A dynamic model of the U.S. cotton market with rational expectations

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A dynamic model of the U.S. cotton market with rational expectations

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A dynamic model of the U.S. cotton market with rational expectations

by

Grace Yueh-Hsiang Tsai

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1 INTRODUCTION

The analysis of agricultural commodity markets includes a long list of attempts to explain the dynamic regularities in the data and to forecast or simulate market behavior using models. Indeed, many earlier economists tried to identify and measure the determinants of supply in various agricultural commodities. Ezekiel (1938) also considered how producers form expectations and what the stability properties of the dynamic models would be. Many reports have shown that estimates of the supply-demand structure of the model were used to examine the feasibility of several policies designed to stabilize farm prices and farmer income. McCallum (1976), Wallis (1980), and Fisher (1982) applied rational expectations to an established econometric model in which expected values of a subset of endogenous variables were included in the system. Fisher also explored the impact of a policy rule change on the model. Eckstein (1985) argued that the observed dynamic of agricultural supply can be explained and measured by a rational expectations equilibrium model as well as by the Nerlove supply response model.

In addition, agricultural production has some specific aspects. Its biological nature separates the planning period from the time output is realized. Agricultural production occurs in dynamic environments. The random influences of weather, disease, and pests join to make agricultural production a risky prospect. Agricultural prices have an important stochastic element independent of any random production influences such as
unexpected shifts in the demand for agricultural products resulting from a shock in the world export market, or unexpected changes in government domestic or trade policy. Government intervention also has had an important influence on the markets of many agricultural products. Although the programs are designed to support prices and incomes and to control production, the presence of price supports and income guarantees modifies and conditions the participant's behavior.

Finally, because of the large number of producers in these markets, production in agriculture follows pure competition. In a pure competition market, agricultural producers behave as price takers.

Cotton is the most important textile fiber in the world, accounting for about 50 percent of all fiber used. In this study the U.S. cotton market is studied since cotton is a major cash crop and an important source of foreign exchange. A theoretical and empirical model is developed for the U.S. cotton market that explains the above observations.

1.1 Issues in the U.S. Cotton Market

The cotton producers and the cotton millers have some planning horizon. At the farm level, the decision consists of specifying the portion of land to be devoted to the alternative crop, corn, before the price of cotton for the new crop year is known. At mills, a representative miller has to decide the quantity of cotton to gin and spin into yarn and the inventory level of cotton to hold at the end of each crop year. Mill stocks averaged more than 1.2 million bales a year and nearly 10-20 percent of cotton mill consumption during the 1955-1986
period, fluctuating from a low of 0.7 million bales in 1985 to a high of 1.9 million bales in 1961. Both producers and millers also make predictions concerning the prices received for output and the prices paid for inputs. These decisions not only are based on the current situation, but also on their expectations for the future. In a model setting, structures expressing the formulation of these expectations are crucial for a precise representation of economic behavior.

Government intervention also is an important factor influencing the producer's decision. Over the past 50 years, various government programs have been designed to achieve a number of goals such as influencing cotton acreage, controlling production, stabilizing prices, and supporting farm incomes. The government has used a complex system of price supports, acreage controls, and direct payments to reach these goals. During 1975-87 government payments to cotton producers ranged from a low of $69 million for the 1977 crop to a high of $1.7 billion in 1987, including payment-in-kind entitlements. Although these programs met the objective of reducing production and stocks, the direct payments resulted in relatively high treasury costs. The decision maker was concerned about the effects of government policy on production decisions and the cost of the programs. An appropriate model should adequately reflect the multitude of policy options available to producers in a number of explanatory variable choices. Houck and Ryan (1972) worked in this area to construct effective price support and paid diversion payment variables. Lee and Helmberger (1985) explicitly considered the effects of government policy on supply response.
In the estimation of dynamic models which include expectations, a problem arises since a series which measures expectations is usually not available in applications. Because expectations are subjective and personal, various proxies have been suggested. The simplest procedure is to assume farmers are unduly naive, following last year's price to form expectations. The more complicated lagged-price procedures following the work of Nerlove (1958) have commonly been used to capture adaptive expectations. The application of a lag distribution to form expectations implies that an agent's best judgment of the future is captured in historical data. In other words, the future behaves as the past. However, there is little indication to suggest that the presumed relations bear a similarity to the way the economy works (Muth 1961). This lag distribution approach performs well for the sample period, but questions arise when one tries to forecast beyond the sample period, since this approach introduces error correlation into the model (Griliches 1967, Gardner 1976). In addition, the distributed lag parameters are an ad hoc approach since the parameter restrictions in the distributed lag are not derived from an optimization process (Griliches 1967, Fisher 1982).

Alternatively, the rational expectations method is used to form expectations. Applications of this approach in the agricultural sector can be found in Goodwin and Sheffrin (1982) and Eckstein (1985). The reduced forms of rational expectations models are distributed lag functions of themselves, but the lag distribution has been generated from the underlying model structure, not ad hoc.

Consideration has more recently focused on the importance of risk in
the decision-making processes. At the time producers have to make decisions on resources, production and price are uncertain. Producers usually are risk averse. Economists should consider of the risk measures when they do econometric analysis. There are a number of applied works on determining the effects of uncertainty on production decisions. It is typical to use an expected utility framework to derive the optimal supply and/or input decisions by using firm-level data such as Just and Pope (1978, 1979) and Collender and Zilberman (1985). Behrman (1968), Just (1974), and Ryan (1977) examined the potential importance of risk in aggregate supply functions by using a distributed lag mechanism to generate measures of resource variance. Recently, the rational expectations hypothesis was extended to incorporate uncertainty (Antonovitz and Green 1987). The rational expectations framework offers a systematic manner in which the effect of uncertainty is included.

1.2 Problems with Econometric Models

In recent years an impressive array of econometric models have been developed for policy evaluation and forecasting. Nevertheless, additional research is needed since most existing agricultural econometric models use ad hoc model specifications. That is, model specifications are not derived from any explicit optimization problem. Economic theory is applied only for comparative statics analysis and as an indicator for selecting variables relevant to the analysis. Although these models may allow the analyst to forecast and evaluate policy, they cannot be used to test formally the implications of economic theory. A more consistent way
is to derive the model from the optimization problem. Recent studies (Weaver 1983; Shumway 1983; and Lopez 1984) used duality theory to estimate consistent sets of supply response and input demand functions. Eckstein (1985) used a rational expectations equilibrium framework to derive a dynamic supply.

Any model should be built for its intended purpose. However, many studies only focus on a segment of the problem and exclude or cursorily treat all other possible considerations. In addition, to simulate the effects of a new policy regime, it would not be appropriate simply to place the new policy regime into the existing model. It is required to reestimate the new stochastic process for the new optimal decision rule.

Lack of theoretical rigor and logical consistency in most models of the agricultural sector often have resulted from poorly developed or incomplete theoretical foundations. A set of theoretical relationships useful for examining the linkages between firm-level and market-level choice functions has developed. These micro-foundations also have been extended to the case where firms are assumed to possess nonlinear utility function (Holt 1987). Holt (1987) attempted to examine the theoretical implications of the rational expectations hypothesis, uncertainty theory, and producer behavior in the presence of government programs by deriving supply response and factor demand equations from an expected utility maximization framework. In Holt's study, a number of special structure assumptions are required such as knowing the prior information for a probability distribution of stochastic price. Although little empirical work has been done in this area, the scope of additional research using
rational expectations in empirical microeconomic models is indicated.

1.3 Objectives

The overall objectives of this research are, first, to describe elements of a simple dynamic equilibrium model and its potential for explaining economic relationships. The task is to provide procedures for combining econometric methods with dynamic economic theory to model and explain dynamic regularities in the data. A dynamic rational expectations equilibrium model is presented that explicitly specifies the market clearing conditions and costs of production of raw cotton and cotton milling. The theory of cotton price and quantity determination when storage is not an option is established. The decision rules are derived entirely from the optimization problems. The equilibrium movements of the commodity prices, production, and mill consumption are solved analytically. The economic issues are discussed. A model that takes account of storage is discussed. The dynamics of price behavior are explored by identifying the appropriate structure for generating rational expectations of the stochastic processes of the prices, as well as deriving estimable supply response, derived demand (mill consumption), and demand (inventory) equations which satisfy the transversality conditions and Euler equations of the cotton millers' and producers' maximization problems.

The second objective is to provide an operational example of the theoretical model for the U.S. cotton market. A joint test of the rational expectations hypothesis and the model specification is provided.
The third objective is to use time-series observations to study the impact of direct payments on the dynamic rational expectations equilibrium model.

In summary, the methodology used to derive the dynamic rational expectations equilibrium model provides a much richer framework for conducting policy analysis than previously available.

The study is organized as follows: Chapter II presents the structure of the cotton industry. The literature review is discussed in Chapter III. Chapter IV outlines the model with and without storage, solves the farmer and miller optimization problems, and computes the equilibrium. Chapter V discusses estimation methods and information requirements. Estimation of expectations models and tests of model specifications are presented in Chapter VI. The effects of different policy regimes on the model are then discussed. Elasticities are defined and derived at the end of the chapter. Finally, Chapter VII contains a brief summary and suggestions for further research.
U.S. cotton production was about 19 percent of world cotton production in 1987, down from 31 percent in 1960. During the 1960-85 period, cotton's share of the world textile fiber market declined from about 70 percent to almost 50 percent. Cotton is grown in about 75 countries; mostly in China, the Soviet Union, and the United States, which accounted for 57 percent of the world's cotton in 1987. Over the past three decades, cotton has faced severe competition from manmade fibers. Nevertheless, demand for cotton and cotton blends has recently increased, due to its breathing ability and absorbency quality.

In 1985, cotton ranked fifth ($4 billion) among the major field crops in value of farm production, following corn ($21.3 billion), soybeans ($10.8 billion), baled hay ($9.7 billion), and wheat ($7.7 billion). The farm value of cotton lint and seed accounted for about 5 percent of the value of all major crops marketed in 1982-85. Cotton acres harvested represented about 3 percent of U.S. total acreage of principal crops harvested.

Over the past fifty years, U.S. farmers have often been plagued by excess production capacity, high stocks, and low product prices. The soundness of the U.S. cotton industry is interdependent with the world economy. Exports of U.S. raw cotton have greatly depended on foreign cotton output and general economic conditions. Since abundant harvests in competing exporting countries caused the U.S. share of the world cotton
exports to drop from 40% in 1960 to 10% in 1986, stocks increased from 7,501 thousand bales in 1960 to 9,348 thousand bales in 1986. U.S. cotton has tended to be a residual supply in world trade which has resulted in price and supply instability. Government farm programs since the 1930s have attempted to support commodity prices and adjust acreage and production to market needs.

Government programs have provided varying degrees of success, such as stabilizing crop prices, improving farm incomes, and slowing the transfer of resources out of cotton production. However, cotton farms keep decreasing in number and increasing in size in response to economic and technological forces such as low production costs and mechanization. In the future, the U.S. cotton industry will still heavily depend on exports while domestic mill consumption may be subject to textile imports and competition from manmade fibers.

This chapter provides an overview of the structure of the U.S. cotton industry to provide a broad perspective on the scope and operation of the U.S. cotton market, as well as to indicate how the government has intervened in this commodity market over time. It briefly introduces the history of cotton, the structure of production, mill use, trade, price, and the history of government programs.

2.1 History of Cotton

Cotton is one of the most important textile fibers in the world, supplying about 50 percent of total world fiber production. Cotton is not only a major cash crop, but also an important source of foreign exchange
earnings. In the United States, cotton was originally grown at Jamestown in the 17th century, but was a minor crop until 1793 when Eli Whitney invented the cotton gin, which spurred production and exports. Cotton is produced in the southern United States, with major concentration in the Delta areas (Mississippi, Arkansas, and Louisiana), the Texas Plains and Rolling Plains, central Arizona, and the San Joaquin Valley of California. The most common type of cotton grown in the United States is *Gossypium hirsutum*, known as American upland cotton, which accounts for about 99 percent of the U.S. cotton crop. Another type of cotton grown in the United States, *Gossypium barbadense*, is known as extra long staple (ELS) cotton, or American pima cotton, and is produced in limited areas such as southwest Texas, New Mexico, and Arizona. The production of ELS cotton is small relative to upland cotton because of its higher costs of production and higher product values.

2.2 The Structure of Production

U.S. cotton production has significantly changed over the past sixty years. Cotton production fluctuated from 7,443 to 17,978 thousand bales over the 1920-1986 period. Acreage planted to cotton declined from 36.9 million acres for the 1920-25 period to 11.9 million acres for the 1981-86 period. Cotton average yield, however, increased from 157 to 570 pounds per acre during the same period. U.S. cotton production has shifted westward. From 1970 to 1987, production in California and Arizona, as a share of total U.S. production, increased from 16 percent to 27 percent, especially in 1985 when it almost doubled. This regional shift is due
primarily to lower unit costs of production, higher net returns in relation to other crops, flat terrain, good soils, available water for irrigation in the Southwest and West, and the elimination of marketing quotas and acreage allotment restrictions.

The number of farms harvesting cotton declined sharply from 1949 to 1982, with fewer and bigger farms controlling cotton production. In 1949, 1.1 million farms were growing cotton; the average cotton farm harvested 24 acres. In 1982, 38,000 farms harvested an average of 256 acres of cotton per farm. Despite this more than tenfold growth in average size, individuals or family businesses today control more than 80 percent of the cotton farms.

Share renting and cash renting of land for cotton production prevail in the United States. In 1982, nearly half of all farms harvesting cotton were operated by part owners, 27 percent by full owners, and 23 percent by tenants. Full ownership becomes less prevalent as size of farm increases, but the proportion of part owners increases with farm size, while the proportion of tenants varies less by size. The corporate form of organization undertaken by farm operators is increasing in order to take advantage of tax policies, limited liability, or property tax provisions. Cotton production, however, has not attracted a substantial inflow of capital investment by nonfarm corporations.

The 38,000 cotton producers scattered across the cotton belt from Virginia to California received about $3.6 billion in 1985/86 from the sale of cotton lint and an added value of $350 million from the sale of cottonseed. Ginning, warehousing, and marketing also provided significant
sources of earnings and employment in local areas. In addition, 
pesticides, fertilizers, and machinery and equipment were involved in 
production services.

Cottonseed also provides a secondary source of earnings for cotton 
producers. Cottonseed can be fed directly to dairy cattle or crushed into 
meal and oil. Seeds also produce linters (small fuzzy fibers) and hulls. 
Hulls and meal, as well as whole seeds, can be fed to cattle as feed 
supplements. Linters are used in paper, upholstery stuffing, dynamite, 
and other lower strength fiber products, and also used as the cellulosic 
material to produce rayon and acetate. As usual, cottonseed provides 
about 12-15 percent of the total farm value of cotton production, with 
lint accounting for the rest of the value. Cottonseed oil accounts for 
about 5 percent of the fats and oils used in edible oil products in the 
U.S., competing with soybean oil, corn oil, and edible tallow.

2.3 Exports

Cotton has been a major export crop for nearly 200 years. The United 
States is competitive in raw cotton, but other cotton-producing countries 
are more competitive as exporters of finished products. In 1850, nearly 
90 percent of U.S. lint production was exported, with earnings offsetting 
the costs of about two-thirds of all goods imported into the United 
States. During 1980-86 exports accounted for about 30 percent of world 
cotton trade except 1985/86. Export earnings averaged nearly $2 billion, 
or about 5 percent of the total value of U.S. agricultural exports. 
Exports also accounted for about 49 percent of total disappearance (mill
consumption plus exports) of U.S. cotton in 1987.

The United States and the Soviet Union are the world's largest cotton exporters, with 1984-87 shares of 24 percent and 13 percent, respectively. The U.S. share has varied substantially since 1960, ranging from 10-40 percent of world exports, and dropping to 10 percent in the 1985/86 marketing season. Much of the variation in market share is explained by relative prices for U.S. cotton and cotton from competing exporters, and from abundant harvests in competing exporting nations.

The major export markets for U.S. raw cotton are Japan, South Korea, Taiwan, Hong Kong, Indonesia, Thailand, and Canada. During 1978-81 China was a major importer, but imports have tapered off dramatically since 1980, and China has become a net exporter since 1983. Japan was the largest export market for the United States during 1982-87, followed closely by Korea. The U.S. lead in exports could be threatened if China follows a strong policy of cotton expansion. China has the potential to export much more, but exports are limited by quality and marketing system problems. Although the Soviet share of world trade almost doubled during 1961-81, Soviet production peaked in 1980, and exports from that country leveled off at about 3 million bales annually during 1983-85.

The United States will likely be a leading exporter of raw cotton. However, its share of world exports will depend on the level of economic growth abroad, the value of the dollar, U.S. versus world cotton prices, and foreign cotton production. The provisions of the Food Security Act of 1985 were designed to enable U.S. cotton and other commodities to compete at world price levels. U.S. cotton exports should rebound under the
marketing provisions of this legislation.

2.4 Imports

The United States imposes an annual import quota on raw cotton of 14.5 million pounds of short-staple cotton (less than 1-1/8 inches in length) and a quota of 45.7 million pounds of long-staple cotton (1-1/8 inches or longer in length). The import quota has been effective since the Food and Agriculture Act of 1977. Special quotas have been triggered twice, in February and September 1980, and the quota did not cause substantial new imports in either case. About 1 million bales were eligible under the special import quotas in 1980, but only 12,000 bales were actually shipped.

2.5 Distribution and End Uses

The path from raw cotton to finished product may take many different forms. End uses of cotton include clothing, household, and industrial products. On average, about 256 pounds of total end uses of an average bale (480 pounds) of cotton are distributed to clothing, 138 pounds are distributed to home furnishings, and 64 pounds are distributed to industrial products. For more discussion of the breakdown of an average bale of cotton among specific applications see The U.S. Cotton Industry.

2.6 Cotton Pricing

Cotton prices are determined by the global cotton supply and demand forces. Major determinants for the annual supply of cotton are (1) the
relative profitability of cotton to alternative crops, (2) domestic and foreign government policies and programs, and (3) the availability of production inputs. On the demand side, important factors are (1) the relative prices of raw cotton to competing fibers, (2) domestic demand for textiles, (3) export demand for raw cotton and processed textiles, and (4) consumer incomes and levels of general economic activity. The price is rapidly responsive for actual and anticipated changes in market forces. Both cash and futures prices provide a broad base for market transactions. Because all major types of cotton can be substituted for each other either directly or indirectly, and all qualities of cotton have a direct market effect, so there is no single price for cotton. Rather, there are many prices depending on the form, type, quality, and location of a particular bale on any particular day. In general, the price of cotton is averaged at four levels of the marketing system: farm, cash market, mill delivered and international priced. Prices are also averaged by state and in designated spot markets. Prices vary by quality and with distance from consuming centers, as well as with time prior to mill use. The New York Cotton Futures Exchange is the major established market for trading cotton futures in the United States and is used by many foreign nations for hedging goals. Prices on the New York Futures Exchange are averaged.

Spot and futures prices have a theoretical relationship. Spot prices should be less than futures prices, with the difference reflecting the costs of storage and delivery. As the contract delivery date approaches, the cost of storage decreases, and the basis should narrow to reflect the costs of delivery and certification that the cotton meets contract
specifications. Prices can vary from the expectation. As forecasts of supply, use, and ending stocks change, the market responds with smaller or larger rewards for the storage of cotton. When current supplies are tight but an expected good harvest holds out the potential for rising stocks, spot prices can go beyond futures prices.

2.7 Textile and Apparel Industry

The textile and apparel industries change raw fiber into finished products. These industries indicate one of the largest sectors of the U.S. economy, offering employment to millions. Consumer purchases of apparel totaled $118 billion during 1984, about 14 percent of all nondurable goods expenditures. The estimated retail value of domestically produced cotton apparel products alone totals between $10-$13 billion a year. Americans used about 70.8 pounds of fiber per capita in 1987, which includes products produced by U.S. mills and the raw fiber content of imported textiles. Consumption of manmade fibers in all uses totaled about 42.2 pounds per capita, compared with cotton at 23.8 pounds and wool at 1.7 pounds.

Cotton is used in the production of clothing, so it can be exchanged as raw cotton, yarn, fabric, or finished apparel. Although cotton accounts for about one-half of total world fiber used, manmade fibers now account for about three-fourths of U.S. mill consumption. Major factors affecting U.S. mill demand consumption are competing fiber prices, consumer income, population growth, varying life styles, volatility of cotton price, fiber features, and trade in textile products. In the long
run, total fiber demand is price inelastic. However, the demand for individual fibers may be less inelastic than the demand for all fibers together. The elasticity of demand for individual fibers is less than one. Per capita fiber consumption rose from about 34 pounds in 1949 to about 56 pounds in 1978. Both total and per capita fiber consumption fell during 1979-82 to 10.5 billion pounds and 45 pounds, but then recovered following the recession to about 13 billion pounds and 70.6 pounds in 1987. Despite the increase in total fiber consumption, domestic consumption of cotton declined from a postwar peak of 9.5 million bales in 1966 to 5.5 million bales in 1982, before rebounding to 7.8 million bales in 1987. Loss of market share to polyester and nylon explains cotton's decline. Cotton accounted for 81 percent of total U.S. fiber consumption in 1940, 53 percent in 1966, and about 31 percent in 1987.

2.8 Government Programs

U.S. farmers have experienced a depressed farm economy (i.e., overproduction, high carryover, and low product prices) since the turn of the century. Government programs since the early 1930s have been designed to deal with support prices and adjust acreage and production to balance with market needs. Cotton programs during 1933-65 included acreage allotments, marketing quotas, parity price supports, and nonrecourse loans. Cotton programs since 1966 have been more market oriented, featuring market price based on world price levels and direct payments to participating producers. These programs have provided some price and income stability, have met the objective of reducing or eliminating
surpluses, and have slowly transferred resources out of cotton production. However, they have not solved the underlying problem of chronic overcapacity of production, loss of market shares to manmade fibers, and loss of domestic market shares to cotton textile imports.

Prior to 1966, the loan rate served as an effective floor price on both U.S. and world cotton prices. However, U.S. farm programs for cotton since 1966 have had little impact on domestic use or U.S. exports of raw cotton because the market prices exceeded U.S. support price levels.

2.8.1 Early programs

From 1933 through the early 1960s, cotton programs included parity-price supports, nonrecourse loans, voluntary acreage reduction, marketing quotas, and acreage allotments. These programs were aimed at controlling production, reducing stocks, stabilizing market prices, and increasing prices. Parity price was announced in the Agricultural Adjustment Act of 1933 for restoring farm purchasing power of agricultural commodities to the 1910-14 average level. Although parity prices failed to reflect varying demand and supply conditions, this concept was used to set minimum levels of price supports through the mid-1960s for cotton. The nonrecourse loans were held through the Commodity Credit Corporation (CCC). Loans were secured by storing commodities in approved facilities either on or off the farm. The nonrecourse loans allowed the CCC to accept the commodity as full repayment of the loan. The loans also allowed producers to gain from any price increase while restricting the producer's downside risk by offering a floor price. The objective of nonrecourse loans was to enhance the cash flow of eligible farmers, to
stabilize market prices, and to support producer income.

The voluntary acreage reduction program was also introduced in the 1933 Act for controlling output and enhancing market prices. In 1934, legislation instituted marketing quotas to restrict the quantity of cotton that each producer could sell without paying a penalty tax; the programs ended in 1970. The other controlling production program was acreage allotments. The size of the national allotments was determined by the amount of acreage that would offer a normal year's consumption and exports, plus an allowance for stocks. These were then assigned to states, counties, and farms on the basis of past production (Cochrane and Ryan 1976). Another feature of the allotment program allowed any other crop to be planted on the withdrawn land.

Acreage allotments, marketing quotas, and price supports based on parity had a great effect during the early years, with the exception of 1943-49, due to the need to expand wartime production. Allotments remained in effect at varying levels from 1950 through 1970.

2.8.2 Cotton programs in the 1960s

In the late 1950s and early 1960s there was large stock carry-over, and existing legislation provided no effective provision to deal with it. The Cotton-Wheat Act of 1964 set payments to domestic handlers or textile miller to bring the U.S. cotton price down to the export price. The act also established a domestic cotton allotment, smaller than the regular allotment. Producers who participated within the domestic allotment received a higher support. The 1964 Act set up a voluntary program for reducing cotton production. The Food and Agriculture Act of 1965 was a
turning point in cotton policy. This act was more market orientated. The market price of cotton was supported at 90 percent of estimated world price levels, which allowed domestic market prices to follow world price levels. Incomes of cotton farmers were maintained through farmer-received payments from joining an acreage-reduction program. Payments were made to producers who complied with the minimum acreage allotments reduction requirements. Cotton producers started to join the diversion acreage programs in 1966. Consequently, cotton production substantially dropped during 1966-68 due to the diversion payments and low yields in 1966 and 1967.

By the end of 1970, the huge CCC stocks of cotton were gone. Although the voluntary program to reduce acreage had met the goal of reducing stocks, the direct payments in excess of $600 million during the late 1960s had resulted in relatively high treasury costs, with large payments going to large cotton producers.

2.8.3 Cotton programs in the 1970s

In the Agriculture Act of 1970, the set-aside programs were provided. Under the set-aside, participants were required to allocate at least 28 percent of their base acreage to approved conserving uses. The set-aside concept gave producers a greater flexibility in crop selection, allocating resources in response to changing economic conditions, because there was no limit on the crop mix on the remaining planted acreage. The 1973 Act set target prices and the disaster payments. The 1977 Act also set target prices based on cost of production. Loan rates were computed by the lower 85 percent of a preceding 3-year average of prices at domestic locations.
or 90 percent of the average price of specified classes of cotton in
northern Europe during the 15-week period beginning July 1 of the year.
No deficiency payments were made through 1977, as the average market price
received exceeded the target price. The 1977 Act facilitated a shift of
cotton production to the west and southwest regions where cotton held a
comparative advantage.

2.8.4 Cotton programs in the 1980s

The Agriculture and Food Act of 1981 was developed to support price
and income. The acreage set-aside programs were not satisfactory since
the programs allowed all planted acreage within the base acreage allotment
to qualify for deficiency payments which resulted in additional
production. Although the set-aside programs gave producers greater
flexibility in crop selection, the programs were not effective in
achieving crop-specific acreage reduction; therefore, the crop-specific
acreage reduction program was introduced. The acreage reduction programs
(ARP) revived the base acreage concept which had been removed in the 1977
Act. The ARP were in effect during 1982-84. In addition, the 1982-85
target prices were established at successively higher levels, such as 81
cents per pound in 1984 and 1985. The 1982 acreage and production dropped
20 percent and 25 percent from 1981.

A worldwide recession reduced both domestic and export needs,
inflation rates declined, and yields hit record levels. Stocks quickly
accumulated, despite acreage reduction programs. The PIK program,
announced to comply with the existing acreage reduction and cash-paid land
diversion programs, was to decrease the burdensome levels of government-
owned stocks. In addition to the 20 percent acreage reduction program, a diversion payment was offered on the additional 5 percent of idled acres. A producer who participated in the ARP had an option to idle an additional 10 to 30 percent of his base acreage and receive in-kind payments on planted acreage equal to 80 percent of the program yield. The PIK program, together with the ARP and paid land diversion, resulted in a drop of planted upland cotton acreage to 7.9 million acres in 1983. Production dropped by 4.2 million bales, and stocks dropped from 7.8 million bales on August 1, 1983, to 3 million bales on August 1, 1984.

No deficiency payments had been paid to cotton producers from 1974 through 1980, but there had been some disaster payments. Large deficiency payments were made during 1981-83, comprising a share of total income from raising cotton from 12 percent to 39 percent. Deficiency payments were raised to $1,706 million in 1986 because the average price was lower than the loan rate.

Falling mill use, lower export expectations, rising stocks, growing textile imports, and low farm income led to the development of new farm legislation. The 1985 Food Security Act established farm policy for five crop years, 1986-90. The act provided for greater market orientation and more flexibility to promote market competitiveness. The act also specified minimum target prices through 1990. Loan rates continued to be based on an average of past market prices with provisions for allowing loans to be repaid at levels below the loan rate if market competitiveness might be impeded by the formula-determined rate. The act specified that the total combined deficiency and diversion payments for each producer
could not exceed $50,000 annually during 1986-90 under one or more programs for wheat, feed grains, upland cotton, ELS cotton, and rice. The limitation of disaster payments per person was up to $100,000. In October 1986, Congress established a new ceiling of $250,000 per person on total farm payments, effective with all 1987 commodity programs, including the $50,000 payment limit for regular deficiency payments and land diversion payments, as well as all other government payments except crop support loans, grain reserve storage programs, and rice marketing certificate payments.

2.8.5 ELS cotton programs

In 1942, ELS cotton was eligible for government loans and price support while a CCC purchase program was in effect. During 1943-49 CCC loans were available for ELS cotton, but acreage allotments were eliminated from upland cotton, and the acreage planted to ELS cotton dropped to less than 15,000 acres. ELS cotton programs since 1952 have included acreage allotments, marketing quotas, and parity price supports.

In short, program specifics have altered substantially during the past 50 years, but a number of common characteristics remain. First, the cotton programs are designed to support price and producer income. The price support loan not only has supported and stabilized market prices, but it has also served to eliminate the downside price risk for complying producers. Second, the price support concept has remained since 1965. It was initially referred to as price support loan payments and later as deficiency payments, with the same intended effect. Income support can be offered to producers which does not require direct market intervention.
In addition, the payment level is determined as the difference between target price and the announced loan rate. The other continuing characteristic of cotton programs is supply control. In the past, if producers idled some proportion of their total crop land they were eligible to receive program benefits. Whether the programs are called allotments, diversions, set-asides, or acreage reduction programs, these acreage reduction programs have had the same goals: to reduce supply and limit budgetary expenses, as well as increase producer prices and income.

This chapter presents an overview of the U.S. cotton industry by providing a broad perspective on the scope and operation of the U.S. cotton market, as well as by indicating how government programs have evolved in this market over time. In so doing, this chapter sets the stage for the model specification and empirical application considered in ensuing chapters and provides important motivation for developing a dynamic model for U.S. cotton under rational expectations.
3 LITERATURE REVIEW

Expectations variables are extensively used in applied econometrics because the optimizing behavior of economic agents depends in part on agent views of the future. In application, a series which measures expectations or anticipations is usually not available, and various forecasting schemes have been suggested. The most popular expectations take the form of extrapolations, where the forecast of a variable is a weighted average of its own past values. However, these "are almost surely inaccurate gauges of expectations..." (Tobin 1955). An alternative device, developed by Muth (1961), specifies that the formulation of rational expectations is based on the premise that expectations represent informed predictions of the future. These informed predictions imply that agents are presumed to use all available information when they make predictions. The rational expectations framework is useful for considering various aspects of economic policy because it provides a model for the common perceptions of economic agents, having observed certain economic phenomena and expecting the impact on the system of the government policy they think will be brought in response to those phenomena.

A number of topics in theoretical models incorporating rational expectations have been discussed (Nelson 1975a, 1975b; McCallum 1976; Shiller 1978; Gallant and Jorgenson 1979; Kennan 1979; Blanchard and Kahn 1980; Wallis 1980; Hansen and Sargent 1982; Hansen and Singleton 1982;
Hansen 1982; Hayashi and Sims 1983). The statistical properties of rational expectations models and the issues of identification and estimation on such models have been addressed in the literature during the last two decades.

This chapter is organized as follows. First, methods for formulating price expectations are reviewed. In particular, this discussion will focus on the price expectations in agricultural economics. Second, the methods used to estimate rational expectations models are discussed. Third, some empirical evidences are presented that provide support for the rational expectations hypothesis. Finally, the issues associated with rational expectations are addressed.

3.1 Price Expectations in Agricultural Economics

In farm planning decision making, agricultural economists have recognized that price expectations are important, but few changes in the way price expectations appear in economic models of agriculture have been made during the past decades. Early expectations theories were based on the premise that decision makers depend on events of the recent past when forming predictions of the future such as Ezekiel's cobweb model (static expectations) and Nerlove's adaptive expectations. Nerlove's adaptive expectations hypothesis is still widely used in empirical analysis (Askari and Cummings 1977) mainly due to the simple operation technique. However, a number of conceptual problems of the hypothesis arise. For example, the specifications of these extrapolative predictions are not the result of an optimization process. Perceiving this contradiction, Muth (1961)
developed the rational expectations hypothesis (REH) based on the assumption that agents use all relevant information when making predictions. Kantor (1979) suggested that the REH is merely a modification of the assumption of optimizing behavior to the use of information.

3.1.1 The formulation of price expectations

Economists are concerned with formulating expectations of how many lag lengths should be needed, how serial correlation would be, and what weights should be used in the model. Distributed lag models typically used in literature are static expectations, extrapolative expectations, and adaptive expectations. Under the static expectations hypothesis, economic agents perceive that the current expected value is the same as last period's value of the relevant variable. The static expectations hypothesis provides the best forecasts if the relevant variable follows a random walk, such as "The Cobweb Theorem" by Ezekiel (1938). The cobweb theorem assumes that expected prices are current prices at the time of making production decisions. For instance, the expected price formed in period t-1 is given by the observed price at period t-1 if the production decision is made at period t-1 for output at period t.

The extrapolative expectations are merely to modify the static expectation by taking into account the most recent trend (change) in the relevant variable; for example, expected price $P_t^e$ formed as $P_t^e = P_{t-1} + \alpha(P_{t-1} - P_{t-2})$, where $P_t^e$ the expected price formed at period t-1; $P_{t-1}$ and $P_{t-2}$ are observed prices in period t-1 and t-2, $\alpha$ is Metzler's coefficient of expectation (Metzler, 1941). The extrapolative expectations approach
has been widely used and modified. For example, the adaptive expectations
are formed as follows: \( P_t^e = P_{t-1}^e + \alpha(P_{t-1} - P_{t-1}^e) \). Expected price is
based on the price expectations in the last period and the difference
between the actual and the expected price in the last period; where \( \alpha \)
reflects the economic agent's perception about the direction of the
expected price. \( P_t^e \) can also be written as
\[
P_t^e = \frac{\alpha}{1 - (1 - \alpha)L} P_{t-1}
\]
or
\[
P_t^e = \alpha \sum_{i=0}^{\infty} (1 - \alpha)^i P_{t-1-i}; \quad 0 < \alpha < 1.
\]
The expected price is explained as the sum of the weighted average of all
past prices. Forecasts under an adaptive model assume that the forecast
values of a variable will act in the future as they did in the past.
However, this assumption is too rigid to reflect any changes that will
occur. As cited by Fisher (1982), the assumption that price expectations
are formed adaptively is ad hoc because the parameter restrictions in the
distributed lag are not the result of an optimization process.

An alternative, the rational expectations approach, is based on the
assumption that agents use information optimally. The REH relies on an
optimizing principle that individuals should not make systematic errors in
predictions of the future. In forming expectations of endogenous
variables, economic agents account for the interrelationships among
variables involved in the appropriate economic system. Importantly, for a
given information set it is possible to test the presumption of rational
expectations on a certain market structure (Wallis 1980). Thus, the REH
model is consistent with the optimizing assumptions of economic theory and
is empirically testable.

3.1.2 Information and rational expectations

When forming expectations, agents also have to be concerned with what kind of information should be used and how it is put together to make estimations about the future. This information consists of a set of all available observations on the variable in question and on related variables at the time when making predictions. The expectations of REH represent informed predictions of the future. That is, they are informed in the sense that agents are presumed to use all available information optimally when making predictions. These informed predictions are the same as the predictions of the relevant economy theory (Muth 1961). REH is based on the premise that information is scarce, and the economic system generally does not waste it (Revankar 1980). The REH implies that economic agents form their expectations as if they know the process which generates the actual outcomes.

Economic agents have perceived that their predictions will be correct. But this REH does not state how economic agents derive or gather the knowledge which they would use to formulate expectations. The question of the informational requirements of rational expectations has led some to suspect the empirical applicabilities of these models, but this seems to be unresolved (Fisher 1982).

