On the Differences Between the Commercial and Economic Profitability of a Project

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Abstract
The economic profitability of a project is derived from its commercial profitability by making two adjustments. First, an adjustment needs to be made for the change in the sum of consumers' surplus and factor rents. Second, an adjustment has to be made for the change in tax revenues and subsidies. Conventional practice ignores the former adjustment and mistakenly assumes that the taxes or subsidies paid by the project

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The economic profitability of a project is derived from its commercial profitability by making two adjustments. First, an adjustment needs to be made for the change in the sum of consumers' surplus and factor rents. Second, an adjustment has to be made for the change in tax revenues and subsidies. Conventional practice ignores the former adjustment and mistakenly assumes that the taxes or subsidies paid by the project constitute a net addition to the transfer payments paid or received by the industry [3, p. 50, 7, p. 19]. We construct a model of a competitive industry producing one output and using two factors of production. We then express the required adjustments as a fraction of the project's commercial profitability. We find that a project inserted into the industry increases the sum of consumers' surplus and factor rents by a second order magnitude measured as triangle EEC in the output and input markets in figures 1, 2, 3 and 4. Given linear demand and supply curves in the neighborhood of the initial competitive equilibrium, the first adjustment can be expressed as the following approximate fraction

\[ \frac{r \Delta X/X^0}{2(1+r)(e^*-e_\lambda)} \]

We measure the size of the project in terms of its physical output \( \Delta X \). We measure its relative size \( \Delta X/X^0 \) by dividing \( \Delta X \) by the initial market clearing level of output \( X^0 \). The commercial rate of profit \( r \) is calculated by dividing the project's revenue net of the tax on output by the project's outlay on the factors of production gross of factor taxes. The negative price elasticity \( e_\lambda \) of demand of the industry demand curve for output is measured at the market clearing level of output given the implementation of the project. The positive price elasticity \( e^*_\lambda \) of the derived output supply
curve representing the output produced by non-project producers is measured at the market clearing level price level given the implementation of the project. Economic profitability calculations may ignore the project induced changes in consumers' surplus and factor rents whenever the industry demand or supply curves are infinitely price elastic, the project's relative size is infinitesimally small, or its commercial rate of return equal to zero.

A project when commercially profitable will tend to increase the industry market clearing level of output, while on balance decreasing the industry outlay on factors of production. The increase in output generates additional tax revenue. The decrease in outlay on the factors of production decreases the sum of tax revenues collected as input taxes. The project induced change in the sum of output and input taxes is therefore indeterminate, even though of necessity the tax revenue paid by the project proper is positive. In figure 4 we measure the gain in output tax revenue as the trapezoid CEIJ. The loss in input tax revenue is measured as the trapezoid BELM. With a linear demand and supply curve for output one can express the project induced change in industry tax revenues as a fraction of the project's commercial profit

\[ \frac{-(\frac{t \cdot e_x}{x} + \frac{t^* \cdot e^*_x}{x})}{e^*_x - e_x} < 0 \]

where \( t \) is the ad-valorem output tax and \( t^* \) the weighted ad-valorem input tax, both calculated with reference to the post project market clearing price for output. The assumption of a competitive industry equilibrium requires that either the output demand curve or derived supply curve be less than infinitely price elastic. Should the demand curve be infinitely price
elastic than industry tax revenues will increase by the output tax fraction \( t \) of the project's commercial profit. There is no industry loss of input tax revenues. With infinitely price elastic factor supply curves the price elasticity \( e^*_X \) of the derived output supply curve will also equal infinity. Industry tax revenues collected from inputs decrease by the fraction \( t^* \) of the project's commercial profit. There is no change in the industry output tax revenue. With uniform ad-valorem rates of taxation on inputs and outputs and equal absolute output demand and price elasticities the project induced change in industry tax revenues equals zero.

The difference between economic and commercial profitability is determined by the sum of the two adjustments and the parameters therein. The latter reflect the special characteristics of the industry, the project, and existing taxes. Note that the commercial rate of return may be negative, as may ad-valorem taxes when treated as subsidies.

Partial differentiation of the sum of (1) and (2) with respect to the relative size \( \Delta X/X^0 \) of the project yields the transparent result that for relatively large projects economic profitability will increasingly differ from its commercial profitability. An increase in the ad-valorem output tax \( t \) increases economic profitability versus its commercial profitability. The opposite holds for an increase in the weighted ad-valorem input tax \( t^* \). Other things being constant, an increasingly price elastic demand for output increases a project's economic profitability relative to its commercial profitability. This is caused by a larger increase in market clearing output and therefore a smaller decrease in the market clearing level of outputs. The opposite conclusion holds for an increasingly price elastic derived supply curve. An increase in the commercial rate of return \( r \) increases the economic profitability of the project. Finally, economic
profitability, calculated correctly, may differ substantially from economic profitability, calculated conventionally. For a portfolio of projects, with substantial variation in project parameters, the two methods will likely produce different rankings.

Economists are rarely involved in the technical design of a project. Not infrequently only one design is submitted and accepted. There is a reasonable possibility that the design includes innovations beyond industry standards. Consider therefore a technically feasible project with a proprietary fixed output-input coefficient structure. The project at arbitrary finite scale is not a solution of the known linearly homogeneous industry production function. The project at a predetermined non-infinitesimal scale is inserted into an industry characterized by competitive output and input markets by an entrepreneur who purchases predetermined amounts of industry homogeneous factor inputs and sells the resulting output to consumers. This will change the prevailing market clearing prices and quantities in all interrelated markets. The project entrepreneur will earn permanent entrepreneurial profits if at the newly established equilibrium prices project revenues (net of taxes) exceed expenditures (gross of taxes). Profits accruing to the remaining entrepreneurs with access to a variable choice of technique based on a representative industry production function will continue to equal zero. Consumers will change their expenditure on industry output and the suppliers of the factors of production will experience changes in factor incomes. Given this background we want to determine the economic profitability of the project using for that purpose a derived money metric in terms of the parameters of stationary demand (3) and factor supply functions (5, 7). We do not accept the implicit conventional assumption that for given market
clearing prices the demand and factor supply functions will simultaneously shift to the right so as to exactly absorb project output and provide for the required increases in project inputs. (See also 8, p. 105, 9, pp. 39-47).

