Abrikosov vortex motion and elementary pinning force in a SNS Josephson junction

Ok-Bae Hyun
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Abrikosov vortex motion and elementary pinning force in a SNS Josephson junction

Hyun, Ok-Bae, Ph.D.

Iowa State University, 1987
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Abrikosov vortex motion and
elementary pinning force
in a SNS Josephson junction

by

Ok - Bae Hyun

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Department: Physics
Major: Solid State Physics

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1. INTRODUCTION

In a bulk type II superconductor, magnetic field is expelled from its interior (Meissner effect) until the field reaches $H_{c1}$ (lower critical field). At this field, it becomes energetically favorable for flux to enter the bulk of the superconductor in the form of quantized flux lines (or fluxons) carrying just one flux quantum, $\Phi_0$ (mixed state) \[1\]. The quantized flux line is called an Abrikosov vortex, named after the theorist who first predicted the existence of these structures, or sometimes a superconducting vortex. In this thesis this will simply be called a vortex. In this vortex superconducting current circles around a central core and generates a spatially varying magnetic field \[2\]. The flux lines generally form a triangular flux line lattice (FLL) with the flux line density of $H_g/\Phi_0$.

This vortex state is a characteristic of type II superconductors. If a transport current is applied to a superconductor containing a flux line lattice, the vortex can be pushed by the Lorentz force due to the current. Vortex flow caused by this force results in a resistive state, even though it is not the true normal state, because it generates a dissipative electric field following the Faraday's law. This vortex flow does not occur for small transport current because the vortices tend to be trapped in such places (pinning centers) of the superconductor as impurities, dislocations, grain boundaries, voids, etc., where the trapping force (pinning force) is stronger than the Lorentz force (depinning force). So the critical current of a type II
superconductor is determined by the maximum pinning force, $F_p = BJ_c/c$. For the use of the current in magnets, for which large supercurrents are desired, strong pinning materials (hard superconductors) is required. On the other hand, one may need to remove all vortices from superconducting devices such as electronic circuits. In this case materials with weak pinning are required (soft superconductors).

The vortex pinning mechanism in a bulk superconductor is not fully understood. The flux pinning problem has two separate aspects: (1) the interaction between a vortex and a single defect leading to the elementary pinning force, $f_p$, and (2) the addition of these forces to form a net volume pinning force on the elastic flux lattice accommodating itself to the presence of the impurities (summation problem). There are several important contributions to these problems. Conventionally, the pinning potential is assumed to be the condensation energy density, $H_c^2/8\pi$, times the volume of a defect or pinning center. In one regime, Campbell and Evetts [3] proposed that the difference of $H_{c2}$ (upper critical field) across the grain boundary, which arises from its anisotropy, may cause vortex pinning. This is a mean free path effect which causes a change in the size of the core of the vortex. In another regime, Larkin and Ovchinnikov [4] explain the flux pinning in terms of the breakdown of the long-range positional order of the flux line lattice [5] due to the interaction with a dense system of randomly distributed pinning centers. Recently, Thuneberg et al. [6] calculated the flux pinning brought about by quasiparticle scattering of small spatial extent defect. Also described in the work is a typical field dependence of core pinning as $f_p \propto (1-b)b^{3/2}$, where $b = B/(\mu_0H_{c2})$. The
grain boundary pinning is mainly caused by the enhanced electron scattering at the grain boundary. Zerweck [7] first worked out the idea, and later it was improved by Yetter et al. [8] and Welch [9]. Experimentally, both the magnitude of local change in $H_{c2}$ and anisotropy of $H_{c2}$ have been proposed as a cause of pinning, as demonstrated experimentally in a Nb bicrystal [10,11]. An excellent review, both theoretical and experimental, on most recent developments has been given by Kes [12].

Numerous works have been devoted to the study of collective pinning and the measurements of the bulk pinning force, for which massive arrays of vortices are moved by the transport current so that the measurement produced average of the pinning forces [13,14]. There have been no reports of a direct measurement of the elementary pinning force, $f_p$, for which the pinning force is associated with an individual pinning site. Cai et al. [15] reported the measurements of the pinning force for a vortex lattice in a Nb bicrystal. In this case, many vortices were involved and the pinning center was a single grain boundary between the two crystals. Here we report the measurements of the elementary pinning force associated with one vortex at a particular pinning center using a SNS Josephson junction.

One of the special aspects of superconductivity is the occurrence of Josephson tunneling [16], in which the supercurrent flows from one superconductor to another through a barrier. One of the most popular geometry of the Josephson junction consists of two superconducting films separated by a thin insulator or normal metal barrier. The Josephson current density is given by the Josephson equation, $J = J_0 \sin(\gamma)$, where
\( J \) is the Josephson current density and \( \gamma \) is the gauge invariant phase difference across the junction. Since the phase is dependent on the local magnetic field, the vortex field induces a phase change which can affect the current density. The prime interest of the present research is the change of the current characteristic of a Josephson junction when it contains vortices. This is a particularly important problem because the junctions are used in SQUIDs and other electronic devices, all of which are vulnerable to the presence of vortices. In addition to the measurement of the elementary pinning force, these practical effects provide another motivation of the present work.

It was predicted theoretically that a thin superconducting film will trap flux quanta (vortices) at a density of \(-H/\phi_0\), when it is cooled through the superconducting transition temperature \( T_C \) while subjected to a magnetic field normal to the plane of the film [17]. Many studies of vortex arrays have been made and research on Josephson junctions containing vortices is not new. Experimental work was started by Band and Donaldson [18] who investigated the superconductor-superconductor tunneling behavior with insulating barrier containing a dilute vortex state. They found that the vortex core to core tunneling is negligibly small compared to the usual superconductor-superconductor tunneling. After that, considerable attention was given to the effect of the vortices trapped in the superconducting electrode on the critical current [19,20]. Fulton et al. [21] first observed discrete quantized increases in the differential conductance of SIS tunnel junctions, while they attributed to the presence of individual vortices in the electrode. Hebard and Eick [22] confirmed that flux trapping reduces the critical
current of Josephson junctions, an effect due to a parallel component of the field in the junction resulting from vortex misalignment. Also Washington and Fulton [23] observed that flux trapping occurred in the field cooling process in which the junction is cooled in the presence of perpendicular magnetic field greater than a minimum threshold value. The collective effects of hundreds of trapped vortices on the differential conductance of SIS junctions was investigated by Uchida et al. [24] with the suppression of the critical current due to an individual trapped vortex inferred. In addition to those vortex effects on tunneling, the trapped flux on the performance of electronic devices such as SQUID, high speed digital logic circuits and so on, is of interest. Chang [25] theoretically investigated the effect of the trapped magnetic flux on the threshold curve of a three junction Josephson interferometer and found severe distortion of the curve in shape by the presence of coupled vortices. Also Bermon and Gheewala [26] studied the effect of vortex in SQUID.

So far, all experiments were related multi-vortex state and did not pay attention to the detailed current distribution. As we shall see later, since the current distribution is uniquely related to the vortex configuration in the junction containing vortices, understanding the electric properties of junctions is closely related to finding the vortex configuration.

In the past few years, there have been remarkable advances in the ability to find the vortex distribution trapped inside the junction via the Josephson current distribution using both (1) laser beams [27,28] and (2) electron beams [29,30] in an electron microscope. In the former
case of (1), a focused laser beam irradiation changes the local temperature which then modifies the maximum current density, London penetration depth, $\lambda_L$, and Josephson penetration depth, $\lambda_J$. In the presence of magnetic field, the change of $\lambda_L$ induces a non-local long range change of the current density. In the latter case of (2), a modulated electron beam is used for direct detection of vortices in a Josephson tunnel junction. By scanning the surface of a current biased tunneling junction in the voltage state with the electron beam, quasielectron injection into the junction causes an intrinsic instability above a certain threshold current, resulting the change of $\Delta V$ of the junction voltage [31]. This effect can be explained by local heating effect by the electron beam [32]. This change of voltage then serves to generate 2-D voltage image of the sample properties such as the spatial distribution of the quasiparticle tunneling current density, energy gap of the superconducting electrodes etc. [Ref. 29]. Once the current density distribution is known, one can readily infer the vortex configuration according to the theory given in the chapter 2 of this thesis.

In addition to these external beam methods, there has been a new procedure for detecting the vortex configuration using the diffraction patterns, $I_C$ vs. $H_\parallel$ (or $H_\perp$), as signatures to specify the location of trapped vortices [33]. Since this diffraction pattern technique requires no external beam, it is a particularly convenient procedure for finding the presence of a vortex in an electronic circuit where it is impractical to use the external beams. J. Clem and K. Biagi developed the theory for vortex induced phase and corresponding current density,
as described in the Chapter 2. Also S. Miller and D. Finnemore first tried to locate a vortex, trapped by the field cooling process, but with only limited success [Ref. 33].

In order for the theory to work, significant magnetic field regions due to trapped vortices must be developed inside the junction. Since the core size of a vortex is negligibly small compared to the size of the junction, an aligned vortex pair located in both superconducting layers can not generate a large region of parallel magnetic field, thus causing only trivial effect on the current density. This is why, for instance, Uchida et al. could not see very large change of the critical current even though the junction trapped many vortices [Ref. 24]. In SIS junction with dielectric barrier such as Nb-NbO$_x$-Pb tunnel junction [34], the barrier thickness is typically 20 ~ 50Å. In this case, as one can easily expect, the coupling force of the two facing poles in both superconductors is so strong due to small separation that the vortices are usually aligned to leave a negligible parallel magnetic field region [35]. In the SNS junction case, on the other hand, the barrier thickness is typically 1000 ~ 10000Å, about 100 times greater than that of a dielectric barrier. Therefore the coupling force could be greatly reduced by the large separation, then allowing possibility of substantial misalignment of the paired vortices when the pinning force of vortex on each superconductor exceeds the coupling force. Thus a significant magnetic field region can be generated by the vortices such that the current density and the critical current are highly dependent upon the vortex configuration.
In contrast to the SIS junction case, for which considerable research have been done, there has been relatively little work to investigate the SNS junction system. Werthamer [36], de Gennes and Guyon [37], and de Gennes [38] have worked out the basic theory for the critical current of a SNS junction based on the proximity effect. Aslamazov et al. [39] have more thoroughly examined the problem of metal barrier junctions, obtaining the critical current assuming low transparency of the SN interfaces. Barone and Ovchinnikov [40] obtained the boundary conditions for the order parameter via the Greens function of the normal metal. As a more recent work, Makeev et al. [41] have obtained the Josephson current of SNS junctions containing paramagnetic impurities.

Experimentally, Clarke [42] investigated the temperature, normal metal thickness and mean free path dependence of the critical current of SNS junctions in the dirty limit. In related experiment Hsiang and Finnemore [43] have studied the critical current in the clean limit. Patterson [44], Niemeyer and Minnigerode [45], and more recently Yang and Finnemore [46] extended to systems where the normal metal is alloyed with magnetic impurities.

The present research is based on the work by S. Miller [47], who widely investigated the electrical properties of cross type SNS junctions fabricated by Pb-Bi(2.5 atomic percent) as superconductor and Ag-Al(4 atomic percent) as normal metal. He showed that excellent Josephson junctions could be made with the normal metal thickness of about 5000Å using the above materials. Also he observed that trapped vortices in the SNS junction by the field cooling process produced a
step-like critical current pattern as a function of the cooling perpendicular magnetic field to the junction.

As mentioned above, there are two main motivations of the present research. First, the effect of vortex on the Josephson junction characteristic should be understood because of its application to SQUIDs, electronic circuits, etc., all of which are very vulnerable to the presence of trapped vortices. Secondly, the elementary pinning force of an individual pinning site can be measured by studying the response of a trapped vortex to an applied force. The long term goal of this work is to build new devices based on the systematic manipulation of vortices within a Josephson junction.

In the present work, the primary emphasis will be put on the study of the Abrikosov vortex motion inside a SNS Josephson junction. In the chapter 2 we discuss the theory. In the chapter 3, we discuss how the sample is fabricated and the measurements are done. This chapter also includes the preliminary examinations to prove excellent quality of the Josephson junction. In the chapter 4, we present main experimental results. A vortex is introduced by the field cooling process, which allows a single vortex pinning. Then the vortex is depinned and moved to other pinning site by the transport current through one leg of superconducting layers. For each pinning the vortex will be located from the diffraction patterns using the theory. The minimum depinning condition will provide the elementary pinning force of an individual pinning site of the superconducting layer. Finally, we will see that an easier way of reading vortex position and controlled vortex pinning can provide an application to generating a digital signal.
2. THEORY AND MODEL CALCULATION

In this chapter we discuss a model to describe the effects of a vortex in an SNS Josephson junction and calculate the electrical properties of the junction. Both fundamental assumptions of the model and some estimate of the limitations are presented.