Taylor (1975) argued that if it takes time for agents to learn the actual essence of monetary policy (or any kind of policy), then the rational expectations technique might not be a good device for studying the immediate effect of a sudden change in policies. If a learning
process is needed, the REH is invalid for the transitory period. Rational expectations might have advantages for a longer time horizon (Langley 1982). To gather the learning process or information formation into rational expectations is often difficult. Several economists have discussed how to best gather available information; i.e., Chavas and Johnson (1982) developed a formula to measure the information available for formulating expectations based on Theil's information theory. Antonovitz and Roe (1984) presented a measure of the value of information under risk. When the costs of gathering and processing information are taken into consideration, autoregressive expectation or adaptive expectation model may be an alternative to the rational expectations model (Feige and Pearce 1976).

3.1.3 The rational expectations equilibrium path

The econometric implications of a dynamic rational expectations model in which agents are supposed to solve linear-quadratic stochastic optimum problems, subject to linear constraints, have been widely used. The resulting solutions of the dynamic optimum problems are a set of equilibrium stochastic processes which reflect the representative firm's optimal decision rules such as supply equations or inventory demand schedules. An equilibrium is defined when all constraints of the stochastic optimization problem are satisfied. If the optimization problem is not in the linear-quadratic framework rather than alternative nonquadratic objective functions, the dynamic rational expectations models will not result in representations for the variables that are as convenient from the standpoint of econometric analysis. As a matter of
fact, many closed-form solutions for the equilibrium time paths of the variables of interest are derived by imposing prior assumptions on the stochastic characteristics of the "forcing variables," and the property of preferences, or the production technology (Hansen and Sargent 1980).

3.2 The Estimation of Rational Expectations Econometric Models

To derive an estimable decision rule from the dynamic optimization problem, a set of stochastic Euler equations and transversality conditions in equilibrium must be satisfied. A possible way to solve the optimization problem is as follows: (1) specify the rest of the economic environment, such as the production technology and the stochastic properties of the forcing variables; (2) derive an equilibrium representation for the endogenous variables expressed as a function of past endogenous variables and current and past forcing variables; (3) estimate the parameters of tastes, technology, and the stochastic process generating the forcing variables by applying a full information procedure such as maximum likelihood method. However, to have a general representation of the forcing variables, to derive an equilibrium representation of the observable variables, and to estimate the parameters of tastes and technology together with the parameters of the forcing processes, appear to be important econometric work.

These Euler equations, nevertheless, suggest a set of orthogonality conditions that rely in a nonlinear way on variables observed by an econometrician and on unknown parameters characterizing preferences, profit functions, etc. Several authors have used stochastic Euler
equations directly to estimate parameters in the context of models constructed from linear-quadratic optimization problems (Kennan 1979; Hayashi 1980). However, the estimators obtained from directly estimating stochastic Euler equations are less efficient, since they ignore theoretical restrictions. Hansen and Sargent (1982) proposed a method that solves the Euler equations, exploits the symmetry between the feedforward and feedback portion of this solution, and imposes restrictions across the feedforward portion of this solution and the stochastic specification of the observable forcing variables.

Hansen and Singleton (1982) constructed nonlinear instrumental variables estimators for nonlinear rational expectations models in the way suggested by Jorgenson and Laffont (1974), Amemiya (1974, 1977), and Hansen (1982) by using sample versions of the orthogonality conditions close to zero in accordance with a certain metric. These obtained estimators were consistent and asymptotically normal, but they had fairly weak assumptions about the stochastic processes governing the observable time series. In addition, more orthogonality conditions are available for use in estimation than parameters to be estimated. Thus, a test on overidentifying restrictions is to examine how sample versions of population orthogonality conditions are close to zero (Hansen 1982).

Hansen and Sargent (1982) used the instrumental variables procedures to estimate the parameters of a dynamic linear rational expectations model. Their instrumental variables procedures can apply to the case where disturbances are serially correlated and instrumental variables are not exogenous. They derived estimators from an underlying set of
orthogonality conditions implied by the econometric model. Their estimators have the same form as the nonlinear instrumental variables estimators considered by Jorgenson and Laffont (1974) and Amemiya (1974, 1977). However, results from the latter's papers do not take into account serial correlation among the disturbance terms, and the error terms may not be uncorrelated with some set of instruments. Therefore, a drawback of the latter's approach is that their construction is not possible to practice in reality. Hansen and Sargent also compared their procedures to some alternative estimators that estimate free parameters from restrictions implied by the Euler equations. They concluded that "the Euler equation approach to estimating dynamic linear rational expectations models is computationally simpler and requires that less be specified a priori."

Although the Euler equations approach need not require an explicit stochastic specification of the observable forcing variables, this does not suggest the resulting instrumental variables estimators will be more sound than other alternatives in a change policy regime. The Euler equation approach implicitly assumes that the projections of the variables onto the instruments have time invariant representations.

Hansen (1982) and Cumby, Huizinga, and Obstfeld (1981) constructed consistent estimators for models which may have serially correlated errors that may be uncorrelated with some set of instrumental variables, but they do not explicitly correct serial correlation in the estimation process. Hansen (1982) obtained the asymptotic distribution for these estimators and proved that they are efficient within a certain class. Hayashi and
Sims (1983) called these estimators "finite-order efficient". They proposed a method to correct serial correlation in such models which make standard theory of employing instrumental variables estimation. Their instrumental variables estimators were constructed by first filtering the disturbance term forward to remove serial correlation. They also assimilated these forward filtered estimators to ones that used a fixed finite number of orthogonality conditions without forward filtering and interpreted some advantages of forward filtering. Hayashi and Sims (1983) also provided the interpretation of the optimal weighing scheme, but they did not explicitly characterize it rather than discuss how to establish optimal estimators.

The estimation scheme described by Hansen and Sargent (1982) and Hayashi and Sims (1983) exploited the serial correlation properties of the disturbances in constructing an optimal estimator. An important advantage of their procedure over maximum likelihood procedures does not strictly require a precise specification of the temporal covariance structure of the instrumental variables and disturbances. Furthermore, the all free parameters can be estimated by numerically searching over a smaller parameter space using this instrumental variables procedure than is required by maximum likelihood procedures.

### 3.3 Testing the REH

In rational expectations models, the expectations are formed as mathematical expectations of variables conditional on all available information up to that time. The coefficients of these variables are
restricted to be certain functions of the parameter in the embedding model. Thus, the rational expectations variables take on the form \( X_t' \), where \( X_t \) is a vector of observable exogenous variables; the coefficient vectors, \( r \)'s, have to satisfy certain restrictions in terms of the parameters of the underlying model. However, the economic agents actually involved in the market may have difficulties in learning the underlying structure of the model (De Canio and Stephen 1979; Friedman 1979). Therefore, the special structure of rational expectations suggests a test of the validity of the restrictions.

The basic concept of testing the validity of the REH was first proposed by Lucas (1972), who employed it in a simple macro model. It was later used by Revankar (1976) in setting up multivariate regressions. Wallis (1980) also discussed this theme, but only briefly. Hoffman and Schmidt (1978) put this idea to work under alternative model specifications.

The choice of the alternative (\( H_1 \)) to the null hypothesis (\( H_0 \)) is optional. Under \( H_1 \), expectations variables can be chosen as any arbitrary functions of the observable predetermined variables, but not necessarily in \( X_t \). For example, adaptive or extrapolative expectations formed from past values of respective endogenous variables are among the obvious candidates. Then, an ordinary likelihood ratio test is formed; taking the unobservables in the form, \( X_t' \), treating \( r \)'s as unconstrained, and the REH restrictions on \( r \)'s. But the likelihood ratio tests are too severe to permit analytical progress with constrained maximum likelihood estimation of the model (under \( H_0 \)) except in simple cases (Taylor 1979; Hansen and
Sargent 1980). On the other hand, unconstrained maximum likelihood estimation (under $H_0$) is much simpler to analyze, and under a broad range of alternative model specifications. The Wald test based on the asymptotic distribution of the unconstrained maximum likelihood estimators of the $r$'s is an alternative to test the REH restrictions.

Another technique, a non-nested test, was suggested by Cox (Gayer and Geisel 1974). Gallant and Jorgenson (1979) proposed a test for restrictions that is the nonlinear three-stage least squares similar to the likelihood ratio test. They used chi-square test (G.J. chi-square test) to test the across-the-equations restrictions implied by rational expectations hypothesis. Also, derivation of the asymptotic distribution of their test statistic could be obtained in the estimation environment by employing Hansen's "Large Sample Properties" (Hansen 1982). Avery, Hansen, and Hotz (1981) used Lemma 4.1 of Hansen's "Large Sample Properties" to derive some alternative specification tests.

3.4 Empirical Microeconomic Research

The succeeding two decades were characterized by growing optimism about the utility and general rational expectations equilibrium modeling. Rational expectations provide a useful variation point for analyzing the implications of various alternative assumptions. Empirical work on rational expectations models in agriculture is quite limited compared to the large number of studies employing the adaptive expectations hypothesis.

One popular way of incorporating rational expectations in the model
is to impose rational expectations on a conventional econometric model in which expected values of a subset of the endogenous variables are included in the economic system. In doing so, the equations in the system implicitly reflect both the optimal decision rules of the agents and the way they involved in each other. Although using decision rules of econometric models may lose some of the structure derived from an optimization problem, the analyst employs restrictions from economic theory by incorporating unobservable anticipations variables in the model. Huntzinger's (1979) was the first attempt to apply the REH in an econometric model of the broiler chicken industry. He used weekly data series and found reasonable estimates of key coefficients, applying an instrumental variables technique with the assumption of rational expectations.

Goodwin and Sheffrin (1982) applied the same basic framework. They used seasonal, autoregressive, moving-average processes to derive forecast values of the relevant exogenous variables and then estimated the full demand and supply system with full-information methods. They also applied their model to test the REH. They found that Muth's concept of rational expectations does characterize the broiler behavior. Their work strongly supports the recent contributions of Wallis (1980) and others (i.e., Friedman 1979 and Hoffman and Schmidt 1981) to the econometrics of rational expectations.

The other way of incorporating the REH is from micro-framework to derive the optimal decision rules for an optimization problem. The representative agent is assumed to be rational. All exogenous variables
whose values are unknown at the time the forecast is formulated are assumed to follow stable stochastic processes. A rational expectations equilibrium has to meet the constraints of the stochastic optimization problem; then the final estimating equations include nonlinear cross-equation restrictions on the parameters. Eckstein (1984, 1985) developed a dynamic model of price expectations in analyzing the impact of prices on agricultural supply and land allocation. He provided some evidence on the actual facts of the dynamics of agricultural supply. He showed that a rational expectations equilibrium model can generate equilibrium movements of prices and output that have the same form as in the cobweb model. He also showed that the rational expectations model with dynamic constraints on land allocations through the cost function is consistent with the data vis-a-vis the Nerlovian model, but leads to different policy implications. Further, he showed that the two models are observational equivalent, having the same reduced form. Finally, he presented a strategy to estimate the supply elasticities with respect to changes in prices.

Langley (1982) presented the theoretical derivation and estimation of the soybean model under the three price expectations regimes—rational, adaptive, and cash-futures. The rational expectations model implies that agents who have a correct perception of market behavior will perform better than those who do not. By insisting that the supply always equal demand, she obtained a dynamic model that determines stochastic processes for the decision variables that clear the soybean market and soybean product markets. However, in her study, the equilibrium stochastic processes did not satisfy the transversality conditions and the Euler
equations of the producer and the miller problems. The nonlinear restrictions imposed by the rational expectations model in her study failed to pass the test of cross-equation restrictions.

Most recently, there has been some extensive work on the REH to include uncertainty (Antonovitz and Green 1987; Holt 1987). Antonovitz and Roe (1984) incorporated agent's risk preferences in a market-level econometric model to estimate the value of information as a function of the mean and variance of a rational expectations forecast. With the assumption that agents have rational expectations about all relevant moments of the equilibrium price distribution, Holt derived supply equations for program and nonprogram acreage response from an expected utility maximization work. Following the truncation effect of price supports on producers' expectations, Holt showed that change in program benefits, such as an increase in the support price, may decrease aggregate acreage as implications of such a policy change on the land set-aside requirement are taken into account.

3.5 Rational Expectations and Policy Evaluation

Lucas's (1976) critique on policy evaluation showed what serious errors can be made in econometric policy analysis if the response of expectations formulation to policy is ignored. Lucas perceived that the implication of dynamic economic theory is that in general all equations can be expected to change following a change in policy regime, not just the equations describing the government policy. And he argued that the parameters of the decision rules are functions not only of parameters in
agent's objective functions and the stochastic processes governing the exogenous variables but also of government policies. If a change in government policy may affect the paths of those exogenous variables considered to be policy instruments, the estimated coefficients of most economic models are not invariant to change in policy rules or regimes. Thus, economists wanting to analyze policy interventions, such as price stabilization schemes or changes in target price formula, should trace the effect of a policy regime through to the relevant decision rule (Fisher 1982). If economic agents use information optimally, then the REH provides a good device to solve this crucial problem.

Many economists such as Mishkin (1979) and Sims (1980) have argued that many policy actions are actually exercises within a stable framework so that equations of econometric models may really be invariant to some styles of policy actions. However, a nontrivial change in policy could not be evaluated along with the model, and structure of the model most likely would not maintain past historical relationships. Thus, estimated decision rules will only provide restricted scope for policy evaluation.

3.6 Rational Expectations and Vector Autoregressions

The influence of Lucas's critique (1976) has created doubts about the validity of the a priori restrictions used to identify many economic models. One response to the spurious nature of the a priori theoretical restrictions has return to a Kepler style which is less dependent on a priori theoretical restrictions and uses time series methods. This evolution is due primarily to Christopher Sims's vector autoregression
In a dynamic setting, decision theory describes an optimal policy choice as a single analytical practice. A complete contingency plan has to reflect policy actions at all future events. However, in practice, economic policymakings do not seem to follow this sort of once-and-for-all analysis. Policymakers usually consider what actions to take in the next few periods or years and reformulate their plans every few periods, and repeatedly employ econometric models to predict the possible effects of alternative actions. Furthermore, optimal policy should be a deterministic function of all available information up to that time, but actual policy seems to contain some unpredictable components.

Vector autoregressions (VAR) approach has been suggested an appropriate way of forming the government policy variables predicted to reflect the most desirable VAR for the economy. When the structure of the economy is unknown, VAR might be a good device for characterizing economic relationships. Since such VAR can be used to summarize the second moments of time-series data, it complies with the recursive decision theory associated with dynamic rational expectations. Of importance, VAR need not impose a priori restrictions.

If the policy rule is affected and cohered to some future time, people will eventually be convinced that it is highly possible for future policy to set and use the scheme. Then, the rational expectations assumption provides a useful tool to analyze the long-term effects of fixed rules. But policy does not always take the form of fixed rules, so it is not reasonable to analyze the effects of permanent shifts in fixed
rules. Thus, the economist's intention is to obtain good practical qualitative advice for formulating new strategies for government actions in the years beyond the sample period. Using VAR to capture the likely effects of various paths for policy variables in order to avoid constructing behavior stories about each individual equation in the model is highlighted in Sims's "Policy Analysis with Econometric Model" (1982). However, use of Sims's VAR methodology to assess the economic forecasts might turn out to be inappropriate if policy is not generated by the previous decision rule. Consequently, the predictable outcomes based on the historical record could not be applied. Additional research on appropriate ways of estimating and utilizing VAR will be required before the VAR approach will be useful as a tool for or applied analysis of decisions under uncertainty.

3.7 Rational Expectations, Econometric Exogeneity, and Causality

The issue of exogeneity in estimating the rational expectations model has been discussed by Sims (1972) and Sargent (1979a, 1979b). To know causality tests, suppose two time series \( \{x_t\} \) and \( \{y_t\} \), the series \( \{y_t\} \) does not Granger-cause \( \{x_t\} \) according to the Granger (1969) test if, in a regression of \( x \) on lagged \( x \) and lagged \( y \), the latter takes on a zero coefficient. In terms of the VAR model, the regression of interest can be written as: \( x_t = \pi_{11}x_{t-1} + \pi_{12}y_{t-1} + \mu_{xt} \), and the coefficient \( \pi_{12} \) takes zero. According to Sims's (1972) test, if, in a regression of \( y \) on lagged \( y \) and future \( x \), the latter takes on a zero coefficient, it implies that \( \{y_t\} \) fails to Granger-cause \( \{x_t\} \).
Another option is to prewhiten the data by using Box-Jenkins methods and to derive the univariate innovations of each series, and then to find out whether the innovations in \( \{x_t\} \) can be forecasted from those in \( \{y_t\} \) according to Granger-Sims tests. This procedure was developed by Pierce and Haugh (1977). Even if there are econometric differences between these tests (Feige and Pearce 1976; Hosoya 1977; Pierce and Haugh 1977; Kohn 1981; Chamberlain 1982; Florens and Mouchart 1982), it is clear that the Granger and Sims tests indicate the same null hypothesis (Jacobs, Leamer, and Ward 1979). If \( y \) fails to Granger-cause \( x \) it is said that \( x \) is exogenous with respect to \( y \). That is to say causality runs only from \( x \) to \( y \) if past \( y \) does not help in predicting current \( x \), given past \( x \). Further, if \( x \) does Granger-cause \( y \), \( x \) is said to be causally prior to \( y \).

Suppose there is a regression form as

\[
y_t = \sum_{i=0}^{\infty} d_i x_{t-i} + v_t, \tag{3.1}
\]

\[
\sum_{i=0}^{\infty} d_i^2 < \infty, \ E v_t = 0, \ E v_t^2 = \sigma_v^2, \text{ for all } t, \text{ and } \\
E(v_t x_s \text{ for all } s) = 0. \tag{3.2}
\]

Where \( E \) is the linear least-square projection operator. The condition (3.2) says \( x \) strictly econometrically exogenous with respect to \( y \). An equivalent statement of (3.2) is

\[
E v_t x_s = 0 \text{ for all } s,t. \tag{3.3}
\]

Sargent (1979b) proved that there exists a family of equations expressing \( y_t \) as a one-sided distributed lag of \( x \) for a given \( (y_t, x_t) \) process in which \( y \) fails to Granger cause \( x \). So Sims's condition that for
x to be strictly exogenous in a particular equation of the form (3.2) is necessary but not sufficient that y fails to Granger cause x.

Hansen and Sargent (1978) applied stronger exogeneity tests to certain rational expectations models by imposing overidentifying, cross-equation restrictions on the lag distribution \( x \) in form (3.1). The test is somewhat important, since the most efficient estimation techniques for distributed lags are invalid unless causality is unidirectional in the Granger sense.

Strict exogeneity does certainly indicate Granger noncausality so that failure of a Granger or Sims test is an indication against strict exogeneity with the ordinary significance criteria, but acceptance of Granger non-causality does not suggest strict exogeneity.

The strict exogeneity, a stronger restriction than predeterminedness, allows the two types of interventions that can be exercised in dynamic models—intercept shifts and altered time paths for one of the variables. Given predeterminedness, the strict exogeneity equals the Granger non-causality. Therefore, it can be formed either by a Granger or Sims test, or simply by executing the relevant simulations and then comparing the results.

But for predeterminedness itself, which is required to verify any of the interventions under discussion, Granger non-causality is not related to the theme nor are the Granger-Sims tests. Clearly, Granger non-causality is neither necessary nor sufficient for predeterminedness. Since predeterminedness is the relevant exogeneity concept for the analysis of interventions, the Granger and Sims tests are irrelevant as to
whether a causal exposition of a conditional correlation is proven right. Furthermore, predeterminedness is the relevant exogeneity concept for econometric estimation so that the Granger and Sims tests are unrelated to the question of whether a model is consistently estimated (Cooley and LeRoy 1985).

The disturbances in general are correlated with future values of the instrumental variables (Hansen and Sargent 1982). The decision variable contains information that marginally helps predict future values of the instrumental variables. So failing a Granger-Sims test for the null hypothesis, y fails to Granger cause x does not necessarily indicate model misspecification. In particular, it has no bearing on whether or not the orthogonality condition is proper.

3.8 Summary

In reviewing the previous studies on rational expectations and associated issues, some themes were selected and emphasized in this chapter. The difficulties connected with parameter identification in the rational expectations models have caused most econometricians, with some justification, to become cautious when explaining them. Identification problems have been addressed by Hansen and Sargent (1980), Wallis (1980), and Pesaran (1981). Usually, a priori restrictions have been imposed if rational expectations models with future expectations are to be identifiable (Hansen and Sargent 1980; Pesaran 1981).

Current economic decisions depend not only on the observable values of variables entering an econometric model, but also on an individual's
expectations about future values of those variables. This implies economic theory cannot be discussed without expectations formulation. Further, alternative models can be related within a general framework as their different expectations assumptions are perceived. In modeling an economic system, *a priori* questions on expectations formation are required.

Economists use data over a long period to explain expectations formation as a stochastic process or a decision rule. But reliable data on current and future are usually not available. Faced with these problems, econometricians should carefully specify the important characteristics of the market when estimating models. For example, the forecasting horizon and time-series properties of the exogenous variables must be explained precisely in order to effect the estimation and to reflect the way the economy works. A model building must be known and could test the implication of the economic theories underlying the economic structure with the empirical data. When the structure of the economy is unknown, VAR gives a useful characterization of economic relationships.
There is growing evidence that rational expectations and uncertainty play useful roles in economic decision making (e.g., Helmberger et al. 1982; Eckstein 1985; Antonovitz and Green 1987; Holt 1987). Considerable effort has been made to examine the effects of uncertainty and expectations on the optimal decisions of the firm. The aim of this chapter is to develop decision rules at the firm level and then to extend them to the market level under uncertainty in the U.S. cotton market. This development draws upon the earlier work of Eckstein and Langley. Eckstein (1985) derived the dynamic agricultural supply for Egyptian cotton. He showed that this rational expectations equilibrium model can explain the observed dynamics of agricultural supply as well as the Nerlovian supply response model. Langley (1982) built a rational expectations equilibrium model for the U.S. soybean market. However, the way stochastic processes are formed within Langley's study is not consistent with rational expectations. One purpose of this chapter is to extend the model to more than one market, including raw cotton and cotton yarn markets.

In addition, this chapter presents the cotton industry model by considering market equilibrium in the presence of rational expectations and government programs (direct payments). It is assumed that producers face a random demand schedule, the sole source of uncertainty. The model is closed with a REH and conditions under which equilibrium is reached.
The chapter is organized as follows: First, a model representing the
decision-making behavior of producers and consumers in the U.S. cotton
market is outlined. In particular, a priori information necessary to
obtain tractable decision rules is discussed. Specific assumptions about
production technology and the structure of the market demand functions
used for the empirical application are presented and discussed. Second,
the models with and without storage are introduced. For processing
decisions, producer and miller optimization problems are solved
individually, and the market equilibriums for the cotton market and the
yarn market are computed simultaneously. More specifically, a framework
is introduced which can be used to evaluate the optimal decision rules.

U.S. cotton producers and consumers have some planning horizon. When
a rational expectations equilibrium model is generated to solve an
optimization problem, it is to use available information in choosing a
sequence of contingency plans. The derived decision rules on acreage
planted, cotton mill consumption, and inventory are solved into the
infinite horizon. That is, the derived decision rules rest on the
parameters underlying the structure of the cotton industry and the
parameters which characterize the exogenous variables.

For deriving observable decision rules, it is required to know the
probability distribution of exogenous and endogenous variables in order to
obtain the future values of the relevant variables. However, the joint
distribution of the relevant variables is not known, and even if it were
known, the resulting formula is often too complicated to be of any
practical value. Therefore, the Wiener and Kolmogorov linear least square
prediction and Wold's decomposition theorem are applied to forecast future values of the relevant variables.

If agents optimize, the forms of such forecasting rules depend on the nature of the exogenous stochastic processes facing them. Since changes in the policy rules alter those processes, the forecasting rules, and, therefore, the parameters of the model, will change with each change in the policy rule. The dynamic nature of the cotton industry and the farmer program policy regime and disturbance shocks characterize the paths of the decision rules.

The rational expectations model relates the two sectors: producers and consumers of U.S. cotton by using a representative cotton farmer endowed with land that is to be allocated between two crops (i.e., cotton and corn). While the cotton consumers buy cotton for milling into yarn, the farmer and the miller also have to decide whether or not inventory is to be carried over for future usage and for speculation, and what level of inventory should be carried over. The rational expectations model is built to solve the firm's decision problem.

4.1 An Equilibrium Model

The model is designed to represent the phenomenon of the U.S. cotton market. The following equations belong to the kth farmer and the qth mill firm at time period t where subscripts "k" and "q" are omitted, and k = 1,2,...,n, and q = 1,2,...,m. All variables are defined in Appendix A. Small letters indicate a representative agent while capital letters are for aggregate U.S. cotton. Under tropical conditions cotton is a
perennial, but it is grown in the United States as an annual from seed planted each year; most U.S. cotton is planted in the second quarter and is harvested in the fourth quarter each year. The time index "t" in the following structural model refers to year and takes values at t = 0, 1, 2, ....

Production of cotton
\[ ct_t = f_{10} + f_{11}a_{1t} + f_{12}y_{1t}; \]  
(4.1)

Production of corn
\[ cr_t = f_{20} + f_{21}a_{2t} + f_{22}y_{2t}; \]  
(4.2)

and \( a_{1t} + a_{2t} \leq a; \)

Cotton sold
\[ cts_t = ctb_t + ctx_t + ctg_t - ctg_{t-1}; \]  
(4.3)

Cotton bought by miller
\[ ctb_t = ctm_t + (ctms_t - ctms_{t-1}); \]  
(4.4)

Identity of cotton
\[ ct_t = cts_t; \]  
(4.5)

Farm price linkage
\[ RPF_t = \beta RPW_t + U_{1t}; \]  
\( 0 < \beta < 1; \)  
(4.6)

Cost of production
\[ c_{1t} = RCTVC_t a_{1t} + \frac{d_1 a_{1t}^2}{2} + d_2 a_{1t} a_{1t-1}; \]  
(4.7)

Cost of miller
\[ c_{2t} = RPW_t * ctb_t; \]  
(4.8)

Adjustment cost of milling
\[ c_{3t} = h_1 ctm_t + h_2 (ctm_t - ctm_{t-1})^2; \]  
(4.9)
Inventory cost of miller
\[ c_{lt} = k_1 ctms_t + \frac{k_2}{2}(ctms_t - ctms_{t-1})^2; \quad (4.10) \]

Cotton mill production
\[ cm_t = ctm_t \times r; \quad (4.11) \]

Mill demand
\[ CTM_t = m_0 + m_1 RPM_t - m_2 RPW_t + U_{2t}; \quad (4.12) \]

The crop production of cotton and/or corn for the \( k \)th representative farmer with a given land, \( a \), is a function of acreage planted, \( a_{1t} \) and/or \( a_{2t} \), and production yields, \( y_{1t} \) and/or \( y_{2t} \), expressed in equation (4.1) and/or equation (4.2). Equation (4.1) and/or equation (4.2) was derived by using a Taylor series expansion and shown in Appendix B. It is assumed both crops could be produced on the same land at the same period.

The total cost of production of cotton is given a linear-quadratic form expressed in equation (4.7). The first term of the cost of production, \( c_t a_{1t} \), is the per-acre variable cost of cotton production. The second term, \( (d_1 / 2)a_{1t}^2 \), reflects decreasing returns to scale over the long run to indicate existing rent on the fixed asset, land. The third term of the cost of production is to capture deterioration in land productivity. If \( d_2 < 0 \), there has been more plot preparation for a given crop in the previous year, the producer will, ceteris paribus, attempt to produce again since the current average productivity of land is increased. But, if \( d_2 > 0 \), growing a given crop continuously on the same plot may be exhausting the productivity of the plot, and the producer will then rotate crops. Hence, land productivity is reduced. If \( d_2 = 0 \), there is no
linkage between the current average productivity of land and past cultivation. It then would be a static farmer maximum problem. For more discussion of soil conservation with the sign of $d_2$, see Eckstein (1981).

At harvest, seed cotton is hauled from farms to cotton gins. At gins, cotton is pressed into lint and seed. Then the cotton lint is assembled, bagged, and tied into 500-pound bales. The bales of lint cotton are offered for sale to the textile industry, exported, or stored. Inventories of cotton are held for anticipated profits or as collateral for the CCC. The data on farm stocks are included in the category "elsewhere" within transit. Moreover, the stocks on farms are a small fraction of total stock and therefore are ignored in the study.

In period $t$, the farmer's total production of cotton should be equal to its disappearance to mill consumption, exports, and changes in on-and-off farm stocks. This expresses in equation (4.5), identity equation.

In addition, the market is confronted with an exogenous, linear demand schedule for the mill consumption, where under market clearing condition, the derived demand is in equation (4.12) and reproduces here

$$CTM_t = m_0 + m_2RPM_t - m_2RPW_t + U_{2t}; m_1, m_2 > 0; \quad (4.13)$$

where $CTM_t$ is the aggregate mill consumption at time $t$, $RPM_t$ the price of spun yarn at time $t$, $RPW_t$ the wholesale price of cotton, and $U_{2t}$ is a shock to demand, the source of uncertainty.

The production of spun yarn is expressed in equation (4.11); it equates spun yarn production to the conversion factor for cotton lint into yarn times the quantity of cotton-baled lint. The representative miller is competitive in the output (yarn) and factor markets (cotton-baled lint)
and thus is a price-taker with respect to the output market equilibrium price as a stochastic process (RPM_t) and also with respect to the factor market price (RPW_t) derived from the cotton equilibrium market.

The inventory of lint cotton at mills is to protect against possible scarcity in the next period. The cost of producing spun yarn includes the cost of raw cotton baled lint (in equation [4.8]), adjustment cost of milling (in equation [4.9]), and inventory cost (in equation [4.10]). The inventory cost of the miller is comprised of storage charges and adjustment costs. Adjustment cost of milling equation (4.8), which reflects the speed of adjustment cost increasing at a decreasing rate. In addition, it is designed for a tractable solution. Equilibrium is also imposed in the cotton yarn market.

The linkage between the cotton price farmers received (RPF_t) and the wholesale price of cotton lint (RPW_t) is expressed in equation (4.6). Price RPF_t determines the farmer's decision to plant cotton, while RPW_t determines the demand for mill consumption. A difference between RPF_t and RPW_t reflects a marketing margin. That is, U_{1t} is an error term to capture unexplained price differences.

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1 A marketing margin may be defined as a difference between the price paid by consumers and that obtained by producers. Thus, a marketing margin is a difference between the primary demand and derived demand curves for a particular product.
4.2 A Model Without Storage

To simplify analysis without loss generality, a zero inventory is assumed in the U.S. cotton market. That is, \( c_{t+s} = c_{t+s-1} = c_{t+g} = c_{t+g-1} = 0 \). And there is no charge on inventory cost, \( c_{it} = 0 \). At equilibrium, supply equals demand. That is, production equals cotton sold, \( c_t = c_{st} \).

This section is organized as follows: First, a representative cotton farmer's optimization problem is described. The firm-level optimal decision is derived and discussed. A miller's optimization case is then presented. The firm-level optimal decision is aggregated, and the market equilibrium is obtained and discussed.

4.2.1 A representative cotton farmer

A representative cotton farmer is assumed to maximize his expected discounted profit by choosing a contingency plan at each period \( t \) for allocating his land, \( a \), to the two crops: cotton and corn.\(^2\) Thus, the farmer's maximum problem is to maximize

\[
V_{1t} = E_t \sum_{t=0}^{\infty} b^t \pi_t. \tag{4.14}
\]

Where \( \pi_t = \text{RPF}_t c_{st} + \text{RCORR}_t c_{rt} - c_{it} + \text{RDP}_t \times \left[ (c_{it} \times a \times \text{Part}) / (1 - (1 - a) \times \text{Part}) \right] \). \( \pi_t \) is the farmer total profit at time \( t \), \( \text{RCORR}_t \) is the

\(^2\) The competitors of cotton for land include soybeans, corn in the Southeast and Delta, grain sorghum and wheat in the Southwest, and wheat, hay crops, and barley in the irrigated Far West. Soybeans are a major competitor of cotton in the Delta, but data on soybeans for the study period are not all available so that corn is chosen for the cotton's competitor in this study.
net return of corn per acre at time \( t \). To simplify things, \( RCORNR_t \) is exogenous to this cotton market. Although net returns of corn play an important role in the determination of the dynamics of the cotton market, we focus on the dynamics that emerge from cotton markets so that we abstract from the role of corn markets. Furthermore, assuming \( RCORNR_t \) follows an \( n \)-th order Markov process

\[
RCORNR_t = \nu_0 + \nu_1 RCORNR_{t-1} + \nu_2 RCORNR_{t-2} \ldots + \nu_n RCORNR_{t-n} + \varepsilon_{3t}, \tag{4.15}
\]

where \( \varepsilon_{3t} \) is a least-square disturbance that satisfies \( E_t \varepsilon_{3t} = E_{t-1} \varepsilon_{3t} | \Omega_{t-1} = 0 \). The condition that \( E \varepsilon_{3t} | \Omega_{t-1} = 0 \) means that \( \varepsilon_{3t} \) is serially uncorrelated and that \( RCORNR_t \) is not caused in Granger's (1969) sense, by \( a_{1t} \). The lack of Granger causality from \( a_{1t} \) to \( RCORNR_t \) is to be detected in the empirical section. The roots of the stochastic process (4.15) are inside the unit circle. The \( n \)-th order process of \( RCORNR_t \) is sufficient to capture a cyclical movement on the corn market, which is assumed to be exogenous to the cotton market.

cs_{st}, cr_{st}, and c_{st} are defined as in the previous section. The last term of the profit function represents approximate direct payments, including deficiency payments, diversion payments, and PIK entitlements. A key feature of the farmer's profit function includes the amount of money received from the government. Direct payments to farmers make up a greater share of their total cotton income since 1970. In agricultural supply response analysis, important structural changes have often reflected the influence of government farm programs that are designed to control production and enhance producer income. Consequently, the
incorporation of farm program changes in crop supply response models has been given considerable attention in the previous empirical research (Houck and Ryan 1972; Lee and Helmberger 1985). Lucas (1976) also argues that the structure of economic models varies directly with government policy rules. Therefore, an important aspect in analyzing the farmer's optimal problem is to incorporate the government farm policy variable in the model. Note that the last term of the profit function will drop out if the farmer does not join the government programs. In the 1980s, over 80 percent of the cotton farmers joined in the programs.

Approximate total direct payments for cotton were calculated in the following manner

\[ RDP_t \times \left( \frac{(ct_t \times \alpha \times Part)}{1 - (1 - \alpha) \times Part} \right) \]

where \( RDP_t \) is direct payments. The maximum direct payments that could be received are based on the difference between the target price and the market price. But if the market price is down below the loan rate, then the direct payments are the difference between the target price and the loan rate. \( \alpha \) is (1 - diversion rate or set-aside rate), Part is the participation rate. \( \left( \frac{(ct_t \times Part \times \alpha)}{1 - (1 - \alpha) \times Part} \right) \) represents the eligible participant's production; it reflects the program payments which are usually computed as (the established program yield) \( \times \) (acreage planted under the program) \( \times \) (an allocation factor). If market price falls below the loan rate, participants could take out a nonrecourse loan and be assured of receiving at least the loan rate price. It is of interest that the target price regime instituted in 1973 has the same function as the price support loan payments offered in the 1960s. The
target price concept was merely a modifying policy instrument used in earlier programs.

If we substitute equation (4.5) for $c_{ts}$, equation (4.1) for $ct$, equation (4.2) for $cr_t$, and equation (4.7) for $c_{t_0}$. Then $\pi_t$ can be written as

$$\pi_t = RPF_t(f_{10} + f_{11}a_{1t} + f_{12}y_{1t}) + RCORNR_t[f_{20} + f_{21}(a - a_{1t}) + f_{22}y_{2t}]$$

$$- RCTVC_t a_{1t} - \frac{d_1 a_{1t}^2}{2} - d_2 a_{1t} a_{1t-1}$$

$$+ RDP_t(f_{10} + f_{11}a_{1t} + f_{12}y_{1t}) \frac{\alpha * Part}{(1 - (1 - \alpha) * Part)}$$

for $t = 0, 1, 2, \ldots$.