(3) \( (X + \Delta X) = x^D(P) \)

(4) \( p^N_1 = P_1 - T \)

(5) \( (X_1 + \Delta X_1) = x^S_1 (P^N_1) \)

(6) \( p^N_1 = P_1 - T_1 \)

(7) \( (X_2 + \Delta X_2) = x^S_2 (P^N_2) \)

(8) \( p^N_2 = P_2 - T_2 \)

Industry output equals the sum of project output \( \Delta X \) and that produced by non-project producers \( X \). It is sold at a uniform price \( P \) to consumers. Because of a specific output tax \( T \), the net price received by producers \( p^N \) equals \( P - T \). The industry quantity supplied of the first factor of production equals the sum of the amounts absorbed by the project \( \Delta X_1 \) and that used by non-project producers \( X_1 \). Producers pay a price \( P_1 \). Because of a specific input tax \( T_1 \), the net factor price \( p^N_1 \) received by factor suppliers equals \( P_1 - T_1 \). Non-project producers are assumed to minimize the cost of producing any given level of output \( X \). With a linearly homogeneous production function \( X(X_1, X_2) \) we may write the following marginal cost and
factor demand functions

(9)  \( F^N = P(P_1, P_2) \)

(10)  \( X_1 = X_1(P_1, P_2, X) \)

(11)  \( X_2 = X_2(P_1, P_2, X) \)

The marginal cost function is dual to the industry production function and linearly homogeneous in factor prices \( P_1 \) and \( P_2 \) paid by producers. (6, p. 306). Above one output-two input model of the competitive industry can be solved for market clearing levels of quantities and prices in terms of prevailing specific tax distortions \( T, T_1, T_2 \) and autonomous project components \( (\Delta \bar{X}, \Delta \bar{X}_1, \Delta \bar{X}_2) \) (4). By treating the price of output \( P \) as a parameter we may suspend the market clearing condition in the output market while maintaining market clearance in the related input markets. The solution of equations 4 through 11 yields a derived supply curve for non-project producers

(12)  \( S^*_X = X^*[P, P_1(P), P_2(P), ...] = X^*[P,...] \)

Along \( S^*_X \) the tax inclusive factor prices \( P_1 \) and \( P_2 \) paid by producers adjust so as to maintain continuous factor market clearance. The dots within brackets indicate that the derived supply is not completely specified until we make additional assumptions as to the inclusion of the factor tax parameters \( T_1, T_2 \) or project input components \( \Delta \bar{X}_1, \Delta \bar{X}_2 \). An increase in any of these four parameters will shift the derived supply curve upwards.
By taking successively \( p_1 \) and \( p_2 \) as parameters we obtain the industry derived demands for the two factors of production by non-project producers

\[
D^*_X = X_1^* [F^N(p_1), p_1, p_2(p_1), \ldots] = X_1^* [p_1, \ldots]
\]

\[
D^*_X = X_2^* [F^N(p_2), p_1(p_2), p_2, \ldots] = X_2^* [p_2, \ldots]
\]

Factor prices are gross of taxes, whereas the price of output is treated as net of the specific tax in that market. The properties of the derived demand curve per se have been extensively studied (2, 5).

In figures 1, 2 and 3 the initial demand and supply curves are drawn such that all taxes and project parameters are set equal to zero. The insertion of the project will cause simultaneous shifts in the industry demand and supply curves of non-project producers in every one of the interrelated markets. The latter now face \((D^*_X - \Delta X)\) in the output market and \((S^*_X - \Delta X_1)\) in the first factor market. With the coming to market of project output \( \Delta X \) the demand curve facing non-project producers will shift to the left by a corresponding amount. At the original market clearing price \( p^0 \) there would exist an excess quantity supplied. Given continuous market clearance the price of output must fall, leading to a decrease in output produced by non-project producers.

There are additional crowding out effects to be considered. The project absorbs \( \Delta X_1 \) of the first factor of production. This at the original factor market clearing price \( p^0_1 \) will create an excess demand by non-project producers for this factor of production. Its price must rise, leading indirectly to an increase in the marginal cost of producing \( X \), i.e. the
Figure 1. The market for output with no tax distortions.
Figure 2. The market for the first factor of production with no tax distortions.
Figure 3. The market for the second factor of production with no tax distortions.
derived supply curve $S_X^*$ will shift upwards. With a downward sloping demand curve equilibrium output supplied by non-project producers must decrease. By analogy the insertion of $\Delta X_2$ will lead to yet a further reduction in output supplied by non-project producers.