Consider a superconducting thin film layer containing a vortex. If this film is covered by a normal metal layer, magnetic flux will penetrate the normal metal and have little effect on the flux lines from the vortex. If, in turn, another superconducting layer is put on the top of the normal metal layer at right angle to the first film, a cross strip Josephson junction is formed as shown in Fig. 2.1. The top superconducting layer expells magnetic flux from the vortex such that the flux lines are confined into the normal metal region in the junction area. As a result of this screening, a parallel magnetic field is generated in the normal metal area. The parallel field has spatial dependence associated with the position of the vortex. The field, in turn, generates gauge invariant phase difference between the two superconductors (top and bottom), so the Josephson junction property is now determined by the vortex configuration. Because the relation between vortex location and the critical current density is unique except geometrical symmetries, we can study the vortex and vortex motion confined in a superconductor of the junction using the junction critical current characteristics. In this chapter we will see how the vortex location is related to a Josephson junction characteristics.
(a) Vortex in a superconducting strip
(b) Normal metal layer is deposited
(c) Second superconducting layer is deposited. The top S layer then expells the flux lines due to the vortex such that the flux lines are confined in the normal metal region

Fig. 2.1. Vortex in a SNS junction
2.1. Josephson Critical Current

The Josephson current density in the z direction for an SNS junction in the x-y plane is expressed in the form

\[ J_z(r) = J_0(r) \sin \gamma(r) \]  

(2.1)

where \( \gamma(r) \) is the gauge invariant phase difference across the junction at the polar coordinate \( r = (x,y) \) and \( J_0 \) is a temperature dependent amplitude. Generally the phase, \( \gamma(r) \), satisfies two-dimensional (2-D) sine-Gordon equation [48]. In steady state the sine-Gordon equation is

\[ \nabla^2 \gamma = \frac{\sin \gamma}{\lambda_J^2} \]  

(2.2)

where \( \lambda_J = \left( \frac{\kappa}{8 \pi \rho_0} \right)^{1/2} \) is the Josephson penetration depth.

This penetration depth is a measure of the length within which d.c. Josephson currents are confined near the edges of the junction. This defines two classes of junctions: "small" and "large junctions. In small junction \( \lambda_J \) is larger than the size of the junction (W) and the current density throughout the junction area is essentially uniform. In large junction where \( \lambda_J < W \), on the other hand, self induced field is not negligible and the currents are confined to the edges of the junction. In the experiments described here, we will concentrate on the small junction limit, where \( J_0 \) has no spatial dependence, except a few cases to observe self field effect.

In the presence of magnetic field, \( H \), inside the junction, the phase \( \gamma \) varies as [49]

\[ \nabla^2 \gamma(r) = \frac{2 \pi \rho_0}{\phi_0} H \times \hat{z} \]  

(2.3)
in the same coordinate system as in Eq. (2.1), where \( \nabla_2 = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \),
a 2-D gradient vector and \( \phi_0 \) is the flux quantum, \( 2.07 \times 10^{-7} \) gauss-cm\(^2\).
The effective thickness, \( d_{\text{eff}} \), is the thickness where the field penetrates and given by [50]
\[
d_{\text{eff}} = d_n + \lambda_1 \tanh\left( \frac{d_1}{2\lambda_1} \right) + \lambda_2 \tanh\left( \frac{d_2}{2\lambda_2} \right)
\]  
(2.4a)
where \( d_n \) is the thickness of the normal metal. The subscripts 1 and 2 denote the bottom and the top superconducting layers of the junction respectively. When the two superconducting layers are of identical material and much thicker than \( 2\lambda \), then the equation reduces to
\[
d_{\text{eff}} = d_n + 2\lambda
\]  
(2.4b)
Here it is useful to see the physical meaning of Eq. (2.3). By integration, we get
\[
\gamma(r) = \left( \frac{2md_{\text{eff}}}{\phi_0} \right) \cdot (xH_y + yH_x) + \text{constant}
\]
\[
= \left( \frac{2\pi}{\phi_0} \right) \mathbf{H} \cdot \hat{\mathbf{r}} d_{\text{eff}} + \text{constant}
\]  
(2.5)
So the phase is the ratio of the total flux normally threading the area of \( d_{\text{eff}}^2 \times r \) to the flux quantum, \( \phi_0 \). As an example, there is \( 2\pi \) change in phase for every \( \phi_0 \) of flux.

As given in the Eq. (2.3), the magnetic field dependent phase, \( \Theta(r) \), is included by rewriting \( \gamma(r) \) as
\[
\gamma(r) = \gamma_0 + \Theta(r)
\]  
(2.6)
with
\[
\Theta(r) = \frac{2\pi}{\phi_0} \hat{\mathbf{r}}(r)
\]  
(2.7)
\( \gamma_0 \) is the field independent phase. \( \Phi(r) = d_{\text{eff}} \int \mathbf{H} \cdot (\mathbf{z} \times d\mathbf{r}) \) is the flux enclosed between the \( z \) axis and the radial coordinate \( r \) in the normal metal plus the penetration layers.

The total current, \( I \), is found by the integration of the Josephson current density over the junction area after inserting Eq. (2.6) into Eq. (2.1).

\[
I = \iint dxdy J_0 \sin(\gamma_0 + \Theta(x,y))
= \sin(\gamma_0) \iint dxdy J_0 \cos(\Theta(x,y)) + \cos(\gamma_0) \iint dxdy J_0 \sin(\Theta(x,y)) \tag{2.8a}
= I_1 \sin(\gamma_0) + I_2 \cos(\gamma_0) \tag{2.8b}
\]

where \( I_1 \) and \( I_2 \) are the cos and sin integrations over the junction area respectively. \( \gamma_0 \) is to be determined such that the current has the maximum value. Maximization of the Eq. (2.8) yields \( I_c \), the critical current. By taking variation with respect to \( \gamma_0 \), we get

\[
\delta(I)I=I_c = (I_1 \cos \gamma_0 - I_2 \sin \gamma_0) \delta \gamma_0 = 0
\]

\[
\tan \gamma_0 = I_1 / I_2
\]

\[
\gamma_0 = \tan^{-1}(I_1 / I_2) \tag{2.9}
\]

which gives the critical current density. Also,

\[
\sin \gamma_0 = (1 + \cot^2 \gamma_0)^{-1}
= I_1 / \sqrt{(I_1 + I_2)} \tag{2.10a}
\]

Similarly,

\[
\cos \gamma_0 = I_2 / \sqrt{(I_1 + I_2)} \tag{2.10b}
\]

Finally, by inserting Eq. (2.9) into Eq. (2.8) and using the expression \( I_0 = \iint J_0 dxdy \), the maximum Josephson current, we obtain the normalized critical current of [Ref. 33]

\[
I_c / I_0 = \{ \langle \sin \Theta(x,y) \rangle^2 + \langle \cos \Theta(x,y) \rangle^2 \}^{1/2} \tag{2.11}
\]

where the brackets \( \langle \ldots \rangle \) denote spatial averages over the junction area.
Fig. 2.2. Schematic of junction geometry and symmetric current configuration
Here we note the ambiguity of the constant term in Eq. (2.5), which is self-adjustable, meaning no absolute reference point where the phase is to be zero. As we shall see later, this ambiguity provides enormous convenience in handling the phases.

Now we are to find the appropriate expression of $\Theta(x,y)$. Consider a square junction, of width $W$ and thickness $d_n$ sandwiched between two crossed superconducting strips, each of thickness $d_s$ as shown in Fig. 2.2. The junction lies in the $x$-$y$ plane and centered at the origin such that the junction extends from $-W/2$ to $+W/2$ in $x$ and $y$ directions respectively and from $-d_n/2$ to $+d_n/2$ in $z$ direction. Frequently we will use the reduced coordinate, in which the junction extends from $-1$ to $+1$ in $x$ and $y$ directions. We also assume small junction limit, $\lambda_j \gg W$, so that self field effects are negligible.

2.2. External Field Dependence

2.2.1. Parallel field

The spatially dependent term for the parallel field of $H_y$ is obtained from Eq. (2.5)

$$\Theta(H_y) = 2m_{\text{eff}} H_y x / \phi_0$$  \hspace{1cm} (2.12)

The normalized critical current to $I_0$ is

$$I_c / I_0 = \frac{1}{W^2} \left[ \int_{-W/2}^{+W/2} \int_{-W/2}^{+W/2} \left( \frac{2m_{\text{eff}} H_y}{\phi_0} x \right)^2 + \int_{-W/2}^{+W/2} \int_{-W/2}^{+W/2} \left( \frac{2m_{\text{eff}} H_y}{\phi_0} x \right)^2 \right]^{1/2}$$
The sine term vanishes by symmetry.

\[
\frac{I_C}{I_0} = \left| \frac{1}{W^2} \int_{-W/2}^{W/2} \int_{-W/2}^{W/2} \cos \left( \frac{2n_{\text{eff}} H_y}{\phi_0} x \right) y^2 \right|
\]

\[
= \left| \frac{\sin \left( \frac{n_{\text{eff}} H_y}{\phi_0} \right)}{\frac{n_{\text{eff}} H_y}{\phi_0}} \right|
\]

\[
= \left| \frac{\sin \left( \frac{\phi}{\phi_0} \right)}{\left( \frac{\phi}{\phi_0} \right)} \right|
\]  \hspace{1cm} (2.13)

where \( \phi = n_{\text{eff}} H_y \), total flux threading the junction parallel to the flat surface. This is the Fraunhofer pattern. The critical current undergoes periodic oscillations in \( H \) and the magnitude decreases as \( 1/H_y \) as \( H_y \) increases.

As specified above, this equation is not true for magnetic field other than a uniform field parallel to junction. If fields include other sources than the parallel one, for instance a vortex, the pattern, \( I_C vs. H \), is no longer the Fraunhofer pattern. If the fields are not very strong, however, the pattern, \( I_C vs. H \), shows that \( I_C \) oscillates and decreases in \( H \), just as it does in a Fraunhofer pattern. By this reason, we will call the pattern, \( I_C vs. H \), as a diffraction pattern.

To simplify the Eq. (2.12), we introduce \( H_1 \) defined by

\[
H_1 = \frac{\phi_0}{n_{\text{eff}}}
\]  \hspace{1cm} (2.14)

which is the field intensity to give the magnetic field threading the junction by one flux quantum and is the first minimum value of \( H \) in the Fraunhofer pattern. So, the phase due to the parallel magnetic field can be rewritten as

\[
\Theta(H_y) = \left( \frac{2n}{W} \right) \left( \frac{H_y}{H_1} \right) x
\]  \hspace{1cm} (2.15)
2.2.2. Perpendicular Field

When a perpendicular magnetic field is applied to a plane of a junction, a complicated distribution of induced screening currents and magnetic fields are produced at the surface of the superconductors. Hebard and Fulton [51] first discussed the perpendicular field problem and showed that these induced screening currents feed into the interior of the junction, where they generate local magnetic field parallel to the plane of the junction. This field then alters the phase, $\gamma(x,y)$, across the junction, thereby producing a spatially varying Josephson current density, $J_z(x,y) = J_0 \sin \gamma(x,y)$. In addition, Miller et al. [Ref. 33] obtained the appropriate expressions for the interior magnetic field and resultant phase across the junction for a cross type SNS Josephson junction. They obtained the linear screening current density, $K$, magnetic field inside the junction, $B$, and the corresponding phase, $\Theta$, in the first approximation as

\begin{align*}
K(x,y) &= \frac{cH_z}{nW} (y\hat{x} + x\hat{y}) \quad (2.16) \\
B(x,y) &= \frac{4H_z}{W} (x\hat{y} - y\hat{x}) \quad (2.17) \\
\Theta(x,y) &= - \frac{8\pi Ud_{\text{eff}}}{\Phi_0} H_z xy \\
&= - \frac{8\pi}{\Phi_0} \left( \frac{H_z}{H_1} \right) xy \quad (2.18)
\end{align*}
Fig. 2.3. Field lines inside the junction and induced current in the top surface (inner surface) of the bottom superconductor due to the perpendicular field along the $+z$ axis. The $x$ and $y$ coordinates are in units of $W/2$. 
since $H_1 = \frac{W_{\text{eff}}}{\Phi_0}$. This solution does not apply for an arbitrary rectangular junction. So a square geometry is highly desired for the use of this result.