The farmer's optimal problem becomes

$$\text{Max } V_{1t} = E_t \sum_{t=0}^{\infty} b^t (RPF_t(f_{10} + f_{11}a_{1t} + f_{12}y_{1t})$$

$$+ RCORNR_t[f_{20} + f_{21}(a - a_{1t}) + f_{22}y_{2t}]$$

$$- RCTVC_t a_{1t} - \frac{d_1 a_{1t}^2}{2} - d_2 a_{1t} a_{1t-1}$$

$$+ RDP_t(f_{10} + f_{11}a_{1t} + f_{12}y_{1t}) \frac{\alpha * \text{Part}}{(1 - (1 - \alpha) * \text{Part})}$$

for $t = 0, 1, 2, \ldots$.

where $f_{10}, f_{11}, f_{12}, f_{20}, f_{21}, f_{22}, d_1, d_2, \alpha, \text{part} > 0$, and $b$ is a discount factor that lies between zero and one. The index "t" takes values at $t = 0, 1, 2, \ldots$ for all variables. The operator $E_t$ is defined by $E_t X = EX|_{\Omega_t}$, where $X$ is a random variable, $E$ the mathematical expectation operator, and $\Omega_t$ an information set available to the firm at time $t$. It is assumed that the farmer's information at time $t$ contains all realizations of all the variables in the markets at time $t, t-1, t-2, \ldots$. That is, $\Omega_t$ includes at
The firm's optimum plan is to choose a stochastic process for $a_{it}$ from the set of stochastic processes that are functions of the information set $\Omega_t$. The optimization is subject to a given level of $a_{t-1}$ and $a_{t-2}$, and a given law of motion for the stochastic processes for $X_t' = (CTX_t, \ldots, RCONR_t, RCTVC_t, RDP_t, RGNP_t, \ldots)'$;

i.e., $V(L)X_t = \epsilon_t$ \hspace{1cm} (4.17)

where $X_t$ is an (n*1) vector wide-sense stationary stochastic process. $\epsilon_t$ is an (n*1) vector of white noise with mean zero and contemporaneous covariance matrix $E\epsilon_t'\epsilon_t = V$, an n*n matrix. We assume $E\epsilon_t\epsilon_{t-s}' = 0_{n\times n}$ for all $s \neq 0$. $V$ is a positive semidefinite matrix. In (4.17), $V(L)$ is an n*n matrix of (finite order) polynomials in the lag operator $L$:

$$V(L) = \begin{bmatrix} V_{11}(L) & V_{12}(L) & \ldots & V_{1n} \\ \vdots & \ddots & \vdots \\ V_{n1}(L) & V_{n2}(L) & \ldots & V_{nn} \end{bmatrix},$$

where each $V_{ij}(L)$ is a finite order polynomial in the lag operator. The vector stochastic process (4.17) is assumed to be of mean exponential order less than $1/b$. That is, the roots of $V(L) = 0$ lie outside the unit circle. The variables in the vector $X_t$ are viewed as being unaffected by the farmer's decisions.

The farmer's optimization problem is solved by using the discrete
time calculus of variations. Differentiating the objective function (4.16) with respect to \( a_{1t} \) for \( t = 0, 1, 2, \ldots \) and then setting the derivative equal to zero to get the system of stochastic Euler equations and a transversality condition. The Euler equations for \( a_{1t} \) is

\[
-b^t \left[ d_1 a_{1t} + RCTV_t + d_2 a_{1t-1} - f_{11} RPF_t + f_{21} RCORNR_t \right] \tag{4.18}
\]

\[
- RDP_t \ast f_{11} \ast \alpha \ast Part \]
\[ \frac{1}{1 - (1 - \alpha) \ast Part} \]

\[- b^{t+1} d_2 E_t a_{1t+1} = 0, \text{ for } t = 0, 1, 2, \ldots \]

Rearranging, we get:

\[
E_t a_{1t+1} + d_1 a_{1t} + \frac{1}{b} a_{1t-1} = \frac{1}{b d_2} \left( f_{11} RPF_t - f_{21} RCORNR_t \right) \right. \tag{4.19}
\]

\[- RCTV_t + f_{11} \alpha_p RDP_t \] ,

where \( \alpha_p = \frac{\alpha \ast Part}{1 - (1 - \alpha) \ast Part} \).

The transversality condition of \( a_{1t} \) is

\[
\lim_{T \to \infty} b V_{10} = 0 \tag{4.20}
\]

That is,

\[
\lim_{T \to \infty} b^T \left[ - d_1 a_{1T} - d_2 a_{1T-1} - RCTV_T + f_{11} RPF_T \right. \right.
\]

\[- f_{21} RCORNR_T + \alpha_p f_{11} RDP_T \] = 0

The transversality condition (4.20) imposes boundedness on the solution for \( a_{1t} \). The limit is true, if \( (a_{1t}), (RPF_t), (RCORNR_t), (RCTV_t), \) and \( (RDP_t) \) are of exponential order less than \( 1/b \). Then, the

---

The conditions of \( (a_{1t}), (RPF_t), (RCORNR_t), (RCTV_t), \) and \( (RDP_t) \) are of exponential order less than \( 1/b \) are not necessary. The sufficient and necessary conditions are
transversality condition is satisfied as \( T \to \infty \).

To solve the Euler equation for the optimum contingency plan, \( a_{1t} \), the left hand side of equation (4.19) is written as a form of factorization:

\[
[1 + \frac{d_1 L + L^2}{b}] = (1 - \mu_1 L)(1 - \mu_2 L),
\]

where
\[
\mu_1 \mu_2 = \frac{1}{b} \quad \mu_2 = \frac{1}{b \mu_1} \quad \mu_1 + \mu_2 = -\frac{d_1}{d_2 b}.
\]

For existence of a real solution, the parameters are constrained by \( |d_1 / d_2| > 1 + b \). Then, \( (\mu_1 / d_2) < 0 \), and \( \mu_1 < 0 \) if \( d_2 > 0 \) or \( \mu_1 > 0 \) if \( d_2 < 0 \), where \( \mu_1 \) is the smaller root in absolute value. It then follows that the solution of the Euler equations that satisfies the transversality condition and the initial conditions \( a_{t-1} \) and \( a_{t-2} \), is given by

\[
a_{1t} = \mu_1 a_{1t-1} - \frac{\mu_1}{d_2} \sum_{i=0}^{\infty} (b \mu_1)^i E_t (f_{11} RPF_{t+1} - RCTVC_{t+1})
\]

\[- f_{21} RCORNR_{t+1} + f_{11} \alpha p RDP_{t+1}), \text{ for } t = 0, 1, 2, \ldots \]

where \( \mu_1 \) is the smaller root in absolute value, that solves (4.22).

Equation (4.23) is derived from the Euler equations (4.19) by solving the unstable root forwards in order to satisfy the transversality condition (4.20). Equation (4.23) also exhibits the certainty equivalence

\[
\sum_{i=0}^{\infty} a_{1t}^2 < \infty, \quad \sum_{i=0}^{\infty} RPF_t^2 < \infty, \quad \sum_{i=0}^{\infty} RCORNR_t^2 < \infty, \quad \sum_{i=0}^{\infty} RCTVC_t^2 < \infty, \quad \sum_{i=0}^{\infty} RDP_t^2 < \infty.
\]
or separation property. That is, the same solution for $a_{lt}$ would emerge if we maximized the criterion formed by replacing $(RPF_{t+1}, RCORN_{t+1}, RCTVC_{t+1},$ and $RDP_{t+1})$ by $(E_t RPF_{t+1}, E_t RCORN_{t+1}, E_t RCTVC_{t+1},$ and $E_t RDP_{t+1})$ and eliminating the operator $E_t$ from outside the sum in the objective function (4.16).

The representation of the solution by equation (4.23) implies that if $d_2 \neq 0$ the demand for $a_{lt}$ depends upon current and all future values of prices received by farmers, the net revenue of competitive crop (corn), the variable cost of cotton, and the direct payments and the past decisions on land. But if $d_2 = 0$, equation (4.19) is just the standard linear demand equation for $a_{lt}$.

For any arbitrary set of expectations, equation (4.23) also implies that

\[ \frac{\delta a_{lt}}{\delta E_t RCORN_t} = \frac{\mu_1 f_{21}}{d_2} < 0; \quad (4.24) \]

and

\[ \frac{\delta a_{lt}}{\delta E_t RCORN_{t+1}} = \frac{\mu_1 (b_1 \mu_1) f_{21}}{d_2} > 0, \quad (4.25) \]

for $d_2 > 0$.

Equation (4.24) is the standard result of the substitution effect that more land is allocated to cotton as the expected net returns of corn decrease. However, equation (4.25) suggests that if farmers expect that in the following year the net returns of corn are going to decrease, they will decrease the quality of current land allocated to cotton. This result implies a rationale for the "cobweb phenomenon" in the model. That is, the inherent dynamics in the production cause cotton and corn
production to oscillate with respect to changes in the expected price. As the adjustment cost of land \( d_2 < 0 \), dynamic models imply the same result for (4.24), but the opposite sign for (4.25).

Equation (4.23) is not yet a decision rule for \( a_{1t} \) because it is a function of future values of the relevant variables, which are unknown at the time planting decisions are made. To derive the explicit decision rule for \( a_{1t} \), it is necessary further to convert future terms into an observable form, which will be discussed later.

4.2.2 A representative cotton miller

A representative cotton mill firm chooses a contingency plan for \( c_{t m_t} \) to maximize its expected present value,

\[
V_{2t} = E_t \sum_{t=0}^{\infty} \delta^t [RPM_t c_{m_t} - c_{2t} - c_{3t}].
\]

where \( c_{m_t} \) represents the cotton yarn sold to the consumers. With the assumption that mill production always equals yarn demand, and substitute equation (4.8) for \( c_{2t} \) and equation (4.9) for \( c_{3t} \), then \( V_{2t} \) can be written as follows:

\[
V_{2t} = E_t \sum_{t=0}^{\infty} \delta^t [RPM_t c_{m_t} - RPM_t c_{t b_t} - h_{ctm_t} - h_2 (ctm_t - ctm_{t-1})^2].
\]  

Next, substitute equation (4.11) for \( c_{t m_t} \), and equation (4.4) for \( c_{t b_t} \), then equation (4.26) becomes

\[
V_{2t} = E_t \sum_{t=0}^{\infty} \delta^t [RPM_t c_{tm_t} * r - RPM_t c_{tm_t}].
\]
subject to $ctm_{t-1}$ and $ctm_{t-2}$ given.

In equation (4.27) $h_1$, $r$, and $h_2$ are each positive constant, while the constant discount factor $\delta$ satisfies $0 < \delta < 1$. With the assumption that cotton mill firms have the same information set as the cotton farmers, it is possible to solve the cotton miller's problem by using the same method, the discrete time calculus of variations. Differentiating the objective function (4.27) with respect to $ctm_t$, for $t = 0, 1, 2, \ldots, T-1$. And setting each derivative equals zero to get the system of stochastic Euler equations for $ctm_t$ as follows:

\[
\delta E_t ctm_{t+1} - (1 + \delta)ctm_t + ctm_{t-1} = (h_2)^{-1}E_t[RPM_t - RPM_t^r + h_1].
\]

The transversality condition for $ctm_t$ is

\[
\lim_{T \to \infty} \delta^T E_0 ctm_T = 0,
\]

that is,

\[
\lim_{T \to \infty} \delta^T [RPM_T^r - RPM_T - h_1 - h_2(ctm_T - ctm_{T-1})] = 0.
\]

Given the assumption about the sign and magnitude of the parameters $h_1$, $h_2$, and $\delta$, it then follows that the solution of the Euler equations that satisfy the transversality conditions and the initial conditions is given by

\[
ctm_t = ctm_{t-1} + \sum_{i=0}^{\infty} \frac{1}{h_2} \sum_{i=0}^{\infty} (\delta)^i E_t RPM_{t+i}^r - RPM_{t+i}^r - h_1). \tag{4.30}
\]

Equation (4.30) implies that a mill firm facing a small adjustment cost $h_2$
will adjust its mill consumption \( (ctm_t) \) more quickly in response to current conditions than a firm facing a larger value of adjustment cost \( h_2 \). Moreover, it gives the firm's rate of mill consumption as a function of future values of the marketing margin (the difference between the output price and the price of cotton).

4.2.3 Equilibrium

Formally, a demand schedule and a supply schedule for cotton have been constructed. We now assume that the cotton market and the cotton yarn market clear at all points in time and solve for the stochastic processes for acreage planted, mill demand, and the cotton prices that always clear the two markets.

A rational equilibrium market for the cotton industry is defined as a stochastic process for \( \{a_t, ctm_t, RPM_t, RPF_t, \text{and } CT_t = CTM_t + CTX_t\} \), where \( CT_t = n * ct_t \), \( CTM_t = m * ctm_t \), \( CTX_t = n * ctx_t \), and \( n \) and \( m \) defined the numbers of the total cotton farmers and millers \( t = 0 \) that satisfies the necessary condition for the maximum problems of the farmer \( (4.16) \) and the cotton miller \( (4.27) \), the production function \( (4.1), (4.2) \), and \( (4.11) \), the mill demand equation \( (4.12) \), cost function \( (4.7), (4.8) \), and \( (4.9) \), and identity \( (4.5) \) and the given stochastic processes of \( CTX_t, y_{1t}, RCORNR_t, RCTVC_t, RDP_t, U_{1t}, \) and \( U_{2t} \).

To construct an equilibrium, first, summing up the Euler equations over all cotton farmers and millers obtains the aggregate cotton acreage planted \( (A_t) \) and the aggregate cotton mill consumption \( (CTM_t) \) as follows:

\[
\begin{align*}
& d_2bE_tA_{1t+1} + d_1A_{1t} + d_2A_{1t-1} \\
= & n(f_{1t}RPF_t - f_{2t}RCORNR_t - RCTVC_t + f_{11}pRDP_t), \text{ for } t = 0, 1, \ldots,
\end{align*}
\]
\[ \delta h_2 E_t CTM_{t+1} - h_2 (\delta + 1) CTM_t + h_2 CTM_{t-1} \]  
\[ = - m (RPM_t - PRW_t - h_1), \text{ for } t = 0, 1, 2, \ldots, \]
where \( A_{1t} = n \cdot a_{1t} \) and \( CTM_t = m \cdot ct_m \). Summing up both the production function (4.1) over farmers and the mill production (4.11) over millers gives

\[ CT_t = F_{10} + F_{11} A_{1t} + F_{12} Y_{1t}, \]  
(4.33a)

and

\[ CM_t = CTM_t \times \tau, \]  
(4.33b)

where \( CT_t = n \cdot ct_t \), \( F_{10} = n \cdot f_{10} \), \( F_{11} = f_{11} \), and \( F_{12} = n \cdot f_{12} \), \( CM_t = m \cdot cm_t \).

Second, for each market to clear at harvest time \( t \), total demand always equals total supply at market prices (\( RPW_t \) and \( RPM_t \)). The market clearing condition for the cotton market is

\[ CTM_t + CTX_t = CT_t, \]  
(4.34a)

and for the cotton yarn market

\[ CM_t = CMD_t. \]  
(4.34b)

From equation (4.11) and (4.12), we obtain

\[ RPM_t = \frac{1}{m_1} CTM_t + \frac{m_2 RPW_t}{m_1} - \frac{U_{2t}}{m_1} - \frac{m_0}{m_1}. \]  
(4.35)

Third, find stochastic processes for \( (A_{1t})_{t=0}^\infty \), \( (CTM_t)_{t=0}^\infty \), \( (RPF_t)_{t=0}^\infty \), \( (RPW_t)_{t=0}^\infty \), and \( (RPM_t)_{t=0}^\infty \) that satisfy the transversality conditions of the cotton industry problems and that satisfy the Euler equations (4.31) and (4.32). To proceed, substitute (4.6) into (4.31) eliminates \( RPF_t \) and obtains

\[ d_2 b E_t A_{1t+1} + d_1 A_{1t} + d_2 A_{1t-1} \]  
(4.36)
- nf₁₁βRPWᵗ + nf₁₁U₁ᵗ - nf₂₁RCORNRᵗ - nRCTVCᵗ + nf₁₁αₚRDPᵗ,

for t = 0, 1,....

Rearrange (4.36) obtains

\[ EₜAₜ₊₁ + \theta Aₜ + (b)^{-1}Aₜ₋₁ \]  \hspace{1cm} \text{(4.37)}

\[ = (bd₂)^{-1}(f₁₁βRPWᵗ + f₁₁U₁ᵗ - f₂₁RCORNRᵗ - RCTVCᵗ + f₁₁αₚRDPᵗ), \]

where \( \theta = d₁ / bd₂ \), for t = 0, 1, 2,....

Substitute (4.35) into (4.32) obtains

\[ \delta h₂EₜCTMₜ₊₁ - \left( \delta h₂ + h₂ - \frac{mr}{m₁} \right)CTMₜ + h₂CTMₜ₋₁ \]  \hspace{1cm} \text{(4.38)}

\[ = \frac{mrU₂ᵗ + m(1 - \frac{rm₂}{m₁})RPWᵗ + mh₁ + mrm₀}{m₁}, \text{ for t = 0, 1,....} \]

Rearrange (4.38) obtains

\[ \delta EₜCTMₜ₊₁ + \phi₁'CTMₜ + CTMₜ₋₁ \]  \hspace{1cm} \text{(4.39)}

\[ = \frac{1}{h₂} \left( \frac{mrU₂ᵗ + m(1 - \frac{rm₂}{m₁})RPWᵗ + mh₁ + mrm₀}{m₁} \right), \]

where \( \phi₁' = - \left( \frac{\delta + 1 - \frac{mr}{m₁h₂}}{m₁h₂} \right) \), for t = 0, 1, 2,....

To eliminate RPWᵗ, we use "(4.38) * nf₁₁β - (4.36) * m(1 - \frac{rm₂}{m₁})" to get

\[ nf₁₁β\delta h₂EₜCTMₜ₊₁ - nf₁₁β\left( \delta h₂ + h₂ - \frac{mr}{m₁} \right)CTMₜ \]

\[ + nf₁₁βh₂CTMₜ₋₁ - d₂bm(1 - \frac{rm₂}{m₁})EₜAₜ₊₁ - m(1 - \frac{rm₂}{m₁})d₁A₁ᵗ \]

\[ - m(1 - \frac{rm₂}{m₁})d₂A₁ᵗ₋₁ \]

\[ = \frac{mrnf₁₁βU₂ᵗ - nf₁₁m(1 - \frac{rm₂}{m₁})U₁ᵗ + nf₂₁m(1 - \frac{rm₂}{m₁})RCORNRᵗ}{m₁}, \]

\[ + nm(1 - \frac{rm₂}{m₁})RCTVCᵗ - nm(1 - \frac{rm₂}{m₁})f₁₁αₚRDPᵗ + nf₁₁βmh₁ \]
From the cotton market clearing condition (4.34a) \( CT_t + CT_x = CT_t \), and the aggregate production function (4.33a) \( CT_t = F + F_{11}A_{1t} + F_{12}Y_{1t} \), we obtain

\[
A_{1t} = F_{11}^{-1}(CT_t + CT_x - F_{12}Y_{1t} - F_{10}).
\] (4.41)

Substitute (4.41) into (4.40) eliminates \( A_{1t} \) and obtains

\[
\left[ \frac{f_{11} \delta h_2 - d_{2} b_{m}}{f_{11}} \left( \frac{1 - \tau m_2}{m_1} \right) \right] E_t CT_{t+1}
\]

\[
- \left[ \frac{f_{11} \delta (h_2 + h_2 - m_r)}{f_{11}} + \frac{d_{1} m}{f_{11}} \left( \frac{1 - \tau m_2}{m_1} \right) \right] CT_t
\]

\[
+ \left[ \frac{f_{11} \delta h_2 - d_{3} m}{f_{11}} \left( \frac{1 - \tau m_2}{m_1} \right) \right] CT_{t-1}
\]

\[
= \frac{be_{t} m}{f_{11}} \left( \frac{1 - \tau m_2}{m_1} \right) E_t (CT_{t+1} - F_{12}Y_{1t+1} - F_{10})
\]

\[
+ \frac{d_{1} m}{f_{11}} \left( \frac{1 - \tau m_2}{m_1} \right) (CT_t - F_{12}Y_{1t} - F_{10})
\]

\[
+ \frac{d_{2} m}{f_{11}} \left( \frac{1 - \tau m_2}{m_1} \right) (CT_{t-1} - F_{12}Y_{1t-1} - F_{10})
\]

\[
+ \frac{f_{11} \delta m}{m_1} \left( h_1 + \tau m_0 \right) + \frac{f_{11} \delta m r U_{2t}}{m_1}
\]

\[
- \frac{f_{11} m}{m_1} (1 - \tau m_2) U_{1t} + \frac{f_{21} m}{m_1} (1 - \tau m_2) RCORNR_t + \frac{nm}{m_1} (1 - \tau m_2) RCRTC_t
\]

\[
- \frac{nm}{m_1} (1 - \tau m_2) \eta RDP_t.
\]

Let \( b = \delta \), and define the right hand side of equation (4.42) as \( Q_t \), then (4.42) can be written as

\[
b E_t CT_{t+1} + WCT_t + CT_{t-1} = \left( \frac{f_{11} \delta h_2 - d_{2} b_{m}}{f_{11}} \left( \frac{1 - \tau m_2}{m_1} \right) \right)^{-1} Q_t,
\] (4.43)

where
\[
W = - \frac{[nf_{11} \beta (\delta h_2 + h_2 - mr)] - d_1 m \left(1 - \frac{\tau m_2}{m_1}\right)}{nf_{11} \delta h_2 - d_1 m \left(1 - \frac{\tau m_2}{m_1}\right)},
\]

for \( t = 0, 1, 2, \ldots \).

From (4.37) the definition of \( \psi \), write

\[
b^\psi = \frac{d_1}{d_2} \frac{\frac{m}{\check{f}_{11}} \left(1 - \frac{\tau m_2}{m_1}\right)}{\frac{m}{\check{f}_{11}} \left(1 - \frac{\tau m_2}{m_1}\right)} = \frac{\psi_n}{\psi_d} > 0, \text{ for } d_2 > 0; \text{ for } d_2 < 0,
\]

then \( \psi_d < 0 \), so \( \psi < 0 \).

From (4.39) the definition of \( \phi_1' \), write

\[
\phi_1' = -\frac{\left[bh_2 + h_2 - mr\right] nf_{11} \beta}{h_2 nf_{11} \beta} = \frac{\phi_{1n}'}{\phi_{1d}} < 0.
\]

Then \( W \) can be written

\[
W = \frac{W_n}{W_d} = \frac{\phi_{1n}'}{\phi_{1d}} - \frac{\psi_n}{\psi_d} - \phi_1' - \phi_1' - b \frac{\psi_d}{\phi_{1d} - \psi_d},
\]

since \( \phi_1' < 0, \psi > 0, \) and \( \psi_d > 0, \phi_{1d}' > 0, \) it follows that

\[
|W| = \min (|\phi_1'|, |b\psi|) \quad \text{and} \quad (4.44)
\]

sign \( W = \text{sign} (\psi_d - \phi_{1d}') \),

for \( d_2 < 0, \psi < 0, \) and \( \phi_1' < 0, \phi_{1d}' > 0, \psi_d < 0, \) then \( W < 0 \).

Condition (4.44), Euler equations for the aggregate levels of \( A_{1t} \) and \( CTM_t \) (4.31) and (4.32), price linkage equation (4.6), and market clearing conditions (4.34a) and (4.34b) combine to ensure that the solution of equation (4.42) can be obtained by applying the same technique, the discrete time calculus of variations.

By proceeding as the earlier analysis of the Euler equations (4.18)
and (4.28), and using the condition (4.44), the equation (4.43) can be written as

$$E_t C T M_{t+1} (1 - \Theta_1 L) (1 - \Theta_2 L)$$

$$- \left[ n f_{11} \delta h_2 - \frac{d_{2m}}{F_{11}} \left( \frac{1 - \tau m_2}{m_1} \right) \right]^{-1} b^{-1} Q_t$$

$$- D^{-1} b^{-1} Q_t,$$

where $D = \left[ n f_{11} \delta h_2 - \frac{d_{2m}}{F_{11}} \left( \frac{1 - \tau m_2}{m_1} \right) \right].$

For existence of two real roots, the parameters are constrained by

$$\left| \frac{1}{1 - \frac{d_{2m}}{n f_{11} \delta h_2} \left( \frac{1 - \tau m_2}{m_1} \right)} \right| > 1 + b.$$  \hspace{1cm} (4.46)

And $\Theta_1 = 1 / (b \Theta_2)$, $\Theta_1 + \Theta_2 = W / b$, then $\Theta_1$ is the smaller root in absolute value. It then follows that the solution of the Euler equations satisfies the transversality condition, and the initial condition is given by

$$CTM_{t+1} = \Theta_1 C T M_t$$

$$- \Theta_1 d_{2m} / F_{11} \left( \frac{1 - \tau m_2}{m_1} \right) \Sigma (b \Theta_1)^i E_t (C T X_{t+1} - F_{12} Y_{t+1} - F_{10})$$

$$- \Theta_1 d_{2m} / F_{11} \left( \frac{1 - \tau m_2}{m_1} \right) \Sigma (b \Theta_1)^i E_t (C T X_{t+1} - F_{12} Y_{t+1} - F_{10})$$

$$- \Theta_1 d_{2m} / F_{11} \left( \frac{1 - \tau m_2}{m_1} \right) \Sigma (b \Theta_1)^i E_t (C T X_{t+1} - F_{12} Y_{t+1} - F_{10})$$

$$- \Theta_1 m r n f_{11} \delta \Sigma (b \Theta_1)^i E_t U_{2t+1} i = 0$$
Equation (4.47) gives the solution stochastic process for \( \{\text{CTM}_t\}_{t=0}^\infty \) as a function of the stochastic processes of the exogenous variables \( \{\text{CTX}_t, y_t, \text{RCORN}_t, \text{RCTVC}_t, \text{RDP}_t\}_{t=0}^\infty \) and demand shock process \( \{U_t\}_{t=0}^\infty \) and the error stochastic process \( \{\epsilon_t\}_{t=0}^\infty \).

To determine the equilibrium stochastic process for \( \{\text{RPW}_t\}_{t=0}^\infty \), we can use the solution (4.47) to eliminate \( \text{E}_t\text{CTM}^{t+1}, \text{CTM}_t, \text{CTM}^{t-1} \) from (4.38). The equilibrium stochastic process for the wholesale price of cotton, \( \text{RPW}_t \), is

\[
\text{RPW}_t = -\frac{\theta_1 d_1 b_1^2 h_1}{D_{t11}} \sum_{i=0}^{\infty} (b\theta_1)^i E_t \left( \text{CTX}_{t+i} - F_{12}y_{t+i} - F_{10} \right)
\]

\[
-\frac{\theta_1 b h_2 d_1}{D_{t11}} \sum_{i=0}^{\infty} (b\theta_1)^i E_t \left( \text{CTX}_{t+i} - F_{12}y_{t+i} - F_{10} \right)
\]

\[
-\frac{\theta_1 b h_2}{D_{t11}} \sum_{i=0}^{\infty} (b\theta_1)^i E_t \left( \text{CTX}_{t+i} - F_{12}y_{t+i} - F_{10} \right)
\]
Equation (4.48) expresses the stochastic process of the wholesale price of cotton in terms of $\text{CTM}_t$ and the future values of the exogenous variables. The wholesale price process is turned out by a competitive cotton market and a competitive cotton yarn market that clear each period. In other words, it interprets as a contingency plan or rule for setting the wholesale price of cotton at time $t$, contingent on all relevant information available up through time $t$.

Substituting (4.48) into the price linkage equation (4.6), the dynamic stochastic process for the equilibrium price of the farmer received is

\[
\text{RPF}_t = -\frac{\theta_t \beta_d d^2 h_2}{D_{11}} \sum_{i=0}^{\infty} (b\theta_1)_i E_t (CT_{t+2} - F_{12} Y_{t+2} - F_{10})
\]
The equilibrium law of motion for aggregate acreage planted is computed by using aggregate production function (4.33a), market clearing condition (4.34a), and the stochastic process of CTM^t (4.47). The stochastic process of acreage planted A_{it} is

\[ A_{it} = \] (4.50)

\[ - \frac{\theta_i \delta bh_d d_{i}}{D_{11}} \sum_{i=0}^{\infty} (b\theta_1)^i E_t (CTX_{t+1+i} - F_{12}Y_{1t+1+i} - F_{10}) \]

\[ - \frac{\theta_i \delta d_{i} bh_2}{D_{11}} \sum_{i=0}^{\infty} (b\theta_1)^i E_t (CTX_{t+1} - F_{12}Y_{1t+1} - F_{10}) \]

\[ - \frac{\theta_i \delta^2 \gamma_{n+1} bh_2}{D_m} \left( \frac{1 - \frac{\tau m_2}{m_1}}{m_1} \right)^{-1} \sum_{i=0}^{\infty} (b\theta_1)^i E_t U_{2t+i+1} \]

\[ + \frac{\theta_i \gamma_{n+1} bh_2}{D} \sum_{i=0}^{\infty} (b\theta_1)^i E_t U_{1t+i+1} \]

\[ - \frac{\theta_i \gamma_{n+1} bh_2}{D} \sum_{i=0}^{\infty} (b\theta_1)^i E_t R{\text{CO}}N{\text{R}}_{t+i+1} \]

\[ + \frac{\theta_i \gamma_{n+1} \delta bh_2}{D} \sum_{i=0}^{\infty} (b\theta_1)^i E_t R{\text{DP}}_{t+i+1} \]

\[ - \frac{n\theta_i bh_2}{D} \sum_{i=0}^{\infty} (b\theta_1)^i E_t R{\text{CTV}}{\text{C}}_{t+i+1} \frac{\tau_1}{1 - \frac{\tau m_2}{m_1}} U_{2t} \]

\[ - \frac{\delta}{D(1 - b\theta_1)} \left( h_1 + \frac{\tau m_2}{m_1} \right) \left( \frac{1 - \frac{\tau m_2}{m_1}}{m_1} \right)^{-1} \]

\[ + \delta \left[ \frac{m}{1 - \frac{\tau m_2}{m_1}} \right]^{-1} \left( bh_2 \theta_1 - bh_2 - h_2 + \frac{mr}{m_1} + h_2 L \right) CTM^t + U_{1t}. \]
Substitute (4.48) into (4.35) obtains the equilibrium law of motion for the cotton yarn price as following

$$RPM_t = \frac{-\Theta_i d_i m}{D_f_{11}} \left(1 - \frac{\tau m_2}{m_1}\right) \sum_{i=0}^{\infty} (\theta_1)^i E_t (CTX_{t+i} - F_{12Y_{1t+i}} - F_{10})$$

$$- \frac{\Theta_i d_i m}{D_f_{11}} \left(1 - \frac{\tau m_2}{m_1}\right) \sum_{i=0}^{\infty} (\theta_1)^i E_t (CTX_{t+1+i} - F_{12Y_{1t+1+i}} - F_{10})$$

$$- \frac{\Theta_i m_{rn}^2}{D_{m_1}} \sum_{i=0}^{\infty} (\theta_1)^i E_t U_{2t+i}$$

$$+ \frac{\Theta_i nm_{rn}}{D} \left(1 - \frac{\tau m_2}{m_1}\right) \sum_{i=0}^{\infty} (\theta_1)^i E_t U_{1t+i}$$

$$- \frac{\Theta_i nm_{rn}}{D_f_{11}} \left(1 - \frac{\tau m_2}{m_1}\right) \sum_{i=0}^{\infty} (\theta_1)^i E_t RCVC_{t+i}$$

$$+ \frac{\Theta_i nm_{rn}}{D} \left(1 - \frac{\tau m_2}{m_1}\right) \sum_{i=0}^{\infty} (\theta_1)^i E_t RDP_{t+i}$$

$$- \frac{\Theta_i n_f_{21} m}{D_f_{11}} \left(1 - \frac{\tau m_2}{m_1}\right) \sum_{i=0}^{\infty} (\theta_1)^i E_t RORNP_{t+i}$$

$$- \frac{\Theta_i n_f_{21} m}{D} \left(1 - \frac{\tau m_2}{m_1}\right) \sum_{i=0}^{\infty} (\theta_1)^i E_t RORN_{t+i}$$

$$+ \frac{1}{F_{11}} (CTX_t - F_{12Y_{1t}} - F_{10}) + \frac{\Theta_i CTM_{t-1}}{F_{11}}$$
The rational expectations equilibrium assumptions in this study mean that the model is solved such that equations (4.16) and (4.27) are maximized subject to the true stochastic processes of $CTX_t$, $y_{1t}$, $RCORN_{R_t}$, $RCTVC_t$, $RDP_t$, $U_{1t}$, and $U_{2t}$ and the price linkage equation (4.6). Moreover, the U.S. cotton market participants are assumed to act as if they know both the underlying structure of the model and the stochastic processes governing the exogenous variables. In this way the agents' behavior in this market, on the average, would be the same, and their perceptions on the aggregate law of motion for the exogenous variables would be near the true expected values of the exogenous variables to be predicted.
4.2.4 The optimal decision rules

Equations (4.47) and (4.50) are decision schedules for setting CTM\_t and A\_t, while equations (4.48), (4.49), and (4.51) are stochastic sequences for prices (RPW\_t, RPF\_t, and RPM\_t). These equations are expressed as functions of lagged CTM\_{t+1} and the sum of the geometric sums of all future values of the exogenous variables (CTX\_t, RCORNR\_t, y\_t, RCTVC\_t, RDP\_t, U\_1t, and U\_2t). They implicitly reflect the optimal decision rules of the agent and the way the optimal decision rules interact with each other.

However, these decision rules are not estimable due to the infinite sum problems. In order to derive explicit decision rules for CTM\_t, A\_t, RPW\_t, RPF\_t, and RPM\_t as functions of the information set Ω\_t, it is necessary to restrict the stochastic processes of CTX\_t, y\_t, RCORNR\_t, RCTVC\_t, RDP\_t, U\_1t, and U\_2t in order to make the infinite sum of the expected variables converge.

Provided that U\_1t and U\_2t each follow a random walk process for which

\[ U\_1t = U\_1t-1 + \epsilon\_1t, \]
\[ U\_2t = U\_2t-1 + \epsilon\_2t, \]

(4.52)

where \( \epsilon\_i \) and \( \epsilon\_2 \) are least-squares residuals with finite variances and \( E \epsilon\_i | \Omega\_t-1 = 0 \), for \( i = 1, 2 \). And \( \epsilon\_1 \) and \( \epsilon\_2t \) are not serially correlated. For the additional discussion of constructing an optimal estimator with serially correlated errors, see Hansen and Sargent (1982) and Hayashi and Sims (1983).

In order to make the infinite sum of the expected variables converge, we assume that all exogenous variables (CTX\_t, y\_t, RCORNR\_t, RCTVC\_t, and
RDP_t) follow an qth-order vector autoregression.

\[ X_t = V_1 X_{t-1} + V_2 X_{t-2} + \ldots + V_q X_{t-q} + \epsilon_t, \]  

where \( V(L) = I - V_1 L - V_2 L^2 - \ldots - V_q L^q \), where \( V_j \) is \( n \times n \), \( (n \) is the number of exogenous variables), and the zeroes of \( \text{det}(Z) \) are assumed to be greater than \((/b)\) in modulus. This condition on the zeroes of \( \text{det}V(Z) \) means that CTX, RCRNR, y, RCTVC, and RDP are of mean exponential order less than \((/b)^{-1}\). Under the condition, the infinite sum converges.