In figure 1 the sum of the indirect effects $\Delta X^*(\Delta X_1) + \Delta X^*(\Delta X_2)$ equals the leftward shift in the derived supply curve, i.e. the distance $X^1 X^2$ on the output axis. With the project in place the market clearing i.e. the distance $X^1 X^2_1$ on the input axis. With the project in place the factor market clearing price equals $P^1_1$. At that price non-project producers purchase $OX^1_1$, the remainder $\Delta X_1$ being purchased by the project entrepreneur. Industry quantity supplied decreases by the distance $X^3_1$ and $X^0_1$. The project has reduced factor absorption by non-project producers equal to the quantity $X^1_1 X^0_1$. The net effect of the project on market clearing factor supply is seen to be the sum of offsetting shifts $\Delta X_1$ in the factor supply curve $S_X^*$ and $\Delta X^*(\Delta X_2) + \Delta X^*(\Delta X)$ in the derived factor demand curve $D_X^*$. The sign of the sum of these shifts is indeterminate.

\[
(16) \quad \Delta X^1_1(\Delta X) + \Delta X^1_1 + \Delta X^*(\Delta X_2) > 0
\]

In figure 2 this sum is negative. With two factors of production the derived demand curve $D_X^*$ for the second factor of production will shift to the left upon implementation of the project. This will tend to decrease the market clearing price in that market. Nevertheless the project may absorb a disproportionately large amount $\Delta X_2$ of that factor such that, as in figure 3, the market clearing price increases.
With (16) and (17) both of negative sign the project would be generally
factor saving in the sense that an increased or equal amount of market
clearing output is obtained with a decrease in market clearing quantities of
both inputs. Factor rents, as well as factor incomes, would decrease for
both factors of production.

We now turn to the calculation of changes in consumers' surplus, factor
rents, tax revenues and entrepreneurial profits and their geometrically
equivalent areas in figures 1, 2 and 3. In figure 1 we begin by calculating
the area enclosed by the output price line and the derived supply curve
given alternative assumptions as to output price, tax and project
parameters.

Following Samuelson (5) consider the following unconstrained
maximization problem

\begin{align*}
(18) \quad PS = \max_{X_1, X_2} & \quad (P \cdot X(X_1, X_2) - T_1 \cdot X_1 - T_2 \cdot X_2 - \int P_1^N(X_1 + \Delta X_1) \, dX_1 \\
& - \int P_2^N(X_2 + \Delta X_2) \, dX_2)\]
\end{align*}

Producers' surplus for non-project producers is defined as the difference
between revenue $P \cdot X$, with $P$ constant, and the sum of factor tax revenues
augmented by the sum of the areas below the upward sloping inverse factor
supply curves $P_1^N(X_1 + \Delta X_1)$ and $P_2^N(X_2 + \Delta X_2)$, given that non-project
producers choose input levels $X_1$ and $X_2$ so as to maximize the expression
within curly brackets.

Partial differentiation of (18) with respect to $X_1$ and $X_2$ yields the
conventional first order optimality conditions for a competitive industry
equilibrium

\begin{align*}
(19) \quad P \cdot \partial X/\partial X_1 &= P^N_1(X_1 + \Delta X_1) + T_1 = P_1 \\
(20) \quad P \cdot \partial X/\partial X_2 &= P^N_2(X_2 + \Delta X_2) + T_2 = P_2
\end{align*}

In above two equations factor supply prices $P^N_1$ and $P^N_2$ are endogenous. The
two equations can be solved simultaneously to yield the optimal use of
factors $X_1^*$ and $X_2^*$ in terms of parameters $P$, $T_1$, $T_2$, $\Delta X_1$, and $\Delta X_2$. We
therefore write the dual or indirect industry producers' surplus function
as

\begin{align*}
(21) \quad PS &= PS[X_1^*(P, T_1, T_2, \Delta X_1, \Delta X_2), X_2^*(P, T_1, T_2, \Delta X_1, \Delta X_2)] \\
&= PS[P, T_1, T_2, \Delta X_1, \Delta X_2]
\end{align*}

The first derivative of (21) with respect to product price $P$ is the derived
supply curve $S_X^*$ in equation (12). Depending on specific parametric
assumptions alternative derived supply curves will be generated. In figure
1 we initially set all shift parameters equal to zero, yielding $S_X^*$. We then
maintain $T_1 = T_2 = 0$ but allow for input absorption $\Delta X_1$ and $\Delta X_2$ by the
project. This will shift the derived supply curve $S_X^*$ to the left by the
distance $\Delta X(\Delta X_1) + \Delta X(\Delta X_2)$. 
Using (21) we can calculate the producers' surplus for triangles $P^0EF$, $P^1AG$ and $P^1BF$ respectively. Non-project producers will always choose that variable technique of production such that the industry equilibrium marginal value product of each input equals its industry equilibrium price. There are three such equilibria in figure 1 characterized by the market clearing price-quantity coordinates of points E, A and B respectively. The latter is the pre-project industry equilibrium which would exist if the industry demand for output would be infinitely price elastic at price $P^1$.

Consider the following line integral derived from (21).

\[
P_1 \hfill T_1 \hfill T_1 \quad T_2 \quad T_2
\]

\[
\int_{C} \, dPS = \int_{P^0}^{P^1} \frac{\partial PS}{\partial P} \cdot dP + \int_{T^0}^{T_1} \frac{\partial PS}{\partial T_1} \cdot dT_1 + \int_{T^2}^{T_2} \frac{\partial PS}{\partial T_2} \cdot dT_2
\]

\[
\Delta x_1 \quad \Delta x_2
\]

\[
+ \int \frac{\partial PS}{\partial \Delta x_1} + \int \frac{\partial PS}{\partial \Delta x_2} \cdot d\Delta x_2
\]

This line integral is path independent because its differential form is the gradient vector of 21 (1, p. 1050). The value of (22) is therefore not affected by the sequence of integration, it being understood that the parameters appearing in $PS$ are adjusted when carrying out the evaluation of the next integral along some specified path $C$.

By setting all tax and project parameters equal to zero the trapezoid $P^0EBP^1$ become the first integral to be evaluated along path $C$ in (22).