The critical current is now calculated by substituting Eq. (2.18) into Eq. (2.11). By symmetry the $\langle \sin \theta(x,y) \rangle$ term drops, leaving

$$I_c/I_0 = \frac{1}{W^2} \int_{-W/2}^{W/2} \int_{-W/2}^{W/2} dx dy \cos \left( \frac{8\pi H_y}{W^2 H_1} xy \right)$$

$$= \frac{1}{W^2} \int_0^W dx' \sin \left[ \frac{(2\pi H_z/H_1)x'}{(2\pi H_z/H_1)x'} \right]$$

Let $\alpha = \frac{2\pi H_z}{H_1}$, and $t = \alpha x'$, then

$$I_c/I_0 = \frac{1}{\alpha} \int_0^\alpha \sin(t) \frac{dt}{t}$$

$$I_c/I_0 = \frac{\text{Si}(\alpha)}{\alpha} \quad (2.17)$$

where $\text{Si}(\alpha) = \int_0^\alpha \frac{\sin(x)}{x} dx$, the sine integral, and $\alpha = \frac{2\pi H_z}{H_1}$.

Expanding $\text{Si}(\alpha)$ in terms of $\alpha$ reveals that $I_c$ decreases quadratically with $H_z$ increase at small field ($\alpha \ll 1$) and as $1/H_z$ for large field ($\alpha \gg 1$). A good fit to this behavior was found by Miller et al. [Ref. 33].
2.3. Vortex Induced Field Dependence

When a vortex is trapped in superconducting layer within the junction area as shown in Fig. 2.1, one pole of the vortex is inside the sandwiched area and the other pole is outside the junction. An interior pole may act as a source (or sink) of magnetic flux with total flux equal to $\Phi_0$, while the outer pole is completely shielded by the superconductor so that it will not have any influence to the current characteristics of the junction. Therefore a vortex may be regarded as a magnetic monopole as long as we are concerned about the Josephson current only.

There are two types of interior vortices, N pole and S pole, depending on the direction of the source field. For example, when $H_z$ is positive and is directed from the bottom layer to the top layer, the bottom layer is supposed to contain only N pole(s), while the top layer contains S pole(s) only. Since the size of a vortex is much smaller than the size of the junction, the magnetic field lines from an N pole in the top layer are same as those of the N pole in the bottom layer. So we do not distinguish in the theory which layer contains the vortex. If the vortex is N pole, we will also call the vortex as a "positive" vortex, or + pole, whichever layer the vortex locates. Similarly, we call a vortex as a "negative" vortex, or - pole, if the vortex is S pole inside the junction.

There may be many vortices within the junction, that is, many positive or negative vortices. They can be in the same layer or in
(a) A misaligned dipole vortex in a SNS junction. t and b denote the top and the bottom superconductor respectively

(b) Theoretically equivalent dipole to (a). The theory treats the flux lines inside the junction only

(c) Linear superposition

(d) Magnetic monopole charge approximation

Fig. 2.4. Theoretical treatment of a vortex
different layer. Also they can be magnetically coupled each other. Regardless of all the above situations, we will assume that any one of the vortices is individual to the others, so that total magnetic field arising from the vortices inside the junction is the linear superposition of the fields from them (Fig. 2.4). Extending the assumption to external fields, we get the total phase difference across the junction by linear superposition of phases contributed by all individual sources.

2.3.1. Vortex dependent field and phase

As discussed early in this chapter, the flux lines due to a vortex are confined inside the junction. The magnetic field intensity due to one vortex decreases radially, and is given by [Ref. 33]

$$B_{\text{vortex}} = \pm \frac{\Phi_0}{2\pi} \frac{(r - r_{+,-})}{|r - r_{+,-}|}$$

where $r_{+,-}$ is the vector from the origin to the vortex. The + sign and the subscript + are associated with a positive vortex, and the - sign and - are associated with a negative vortex. Magnetic field lines due to a positive vortex are shown in Fig. 2.5. We know that the film confined magnetic flux between the $z$ axis and $r$ arising from a positive vortex alone at $r_+$ is $\frac{\Phi_0}{2\pi} \Theta_+(r)$, where $\Theta_+$ is the angle between $-r_+$ and $r - r_+$ as shown in Fig. 2.5. Similarly, the flux due to a negative vortex is $-\frac{\Phi_0}{2\pi} \Theta_-(r)$. As in Eq. (2.7), the total flux $\Phi(r)$ is the sum
of all contributions, which gives total phase difference across the
junction, using the coordinates \( r = (r, \theta) = (x, y) \) and \( r_{+,-} = (r_{+,-}, \theta_{+,-}) \)

\( = (x_{+,-}, y_{+,-}) \), of

\[ \Theta_v(r) = \Sigma_{+} \Theta_+(r) - \Sigma_{-} \Theta_-(r) \]  

(2.19)

with

\[ \Theta_{+,-}(r) = \cos^{-1} \left[ \frac{r_{+,-} - r \cos(\theta - \theta_{+,-})}{|r - r_{+,-}|} \right] \]  

(2.23)

and

\[ |r - r_{+,-}| = \left( (x - x_{+,-})^2 + (y - y_{+,-})^2 \right)^{1/2} \]

where \( 0 < \theta_{+,-} < \pi \) when \( 0 < \theta - \theta_{+,-} < \pi \) and \( -\pi < \theta_{+,-} < 0 \) when \( -\pi < \theta - \theta_{+,-} < 0 \). Here, the reference point of zero phase is the origin.

Now we take advantage of the freedom of choosing the arbitrary
constant in the phase. For simplicity, let the vortex be a positive
vortex at \( r_{+} = (x_0, y_0) \). After simple manipulation of Eq. (2.23) or
from Fig. 2.5, we find the phase at \( r = (x, y) \) due to the vortex as, in
the \( x-y \) coordinates,

\[ \Theta_+(x, y) = \pi + \tan^{-1}(y_0/x_0) - \tan^{-1}\left[ (y-y_0)/(x-x_0) \right] \]

The first two terms are constants, so we rewrite the equation as

\[ \Theta_+(x, y) = -\tan^{-1}\left[ (y-y_0)/(x-x_0) \right] \]  

(2.24)

where \( \pi + \tan^{-1}(y_0/x_0) \) is omitted because \( y_0 \) in Eq. (2.6) will contain
it. This is equivalent to take the reference point of zero phase to be
\( x \to -\infty \). Eq. (2.24) reduces the complexity of Eq. (2.23) enormously.

In deriving Eq. (2.23), we assumed the core effect to be
insignificant, since their influences on \( I_C \) scales with the ratio of
the core area to the junction area, which is negligibly small in the
Fig. 2.5. Construction used to calculate the phase at \((x,y)\) due to a positive vortex at \((x_0, y_0)\). The phase is the angle between 
\(-r_0\) and \(r-r_0\), \(\Theta_1\), which includes the constant term of \(\Theta_0\).
So the phase can be reduced to \(\Theta_2\), or equivalently \(\Theta_3\) 
\[= -\tan^{-1}\left(\frac{y-y_0}{x-x_0}\right)\]
experiment. In addition, no vortex image were included, and any vortex generated flux leaking out of the junction was not considered. As a result, the accuracy of Eq. (2.23) is questionable when individual vortex is near an edge. We will see later that the experimental data well fits the theory only after the image correction is included for a vortex near the edge.

2.3.2. Correction with images

When a SNS junction contains a vortex inside, screening current are generated to exclude flux lines from the S layers which then confine the flux lines due to the vortex in the junction region. The screening current is circular around the vortex, resulting the magnetic field direction to be radial in the x-y plane. However, the current is to be parallel to the edge at the boundary of the junction, so that field lines are perpendicular to the edge. This specific flux line configuration can be achieved by introducing image vortices outside the junction area.

The images play little role when the vortex is in the middle of the junction, but a significant role when it is near the edge. By assuming the vortex to be a magnetic monopole charge, the problem becomes mathematically same as the 2-D electrostatic problem, where an electric charge is in a grounded rectangular box. The charge generates infinite number of image charges outside the box. Moreover, the images distribute all over the x-y plane to form a periodic lattice, as
Fig. 2.6. Image vortex lattice
schematically shown in Fig. 2.6. Each lattice contains four images, two positive poles and two negative poles crossing over each other. There are two methods to estimate the phase due to the vortex and the images.

1) Direct summation of phases:
This is a somewhat primitive way of estimating the phase arising from the vortex and all images. In this scheme, finite number of images within a pre-set boundary are selected since the images far away from the junction will have little effect. For the real vortex and those selected image vortices, the phase for each vortex is calculated according to Eq. (2.23) or Eq. (2.24), then summed up to give the total phase.

$$\Theta_{\text{vortex}} = \Theta_{\text{real vortex}} + \Theta_{\text{all images}}$$ (2.25)

As the number of images grows, the result of Eq. (2.25) becomes closer to the exact solution. Practically, 24 images came out to be good approximation (Fig. 2.7a).

2) Exact solution - infinite number of images:
Consider a rectangular box in complex $z = x + iy$ plane containing a positive vortex at $z_0 = x_0 + iy_0$. We define a function $F$ as

$$F = \Psi + i\Gamma$$

$$= -\ln(z-z_0)$$

$$= -(1/2)\ln[(x-x_0)^2 + (y-y_0)^2] - \tan^{-1}[(y-y_0)/(x-x_0)]$$ (2.26)

Thus,

$$\Psi = \Re(F) = -(1/2)\ln[(x-x_0)^2 + (y-y_0)^2]$$ (2.27)

$$\Gamma = \Im(F) = -\tan^{-1}[(y-y_0)/(x-x_0)]$$ (2.28)

Then $\Gamma$ is found to be the phase of the vortex. In the same way,
utilizing the Schwartz-Christoffel transformation, the exact solution of estimating the phase from all images was obtained by J. Clem [52]. In this calculation, the box in the $z$ plane is transformed into the upper half part of the $w = u + iv$ plane. Then the vortex at $v_o$ (corresponding to $z_o$) has only one image at $v_o^* = u_o - iv_o$. So we define the function $F$ as

$$F = \ln[(w - w_o^*)(w - v_o)]$$

$$= Y + i\Gamma \tag{2.29}$$

Then $Y$ is the magnetic potential and $\Gamma$ is the phase of the two vortices.

$$Y = (1/2)\ln\left[\frac{(u-u_o)^2 - (v+v_o)^2}{(u-u_o)^2 - (v-v_o)^2}\right] \tag{2.30}$$

$$\Gamma = -\tan^{-1}\left[\frac{v-v_o}{u-u_o}\right] + \tan^{-1}\left[\frac{v+v_o}{u-u_o}\right] \tag{2.31}$$

The transformation back to the $z$ plane is [53]

$$w = \text{sn}^2(Kz/W, k) \tag{2.32}$$

where $\text{sn}$ is the Jacobi elliptic function of [54]

$$\frac{1}{\text{sn}(u,k)} = \int_0^u \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}}^{1/2}$$

$$= \frac{\pi}{2K} \cosec\left(\frac{\mu}{2K}\right) + \left(\frac{2\pi}{K}\right) \sum_{m=1}^\infty \frac{\sin(\mu/2K) \cdot \cosh(nm'/K)}{\cosh(2nm'/K) - \cos(\mu/K)} \tag{2.34}$$

So the phase of the positive vortex at $z_o = x_o + iy_o$ is given by the imaginary part of the function $F$ as

$$\Theta_+(\text{vortex}) = \Gamma = \text{Im}\{\ln[(w-v_o^*)(w-w_o)]\}$$
Fig. 2.7a. Theoretical diffraction patterns, $I_c$ vs. $H_y$, with and without image vortices. Twenty-four images turn out to be good approximation to the exact solution. The assumed vortex is positive one at $(0.4, -0.1)$ in reduced coordinates.
Fig. 2.7b. Theoretical diffraction patterns, $I_c$ vs. $H_z$, with and without image vortices for the same vortex as in Fig. 2.7a.
\[ \text{Im} \ln \left[ \frac{\text{sn}^2(Kz/W, k) - \text{sn}^2(Kz/W, k)}{\text{sn}^2(Kz/W, k) - \text{sn}^2(Kz/W, k)} \right] \quad (2.33) \]

For a square junction, \( K = K' = 1.854075 \) and \( k = \sqrt{1/2} = 0.707107 \) [54], putting \( u = Kz/W \), we have

\[ \frac{1}{\text{sn}(z)} = \frac{\pi}{2K} \left( \csc \left( \frac{\pi z}{2W} \right) + 4 \sin \left( \frac{\pi z}{2W} \right) \sum_{n=1}^{\infty} \frac{\cosh(n\pi)}{\cosh(2n\pi) - \cos(\pi z/W)} \right) \quad (2.35) \]

If the vortex is a negative one, the phase is to be multiplied by -1.