Assume \( E(\epsilon_t | X_{t-1}, X_{t-2}, \ldots, A_{1t-1}, A_{1t-2}, \ldots, CTM_{t-1}, CTM_{t-2}, \ldots, RPM_{t-1}, RPM_{t-2}, \ldots, RPF_{t-1}, RPF_{t-2}, \ldots, RPW_{t-1}, RPW_{t-2}, \ldots) = 0 \), for all \( t \), and \( E\epsilon_t \epsilon_t = \nu \), where \( \nu \) is a positive semidefinite matrix for all \( t \) in order to ensure the criterion objective function well defined as \( t \rightarrow \infty \).

Before deriving the explicit decision rules, equation (4.47) can also be written as

\[ \text{CTM}_t = \Theta_1 \text{CTM}_{t-1} + \frac{\Theta_1 \text{bd}_m}{DF_{11}} \left( \frac{1 - \tau_m}{m_1} \right) E_t \text{CTX}_{t+1} + \sum_{i=0}^{\infty} \left( \frac{1}{b_\Theta_1} \right)^i E_t \text{CTX}_{t+1} \]

\[ - \frac{1}{b_\Theta_1} \text{CTX}_t \]

\[ = \Theta_1 \frac{d_m}{DF_{11}} \left( \frac{1 - \tau_m}{m_1} \right) \text{CTX}_{t-1} + b_\Theta_1 \sum_{i=0}^{\infty} \left( \frac{1}{b_\Theta_1} \right)^i E_t \text{CTX}_{t+1} \]

\[ + \frac{\Theta_1 \text{bd}_m}{DF_{11}} \left( \frac{1 - \tau_m}{m_1} \right) F_{12} \left( \frac{1}{b_\Theta_1} \right)^i E_t y_{1t+1} - \frac{1}{b_\Theta_1} y_{1t} \]

\[ + \frac{\Theta_1 \text{bd}_m}{DF_{11}} \left( \frac{1 - \tau_m}{m_1} \right) F_{12} \sum_{i=0}^{\infty} \left( \frac{1}{b_\Theta_1} \right)^i E_t y_{1t+1} \]

\[ + \frac{\Theta_1 d_m}{DF_{11}} \left( \frac{1 - \tau_m}{m_1} \right) F_{12} (y_{1t-1} + b_\Theta_1 \sum_{i=0}^{\infty} \left( \frac{1}{b_\Theta_1} \right)^i E_t y_{1t+1}) \]
By applying Hansen and Sargent's (1980) extension of Wiener-Kolmogorov prediction formula, the geometric sum in expected variables can be written as

\[ \sum_{i=0}^{\infty} (\theta_i b)^i E_t X_{t+i} \]  \hspace{1cm} (4.55)

where \( X_t \) represents the exogenous variables (\( CTX_t, y_{1t}, RCORNR_t, RCTVC_t, \) and \( RDP_t \)) a vector matrix. \( \phi_i \) is the raw vector \((1,0,...,0)\) with 1 in the \( i \)th collum, and \( V(\theta_i b)^{-1} = (I - V_1(\theta_i b) - V_2(\theta_i b)^2 - ... - V_q(\theta_i b)^q)^{-1} \).

Substitution of (4.55) into (4.54) gives the equilibrium law of motion for aggregate mill consumption \( CTM_t \) as follows
CTM_t = \Theta_1 CTM_{t-1} - \frac{m}{DF_{11}} \left( 1 - \frac{\tau m_2}{m_1} \right) \left( d_2 + \Theta_1 d_1 + b\Theta_1^2 d_2 \right) \phi_4

* V(\Theta_1 b)^{-1}(I + \sum \left( \sum (\Theta_1 b)^{i-r} V_1 \right) L^r) X_t

\begin{align*}
&+ \frac{d_{1m}}{DF_{11} D} \left( 1 - \frac{\tau m_2}{m_1} \right) \phi_1 X_t - \Theta_1 d_{2m} \left( 1 - \frac{\tau m_2}{m_1} \right) \phi_1 X_{t-1} \\
&+ \frac{F_{12m}}{DF_{11}} \left( 1 - \frac{\tau m_2}{m_1} \right) \left( d_2 + \Theta_1 d_1 + b\Theta_1^2 d_2 \right) \phi_2

&* V(\Theta_1 b)^{-1}(I + \sum \left( \sum (\Theta_1 b)^{i-r} V_1 \right) L^r) X_t

\begin{align*}
&+ \frac{d_{2m}}{DF_{11} D} \left( 1 - \frac{\tau m_2}{m_1} \right) \phi_2 X_t + \Theta_1 d_{2m} \left( 1 - \frac{\tau m_2}{m_1} \right) \phi_2 X_{t-1} \\
&- \frac{\Theta_1 n f_{11} \epsilon_{21}}{D(1 - b\Theta_1)} \epsilon_{21} + \frac{\Theta_1 n f_{11} \epsilon}{D(1 - b\Theta_1)} \left( 1 - \frac{\tau m_2}{m_1} \right) \epsilon_{11}

&+ \Theta_1 m \frac{1}{D} \left( 1 - \frac{\tau m_2}{m_1} \right) n^2 f_{21} \phi_3

&* V(b\Theta_1)^{-1}(I + \sum \left( \sum (b\Theta_1)^{i-r} V_1 \right) L^r) X_t

\begin{align*}
&+ \frac{\Theta_1 m}{D} \left( 1 - \frac{\tau m_2}{m_1} \right) n \phi_4

&* V(\Theta_1 b)^{-1}(I + \sum \left( \sum (\Theta_1 b)^{i-r} V_1 \right) L^r) X_t

\begin{align*}
&+ \frac{\Theta_1 m}{D} \left( 1 - \frac{\tau m_2}{m_1} \right) n f_{11} \phi_5

&* V(b\Theta_1)^{-1}(I + \sum \left( \sum (b\Theta_1)^{i-r} V_1 \right) L^r) X_t + C

\begin{align*}
&+ \frac{\Theta_1 m}{D} \left( 1 - \frac{\tau m_2}{m_1} \right) \left( bd_2 + d_1 + d_2 \right) - n f_{11} \phi \left( h_1 + \frac{\tau m_0}{m_1} \right)

where \ C = \frac{\Theta_1 m}{D(1 - b\Theta_1)} \left[ F_{10} \left( 1 - \frac{\tau m_2}{m_1} \right) (bd_2 + d_1 + d_2) - n f_{11} \phi \left( h_1 + \frac{\tau m_0}{m_1} \right) \right].

Rearrange and collect the terms of equation (4.56), the equation can be
written as

\[
\text{CTM}_t = \theta_1 \text{CTM}_{t-1} + \frac{1}{Df_{11}} \left[ \frac{1 - \tau_2}{m_1} \right] \left[ (d_2 + \theta_1 d_1 + b\theta_2)(\phi_1 - F_{12}\phi_2) \right.
\]

\[
+ n\theta_1 f_{11}(f_{21}\phi_3 + \phi_4 - f_{11}a_2) + v_1 \sum_{r=1}^{q-1} \sum_{i=r+1}^{q} (D_{11} - b\theta_1) X_t
\]

\[
+ \Delta \frac{1}{Df_{11}} \left[ \frac{1 - \tau_2}{m_1} \right] (\phi_1 - F_{12}\phi_2) \right) X_t
\]

\[
- \theta_1 d_2 m \frac{1 - \tau_2}{m_1} (\phi_1 - F_{12}\phi_2) X_t
\]

\[
+ a_t + C,
\]

where \( a_t = -\frac{\theta_1 m f_{11} m \tau_2}{Dm_1(1 - b\theta_1)} \epsilon_{2t} + \frac{\theta_1 m f_{11}}{D(1 - b\theta_1)} \left[ \frac{1 - \tau_2}{m_1} \right] \epsilon_{1t}, \)

and C as defined in equation (4.56).

Equation (4.57) is a closed form expression for \( \text{CTM}_t \). Under the assumptions that \( X_t \) has a qth order vector autoregressive representation and \( U_{it} \) a first-order univariate autoregressive representation, the closed form equation (4.57) reflects the restrictions imposed across the decision rule and the parameters of the stochastic processes for \( X_t \) and \( U_{it} \). The equation expresses the optimal choice of \( \text{CTM}_t \) as a function of last period of \( \text{CTM}_t \) and (q-1) lagged values of \( X \). The current and (q-1) lagged values of \( X_t \) appear in the decision rule, because they help predict future exogenous variables. Furthermore, any stochastic processes that both Granger cause the relevant variables and that are in the firm’s information set should be included in the decision rule, \( \text{CTM}_t \). The issue will be discussed in more detail in the empirical model.
With the assumptions (4.52) and (4.53), equation (4.48) can be written as

\[
\text{RPW}_t = \frac{\theta_1 d_2 b^2 h_2}{Df_{11}} \left[ \frac{1}{(b_0)} \sum_{i=0}^{\infty} (b_{\theta})^i E c \text{CTX}_{t+i} \right] - \frac{1}{(b_0)} \text{CTX}_{t} - \frac{1}{(b_0)} \text{CTX}_{t+1}
\]

\[+ \frac{\theta_1 d_2 b^2 h_2}{Df_{11}} \left[ \frac{1}{(b_0)} \sum_{i=0}^{\infty} (b_{\theta})^i E c y_{1t+i} \right] - \frac{1}{(b_0)} y_{1t} - \frac{1}{(b_0)} y_{1t+1}
\]

\[+ \frac{\theta_1 b h d_1}{Df_{11}} \left[ \frac{1}{(b_0)} \sum_{i=0}^{\infty} (b_{\theta})^i \text{CTX}_{t+i} - \frac{1}{(b_0)} \text{CTX}_{t} \right]
\]

\[+ \frac{\theta_1 b h d_1}{Df_{11}} \left[ \frac{1}{(b_0)} \sum_{i=0}^{\infty} (b_{\theta})^i y_{1t+i} - \frac{1}{(b_0)} y_{1t} \right]
\]

\[+ \frac{\theta_1 d_2 b h^2}{Df_{11}} \left[ \sum_{i=0}^{\infty} (b_{\theta})^i \text{CTX}_{t+i} \right]
\]

\[+ \frac{\theta_1 d_2 b h^2}{Df_{11}} \left[ \sum_{i=0}^{\infty} (b_{\theta})^i y_{1t+i} \right]
\]

\[+ \frac{\theta_1 b h^2(F_{12} + b d_2 + d_1 + d_2)}{Df_{11}(1 - b_0)}
\]

\[+ \frac{\theta_1 r w_{11} b h_2}{Dm_1(1 - b_0)} \left( \frac{1 - r m_2}{m_1} \right)^{-1} \epsilon_{2t} + \frac{\theta_1 x F_{11} b h_2}{D(1 - b_0)} \epsilon_{1t}
\]

\[+ \frac{\theta_1 b h_2}{D} \left[ \frac{1}{(b_0)} \sum_{i=0}^{\infty} (b_{\theta})^i E c \text{RCORNR}_{t+i} - \frac{1}{(b_0)} \text{RCORNR}_{t} \right]
\]

\[+ \frac{n \theta_1 b h_2}{D} \left[ \frac{1}{(b_0)} \sum_{i=0}^{\infty} (b_{\theta})^i E c \text{RCTVC}_{t+i} - \frac{1}{(b_0)} \text{RCTVC}_{t} \right]
\]

\[+ \frac{\tau}{m_1} \left( \frac{1 - r m_2}{m_1} \right)^{-1} \epsilon_{2t}
\]
With the substitution (4.57) for $CTM_t$ and the prediction formula equation (4.55), the closed form for $RPW_t$ is

$$RPW_t = \theta_t RPW_{t-1}$$

$$+ \frac{h_2}{D_{F_{11}}^2} \left( b\theta_1 + \theta_1 - 1 - \frac{mr}{m_1h_2} - b\theta_1^2 \right)$$

$$\times \left[ (d_2 + \theta_1d_1 + b\theta_1^2d_2)(\phi_1 - F_{12}\phi_2) + nf_{11}\theta_1(f_{21}\phi_3 + \phi_4 - f_{11}\alpha_p\phi_5) \right]$$

$$\times V(b\theta_1)^{-1} \left[ I + \sum_{q=1}^{q-1} \sum_{i=r+1}^q \left( b\theta_1 - 1 - \frac{mr}{m_1h_2} - bV - b\theta_1 \right) \right]$$

$$+ \left[ \phi_1 - F_{12}\phi_2 \right] \left( d_2 + d_1d_1 - d_2d_1 \left( b + 1 - \frac{mr}{m_1h_2} - bV - b\theta_1 \right) \right)$$

$$+ nf_{11}\theta_1(f_{21}\phi_3 + \phi_4 - f_{11}\alpha_p\phi_5) X_t$$

$$+ \frac{h_2}{D_{F_{11}}^2} \left[ (\phi_1 - F_{12}\phi_2) \left( \theta_1d_1 \left( b + 1 - \frac{mr}{m_1h_2} - bV - b\theta_1 \right) \right) - d_1d_1 \right]$$

$$+ nf_{11}\theta_1(f_{21}\phi_3 + \phi_4 - f_{11}\alpha_p\phi_5) X_{t-1}$$

$$+ \frac{h_2d_2}{D_{F_{11}}} \left( \phi_1 - F_{12}\phi_2 \right) X_{t-2}$$

$$+ a_t + C,$$

where

$$C = \theta_t F_{10} \left( bd_2 + d_1 + d_2 \right) \left[ 2bh_2(l - 1 - \theta_1) - \frac{mr}{m_1} \right] \left( h_1 + \frac{mr}{m_1} \right) \left( l - \frac{mr}{m_1} \right)^{-1}$$

$$\times \left[ \frac{\theta_t nf_{11}b}{D(l - b\theta_1)} \left[ 2bh_2(l - 1 - \theta_1) - \frac{mr}{m_1} \right] + (l - \theta_1) \right],$$

and
The closed form for \( RPF_t \) is

\[
\begin{align*}
RPF_t &= \Theta_t RPF_{t-1} + \frac{\beta h_2}{D \Theta_1 f_{11}} \\
&\quad \times \left\{ (b \Theta_1 + \Theta_1 - 1 - \frac{mr \Theta_1}{m_1 h_2}) \left[ (d_2 + \Theta_1 d_1 + b \Theta_1^2 d_2) \right] \\
&\quad \times v(b \Theta_1)^{-1} (I + \sum_{r=1}^{q-1} \sum_{i=r+1}^{q} \frac{1}{v_i} L^r) \\
&\quad + [(\phi_1 - F_{12} \phi_2) [d_2 + d_1 \Theta_1 - d_2 \Theta_1 \left( b + 1 - \frac{mr}{m_1 h_2} - b \Theta_1 - b \phi_1 ] \\
&\quad + n f_{11} \Theta_1 (f_{21} \phi_3 + \phi_4 - f_{11} \alpha_p \phi_5)] X_t \\
&\quad + \frac{\beta h_2}{D f_{11}} [(\phi_1 - F_{12} \phi_2) (d_2 \Theta_1 \left( b + 1 - \frac{mr}{m_1 h_2} - b \Theta_1 - b \phi_1 ] - d_2 \Theta_1 ) \\
&\quad - n f_{11} \Theta_1 (f_{21} \phi_3 + \phi_4 - f_{11} \alpha_p \phi_5)] X_{t-1} \\
&\quad - h_2 \Theta d_2 \beta (\phi_1 - F_{12} \phi_2) X_{t-2} \\
&\quad + a_t + c,
\end{align*}
\]

where

\[
\begin{align*}
a_t &= \frac{\beta r}{m_1} \left[ \Theta_t n f_{11} \beta \left( b \Theta_1 h_2 - h_2 + \frac{mr}{m_1} \right) \left( 1 - \frac{r m_2}{m_1} \right)^{-1} \\
&\quad + \left( 1 - \frac{r m_2}{m_1} \right) \varepsilon_{2t} + \left( 1 - \frac{r m_2}{m_1} \right) \varepsilon_{2t-1}
\end{align*}
\]
The closed form for $A_{1t}$ is

$$A_{1t} = \Theta_1 A_{1t-1} - \frac{1}{m_1} \left[ \frac{d_2(\phi_1 - F_{12}\phi_2)}{F_{11}} - \frac{1}{F_{11}} \left( \phi_1 - F_{12}\phi_2 \right) \right] X_t$$

$$+ \frac{\Theta_1 r_1}{m_1} \left[ \Theta_1 \left( \phi - F_{12}\phi_2 \right) \right]$$

$$+ \frac{\Theta_1 n f_{11}}{m_1} \left( f_{21}\phi_3 + \phi_4 - f_{11}\alpha_p\phi_5 \right)$$

where $C = \frac{1}{F_{11}} \left[ \Theta_1 m \left( b d_2 + d_1 + d_2 \right) / \left( 1 - r m_2 \right) + n f_{11} \left( h_1 + r m_0 \right) \right]$$

$$- (1 - \theta_1) F_{10},$$

and $a_t = - \frac{\Theta_1 n f_{11} m r}{m_1 D m_1 (1 - b \theta_1)} \epsilon_{2t} + \Theta_1 m n \left( 1 - r m_2 \right) / \left( m_2 \right) \epsilon_{1t}.$

The closed form for $R P M_t$ is

$$R P M_t = \Theta_1 R P M_{t-1} + \frac{1}{m_1 D f_{11}} \left[ \left( d_2 + \theta_1 d_1 + b \theta_1^2 d_2 \right) \left( \phi_1 - F_{12}\phi_2 \right) \right]$$

$$+ n \Theta_1 f_{11} \left( f_{21}\phi_3 + \phi_4 - f_{11}\alpha_p\phi_5 \right)$$

$$+ ( \Theta_1 n f_{11} h_2 + 1 ) \epsilon_{1t-1}. \]
\[
* \left[ \frac{m_2 h_2}{\theta_1} \left( b \theta_1 + \theta_1 - \frac{m \theta_1 - b \theta_1^2}{m_1 h_2} \right) - \frac{m (1 - \frac{r m_2}{m_1})}{m_1 h_2} \right] \\
q_1 \sum_{i=r+1}^{q-1} \frac{V(b \theta_1)^i}{(1 + \sum_i (b \theta_1)^{i-r} v_i)} L_i^r \\
+ \left[ \frac{m_2 h_2}{\theta_1} \left( (\phi_1 - F_{12} \phi_2) \left( d_2 + d_2 b V \theta_1 + d_1 \theta_1 - b d_2 \theta_1 - d_2 \theta_1 \right) + m \frac{(1 - r m_2)}{m_1} d_2 \right) \right] X_t \\
+ \frac{1}{D F_{11} m_1} \left[ m_2 h_2 [(\phi_1 - F_{12} \phi_2) \theta_1 \left( d_2 b (1 - \theta_1 - V) + 1 \right) - \frac{m \phi_r}{m_1 h_2} - d_1 \right) - n f_{11} \theta_1 [f_{21} \phi_3 + \phi_4 - f_{11} \phi_5] \right] \\
- \theta_1 m d_2 \left( \frac{1 - \frac{r m_2}{m_1}}{m_1} \right) \left( \phi_1 - F_{12} \phi_2 \right) X_{t-1} \\
- \frac{m_2 \theta_1 d_2 h_2 (\phi_1 - F_{12} \phi_2)}{m_1 D F_{11}} X_{t-2} \\
+ a_t + C,
\]

where 
\[
C = \frac{1}{m_1} \left( \frac{\theta_1 F_{10} (b d_2 + d_1 + d_2)}{D F_{11} (1 - b \theta_1)} [m_2 (2 b h_2 (1 - \theta_1) - m r) + m (1 - \frac{r m_2}{m_1})] \\
- \frac{\theta_1 n f_{11} \phi_1}{D (1 - b \theta_1)} h_1 + \frac{m_0}{m_1} \left( \frac{1 - \frac{r m_2}{m_1}}{m_1} \right)^{-1} \frac{2 b h_2 (1 - \theta_1) - m r}{m_1} + m \\
+ (1 - \theta_1) \left( \frac{h_1 + m_0}{m_1} \right) \left( \frac{1 - \frac{r m_2}{m_1}}{m_1} \right)^{-1} + m_0 \right),
\]

and 
\[
a_t = \frac{1}{m_1} \left( \frac{r \left( \frac{\theta_1 n f_{11} \phi_1}{D (1 - b \theta_1)} (m + (b \theta_1 h_2 - h_2 + m r) (1 - \frac{r m_2}{m_1}))}{m_1} \right) \right) + \frac{\theta_1 m r}{m_1 m_1} \left( \frac{n f_{11} \phi_1 h_2}{D (1 - b \theta_1)} (\theta_1 b - 1) \left( \frac{1 - \frac{r m_2}{m_1}}{m_1} \right)^{-1} + \left( \frac{1 - \frac{r m_2}{m_1}}{m_1} \right) + 1 \right) \varepsilon_{2t} \\
+ \frac{\theta_1 m r}{m D (1 - b \theta_1)} \left( \frac{m_2 (b h_2 \theta_1 - h_2 + m r) + m (1 - \frac{r m_2}{m_1}) \varepsilon_{1t}}{m_1} \right) \\
+ \theta_1 m r \left( \frac{n f_{11} \phi_1 h_2}{D (1 - b \theta_1)} (\theta_1 b - 1) \left( \frac{1 - \frac{r m_2}{m_1}}{m_1} \right)^{-1} + \left( \frac{1 - \frac{r m_2}{m_1}}{m_1} \right) + 1 \right) \varepsilon_{2t-1} \\
+ \frac{\theta_1 m r}{m D (1 - b \theta_1)} \left( \frac{m_2 (b h_2 \theta_1 - h_2 + m r) + m (1 - \frac{r m_2}{m_1}) \varepsilon_{1t}}{m_1} \right)
\]
The equilibrium processes for acreage planted, mill consumption, and prices show dynamic interaction with their lagged values and current and lagged exogenous variables. The model predicts that the paths of $A_{1t}$, $CTM_t$, $RPF_t$, $RPW_t$, and $RPM_t$ from an arbitrary initial allocation toward the steady state, all are characterized by the same dynamic properties. The sign of the first-order serial correlation in acreage planted, mill consumption, and prices is determined by the values of the dynamic cost parameters $d_2$ and $h_2$.

Under the assumption that $U_{1t}$ follows a random walk process for $i = 1, 2$, it turns out that the error terms in all the optimal decision rules for the prices ([4.59], [4.60], and [4.62]) are a first-order moving average process and for mill demand and acreage planted decision rules ([4.57] and [4.61]) are white noise with the restrictions imposed on the coefficients of the error terms. The parameters of the error terms are determined by the parameters of their own processes (4.52) and the parameters of the stochastic processes for $X_t$ and the parameters of the objective functions ([4.16] and [4.27]).

4.3 Model with Storage

Cotton can be stored at least some period of time. This section introduces inventory into the previous model to make a new equilibrium which does not require that current production equals current consumption. Moreover, inventories can be held in anticipation of an increase in price.
in the future, and thus, speculative motives are introduced in the model.

Helmberger, Weaver, and Haygood (1982) developed a theory of competitive pricing and storage under conditions of uncertainty assuming that producers and arbitrageurs are guided by rational expectations. Although the way price expectations are formed within their theory is consistent with rational expectations, their theoretical development is limited to a two-period case with the terminal condition for inventory setting zero, \( I_{t+1} = 0 \). They briefly described an algorithm for solving models with horizons extending beyond two periods, but the form of the density function for \( Z_t \) (the difference between the demand and supply shocks) was still left open. This section extends the previous section by considering market equilibrium in the presence of rational expectations and an inventory option. Optimal firm-level and aggregate decision rules are derived, and the new market equilibrium is discussed.

4.3.1 A representative cotton farmer

The representative cotton farmer is the same as the one discussed in section 1, the model without storage. We will not incorporate the stocks decision in the farmer's maximizing present value problem since data on farmers's stocks are not available, and most cotton farmers sold out their cotton to millers at the harvest. The representative cotton farmer's maximizing problem is

\[ V_{1t} = E_t \sum_{i=0}^{\infty} b^t \pi_t, \]

where \( \pi_t \) is the same as the earlier analysis in the previous model. That is,
\[ \pi_t = \text{RPF}_t c_{st} + \text{RCORNR}_t c_{rt} - \text{RCTVC}_t + \frac{\text{RDP}_t * c_{rt} * \alpha * \text{Part}}{1 - (1 - \alpha) \text{Part}}. \]

The farmer chooses contingency plan for \( a_{1t} \) to maximize its expected real present value. Substitute (4.5) for \( c_{st} \), (4.1) for \( c_{rt} \), (4.2) for \( c_{rt} \), (4.7) for \( c_{1t} \), then \( V_{1t} \) can be written as

\[ V_{1t} = E_t \sum_{t=0}^{\infty} b^t (\text{RPF}_t(f_{10} + f_{11}a_{1t} + f_{12}y_{1t} - c_{tf_t} + c_{tf_{t-1}})
+ \text{RCORNR}_t[f_{20} + f_{21}(a - a_{1t}) + f_{22}y_{2t}]
- [\text{RCTVC}_t a_{1t} + d_1a_{1t}^2 + d_2a_{1t}a_{1t-1}]
+ \frac{\text{RDP}_t * (f_{10} + f_{11}a_{1t} + f_{12}y_{1t}) * \alpha * \text{Part}}{1 - (1 - \alpha) \text{Part}}), \]

\( f_{10}, f_{12}, f_{11}, f_{20}, f_{21}, f_{22}, d_1, d_2, \alpha, a, \text{Part} > 0, 0 < b < 1 \), where \( a_{1t-1} \), and \( a_{1t-2} \) are given. \( b \) is a real discount factor that lies between zero and one. At time \( t \) the firm has an information set \( \Omega_t \) which is defined in the previous section, model without storage, but with additional information on stocks.

The Euler equations for \( \{a_{1t}\} \) are the same as that derived from without the storage model (4.19).

Rewrite (4.19) is

\[ E_t a_{1t+1} + \frac{d_1a_{1t} + a_{1t-1}}{b} \]

\[ = \frac{1}{b d_2} (f_{12} \text{RPF}_t - f_{22} \text{RCORNR}_t - \text{RCTVC}_t + f_{11} \alpha_p \text{RDP}_t) \]

where \( \alpha_p = \frac{\alpha * \text{Part}}{1 - (1 - \alpha) * \text{Part}}. \)

for \( t = 0, 1, 2, \ldots \)

The transversality conditions are
Given the assumptions about the signs and magnitudes of the parameters composing $f_{11}, f_{21}, \alpha, \text{Part}, b, d_1, \text{and } d_2$, it then follows that solution of the Euler equations that satisfy the transversality conditions and the initial conditions is given by
\[
a_{st} = \mu_1 a_{st-1} - \frac{\mu_1}{d_2} \sum_{i=0}^{\infty} (b\mu_1)^i [E_t(f_{11} \text{RPF}_{t+1}^{\text{RPF}} + f_{21} \text{RGRNR}_{t+1} - \text{RCTVC}_{t+1} + f_{11} \alpha_p \text{RDP}_{t+1})],
\]
for $t = 0, 1, 2, \ldots$

Equation (4.66) gives the farmer's planted acreage as a function of last period acreage planted and future values of the exogenous variables, alternative crop net return, the direct payments, and the farmer received price.

4.3.2 A representative cotton miller

The representative cotton miller's maximum problem incorporated the inventory decision. The representative firm faces the problem of calculating a contingency plan for its mill demand $c_{tm}$ and inventory $ctms_t$ so as to

maximize
\[
V_{2t} = E_t \sum_{t=0}^{\infty} \delta^t [\text{RPM}_t \text{cmd}_t - c_{3t} - c_{4t} - c_{5t}],
\]
subject to $ctm_{t-1}, ctm_{t-2},$ and $ctms_{t-1}, ctms_{t-2}$ given.

With the assumption that cotton yarn demand always equals cotton yarn production $\text{cmd}_t = \text{cm}_t$, substitute equation (4.9) for $c_{3t}$, and equation (4.10) for $c_{4t}$, then $V_{2t}$ can be written as follows
With the use of equations (4.12) and (4.4) for \( c_{m_t} \) and \( c_{tb_t} \), the above equation becomes

\[
V_{2t} = E_t \sum_{t=0}^{\infty} \delta^t (RPM_t c_{m_t} - RPM_t c_{tb_t} - h_2 c_{m_{t-1}} \frac{(ctm_t - ctm_{t-1})^2}{2} - k_1 c_{ms_t} \frac{(ctms_t - ctms_{t-1})^2}{2}).
\]

where \( ctm_{t-1}, ctm_{t-2} \), and \( ctms_{t-1}, ctms_{t-2} \) are given.

By proceeding the same previous analysis, using the discrete time calculus of deviations, the Euler equations for \( (ctm_t) \) and \( (ctms_t) \) are

\[
\delta E_{t} ctm_{t+1} + (1 + \delta) ctm_t + ctm_{t-1}
\]

\[
= (h_2)^{-1} E_t (RPM_t - RPM_{t-1} + h_1);
\]

\[
\delta E_{t} ctm_{t+1} + (1 + \delta) ctm_t + ctm_{t-1}
\]

\[
= (k_2)^{-1} (RPM_t - \delta RPM_{t-1} + k_1),
\]

Note that (4.69) is the same as (4.28).

The transversality conditions for \( ctm_t \) and \( ctms_t \) are

\[
\lim_{T \to \infty} \delta^T E_0 ctm_T = \lim_{T \to \infty} \delta^T E_0 ctms_T = 0.
\]

Given the assumptions about the signs and magnitudes of the
parameters composing $\delta$, $h_1$, $h_2$, $k_1$, $k_2$, it then follows that the solutions of the Euler equations that satisfy the transversality conditions and the initial conditions are given by

\begin{equation}
ctm_t = ct_{m,t-1} + \frac{1}{h_2} \sum_{i=0}^{\infty} (\delta)^i E_t (RPM_{t+i} - RPW_{t+i} - h_1), \tag{4.72}
\end{equation}

\begin{equation}
ctms_t = ct_{ms,t-1} + \frac{1}{k_2} \sum_{i=0}^{\infty} (\delta)^i E_t (\delta RPM_{t+i} - RPW_{t+i} - k_1). \tag{4.73}
\end{equation}

Equation (4.72) which is the same as (4.30) gives that the firm's rate of mill consumption decision at $t$ depended on the entire future sequences of marketing margin $(RPM_t - RPW_t)$. Equation (4.73) provides that the firm's rate of inventory is a function of future values of wholesale prices of next year and the current year.

4.3.3 Equilibrium

We have constructed the decision rules of acreage planted, mill consumption, and commercial stocks. In each decision rule an entire "contingency plan" explains the decision variable as a function of its initial values and the stochastic processes of the exogenous variables. Provided that the cotton market and the cotton yarn market clear at all points in time, the stochastic processes for acreage planted, commercial stocks, consumption, the farm price of cotton, the wholesale price of cotton, and the cotton yarn price that clear the two markets, the cotton market and the yarns cotton market are derived. The model is thus a dynamic, stochastic version of the model discussed in the previous section.

To construct an equilibrium, first, get the Euler equations of the
aggregate level of \( a_{1t} \), \( ctm_t \), and \( ctms_t \) by summing up the Euler equations over all cotton farmers and millers. The Euler equations for \( A_{1t} \), \( CTM_t \), and \( CTMS_t \) are

\[
\begin{align*}
\text{(4.74)} & \quad d_2 b E_t A_{1t+1} + d_1 A_{1t} + d_2 A_{1t-1} \\
& = n f_{11t} P F_t - n f_{21t} R CORN R_t - n R C T V C_t + n f_{11t} \alpha_p R D P_t , \\
\text{(4.75)} & \quad \delta h_2 E_t C T M_{t+1} - (\delta h_2 + h_2) C T M_t + h_2 C T M_{t-1} \\
& = - m \tau R P M_t + m R P W_t + m h_1 , \\
\text{(4.76)} & \quad \delta k_2 E_t C T M S_{t+1} - (\delta k_2 + k_2) C T M S_t + k_2 C T M S_{t-1} \\
& = m R P W_t - m \delta R P W_{t+1} + m k_1 ,
\end{align*}
\]

for \( t = 0, 1, 2, 3, \ldots \)

where \( A_{1t} = n \times a_{1t} \), \( CTM_t = m \times ctm_t \), \( CTMS_t = m \times ctms_t \), and \( n \) and \( m \) define the number of the cotton farmers and the cotton millers. Next, summing up the production functions (4.11) over the cotton farmers and the cotton mill firms gives the same aggregate production function in the model without the storage component. Reproduce the aggregate production functions ([4.33a], [4.33b]) as following

\[
\begin{align*}
\text{(4.77)} & \quad C T_t = F_{10} + F_{11} A_{1t} + F_{12} Y_{1t} , \\
\text{(4.78)} & \quad C M_t = C T M_t \times \tau ,
\end{align*}
\]

where \( C T_t = n \times c t_t \), \( F_{10} = n \times f_{10} \), \( F_{11} = f_{11} \), \( F_{12} = n \times f_{12} \) and \( C M_t = m \times cm_t \).

In equilibrium, where total demand always equals total supply at market equilibrium prices (\( RPW_t \) and \( RPM_t \)) at each harvest time (\( t \)), the cotton market and cotton yarn market clear at each harvest. The market clearing conditions for the two markets are
\[ CTM_t + CTX_t + CTMS_t + CTG_t = CT^t + CTMS_{t-1} + CTG_{t-1}, \quad (4.79) \]
\[ CMD_t = CM_t, \quad (4.80) \]

where \( CTG_t = n \times cgt_t \), \( cgt_t \) is the CCC stock at time \( t \).

