APPLICATION OF THE RFC/NDE SYSTEM TESTING RESULTS

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INTRODUCTION

The evaluation of the RFC/NDE System has produced capability characteristics in the format of \( \text{a vs. a} \) or apparent cracksize vs. actual cracksize. Although crack sizing has long been a major concern of NDE practitioners and theoreticians, the analysis procedure which permits straightforward conversion of these \( \text{a vs. a} \) to POD vs. \( a \), or Probability of Detection vs. cracksize, is a recent development [1]. This paper will compare these two descriptions of NDE system capability by comparing their influence on the Retirement for Cause (RFC) process.

The life cycle of a gas turbine component which is fatigue limited can be conceptualized as being comprised of two phases: (1) an initiation phase, during which material undergoes cyclic loading and thus accumulates fatigue damage; and (2) a propagation phase as the initiated crack actively grows until it is either detected and removed during nondestructive evaluation or reaches critical dimensions and fails in service. This simplified representation is illustrated graphically in Fig. 1, where cracksize is plotted as a function of time (and therefore accumulated fatigue cycles). The figure shows that cycles required to initiate a crack vary according to some statistical distribution; once initiated, cracks proceed according to the laws of fracture mechanics [2,3,4].

The Probabilistic Life Analysis Technique, PLAT [5] embodies a Monte Carlo simulator; it is a statistical description of the gas turbine life cycle. It uses distributions of the independent variables rather than the more familiar single-value input functions. Because the input can assume a range of values, the simulator output is also multivalued and is presented in statistical terms, such as expected outcome or rates of occurrence. To do this, the simulator selects from distributions of the independent variables and combines their effects according to the physical laws of the system being modeled. Each "pass" through the simulator will result in one outcome based on one sample from each of the controlling variables. After many -- sometimes tens of thousands of passes -- these individual results are collected and analyzed statistically.
The PLAT is an application of statistical methods to component life analysis. Previous life analyses were based on a deterministic criterion with the worst case establishing the usable life for all parts. If the worst case occurred only once in a thousand times, then 99.9 percent of the parts (gas turbine disks in this case) were being retired prematurely. PLAT is a methodology for establishing the statistical behavior of all the disks which comprise an entire fleet of engines. Instead of single-valued functions, PLAT input consists primarily of information about statistical distributions of life-controlling parameters. These include Initial Material Quality (the distribution of expected sizes of microstructural anomalies such as voids and inclusions), crack initiation behavior in the form of a stress vs. cycles (s-N model), mission severity, stress variability, and a transition model of behavior from IMQ through crack initiation, to the propagation phase. Also required is a life prediction model, describing the expected longevity of a component after a crack has initiated. Finally, A Nondestructive Evaluation/Return-to-Service model is required.

\( \hat{a} \) vs. \( a \) ANALYSIS

Actual cracksize (\( a \)) and indicated cracksize (\( \hat{a} \)) -- often an eddy current signal voltage or fluorescent penetrant brightness -- are recorded for each observation and a threshold level \( \hat{\delta}_{\text{th}} \) is determined. This threshold is considered to be the smallest signal which represents a crack; any signal below this level is defined as "noise".

Figure 2 illustrates the approach to be used in defining POD(\( a \)) as a function of cracksize by considering the probability that a crack will appear large enough to be detected [5]. Of course this probability is itself a function of actual cracksize. The basic model for \( \hat{a} \) vs. \( a \) data is given by:

\[
\hat{a} = f(a) + c + e \quad (1)
\]
where \( f(a) \) represents the overall trend in \( \hat{a} \) as a function of \( a \), \( c \) represents flaw to flaw variation, and \( e \) represents the variation from inspection to inspection of the same flaw. The function \( f(a) \) is fixed, while the variables \( c \) and \( e \) are random variables (the variance components of the model) with means of zero.

There are many data analysis methods based on the foregoing equation, and the appropriate method depends on the form of \( f(a) \). One method is to convert \( f(a) \) to a linear relationship through transformations of \( a \) and \( a \). For example, \( \ln(a) \) and \( \ln(a) \) are often observed to be linearly related. The basic equation is then given by:

\[
\ln(\hat{a}) = a + \beta \ln(a) + c + e
\]  

(2)

Again, \( c \) and \( e \) are random variables with means of zero, but not identical with those in equation (1). Assuming these variance components to have normal distributions leads to the POD function:

\[
\text{POD}(a) = P(\hat{a} > \hat{a}_{th})
\]

(3)

\[
= P( \ln(\hat{a}) > \ln(\hat{a}_{th}))
\]

(4)

\[
= 1 - \Phi \left( \frac{\ln(\hat{a}_{th}) - (\alpha + \beta \ln(a))}{S} \right)
\]

(5)

where \( S^2 \) is the total variance and \( \Phi \) is the standard normal distribution function. Since the area under the standard normal curve is one:

\[
\text{POD}(a) = \Phi \left[ - \left( \ln(\hat{a}_{th}) - (\alpha + \beta \ln(a))S \right) \right]
\]

(6)

Dividing numerator and denominator by \( \beta \) then gives:

\[
= \Phi \left( \frac{\frac{\beta \ln(a) \ln(\hat{a}_{th}) + \alpha}{\beta}}{\frac{S}{\beta}} \right)
\]

(7)
\[ = \Phi \left( \frac{\ln(a) - \ln(\hat{a}_{th}) - \alpha}{\beta} \right) \]  
\[ \text{(8)} \]

which is observed to be a lognormal distribution in \( a \) with mean and standard deviation given by:

\[ \mu = \frac{\ln(\hat{a}_{th}) - \alpha}{\beta} \]  
\[ \text{(9)} \]

\[ \sigma = \frac{S}{\beta} \]  
\[ \text{(10)} \]

For simplicity and computational efficiency, we may wish to approximate this lognormal distribution by a log logistic distribution [5] with the same mean and standard deviation. Estimates of the scale and location parameters (\( t \) and \( w \)) of the log logistic distribution are given by:

\[ t = \frac{\pi}{\sigma \sqrt{3}} \]  
\[ \text{(11)} \]

\[ w = \exp[-\mu t] \]  
\[ \text{(12)} \]

The form of the resulting POD model is then:

\[ POD = \frac{(wa^{\dagger})}{1 + (wa^{\dagger})} \]  
\[ \text{(13)} \]

which is computationally more straightforward than a lognormal distribution.