Given undistorted input markets, as assumed, the second and third integrals will both be zero. The negative sum of the fourth and fifth integrals, evaluated at $P^1$, equals the trapezoid $ABFG$. But this measure is exactly
offset by the increase in producers' surplus attributable to input absorption $\Delta X_1$ and $\Delta X_2$ by the project, because latter measure, with $T_1 = T_2 = 0$, equals

$$
\int_0^P \frac{\partial \psi}{\partial P} \, dP + \int_0^{\Delta X_1} \frac{\partial \psi}{\partial \Delta X_1} \, d\Delta X_1 + \int_0^{\Delta X_2} \frac{\partial \psi}{\partial \Delta X_2} \, d\Delta X_2
$$

On balance producers' surplus decreases by the trapezoid $P^0EBF^1$. Market clearing output with the project equals $OX^3$. Consumers' surplus therefore increases by the trapezoid $P^0ECP^1$. With undistorted markets the project increases the equally weighted sum of consumers' and producers' surplus by the triangle $EBC$. This second order effect is always positive. For example with a project producing no output, i.e. $\Delta X = 0$ and positive absorption of inputs, $\Delta X_1 > 0; \Delta X_2 > 0$, the increase in producers' surplus will be greater than the decrease in consumers' surplus. If entrepreneurial gains or losses are not borne by consumers or factor suppliers they will always derive second order gains from entrepreneurial efforts.

The project embodies an autonomous choice of technique such that the first order optimality conditions (19) and (20) do not apply to the behavior of the project entrepreneur. Consequently project revenue, at post-project prices, does not have to equal the sum of the expenditure on project inputs

$$
P_1^1 \cdot \Delta X_1 + P_2^1 \cdot \Delta X_2 \geq P^1 \cdot \Delta X
$$

The sum of project expenditures can be measured as rectangle $AXB^2X_1^1$ in Figure 1. With project revenue equal to $P^1 \cdot \Delta X$, entrepreneurial profits equal rectangle $BCX^2X_3$. Economic profitability exceeds financial
profitability by a second order difference, i.e. triangle EBC. With undistorted markets a project will be financially as well as economically profitable if the sum of the offsetting shifts in the ordinary demand curve $D_X$ and derived supply curve $S_X^*$ induced by the project is positive.

Triangle EBC in figure 1 has an exact geometric equivalent in either one of the two factor markets. Consider the following unconstrained maximization problem:

$$M = \max_{X_1, X_2} \left\{ \int P(X + \Delta X) \, dX - \int P_2^N(X_2 + \Delta X_2) \, dX_2 - P_1^N \cdot X_1 \right\}$$

The first integral measures the area under the ordinary demand curve $(D_X - \Delta X)$ in figure 1. The second integral measures the area under the inverse upward sloping factor supply curve $P_2^N(X_2 + \Delta X_2)$ or $(S_{X_2} - \Delta X_2)$ in figure 2. With $P_1^N$ taken as constant the third term in (25) measures the outlay by non-project producers on $X_1$, net of the specific tax in that market. The last three terms measure taxes paid by non-project producers. $M$ therefore represents the equally weighted sum of consumers' surplus and factor rent earned in the second market assuming, first, an infinitely elastic factor supply curve for the first factor of production, second, that non-project producers choose input levels $X_1$ and $X_2$ so as to maximize $M$.

Partial differentiation of (25) yields the following first order conditions for the competitive industry equilibrium at point A in figure 2.

$$[P(X + \Delta X) - T] \cdot \frac{\partial X}{\partial X_1} = P_1^N + T_1$$
In above equations the prices of output and of the second factor of production are endogenous. The two equations can be solved simultaneously to yield the optimal use of factors $X_1^*$ and $X_2^*$ in terms of the net of tax price of the first factor of production and the listed tax and project parameters. We therefore write the dual or indirect surplus function as

\[(28) \quad M = M(P_1^N, T, T_1, T_2, \Delta X, \Delta X_2)\]

Setting all tax and project parameters equal to zero and taking the first derivative with respect to the first factor price one obtains the derived factor demand curve $D_X^*$ in figure 2. Alternative parametric assumptions can be made so as to calculate the area between the factor price line and the relevant derived demand curves.

Consider next the path independent line integral based on the gradient vector of $M$

\[(29) \quad \int \frac{\partial M}{\partial P_1} \cdot dP_1 + \int \frac{\partial M}{\partial T} \cdot dT + \int \frac{\partial M}{\partial T_1} \cdot dT_1 + \int \frac{\partial M}{\partial T_2} \cdot dT_2\]

\[\int \frac{\partial M}{\partial \Delta X} \cdot \Delta X_2 + \int \frac{\partial M}{\partial \Delta X} \cdot \Delta X + \int \frac{\partial M}{\partial \Delta X_2} \cdot d\Delta X_2 + \int \frac{\partial M}{\partial \Delta X} \cdot d\Delta X\]

Setting all tax parameters equal to zero consider the following path of integration
The first integral in (29) equals the trapezoid $P_1^0EBP_1^1$ in figure 2. The sum of the fourth and fifth integrals represent the trapezoid $ABFG$. But this measure is exactly offset by the increase in consumers' surplus and factor rents in the second market attributable to project output $\Delta X$ and input absorption $\Delta X_2$. The sum of these project effects, with all taxes equal to zero, equals

$$\pi$$

$$\frac{P_1^0}{\partial M} + \frac{\partial M}{\partial P_1} + \int \frac{\partial M}{\partial \Delta X_2} \cdot d\Delta X_2 + \int \frac{\partial M}{\partial \Delta X} \cdot d\Delta X$$

On balance the sum of consumers' surplus and factor rents in the second market increases by the trapezoid $P_1^0EBP_1^1$. On the other hand factor rents earned by the first factor of production decrease by the trapezoid $P_1^0BCP_1^1$.