The use of equation (4.12) obtains
\[ RPM_t = \frac{1}{m_1} CTM_t + \frac{m_2}{m_1} RPW_t - \frac{1}{m_1} U_{2t} - m_2. \quad (4.81) \]

Our objective is to find stochastic processes for \{A_{it}\}_{t=0}^T, \{CTM_t\}_{t=0}^T, \{CTMS_t\}_{t=0}^T, \{RPW_t\}_{t=0}^T, \{RPM_t\}_{t=0}^T, \text{ and } \{RPF_t\}_{t=0}^T \text{ that satisfy the}

transversality conditions of the cotton industry problems and that satisfy

the Euler equations (4.74-76). To proceed, substitute (4.81) into (4.75)

to obtain
\[ \delta h_2 E_t CTM_{t+1} - \left( \delta h_2 + h_2 - \frac{mr}{m_1} \right) CTM_t + h_2 CTM_{t-1} \quad (4.82) \]
\[ = mh_1 + m_{2} RPW_t - \frac{m_{2}}{m_1} m_{2} RPW_t + m_{2} U_{2t}. \]

Rewrite (4.82) as
\[ RPW_t = \left[ m \left( 1 - \frac{rm_2}{m_1} \right) \right]^{-1} \left( \delta h_2 E_t CTM_{t+1} - \left( \delta h_2 + h_2 - \frac{mr}{m_1} \right) CTM_t \right) \quad (4.83) \]
\[ + h_2 CTM_{t-1} - mh_1 - \frac{rm_2}{m_1} U_{2t} - m_{2} m_{2} \} \right). \]

With the substitution of equation (4.83) for \( RPW_t \), equation (4.76)

becomes
\[ \delta h_2 E_t (CTMS_{t+1} - CTMS_t) - k_2 (CTMS_t - CTMS_{t-1}) \quad (4.84) \]
\[ + \delta h_2 \left( l - \frac{rm_2}{m_1} \right)^{-1} CTM_{t+2} - \left( l - \frac{rm_2}{m_1} \right)^{-1} \left( \delta h^2 + 2 \delta h_2 - \frac{mr}{m_1} \right) CTM_{t+1} \]
\[ + \left( l - \frac{rm_2}{m_1} \right)^{-1} \left( 2 \delta h_2 + h_2 - \frac{mr}{m_1} \right) CTM_t - \left( l - \frac{rm_2}{m_1} \right)^{-1} h_2 CTM_{t-1} \]
\[ = - \frac{mr}{m_1} \left( l - \frac{rm_2}{m_1} \right)^{-1} U_{2t} + \left( l - \frac{rm_2}{m_1} \right)^{-1} mr \delta U_{2t+1} + m_k \]
\[-(1 - \frac{r_m}{m_1})^{-1}m(h_1 + \frac{r_m}{m_1}) + \left(1 - \frac{r_m}{m_1}\right)^{-1}m_1(1 - \delta_1)\left(h_1 + \frac{r_m}{m_1}\right).\]

Use aggregate production (4.77) and market clearing condition (4.79), then (4.84) can be written as

\[
\delta k_2(f_{11}A_{1t} + F_{12}y_{1t+1} + F_{10} - CTM_{t+1} -CTX_{t+1} - CTG_{t+1} + CTG_t) \\
- k_2(f_{11}A_{1t} + F_{12}y_{1t} + F_{10} - CTM_t - CTX_t - CTG_t + CTG_{t-1}) \\
+ \delta^2 h_2 \left(1 - \frac{r_m}{m_1}\right)^{-1}CTM_{t+2} - \left(\delta^2 h_2 + 2\delta h_2 - \frac{m_0}{m_1}\right) \left(1 - \frac{r_m}{m_1}\right)^{-1}CTM_{t+1} \\
+ \left(1 - \frac{r_m}{m_1}\right)^{-1}(2\delta h_2 + h_2 - \frac{mr}{m_1}) CTM_t - \left(1 - \frac{r_m}{m_1}\right)^{-1}h_2CTM_{t-1} \\
- \frac{mr}{m_1}(1 - \frac{r_m}{m_1})^{-1}U_{2t} + \frac{mr}{m_1}(1 - \frac{r_m}{m_1})^{-1}U_{2t+1} + mk_1 \\
- \left(1 - \frac{r_m}{m_1}\right)^{-1}m(h_1 + \frac{r_m}{m_1}) + \left(1 - \frac{r_m}{m_1}\right)^{-1}m_1(1 - \delta_1)\left(h_1 + \frac{r_m}{m_1}\right).
\]

Rearranging the above equation gives

\[-k_2f_{11}(1 - \delta L^{-1})A_{1t} \quad (4.85)\]

\[= \delta h_2 (1 - \delta L^{-1})(1 - \frac{r_m}{m_1})^{-1}CTM_{t+1} \]

\[-k_2 (1 - \delta L^{-1})CTM_t - \left(\delta h_2 + h_2 - \frac{mr}{m_1}\right)(1 - \frac{r_m}{m_1})^{-1}(1 - \delta L^{-1})CTM_t \]

\[+ \frac{h_2}{1 - \frac{r_m}{m_1}}(1 - \delta L^{-1})CTM_{t-1} + k_2F_{12}(1 - \delta L^{-1})y_{1t} \]

\[-k_2 (1 - \delta L^{-1})CTX_t - k_2 (1 - \delta L^{-1})CTG_t + k_2 (1 - \delta L^{-1})CTG_{t-1} \]

\[-\frac{mr}{m_1}(1 - \frac{r_m}{m_1})^{-1}(1 - \delta L^{-1})U_{2t} + k_2F_{10}(1 - \delta) + mk_1 \]

\[-\left(1 - \frac{r_m}{m_1}\right)^{-1}m(h_1 + \frac{r_m}{m_1})(1 - \delta).\]

To simplify the above equation, operate on both sides of this equation with the inverse of \((1 - \delta L^{-1})\) to get
\[ A_{1t} = - \frac{1}{k_2 f_{11}} \left( \delta h_2 \left( 1 - \frac{r m_2}{m_1} \right) \right)^{-1} C T M_{t+1} - \left[ k_2 + \left( l - \frac{r m_2}{m_1} \right) \right] \] 

\[ \times \left( \delta h_2 + h_2 - \frac{m r}{m_1} \right) C T M_t + h_2 \left( 1 - \frac{r m_2}{m_1} \right) C T M_{t-1} \]

\[ + k_2 F_{12} y_{1t} + k_2 F_{10} - k_2 C T X_t - k_2 C T G_t - k_2 C T G_{t-1} \]

\[ - \left( 1 - \frac{r m_2}{m_1} \right)^{-1} (m U_{2t} - \left( 1 - \frac{r m_2}{m_1} \right)^{-1} m (h_1 + \frac{r m_0}{m_1}) + m k_2 \} \]}

Use (4.83), (4.6), and the lag operator "L" the equation (4.74) can be written as

\[ (b d_2 L^{-1} + d_1 + d_2 L) A_{1t} \]

\[ = n f_{11} B \left[ m \left( 1 - \frac{r m_2}{m_1} \right) \right]^{-1} \delta h_2 C T M_{t+1} \]

\[- n f_{11} B \left[ m \left( 1 - \frac{r m_2}{m_1} \right) \right]^{-1} \left( \delta h_2 + h_2 - \frac{m r}{m_1} \right) C T M_t \]

\[ + n f_{11} B \left[ m \left( 1 - \frac{r m_2}{m_1} \right) \right]^{-1} h_2 C T M_{t-1} \]

\[- n f_{11} B \left[ m \left( 1 - \frac{r m_2}{m_1} \right) \right]^{-1} m r U_{2t} + n f_{11} U_{1t} - n f_{12} C R N R_t - n R C T V C_t \]

\[ + n f_{11} \alpha_p \Delta P_t - n f_{11} B \left( 1 - \frac{r m_2}{m_1} \right)^{-1} \left( h_1 + \frac{r m_0}{m_1} \right) \]

The use of (4.86) and (4.87) eliminates \( A_{1t} \) and obtains

\[ \frac{b d_2 \delta h_2 \left( 1 - \frac{r m_2}{m_1} \right) \right)^{-1} C T M_{t+2} \]

\[ - (l - \frac{r m_2}{m_1}) \left( b d_2 \delta h_2 + n f_{11} B \delta h_2 - \frac{b d_2}{k_2 f_{11}} \left( \delta h_2 + h_2 - \frac{m r}{m_1} \right) \right) \frac{b d_2}{f_{11}} \]

\[ - (l - \frac{r m_2}{m_1}) \left( b d_2 \delta h_2 + b d_2 h_2 - n f_{11} B \delta h_2 + h_2 - \frac{m r}{m_1} \right) \frac{b d_2}{k_2 f_{11}} \]

\[ - \frac{d_1}{k_2 f_{11}} \left( \delta h_2 + h_2 - \frac{m r}{m_1} \right) \frac{d_1}{f_{11}} \]

\[ + (l - \frac{r m_2}{m_1}) \left( b d_2 \delta h_2 + h_2 - \frac{m r}{m_1} \right) \frac{d_1 h_2 - n f_{11} \delta h_2}{k_2 f_{11}} \frac{d_2}{f_{11}} \]
- \frac{d_2}{k_2 f_{11}} \left( \frac{1 - r_{m_2}}{m_1} \right)^{-1} C Y_{t-2}^{m_1}

= \frac{b d_2}{f_{11}} Y_{t+1}^{m_1} + \frac{d_1}{f_{11}} Y_{t}^{m_1} + \frac{d_2}{f_{11}} Y_{t-1}^{m_1} - \frac{b d_2}{f_{11}} C Y_{t+1}^{m_1} - \frac{d_1}{f_{11}} C Y_{t}^{m_1}

- \frac{d_2}{f_{11}} C Y_{t-1}^{m_1} - \frac{b d_2}{f_{11}} C Y_{t}^{m_1} - \left( \frac{d_1 - b d_2}{f_{11}} \right) C Y_{t}^{m_1} - \left( \frac{d_2 - d_1}{f_{11}} \right) C Y_{t-1}^{m_1}

+ \frac{d_2}{f_{11}} C Y_{t-2}^{m_1} \frac{b d_2}{k_2 f_{11}} \left( \frac{1 - r_{m_2}}{m_1} \right)^{-1} m r U_{2t+1}^{m_1}

- \frac{r}{m_1} \left( \frac{1 - r_{m_2}}{m_1} \right)^{-1} \left( \frac{d_1 m}{k_2 f_{11}} \right) U_{2t}^{m_1} \frac{m r d_2}{k_2 f_{11} m_1} \left( \frac{1 - r_{m_2}}{m_1} \right)^{-1} U_{2t-1}^{m_1}

+ n f_{11} U_{1t}^{m_1} - n f_{2t} R C O N R_{t}^{m_1} - n R C T V C_{t}^{m_1} + n f_{11} a P R D P_{t}^{m_1}

- \left\{ \left( \frac{1 - r_{m_2}}{m_1} \right)^{-1} \left( \frac{h_1 + r_{m_2}}{m_1} \right) n f_{11} + (b d_2 + d_1 + d_2) \frac{m}{k_2 f_{11}} \right\}

- \left( b d_2 + d_1 + d_2 \right) m k_1^{m_1} \frac{1}{f_{11} (1 - \delta)^{m_1}}

Equation (4.88) is a fourth-order difference equation. The higher-order difference equation has resulted from the second-order adjustment costs of commercial stocks and the interrelation among the decision variables. To solve this difference equation, write it as a form of factorization for finding the roots of the polynomials. Use of the method of partial fractions enables us to express the fourth-order polynomials as

\[ A(L) = \frac{F(L)}{G(L)} = \frac{A_1}{1 - \theta_1 L} + \frac{A_2}{1 - \theta_2 L} + \frac{A_3}{1 - \theta_3 L} + \frac{A_4}{1 - \theta_4 L}, \]

where

\[ A_1 = \frac{\theta_3^3}{(\theta_1 - \theta_2)(\theta_1 - \theta_3)(\theta_1 - \theta_4)}, \]

\[ A_2 = \frac{\theta_2^3}{(\theta_2 - \theta_1)(\theta_2 - \theta_3)(\theta_2 - \theta_4)}, \]

\[ A_3 = \frac{\theta_3^3}{(\theta_3 - \theta_1)(\theta_3 - \theta_2)(\theta_3 - \theta_4)}, \]

\[ A_4 = \frac{\theta_4^3}{(\theta_4 - \theta_1)(\theta_4 - \theta_2)(\theta_4 - \theta_3)}. \]
\[ A(L) = 1 + \left( \frac{1}{bd_2 \delta h_2} \left[ \frac{d_1 \delta h_2 + \frac{nf_{11}^2 \delta h_2 k_2}{m}}{\delta h_2 + h_2 - \frac{m r}{m_1}} \right] \right) L \]

\[ + \left( \frac{1}{bd_2 \delta h_2} \left[ d_2 \delta h_2 + \frac{bd_2 h_2 - \frac{nf_{11}^2 k_2}{m}}{\delta h_2 + h_2 - \frac{m r}{m_1}} \right] \right) L^2 \]

\[ + \left( \frac{1}{bd_2 \delta h_2} \left[ d_2 \delta h_2 + \frac{bd_2 h_2 - \frac{nf_{11}^2 k_2}{m}}{\delta h_2 + h_2 - \frac{m r}{m_1}} \right] \right) L^3 \]

\[ + \frac{1}{b_0} L^4 , \]

and \( F(L) = 1, \ G(L) = (1 - \theta_1 L)(1 - \theta_2 L)(1 - \theta_3)(1 - \theta_4 L). \)

Then

\[ A(L) = \frac{\theta_3^3}{(\theta_1 - \theta_2)(\theta_1 - \theta_3)(\theta_1 - \theta_4)(1 - \theta_1 L)} \]

\[ - \frac{\theta_3^3}{(\theta_1 - \theta_2)(\theta_2 - \theta_3)(\theta_2 - \theta_4)(1 - \theta_2 L)} \]

\[ + \frac{\theta_3^3}{(\theta_1 - \theta_3)(\theta_2 - \theta_3)(\theta_3 - \theta_4)(1 - \theta_3 L)} \]

\[ - \frac{\theta_3^3}{(\theta_1 - \theta_4)(\theta_2 - \theta_4)(\theta_3 - \theta_4)(1 - \theta_4 L)} \]

Denote the left hand side of (4.88) \( Q_6 \) and use \( A(L), \ (4.88) \) can be written as

\[ C_{TM_{b+2}} = \frac{(Q_6)}{(1 - \theta_1 L)(1 - \theta_2 L)(1 - \theta_3 L)(1 - \theta_4 L)} \]

\[ = \frac{\theta_3^3}{(\theta_1 - \theta_2)(\theta_1 - \theta_3)(\theta_1 - \theta_4)(1 - \theta_1 L)} (Q_6) \]

\[ - \frac{\theta_3^3}{(\theta_1 - \theta_2)(\theta_2 - \theta_3)(\theta_2 - \theta_4)(1 - \theta_2 L)} (Q_6) \]
We solve the stable roots backward if $|\text{root}| < 1$, and the unstable roots forward if $|\text{root}| > 1$. Knowing the roots, we apply Wiener-Kolmogorov formula to derive the closed form solutions for the decision rules as we discussed in the previous section, the model without storage. Unfortunately, the fourth-order polynomials with highly nonlinear parameters do not allow us to derive the roots. But the equation (4.88) shows that the introduction of inventory would lead to a different set of estimates. Although the optimal decision rules cannot be drawn, we would expect that the effects of demand disturbances are smoothed out. With inventory option, millers will have a choice of buying additional harvest and carrying it into the future. Because a good harvest makes prices drop temporarily, profits can be gained by carrying stock into the future to sell when price is higher. This has the effect of dampening price fluctuation, therefore reducing the variances in prices. If millers are intuitive to price differentials, their activity will tend to remove predictable movements in prices. Consequently, price changes will tend to be random over short periods; that is, the stochastic process of price is close to a random walk. This implies, using our model, that the coefficient on lagged prices is close to 1.
4.4 Summary

To this point, a dynamic rational expectations model is built for the U.S. cotton industry with zero storage assumption. The model presented in this chapter explicitly specifies the market conditions and costs of production for cotton and cotton yarn. The dynamic decision rules for acreage planted, mill consumption, and prices (\( \text{RPF}_t \), \( \text{RPW}_t \), and \( \text{RPM}_t \)) derived from the farmer and miller optimization problems, market clearing conditions, cotton and cotton yarn productions, and land allocations are solved analytically. Moreover, the optimal decision rules are obtained under the assumptions that \( X_t \) (including the relevant exogenous variables) had an \( q \)th order vector autoregressive representation and \( a_t \) has a first-order Markov process. The exogenous variables and their lagged values and the error terms appear in the optimal decision rules. Since they help predict futures values of the relevant exogenous variables.

A feature of the rational expectations model is that it imposes restrictions across parameters in a firm's decision rules and the parameters of stochastic processes that firms face passively. These cross-equation restrictions introduce a test on validity of the rational expectations model - for example, to perform a ratio-test on overidentifying cross-equation restrictions against alternative models. The problem of the identification of the structural parameters of rational expectations models have been discussed in the literature (Wallis 1980; Hansen and Sargent 1980, 1981; Pesaran 1981).

For the model without storage, the optimal decision rules were left
open. One way of proceeding is to postulate the roots of the fourth-order polynomials, then solve backward for $|\text{root}| < 1$ and forward for $|\text{root}| > 1$, and use the Wiener-Kolmogorov Prediction formula to derive the optimal decision rules as described in the previous section. Comparing the resulting formula with the associated zero storage model would provide valuable insight into the rule of storage in market economies.

Finally, the very simple model designed to represent the phenomenon of the U.S. cotton market used here gives rise to complicated nonlinear decision rules.
5 ESTIMATION METHODS AND INFORMATION REQUIREMENTS

This chapter describes in detail the data and the estimation procedure employed for empirical investigation of a dynamic model in the U.S. cotton market. A number of modifications on data are required to make econometric estimation acceptable. For example, data on the variable costs for the 1956-1963 period were not available, but a series of cost per unit of production for cotton are reported in Statistic Bulletin. With the cost per unit of production series, it was possible to convert the series to a variable costs series by using the 1944 base $49.70. The optimal decision equations are highly nonlinear in parameters. Following this nonlinear effect, it is necessary to use nonlinear estimation techniques to estimate the structural parameters.

This chapter is organized as follows: first, the closed form equations for the U.S. cotton market are summarized; second, time series are discussed, particularly, a convenient way to reflect the economic relationships among variables using a vector autoregressions approach, which is also discussed; third, the nonlinear LS estimation methods and algorithms for solving the nonlinear least squares problems are reviewed and discussed, and some issues related to the computing programs are also addressed; finally, data sources and other relevant considerations will be discussed.
5.1 Summary of the Closed Form Equations for the U.S. Cotton Market

The closed forms of the system of equations derived in Chapter IV are summarized as follows:

$$CTM_t = \Theta_1 CTM_{t-1}$$

\[ \begin{align*}
- \frac{m}{Df_{11}} \left( 1 - \frac{r_2}{m_1} \right) & \left[ (d_2 + \Theta_1 d_1 + \Theta_1^2 d_2) (\phi_1 - F_{12} \phi_2) \\
+ n \Theta_1 f_{11} (f_{21} \phi_3 + \phi_4 - f_{11} \alpha \phi_5) \right] \\
* V(b \Theta_1)^{-1} \left[ I + \sum_{r=1}^{q-1} \sum_{i=r+1}^{q} (b \Theta_1)^{i-r} V_i \right] L^r & X_t \\
+ \frac{d_2 m}{Df_{11}} \left( 1 - \frac{r_2}{m_1} \right) (\phi_1 - F_{12} \phi_2) X_t \\
- \frac{\Theta_2 d_2 m}{Df_{11}} \left( 1 - \frac{r_2}{m_1} \right) (\phi_1 - F_{12} \phi_2) X_{t-1} \\
+ a_{1t} + C_1,
\end{align*} \]

where 

\[ a_{1t} = - \frac{\Theta_1 n f_{11} \beta m r}{D \Theta_1 (1 - b \Theta_1)} \epsilon_{2t} + \frac{\Theta_1 n f_{11}}{D \Theta_1 (1 - b \Theta_1)} \left[ 1 - \frac{r_2}{m_1} \right] \epsilon_{1t}, \]

and 

\[ C_1 = \frac{\Theta_2 m}{D (1 - b \Theta_1)} \left[ F_{10} \left( 1 - \frac{r_2}{m_1} \right) (b d_2 + d_1 + d_2) - n f_{11} \beta \left( h_1 + \frac{r m_0}{m_1} \right) \right], \]

$$RPW_t = \Theta_1 RPW_{t-1}$$

\[ \begin{align*}
+ \frac{h_2}{D \Theta_1 f_{11}} & \left( \left( b \Theta_1 + \Theta_1 - 1 - m r \Theta_1 - b \Theta_1^2 \right) \\
* V(b \Theta_1)^{-1} \left[ I + \sum_{r=1}^{q-1} \sum_{i=r+1}^{q} (b \Theta_1)^{i-r} V_i \right] L^r & X_t \\
+ \left[ (\phi_1 - F_{12} \phi_2) (d_2 + d_1 \Theta_1 - d_2 \Theta_1 \left( b + 1 - \frac{m r}{m_1 h_2} - b V - b \Theta_1 \right) \right] \\
+ n f_{11} \Theta_1 (f_{21} \phi_3 + \phi_4 - f_{11} \alpha \phi_5) \right) X_t
\end{align*} \]
\[
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\]
\[
+ h_2 \left[ (\phi_1 - F_{12}\phi_2) \left( \theta_1 d_2 \left( b + 1 - \frac{mr}{m_1 h_2} - b\theta_1 - bV \right) - d_1 \theta_1 \right) \right] \frac{Df_{11}}{Df_{11}}
\]
\[
- nf_{11} \theta_1 (f_{21}\phi_3 + \phi_4 - f_{11}\alpha_p\phi_5) \right] X_{t-1}
\]
\[
- h_2 \theta_2 d_2 (\phi_1 - F_{12}\phi_2) \frac{Df_{11}}{Df_{11}}
\]
\[
+ a_{2t} + c_2,
\]
where
\[
C_2 = \frac{\theta_1 F_{10} (bd_2 + d_1 + d_2)}{Df_{11}(1 - b\theta_1)} \left[ 2b h_2 (1 - \theta_1) - \frac{mr}{m_1} \right] \left( h_1 + \frac{r m_0}{m_1} \right) \left( 1 - \frac{r m_2}{m_1} \right) - 1
\]
\[
* \left[ \frac{\theta_1 n f_{11} \theta_1}{D(1 - b\theta_1)} \left( 2b h_2 (1 - \theta_1) - \frac{mr}{m_1} \right) + (1 - \theta_1) \right]
\]
and
\[
a_{2t} = \frac{- r}{m_1} \left[ \frac{\theta_1 n f_{11} \theta_1}{D(1 - b\theta_1)} \left( b\theta_1 h_2 - h_2 + \frac{mr}{m_1} \right) \left( 1 - \frac{r m_2}{m_1} \right) \right] \epsilon_{2t}
\]
\[
+ \left( 1 - \frac{r m_2}{m_1} \right) \epsilon_{2t-1}
\]
\[
+ \frac{\theta_1 r}{m_1} \left[ \frac{n f_{11} s h_2}{D(1 - b\theta_1)} (\theta_1 b - 1) \left( 1 - \frac{r m_2}{m_1} \right) + \left( 1 - \frac{r m_2}{m_1} \right) \right] \epsilon_{2t-1}
\]
\[
+ \frac{\theta_1 n f_{11} \theta_1}{D(1 - b\theta_1)} \left( b h_2 \theta_1 - h_2 + \frac{mr}{m_1} \right) \epsilon_{1t}
\]
\[
+ \frac{\theta_1 n f_{11} h_2}{D} \epsilon_{1t-1}
\]
\[
R P F_t = \theta_1 R P F_{t-1} + \frac{s h_2}{D \theta_1 f_{11}} (5.3)
\]
\[
* \left( b\theta_1 + \theta_1 - 1 - \frac{mr \theta_1}{m_1 h_2} - b\theta_2^2 \right) \left[ \left( d_2 + \theta_1 d_1 + b\theta_1^2 d_2 \right) \right]
\]
\[
* (\phi_1 - F_{12}\phi_2) + nf_{11} \theta_1 (f_{21}\phi_3 + \phi_4 - f_{11}\alpha_p\phi_5)
\]
\[
* V(\theta_1)^{-1} \left( I + \sum_{r=1}^{q-1} \sum_{i=r+1}^{q} \left( \theta_1 \right)^{i-r} V_i \right) L^r
\]
\[
+ [ (\phi_1 - F_{12}\phi_2) (d_2 + d_1 \theta_1 - d_2 \theta_1 \left( b + 1 - \frac{mr}{m_1 h_2} - b\theta_1 - bV \right) )
\]
\[
+ nf_{11} \theta_1 (f_{21}\phi_3 + \phi_4 - f_{11}\alpha_p\phi_5) ] X_t
\]
\[ \begin{align*}
&+ \frac{\beta h_2}{D_{f_{11}}} \left( \phi_1 - F_{12}\phi_2 \right) \left( d_2 \Theta \left( b + l - \frac{mr}{m_1} - b\theta_1 - bV \right) - d_1 \Theta_1 \right) \\
&- n \frac{f_{11} \Theta_1 (f_{21} \phi_3 + \phi_4 - f_{11} \alpha \phi_5)}{D_{f_{11}}} X_{t-1} \\
&- \frac{b_2 \Theta_2 b_3}{D_{f_{11}}} \left( \phi_1 - F_{12}\phi_2 \right) X_{t-2} \\
&+ a_{3t} + C_3,
\end{align*} \]

where
\[ a_{3t} = - \frac{\beta r}{m_1} \left[ \frac{\Theta_1 n f_{11} \beta}{D(1 - b\theta_1) \delta_2} \right] \left( b h_2 - h_2 + \frac{mr}{m_1} \right) \left( \frac{1 - \tau m_2}{m_1} \right) \]
\[ \epsilon_{2t} \]
\[ + \left( \frac{1 - \tau m_2}{m_1} \right) \epsilon_{2t-1} \]
\[ + \frac{\beta \Theta_1}{m_1} \left[ \frac{n f_{11} \beta h_2}{D(1 - b\theta_1)} \right] \left( \Theta_1 - 1 \right) \left( \frac{1 - \tau m_2}{m_1} \right) \left( \frac{1 - \tau m_2}{m_1} \right) \epsilon_{2t-1} \]
\[ + \left( \frac{\Theta_1 n f_{11} \beta}{D(1 - b\theta_1)} \right) \left( b h_2 - h_2 + \frac{mr}{m_1} \right) \left( 1 \right) \epsilon_{1t} \]
\[ + \Theta_1 \left( b n f_{11} h_2 + 1 \right) \epsilon_{1t-1}, \]

\[ C_3 = \frac{\beta \Theta_1}{D_{f_{11}}} \left( b d_2 - d_1 + d_2 \right) \left( 2 b h_2 \left( 1 - \theta_1 \right) - \frac{mr}{m_1} \right) - \Theta_1 \left( h_1 + \frac{\tau m_2}{m_2} \right) \]
\[ \left( \frac{1 - \tau m_2}{m_1} \right)^{-1} \left[ \frac{\Theta_1 n f_{11} \beta}{D(1 - b\theta_1)} \right] \left( 2 b h_2 \left( 1 - \theta_1 \right) - \frac{mr}{m_1} \right) + \left( 1 - \theta_1 \right). \]

\[ A_{1t} = \Theta_1 A_{1t-1} \]
\[ - \frac{m}{D_{f_{11}}^2} \left( 1 - \frac{\tau m_2}{m_1} \right) \left\{ \left( d_2 + \Theta_1 d_1 + b\theta_1^2 d_2 \right) \left( \phi_1 - F_{12}\phi_2 \right) \\
+ n \frac{f_{11} \Theta_1 (f_{21} \phi_3 + \phi_4 - f_{11} \alpha \phi_5)}{D_{f_{11}}} \right\} \]
\[ \times V(\Theta_1 b)^{-1} \left\{ I + \sum_{r=1}^{q} \sum_{i=r+1}^{q} (b \theta_1)^{i-r} V_1 \right\} \]
\[ \times \left( \phi_1 - F_{12}\phi_2 \right) \left( 1 - \frac{\tau m_2}{m_1} \right) + 1 \right) X_{t-1} \]
\[ - \frac{\Theta_1}{F_{11}} \left( \phi_1 - F_{12}\phi_2 \right) \left( \frac{d_2}{F_{11}} \left( 1 - \frac{\tau m_2}{m_1} \right) + 1 \right) X_{t-1} \]
\[ + a_{4t} + C_4. \]
where $C_4 = \frac{1}{F_{11}} \left( \frac{\Theta_{11}}{D(1 - b\Omega_1)} \right) \left[ F_{10}(bd_2 + d_1 + d_2) \left( 1 - \frac{\tau m_2}{m_1} \right) - nf_{11}\hat{s}h_1 + \frac{\tau m_2}{m_1} \right]$

- $\left( 1 - \Theta_1 \right) F_{10}$,

and $a_{4t} = -\frac{\Theta_{11} m \hat{s}r_m}{D m_1(1 - b\Omega_1)} \epsilon_{2t} + \frac{\Theta_{11} m \hat{s}r_m}{D(1 - b\Omega_1)} \left( 1 - \frac{\tau m_2}{m_1} \right) \epsilon_{4t}$.

$RPM_t = \Theta_1 RPM_{t-1} + \frac{1}{m_1 D F_{11}} \left[ \left( d_2 + \Theta_1 d_1 + b\Omega_1^2 d_2 \right) (\phi_1 - F_{12}\phi_2) \right. (5.5) + n\Theta_1 f_{11}(f_{21}\phi_3 + \phi_4 - f_{11}\alpha_2\phi_5)]$

$\times \left[ \frac{\Theta_{11} m \hat{s}r_m}{D m_1(1 - b\Omega_1)} \left( 1 - \frac{\tau m_2}{m_1} \right) \hat{e}_{2t} + \frac{\Theta_{11} m \hat{s}r_m}{D(1 - b\Omega_1)} \left( 1 - \frac{\tau m_2}{m_1} \right) \hat{e}_{4t} \right.$

$\left. + \frac{m_2 h_2}{\Theta_1} \left( b\Omega_1 + \Theta_1 - \frac{mr\Theta_1}{m_1 h_2} - b\Omega_1^2 - 1 \right) - m \left( 1 - \frac{\tau m_2}{m_1} \right) \right]$ $= \left( I + \sum_{r=1}^{q-1} \sum_{i=r+1}^{q} \Sigma \left( b\Omega_1 \right)^i V_i \right) L^{-1} X_t$

$\times \left( 1 + \sum_{r=1}^{q-1} \sum_{i=r+1}^{q} \Sigma \left( b\Omega_1 \right)^i V_i \right) L^{-1} X_t$

$+ \frac{1}{m_1 D F_{11}} \left[ m_2 h_2 (\phi_1 - F_{12}\phi_2) \Theta_1 [d_2 + d_2 b\Omega_1 + d_1 \Theta_1 - bd\Omega_1 - d_2 \Theta_1$ $+ m \left( 1 - \frac{\tau m_2}{m_1} \right) d_2 (\phi_1 - F_{12}\phi_2) ] X_{t-1}$

$+ \frac{1}{m_1 D F_{11}} \left[ m_2 h_2 (\phi_1 - F_{12}\phi_2) \Theta_1 [d_2 + d_2 b\Omega_1 + d_1 \Theta_1 - bd\Omega_1 - d_2 \Theta_1$ $+ m \left( 1 - \frac{\tau m_2}{m_1} \right) d_2 (\phi_1 - F_{12}\phi_2) ] X_{t-1}$

$- \frac{mr}{m_1 h_2} - d_1] - nf_{11}\Theta_1 (f_{21}\phi_3 + \phi_4 - f_{11}\alpha_2\phi_5)]$

$- \frac{\Theta_{11} m \hat{s}r_m}{D m_1(1 - b\Omega_1)} \left( 1 - \frac{\tau m_2}{m_1} \right) (\phi_1 - F_{12}\phi_2) ] X_{t-2}$

$- \frac{m_2 \Theta_1 d_2 h_2 (\phi_1 - F_{12}\phi_2)}{m_1 D F_{11}} X_{t-2}$

$+ a_{5t} + C_5,$

where $C_5 = \frac{1}{m_1} \left( \Theta_{11} F_{10}(bd_2 + d_1 + d_2) [m_2 \left( 2b h_2 (1 - \Theta_1) - \frac{mr}{m_1} \right) + m \left( 1 - \frac{\tau m_2}{m_1} \right) \right]$
and 

\[ a_{st} = \frac{1}{m_0} \left[ (1 - \theta_1) \left( m + \left\{ bh_2 \theta_1 + \frac{m \rho - h^2}{m_1} \right\} (1 - \frac{m \rho}{m_1})^{-1} \right) \right. \]

\[ + \left. m_2 \left( 1 - \frac{m \rho}{m_1} \right) + 1 \right) \epsilon_{2t} \]

\[ + \theta_1 \left( \frac{m_2 \tau}{m_1} \right) \left[ \frac{nf_{11} \delta h_2 (\theta b - 1)}{D(1 - b \theta_1)} \left( 1 - \frac{m \rho}{m_1} \right)^{-1} + \left( 1 - \frac{m \rho}{m_1} \right) \epsilon_{2t-1} \right) \]

\[ + \frac{\theta_1 nf_{11}}{m_1 D(1 - b \theta_1)} \left[ \frac{m_2 (bh_2 \theta_1 - h^2 + \frac{m \rho}{m_1} + m \left( 1 - \frac{m \rho}{m_1} \right) \epsilon_{1t} \right) \]

\[ + \frac{m_2 \theta_1 h_2 n f_{11}}{m_1 D} \epsilon_{1t-1} \]

with the restriction

\[ |W| \geq \min (\mid \phi_1 \mid, \mid b \psi \mid), \quad (5.6) \]

where

\[ W = \frac{-[nf_{11} \delta h_2 (\theta b - \frac{m \rho}{m_1}) - \frac{d_m}{f_{11}} (1 - \frac{m \rho}{m_1})]}{nf_{11} \delta h_2 - \frac{d_m}{f_{11}} (1 - \frac{m \rho}{m_1})}. \]

These equations are an alternative representation of the model embodied in (4.30) through (4.34), also known as the closed form or the reduced form of the model. They reflect the restrictions imposed across the decision rule and the parameters of the stochastic processes for \( X_t \) and \( U_{it} \), for \( i = 1, 2 \). The feature of the closed form equations is that the system equations have been solved to express the current values of the decision variables as functions of their own lagged values and current and lagged values of all the relevant state variables in the information set. In addition, the nonlinear restriction implied by equation (5.6) is a direct result of REH.
Furthermore, these equations are stochastic since they are derived from the dynamic optimization problem. Also, these equations impose substantial structure on their error processes. That is, the error terms in the optimal decision rules for the prices ([5.2], [5.3], and [5.5]) are first-order autoregressive processes of $\epsilon_{1t}$ and $\epsilon_{2t}$ and the error terms in the decision rules for the acreage planted and the mill demand are composite terms of $\epsilon_{1t}$ and $\epsilon_{2t}$. The parameters of the error terms are determined by the parameters of the objective function (4.16) and (4.27), with the assumptions that $U_{1t}$ and $U_{2t}$ follow a random walk process.

These equations resemble expressions, which are functions of the information they possess. And it is appropriate to use a simultaneous equation estimation framework. It would be possible to use a limited information estimation such as NL2SL, but the estimators derived from NL2SL are inefficient. To make the estimates more efficient, and to take advantage of the cross-equation restrictions, it is necessary to use a full information estimator such as NL3SL or maximum likelihood method. The method of ML is the most popular method for multivariate nonlinear model. Also, under some regularity conditions, ML estimators have well-known asymptotic properties; they are consistent and have the asymptotic distribution with the smallest variance.

Estimating the underlying parameters of the model is crucial in understanding supply response, the land allocation decision, mill consumption, and the price processes. Attempts to estimate the whole system of equations were unsuccessful, because the form of the whole system of equations is complicated and the size of the model is
substantial. Further, the current computer programs are of limited use when it comes to the substantial size of the nonlinear model. Since the goal is to obtain the underlying parameters \( \theta = (f_{11}, f_{12}, f_{21}, d_1, d_2, \beta, h_2, m_1, m_2, \text{ and } \Theta) \), subject to equation (5.6), a single equation CTM, is chosen, instead of estimating the whole system.

The CTM equation (5.1) is a regression equation. The equation includes more information than the traditional mill demand model (Sanford 1988), because the traditional mill demand does not jointly estimate the dynamics of the production process and the dynamics of the prices of cotton and cotton yarn that millers observe. The mill demand equation emphasizes the important role of the dynamic structure of the cost function, the information millers have at the time cotton is consigned to produce cotton yarn, and the way cotton and cotton yarn prices are moving over time in the determination of millers' response to changes in incentives.

Directly maximizing the log likelihood function of the CTM equation is difficult computationally since the coefficients of the error terms in the CTM equation are not only imposed by the underlying structural parameters but also by a composite term, \( \epsilon_{1t} \) and \( \epsilon_{2t} \). If such an estimation strategy is to be used, then it must have a constant variance in the CTM equation. To establish this requirement, it is convenient to assume the effect of the structural parameters on the error term of the CTM equation is trivial. By ignoring the presence of restrictions on the error term of the CTM equation, the CTM equation can be estimated by the method of nonlinear LS. Provided that the error term of the CTM equation
has zero mean and is independently identically distributed with variance \( \sigma^2 \), the nonlinear LS estimator is consistent.

5.2 Estimation Methods

5.2.1 Time series method

There are various methods used to obtain the future values of certain variables in which we are interested. One approach is to build an econometric model, estimate its parameters from the available data, and then employ this model to obtain forecast values of the variables of interest. An alternate approach is to use only the past values of variables to predict their future values. The latter method does not use economic knowledge about the processes that have generated the values for the particular variables.

Clearly, a decision will have to be made as to which technique should be used in order to best make a forecast. This decision will depend on how much knowledge about the workings of the real world process is available and how much time and energy can be spent on the modeling process. Although the construction of a single regression equation may not be difficult, the development of a multi-equation simulation model may require much time and energy. The benefit gained from this work is a better understanding of the relationships and structure involved as well as better performance of the forecast. In some cases, these gains may not be significant, but they are associated with heavy costs. A time series method is a better choice for cases where little information about the relationships of variables and a sufficient amount of data are available.
To develop a time series model, it must be known whether or not the underlying stochastic process that generated the series can be assumed to be invariant through time. If the process is nonstationary, it will often be difficult to express the time series over past and future intervals by a simple algebraic model. Provided that the process is fixed in time, we can model the process via an equation with fixed coefficients estimated from past data. This is similar to the regression model in which one economic variable is explained by other economic variables with coefficients estimated under the assumption of the fixed structure. Although time-series models do not depend on economic relationships, it is much easier to express a stochastic process if characteristics of that process do not change over time. Thus, stationariness is an important characteristic of the stochastic processes to be modeled.