For the critical current calculations we will take either way of estimating the phases, Eq. (2.25) or Eq. (2.33), depending on the situation. Both methods should produce exactly the same current density distribution. According to Eq. (2.9), \( \gamma_0 \) is found to be \( \pi/2 \) if the origin is the reference point of zero phase, while \( 3\pi/2 + \tan^{-1}(\gamma_0/x_0) \) if \( x \to -\infty \) is the reference point. For the current density distribution in the rest of this chapter, the origin will be taken to be zero phase point because it provides a better way of understanding.

2.3.3. Vortex dependent current density

The critical current density \( J_C(r)/J_0 \) corresponding to Eq. (2.25) is shown in Fig. 2.8a for a single positive vortex at \( r_+ = (+0.01,+0.01) \) in reduced coordinates, near the center of the square junction. We know that the phase due to the vortex changes discontinuously from \(-\pi\) to \(+\pi\) as the observing point crosses the diagonal of the 1st quadrant. In addition, the phase is 0 around the diagonal line in the 3rd quadrant. Since the constant term in phase, \( \gamma_0 \), is found to be \( \pi/2 \) in this case,
Fig. 2.8a. Josephson critical current density, \( J_c \), at zero field due to a single positive vortex at (0.01, 0.01) in the unit of W/2. All images are included in the calculation.
Fig. 2.8b. $J_c$ due to a single positive vortex at (0.5, 0.5)
Fig. 2.8c. $J_c$ due to a single positive vortex at (0.5, 0.0)
Fig. 2.8d. $J_c$ due to a single positive vortex at (0.4, 0.0). No images are included. The flux lines do not cross the edges at right angle, but oblique angles
the current distribution has the lowest value \((-J_0)\) at the right-upper corner, and the highest value \((+J_0)\) at the left-lower corner. Along the diagonal of the 1st quadrant, it makes a gradually narrowing valley and abruptly rising up to \(+J_0\) near the center. It also shows a ridge gradually narrowing along the diagonal of the 3rd quadrant and dropping off rapidly to \(-J_0\) near the center, while remaining slowly varying current density in the 2nd and 4th quadrant with negative value near the 1st quadrant and positive near 3rd quadrant of the junction. This distribution is perfectly symmetric with respect to the diagonal line passing through the 2nd and 4th quadrant along which the current density is zero. So the total current is zero (with no external field). The positive contribution to the Josephson current grows as the vortex moves toward the right-upper corner along the diagonal line as shown in Fig. 2.8b and Fig. 2.8c, increasing the net current. Also one can see that the negative current contribution increases as the vortex moves to the 3rd quadrant, for instance \(r_c = (-0.1, -0.1)\), resulting negative total current. Here one must note that only the magnitude of the total current has physical meaning because the current characteristics is to be same in both \(+\) and \(-\) z directions. So the current distribution as a function of the vortex position has four fold symmetry in a square Josephson junction as the junction geometry does.

The magnetic field distribution inside the junction is of interest. The magnetic flux from a monopole (a + pole only) spreads out radially, influencing over the junction area. The field lines are passing the boundaries with right angles due to image vortices, meaning parallel screening current at the edges.
With two vortices involved, there are two kinds of combinations of them, a positive-negative dipole case and a positive-positive (or negative-negative) vortex pair case. A positive-negative dipole is particularly of interest since it is energetically more favored than a single vortex or a pair of same vortices. Fig. 2.9a shows $J_c/J_0$ for a dipole consisting of a primary at $(x_+, y_+) = (0.2, 0.2)$ in the units of $W/2$ and a secondary at $(x_-, y_-) = (-0.2, -0.2)$. The current distribution drops off rapidly to a negative value with minimum of $-J_0$ in the central region between the vortices while retaining fairly uniform elsewhere in the junction, saying a dipole has small effect on the current density at $r$ far from the dipole, or $|r - r_+| \gg |r_+ - r_-|$, where $|r_+ - r_-|$ is the dipole strength or misalignment distance. So, if a dipole is aligned, or $|r_+ - r_-| = 0$, the current distribution is uniform throughout the junction area (neglecting core effect) and $I_c = I_0$. On the other hand, an increased dipole strength enhances the negative contribution to the Josephson current and suppresses the positive contribution, reducing the zero field current.

The resulting magnetic field distribution inside the junction is shown in Fig. 2.9b for the same dipole as discussed above. The magnetic field is strong between the two poles, and weak outside the dipole, differing from that of a monopole. Note that, although $\Theta_r \to 0$ for $r$ far from the dipole, $\Theta_r = \pi$ along the line connecting the two vortex axes, such that $J_c(r) = -J_0$ there. For this reason, $I_c/I_0$ is roughly approximated by $1 - 2(\delta/W)^2$, where $\delta = |r_+ - r_-|$, the dipole strength, and $W^2$ is the junction area [Ref. 26].
Fig. 2.9a. $J_e$ of a dipole vortex at +(0.2, 0.2) and -(0.2, -0.2)
Fig. 2.9b. Magnetic field lines due to the same dipole vortex as in Fig. 2.9a. The field lines are same as the equi-current density lines.
Now we have a simple rule about the current distribution due to vortices in a Josephson junction. The critical current density is expected to follow the magnetic field distribution pattern. Since the phase due to a vortex changes by $\pi$ at the vortex position, the critical current density has a discontinuity varying from a minimum ($-J_0$) to a maximum ($+J_0$) or vice versa at the vortex position. If more vortices are involved, the current density does not change from $-J_0$ to $+J_0$, but varies discontinuously between intermediate values because of the presence of other vortices.

Even though it is not energetically favored, a pair of same poles, say two positive vortices or two negative vortices, is worth investigating. The Josephson current distribution with two $+$ poles at $r_+ = (+0.2, +0.2)$ and $(-0.2, -0.2)$ are shown in Fig. 2.10. The phase is $0$ along the straight line connecting the vortices and $+\pi$ or $-\pi$ outside the vortices along the diagonal line in 1st and 3rd quadrants. The current distribution has a ridge through the 2nd and 4th quadrants and two discontinuous drops at the vortex positions.

Experimental results are neither the field lines, nor the Josephson current density, but the maximum zero voltage current which is identified to be the Josephson critical current, $I_C$. Figs. 2.11a and 2.11b show the theoretical distribution of the zero field critical current, $I_C/I_0$, corresponding to a primary monopole at various positions in the junction. The critical current is zero at $r_+ = (0,0)$, and increases monotonically as the vortex moves away from the origin. At an edge, $r_+ = (0.95,0)$, $I_C$ recovers up to $-98\%$ of $I_0$, and $-99\%$ at the
Fig. 2.10. $J_c$ due to two plus poles at $+(0.2, 0.2)$ and $-(-0.2, -0.2)$
Fig. 2.11a. The reduced critical current, $I_c/I_{CO}$, as a function of single positive vortex position. $I_c/I_{CO}$ is close to 1.0 along the perimeters primarily due to images.
Fig. 2.11b. 3-D version of the map of Fig. 2.11a
corner, \( r_\perp = (0.95,0.95) \). Since the critical current has a specific dependence on the position of the vortex, the crosssection of the topography can be used to locate the vortex. This provides a very easy way of reading vortex motions without investigating diffraction patterns. This will be discussed in Chapter 4.

2.4. Superposition of Phases due to Vortex and External Field

In the previous section, we calculated the phase due to the applied field \( H_y \) (or \( H_z \)) and the vortex inside the junction. Those phases are assumed to be linearly superposed to give the total phase, \( \gamma(x,y) \), as

\[
\gamma(x,y) = \Theta(H) + \Theta(\text{vortex})
\]

(2.36)

where \( \Theta(H) \) is given by Eq. (2.15) for \( H_y \) and Eq. (2.16) for \( H_z \). Also \( \Theta(\text{vortex}) \) is estimated in Eq. (2.35). Because of the \( \Theta(\text{vortex}) \) term, the diffraction pattern \( I_c/I_0 \) vs. \( H_y \) is no longer the Fraunhofer pattern, but a severely distorted one. Since this distortion is uniquely dependent on the vortex configuration, except geometrical symmetries, we can use it to locate the vortex inside the junction.

2.4.1. Diffraction pattern distortion

Distortion of a diffraction pattern is related to the magnetic field distribution from the vortex. A single vortex tends to distort a diffraction pattern significantly because its fields affect all over the
junction area, if the vortex is not close to the edge. A dipole has less effect than a single pole on the distortion because the magnetic fields from the vortices are localized approximately between the two opposite poles. On investigating the change of diffraction pattern, we put emphasis on the single vortex case, partly because it has less varieties and partly because its distortion is more distinguished than a dipole case. It also is the situation studied most extensively in this work.

When a single vortex is near the edge, its effect is small because the magnetic field due to the vortex is well localized between the vortex and the nearest image. If the vortex is near the center, however, the image effect becomes negligible, so the change in the diffraction pattern is significant. Figs. 2.12a and 2.12b schematically demonstrate the change of diffraction patterns according to the various positions of a single positive vortex. As the vortex moves from the edge to the center along the x axis, $I_C(H=0)$ becomes more suppressed and finally the central peak of $I_C$ vs. $H_y$ splits into two parts. Especially, $I_C$ at zero field diminishes when the vortex is at the center. For $I_C$ vs. $H_Z$ pattern, it is symmetric in $H_Z$ without the vortex. With the single vortex, however, the pattern is no longer symmetric. Surprisingly, $I_C$ is zero for all $H_Z$'s when the vortex locates at the center.

For both patterns, $I_C$ vs. $H_y$ and $I_C$ vs. $H_Z$, applied fields play dominant role in the patterns over the vortex. On the other hand, the vortex is dominant when the applied fields are weak. Thus we will focus on the weak field region for data analysis.
Fig. 2.12a. Diffraction patterns, $I_c$ vs. $H_y$, for various single positive vortex positions. The vortex moves from the center (#1) to the edge (#5) of the junction. #6 is the full Fraunhofer pattern.
Fig. 2.12b. Diffraction patterns, $I_C$ vs. $H_z$, for various single positive vortex positions. If the vortex locates at the center of the junction, $I_C$ is zero for all $H_z$'s.
2.4.2. Symmetry and symmetry breaking

Because of the geometrical symmetries of the junction, different vortex configurations can generate the same diffraction pattern. Asymmetry of the pattern, $I_C$ vs. $H_y$, is caused by different cancellation and addition regions of field. For instance, for a positive vortex at $(x_0, y_0 > 0)$, $H_y > 0$ is cancelled in larger area of $y < y_0$, while added in smaller area of $y > y_0$. Thus, the peak value of $I_C$ is in $H_y > 0$ region. Likewise, the peak of $I_C$ is in $H_y < 0$ if the vortex is at $(x_0, y_0 < 0)$. By this reason, changing the field direction is equivalent to changing the vortex position from $(x_0, y_0)$ to $(x_0, -y_0)$. The above effect is true for $H_x$ if $x$ and $y$ are exchanged each other. Noting that the parallel field is $H_y$ only, the symmetries are as follows;

1) The diffraction patterns ($I_C$ vs. $H_y$) of two single vortices at $(x_0, y_0)$ and $(x_0, -y_0)$ are the same, but with reversed $H_y$. In addition, there is a two fold symmetry in the vortex positions with respect to $x$ axis. So reversing the vortex position from $x_0$ to $-x_0$, while retaining $y_0$, does not alter the diffraction patterns.

2) A dipole brings a much more complicated symmetry property. As an example, $r_+ = (x_0, y_0)$ and $r_- = (-x_0, -y_0)$ gives same pattern as the dipole at $r_+ = (-x_0, y_0)$ and $r_- = (x_0, -y_0)$. We will not go further.