With undistorted markets the project increases the equally weighted sum of consumers' surplus and factor rents by the triangle $EBC$. This second order effect was previously measured in the output market. In principle it can be measured as a congruent triangle in any one of a system of interrelated markets.

The price-quantity coordinates of points A and B in figure 1 represent alternative industry competitive equilibria. At point A, with inputs levels $X_1^1$ and $X_2^1$, the following first order optimality conditions hold

$$p_1^1 \cdot \frac{\partial X(X_1, X_2^1)}{\partial X_1} = p_1^1$$
(33) \[ p_1 \cdot \Delta X(X_1, X_2) \big/ \Delta X_2 = p_2 \]

With a linear homogeneous industry production function \( X(X_1, X_2) \) we obtain, using Euler's theorem and above equations

(34) \[ p_1 \cdot X_1 = p_1^1 \cdot X_1^1 + p_2^1 \cdot X_2 \]

At point B the successive levels of the variables held constant in the partial derivatives are \((X_1^1 + \Delta X_1^1)\) and \((X_2^1 + \Delta X_2^1)\). Applying Euler's identity for these input levels one obtains

(35) \[ p_1 \cdot X_2 = p_1^1 (X_1^1 + \Delta X_1) + p_2 (X_2^1 + \Delta X_2) \]

By subtracting one finds that

(36) \[ p_1 (X_2 - X_1) = p_1^1 \cdot \Delta X_1 + p_2 \Delta X_2 \]

This equality allows one to measure the sum of project outlays on factors of production exactly as the rectangle \( ABX_2 X_1 \) in figure 1. Project revenue \( p_1 \cdot \Delta X \) minus project expenditures yields entrepreneurial profits measured as rectangle \( BCX_2 X_2 \) in figure 1. The latter can be imputed exactly to the sum of the factor (dis)savings rectangles \( BCX_1 X_1 \) and \( BCX_2 X_2 \) in Figures 2 and 3. With a perfectly price elastic factor supply curve \( S_{X_1} \) at factor price level \( p_1 \) and with a stationary demand curve \( D_X \) and factor supply curve \( S_{X_2} \), the competitive industry equilibrium is characterized by output coordinate
OX^3 in figure 1 and input coordinates OX^1 and OX^2 in figures 2 and 3 respectively. At point B on the derived demand curve D^* in figure 2 the following first order optimality conditions hold.

\[(37) \quad p^1 \cdot \frac{\partial x(x_1, x_2)}{\partial x_1} = p^1_1\]

\[(38) \quad p^1 \cdot \frac{\partial x(x_1^*, x_2^*)}{\partial x_2} = p^1_2\]

Substitution in Euler's identity yields

\[(39) \quad p^1 \cdot x^3 = p^1_1 \cdot x^1_1 + p^1_2 \cdot x^2_2\]

Subtracting the expression in 35 from 39 confirms the proposition

\[(40) \quad p^1 \cdot (x^3 - x^2) = p^1_1 \cdot (x^1_1 - x^1_1) + p^1_2 \cdot (x^2_2 - x^2_2) < 0\]

For a project to be commercially profitable the sum of factor savings must be positive. Note that in figure 2, the industry, after insertion of the project, will use less of the first factor of production. The opposite assumption is made for the second factor of production in figure 3.

Define the commercial rate of return of the project r as profits divided by the sum of the outlay on the factors of production. In figure 1 the project drives a wedge between the quantity OX^2 that will be supplied at price P^1 barring project factor savings and the quantity OX^3 allowing for factor savings. The output wedge BC reflects the size and rate of return of the project.
Market clearing output price $P$ can be written as an implicit function of $BC$

\[(42) \quad f(P, BC) = 0\]

After total differentiation one obtains the rates of change in market clearing price as an explicit function of $BC$:

\[(43) \quad \frac{dP}{d(BC)} = \frac{1}{\frac{\partial D_X}{\partial P} - \frac{\partial S_X^*}{\partial P}} < 0\]

With a linear output demand and supply curve around the pre-project equilibrium point one may calculate in figure 1 the positive area of triangle EBC as

\[(44) \quad EBC = \frac{r \cdot \Delta X (P^0 - P^1)}{2(1+r)}\]

This triangle, the net change in the sum of consumers' surplus and factor rents, as a fraction of entrepreneurial profits equals

\[(45) \quad \frac{EBC}{(r/(1+r)) \cdot P^1 \cdot \Delta X} = \frac{r \cdot \Delta X}{(1+r)[S_X^* e_X^* - D_X e_X]}\]

where the supply price elasticity $e_X^*$ and output produced by non-project producers $S_X^*$ are evaluated at point B in figure 1. On the other hand the
price elasticity of demand $e_X$ and the post-project industry clearing level of output $D_X$ is evaluated at point C.

Above expression simplifies to (1) if we assume that for small projects the post project market clearing ratios $X^2/X_0$ and $X^3/X_0$ equal unity. For small projects with moderate rates of return and large absolute demand and supply price elasticities above ratio is quite small. Financial and economic profitability virtually coincide. Project evaluation in these circumstances need not give high priority to the calculation of the net change in the sum of consumers' surplus and factor surpluses unless redistributive considerations are of paramount importance.