Forecasting is one of the objectives of the analysis, and it is better to use as much information as possible. That is, instead of using only the information in the past values of a single variable, the information set on which the forecasts are based is extended to contain the information in the past values of other variables as well. In order to use the largest possible information base for computing forecasts, a simultaneous time series analysis of all of them should be done. Sims (1980) developed a VAR method to do policy analysis and to evaluate the plausibility of the Reagan administration's economic prediction. The VAR approach does not require knowing the structure of the model. This approach provides a convenient way of characterizing economic relationships and allows relationships among the variables to be checked.
without forcing \textit{a priori} restrictions regarding exogeneity. The VAR technique will be applied to solve the forecasting problem in the U.S. cotton model.

5.2.2 Nonlinear LS method

In econometric models, the nonlinear form can enter in both parameters and variables or only in parameters. If only the variables are nonlinear, the model can be treated as in the linear framework. Several estimation methods have been developed to solve the nonlinear regression models, including maximum likelihood, linearizing transformation, Bayesian estimation, nonlinear instrumental variables, nonlinear two-stage least squares, and nonlinear three-stage least squares.

A number of nonlinear regression models have been proposed in the literature. Moreover, various specialized methods have been developed to explain the consistency of the suggested estimators, e.g., Jennrich (1969) and Malinvaud (1970) proved the consistency of a nonlinear least squares estimator and derived its asymptotic distribution.

Systems of nonlinear equations do arise in practice. The problems faced in estimation and inference of systems of nonlinear equations are worse. Because of the high number of parameters and a highly nonlinear objective function, the practical difficulties can become substantial and may be intractable. Full information maximum likelihood (FIML) estimation is one way to overcome this problem. The FIML method has been widely used (Sargent 1978; Eckstein 1984; Eichenbaum 1981). If maximum likelihood estimates (MLE) can be found, they have desirable properties that allows us to do some tests. However, the choice of the method for estimation is
determined by the error term of the model.

By ignoring the presence of restrictions on the mill demand optimal decision rule (5.1), the decision rule can be written as

$$y = f(X, \theta) + e,$$

where $y$ is a $(t * 1)$ vector of the endogenous variable (CTM), $X$ is a $(t * k)$ matrix of $t$ observations on the exogenous variables, $\theta$ is a $(h * 1)$ vector of parameters, and $e$ is a $(t * 1)$ vector of independently and identically distributed random variables, i.e., $E(e_u) = 0$ and $E(e_u^2) = \sigma^2$, for all $u$. The evaluation used for determining the estimated values for the parameter vector $\theta$ is the same as that for linear models: to minimize the sum of squared errors. The sum of squared errors for the nonlinear model can be written as

$$s(\theta) = e_u' e_u,$$

or

$$\phi(\theta) = \sum_{u=1}^{t} e_u^2(\theta)$$

where $e_u = y - f(x, \theta) = U_{1u} + U_{2u}$.

Concerning the properties of nonlinear LS estimator: consistency and asymptotically normal distribution, the following conditions should be satisfied:

1. The sequence of independent variables $X_t$, $t = 1, 2, \ldots, T$ is bounded and well behaved as $T \to \infty$.

2. The function $f(X, \theta)$ is at least twice continuously differentiable with respect to $\theta$.

3. The $e_t$ are assumed to be independently, identically distributed with zero mean and variance $\sigma^2$. 
5.2.3 Computation of the estimates in unconstrained cases

The technique for solving the nonlinear equations is closely related to unconstrained minimization problems, since the nonlinear least squares problem is merely a special case of unconstrained minimization. The modification of unconstrained minimization techniques for nonlinear least squares problems can produce better algorithms due to the advantage of the structure of the nonlinear least squares problem.

Several methods have been developed to compute the estimates of the nonlinear normal equations. The constraints often arise from a priori information concerning the values of the parameters. The presence of constraints, especially in the inequality cases, often has an impact on the convergence of an optimization algorithm. For the constrained problems, the analyses are similar to the unconstrained problem, but more complex. Only the methods for unconstrained optimization will be discussed.

All of these numerical methods for computing the nonlinear estimators, including iterative method, grid search method, and direct optimization method, are based on the following justifications:

1. Set \( n = 1 \) with the initial guess \( \theta_1 \) given.
2. Choose a step direction, \( v_n \), in the \( n \)th step.
3. Choose a step length \( r_n \) such that

\[
\theta_{n+1} = \theta_n + r_n v_n \tag{5.11}
\]

is acceptable, which is to require that \( r_n \) be chosen so that \( \phi_{n+1} < \phi_n \) holds.
4. Test whether the termination criterion, \(|\theta_{n+1} - \theta_n| \leq \epsilon\), (where \(\epsilon\) is a quite small number) is met. If not, increase \(n\) by one and return to step 2. If yes, accept \(\theta_{n+1}\) as the value of \(\theta^*\).

5.2.3.1 Iterative method

Given an initial estimate, say \(\theta_0\), a new estimate can be calculated from \(\theta_1 = [x(\theta_0)'x(\theta_0)]^{-1}x(\theta_0)y(\theta_0)\) if the nonlinear problem can be solved by least squares. The calculation of \(\theta_1\) is called the first iteration, and the point \(\theta_1\) the first iterate, and \(\theta_1 - \theta_0 = \sigma_1\), the \(\sigma_1\) is called the first step. If \(\phi_1 < \phi_0\), the step is acceptable, if not, repeat iteratively. This process is repeated to generate a sequence of points \(\theta_1, \theta_2, \ldots\) until the estimated sum of squared errors converge.

5.2.3.2 Gradient method

An alternative way to calculate the iteration step is through the gradient methods. The general equation of the iteration step in all of the gradient methods takes the following formulation

\[
\theta_{n+1} = \theta_n - \tau_n R_n q_n, \tag{5.12}
\]

where \(\tau_n\) is the steplength in the nth step, \(R_n\) is some positive definite (direction) matrix in the nth step, and \(q_n\) is the gradient vector of the objective function at the current estimates \(\theta_n\). The direction matrix in this case is written as \(v_n = -R_n q_n\). Various gradient methods differ in how they choose \(\tau_n\) and \(R_n\). For example, the update parameter in the Newton (Newton-Raphson) method is computed as follows:

\[
\theta_{n+1} = \theta_n - H_n^{-1} q_n, \tag{5.13}
\]

with \(\tau_n = 1\), \(R_n = H_n^{-1}\).
where $H_n$ is the Hessian matrix of $\phi$ evaluated at the current vector $\theta = \theta_n$. Note the Hessian matrix $H(\theta)$ of the function $\phi(\theta)$ is the matrix of second partial derivatives, i.e.,

$$H_{ab} = \frac{\partial^2 \phi}{\partial \theta_a \partial \theta_b}.$$

In a single equation case, $H_{ab}$ can be written as:

$$H_{ab} = \frac{\partial^2 \phi}{\partial \theta_a \partial \theta_b} = -2 \sum_{u=1}^{t} \frac{\delta f_u}{\partial \theta_a} \left( \frac{\delta f_u}{\partial \theta_b} \right) + 2 \sum_{u=1}^{t} \frac{\delta f_u}{\partial \theta_a} \frac{\delta f_u}{\partial \theta_b}.$$

Some other algorithms use approximations to the Hessian or its inverse such as Gauss method, which ignores the first term of the Hessian matrix $H(\theta)$ in the Newton method. That is,

$$N_{ab} = 2 \sum_{u=1}^{t} \frac{\delta f_u}{\partial \theta_a} \frac{\delta f_u}{\partial \theta_b}.$$

The $n$th step direction in the Gauss method is to choose

$$\nu_n = -N_n^{-1} q_n,$$

where $N_n = R_n$, and $N$ is positive definite.

However, application of the Newton or Gauss algorithm can cause problems. For example, the Newton method will usually converge in a few iterations, but it may not converge to a minimum of the objective function. Other problems associated with both the Newton and Gauss methods are their use of derivatives of the objective function and their Hessian matrix $H(\theta)$ and approximate inverse Hessian matrix $N^{-1}$ are to be positive definite. In some cases, it is difficult or even impossible to obtain the required derivatives analytically, and $H(\theta)$ and $N^{-1}$ are not positive definite at the estimate $\theta$. Therefore, modifications such as the Marquardt-Levenberg algorithm and the Davidon-Fletcher-Powell algorithm...
have been developed to avoid or remove these problems.

The DFP inverse positive definite secant update algorithm is a modification of the Newton method. The DFP algorithm uses the gradient vector of the objective function (the negative of the sample log likelihood) to derive the approximation of the inverse Hessian matrix. DFP is an optimization algorithm. Although other algorithms may perform better than DFP, the algorithm has proven to be efficient in many practical cases (Dennis and Schnabel 1983). A rank two update is used, which makes the computation faster. The DFP method will be used in estimating the U.S. cotton model.

Some econometric software packages such as Gauss have the ability to change algorithms during the computing process. The results derived from the mixed procedures show that they are outperformed when compared to pure algorithms. Furthermore, they not only take less time per iteration, but also take fewer iterations than pure algorithms.

The computation of the gradient is sometimes the most time-consuming step in each of the algorithms used. Efficiency and accuracy in this step is important. If the gradient is computed with insufficient accuracy, it may cause the step direction to fail in reducing the value of the objective function, \( \phi \), or make the matrix \( R_i \) insufficiently positive definite.

5.2.4 Issues in estimation

Some issues arise when computing the nonlinear estimates. The first issue is bad scaling which occurs when dependent and independent variables vary greatly in magnitudes. Failure to rescale can produce round-off
error and may lead the program to converge erroneous roots or cause it to

Thus, the impact of ignoring the scaling problem can degrade the

performance of the nonlinear algorithms. A remedy for this problem is to

rescale variables; that is, change their units so that each component will

have roughly the same magnitude. Although rescaling variables will not

affect the Newton step, it will affect the steepest-descent direction

because determining which direction is "steepest" depends on what is a

unit step in each direction (Dennis and Schnabel 1983).

The second issue concerns good starting values. Good starting values

can reduce the number of iterations. A poor choice of starting values, \theta,

can cause the algorithm to search a region of the parameter values far

from a solution.

If \theta_n is a solution to the normal equations, it does not explain why

this vector is the global minimum of s(\theta). Because the normal equations

may possess different solutions and different starting values, \theta may

result in different solutions. Thus, using different starting values is

one way to find the global minimum of s(\theta). If different solutions to the

normal equations are found, the one with the smallest sum of squared

errors is chosen.

The third issue is the stopping criterion. Stopping the iterative

search for the minimum of \phi(\theta) is a somewhat ad hoc process. It may seem

natural to stop the program when the gradient vanishes, due to convergence

to a stationary point of \phi, but rounding errors and poor scaling may make

the goal of a vanishing gradient unattainable. In many practical cases,

the computer may come up with parameter values very near the point of the
minimum, but the gradient is still not small enough. Furthermore, if the algorithm does not approach convergence at all, a termination rule based wholly on the gradient allows the program to iterate infinitely.

In practice, the iteration is stopped as soon as further iterations do not change the parameter values significantly. That is, given a set of small numbers \( \varepsilon_\alpha (\alpha = 1, 2, \ldots, h); \) the number of estimates), a set of \( \theta_{n+1} \) is accepted as the solution \( \theta^* \) is

\[
|\theta_{n+1,\alpha} - \theta_{n,\alpha}| \leq \varepsilon_\alpha, \quad (\alpha = 1, 2, \ldots, h),
\]

where \( \theta_{n,\alpha} \) is the \( \alpha \)th component of \( \theta_n \). The number \( \varepsilon_\alpha \) may either be a prespecified small number, or may be computed by the program.

A common stopping criterion is to define the relative gradient of \( f \) at \( \theta \) by

\[
\text{relgrad}(\theta)_n = \frac{\text{relative rate of change in } f}{\text{relative rate of change in } \theta_n}
\]

\[
= \lim_{\delta \to 0} \frac{f(\theta + \delta \theta_n) - f(\theta)}{\delta} \frac{\delta}{\theta_n}
\]

\[
= \frac{\nabla f(\theta_n) \theta_n}{f(\theta)}
\]

and test \(|\text{relgrad}(\theta)| < \text{gradtol}\),

where \( \text{relgrad} \) is the elasticity of the objective function with respect to the parameters, \( \text{gradtol} \) is the convergence tolerance for the relative gradients. The drawback of this approach is that the idea of relative change in \( \theta_n \) or \( f \) breaks down if \( \theta_n \) or \( f(\theta) \) is near 0. The problem can be fixed by replacing \( \theta_n \) and \( f \) in (5.16) by \( \max(\max(\theta_n), \text{typ}_\theta) \) and \( \max(|f(\theta)|, \text{typ}_f) \), respectively, where \( \text{typ}_\theta \) is a vector specifying the "typical" magnitude expected for the estimated parameters. Usually, a
scaler 1 is used to do a "fix-up" in the convergence test if the parameter values fall below typθ in absolute value. This is very useful when any of the parameters are close to 0. Typf is the "typical" magnitude expected for the objective function at the optimum. A scaler 1 is used to do a "fix-up" in the convergence test, if the value of the objective function falls below typf in absolute value. A fix-up is required if the value of the objective function gets too close to 0.

The resulting test

\[ \max \left[ f(\Theta) \max \left( |\Theta_n|, \text{typ}\Theta_n \right) \right] \leq \text{gradtol} \quad (5.16) \]

is the one used in computing the U.S. cotton model.

The relative change in \( \Theta_n \) is measured by

\[ \text{rel} \Theta_n = \max \left( |(\Theta_n)_{n-1} - (\Theta_n)|, \text{typ} \Theta_n \right) \quad (5.17) \]

and test

\[ \text{rel} \Theta < \text{eto}l, \quad (5.18) \]

where \( \Theta_n = \Theta_{c} - f(\Theta_{c}) / f'(\Theta_{c}) \), \( \Theta_{c} \) initial guess, and \( f'(\Theta_{c}) \) the first derivative of \( f \) evaluated at \( \Theta_{c} \). \( \text{eto}l \) is the convergence tolerance for \( \Theta \).

So, if \( \text{rel} \Theta < \text{eto}l \), and \( \text{relgrad} < \text{gradtol} \) are both true, then the iteration terminates. But the above criterion does not guarantee that the process terminates in a finite number of steps. If the objective function has a finite minimum, then termination can be assumed that the program stops whenever \( \phi_{n-1} - \phi_n < \epsilon \), where \( \epsilon \) a small prespecified positive number. That is, the iteration stops as soon as no significant change in the successive values of the objective function occur.
An upper bound may be placed on the number of iterations in order to allow the maximum number. None of these methods can guarantee convergence to the global minimum. If a solution has been computed, but we suspect it may not be the global minimum, we can restart the calculation from a radically different initial guess and repeat the process until convergence to the same $\Theta^*$ occurs, or until we are satisfied with the result.

5.3 Information Requirements

5.3.1 Exchange rates

U.S. exports of raw cotton during 1955-1975 accounted for about one-third of total cotton disappearance (mill demand plus exports), but in the 1980s accounted for about one-half. Several times during the late 1970s and 1980s, U.S. exports exceeded domestic mill use, meaning that the U.S. cotton industry depended heavily on exports. The major export markets for U.S. cotton have been Japan, South Korea, Taiwan, Hong Kong, Indonesia, Thailand, and Canada. Many times during the 1956-1986 period Japan was the largest single export market for U.S. cotton.

During the 1970s, fluctuating foreign exchange rates have made the prediction of U.S. cotton exports more difficult. After many years of fixed exchange rates under the Bretton Woods Agreement since 1944, the U.S. moved to the floating exchange rate system in 1973. Moreover, large devaluations in 1972 and 1973 have caused rising agricultural prices during the same period. Consequently, these actions have sparked a controversy over the extent to which the departure from fixed exchange rates has impacted domestic agricultural markets and trade.
The value of the dollar in the 1980s as compared to foreign currencies has varied substantially. But the dollar gained strength through the early 1980s, attained a peak in 1985, and has declined since.

Under a floating rate regime, an exchange rate is the price of one unit of foreign currency, which is determined by the market demand for and supply of the currency. The price of the International Monetary Fund's Special Drawing Right is used as a measure of the general convertibility of a dollar.

The theoretical impact on U.S. trade of a weakening dollar has been explained this way: a weaker dollar makes U.S. exports cheaper than other exporting countries, therefore increasing export demand. A weaker dollar also raises the price of foreign goods, and as less is demanded, domestic producers gain comparative advantage. On the other hand, a strengthening dollar would be expected to produce the opposite results. In general, U.S. trade patterns have supported these observations, and some studies have shown that the real value of the dollar contributed to a reduced volume of U.S. farm exports in the 1980s (Batten and Belongia 1986).

In analyzing exports of U.S. cotton, the exchange rate plays an important role because it can reflect a way of transferring prices into a currency relevant to producers and purchasers of goods in the country. Schuh (1974) was the first to address the issue of the exchange rate effects on agriculture. From that time on, a series of studies on the effects of exchange rate on U.S. agriculture have been done.

Exchange rates should not be neglected in the empirical model since it is an important factor for influencing exports. Moreover, if other
more informative a priori information was available for the processes generating predetermined variables, it would be included by expanding the dimensions of the reduced form model. In order to make a simple model, only exchange rates of Japan are considered in the empirical model. For 1956-1980, data on the exchange rates for Japan were obtained from the publication *International Financial Statistics Supplements* (No.1, 1981). For 1981-1986, exchange rates were from various issues of the publication *International Financial Statistics*. Data on exchange rates were converted to 1980 U.S. dollars by deflating with the ratio of the consumer price index for Japan and the consumer price index for the United States.

5.3.2 Direct payments

As described in Chapter II, producer price supports linked with acreage allotments and other supply controls have been the cornerstone of the cotton crop policy over the past three decades. A key feature of the crop policy programs is its voluntary nature by which each producer has the option to participate in the program to be eligible for price support loans, deficiency payments, and associated benefits. From a producer standpoint, the direct payments are the relevant policy variable since they reflect the effective income supports on planted acreages. The direct payments are considered in the theoretical development.

In the empirical study, the direct payments include deficiency payments, diversion payments, and payment-in-kind. The maximum deficiency payments that could be received are based on the difference between the target price and the higher of the calendar year average price or the base loan rate. For 1964-1970, price support payments were available on the
domestic allotment (67% of total in 1964, 65% in 1965-1970). Loans were also available on the entire production within the allotment. For 1971-1973, the direct payments reflect the minimum payment rate available on the entire base acreage allotment. Payments in 1971-1972 were contingent on participation in the cropland set-aside program. No set-aside requirement was placed for 1973. For 1974-1980, no deficiency payments data were reported since prices received were higher than target prices. However, large deficiency payments were made for 1981-1986, as the average market prices dropped below the rising target prices. Diversion payments were made only two years since 1968: 1978 and 1983.

The direct payments data were obtained from the USDA publication Cotton: Background for 1985 Farm Legislation for 1956-1983. For 1984-1986 the direct payments were from USDA Cotton and Wool Situation and Outlook. Then the direct payments were converted to per pound units by dividing the direct payments by production of cotton for the 1984-1986 period in order to be consistent with the previous period data.

5.3.3 The cost of production

The variable cost used in cotton production includes costs of preparing and planting (hauling and spreading manure), cultivating and hoeing, harvesting (picking and snapping cotton, hauling to gin, and hauling lint and cottonseed to local markets), fertilizer and manure, seed, ginning, and miscellaneous (irrigation, overhead, etc.). The variable cost of cotton was obtained from various issues of the publication Agricultural Statistics for the period 1956-60. However, the variable cost of cotton for 1961-1963 is not explicitly reported in any
publication. Fortunately, the index of the cost per unit of production for cotton is reported in Statistic Bulletin (No 535). With the data on the cost per unit of production available during the 1960-1963 period, it is possible to infer from the index values for the 1961-63 period by using the 1960 cotton cost data, 20.7 cents per pound, as the base. The variable costs for 1964-1986 were from Missouri data reports. However, because the land rent was included in the 1956-63 period, it made the series inconsistent with the variable definition. Therefore, an adjustment was made by excluding rent for the 1956-1963 period. Land rent series were obtained by multiplying the index of price paid by farmers for production items on a 1957-1959 base with the 1944 land rent, $7.49 (per acre).

5.3.4 Alternative crops variables

The rapid rise in production costs after the 1960s significantly changed the competitive relationships among cotton and other crops. In the Southeast and Delta, cotton's primary competitor is soybeans and, to a lesser extent, corn. In the Southwest the alternative crops for cotton are grain sorghum and wheat. However, in the irrigated Far West, the substitute crops move to wheat, hay crops, and barley. In addition, U.S. cotton production has moved westward. In 1986, the West (California, Arizona, and New Mexico) accounted for about 31 percent of U.S. output, up from 18 percent in 1970. On the contrary, the southeastern share has reduced to 7 percent of the total. The Southwest (Texas and Oklahoma) and the West accounted for 60 percent of U.S. cotton production. From the regional shares of U.S. production, soybeans are an obvious competitor for
cotton, but data on soybean's variable cost are not available prior to 1960. Thus, wheat and corn are chosen for cotton's competitors in the empirical study.

The net returns for selected corn were determined by subtracting direct costs from returns generated by the corresponding crop. The direct costs used in corn include seed, fertilizer, pesticides, materials for chemical weed control, irrigation water, machinery operating expenses, custom work hired, and operator, family, and hired labor. Data on direct costs for corn were obtained from Missouri data reports for the 1964-1986 period. Prior to 1964 data were computed by multiplying the index of operating expense per unit of production with the 1964 direct cost, $50.36 per acre. Operating expense per unit of production is current cash expenditures plus net depreciation on service buildings (excluding operator's dwelling), machinery, and equipment divided by current gross farm production. The index of operating expense per unit of production is derived by dividing operating expense expressed as ratios by the index of gross farm production.

Average farm received prices for corn can be found in various issues of the annual publication Agricultural Statistics. Corn yield data were also from various issues of Agricultural Statistics. By multiplying prices and yields for each specified crop, returns can be obtained. The returns minus corresponding direct costs represent the net returns or the net losses of corn.
5.3.5 The mill demand shifters

Major factors influencing U.S. mill consumption are competing fiber prices, consumer income, cotton price inconstancy, fiber characteristics, changing lifestyles, and trade in textile products. In the empirical model, the gross national product is treated as a proxy for consumer income. For 1956-1984, data on the gross national product were obtained from the publication *Business Statistics*. For 1985-1986, data for the two years were from various issues of *Survey of Current Business*. Then the quarterly series of the gross national product were converted to the crop year series data.

Polyester prices are considered cotton's competing fiber prices. For 1956-1959, data on polyester price were obtained from *Wool Statistics and Related Data 1920-1964: reported in Statistic Bulletin (No 363)*. Polyester prices for 1960-1986 were from various issues of *Cotton and Wool Situation and Outlook*.

The other variables in the mill demand consumption equation for cotton are yield and exports. Information pertaining to mill demand consumption, yield, and exports were from various issues of *Cotton and Wool Situation and Outlook*. In addition, data on cotton variable costs, the prices, and direct payments were converted to 1980 dollars by deflating with the producer price index for all commodities.

5.4 Summary

This chapter describes the data and estimation procedures used for empirical investigation of the dynamic model in the U.S. cotton market. A
number of modifications on data were required, including variable costs for cotton and corn, the gross national product, and direct payments. The model developed in Chapter IV required more information than the traditional econometric model since production, mill demand, and market prices were determined simultaneously.

The reduced form models discussed in Chapter IV are derived from the theoretical specifications with the assumption of rational expectations. Coefficients of the reduced form models are highly nonlinear in the underlying structural parameters. Because parameters are both highly nonlinear and of large number, the practical difficulties can become substantial and intractable. By ignoring the restrictions on the error term and satisfying some conditions which are discussed in section 5.2, the CTM equation can be estimated by the method of nonlinear LS.
6 ESTIMATION OF U.S. COTTON MARKET

In Chapter IV, a theoretical model for the U.S. cotton industry was derived in which storage is not an option, and the case of holding inventories was briefly discussed. A particular feature of the model included rational expectations. A set of decision rules was derived under the rational expectations hypothesis. The decision rules represent the optimal time paths for cotton acreage planted, cotton mill demand, and prices. Also, they imply current and expected future profit opportunities for the decision makers. Coefficients of the decision rules are highly nonlinear in the underlying structural parameters. The structural parameters \((f_{11}, f_{12}, f_{21}, d_1, d_2, r, h_2, m_1, \text{ and } m_2)\) and the Markov processes governing all the exogenous variables are to be estimated. The primary goal of this chapter is to estimate these parameters using all available information and cross-equation restrictions in the model as discussed in Chapter IV. Moreover, exogenous variables which appear in the objective functions and/or which help to predict the variables should be included in the decision rules.

In this chapter, the model is estimated without the storage component. To estimate the model, a two-step procedure was considered. The two-step procedure includes forecasting and estimation. This procedure yielded consistent estimates, but was not fully efficient due to the loss of some information on the cross-equation restrictions. However, it is intractable to estimate the agent's decision rules jointly with
models for the stochastic processes they face, subject to the cross-equation restrictions implied by the rational expectations hypothesis in the rational expectations model. Because these cross-equation restrictions are complicated, highly nonlinear parameters cause analytical problems, and the large number of estimates make computation burdensome. The estimates obtained in the two-step procedures may be inefficient, but one can envision situations in which nothing better can be done. Thus, we separate the estimation process into forecasting and estimation to make estimation more tractable.

The following chapter is organized as follows: first, an exogeneity test is discussed; second, a vector autoregression approach is employed for solving the prediction problem, and the coefficients and error decompositions of the vector autoregressions are presented; third, the estimates of the coefficients of the structural model are presented and assessed. In particular, the structural parameter estimates will be examined for their implications in comparison with the economic theory. Finally, forecasting and policy analysis is addressed.

6.1 Exogeneity

Estimation of mill demand for cotton was shown to be influenced by the prices of cotton and competing fibers (polyester and rayon), yarn prices, income, and other demand shifters (Sanford 1988). This was a much stronger indication for Granger (1969) causality flowing from exogenous variables to decision variables than for Granger causality in the reverse direction, by utilization of the concept of Granger causality.
To detect Granger causality, the dynamic statistical properties of the data were initially analyzed by the estimation of vector autoregression that included cotton cropped area, mill demand for cotton, farm, wholesale and output prices, exports, yield, variable costs of cotton, net return of a substitute crop (corn) and direct payments. A block exogeneity test was performed under the null hypothesis that the lags of one set of variables do not enter into the equations for the remaining variables.

With the assumption that the order of the vector autoregression is chosen as one, the vector autoregressions computed were of the form

$$ Y_t = \sum_{i=1}^{m} \alpha_i Y_{t-i} + \sum_{h=1}^{n} \beta_h X_{t-h} + \text{residuals.} \quad (6.1) $$

The null hypothesis that $X$ fails to Granger cause $Y$ is tested by testing the null hypothesis $\beta_1=\beta_2=\ldots=\beta_m=0$. Tables 6.1 - 6.5 report the results of testing for a block exogeneity over the period 1957-86. Tables 6.1 - 6.4 report two sets of estimates of first-order vector autoregression and their marginal significance levels associated with the $\chi^2$-statistic pertinent for testing this null hypothesis are presented in Table 6.5.¹

In case 1, the null hypothesis says that in the set of variables RPM,

¹The marginal significance level for the likelihood ratio test is defined as follows: let $X$ be a $\chi^2$ random variables with $q$ (the number of restrictions) degrees of freedom and let $x$ be the computed value of the test statistic. Then the marginal significance level is $\text{prob}(X>x)$ under the null hypothesis. Small values of the marginal significance level indicate that the null hypothesis is doing badly.
Table 6.1. Regression of $Y_t = \sum_{i=1}^{5} \alpha_i Y_{t-1} + \sum_{h=1}^{5} \beta_h X_{t-1}$, $^{a}$

<table>
<thead>
<tr>
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<th>RPW,1</th>
<th>RPF,1</th>
<th>AP,1</th>
<th>CTM,1</th>
<th>CTX,1</th>
<th>$Y_{t-1}$</th>
<th>RCORNR,1</th>
<th>RCTVC,1</th>
<th>RDP,1</th>
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<th>D-W$^{d}$</th>
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<td>.059</td>
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<th>RPF,1</th>
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$^{a}$Y$_t$ is defined as dependent variables, including RPM, RPW, RPF, AP, and CTM, while X$_t$ is defined as exogenous variables, including CTX, $Y_{t-1}$, RCORNR, RCTVC, and RDP.

$^{b}$Standard errors for coefficients appear in parentheses below relevant coefficients.

$^{c}$\sigma is the standard error of estimate.

$^{d}$D-W is the Durbin-Watson statistic.
Table 6.2. Regression of $Y_t = \sum_{i=1}^{5} a_i Y_{t-i} + \sum_{h=1}^{5} b_{h} X_{t-1}$.  

<table>
<thead>
<tr>
<th>CTX</th>
<th>Y-1</th>
<th>RCORNR-1</th>
<th>RCTVC-1</th>
<th>RDP-1</th>
<th>RPM-1</th>
<th>RPW-1</th>
<th>RPF-1</th>
<th>AP-1</th>
<th>CTM-1</th>
<th>$\sigma$</th>
<th>D-W</th>
<th>R²</th>
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<tbody>
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<td>(3.57)</td>
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<td>.247</td>
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<td>(.13)</td>
<td>(.39)</td>
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<th>RCTVC-1</th>
<th>RDP-1</th>
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</thead>
<tbody>
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<td>(.06)</td>
<td>(.06)</td>
<td>(.19)</td>
</tr>
<tr>
<td>Y₁</td>
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<td>.012</td>
<td>-.315</td>
<td>.413</td>
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<td>(.23)</td>
<td>(.68)</td>
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<td>.502</td>
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<td>(.57)</td>
<td>(.17)</td>
<td>(.20)</td>
<td>(.59)</td>
</tr>
<tr>
<td>RCTVC</td>
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<td>-.041</td>
<td>-.109</td>
<td>-.122</td>
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<tr>
<td></td>
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<td>(.05)</td>
<td>(.05)</td>
<td>(.15)</td>
</tr>
<tr>
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<td>-.101</td>
<td>.621</td>
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<tr>
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<td>(.44)</td>
<td>(.13)</td>
<td>(.15)</td>
<td>(.45)</td>
</tr>
</tbody>
</table>

* $Y_t$ is defined as dependent variables, including RPM, RPW, RPF, AP, and CTM, while $X_t$ is defined as exogenous variables, including CTX, $y_1$, RCORNR, RCTVC, and RDP.

* Standard errors for coefficients appear in parentheses below relevant coefficients.

* $\sigma$ is the standard error of estimate.

* D-W is the Durbin-Watson statistic.
Table 6.3. Regression of $Y_t^a = \sum_{i=1}^{2} \alpha_i Y_{t-1}^i + \sum_{h=1}^{5} \beta_h X_{t-1}^h$, \(^b\)

<table>
<thead>
<tr>
<th></th>
<th>AP.1</th>
<th>CTM.1</th>
<th>CTX.1</th>
<th>Y.1</th>
<th>RCORNR.1</th>
<th>RCTVC.1</th>
<th>RDP.1</th>
<th>$\sigma^c$</th>
<th>D-W(^d)</th>
<th>R(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AP</td>
<td>.276</td>
<td>.465</td>
<td>.092</td>
<td>-.105</td>
<td>-.131</td>
<td>.116</td>
<td>-.111</td>
<td>16.3</td>
<td>1.7</td>
<td>.43</td>
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<tr>
<td></td>
<td>(.20)</td>
<td>(.53)</td>
<td>(.22)</td>
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<td>(.08)</td>
<td>(.22)</td>
<td>(.07)</td>
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<tr>
<td>CTM</td>
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<td>4.9</td>
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\(^a\) \(Y_t\) is defined as dependent variables, including AP, and CTM, while \(X_t\) is defined as exogenous variables, including CTX, \(Y_1\), RCORNR, RCTVC, and RDP.

\(^b\) Standard errors for coefficients appear in parentheses below relevant coefficients.

\(^c\) \(\sigma\) is the standard error of estimate.

\(^d\) D-W is the Durbin-Watson statistic.
Table 6.4. Regression of $Y_t = \Sigma a_i Y_{t-1} + \Sigma b_h X_{t-1}$

<table>
<thead>
<tr>
<th></th>
<th>CTX</th>
<th>$y_1$</th>
<th>RCORNR</th>
<th>RCTVC</th>
<th>RDP</th>
<th>AP</th>
<th>CTM</th>
<th>$\sigma^c$</th>
<th>D-W$^d$</th>
<th>$R^2$</th>
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</thead>
<tbody>
<tr>
<td>CTX</td>
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<td>-.113</td>
<td>.006</td>
<td>.346</td>
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<td>1.7</td>
<td>.31</td>
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<td>(.07)</td>
<td>(.20)</td>
<td>(.06)</td>
<td>(.19)</td>
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<td>(.71)</td>
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<td>RCORNR</td>
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<td>.457</td>
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<td>(.19)</td>
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<td>(.16)</td>
<td>(.05)</td>
<td>(.15)</td>
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<td>.77</td>
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<td>(.16)</td>
<td>(.46)</td>
<td>(.15)</td>
<td>(.43)</td>
<td>(1.13)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>CTX</th>
<th>$y_1$</th>
<th>RCORNR</th>
<th>RCTVC</th>
<th>RDP</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CTX</td>
<td>-.184</td>
<td>-.092</td>
<td>-.154</td>
<td>.257</td>
<td>-.097</td>
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</tr>
<tr>
<td></td>
<td>(.19)</td>
<td>(.06)</td>
<td>(.06)</td>
<td>(.19)</td>
<td>(.05)</td>
<td></td>
</tr>
<tr>
<td>$y_1$</td>
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<td>(.20)</td>
<td>(.23)</td>
<td>(.68)</td>
<td>(.18)</td>
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</tr>
<tr>
<td>RCORNR</td>
<td>.087</td>
<td>-.083</td>
<td>.502</td>
<td>-.658</td>
<td>-.006</td>
<td>44.1</td>
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<td>(.17)</td>
<td>(.20)</td>
<td>(.59)</td>
<td>(.16)</td>
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<tr>
<td>RCTVC</td>
<td>-.457</td>
<td>-.041</td>
<td>-.109</td>
<td>-.122</td>
<td>-.139</td>
<td>11.5</td>
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<td>(.05)</td>
<td>(.05)</td>
<td>(.15)</td>
<td>(.04)</td>
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<tr>
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<td>(.13)</td>
<td>(.15)</td>
<td>(.45)</td>
<td>(.18)</td>
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</tbody>
</table>

$^a$Y$_t$ is defined as dependent variables, including AP and CTM, while $X_t$ is defined as exogenous variables, including CTX, $y_1$, RCORNR, RCTVC, and RDP.

$^b$Standard errors for coefficients appear in parentheses below relevant coefficients.

$^c\sigma$ is the standard error of estimate.