3) The patterns, $I_C$ vs. $H_x$, have more symmetries. As one can see from the B field distribution inside the junction, a single
vortex has 2 fold symmetries, with respect to x and y axes and antisymmetries with respect to two diagonal lines.

4) Since the vortex field region is well localized when the vortex is near the edge, small change of vortex position does not significantly alter the pattern, \( I_c \) vs. \( H_y \). Strictly speaking, they must be different, but we see practically almost same \( I_c \) vs. \( H_y \) patterns for the group of single vortices in the shaded region (Fig. 2.13).

5) Reversing the vortex type simply results in same pattern with reversed \( H \). This reversal in \( H \) is general for all vortex configuration.

Because of the symmetries, ambiguity arises in determining the vortex position, even in the case that the comparison between the data and theory is exact. The symmetry breaking is according to the response of the vortex to external forces such as those caused by \( I_p \) or \( H_z \).

a) Under \( I_p > 0 \), a positive vortex moves in +x direction, otherwise -x direction.

b) In the presence of \( H_z \), the force to the positive vortex is along the field line, while against for a negative vortex.

c) The vortex source field, that causes vortex pinning, determines the type of vortex for each layer as described at the page 21 of this thesis.
Fig. 2.13. Symmetry of diffraction patterns
3. EXPERIMENTAL TECHNIQUES AND CONDITIONS

In this chapter we present the sample fabrication, techniques on data acquisition and analysis. In the second section, special attention is given to the junction quality because it is essential for a proper interpretation of the data.

3.1. Experimental Procedure

3.1.1. Sample preparation

Once the materials are chosen, the procedure of fabrication of a SNS junction is important to ensure a good proximity junction. A cross type superconductor - normal metal - superconductor Josephson junction was prepared by successive evaporations in high vacuum using the materials, Pb-Bi(2.5 a/o) for the S layer and Ag-Al(4 a/o) for the N layer. The process is as follows: first of all, a substrate, Corning glass, 1.0" long, 0.5" wide and 0.048" thick, was cleaned in HCl, hot microcleaning solution and aceton-ultrasonic cleaner. The substrate then was attached to a copper block (1" x 0.5" x 0.5") with Apizon-N grease. The copper block will act as a heat reservoir during evaporation. The mask for a superconducting layer was a slit of 46 μm wide in 25 μm thick steel plate. The distance between the mask and the substrate was ~25 μm giving a sharp fall off in thickness at the edge.
The alloys were also cleaned before the use. The solution of acetic acid and \( \text{H}_2\text{O}_2 \), mixed by 5:1 in volume and diluted with distilled water, was used for Pb(Bi) cleaning. The solution oxidizes the alloy vigorously. An ultrasonic cleaner drops off the oxides to leave shiny material. For Ag(Al) alloy \( \text{NH}_3\text{OH} + \text{H}_2\text{O}_2 \), mixed by 5:1 in volume and diluted with distilled water, was used to clean. Then the alloys were cut into appropriate sizes using non-magnetic (brass) scissors.

As mentioned in chapter 2, a square junction was highly desired for ease in the interpretation of the data. For this purpose, a carefully designed evaporator (substrate and mask holder) was employed to rotate the mask by 90 degree for crossed layers having exactly same width. During rotation of the mask, the substrate was lifted to avoid scratching pre-deposited films.

The alloys were evaporated from electrically heated molybdenum boats at the initial pressure of \( 2 \times 10^{-8} \) torr. The bottom superconducting layer was formed first by evaporating to completion the Pb(Bi) alloy chunk from the boat #1 at a deposition rate of about 50 \( \text{Å/sec} \). To ensure a homogeneous normal layer, the Ag-Al alloy was cut into 50 ~ 100 pieces of pellets. The pellets were dropped into the second boat and evaporated completely, just one by one until all were evaporated. Once a pellet was dropped into the boat, it was evaporated rapidly in order to allow little chance of separate evaporation of Al and Ag. In the worst case, this may form Al-Ag-Al-Ag-Al------, alternating layers with a period of \( -50 \text{Å} \). Even so, the period is much smaller than the coherence length of the normal metal, \( -500 \text{Å} \). Finally, the top superconducting layer was deposited through the same
mask, now rotated by 90 degree, as for the bottom layer to form a complete SNS junction. The pressure was kept at $2 \times 10^{-8}$ torr during the first Pb-Bi layer deposition. It rose to $10^{-7}$ torr during normal metal deposition. The time interval between two evaporations was about 2 minutes, so at most one layer of molecules would be formed at the interface. Two samples were made simultaneously. One is for SNS junction, the other one consists of only a normal metal strip to be used for resistivity measurement of the normal metal.

The evaporator was mounted right underneath a liquid nitrogen tank inside the vacuum chamber. All through the evaporations, the tank retained liquid nitrogen to ensure the stability of the substrate temperature. Samples were deposited on cold substrate in order to maintain a sharp and well defined interface between the superconductor and normal metal layers.

The samples were slowly warmed to room temperature to avoid cracking of the layers due to different thermal expansion rate of the metals and glass. The thickness of the sample were measured using a Dektak thickness profiler and assisted by optical method.

3.1.2. Cryostat

The sample was mounted inside a He$^4$ cryostat allowing measurement between 0.4 K and 10 K. An indium "O" ring sealed vacuum can was installed for thermal shield because the operating temperature was higher than 4.2 K. The SNS junction was attached to a copper block sample holder with a thin layer of Apiezon-N grease, on the back of
which was a calibrated germanium thermometer. A temperature controller (Lake Shore Cryotronics, Model DTC 500-SP) was used to keep the sample temperature constant to a precision of less than a millikelvin. A manually operated heater was also mounted on the copper block.

Electrical connections to the sample were made using superconducting lead wires. The currents and voltage leads were soldered to the films with a Pb-Bi (50 atomic percent each, eutectic) alloy which has a significantly higher critical temperature than that of the junction. A symmetric Josephson current feed was necessary to retain uniform current flows in the junction, but a superconducting closed circuit had to be avoided because of possible induced supercurrent on it arising from changing magnetic fields. Two pairs of 70 ± 0.5 mm copper wires(#18) were inserted in the current leads in order to prevent supercurrent. The joule heat arising from the Josephson current, typically 1 mA or less, was negligibly small (Fig. 3.1).

The voltage output in a SNS junction is extremely small, say $10^{-11}$ - $10^{-10}$ volt at the Josephson current, $I_J$, of twice of the critical current, $I_C$. A rf SQUID (S.H.E. Model 330) was used for measurement of such small voltages. The use of the SQUID limits the experimental conditions seriously. First of all, the SQUID sensor must reside in a magnetically stable environment. This is accomplished by surrounding the part of the vacuum can with a superconducting lead cylinder mounted just inside the helium dewar. The helium dewar was also surrounded by three double folded concentric conetic cylinders, so that the residual field was reduced to a few milligauss in the sample prior to cooling the
Fig. 3.1. Symmetric current feed arrangement accomplished via insertion of registers in series with the superconducting leads
Fig. 3.2. Longitudinal section of cryostat
lead cylinder. In addition, electric and magnetic noise from outside has to be completely shielded. For this purpose, all currents to the sample were passed through wide band rf filters and all current wires were completely shielded with concentric cables. A superconducting shield over the wires also was used where possible.

Two orthogonal magnetic fields could be applied to the sample using two pairs of properly oriented Helmholtz coils. The coils were mounted on the vacuum can surrounding the sample and inside the superconducting shield. Even with distortions of magnetic field by the lead cylinder, lead wires and soldering blocks around the sample, the experimental results showed that parallel and perpendicular fields with good quality were produced by the coils. The calibration constants of the magnets may not be very accurate with the error range of 2 - 3 percents. This caused the analysis to be somewhat difficult, and will be discussed at the end of this chapter.

3.1.3. Data acquisition

Electrical measurements were semi-automated with an Apple II+ microcomputer and IEEE-488 bus. The schematic diagram is shown in Fig. 3.4. Data to be taken and recorded were mostly the critical currents as a function of magnetic fields and temperatures.

The critical current of the sample was not measured directly. Any power driven commercial current supplier introduced rf noise which caused the SQUID operation to be deteriorated. So a battery operated constant current supplier (S.H.E. Model CCS), electrically shielded and
even floated from the ground, was used to feed the currents, $I_J$ and $I_p$, to the sample. For reading $I_J$, the control heliopot of the CCS was mechanically connected via a stepper motor to a potentiometer while the current itself was connected to the sample. The stepper motor was operated by a hand held control. The potentiometer voltage was displayed on the x axis on the X-Y recorder as well as being digitized by the computer using a high speed voltmeter (HP 3437A). The current calculated from the voltage differed from the actual value by less than 2%. The voltage output via SQUID was transferred to the y axis on the X-Y recorder and also digitized as needed.

Temperature measurement was completely automated. The resistance of the thermometer was determined by reading the voltage across the thermometer and a standard resistor with a nanovoltmeter (Keithley 181), reversing the current and repeating the measurements. Thermal emf's were averaged out by the reverse current measurement. The temperature was calculated from the resistance to a precision of a millikelvin.

The magnitude of the applied magnetic field was determined by reading the current through the coils. This was done by measuring the voltage across a standard resistor in series with the circuit. A Constant Current Supplier (CCS) was used as a current source to the magnets to keep the stability of the current maximum.

The following sequence of steps occurred during the acquisition of each data point. As an operator slowly increases the Josephson current, $I_J$, via the stepper motor, a voltage increase across the junction suddenly appears in the X-Y recorder. Then the operator picks up the instant value of x axis for the current. Better accuracy was achieved
Fig. 3.3. Schematic of junction V - I characteristic measurement system
Fig. 3.4. Data acquisition system
in manual operation, but sometimes this was done automatically. The temperature was then measured immediately, and finally the magnetic field was determined. The temperature reading takes about 4 seconds primarily the time allowed for the voltmeter to settle. But this time interval is not important since the temperature stability was within 1 millikelvin.

The data were then printed out and stored on a floppy disk in the form of a random access text file. The designed change in experimental parameters was made and the entire process was repeated for the next point.

3.1.4. Data analysis - curve fitting

Most of the data analysis task involves curve fitting to determine the vortex configuration in the junction using VAX 11/780. The Nelder and Mead algorithm of Simplex was adopted for the curve fitting [55]. This algorithm neither needs the derivatives of a function, nor diverges. For many vortices the program may not work because of mainly two reasons. First, there are too many varieties of the vortex configuration to give a unique result within the experimental error. Secondly, many vortices require long CPU times for a large number of iterations.

To fit the patterns, one normally starts with one vortex of two variables, x and y coordinates of the vortex. If this trial is not successful, then two vortices of 4 variables are tried. One can also go 3 vortices, and so on. As a matter of fact, difficulty usually arose
for more than two vortices in which it was not possible to produce a reliable fitting in the given experimental condition. Three vortex fitting was possible only in limited cases.

With the measured data transferred from AppleII+ micro-computer, $I_c$ vs. $H_y$ (or $H_z$), the program calculates theoretical critical current, $I_c^{th}$, for each experimental $H_y$ (or $H_z$). The program then sums up the differences between $I_c^{exp}$ and $I_c^{th}$ which is to be minimized. There is a danger of finding local minima instead of absolute minimum condition. So several trials were made with different initial conditions.

In this analysis, we are implicitly assuming that the $x$ axis values, $H_y$ (or $H_z$), are infinitely accurate. Actually, the accuracy of $H_y$ was even less than that of $I_c$. In the measurements, the currents to the magnets via a standard resistor were measured for $H_y$ with high accuracy. $H_y$ was calculated by multiplication of a calibration constant, say $C_y$, as $H_y = C_y \cdot I_y$. The most important quantity here is $H_1$, the field intensity at the first minimum of $I_c$ vs. $H_y$ of the junction. We write as $H_1 = C_y \cdot I_y$. The phase for an $H_y$ is given by

$$\Theta(H_y) = \pi \cdot x \cdot (H_y/H_1)$$
$$= \pi \cdot x \cdot (I_y/I_y)$$

So $C_y$, the calibration constant, is dropped out, and the analysis should work. For $H_z$ case,

$$\Theta(H_z) = 2\pi \cdot x \cdot y \cdot (H_z/H_1)$$
$$= 2\pi \cdot x \cdot y \cdot (I_z/I_y) \cdot (C_z/C_y)$$

Because of the calibration constants term, which is not very accurate, the data $I_c$ vs. $H_z$ were not directly used in curve fitting, but used to insure the quality of the curve fitting with $I_c$ vs. $H_y$. 
3.2. Preliminary Results

- Investigation of Junction Quality

In this section we investigate the basic properties of the proximity induced SNS Josephson junction to insure the quality of the junction.