In figure 1 an increasingly price elastic demand curve $D_X$ would rotate counterclockwise around the initial equilibrium point E. The resulting market clearing level of output will follow the path $CC^1$. The output produced by non-project producers follows the line segment $AA^1$. If the demand for output were infinitely price elastic the net change in consumers' surplus and factor rents would equal zero. One may confirm this result by setting $e_X$ equal to minus infinity in (44). The project's commercial profitability increases by the parallelogram $BCCE^1$ or twice the triangle $EBC$. This calculation is based on the industry demand and supply price elasticities. The project entrepreneur as a single competitive buyer and seller will face substantially more price elastic demand and supply curves. This may lead to price elasticity optimism and an overestimate of the project's true commercial profitability.

Given that the demand for output is infinitely price elastic the net expression in market clearing output $EC^1$ is nevertheless less than project output $\Delta X$. The project crowds out production by non-project producers as
measured by the line segment $A E$. The conventional assumption in project evaluations is the absence of this possibility, i.e. the industry demand curve $D_X$ shifts to the right so as to exactly absorb project output $\Delta X$. There is no justification for such a coincident income effect.

Consider now the context of increasing price elastic factor supply curves. The derived supply curve $S_X^*$ in figure 1 will then rotate clockwise around the initial point of equilibrium $E$. With a stationary demand curve $D_X$, market clearing output will follow the path $CE$. Output produced by non-project producers follows the path $AA'$ along the stationary demand curve $(D_X - \Delta X)$. With infinitely price elastic factor supplies the project cannot increase the initial equilibrium level of output. The project fully displaces output previously produced by non-project producers.

Entrepreneurial profits increase by the parallelogram $BB'EC$. Profits as before can be measured as the sum of factor savings in the input markets. Not all of factor savings need to be positive for profits to be positive, regardless the size of the price elasticities $e_X$ and $e_X^*$.

Consider now a situation where specific taxes $T_1 T_2$ distort the single output and two input markets prior to the insertion of the project. Such taxes drive a wedge between prices paid and received. In figure (4) the introduction of input taxes $T_1$ and $T_2$, for given price $P^0$, shift the derived supply curve $S_X^*$ to the left by the distance $\Delta X(T_1) + \Delta X(T_2)$. Equivalently the marginal cost of producing output $X^0$ increases by $\Delta P(T_1) + \Delta P(T_2)$. Numerical approximations of these shifts can be based on the derived supply curve in (12). The introduction of the output tax $T$ for given price $P^0$, will shift the demand curve $D_X$ to the left by the distance
\( \Delta X(T) \). Equivalently for given output \( X^0 \) the price received by producers decreases by \( T \). The total tax wedge measured as \( IL \) in the output diagram therefore equals \( T + \Delta P(T_1) + \Delta P(T_2) \). The latter, multiplied by the market clearing level of output \( X^0 \) equals total initial tax revenue. The sum of the deadweight losses in the output and factor markets can be measured as triangle IHL in figure 4. The latter equals the area under the gross of tax demand curve \( D_X \) minus the area under net of input taxes derived supply curve \( S_X^* \) between the undistorted and distorted market clearing levels of output \( X^4 \) and \( X^0 \). With a stationary demand curve the introduction of taxation decreases the area under \( D_X \) as measured by the following integral

\[
\int_{X^4}^{X^0 + \Delta X(T) + \Delta X(T_1) + \Delta X(T_2)} P(X) \, dX < 0
\]

With stationary factor supply curves the area under the derived supply curve \( S_X^* \) equals the sum of the areas under the factor supply curves. The introduction of taxation decreases the area under \( S_X^* \) as measured by the sum of the following integrals

\[
\int_{X_1^*}^{X_1^* + \Delta X_1(T) + \Delta X_1(T_1) + \Delta X_1(T_2)} P_1(X_1) \, dX_1 > 0
\]

\[
\int_{X_2^*}^{X_2^* + \Delta X_2(T) + \Delta X_2(T_1) + \Delta X_2(T_2)} P_2(X_2) \, dX_2 > 0
\]
Figure 4. The market for output with tax distortions.

\[ S_X^* + \Delta X(T_1) + \Delta X(T_2) + \Delta X(\Delta X_1) + \Delta X(\Delta X_2) \]
The upper and lower limits appearing in these integrals can be obtained by solving the model contained in equations 3 through 11 for the relevant initial and final values of the tax parameters.

Consider now the introduction of a project given unchanged specific tax levels and stationary demand and factor supply curves. Entrepreneurs pay specific taxes on project output and inputs, but not on profits. In figure 4 entrepreneurial profits equal the rectangle $BCX^3x^2$. As before the net change in the sum of consumers’ surplus and factor rents equals the triangle $EBC$. The parallelogram $CBEJ$ measures the increase in industry tax revenue determined by the increase in the market clearing level of output. With profits being positive the sum of factor savings must also be positive. The post-project reduction in industry outlay on factors of production, gross of taxes, implies a reduction in the sum of input tax revenues. This negative amount in figure 4 is measured by the parallelogram $BELM$. When augmented by the increase in output tax revenue $CEIJ$, the sign of the net increase in tax revenue, collected from project and non-project producers alike, is indeterminate. The net increase in tax revenue is less than the amount of taxes paid by the project. The latter will always be positive whereas the former may be negative. Current practice calculates the economic profit of a project by adding the full amount of taxes paid by the project to its commercial profit. This erroneous procedure leads to a systematic overestimation of the economic profitability of a project. For a portfolio of projects it is possible to calculate the taxes paid by each project as well as the net change in tax revenue because of crowding out effects. If levels of taxation vary between industries the conventional calculations of economic profitability will likely rank projects erroneously.
The net change in tax revenue induced by the project in this model is measured by the sum of the integral elements of a 3x3 matrix

$$\Delta TR = \sum_{j=1}^{3} \left( \sum_{i=1}^{3} \int_0^T T_i \cdot \frac{\partial X_i}{\partial \Delta X_j} \cdot d\Delta X_j \right) > 0$$