$^d$D-W is the Durbin-Watson statistic.
Table 6.5. Results of block exogeneity tests, 1957-1986

<table>
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<tr>
<th>Test</th>
<th>Objective</th>
<th>Test Statistic</th>
<th>Marginal Sig. Lev.</th>
<th>Conclusion*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\beta_1 - \beta_2 - \beta_3 - \beta_4 - \beta_5 = 0$</td>
<td>19.43 17.51</td>
<td>38.30(_{(25)})</td>
<td>.04</td>
<td>reject Ho.</td>
</tr>
<tr>
<td>2. $\alpha_1 - \alpha_2 - \alpha_3 - \alpha_4 - \alpha_5 = 0$</td>
<td>31.11 29.42</td>
<td>33.84(_{(25)})</td>
<td>.11</td>
<td>accept Ho.</td>
</tr>
<tr>
<td>3. $\gamma_1 - \gamma_2 - \gamma_3 - \gamma_4 - \gamma_5 = 0$</td>
<td>8.96 8.11</td>
<td>19.46(_{(10)})</td>
<td>.03</td>
<td>reject Ho.</td>
</tr>
<tr>
<td>4. $\alpha_1 - \alpha_2 = 0$</td>
<td>31.11 30.96</td>
<td>3.63(_{(10)})</td>
<td>.96</td>
<td>accept Ho.</td>
</tr>
</tbody>
</table>

*Critical values: \(\chi^2\_{.05,25} = 37.65; \chi^2\_{.05,10} = 18.31.\)
RPW, RPF, AP, CTM, CTX, y, RCORNR, RCTVC, and RDP lags of neither of the last five affect the first five. Two systems for RPM, RPW, RPF, AP, and CTM are estimated; the restricted one omits the lags of CTX, y, RCORNR, RCTVC, and RDP; the restricted one includes them. The result of case 1 is reported in Table 6.1. The $\chi^2$-statistic pertinent for testing the null hypothesis that lagged values of (CTX, y, RCORNR, RCTVC, and RDP) have zero coefficients in the vector autoregression for (RPM, RPW, RPF, AP, and CTM) has a marginal significance level 0.043266 (see Table 6.5).

In case 2, the null hypothesis says that in the set of variables RPM, RPW, RPF, AP, CTM, CTX, y, RCORNR, RCTVC, and RDP lags of neither of the first five affect the last five. Two systems for CTX, y, RCORNR, RCTVC, and RDP are estimated, the restricted one omits the lags of RPM, RPW, RPF, AP, and CTM, the unrestricted one includes them. The $\chi^2$-statistic pertinent for testing the hypothesis that lagged values of (RPM, RPW, RPF, AP, and CTM) have zero coefficients in the vector autoregression for (CTX, y, RCORNR, RCTVC, and RDP) has a marginal significance level of 0.11144 (see Table 6.5). In cases 3 and 4 eliminate price variables. The results of applying Granger's test for case 3 and case 4 are shown in Tables 6.3 - 6.5. The estimates in Tables 6.1 - 6.4 came from the data that are residuals from regressions on constant, trend, and trend squared. The data were described more in section 6.2 below. These results shown in Table 6.5 indicate that (CTX, y, RCORNR, RCTVC, and RDP) seems not to be Granger caused by (RPM, RPW, RPF, AP, and CTM) while (RPM, RPF, RPW, AP, and CTM) is Granger caused by (CTX, y, RCORNR, RCTVC, and RDP). The exogeneity results suggest that (CTX, y, RCORNR, RCTVC, and
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DRP_t) are econometrically exogenous. It is equivalent to say \((\text{CTX}_t, y_t,\ RCONRR_t, \ RCTVC_t, \text{and RDP}_t)\) are orthogonal to the error \(a_s\) at all \(t\) and \(s\).

6.2 A VAR Model

Before estimating the model, all the data series were detrended by regressing them on a constant, linear trend and trend squared, or any two of them, in accordance with their statistics. The reason for detrending in this way prior to fitting the model is that the model neglects the effects of inventory on production and mill demand consumption, except to the extent that this can be captured by the demand shock \((u_{1t})\). Further, the implication of the prediction theory is any deterministic components of the jointly covariance stationary processes will not refer to the same distributed lag model as are their indeterministic parts (Sargent 1978a). Thus, detrending prior to estimation is to remove the deterministic components from these processes for implementing the Wiener-Kolmogorov prediction formula.

The residuals derived from the trend regressions are used as the data for the estimation model. The estimates and residuals from regressions on constant, trend and trend squared are reported in Tables 6.6 and 6.7.

For implementing the first-step estimation process, Sims' VAR approach is employed to generate forecasting elements (the parameters of \(v\)'s in equation [5.1]). The VAR approach has the advantage of being capable of reflecting persistence effects in the data, i.e., trends, cycles, etc.

Let \(X_t\) be a \((q\times 1)\)-vector stationary stochastic process governed by
Table 6.6. Estimates of the detrending model of the U.S. cotton industry, 1955-1986$^a$, $^b$

<table>
<thead>
<tr>
<th>Model</th>
<th>Equation</th>
<th>$R^2$</th>
<th>SEE</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGNP</td>
<td>$448.04 + 6.29$ Trend</td>
<td>0.99</td>
<td>5.88</td>
<td>0.98*</td>
</tr>
<tr>
<td></td>
<td>(103.13) (55.88)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RWHNR</td>
<td>$-71.37 - 10.82$ Trend $- 0.17$ Trendsq</td>
<td>0.24</td>
<td>37.75</td>
<td>1.27**</td>
</tr>
<tr>
<td></td>
<td>(-0.6) (-1.64) (-1.90)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RCORNR</td>
<td>$-208.50 - 23.54$ Trend $- 0.37$ Trendsq</td>
<td>0.51</td>
<td>50.11</td>
<td>0.98*</td>
</tr>
<tr>
<td></td>
<td>(1.32) (-2.68) (-3.19)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RCTVC</td>
<td>$236.03$</td>
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<td>14.33</td>
<td>2.17</td>
</tr>
<tr>
<td></td>
<td>(93.19)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RPM</td>
<td>$88.95 - 2.40$ Trend</td>
<td>0.81</td>
<td>11.16</td>
<td>1.44**</td>
</tr>
<tr>
<td></td>
<td>(10.78) (-11.26)</td>
<td></td>
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</tr>
<tr>
<td>RPW</td>
<td>$33.62 - 1.21$ Trend</td>
<td>0.32</td>
<td>16.73</td>
<td>1.34**</td>
</tr>
<tr>
<td></td>
<td>(2.72) (-3.77)</td>
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</tbody>
</table>

$^a$T-statistics for coefficients appear in parentheses below relevant coefficients.

$^b$DW is based on the Durbin-Watson statistic (d). Reject the nonautoregressive hypothesis if $d < d_L$; indeterminate solution if $d_L < d < d_U$, where $d_L$ and $d_U$ are the lower and upper limits for the significant levels of d.

*DW < $d_L$.

**$d_L < DW < d_U$. 

°D-statistics for coefficients appear in parentheses below relevant coefficients.
Table 6.6. (Continued)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equation</th>
<th>R²</th>
<th>SEE</th>
<th>DW</th>
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</thead>
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<tr>
<td>RPF</td>
<td>24.36 - 1.33 Trend</td>
<td>0.54</td>
<td>11.60</td>
<td>0.96*</td>
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<tr>
<td></td>
<td>(2.85) (-5.99)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RDP</td>
<td>-353.51 - 24.78 Trend</td>
<td>0.19</td>
<td>64.42</td>
<td>0.33*</td>
</tr>
<tr>
<td></td>
<td>(-1.74) (-7.20) (-2.33)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AP</td>
<td>98.62 + 0.02 Trend²</td>
<td>0.39</td>
<td>19.43</td>
<td>0.96*</td>
</tr>
<tr>
<td></td>
<td>(12.15) (4.42)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>YD</td>
<td>629.18 + 3.87 Trend</td>
<td>0.36</td>
<td>49.06</td>
<td>1.80</td>
</tr>
<tr>
<td></td>
<td>(17.34) (4.12)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CTX</td>
<td>134.70 + 4.46 Trend + 0.06 Trend²</td>
<td>0.08</td>
<td>17.59</td>
<td>1.88</td>
</tr>
<tr>
<td></td>
<td>(2.43) (1.45) (1.35)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CTM</td>
<td>32.06 - 1.19 Trend</td>
<td>0.69</td>
<td>7.50</td>
<td>0.71*</td>
</tr>
<tr>
<td></td>
<td>(5.78) (-8.26)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RERJ</td>
<td>122.94 + 0.16 Trend²</td>
<td>0.91</td>
<td>37.28</td>
<td>0.39*</td>
</tr>
<tr>
<td></td>
<td>(7.89) (17.22)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RPOLY</td>
<td>522.58 + 34.58 Trend + 0.64 Trend²</td>
<td>0.97</td>
<td>25.57</td>
<td>0.44*</td>
</tr>
<tr>
<td></td>
<td>(6.48) (7.72) (10.79)</td>
<td></td>
<td></td>
<td></td>
</tr>
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</table>
Table 6.7. Residuals of the detrending model of the U.S. cotton industry, 1955-1986

<table>
<thead>
<tr>
<th>YEAR</th>
<th>RGNP</th>
<th>RCORN</th>
<th>RCTVC</th>
<th>RPM</th>
<th>RPW</th>
<th>RPF</th>
<th>RDP</th>
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<tbody>
<tr>
<td>1956</td>
<td>-1.0319</td>
<td>53.4827</td>
<td>28.3741</td>
<td>2.9787</td>
<td>2.7316</td>
<td>0.4672</td>
<td>7.6982</td>
</tr>
<tr>
<td>1957</td>
<td>-5.4821</td>
<td>1.2541</td>
<td>-49.9264</td>
<td>-4.0988</td>
<td>3.8457</td>
<td>-6.6919</td>
<td>-3.4290</td>
</tr>
<tr>
<td>1959</td>
<td>-0.3700</td>
<td>-29.0775</td>
<td>3.4276</td>
<td>2.8498</td>
<td>-2.3904</td>
<td>0.2079</td>
<td>-23.5919</td>
</tr>
<tr>
<td>1964</td>
<td>-2.2420</td>
<td>-43.2547</td>
<td>-4.7273</td>
<td>-7.2579</td>
<td>0.5008</td>
<td>5.2332</td>
<td>-14.0717</td>
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<tr>
<td>1967</td>
<td>4.5230</td>
<td>49.9411</td>
<td>-0.8728</td>
<td>0.6286</td>
<td>-16.4270</td>
<td>-7.0736</td>
<td>72.4168</td>
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<tr>
<td>1971</td>
<td>-2.5850</td>
<td>5.0758</td>
<td>-8.6049</td>
<td>-10.4713</td>
<td>-4.0001</td>
<td>-7.0220</td>
<td>83.8028</td>
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<tr>
<td>1972</td>
<td>2.0166</td>
<td>-7.3105</td>
<td>-11.2260</td>
<td>7.3250</td>
<td>-2.3229</td>
<td>-10.5770</td>
<td>75.8498</td>
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<tr>
<td>1973</td>
<td>7.1400</td>
<td>-27.7925</td>
<td>-17.4513</td>
<td>18.0230</td>
<td>58.0098</td>
<td>18.1487</td>
<td>57.0615</td>
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<td>1974</td>
<td>-0.5411</td>
<td>105.2840</td>
<td>1.9333</td>
<td>24.2483</td>
<td>-4.6465</td>
<td>2.5567</td>
<td>-85.9267</td>
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<tr>
<td>1975</td>
<td>-9.6220</td>
<td>144.8930</td>
<td>11.4428</td>
<td>-19.0741</td>
<td>15.7104</td>
<td>10.7080</td>
<td>84.5051</td>
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<tr>
<td>1976</td>
<td>-4.6622</td>
<td>72.6525</td>
<td>8.6355</td>
<td>23.5273</td>
<td>31.7859</td>
<td>27.2403</td>
<td>82.3864</td>
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<tr>
<td>1977</td>
<td>0.3906</td>
<td>37.0040</td>
<td>-1.0895</td>
<td>19.3303</td>
<td>1.9594</td>
<td>6.8808</td>
<td>-79.5706</td>
</tr>
<tr>
<td>1979</td>
<td>7.7744</td>
<td>-47.4575</td>
<td>-2.1757</td>
<td>-0.0097</td>
<td>12.9278</td>
<td>8.4483</td>
<td>71.8474</td>
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<tr>
<td>1981</td>
<td>0.2024</td>
<td>18.1088</td>
<td>19.2729</td>
<td>4.6949</td>
<td>-10.8177</td>
<td>-10.4697</td>
<td>-27.5904</td>
</tr>
<tr>
<td>1984</td>
<td>1.8700</td>
<td>30.4733</td>
<td>5.4823</td>
<td>-7.1467</td>
<td>-10.1921</td>
<td>-6.1666</td>
<td>3.3349</td>
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<tr>
<td>1986</td>
<td>5.3151</td>
<td>-4.7649</td>
<td>3.9210</td>
<td>-7.3226</td>
<td>-12.5326</td>
<td>-6.7790</td>
<td>120.4550</td>
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Table 6.7. (Continued)

<table>
<thead>
<tr>
<th>YEAR</th>
<th>AP</th>
<th>YD</th>
<th>CTX</th>
<th>CTM</th>
<th>RERJ</th>
<th>RPOLY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959</td>
<td>7.3892</td>
<td>21.2363</td>
<td>24.8578</td>
<td>0.1197</td>
<td>32.7606</td>
<td>17.8580</td>
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<tr>
<td>1960</td>
<td>11.9730</td>
<td>2.3706</td>
<td>20.4249</td>
<td>-5.4152</td>
<td>35.5886</td>
<td>17.1358</td>
</tr>
<tr>
<td>1963</td>
<td>5.6828</td>
<td>61.7736</td>
<td>11.6626</td>
<td>1.5701</td>
<td>4.5122</td>
<td>59.1849</td>
</tr>
<tr>
<td>1965</td>
<td>2.6080</td>
<td>64.0423</td>
<td>-14.9187</td>
<td>12.9403</td>
<td>0.6943</td>
<td>16.2365</td>
</tr>
<tr>
<td>1969</td>
<td>-12.9444</td>
<td>-44.4203</td>
<td>-16.1788</td>
<td>2.8608</td>
<td>34.0027</td>
<td>-34.5769</td>
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<tr>
<td>1970</td>
<td>-10.6365</td>
<td>-44.2859</td>
<td>-6.1879</td>
<td>4.9459</td>
<td>40.4922</td>
<td>-35.3085</td>
</tr>
<tr>
<td>1973</td>
<td>-0.5142</td>
<td>26.1171</td>
<td>14.8110</td>
<td>1.1812</td>
<td>-44.6420</td>
<td>-24.2494</td>
</tr>
<tr>
<td>1976</td>
<td>-4.5741</td>
<td>-40.4798</td>
<td>0.8357</td>
<td>-3.2435</td>
<td>-31.6997</td>
<td>5.3436</td>
</tr>
<tr>
<td>1978</td>
<td>15.5180</td>
<td>-93.2111</td>
<td>11.0668</td>
<td>-4.0932</td>
<td>-83.3556</td>
<td>7.5955</td>
</tr>
<tr>
<td>1979</td>
<td>22.8337</td>
<td>29.9232</td>
<td>40.3620</td>
<td>-1.3681</td>
<td>-51.9233</td>
<td>9.9665</td>
</tr>
<tr>
<td>1982</td>
<td>0.0993</td>
<td>61.3262</td>
<td>-4.1059</td>
<td>-7.7528</td>
<td>37.4868</td>
<td>12.0942</td>
</tr>
<tr>
<td>1984</td>
<td>0.2784</td>
<td>63.5949</td>
<td>2.5890</td>
<td>-5.1026</td>
<td>49.9188</td>
<td>6.3954</td>
</tr>
<tr>
<td>1985</td>
<td>-3.2974</td>
<td>89.7293</td>
<td>-41.8195</td>
<td>4.6726</td>
<td>62.5046</td>
<td>-8.6266</td>
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<td>1986</td>
<td>-8.7168</td>
<td>7.8636</td>
<td>3.2114</td>
<td>15.8677</td>
<td>-0.2478</td>
<td>-16.1926</td>
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</table>
the following difference equations

\[ V(L) X_t = \epsilon_t, \quad (6.2) \]

where \( \epsilon_t \) is a (q*1) vector of white noises with mean zero and contemporaneous covariance matrix \( E\epsilon_t\epsilon_t' = \nu \), where \( \nu \) is a q*q matrix. And assume \( E\epsilon_t\epsilon_{t-s} = O_{q,q} \) for all \( s \neq 0 \). In (6.7), \( V(L) \) is a q*q matrix of (finite order) polynomials in

\[
V(L) = \begin{bmatrix} v_{11}(L) & v_{12}(L) & \ldots & v_{1q}(L) \\ v_{21}(L) & v_{22}(L) & \ldots & v_{2q}(L) \\ \vdots & \vdots & \ddots & \vdots \\ v_{q1}(L) & v_{q2}(L) & \ldots & v_{qq}(L) \end{bmatrix},
\]

where each \( v_{ij}(L) \) is a finite order polynomial in the lag operator. The vector stochastic difference equation \( V(L) X_t = \epsilon_t \) is to be called an autoregressive representation for the vector process \( X_t \).

VAR are projection equations, they need not require structural relations, since VAR can allow us to examine economic relationships among the variables. Also, the approach can capture a representation of the actual processes followed by the economic time series.

In the VAR analysis, the eight variables \( \{CTX_t, y_{1t}, RCORNR_t, RCIVC_t, RDP_t, RPOLY_t, RGP_t, \text{ and } RERJ_t \} \) are included to capture the information available to the firms at their planting and milling decisions. The value of each of the eight variables reflects the response of its own past, the past realizations of the other key variables, and a current period shock.

The first-order process is considered in the VAR analysis, since the number observations over the period, 1955-1986, is inadequate for employing longer lag lengths due to degree of freedom problems. Although
the number observations can be expanded to a longer period, we found that the behavior of the data over the 1920-1986 period do not have a constant structure. The results suggest the data series are not stationary. Especially, data on the exchange rates for Japan after the Second World War have an obviously shift in its structure. In order for the time series method to be applicable, a shorter period after World War II is chosen. Usually, the first lag is the most important factor to respond to the nature of economic variables. Therefore, only one lag is conducted for each variable to maintain a fairly thrift specification in the VAR model.

Table 6.8 reports the estimates of the VAR model. Casual examination of the estimates obtained from the vector autoregressive system shows that much of the variation in a variable is explained by the past realizations of that variable, except for $y_{1t}$ and $RCTV_{Ct}$ which have t-statistics around 1.2. These t-statistics can be interpreted to mean that past realizations of the variable are mildly though not powerfully significant in explaining its own variation. Another reason is that most economic variables are related so that multicollinearity causes an insignificant coefficient problem.

The coefficient estimates in a VAR model are difficult to interpret, but a moving average representation (MAR) generated by a VAR makes it easy to analyze the dynamic interactions among the variables in the model. A MAR is generated by impulse responses of a VAR. Impulse response functions reflect the dynamic response of each endogenous variable to a shock to the system. An alternative, the decomposition of variance for a
Table 6.8. Estimates of the VAR model

<table>
<thead>
<tr>
<th>Variables</th>
<th>CTX</th>
<th>$y_1$</th>
<th>RCORNR</th>
<th>RCTVC</th>
<th>RDP</th>
<th>RPOLY</th>
<th>RGNP</th>
<th>RERJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CTX_{t-1}$</td>
<td>-0.390*</td>
<td>-0.075</td>
<td>-0.101</td>
<td>-0.372*</td>
<td>-0.704</td>
<td>0.027</td>
<td>-0.075</td>
<td>0.605*</td>
</tr>
<tr>
<td>$y_{1t-1}$</td>
<td>-0.024</td>
<td>-0.247</td>
<td>-0.056</td>
<td>-0.089*</td>
<td>-0.050</td>
<td>0.002</td>
<td>0.005</td>
<td>-0.103</td>
</tr>
<tr>
<td>RCORNR$_{t-1}$</td>
<td>-0.254**</td>
<td>0.104</td>
<td>0.497*</td>
<td>-0.031</td>
<td>0.019</td>
<td>-0.094</td>
<td>-0.012</td>
<td>0.082</td>
</tr>
<tr>
<td>RCTVC$_{t-1}$</td>
<td>0.371*</td>
<td>-0.080</td>
<td>-0.611</td>
<td>-0.182</td>
<td>0.551</td>
<td>-0.068</td>
<td>-0.067</td>
<td>0.148</td>
</tr>
<tr>
<td>DP$_{t-1}$</td>
<td>-0.171</td>
<td>0.325</td>
<td>0.176</td>
<td>-0.018</td>
<td>0.624**</td>
<td>-0.211**</td>
<td>0.017</td>
<td>0.081</td>
</tr>
<tr>
<td>RPOLY$_{t-1}$</td>
<td>-0.294</td>
<td>1.381*</td>
<td>0.265</td>
<td>0.341*</td>
<td>0.078</td>
<td>0.391**</td>
<td>0.031</td>
<td>0.264</td>
</tr>
<tr>
<td>RGNP$_{t-1}$</td>
<td>0.020</td>
<td>-1.186</td>
<td>-0.471</td>
<td>0.083</td>
<td>2.846*</td>
<td>-0.667</td>
<td>0.424*</td>
<td>0.354</td>
</tr>
<tr>
<td>RERJ$_{t-1}$</td>
<td>-0.125</td>
<td>0.285</td>
<td>-0.371</td>
<td>-0.031</td>
<td>0.632*</td>
<td>0.027</td>
<td>-0.031</td>
<td>0.914**</td>
</tr>
</tbody>
</table>

*Significant at 5% level.
**Significant at 1% level.
VAR model, provides information equivalent to that contained in the VAR estimates, but easier to interpret.

Figures 6.1.1 - 6.1.8 show the vector MAR implied by the VAR model. Tables 6.9 and 6.10 show the corresponding covariance/correlation matrix of the innovations and a decomposition of variance of the 30-year forecast error variance. Variance decomposition explains the total proportion of the prediction error variance that is attributable to each variable in the system. One problem with applying the decomposition is that there is no unique way in ordering the variables. For illustrative purposes, we present the results with the ordering implied by the CTM equation (5.1); the orthogonalization order is CTX, y₁, RCRNR, RCTVC, RDP, RPOLY, RGNP, and RERJ.

Figures 6.1.1 - 6.1.8 show that the response to its own innovation generally yields a strong, sustained decrease in its value and a relatively small, sustained change in other variables. For example, a one-standard deviation innovation in CTX yields a sizable, sustained decrease in CTX and the other variables except the RCRNR variable change slightly and gradually converge to some small values. The reason is that exports of cotton have a closely negative relationship with the net returns of corn. Thus, a one-standard deviation shock in exports of cotton will have cause a significant decrease in the net return of corn that conforms with the pertinent theory.

Upon examining the response of the system equations to a one-standard deviation shock in the three variables RPOLY, RGNP, and RERJ individually, the dynamic responses of these variables to the shock were found in a
Figure 6.1.1. Responses to one-standard deviation shock in CTX
Figure 6.1.2. Responses to one-standard deviation shock in $y_1$
Figure 6.1.3. Responses to one-standard deviation shock in RCORNR
Figure 6.1.4. Responses to one-standard deviation shock in RCTVC
Figure 6.1.5. Responses to one-standard deviation shock in RDP
Figure 6.1.6. Responses to one-standard deviation shock in RPOLY
Figure 6.1.7. Responses to one-standard deviation shock in RGDP
Figure 6.1.8. Responses to one-standard deviation shock in RERJ
Table 6.9. Covariance / correlation matrix of the VAR model, 1957-1986

<table>
<thead>
<tr>
<th>Variables</th>
<th>CXT</th>
<th>$y_1$</th>
<th>RGNR</th>
<th>RCTVC</th>
<th>DP</th>
<th>RNPY</th>
<th>RGNP</th>
<th>RERJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>CXT</td>
<td>139.06</td>
<td>-0.03</td>
<td>-0.47</td>
<td>-0.20</td>
<td>0.05</td>
<td>0.17</td>
<td>0.22</td>
<td>-0.28</td>
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<tr>
<td>$y_1$</td>
<td>-15.79</td>
<td>1570.40</td>
<td>-0.08</td>
<td>-0.19</td>
<td>0.15</td>
<td>0.19</td>
<td>0.20</td>
<td>0.18</td>
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<tr>
<td>RGNR</td>
<td>-221.00</td>
<td>-127.98</td>
<td>1558.20</td>
<td>0.16</td>
<td>-0.39</td>
<td>-0.11</td>
<td>-0.17</td>
<td>0.06</td>
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<td>RCTVC</td>
<td>-22.01</td>
<td>-69.87</td>
<td>61.65</td>
<td>90.22</td>
<td>-0.22</td>
<td>0.20</td>
<td>-0.43</td>
<td>0.31</td>
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<tr>
<td>DP</td>
<td>15.44</td>
<td>162.64</td>
<td>-406.25</td>
<td>-54.97</td>
<td>710.88</td>
<td>-0.40</td>
<td>0.29</td>
<td>0.13</td>
</tr>
<tr>
<td>RNPY</td>
<td>17.42</td>
<td>64.28</td>
<td>-36.97</td>
<td>16.42</td>
<td>-93.78</td>
<td>76.09</td>
<td>-0.22</td>
<td>-0.39</td>
</tr>
<tr>
<td>RGNP</td>
<td>11.01</td>
<td>33.51</td>
<td>-28.18</td>
<td>-17.37</td>
<td>32.92</td>
<td>-8.21</td>
<td>18.05</td>
<td>-0.39</td>
</tr>
<tr>
<td>RERJ</td>
<td>-64.23</td>
<td>138.96</td>
<td>45.24</td>
<td>56.79</td>
<td>65.20</td>
<td>9.81</td>
<td>-31.62</td>
<td>366.69</td>
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</table>
Table 6.10. Variance decompositions for forecast errors implied by the VAR model for (CTX, y, RCORNR, RCTVC, DP, RPOLY, RGNP, RERJ), 1957-1986*

<table>
<thead>
<tr>
<th></th>
<th>CTX</th>
<th>y</th>
<th>RCORNR</th>
<th>RCTVC</th>
<th>RDP</th>
<th>RPOLY</th>
<th>RGNP</th>
<th>RERJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orthogonalization</td>
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<tr>
<td>x = CTX</td>
<td>58.44</td>
<td>2.33</td>
<td>17.88</td>
<td>4.50</td>
<td>6.30</td>
<td>2.44</td>
<td>0.42</td>
<td>7.88</td>
</tr>
<tr>
<td>x = y</td>
<td>0.46</td>
<td>73.18</td>
<td>0.26</td>
<td>4.46</td>
<td>4.52</td>
<td>5.97</td>
<td>3.94</td>
<td>6.21</td>
</tr>
<tr>
<td>x = RCORNR</td>
<td>19.28</td>
<td>1.45</td>
<td>66.35</td>
<td>4.48</td>
<td>2.12</td>
<td>0.25</td>
<td>0.83</td>
<td>5.24</td>
</tr>
<tr>
<td>x = RCTVC</td>
<td>9.52</td>
<td>10.20</td>
<td>3.46</td>
<td>56.41</td>
<td>8.64</td>
<td>3.88</td>
<td>1.50</td>
<td>6.38</td>
</tr>
<tr>
<td>x = DP</td>
<td>4.37</td>
<td>2.63</td>
<td>6.05</td>
<td>4.85</td>
<td>37.10</td>
<td>1.87</td>
<td>3.06</td>
<td>40.07</td>
</tr>
<tr>
<td>x = RPOLY</td>
<td>6.11</td>
<td>3.16</td>
<td>1.31</td>
<td>6.61</td>
<td>39.23</td>
<td>11.57</td>
<td>7.59</td>
<td>24.42</td>
</tr>
<tr>
<td>x = RGNP</td>
<td>3.43</td>
<td>7.28</td>
<td>2.02</td>
<td>17.76</td>
<td>6.03</td>
<td>1.43</td>
<td>59.48</td>
<td>2.59</td>
</tr>
<tr>
<td>x = RERJ</td>
<td>4.35</td>
<td>1.43</td>
<td>2.99</td>
<td>15.23</td>
<td>4.09</td>
<td>1.35</td>
<td>6.87</td>
<td>63.71</td>
</tr>
</tbody>
</table>

*Percentage of 30-year forecast error variance in x accounted for by "orthogonalized innovation" in CTX, y, RCORNR, RCTVC, DP, RPOLY, RGNP, RERJ.
significant pattern. Also, the magnitude of the response to the shock is of substantial economic significance. The finding suggests that the three variables help predict the relevant state variables. The evidence is consistent with economic theory that cotton mill consumption is influenced by the price of the competing fibers, income, and exports. Exchange rates are also an important factor influencing exports. Thus the three variables RPOLY, RGNP, and RERJ should be included in the CTM decision rule.

Table 6.10 shows that the largest proportion of the forecast error variance appears along the diagonal. It indicates that most of the forecast error variance in a variable is explained by that variable's own past, except RDP and RPOLY. The decomposition of variance in RDP shows that only 37 percentage of the variance is explained by its own innovations at thirty steps, 40 percentage of the 30-year forecast-error variance accounted for by RERJ, and the other variables accounted for in the range of 1.9% to 6.0%. Moreover, only 11.6% of the 30-year forecast error variance in RPOLY is explained by its own innovations, but 40% forecast error explained by the innovation in DP and 24% by the innovation in RERJ. Also, the decomposition of variances in RGNP and RERJ shows the forecast errors are explained by their own innovation around 60% for RGNP and 63% for RERJ and 18% and 15% of the variances are accounted for by the innovation in RCTVC. Clearly, these variables are not exogenous due to the proportion of their forecast errors not only explained by their own innovation but also by other variables.

Tables 6.9 and 6.10 suggest that there is a significant dynamic
interaction among the structural variables and the nonstructural variables. In order to confirm this finding, a block exogeneity test was performed. The null hypothesis is that in the set of variables CTX, y1, RCORN, RCTVC, RDP, RPOLY, RG, and RERJ lags of neither of the last three affect the first five. Two systems for CTX, y1, RCORN, RCTVC, and RDP are estimated; the restricted one omits the lags of RPOLY, RG, and RERJ; the unrestricted one includes them. The likelihood ratio statistic pertinent for testing the null hypothesis that lags of neither of the three variables (RPOLY, RG, and RERJ) have zero coefficients is 42.05283. Since this statistic is asymptotically distributed as $\chi^2$ with twenty-one degrees of freedom under the null hypothesis, the marginal significance level is 0.0041. This $\chi^2$-statistic is consistent with the economic theory and indicates that the data contain strong evidence of Granger causality flowing from the last three variables to the first five variables. In other words, the last three variables help predict the future movements of the first five variables.

The estimated VAR model presented in Table 6.8 appears to significantly reflect an economy-wide cyclical movement that is assumed to be exogenous to the U.S. cotton market. The stability of the VAR model is examined by the roots of $|V - mI| = 0$ are less than one in the absolute value, where $V$ is the VAR estimates and $m$ is the characteristic roots of $V$. If all the roots of $|V - mI| = 0$ are less than one in the absolute value, it indicates all the variables in the VAR model are stationary. The characteristic roots for the VAR model are -0.0332, -0.2129, -0.2129,
0.2626, 0.4514, 0.811, and 0.811. The roots are less than one in the absolute value and indicate these relevant state variables are stationary.

6.3 Estimation of the Dynamic Mill Demand Model

Ignoring the restrictions on the error term of the demand equation (5.1), the equation can be written in a compact form as

$$CTM(t) = \theta_1 CTM(t-1) + D(L)X(t) + e(t), \quad (6.3)$$

where $e(t) = \epsilon_1 + \epsilon_2$, $D(L) = D_1 - D_2 L$, and

$$D_1 = -m \frac{1 - \tau m_2}{DF_{11}} \left[ (d_2 + \theta_1 + \theta_2^2 d_2)(\phi_1 - F_{12} \phi_2) \right.$$

$$+ n \theta_1 f_{11}(f_{21} \phi_3 + \phi_4 - f_{11} \phi_5) \right]$$

$$\times V(b \theta_1)^{-1}(I + \sum_{r=1}^{q-1} \sum_{i=r+1}^{q} (b \theta_1)^{i-r} v_{11} L^r)$$

$$\left. + \frac{d_{2m}}{DF_{11}} \frac{1 - \tau m_2}{m_1} (\phi_1 - F_{12} \phi_2); \right.$$}

$$D_2 = \frac{\theta_1 d_{2m}}{DF_{11}} \frac{1 - \tau m_2}{m_1} (\phi_1 - F_{12} \phi_2).$$

With the assumptions that $E(e_t) = 0$ and $E(e_t e_t') = \sigma^2 I$, the equation is nonlinear in its parameters and can be estimated by using the method of nonlinear least squares. The method of maximum likelihood is not appropriate for equation (6.3) since the error term of the equation is not normally distributed. Under the conditions on the series $X_t$, on the function $f(X_t, \Theta)$, and for independently, identically distributed the $e_t$ with zero mean and constant variance, it can be shown that the nonlinear LS estimator is consistent and asymptotically normally distributed even if the error distribution is nonnormal (Judge, G. G. et al., 1982). The
literature contains different sets of sufficient conditions to assure the consistency and asymptotic normality of nonlinear LS estimates (Malinvaud 1970; Fuller 1976).

The criterion used for determining the estimated values for the parameter vector $\theta$ is the same as that for linear models: minimization of the sum of squared errors. The sum of squared errors for the nonlinear CTM model can be written as

$$s(\theta) = [\text{CTM}_t - (\theta_1 \text{CTM}_{t-1} + D(L)X_t)]' [\text{CTM}_t - (\theta_1 \text{CTM}_{t-1} + D(L)X_t)].$$

(6.4)

For minimization of (6.4), a problem arises in that the nonlinear parameters make analytical solutions intractable.

Several approaches can be used to obtain a numerical approximation to the solution of the nonlinear normal equation. In this study, the "forward difference" method was used to compute gradients with a Davidon-Fletcher-Powell algorithm for updating the Hessian from the GAUSS package. The complicated nonlinear structure of the mill demand model makes the analytic gradients burdensome, so the derivative-free method with a DFP algorithm was used to update the Hessian matrix, rather than use Newton or Gauss algorithms. The algorithm for computing "forward difference" of gradients can be found in "Numerical Methods for Unconstrained Optimization and Nonlinear Equations" authored by Dennis and Schnabel (1983).

To achieve convergence, the tolerance levels on the coefficient estimates and the gradient were set $10^{-4}$ in GAUSS. If the relative change in successive estimates of the parameters is less than $10^{-4}$, then the program tests if relative gradients < $10^{-4}$. If both tests are true, then convergence is assumed, and the program stops.
For the purpose of making estimation tractable, the parameters b, n, 
m, \( \alpha \), and part were fixed \textit{a priori}. The discount factor was fixed at 0.95 
by assuming a constant interest rate of 5.26%. The number of farms 
harvesting cotton declined dramatically from 89,536 in 1974 to 52,638 in 
1978 and to 38,266 in 1982 (U.S. Cotton Industry). The number of farms is 
obtained from the average number of cotton farms for the three years since 
the limit of data available. The number of textile mills is based on the 
average of two year data 7,794 in 1977 and 6,630 in 1982. The set-aside 
or diversion program is fixed at a constant value 0.2. Participation rate 
in the cotton program was fixed at 0.75. Thus, \( \alpha_p \) can be computed as 
following

\[
\alpha_p = \frac{\alpha \times \text{part}}{1 - (1 - \alpha) \times \text{part}} = 0.71.
\]

Before estimation, some transformations were made to convert 
constrained variables into unconstrained ones. For instance, to get \(|\theta_1| < 1\), use the transformation

\[
\theta_1 = \frac{1}{1 + \theta_{1e} \times \theta_{1e}};
\]

where \( \theta_{1e} \) is an unconstrained parameter. Another change made was to 
constrain all the parameters to be positive except \( d_1 \) and \( d_2 \); this is done 
by making the parameter equal to the square of the parameter actually 
computed.

Substituting the estimates of VAR (see Table 6.8) into the mill 
demand decision rule (6.3) for \( V \), then the decision rule was estimated by 
using aggregate data from U.S., 1957-1986. The results were 
unsatisfactory from magnitudes of the estimated coefficients, but they
were statistically significant with the proper sign except $d_1$ and met the restrictions imposed on the RE model.