3.2.1. Voltage current characteristics

The SNS Josephson junction, SNS10, shows a well defined reversible I-V characteristics, as shown on Fig. 3.5 at 6.9K. The curve well fits to the Restively Shunted Junction (RSJ) model [56], which predicts the time average voltage as

\[ V = R_n(I_J^2 - I_C^2)^{1/2} \quad \text{for} \quad I_J > I_C \]
\[ = 0 \quad \text{for} \quad I_J < I_C \]  

with \( R_n \) of \( 8.5 \times 10^{-6} \Omega \), the normal metal resistance and \( I_C \) of 0.176 mA, the critical current. This resistance is equal to the differential resistance, \( \Delta V/\Delta I \), for large \( I_J \).

It is important to point out that \( R_n \) in the Eq.(3.1) has not been clearly understood for SNS junctions. The true normal metal resistance can be calculated from separately measured resistivity \( \rho_n \) as

\[ R_n = \rho_n d_n / W^2 \]
\[ = 13 \times 10^{-6} \Omega \]

at 4.2K. The true normal metal resistance tends to be approximately constant in the temperature range of a few kelvin above 4K. The
differential resistance, however, is not. It has been shown by Hsiang and Clarke [57] that the asymptotic slope, $\Delta V/\Delta I$, of the I-V curve is strongly dependent on temperature particularly near $T_c$. For $I_J > I_C$, the Josephson current includes the passage of quasiparticles through the NS boundary. Since the quasiparticles do not immediately relax to the ground state at the boundary, the effective normal metal region includes part of the superconducting region, resulting in higher junction resistance. Even so, $\Delta V/\Delta I$ is smaller than $R_n$. At low temperatures, the induced energy gap in N layer would provide a potential barrier for quasiparticles to undergo Andreev reflection inside the N layer [58], reducing the differential resistance of the junction. The temperature dependence of the differential resistance, $R_n(T)$, is clearly shown in Fig. 3.5 for both samples SNS8 and SNS10. This behavior was observed by Miller also [Ref. 47].

The presence of magnetic field does not alter the I-V characteristic of Eq. (3.1) with changed $I_c(B)$. For example, $I_c = 0$ for $H_y = 0.365$ gauss at 6.9K. I-V curve is a straight line with the slope of 8.5 $\mu$s. The presence of vortex also does not alter the I-V characteristic.

The critical current is to be defined by the maximum zero voltage current in the I-V curve. The transition from zero voltage state to voltage state is somewhat rounded. It is well defined with large $I_c$ at low temperatures, say below 6.5K for SNS10. At higher temperatures the onset of voltage state is not clear as shown in Fig. 3.6. This behavior becomes more significant as the sample temperature gets closer to $T_c$. Errors can be on the order of about 3% of $I_c$ at 7.0K, for example. No
### Table 3.1. Parameters

<table>
<thead>
<tr>
<th>d_n (nm)</th>
<th>d_s (nm)</th>
<th>( \rho_n(4.2^\circ K) ) (( \mu \Omega \cdot \text{cm} ))</th>
<th>( \rho_s(4.2^\circ K) ) (( \mu \Omega \cdot \text{cm} ))</th>
<th>l_n (nm)</th>
<th>l_s (nm)</th>
<th>( \xi_n(6.9^\circ K) ) (nm)</th>
<th>( \xi_{GL}(6.9^\circ K) ) (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>740</td>
<td>600</td>
<td>4.5</td>
<td>2.5</td>
<td>26^a</td>
<td>42^b</td>
<td>46^c</td>
<td>170^d</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \lambda_L(0) ) (nm)</th>
<th>( \lambda_L(6.9^\circ K) ) (nm)</th>
<th>( T_c(\text{SNS}) ) (^\circ K)</th>
<th>( T_c(\text{SNS}) ) (^\circ K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>104</td>
<td>230</td>
<td>7.30</td>
<td>7.35</td>
</tr>
</tbody>
</table>

\[^a\] \( \rho_n l_n \)^{-1} = 8.6 \times 10^{10} /\Omega \cdot \text{cm}^2 \text{ [Ref. 59].} \\
\[^b\] \( \rho_n l_n \)^{-1} = 9.4 \times 10^{10} /\Omega \cdot \text{cm}^2 \text{ [Ref. 60].} \\
\[^c\] \xi_n = (\hbar v_{fn} l_n / (6 \pi k_B T))^{1/2}, \text{ where } v_{fn} \text{ of Ag was used [Ref. 60].} \\
\[^d\] \xi_{GL} = 0.855 \xi_l \delta (1 - T/T_{cs})^{-1/2}, \quad \xi_o = 510 \AA. \\
\[^e\] \text{Dirty limit case.}
Fig. 3.5. Differential resistance, $\Delta V/\Delta I$, as a function of the sample temperature. Above 7.2K (or 7.1K for SNS8) the data values are not reliable because of cease of bulk superconductivity. The graph indicates that the superconductivity of S layer extends into the N layer, reducing the effective N layer thickness.
Fig. 3.6. Fitting to RSJ model (Eq. 3.1)
matter what the reason is, the critical current, \( I_c \), was defined in accordance with the RSJ model, in which the voltage rises rapidly at \( I_c \). The largest slope part of the I-V curve is extrapolated to the zero voltage line. \( I_c \) was defined by the intercept, giving \( I_c = 0.176 \, \text{mA} \) and \( J_c = 8.3 \, \text{Amp/cm}^2 \) at 6.9K.

### 3.2.2. Temperature dependence of \( I_c \)

The critical current spans many decades as a function of temperature (Fig. 3.7). The most widely used form for the temperature dependence of \( I_c \) is [Ref. 43]

\[
J_c \propto (1 - t^2)\exp(-K_n d_n) \tag{3.2}
\]

where \( K_n^{-1} = \xi_n(T) \) for Ag(Al) layer, and \( t = T/T_c \), reduced temperature. The contribution of the exponential term is too big to fit the data of the junction, where the normal layer is thick. Instead, the data fits well to [42, 61]

\[
J_c \propto (1 - t^2) \tag{3.3}
\]

for the temperatures above 6.6K, where \( \lambda_J \geq W/2 \), producing uniform current density over the junction area. Below 6.6K, the current density tends to be confined to the perimeters, so self-field correction should be made [62]. Another fact from \( I_c(T) \) is that \( I_c \) falls off more rapidly as \( T \) increases above 7.2K. This is because the coherence length of the Pb(Bi) film becomes comparable with the thickness (600 nm) and bulk superconductivity ceases [61]. Excluding the temperatures above 7.2K, the linear part of \( (J_c)^{1/2} \) vs. \( T \) of data are extrapolated to \( J_c = 0 \) to give \( T_c = 7.3 \text{K} \) as shown in Fig. 3.7.
Fig. 3.7 $\Delta I_C$ vs. Temperature. The graph is linear up to 7.25K, above which $I_C$'s drop rapidly off the line, meaning cease of bulk superconductivity. The line is extrapolated to zero $I_C$ line to determine $T_{c(SNS)}$ to be 7.3K.
Experimental data and theory comparison for temperature (K) vs. current ($mA$).
3.2.3. External field dependence of $I_c$

Fraunhofer oscillations of regular periodicity were produced by the SNS junction in the presence of an externally applied parallel magnetic field. Fig. 3.8 is the experimental $I_c$ vs. $H_y$ curve for SNS10 at 6.9K, showing an excellent fit to the ideal Fraunhofer formula of

$$\frac{I_c}{I_{co}} = \left| \sin(\Theta)/\Theta \right|$$  \hspace{1cm} (2.13)

where $\Theta = nH_y/H_1$ with $H_1 = 0.365$ gauss, the period of oscillations at 6.9K. Good patterns were obtained at the temperatures higher than 6.7K, where $\lambda_J = 0.74W$. This length provides nearly uniform current density throughout the junction area with symmetric current feed [60]. At higher temperatures $\lambda_J$ becomes larger such as $\lambda_J(6.9K) = 1.1W$, $\lambda_J(7.0K) = 1.5W$ and so on. Near $T_c$, $I_c$ is very small and inaccurate (Fig. 3.6). For this reason, the vortex pinning-depinning experiment was done in the temperatures between 6.8K and 7.0K, particularly at 6.9K.

With the perpendicular magnetic field to the junction surface, the diffraction pattern, $I_c$ vs. $H_z$ at 6.9K, provides another nice fit with the theory of

$$\frac{I_c}{I_{co}} = \left| \sin(\alpha)/\alpha \right|$$  \hspace{1cm} (2.17)

where $\alpha = -2nH_z/H_1$ and $H_1 = 0.365$ gauss at 6.9K (Fig. 3.9). The theoretical curve is a little higher than that of the data. The lack of smoothness of the theoretical curve is considered to be the result of approximation in deriving Eq.(2.17).

Conclusively, the magnetic field dependence of $I_c$'s proves that the junction is strictly subject to the the Josephson equation, $J(r) = J_0\sin(\Theta(r))$, and in good quality for temperatures above 6.7K.
Fig. 3.8. Experimental data of $I_c/I_{co}$ vs. $H_y$ with $I_{co} = 0.176$ mA at 6.9K. The solid line is $|\sin(nH_y/H_1)/(nH_y/H_1)|$, the Fraunhofer pattern, with $H_1 = 0.365$ gauss
Fig. 3.9. Experimental data of $I_c/I_{co}$ vs. $H_z$. The solid line is $|\text{Si}(\alpha)/\alpha|$. 
Fig. 3.10. Determination of field penetration depth. The equations are: 
\[ \phi_0 = \Phi_{\text{eff}} H_1. \]
So 
\[ \frac{1}{H_1(T)} = \frac{\Phi_n + 2\lambda}{\phi_0}, \]
and 
\[ \lambda = \lambda_0 (1 - t^4)^{-1/2}. \]
From the linear graph, \( \lambda_0 = 1040 \, \text{Å} \), \( \lambda(6.9K) = 2300 \, \text{Å} \) and \( \Phi_n = 7600 \, \text{Å} \) are obtained.
3.2.4. The Pb(Bi) film

The Pb(Bi) superconducting films used in SNS junctions showed some irreproducible characteristics in the critical current. For example, the critical current of SNS junctions was highly dependent on how long the sample was kept in high temperature such as room temperature. When the sample was cycled between 4.2K and 300K, the reduction of $I_c$ by ~10% was observed. Also the step-like pattern of $I_c$ vs. $H_c^2$ was significantly changed after temperature cycling to the room temperature. For this reason, the sample was mounted as quickly as possible after it was taken out of the evaporator and kept below 15K for the entire series of measurements.

Crystallization of Pb layer is suggested to explain the reduction of $I_c$. The change of $I_c$ vs. $H_c^2$ means that the configuration of pinning centers was reformed significantly, and supports reshaping of grain boundaries due to continuous crystallization. At the moment of film deposition at the liquid nitrogen temperature in high vacuum, the Pb(Bi) layer would be in highly disordered state. It can be annealed at room temperature. Consequently, it forms large grains of perhaps ~1um and leaves vacancies at the grain boundaries. This gap allows oxygen or other molecules to penetrate and spoil the interfaces between Pb and Ag layers. No attempts were made to identify the quality deterioration. Instead, it was avoided by keeping the sample at low temperature. The sample showed consistent electrical property during the experiment for more than five months, while it was kept in the temperatures below 15K.

One more fact observed was drop-like particle formation on the
surface of the Pb(Bi) film after significant annealing at room
temperature as Fig. 3.10a shows. When the sample was taken out of the
evaporator, the surface, as seen in the optical micrograph, was clean.
After the experiment was completed, the same sort of particle formation
was observed for SNS10 as well as other samples not mentioned here. The
size of the particles was typically 1µm or less. The study by Auger
spectroscopy showed that the surface was covered by 10 ~ 20Å of lead-
oxide layers, followed by Pb only state. Small amount of Bi was not
clearly detected by Auger spectroscopy.