In above integral the market clearing levels of output $X$ and inputs $X_1, X_2$ are seen to be functions of the inserted project components $\Delta X, \Delta X_1, \Delta X_2$. Current practice assumes a context such that the partial derivatives on the leading diagonal equal positive unity, while all off-diagonal partial derivatives equal zero. With a stationary demand curve the partial derivative $\partial X/\partial \Delta X$, with $\Delta X_1 = \Delta X_2 = 0$, equals the ratio $VW/UW$ with the numerator measuring the increase in market clearing output $VW$ and the denominator measuring project output $UW$. The value of this ratio lies between zero and positive unity. Similarly with stationary factor supply curves the partial derivatives $\partial X_1/\partial \Delta X_1$ and $\partial X_1/\partial \Delta X_2$ will have values that lie between negative unity and zero. No general conditions as to the signs of the off-diagonal integrals can be laid down.

The change in tax revenue derived from the change in market clearing output is measured by the following integral of indeterminate sign

$$\Sigma \int_0^T T \cdot \frac{\partial X}{\partial \Delta X} d\Delta X_j > 0$$

Parallelogram ECJI represents this integral under the assumption that the project is commercially profitable. It's area using the demand price elasticity $e_X$ and derived supply price elasticity $e_X^*$ equals
where \( S_X \) equals the quantity of output supplied by non-project producers in post-project equilibrium. This quantity, augmented by project output \( \Delta X \), yields the market clearing output at post-project price \( P^1 \). With a positive rate of profit \( r \) above expression will be positive. On the other hand an unprofitable project cannot generate a net increase in output tax revenue. This contrasts rather sharply with conventional practice where the net increase in output tax revenue equals \( T \cdot \Delta X \) regardless as to the project's profitability. With an infinitely price elastic demand for output, tax revenue increases by the fraction \( r/(1+r) \) of project output. With infinitely price elastic factor supply curves, the price elasticity \( e_X^* \) of the derived supply curve \( S_X \) will equal infinity. The project will not generate any additional tax revenue. In fact factor savings will lead to a reduction in input tax revenues.

The change in tax revenues in the two factor markets is measured by the following integrals

\[
\begin{align*}
(52) \quad & \sum \int_0^{\Delta X_j} T_1 \cdot \frac{\partial X_1}{\partial \Delta X_j} \cdot d\Delta X_j > 0 \\
(53) \quad & \sum \int_0^{\Delta X_j} T_2 \cdot \frac{\partial X_2}{\partial \Delta X_j} \cdot d\Delta X_j < 0
\end{align*}
\]

The assumed negative sum of the revenue effects in input markets is represented as parallelogram BELM in figure 4. Its area equals
In above expression $T^*$ represents the post-project tax wedge $EL$ in figure 4. A commercially profitable project creates factor savings, which on balance leads to a loss in input tax revenues. On the other hand with a negative rate of return input tax revenues increase. If the demand for output is infinitely price elastic the loss in input tax revenues equals zero. Conventional practice, given that assumption, overestimates the increase in input tax revenue by $T_1 \cdot \Delta \bar{X}_1 + T_2 \cdot \Delta \bar{X}_2$, or the area $T^* \cdot (1-r) \Delta \bar{X}$ in figure 4.

The change in output and input tax revenues can be expressed as a fraction of entrepreneurial profits at the post-project market clearing price $P^1$.

\[
(54) \quad -T \cdot \frac{e^* S_X}{e_X S_X - e_X D_X} \cdot \frac{T^* \Delta \bar{X}}{1+r} < 0
\]

For a relatively small project $S_X^*/D_X$ will be close to unity. We then obtain the expression (2) used on page 2. With a uniform ad-valorem tax rate on outputs and inputs and equal absolute output demand and supply price elasticities, above ratio equals zero. On the other hand with input subsidies and output taxation the project will increase tax revenues.

The economic profitability of a project can be derived from its commercial profitability by making two adjustments. First, an adjustment has to be made for the change in the sum of consumers' surplus and factor rents. Second, an adjustment has to be made for the change in tax revenues,
given that taxes or subsidies constitute transfer payments, between members of the same society. The two adjustments can be expressed as a fraction of the project's commercial profitability

\[
\frac{r \Delta X - 2(1+r) \left( tD_X e_X + t^* S_X e_X^* \right)}{2(1+r)(S_X e_X^* - D_X e_X)} > 0
\]

where the parameters \( t \) and \( t^* \) are the implied ad valorem tax rates (\( T/P_1 \)) and \( T^*/P_1 \) on output and inputs respectively. The sign of above expression is indeterminate for positive tax and profit rates. Note that these parameters, reflecting the special characteristics of the industry and the project, may also take on negative values. The expression in (56) reflects the special characteristics of the project, existing taxes and the industry. Partial differentiation with respect to these characteristics yields the generally transparent rules commented on page 4.

The heavily lined trapezoid in figure 4 is the geometric equivalent of the economic profitability of the project. This suggests the following classification of the net benefits of the project. First, a decrease in efficiency losses in output and input markets as measured by the trapezoid IJKL. Second, factor savings, evaluated at variable opportunity cost, net of input taxes equal to the trapezoid KMX^2X^3. The first trapezoid, using (51) and expressed as a fraction of commercial profits equals

\[
\frac{-r \cdot (t+t^*) \cdot D_X e_X}{(1+r)(S_X e_X^* - D_X e_X)} > 0
\]

Only commercially profitable projects yield efficiency gains. In either
case they are proportional to the cumulative ad-valorem tax wedge \((t+t^*)\).