To improve estimation and obtain reasonable estimated coefficients, the mill demand model was estimated as follows: first, $f_{11}$, $f_{12}$, $\beta$, $m_1$, and $m_2$ were obtained by estimating the cotton aggregate production equation (4.32a), the price linkage equation (4.6), and the derived mill demand equation for cotton (4.12) separately, using the 2SLS method. Second, $f_{11}$, $f_{12}$, $\beta$, $m_1$, and $m_2$ were substituted into the closed form of the mill demand equation and the nonlinear LS method was used to obtain the remaining estimates with a priori fixed parameters ($b = 0.95$, $n = 60143$, $m = 7212$, $\alpha = 0.80$, part = 0.75, and $r = 0.81$) and the parameters of VAR. The two-step estimators are appropriate and consistent on the assumption of the mill demand model (6.3). Although the estimated coefficients are not fully efficient, they make sense and are of reasonable magnitudes.

The estimated parameters (see Table 6.11) satisfy the restrictions that the model imposed on the agent's optimization problems; that is $|\theta_1| < 1$, $|w| \geq \min(|\phi_1'|, |bw|)$, and the roots of $|V - mI| = 0$ lie outside the unit circle. The values of $\theta_1$ and $d_2$ derived from the estimation are consistent with the adjustment cost effects in production. The negative sign on $d_2$ suggests that the producer will continue to grow the crop since the average productivity of land is increasing due to the plot preparation for cotton in the previous year. Furthermore, the restriction $|d_1 / d_2| > 1 + b$ on the farm level was substituted into the demand equation before estimating the equation. The restriction (5.6) derived from REH were too complicated to carry out on the mill demand estimation. Consequently, the
Table 6.11. First solution of the mill demand equation, 1957-1986

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>First-step (two-stage least square)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f_{11} )</td>
<td>1.0368</td>
<td>17.4717</td>
</tr>
<tr>
<td>( f_{12} )</td>
<td>0.2646</td>
<td>17.0736</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.6756</td>
<td>7.0042</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>-0.4147</td>
<td>-1.7031</td>
</tr>
<tr>
<td>( m_1 )</td>
<td>0.1474</td>
<td>0.4013</td>
</tr>
<tr>
<td>Second-step (nonlinear LS method)(^a, b, c)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>0.4418</td>
<td>3.8688</td>
</tr>
<tr>
<td>( h_2 )</td>
<td>7.2810</td>
<td>5.8714</td>
</tr>
<tr>
<td>( d_2 )</td>
<td>-10.0359</td>
<td>-7.6077</td>
</tr>
<tr>
<td>( f_{21} )</td>
<td>0.000065</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

\( ^a \)The sum of squared errors = 610.6221; \( R^2 = 0.6261 \); the standard deviation of CTM = 4.8462.

\( ^b \)\( \alpha = .80 \), part = .75, \( n = 60143 \), \( m = 7212 \), \( b = .95 \), \( r = .81 \) imposed a priori.

\( ^c \)The starting values: (1.0, 3.0, -20.0, 0.3).
solutions satisfy the restriction.

The estimates reported in Table 6.11 seem reasonable from a theoretical point of view and have acceptable magnitudes.

If another set of initial values $\theta_0$ yields another set of solutions of the normal equation (see Tables 6.12 and 6.13), multiple minimum are present. Therefore, the solution presented in Table 6.11 does not guarantee that this vector is the global minimum of $s(\theta)$. One way to check is to repeat the estimation starting with a different set of initial guesses for the coefficients.

The finding of the presence of multiple minima of the nonlinear LS equation suggests that the test statistics reported must be interpreted with caution. The estimates of Table 6.11 and Table 6.13 differ considerately while the residual sum of squares is nearly different.

6.4 REH Testing

To test the restrictions implied by the REH one can use either a likelihood ratio test comparing the likelihoods of RE model with an unrestricted model or a Wald test. The two procedures are discussed in Revankar (1980) and Hoffman and Schmidt (1981). A likelihood ratio test was used to test the restrictions implied by the REH, because the ratio test is the most popular device for hypothesis testing. The unrestricted model was the following equation

$$CTM_t = \theta_1 CTM_{t-1} + \pi_1 X_t + \pi_2 X_{t-1} + U_t, \quad (6.5)$$

where $\pi_1 = [\pi_{11} \pi_{12} \pi_{13} \pi_{14} \pi_{15} \pi_{16} \pi_{17} \pi_{18}]$, $\pi_2 = [\pi_{21} \pi_{22} \pi_{23} \pi_{24} \pi_{25} \pi_{26} \pi_{27} \pi_{28}]$. 
Table 6.12. Second solution of the mill demand equation, 1957-86

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{11}$</td>
<td>1.0368</td>
<td>17.4717</td>
</tr>
<tr>
<td>$f_{12}$</td>
<td>0.2646</td>
<td>17.0736</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.6756</td>
<td>7.0042</td>
</tr>
<tr>
<td>$m_2$</td>
<td>-0.4147</td>
<td>-1.7031</td>
</tr>
<tr>
<td>$m_1$</td>
<td>0.1474</td>
<td>0.4013</td>
</tr>
</tbody>
</table>

First-step (two-stage least square)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$</td>
<td>0.4418</td>
<td>3.8704</td>
</tr>
<tr>
<td>$h_2$</td>
<td>7.1523</td>
<td>5.3137</td>
</tr>
<tr>
<td>$d_2$</td>
<td>-9.8585</td>
<td>-4.5651</td>
</tr>
<tr>
<td>$f_{21}$</td>
<td>0.00016</td>
<td>-0.0009</td>
</tr>
</tbody>
</table>

Second-step (nonlinear LS method)$^a$, $^b$, $^c$

$^a$The sum of squared errors = 610.6252; $R^2 = 0.6261$; the standard deviation of CTM = 4.8462.

$^b\alpha = .80, \text{ part} = .75, n = 60143, m = 7212, b = .95, r = .81$

imposed a priori.

$^c$The starting values: (0.5, 1.4, -8.0, 0.5).
### Table 6.13. Third solution of the mill demand equation, 1957-86

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>First-step (two-stage least square)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{11}$</td>
<td>1.0368</td>
<td>17.4717</td>
</tr>
<tr>
<td>$f_{12}$</td>
<td>0.2646</td>
<td>17.0736</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.6756</td>
<td>7.0042</td>
</tr>
<tr>
<td>$m_2$</td>
<td>-0.4147</td>
<td>-1.7031</td>
</tr>
<tr>
<td>$m_1$</td>
<td>0.1474</td>
<td>0.4013</td>
</tr>
<tr>
<td>Second-step (nonlinear LS method)$^a$, $b$, $c$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>0.4420</td>
<td>3.8965</td>
</tr>
<tr>
<td>$h_2$</td>
<td>4.3761</td>
<td>6.3989</td>
</tr>
<tr>
<td>$d_2$</td>
<td>-6.0370</td>
<td>-7.5404</td>
</tr>
<tr>
<td>$f_{21}$</td>
<td>0.00012</td>
<td>-0.0011</td>
</tr>
</tbody>
</table>

$^a$The sum of squared errors = 610.7365; $R^2 = 0.626$; the standard deviation of CTM = 4.8466.

$^b\alpha = .80$, part = .75, $n = 60143$, $m = 7212$, $b = .95$, $r = .81$

imposed a priori.

$^c$The starting values: (0.7, 1.8, -12.0, 0.5).
and \[ X'_t = [\text{CTX}_t \ y_{1t} \ \text{RCORN}_t \ \text{RCTVC}_t \ \text{RDP}_t \ \text{RPOLY}_t \ \text{RGNP}_t \ \text{RERJ}_t]' \],

with the assumption that \( U_t \) has zero mean and constant variance.

The unrestricted model containing 17 parameters was estimated by OLS and presented in Table 6.14. Comparison of the unrestricted estimates (Table 6.14) and the restricted estimates (Table 6.11) is presented in Table 6.15. The estimated coefficients for the unrestricted model and the restricted model are significantly different. The different results are the response of the rational expectations hypothesis on the restricted model imposed restrictions across parameters in the decision equation. The table also showed that the sum of squared errors for the unrestricted model is less than that for the unrestricted model.

Likelihood ratio statistic provides a test on the rational expectations hypothesis. Let \( L_r \) be the value of log likelihood function of the RE model and \( L_u \) be the value of the log likelihood of an estimated unrestricted version of the RE model. The test statistic

\[
-2[L_r - L_u] \tag{6.6}
\]

is used, which is asymptotically distributed as \( \chi^2(q) \), where \( q \) is the number of restrictions imposed and \( q = q_u - q_r \), where \( q_u \) is the number of parameters to be estimated without restrictions and \( q_r \) is the number of parameters to be estimated with restrictions imposed. A high value of the likelihood ratio indicates a failure of the null hypothesis that the rational expectations restrictions are correct.

The likelihood ratio (6.6) could be computed from

\[
T(\log V_r - \log V_u), \tag{6.7}
\]

where \( V_r \) and \( V_u \) are the restricted and unrestricted estimates of \( s(\theta) \), and
Table 6.14. Estimates of the unrestricted model (OLS)\(^a\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_1 )</td>
<td>0.3027</td>
<td>1.1108</td>
</tr>
<tr>
<td>( \pi_{11} )</td>
<td>-0.0031</td>
<td>-0.0374</td>
</tr>
<tr>
<td>( \pi_{12} )</td>
<td>-0.0020</td>
<td>-0.0723</td>
</tr>
<tr>
<td>( \pi_{13} )</td>
<td>0.0121</td>
<td>0.4524</td>
</tr>
<tr>
<td>( \pi_{14} )</td>
<td>-0.2081</td>
<td>-2.0783</td>
</tr>
<tr>
<td>( \pi_{15} )</td>
<td>0.0764</td>
<td>1.8611</td>
</tr>
<tr>
<td>( \pi_{16} )</td>
<td>-0.0332</td>
<td>-0.2571</td>
</tr>
<tr>
<td>( \pi_{17} )</td>
<td>-0.2229</td>
<td>-0.8027</td>
</tr>
<tr>
<td>( \pi_{18} )</td>
<td>-0.0067</td>
<td>-0.1314</td>
</tr>
<tr>
<td>( \pi_{21} )</td>
<td>-0.2063</td>
<td>-2.1845</td>
</tr>
<tr>
<td>( \pi_{22} )</td>
<td>0.0043</td>
<td>0.1827</td>
</tr>
<tr>
<td>( \pi_{23} )</td>
<td>-0.0010</td>
<td>-0.0275</td>
</tr>
<tr>
<td>( \pi_{24} )</td>
<td>-0.1664</td>
<td>-2.2917</td>
</tr>
<tr>
<td>( \pi_{25} )</td>
<td>-0.0463</td>
<td>-0.7606</td>
</tr>
<tr>
<td>( \pi_{26} )</td>
<td>0.1910</td>
<td>1.7577</td>
</tr>
<tr>
<td>( \pi_{27} )</td>
<td>0.0724</td>
<td>0.3187</td>
</tr>
<tr>
<td>( \pi_{28} )</td>
<td>-0.0005</td>
<td>-0.0081</td>
</tr>
</tbody>
</table>

\(^a\)The sum of squared errors of CTM = 233.52677; the standard deviation of CTM = 4.2383; \( R^2 = 0.68 \)
Table 6.15. Comparison of restricted and unrestricted model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameters of reduced form</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Restricted</td>
</tr>
<tr>
<td>CTXₜ</td>
<td>0.000015</td>
</tr>
<tr>
<td>Yₜ</td>
<td>-0.000005</td>
</tr>
<tr>
<td>RCONRNₜ</td>
<td>-0.000031</td>
</tr>
<tr>
<td>RCTVCₜ</td>
<td>0.000074</td>
</tr>
<tr>
<td>DPₜ</td>
<td>-0.000065</td>
</tr>
<tr>
<td>RPOLYₜ</td>
<td>0.000007</td>
</tr>
<tr>
<td>RGNTₜ</td>
<td>-0.000004</td>
</tr>
<tr>
<td>RERJₜ</td>
<td>0.000009</td>
</tr>
<tr>
<td>CTXₜ₋₁</td>
<td>-0.181288</td>
</tr>
<tr>
<td>Yₜ₋₁</td>
<td>0.047970</td>
</tr>
<tr>
<td>RCONRNₜ₋₁</td>
<td>0.000000</td>
</tr>
<tr>
<td>RCTVCₜ₋₁</td>
<td>0.000000</td>
</tr>
<tr>
<td>DPₜ₋₁</td>
<td>0.000000</td>
</tr>
<tr>
<td>RPOLYₜ₋₁</td>
<td>0.000000</td>
</tr>
<tr>
<td>RGNT₋₁</td>
<td>0.000000</td>
</tr>
<tr>
<td>RERJ₋₁</td>
<td>0.000000</td>
</tr>
<tr>
<td>SSE*</td>
<td>610.622075</td>
</tr>
<tr>
<td>Degrees of freedom</td>
<td>26</td>
</tr>
</tbody>
</table>

*SSE represents the sum of squared errors of the CTM equation.
T represents the number of observations since \( L_r = \frac{T}{2} \) \* log\( V_r \) and
\( L_u = \frac{t}{2} \) \* log\( V_u \) in this case. The test statistic \( T(\log V_r - \log V_u) \) is biased, but the bias can be fixed by replacing \( T \) with \( (T - k) \), where \( k \) is the number of parameters in a unrestricted equation (Sargent 1978b).

The resulting test did not lead to rejecting the null hypothesis that the nonlinear restrictions were valid, since the value of the statistic, 12.493 with 13 \((17 - 4)\) degrees of freedom, is less than the critical region \( \chi^2_{13, .025} = 24.736 \). The result provides no strong evidence for rejecting the RE restrictions. In the case, using \( T \) instead of \( (T - k) \) to define the test statistic would affect the conclusion that the model reject the likelihood test.

### 6.5 Elasticity

When constructing the U.S. cotton model, it is of interest to note the description and the prediction of the dynamic response of the cotton industry. One way to quantify the statement is to calculate dynamic multipliers associated with the model's exogenous variables or to calculate dynamic elasticities examining how an expected or unexpected change in one of the uncontrolled variables alters the decision variables throughout the rational equilibrium model.

Let \( Z \) be one of the relevant state variables (CTX, \( y_1 \), RCORNR, RCTVC, or RDP) and \( Y \) be one of the decision variables (\( a_{1t} \) or \( ctm_t \)). Dynamic elasticity for land (mill demand) is calculated with respect to the unconditional means of land (mill demand) and \( Z \). The long-run elasticity for cotton land demand (cotton mill demand) is defined as
\[ e_z = \frac{\partial E(Y) E(Z)}{\partial E(Z) E(Y)} ; \]

e_z is thus a measure of the effect of the expected mean change in Z on the mean change in area (cotton mill demand). The short-run elasticity for cotton land demand (cotton mill demand) is defined as

\[ e_z^k = \frac{\partial E_k(Y_{t+k}) E(Z)}{\partial E_k(Z_{t+k}) E(Y)} ; \]

\( e_z^k \) measures the effect of the expected change in Z, k periods ahead, with condition on current information on the current change in area (cotton mill demand). A medium-run elasticity is defined in the same way, with

\[ e_z^k(r) = \frac{\partial E_k(Y_{t+r}) E(Z)}{\partial E_k(Z_{t+r}) E(Y)} , \text{ for } k > r; \]

\( e_z^k(r) \) measures the effect on area (cotton mill demand), r period ahead, from a change in conditional expected Z, k periods ahead.

Some elasticities are calculated by using the estimated parameters (see Table 6.11) and shown in Table 6.16. The magnitudes of the long-run and short-run elasticity are within a reasonable range except the net return of the corn which is very small since the coefficient \( f_{z1} \) is insignificant and close to zero.

6.6 Forecasting and Policy Evaluation

The estimated mill demand model presented in Table 6.11 adequately reflects the structure of the U.S. cotton mill demand. The \( R^2 \) and standard error value of the estimated CTM equation were presented in Table
Table 6.16. Elasticities at mean level

<table>
<thead>
<tr>
<th>Dependent Variables</th>
<th>Right hand side variables&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RPF</td>
</tr>
<tr>
<td>SR</td>
<td>LR</td>
</tr>
<tr>
<td>SR</td>
<td></td>
</tr>
<tr>
<td>SR</td>
<td></td>
</tr>
</tbody>
</table>

AP

| AP   | 0.03 | 0.08 | 0.000001 | 0.000009 |

CTM

| CTM  | -0.13 | 0.30 |

<sup>a</sup>All the mean levels are the average of the residuals. The average levels as follows: mean of RPF = 72.71557, mean of RCORNR = 125.5969, mean of RPW = 77.39565, mean of RPM = 176.3224, mean of AP = 128.1737, mean of CTM = 75.6453.
6.11. The calculated $R^2$ and standard deviation are 0.626 and 4.846.\footnote{$R^2$ is calculated by the formula: one minus the ratio of the residual sum of squares to the total sum of squares for the CTM equation. Standard deviation is computed by the square root of the sum of squared errors adjusted by the degree of freedom ($T-h$), where $T$ is the number of observations, $h$ is the number of estimates in the equation.}

Although the $R^2$ and standard deviation provide an indication of the model fit, they must be interpreted with caution. This is because the estimated equation does not include the interpret term and the error term is not normally distributed due to ignoring imposed restrictions. This section discusses the evaluation of the model's simulation performance. Finally, the estimated coefficients in Table 6.11 were used to perform historical simulation and policy analysis.

The reasons for performing a model simulation include testing the model validity, policy analysis, and forecasting. Usually, the time horizon over which the simulation is performed depends on the purpose of the simulation. For example, a better indication of the model performance can be obtained by a historical simulation of the model (Pindyck and Rubinfeld 1981).

In this study, the historical simulation was performed by using the estimates of the decision equation (Table 6.11) from the sample data, 1957-1986. The performance of the historical simulation is evaluated by using three statistics: Theil's inequality coefficient ($0 < u < 1$), mean absolute error (MAE), and root mean square error (RMSE) measures.

Theil's inequality coefficient is a function of the RMSE and can be decomposed into three components, $U^m + U^n + U^o = 1$. $U^m$ is called the bias
proportion to reflect the relative deviations of the means of the simulated and actual series; \( V \) is called the variance proportion which measures the relative deviation of the variance of the simulated and actual series; and \( U^c \) is called the covariance proportion which measures the relative residual error between the actual and simulated series. With relatively small values of \( U^b \) and \( V \) and large values of \( U^c \) is indicative of a good model fit.

Mean absolute error measures the average of the absolute difference between the historical and simulated series. A large value of MAE suggests a poor simulation performance. Another simulation statistic is the RMSE which measures the positive root of the sum of the squared deviations between the actual and simulated series. Like the MAE, large values of the RMSE indicate a poor model performance.

Ex post simulation includes a simulation of the model forward in time beyond the estimation period. This type of simulation is called ex post forecasts. Ex post forecasts are not only useful for predictive purposes but also for examining and comparing what might have taken place as a change in the values of policy parameters or letting exogenous policy variables follow different processes. To generate a forecast, the entire forecast period for all the exogenous variable should be known a priori.

Chapter IV has shown that the parameters of the decision rules (4.53, 4.55 - 4.58) are functions not only of parameters in agent's objective function and the stochastic processes that govern the exogenous variables, but are also functions of government policies. The latter has an effect on the paths of those exogenous variables considered to be policy
Instruments. If so, the estimated coefficients of most economic models are not fixed as a nontrivial change in policy regimes occurs. Therefore, to analyze policy interventions, such as a change in target price formula, it is necessary to trace the effect of a change in policy regime through the relevant decision rule.

In the modeling practice, parameters are reserved for measures of tastes and technology, and different policies are modeled as different realizations of a random process. Suppose that the direct payments are made to follow a new process, say random walk, instead of first-order Markov process and the new rule will be active from 1987. Firms are aware of this new policy rule and add this information into the existing model. The proper way to evaluate the effects of a change in direct payments rule would be as follows. Those estimated coefficients for measuring tastes and technology derived from equation (6.3) are reserved, the parameters of the new proposed direct payments rule replace the corresponding estimates of VAR in equation (6.3), and then the new observable mill demand schedule can be derived.

A historical simulation and ex post forecasts were performed in order to justify the model's ability to replicate the actual data.\(^3\) The summary measures for the mill demand decision equation are reported in Table 6.17. The table shows that \(U^m + U^v\) is small relative to \(U^c\). The equation simulated well. In addition, actual and simulated values for the CTM

\(^3\)All simulations are dynamic, in the sense that simulated rather than actual values for the endogenous variable in a given period are used as an input when the model is solved for the future periods.
Table 6.17. Simulation performance of the mill demand model, 1957-1986

<table>
<thead>
<tr>
<th>Variable</th>
<th>Measure (^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(U^m)</td>
</tr>
<tr>
<td>CTM</td>
<td>0.06</td>
</tr>
</tbody>
</table>

\(^a\)The U values are the decomposition of Theil's inequality coefficient into (1) bias proportion, \(U^m\), (2) variance proportion, \(U^v\), and (3) covariance proportion, \(U^c\). MAE is mean absolute error and RMSE is root mean square error.
variable are plotted in Figure 6.2 for the historical simulation that begins in 1957 through 1988, the last two simulated values are obtained by using the predicted values of all the exogenous variables. These results indicate that the model does a good job of simulating historical mill demand in the U.S. cotton market.

Furthermore, the model was used to evaluate the new direct payments policy rule and simulated 2 years into the future, beginning in 1987. The results of the new policy rule are presented in Table 6.18 and Figure 6.2. The results are not at all surprising. The forecast values for the new policy regime are slightly different from those derived from the old policy rule. This is because the reduced form parameters for the mill demand model are quite small whether or not the policy rule follows an old or a new rule and the forecast values are shown in Table 6.19.

In summary, this chapter has focused on estimating a dynamic rational expectations equilibrium model for the U.S. cotton market using nonlinear LS method. For the purpose of implementing the nonlinear LS method, the DFP algorithm was used for evaluating the least squares equation. The algorithm has a speed advantage over the other algorithms and can solve for a nonpositive definite Hessian matrix.

The attempts to estimate the whole system of optimal decision rules were unsuccessful. The model could not converge. The reasons might be the data problem and the poor initial values. Since the purpose of this study is to obtain the structural parameters, the mill demand optimal decision rule was chosen for achieving the goal. Although estimates obtained from a single-equation might be unsatisfactory, one can envision
Table 6.18. Ex post forecast results of the mill demand equation under two different policy regimes

<table>
<thead>
<tr>
<th></th>
<th>Actual(^a)</th>
<th>Policy 1</th>
<th>Policy 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1987</td>
<td>19.2223</td>
<td>5.3220</td>
<td>5.3186</td>
</tr>
<tr>
<td>1988</td>
<td>17.2378</td>
<td>5.3227</td>
<td>5.3184</td>
</tr>
</tbody>
</table>

\(^a\)Actual data are computed by subtracting trend from the original data.

\(^b\)In policy 1 the direct payments follow a first-order Markov process. In policy 2 the direct payments follow a random walk.
Figure 6.2. Simulation of cotton mill demand
Table 6.19. The estimates of the reduced forms of the mill demand with the two different policy regimes, 1957-1986

<table>
<thead>
<tr>
<th>Variables</th>
<th>Policy 1</th>
<th>Policy 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTM_{t-1}</td>
<td>0.4418</td>
<td>0.4418</td>
</tr>
<tr>
<td>CTX_t</td>
<td>0.000015</td>
<td>0.000018</td>
</tr>
<tr>
<td>Y_{1t-20}</td>
<td>-0.000005</td>
<td>-0.000008</td>
</tr>
<tr>
<td>RCORNR_t</td>
<td>-0.000031</td>
<td>-0.000033</td>
</tr>
<tr>
<td>RCTVC_t</td>
<td>0.000074</td>
<td>0.000074</td>
</tr>
<tr>
<td>DP_t</td>
<td>-0.000065</td>
<td>-0.000101</td>
</tr>
<tr>
<td>RPOLY_t</td>
<td>0.000007</td>
<td>0.000011</td>
</tr>
<tr>
<td>RGNP_t</td>
<td>-0.000004</td>
<td>-0.000004</td>
</tr>
<tr>
<td>RERJ_t</td>
<td>0.000009</td>
<td>0.000009</td>
</tr>
<tr>
<td>CTX_{t-1}</td>
<td>0.181289</td>
<td>0.181289</td>
</tr>
<tr>
<td>Y_{t-1}</td>
<td>-0.047974</td>
<td>-0.047974</td>
</tr>
</tbody>
</table>

*In policy 1 the direct payments follow a first-order Markov process. In policy 2 the direct payments follow a random walk.*
situations in which nothing better can be done, so the second best theory is applied here.

The results of estimating the mill demand model are consistent with economic theory that the mill demand is influenced by the prices of cotton and cotton yarn, substitute fiber (polyester), and income. All the estimated parameters of the model have signs consistent with the assumptions of the model and the magnitudes of the long-run and the short-run elasticities are within a reasonable range (except the net returns of corn). In addition, the result of likelihood ratio test provides support to the rational expectations hypothesis. The methodology adopted here requires further improvements in considering the restrictions imposed on the error terms that have been ignored previously.
7 CONCLUSIONS

The objective of this study was to develop a complete conceptual framework useful for analyzing the behavior of competitive agents at firm and market levels under uncertainty. In doing so, the simultaneous determination of industry equilibrium was considered in addition to the individual firm's optimization behavior. Also, this study has attempted to strengthen the relationships between economic theory of expectations and econometric practices. The VAR approach in this study is a new breed in modeling agricultural models.

The present theoretical model incorporated the rational expectations hypothesis. Also, the environment and agent's decision rules were modeled as time invariant linear stochastic difference equations, because such setups allow the utilization of the dynamic stochastic optimization theory and time series methods. There are, however, drawbacks encountered in this study. First, attempts to estimate the whole system of equations were unsuccessful since the form of the whole system of equations is complicated, the size of the model is substantial, and the current computer programs are of limited use when it comes to the substantial size of the nonlinear model. Since the purpose of the study is to obtain the underlying parameters for understanding supply response, land allocation decisions, mill consumption, and price processes, the mill demand optimal decision rule was chosen, instead of estimating the whole system equations.
Second, it is intractable to estimate agents' decision rules jointly with models for the stochastic processes they face, subject to the cross-equation restrictions implied by the rational expectations hypothesis in the rational model. Because these cross-equation restrictions are complicated, highly nonlinear parameters cause analytical problems, and the large number of estimates make computation burdensome. Thus, a two-step procedure including forecasting and estimation was used to estimate the model. Although the two-step procedure was not fully efficient due to the loss of some information on the cross-equation restrictions, it yielded consistent estimates.

The theoretical model presented provides a rich structure for applicability to any commodity market. To keep the model analytical and concrete, a number of specific and simplifying assumptions were made such as quadratic costs of adjustment, demand uncertainty, and the stochastic processes for the relevant state variables.

Using the above assumptions, the theoretical model was developed for the U.S. cotton industry. The cotton market was a candidate for empirical investigation since cotton is a major cash crop and an important source of foreign exchange. Also, government programs, including both price support and acreage restriction mechanisms, have historically been important roles in this market. Direct payments were used in this model to reflect the policy regimes.

Some compromises were made as addressed previously, so the mill demand equation (5.1) was estimated to obtain the underlying structural parameters, instead of estimation of the whole system. A two-step
procedure including forecasting and estimation was conducted to estimate the model by ignoring the restrictions on the error term of the mill demand model a priori. The VAR approach was used to solve the forecasting problem. With the additional assumptions that the series $X_t$ is bounded and well defined, the function $f(X_t, \theta)$ is continuous and differentiable, and the $e_t$ is independently, identically distributed, the equation (6.3) can be estimated by using the method of nonlinear least squares.

The resulting estimates are consistent with the model; the estimated coefficients are significant and have acceptable signs and magnitudes (except the net return of corn variable). Also, the estimated parameters (see Tables 6.8-10) of the mill demand model satisfy the restrictions that the model implied on the industry's problem: $|\theta_1| < 1$, $|W| > \min(|\phi_1|, |\beta_1|)$, the roots of $|V - mI| = 0$ lie outside the unit circle, and the sign of $d_2$ is opposite to the sign of $\theta_1$. Further, the values of $\theta_1$ and $d_2$ are consistent with adjustment cost effects in the cotton production. Using the estimated parameters (Table 6.11), the elasticities are calculated and reported in Table 6.16. In the mill demand model, the elasticities are functions of the underlying parameters, such as adjustment cost parameters, discount factors, and technology. The magnitudes of the long-run and short-run elasticities are within a reasonable range, except for the net returns of corn variable.

Given the mill demand model, a dynamic simulation is performed over the sampling period (1957-1986) plus two forward periods. Figure 6.2 is the result of the predicted values of the cotton mill consumption from the dynamic simulation of the rational expectations model. The result of the
simulation indicates a good performance for the mill demand variable.

Furthermore, the estimated coefficients (Table 6.11) were used to evaluate a new direct payments policy rule and simulated 2 years into the future, beyond in 1986. The forecast values for the two different policy regimes are slightly different. The reduced form parameters change as different policy regimes are taken. Consequently, the empirical results provide some support for the specific model. It is important to emphasize that the results of this study are satisfied (i.e., estimates make sense, a good simulation performance, and passed the likelihood ratio test) since the orders of the adjustment cost processes and the Markov processes governing the state variables and disturbances are imposed \emph{a priori}, and some compromises were made in estimation process.

The present model represents an important contribution to the applied literature, linking estimation and two interrelated markets (cotton and cotton yarn) under uncertainty directly to a coherent theory of individual decision and market outcomes. Some further developments in this study are needed to improve the formulation and estimation of the model including the following:

First, it is desirable to reduce the size of state variables in the model for keeping the model simple and concrete.

Second, it is desirable to estimate agents' decision rules jointly with models for the stochastic processes they face, subject to the cross-equation restrictions implied by the rational expectations hypothesis. Because the presence of restrictions across the parameters of the processes governing the exogenous and endogenous variables is the
reason that joint estimation of the parameters of the exogenous and endogenous processes is necessary for statistical efficiency.

Third, it is important to test the rational expectations in several alternative specifications, since the rational expectations hypothesis is controversial (Fisher 1982).

Fourth, it is desirable to develop a procedure for explicitly deriving the optimal decision rules in the model including the storage component, estimating the model if possible, and using the results to compare the full economic effects of the storage model against the model without the storage component.

Fifth, in general, it is assumed that information is costless and that there is instantaneous learning by agents. But it is unrealistic to assume this. So it is desirable to extend the model including the effects of the cost of information.


Friedman, Benjamin M. 1979. "Optimal Expectations and the Extreme Inflation


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10 APPENDIX A. DEFINITION OF VARIABLES
(capital letters refer to aggregate notations while small letter cases
reserved for an agent notations)

<table>
<thead>
<tr>
<th>Notations</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Total acreage of the representative farmer</td>
<td>acres</td>
</tr>
<tr>
<td>a&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Cotton acreage planted by the representative farmer at time t</td>
<td>acres</td>
</tr>
<tr>
<td>a&lt;sub&gt;2t&lt;/sub&gt;</td>
<td>Soybeans acreage planted by the representative farmer at time t</td>
<td>acres</td>
</tr>
<tr>
<td>A&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Total U.S. cotton acreage Planted at time t</td>
<td>ten mil. acres</td>
</tr>
<tr>
<td>c&lt;sub&gt;1t&lt;/sub&gt;</td>
<td>Cost of production of cotton per acre at time t</td>
<td>dollars</td>
</tr>
<tr>
<td>c&lt;sub&gt;2t&lt;/sub&gt;</td>
<td>Cotton bought by the miller at time t</td>
<td>dollars</td>
</tr>
<tr>
<td>c&lt;sub&gt;3t&lt;/sub&gt;</td>
<td>Adjustment cost of milling at the representative mill firm at time t</td>
<td>dollars</td>
</tr>
<tr>
<td>c&lt;sub&gt;4t&lt;/sub&gt;</td>
<td>Inventory cost of miller at time t</td>
<td>dollars</td>
</tr>
<tr>
<td>CMD&lt;sub&gt;t&lt;/sub&gt;</td>
<td>U.S. total demand for cotton yarn</td>
<td>pounds</td>
</tr>
<tr>
<td>cm&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Cotton yarn production of the representative miller</td>
<td>pounds</td>
</tr>
<tr>
<td>CM&lt;sub&gt;t&lt;/sub&gt;</td>
<td>U.S. total cotton yarn production</td>
<td>pounds</td>
</tr>
<tr>
<td>c&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Cotton production of the representative farmer</td>
<td>bales</td>
</tr>
<tr>
<td>GT&lt;sub&gt;t&lt;/sub&gt;</td>
<td>U.S. total cotton production</td>
<td>bales</td>
</tr>
<tr>
<td>c&lt;sub&gt;tG&lt;/sub&gt;</td>
<td>Cotton stocks on CCC by the representative farmer</td>
<td>bales</td>
</tr>
<tr>
<td>CTG&lt;sub&gt;t&lt;/sub&gt;</td>
<td>U.S. cotton stocks on CCC.</td>
<td>1000 bales</td>
</tr>
<tr>
<td>ctm&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Total cotton milled by the representative miller</td>
<td>bales</td>
</tr>
<tr>
<td>CTM&lt;sub&gt;t&lt;/sub&gt;</td>
<td>U.S. total cotton mill consumption</td>
<td>ten mil. bales</td>
</tr>
</tbody>
</table>
\( c_{\text{tms}_t} \) Cotton stocks at the representative miller's firm
\( \text{bales} \)

\( \text{CTMS}_t \) U.S. total commercial stocks
\( \text{mil. bales} \)

\( c_{\text{ctx}_t} \) Cotton sold exports by the representative farmer
\( \text{bales} \)

\( \text{CTX}_t \) U.S. cotton exports
\( \text{ten mil. bales} \)

\( R_{\text{CTVC}_t} \) U.S. cotton variable cost per acre
\( \text{dollars} \)

\( \text{RDP}_t \) U.S. cotton direct payments per farmer
\( \text{dollars} \)

\( \text{RERJ}_t \) Japan exchange rates in terms of U.S. dollars
\( \text{dollars} \)

\( \text{RPOLY}_t \) U.S. wholesale price of polyester
\( \text{cents/lb} \)

\( \text{RPF}_t \) U.S. cotton price received by farmers
\( \text{cents/lb} \)

\( \text{RPM}_t \) U.S. wholesale price of cotton yarn
\( \text{cents/lb} \)

\( \text{RFW}_t \) U.S. wholesale price of cotton
\( \text{cents/lb} \)

\( r \) Cotton convert factor (convert cotton into cotton yarn)

\( c_{\text{rt}_t} \) Corn production of the representative farmer
\( \text{bushels} \)
Total aggregate production for the U.S. cotton farmers can be computed as follows:

\[ CT_t = y_{1t} \times (A_p + A_{np}) \]
\[ = y_{1t} \times [(\alpha \times A_b \times \text{Part}) + (A_b \times (1 - \text{Part}))] \]
\[ = y_{1t} \times (A_b \times [\alpha \times \text{Part} + (1 - \text{Part})]) \]
\[ = y_{1t} \times A_b \times [1 - (1 - \alpha) \times \text{Part}]. \quad (*) \]

Where \( y_{1t} \) = yield per acre; \( A_p \) = acreage planted for the participants; \( A_{np} \) = acreage planted for the nonparticipants; \( A_b \) = acreage base; \( \alpha = (1 - \text{diversion or set-aside rate}) \). Then the participant's production is

\[ Y_{1t} = A_b \times \alpha \times \text{Part} = \frac{(CT_t \times \alpha \times \text{Part})}{[1 - (1 - \alpha) \times \text{Part}}. \]

Taking Taylor Expansion on equation (*) obtains

\[ CT_t = F_{10} + F_{11} A_{1t} + F_{12} y_{1t}, \]

where

\[ F_{10} = -A_0 \times y_0 \times (1 - (1 - (1 - \alpha) \times \text{Part}); \]
\[ F_{11} = y_0 \times (1 - (1 - \alpha) \times \text{Part}); \]
\[ F_{12} = A_0 \times (1 - (1 - \alpha) \times \text{Part}); \]

\( A_0 \) is the initial value of the acreage planted for the participant;
\( y_0 \) is the initial yield per acre.