In conjunction with the vortex pinning-depinning experiment, the
grain boundaries, presumable pinning centers, were investigated using
scanning electron microscopy (SEM). After the sample was kept at room
temperature and in the vacuum of ~10^-6 torr for one week, the grain size
was turned out to be typically 1 ~ 3 µm (Fig. 3.10b). So during the
experiment, the grain size is expected to be less than that. This small
size of grains provides large number of pinning centers.
Fig. 3.11. Scanning Electron Micrograph (SEM) of (a) particles formed on the surface of the superconducting layer and (b) Grains of the superconducting layer
4. EXPERIMENTAL RESULTS AND DISCUSSIONS

This chapter presents the experimental results of vortex pinning, depinning and measurement of elementary pinning force.

In the first section, we study the free energy associated with a vortex in a SNS junction. Because there is no established theory, we will try a qualitative treatment of the energies involved to see that the junction is not energetically favorable for a vortex without pinning potential. In the second section we discuss problem of the vortex nucleation at the edge of the films by both transport current and applied perpendicular magnetic field with limited success. In the third section, we present a remarkable success on the single vortex pinning by Field Cooling Process (FCP), depinning by the transport current, \( I_p \), and locating the vortex according to the theory given in the chapter 2. As a result, we report the first measurement of the elementary pinning force, \( f_p \), associated with a particular pinning site. Also from the experiment we suggest that the temperature dependence of the pinning force to be \( f_p \sim (1 - T/T_c)^{3/2} \). Finally, in the fourth section, controlled vortex pinning and its application will be discussed.
4.1. Free Energy Associated with a Vortex

4.1.1. Vortex energy in a single superconducting strip

There is well-established theory about the flux line formation for a type II superconducting strip in an applied magnetic field, for which the edge effect plays an important role. The Gibbs free energy of a flux line per unit length is given by [63], for a semi-infinite superconducting slab,

\[
\frac{G}{L} = \frac{\phi_0}{4\pi} \left[ H_a \exp\left(-\frac{x}{\lambda}\right) - \frac{1}{2} K_0\left(\frac{2x}{\lambda L}\right) - (H_a - H_{c1}) \right]
\]

(4.1)

where \( x \) : distance from the edge,

\( H_a \) : perpendicular magnetic field,

\( K_0 \) : zeroth order modified Bessel function, and

\( H_{c1} \) : lower critical field.

The first term contains the interaction between the vortex line and the external field, representing a repulsive force against the edge. The second term described the attractive interaction between the flux line and its image lines. The third term represents the energy of the line inside the superconductor far away from the surface. For \( H_a < H_{c1} \) the Gibbs free energy is positive and the flux line inside the superconductor is unstable. For \( H_{c1} < H_a < H_{en} (= \phi_0/(4\pi\lambda\xi_a)) \), an energy barrier is developed near the edge, essentially due to the first two terms. As \( H_a \) increases, the barrier shrinks and finally disappears at \( H_a = H_{en} \). For larger fields, the flux line attains its lowest energy...
far inside the superconductor.

For a superconducting strip with finite width, the edge effect becomes important. Especially in the field cooling process (FCP), the entry field, $H_{en}$, may have no effect since the strip traps a vortex as it is cooled down and the vortex does not have to move through the edge region. The Gibbs energy barrier, however, plays an important role to keep the vortex in it.

Besides the theoretical work by Tinkham [Ref. 17], vortex pinning by the field cooling process in a superconducting strip with finite width has been studied by S. Bermon and J. Clem [64]. They showed that the Gibbs free energy variation in the presence of perpendicular magnetic field causes a vortex to be formed in the middle of the strip. As in Eq. (4.1), the energy is positive with $H_a = 0$. As the field grows, a valley of the Gibbs free energy appears in the middle of the strip and two barriers on both sides are developed. They also showed that there is a characteristic field intensity $H'$, at which the Gibbs free energy reaches zero at the middle, as

$$H' = \frac{\phi_0}{\mu^2}$$  \hspace{1cm} (4.2)

For $H_a > H'$, the energy becomes more negative and the energy barriers shrink and finally disappear. By this reason, a vortex is most likely to be formed in the middle of the strip when $H_a$ is greater than $H'$. 
4.1.2. Vortex energy in a SNS Josephson junction

When we try to extend the above idea to the SNS junction system, we encounter very different geometrical effects such as superconducting coupling, magnetic field squeezing, magnetic coupling between two poles, and so on. No theoretical work has been reported with the cross type SNS geometry. Here we investigate various energies in qualitative and somewhat primitive way.

The first two terms in the Gibbs free energy of Eq. (4.31) are retained qualitatively, but the function forms must be different because of different geometry. The most important term is the magnetic field energy due to the vortex inside the junction, $\int J_d x dy dz B^2 / 8 \pi$. For a single vortex this energy can be regarded as the interaction of the vortex with all images. For multiple vortex case, particularly a dipole, this energy includes the vortex-image interaction as well as the vortex-vortex interactions (dipole coupling). The free energy also contains Josephson coupling energy. Other perturbations such as field interaction outside the junction may be same as before.

1) Vortex - image interaction: This interaction energy of a vortex inside a junction is much stronger than that of the single strip, because the magnetic field due to the vortex decreases as $1/r$ inside the junction. This energy is highest when the vortex is at the center and decreases slowly as the vortex moves off the center. Near the edge the energy drops rapidly and becomes zero when the vortex is removed from the junction, causing strong attractive force to the edge. The force is given by
\[ f_m = H(\text{images}) \cdot q_m(\text{vortex}) \]
\[ = \frac{\phi_0}{4\pi} \frac{\phi_0}{2\pi d_{\text{eff}}} \sum \frac{r - r_0}{|r - r_0|^2} \cdot p \]  
\[(4.3)\]

where \( r_0 = (x_0, y_0) \), vortex position, and \( r = (x, y) \), positions of the images. \( p = +1 \) for positive images and \(-1\) for negative images. The summation is all over the images. We will see later that this term is much smaller than the real pinning force if the vortex is not near the edge.

2) Dipole case: Each pole of the dipole is under the vortex-image interaction. In addition, there is magnetic coupling energy of the dipole. The field energy can be lowered by reducing the dipole strength, that is, by moving the poles closer together. So there will be competition between the vortex-image interaction and the dipole coupling. If the dipole locates near the middle of the junction and its separation, \( \delta \), is not very large, image effect must be negligible. Provided that \( \delta > d_{\text{eff}} \), we can roughly estimate the coupling force as, with \( d_{\text{eff}} = 1200 \) nm,
\[ f_c = \left( \frac{\phi_0}{4\pi} \right) \cdot \left( \frac{\phi_0}{2\pi d_{\text{eff}}} \right) \]
\[ = (7.5 \times 10^{-9} \ \text{dyne}) \cdot \left( d_{\text{eff}}/\delta \right) \]  
\[(4.4)\]

Rigorous treatment of the coupling force was made by J. Clem [65]. The present sample, SNS10, traps a dipole of strength \( \delta = 2d_{\text{eff}} \), giving coupling force of \( 4 \times 10^{-9} \) dyne. As we shall see later, this force is much smaller than the measured pinning force (~ \( 10^{-8} \) dyne) in the present experiment.

3) External field energy: Because the applied perpendicular field generates the parallel fields inside the junction area according to Eq.
(2.17) (Fig. 2.3), the interaction between the vortex and the external field can be attractive with respect to the origin as well as repulsive, depending upon the vortex type and its position. As an example, for \( H_x > 0 \), a vortex at \( r = (x,0) \) will be under magnetic field interaction attractive for the origin if it is in the bottom layer (plus pole), and repulsive if in the top layer (minus pole). The interaction of the same field with the vortex at \( r = (0,y) \) will be reversed. If the vortex is around the origin, this interaction may be negligible.

4) Josephson coupling energy: \( E_J = (MI_c/2e)(1 - \langle \cos \Theta \rangle) \) \([66]\). The contribution may be at most of

\[
E_J < MI_c/2e \\
= 5.8 \times 10^{-13} \text{ erg}
\]

with \( I_c = 0.176 \text{ mA} \). The average force is \( E_J/(W/2) < 2.5 \times 10^{-10} \text{ dyne} \), which is really ignorable.

5) There are other perturbation terms such as magnetic field interaction with fields outside the junction, etc. They are same as before, and will be ignored.

6) Pinning potential energy associated with individual pinning site, which is assumed to be short ranged, approximately within the coherence length.

7) According to the above terms, the Josephson junction system looks to be energetically unfavorable for vortex presence. But vortex can be trapped particularly by the field cooling process. Moreover, a single vortex could be trapped even though its energy tends to be higher than that of a dipole.

8) So the pinning potential must play dominant role for vortex
pinning. A Josephson junction can trap vortex, particularly a single vortex, only when the potential is strong enough. The net pinning force is the spatial gradient of such energies as the negative pinning potential, positive magnetic field energy and so on.

9) As predicted by Bermon and Clem [64], it is suggested that a single vortex (or a dipole) is likely to be formed in the middle of the junction area when the junction is cooled in \( H_2 \). This assumption agrees with the present experiment, in which the first vortex trapped by field cooling process is always trapped not near the edge, but around the middle of the junction.

Later we will estimate those forces associated with the above energies using the concept of magnetic charge, instead of rigorous treatment of the flux line. Even though there may be non-trivial competitions between various forces, we will see that the pinning force associated with an individual pinning site is dominant.

4.1.3. Vortex nucleation and vortex entry fields

Vortex nucleation can occur either in intermediate state of type I superconductor or in mixed state of type II superconductor. For the low \( \kappa (= 1.8) \) type II material and Josephson junction geometry, there is no theory on vortex nucleation by \( I_p \) and entry into the junction area. As a first approximation, however, we will compare the experimental results with the theory which is good for type I single superconducting strip.

When a superconducting strip carries transport current, \( I_p \), a strong transverse field is built up at the edge. At the small current
values, the current flows predominantly along the edge of the strip, showing Meissner state. When the edge field reaches a value of the order of the critical field, $H_c$ for type I material and $H_{c1}$ for type II material, a normal region will be generated locally at the edge. The normal region indicates the entry to the vortex state. The vortex nucleated at the edge may be pulled into the junction by the transport current itself, and trapped at a pinning center. This allows the junction retain a vortex even after $I_p$ is removed. We are interested in this residual vortex state other than vortex flow state.

The magnetic field around the superconducting strip is calculated in simple forms for the transport current as well as for the applied field. For a superconducting strip of thickness $d_s$ and width $W$, the transport parallel current, $I_p$, produces the magnetic field of [67]

$$H_{||}(\text{surface}) = 0.8I_p/(d_s + W) \text{ (gauss-cm/Amp)} \quad (4.5a)$$

$$H_{\perp}(\text{edge}) = 0.4I_p/d_s \text{ (gauss-cm/Amp)} \quad (4.5b)$$

Also the applied perpendicular field, $H_a$, produces the edge field of

$$H_{\perp}(\text{edge}) = H_a(W + d_s)/d_s$$

$$= H_a(W/d_s) \quad (4.6)$$

when $W \gg d_s$, as in the present sample.

Although the transverse field at the edge, $H(\text{edge})$, reaches the critical field of the superconductor, $H_c$, the strip may not generate vortex nucleation. This is due to Gibbs free-energy barrier given in Eq. (4.1) against magnetic flux entry into the superconductor near a planar surface oriented parallel to the magnetic field. Including this Gibbs energy barrier, the minimum flux entry field at the edge is given by [68]
where we assume a circular flux tube of radius $a_n$. So the critical entry field is much higher than $H_c$ since $a_n W \gg d_S^2$ typically.

Eq. (4.5a) and Eq. (4.6) apply in type II superconductors to calculate the edge field. In Eq. (4.7) we use $a_n = \lambda$, the field penetration depth, because a flux tube contains only one flux quantum in type II film. So the enhancement due to the second term in Eq. (4.7) is expected to be less significant.

