a pixel, $v(k)$ is an additive measurement white-noise sequence, $s(k)$ is a "true image" process, $w(k)$ is a residual sequence, $H$ is a $(1 \times m)$ measurement vector and $s(k)$ is an $(m \times 1)$ vector of $s(k)$. $v(k)$ and $w(k)$ are independent and uncorrelated with $E[v(k)] = 0$, $E[w(k)] = 0$, $E[v(k)w(h)] = 0$, $E[v(k)v(k-h)] = R\delta(h)$, and $E[w(k)w(k)] = Q\delta(h)$. $\phi$'s are coefficients to be estimated. There are a number of ways to estimate $\phi$'s such as maximum likelihood or least squares approaches [6]. "Marquardt's compromise" [7,8] and Yule-Walker equation [6] methods are often used in practice. We estimate the $\phi$'s by solving the Yule-Walker equation

$$r_0 \phi = r_1$$

(5)

where

$$\phi = [\phi_1 \phi_2 \phi_3 \cdots \phi_p]^T$$

$$r_1 = [r_1 r_2 r_3 \cdots r_p]^T$$

$$r_0 = \begin{bmatrix}
  r_0 & r_1 & \cdots & r_{p-1} \\
  r_1 & r_0 & & \\
  r_{p-1} & \cdots & r_0
\end{bmatrix}$$

The $r$'s are autocorrelation coefficients of $s(k)$. Given the measured $z(k)$'s and the variance $R$ of $v(k)$, the $r$'s can be found by taking the expectation of (4). The semi-positive definite property of the $r$'s must be considered when the $r$'s are calculated [9].

After a state-space form of (3) is obtained [10], we are ready to apply the Kalman filter.

KALMAN FILTERING

The Kalman filter is an optimal filter that can separate two or more stochastic process. The Kalman filter theory and applications can be found in many sources [11,12,13].

Since all quantities required for Kalman filtering have now been found, the optimal estimates of pixels are obtained by the following recursive procedure:
1. Enter the recursive loop with the initial values of the a priori estimated \( n \times 1 \) vector \( s(k|k-1) \) and its error covariance matrix \( P(k|k-1) \).

2. Compute the Kalman gain

\[
K(k|k) = P(k|k-1)H^T(HP(k|k-1)H^T + R)^{-1}
\] (6)

3. Estimate a pixel

\[
s(k|k) = s(k|k-1) + K(k|k)(z(k) - Hs(k|k-1))
\] (7)

4. Compute the error covariance matrix

\[
P(k|k) = (I - K(k|k)H)P(k|k-1)
\] (8)

5. Predict

\[
s(k+1|k) = \Phi s(k|k)
\] (9)

\[
P(k+1|k) = \Phi P(k|k)\Phi^T + Q
\] (10)

The process is repeated for the next pixel \( z(k+1) \) from step 2 until all pixels are processed. One should be careful that the Kalman equations are simplified due to the scalar modeling.

RESULTS

By applying the above procedure to low-contrast X-ray images, we have produced enhanced images. One example of a processed image is shown here. It is an 88x88 pixel subimage of an X-ray image of a casting. There is a flaw located near the center area of the image. The flaw is not obvious in the original image which is very dense and has low contrast. The contour map of the original image is shown in Fig. 1(a). The variance of the disturbance fluctuation measured from a flat area in the original image is 1.6. The result from filtering is shown in the contour map in Fig. 1(b). Since the dynamic ranges of the images are too small (about 20 to 30), histogram equalization with an exponential transformation function was applied to both the original and filtered images. Ruled surface plots of the original and the transformation results are shown in Fig. 2(a) and Fig. 2(b), respectively. The dynamic range in Fig. 2 has increased to 128. In the original image, the flaw region is broken into many spikes which make flaw detection difficult. The filtered image shows a bigger concentration of intensity within a region that can be defined by a single boundary. Compared with the original image, the filtered one has a lower and smoother background. Thus the flaw in the filtered image is easily detected now.

The experiment was done on an ISU VAX 11/780 computer. The CPU time for running the Kalman filter part was about 8 minutes (88x88 pixels) using a moving window. The modeling and filtering were applied to a 7x7 moving window and the 3x3 pixels in the center of the window were saved each time.
Fig. 1. Contour plots of intensity before (a) and after (b) filtering. Contour levels are the same in both plots. The flaw in the filtered map (b) clearly stands out above the background.

Fig. 2. Ruled surface plots before (a) and after (b) filtering. In the filtered map (b), the background noise is lowered and smoothed, and the power in the flaw is more centralized and less "spikey" than in the original map.
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REFERENCES