With infinitely price elastic factor supply curves, such that \(S^*_X\) equals infinity, the project has no associated gains or losses in efficiency. On the other hand if the demand for output is infinitely price elastic, as for an open economy industry, and if the cumulative tax wedge is proportionately large, then efficiency gains may equal commercial profits.

The sum of factor savings and efficiency gains is distributed in the following manner. Factor suppliers lose an amount equal to the trapezoid \(\text{P}^0\text{EBP}'\). Consumers gain an amount equal to the trapezoid \(\text{P}^0\text{ECF}'\). Project entrepreneurs gain the rectangle \(BCX^3X^2\). Non-project producers continue to earn zero profits. The public sector gains a net tax revenue equal to the difference between the trapezoids \(\text{EIJC}\) and \(\text{BELM}\).

It is desirable to have an alternative exact integral measure for triangle EBC in figure 4. Consider the change in consumers' surplus, using \(P\) as the integrating variable and \(X(P)\) as the representation of the demand curve for \(X\).

\[
\Delta CS = \int_{P_0}^{P_1} X(P) \, dP 
\]

The upper limit \(P_1\) and market clearing output \(X_1\) are systematically related to project output \(\Delta X\), i.e. \(P = f(\Delta X)\). Substitution of variables yields an expression for the change in consumers' surplus with \(\Delta X\) as the integrating variable.

\[
\Delta CS = \int_{0}^{\Delta X} X_1(\Delta X) \cdot \partial P/\partial \Delta X \cdot d \Delta X 
\]
Similar expressions may be developed for the change in the two factor rents

\[ \Delta X \]

(60) \[ \Delta F S_1 = \int_0^{\Delta X} X_1(\Delta X) \cdot \frac{\partial P_1}{\partial \Delta X} \cdot d \Delta X \]

(61) \[ \Delta F S_2 = \int_0^{\Delta X} X_2(\Delta X) \cdot \frac{\partial P_2}{\partial \Delta X} \cdot d \Delta X \]

The summation of these three integrals determines the ceteris paribus welfare effect of \( \Delta X \) on consumers and factor suppliers

\[ \Delta X \]

(62) \[ \Delta M(\Delta X) = \sum_{i} \int_0^{\Delta X} X_i(\Delta X) \cdot \frac{\partial P_i}{\partial \Delta X} \cdot d \Delta X \]

By analogy we may write out the ceteris paribus welfare effects for factors of production \( X_1 \) and \( X_2 \)

\[ \Delta X_1 \]

(63) \[ \Delta M(\Delta X_1) = \sum_{i} \int_0^{\Delta X_1} X_i(\Delta X_1) \cdot \frac{\partial P_i}{\partial \Delta X_1} \cdot d \Delta X_1 \]

(64) \[ \Delta M(\Delta X_2) = \sum_{i} \int_0^{\Delta X_2} X_i(\Delta X_2) \cdot \frac{\partial P_i}{\partial \Delta X_2} \cdot d \Delta X_2 \]

The sum of these three integrals equals triangle EBC in figure 4. We previously developed an integral representation of the net change in tax revenue. The difference between commercial and economic profitability is
therefore measured by the sum of the following integrals

\[ \sum_{j} \Delta X_{j} \]

\[ (65) \quad \sum_{j} \Delta M(\Delta X_{j}) = \sum_{j} \left\{ \int_{0}^{\Delta X_{j}} \left[ x_{i}(\Delta X_{j}) \frac{\partial P_{i}}{\partial \Delta X_{j}} + T_{i} \frac{\partial x_{i}}{\partial \Delta X_{j}} \right] d \Delta X_{j} \right\} \]

The calculation of above expression is conventionally invalidated by assuming a priori that the project components \( \Delta X, \Delta X_{1}, \Delta X_{2} \) cause a corresponding net increase in all market clearing levels at initially prevailing prices. Given this \( \Delta X_{i} = \Delta X_{i} \) and \( \partial P_{i}/\partial \Delta X_{j} = 0 \). No indirect tax revenue effects need to be considered and therefore \( \Delta TR = T \cdot \Delta X + \Delta X_{1} + T_{2} \Delta X_{2} \).
Footnotes

1In figure 1 the post-project market clearing price $P$ is an implicit function of the output wedge $BC = (r/l+r) \Delta X$. Total differentiation of the implicit function of $f(P, BC) = 0$ yields

$$\frac{\partial f}{\partial P} \cdot dP + \frac{\partial f}{\partial BC} \cdot dBC = 0$$

We therefore have

$$\frac{dP}{dBC} = -\frac{\frac{\partial f}{\partial BC}}{\frac{\partial f}{\partial P}} = \frac{1}{\frac{\partial s_X^*}{\partial P} - \frac{\partial D_X}{\partial P}} < 0$$

Starting with the pre-project market clearing price $P^0$ and output wedge $BC = 0$ one can rewrite above expression as

$$(P^0 - P^1) = \frac{1}{\frac{\partial s_X^*}{\partial P} - \frac{\partial D_X}{\partial P}} \cdot BC$$

Triangle $BEC$ as a fraction of commercial profit $P^1 \cdot r/(l+r) \cdot \Delta X$ then equals

$$\frac{r \cdot \Delta X}{2(l+r)P^1} \cdot \frac{1}{\frac{\partial s_X^*}{\partial P} - \frac{\partial D_X}{\partial P}}$$

Evaluation of the price elasticities of demand $e_X$ and derived supply $s_X^*$ at point C and B in figure 1 yields the expression (45) used in the text.
References Cited