PLOTTING \( \hat{a} \) VS. \( a \) DATA ON THE POD VS. \( a \) CURVE

Cracks of the same physical size (equal \( a \)) often exhibit different probabilities of detection because they appear to be different sizes (have different \( \hat{a} \)). Here, we assume that all cracks which appear to be the same size (i.e., have equal \( \hat{a} \)) have equal probabilities of detection, regardless of their actual size. We can now map any point in the \( \hat{a} \) vs. \( a \) plane to a corresponding point in POD vs. \( a \) space. Although they are not used computationally, individual \( \hat{a} \) vs. \( a \) observations can now be plotted in the POD vs. \( a \) space using several simple assumptions. Examining Fig. 3, we choose a particular observation (the circle) in the \( \hat{a} \) vs. \( a \) plane. Given no other information than the apparent crack size and our linear regression \( \hat{a} \) vs. a model (equation 2), we would assign a ”most probable” cracksize \( (a^{\star}) \) based on the mean regression line (shown). We could assign a ”most probable” POD (POD*) based on the POD vs. a regression (equation 11) and \( a^{\star} \). This process is represented schematically by the short dashed line. We do, however, know the true value of \( a \). Our \( \hat{a} \) vs. \( a \) observation can now be represented in the POD vs. a plane as shown by the circle in the right hand plot. The point is plotted at its actual cracksize, \( a \), and the POD* associated with an average apparent cracksize of \( a^{\star} \).
CONFIDENCE BOUNDS FOR POD VS. \( \alpha \) CURVES

Dr. A. P. Berens [5] has adapted a method described by Cheng and Iles [6] for calculating confidence bounds on cumulative distributions. Confidence bounds on the mean POD are then given by:

\[
POD(\alpha) = \Phi(Z_L), \text{ where}
\]

\[
Z_L = Z - \frac{\lambda}{\sqrt{n}} \left( \frac{\hat{\lambda}^2}{2} + \frac{(\bar{x} - \hat{x})^2}{SSX} + 1 \right)
\]  

and

\[
SSX = \sum_{i=1}^{n} x_i^2 - \left( \frac{\sum_{i=1}^{n} x_i}{n} \right)^2
\]  

In the above equations, \( n \) is sample size, \( \lambda \) is the \( p \)th percentile of a \( \chi^2 \) distribution with two degrees of freedom, and

\[
\hat{\lambda} = \frac{X - \mu}{\sigma}
\]  

Figure 4 illustrates the \( \hat{\alpha} \) vs. \( \alpha \) regression. The corresponding POD vs. \( \alpha \) plot, Fig. 5, presents the mean capability, as determined using the log logistic approximation (equation 13) with the 95% confidence limit (equation 14). Data are plotted as described in the preceding section.
COMPARISON OF \( \hat{a} \) VS. \( a \) AND POD VS. \( a \)

Since \( \hat{a} \) vs. \( a \) and POD vs. \( a \) are different descriptions of the same phenomenon, they should produce identical results when used in PLAT analysis. This is intuitively obvious. The investigation was conducted because the Monte Carlo simulation is different for each, and therefore the study would uncover any unexpected differences. Simulating NDE performance using \( \hat{a} \) vs. \( a \) is straightforward: knowing the true size, \( a \), the distribution of \( \hat{a} \) is sampled, and an individual \( \hat{a} \) is determined. If this value exceeds the threshold cracksize, \( \hat{a}_{th} \), then it is "detected", and the simulator proceeds accordingly. The simulation of POD vs. \( a \) is another matter. Knowing true size, \( a \), the POD vs. \( a \) function is evaluated to determine a POD. A random variable uniformly distributed on the 0,1 interval is generated and compared with POD. If POD is greater than this number, the crack is "detected".
To evaluate any differences in performance, the life cycle of a hypothetical advanced compressor disk lug attachment was simulated using the PLAT. This case was realistic in all respects except two: the crack propagation life was inadequate, and the NDE behavior represented worst-case capability. This was done deliberately. If adequate propagation margin and NDE capability had been simulated, there would be zero failures. The more realistic design therefore would illustrate nothing. By simulating a less-than-optimum situation, we are able to observe failures and removals, and therefore have a basis of comparison.

RESULTS

The failures using each NDE representation are plotted vs. inspection interval and presented in Fig. 6. A similar plot illustrating NDE removals is presented in Fig. 7. As can be seen, these are essentially (but not identically) equal. The small differences are caused by the random nature of a Monte Carlo simulator. The influence of different random number streams has been investigated and quantified and reported [5].

CONCLUSIONS

NDE capability can be described using \( \hat{\alpha} \) vs. \( \alpha \) and POD vs. \( \alpha \), and they result in the same failure and replacements rates when used in a simulation of NDE behavior.

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Figure 6. Comparison of POD vs. \( \alpha \) and \( \hat{\alpha} \) vs. \( \alpha \) Models -- Predicted Failures.
The mean POD and associated confidence limit are easily determined
from the $\hat{a}$ vs. $a$ behavior.

Individual $\hat{a}$, $a$ observations can be plotted as POD, $a$ points for
illustrative purposes.

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