Combining Eq. (4.7) with Eq. (4.5a) and Eq. (4.6), we get the minimum flux entry current, $I_{p1}^n$, and flux entry field, $H_{E1}^n$, as

$$I_{p1}^n = \frac{H_c(T)}{0.4} \left( d_s + \frac{(2\lambda W)^{1/2}}{4} \right) \text{ (Amp/gauss-cm)} \tag{4.8}$$

$$H_{E1}^n = \frac{H_c(T)}{W} \left( d_s + \frac{(2\lambda W)^{1/2}}{4} \right) \tag{4.9}$$

where the superscript $n$ denotes that $I_p$ (or $H_z$) is used for vortex nucleation, and the subscript 1 denote the minimum condition for the event. There are two temperature dependent terms in Eqs. (4.8) and (4.9), $H_c(T)$ and $\lambda(T)$, but $H_c(T)$ is dominant. Using the parameters of SNSIO, namely $W = 46$ $\mu$m, $d_s = 600$ nm and $\lambda = 230$ nm at 6.9K, we obtain

$I_{p1}^n = 37.5$ mA

$H_{E1}^n = 3.26$ gauss,

where $H_c(0) = 803$ gauss of pure Pb is used. This value may be good approximation because $T_{CS} = 7.35$K of Pb(Bi) film is close to $T_C = 7.25$K of pure Pb. These are in good agreement with the experiment as we shall see later. Use of Eqs. (4.5b) and (4.6) gives $H$ (edge) = 250 gauss from both sources, local edge field necessary for vortex nucleation.
There are limitations on the use of Eqs. (4.8) and (4.9). \( H_{c1} \) is not known for the present material, but must be smaller than \( H_c \). Furthermore, the transverse field by \( I_p \) is expected to be strongest at the corner of the junction area, suggesting initial corner pinning.

4.1.4. Vortex pinning by field cooling process

In contrast to the multi-vortex pinning by a transport current, the Field Cooling Process (FCP) turned out to be an ideal method to trap a single vortex (or a dipole) in the SNS Josephson junction. In this process the junction is cooled down through the transition temperature, \( T_{cs} \), in the presence of a perpendicular magnetic field, \( H_{c2} \), where the superscript \( c \) denotes the usage (field cooling) of the quantity as in \( I_p \). The field threads the junction area uniformly for \( T > T_{cs} \). As the temperature goes down through \( T_{cs} \), the films develop superconductivity with type II property. So the superconducting layers form vortices inside as well as outside the junction area. The vortex becomes well defined at lower temperatures. When the temperature is just below \( T_{cs} \), it may migrate around the middle of the the junction area, mostly due to thermal excitation, and be trapped in a nearby grain boundary or a defect, called as a pinning center, whatever the pinning mechanism is.

Differing from the vortex nucleation by \( I_p \), the vortex by the field cooling process can locate either in the bottom superconducting layer, in the top superconducting layer or in both. Moreover, the poles are opposite when both layers contain vortex. If only one layer traps a
vortex, it is a single vortex, while it is a dipole if both layers contain one vortex each. In the latter case, the dipole is very easily misaligned since the distributions of the pinning centers of two layers are independent. They are not completely independent of each other, however. The magnetic coupling of the dipole tends to attract the poles so that they become aligned [65]. This magnetic coupling energy can be greatly reduced by the large thickness of the normal metal layer.

Since large number of pinning centers are assumed in the junction area, the number of vortices trapped in the junction can be controlled by the external field strength, $H_2$. Very weak field must not be able to form a vortex inside the junction area, meanwhile strong field can generate many vortices. There is a threshold field intensity that generates only one vortex (or one dipole) in the junction area. This minimum field may not necessarily be $H' = \phi_0/H^2$, instead it turned out to be larger than that. As an example, the threshold field was 16 mG, meanwhile $\phi_0/H^2 = 10$ mG in the SNS10.

4.2. Vortex Nucleation

When a thin film superconductor carries a transport current above a certain value, it has been shown that vortex can be nucleated on an edge. It propagates into the film by the Lorentz force due to the transport current itself, and stops (pinning) at a grain boundary or defect, called as pinning center, where the pinning force exceeds the Lorentz force [Ref. 13]. This section deals with both the vortex nucleation by a transport current as well as by an applied perpendicular
magnetic field in a Pb(Bi) film of the SNS junction. The vortex nucleation theory for the type I superconductor described in the previous section will be cautiously applied. We will also try to locate the residual vortex using the theory given in Chapter 2.

4.2.1. Vortex nucleation by the transport current, $I_p$

For this experiment the transport current, $I_p$, was fed through the bottom superconducting leg of the SNS junction, and the voltage was measured between the top and the bottom superconducting layers using SQUID as schematically shown in Fig. 3.7. So the SQUID may detect voltage due to vortex flow in the lower half of the bottom superconducting layer, including the junction area. The Josephson junction characteristics with zero $I_p$ will tell if a vortex locates inside the junction area.

First the sample temperature is raised above 9K to eliminate possibly pinned vortices in the junction and subsequently cooled down below $T_c$. At 6.9K $I_p$ is applied and increased to a certain value through the bottom leg of the junction, then decreased to zero. If the vortex state is generated by $I_p$, it may subsequently move under the Lorentz force. The vortex then will be caught and pinned at a pinning center where the pinning force exceeds the Lorentz force, so that the vortex can be held inside the layer even after $I_p$ is removed. The $I_c$ of the junction then is measured to see if the superconducting layer traps a vortex inside the junction area. The above processes are repeated
with higher $I_p$'s (or $I_p$'s in the opposite direction if necessary) and consequent $I_c$'s are recorded. The experimental results are as follows:

1) It is found that there are well defined and reproducible $I_p^n$'s for given temperature at which a vortex is trapped. The superscript $n$ of $I_p^n$ denotes that the host quantity, here $I_p$, is used for vortex nucleation and is turned off for $I_c$ measurement.

2) The junction does not trap vortex for $I_p$'s smaller than $I_p^{n1}$, which is 36.7 mA at 6.9K. Neither $I_c$ change nor resistive state were observed until $I_p < I_p^{n1}$.

3) $I_c$ is monotonically suppressed as $I_p^n$ grows above $I_p^{n1}$ (Fig. 4.1). At higher temperatures, say above 7.0K, the $I_c$ suppression is step like. Below 7.0K it appears to be continuous.

4) Vortex nucleation appears at the edge first of the superconducting layer as determined from the diffraction patterns. It moves toward the center under the Lorentz force.

5) $I_c$ is recovered, but not completely, by application of negative $I_p$. This means that $-I_p^n$ permits one to push the vortex toward the edge of the junction, even though the return does not seem to be complete. The return path also is not the same as the entry one.

6) There is a second characteristic current $I_p^{n2}$, above which the SQUID picks up voltage of typically 1 - 100 nano-volts depending on applied $I_p$ and temperature. This voltage is believed to be the indication of the resistive state of the superconducting...
Fig. 4.1. $I_c$ vs. $I_p^n$ for vortex nucleation. $I_c$ was measured with $I_p = 0$
layer.

7) The minimum vortex nucleation current, $I_{p1}^n$, at 6.9K is 36.7mA. This is close agreement with the prediction given by Eq. (4.8).

Although the above qualitative understanding was made, attempts to find a unique vortex configuration using the theory described in chapter 2 which fits the data well were not successful. The failure might indicate that many vortices are involved for each vortex pinning by nucleation. Or it might be due to something else that is not known yet.

The temperature dependence of vortex nucleation is of interest. Fig. 4.2 illustrates the experimental results of the minimum vortex nucleation current, $I_{p1}^n$, as a function of temperature. There seems to be three phases divided by $\approx 6.6K$, below which the self field becomes significant, and by 7.15K, above which it rises up rapidly. Except the small bump around 6.6K and the strange increase above 7.15K, the curve provides somewhat reasonable agreement with the theory of Eq. (4.8), which is for type I superconductor. The detailed structures of the curve are clearly different from the theory.

By adding $I_{p2}^n$ data to $I_{p1}^n(T)$ curve, as shown in Fig. 4.2, we can infer the window in which vortices can be nucleated at the edge without inducing normal region in the film. The two curves are very close between 6.7K and 7.15K. They also crosses at about 7.05K. At the temperatures below 6.7K, the pinning force is so strong that vortex flow can not occur until the current reaches $I_{p2}^n$, that is much higher than $I_{p1}^n$. 
Fig. 4.2. Minimum vortex nucleation current, $I_{p1}^m$, as a function of temperature
4.2.2. Vortex nucleation by $H_z$

When a superconducting strip is in external magnetic field perpendicular to the surface, vortex state may occur on the edge even though the strip is in Meissner state as stated earlier. The perpendicular field, $H_z$, induces a screening current density, $K(x,y)$, against the external field on the surface inside the junction area according to Eq. (2.10). This current exerts Lorentz force on a vortex and introduces it inside the junction. Experimental procedures are same as those for $I_p$ except that $I_p$ is replaced by the perpendicular field, $H_z$.

This experiment shows that the vortex pinning behavior by the perpendicular field is very similar to that of the transport current. If $H_z^* < H_z^{*1}$, a characteristic value, the junction has no trapped vortices. Otherwise, the critical current of the junction at zero field, $I_c(0)$, is monotonically and rapidly decreased as $H_z$ increases, as is by $I_p$. Also the application for a negative $H_z$ recovers $I_c$ up to almost $I_c(0)$ (Fig. 4.3). This indicates that the vortex has retraced its path. In contrast to the $I_p$ case, no resistive state was observed with $H_z$ up to 3 gauss. Even with the advantage of no resistive state, search for the vortex configuration has not been successful except one case.

The equivalence of $H_z^*$ with $I_p$ permits one to linearly superpose their fields on the edges. In the presence of $I_p < I_p^{*1}$, $H_z^*$ is increased as before for vortex nucleation. Repeated measurements with different $H_z^*$ and $I_p$ produce $I_p^{*1}$ vs. $H_z^{*1}$, as shown by Fig. 4.4.
Fig. 4.3. Vortex nucleation by external perpendicular field, $H_Z^\parallel$. $I_c$ is recovered by opposite $H_Z^\parallel$. 

6.9 K
The center is shifted by $H^0_2 = 0.3$ gauss. We interpret the graph as follows:

1) The two fields generated by $I_p$ and $H_z$ add constructively on one edge and cancel each other on the other edge.

2) There is one favorable pinning center on each side. Geometrically the most favorable pinning center (site 1) is in the right side edge of the 1st quadrant, while the next favorable pinning site (site 2) is on the left edge of the 3rd quadrant of the junction.

3) When $H_z > 0$ and $I_p > 0$, two fields cancels each other on the site 1, but add on the site 2. So the vortex pinning is likely to be on the site 2. On the other hand, when $H_z < 0$ and $I_p > 0$, the site 1 is more favored by the same reason as above. The transport current, $I_p$, and the screening current, $K(x,y)$, also add or cancel in the same way as above.

It is interesting to see the minimum edge field for vortex nucleation. To pin a vortex on Site 1, $I_p$ only needs 36.5 mA, while $H_z$ only needs 2.4 gauss. Using Eq. (4.5b) and (4.6), we have

$$H_{L}(edge, I_p) = 240 \text{ Gauss}$$
$$H_{L}(edge, H_z) = 190 \text{ Gauss}$$

These values are comparable but the difference is not negligible.

There is only one successful trial to locate the vortex in the junction. This appears for $I_p^n = 22$ mA and $H^0_2 = 2.0$ gauss. One vortex fitting is excellent for $I_c$ vs. $H_y$ case, but $I_c$ vs. $H_z$ is not. The vortex turns out to be a positive pole locating $r_+ = (-0.81, 0.57)$, the site 2, in reduced coordinate (Fig. 4.5). As $I_p$ increases to 23 mA,
Fig. 4.4. Combination of $H_D$ and $I^N_P$ for minimum vortex nucleation condition.
Fig. 4.5a. Diffraction patterns for a single vortex nucleation. \( I_p^* \) was increased to 22 mA while \( H_{2}^* = 2.0 \) gauss
Fig. 4.5b. Diffraction patterns for a single vortex nucleation. $I_p$ is 23 mA and $H_2^N = 2.0$ gauss.
the vortex moves into new location of $r_+ = (-0.60, 0.51)$ without introducing new vortex (Fig. 4.6). At $I_p = 24$ mA, one vortex fitting no longer works, saying that rather complicated vortex configuration appears.

4.3. Single Vortex Pinning, Depinning and Elementary Pinning Force

In the previous section we investigated the vortex nucleation behavior in a SNS Josephson junction. A transport current (or applied magnetic field or both) initially nucleates vortex on the edge of the superconducting layer and propagates it into the junction. Even though progress has been made in understanding on vortex nucleation in this experiment, locating the pinned vortex and tracing its motion were not successful except in a few cases, perhaps meaning that there is complex array fo vortices of this type of nucleation. There is technical difficulty with the application of the theory to more than a few vortices, namely two, as described in the chapter 2. Thus a single vortex pinning is highly desired in the experiment. This section presents that a remarkable success has been made on the single vortex pinning and depinning experiment, leading to a breakthrough on the measurement of the elementary pinning force associated with a particular pinning center.

There are two main ingredients for success of the experiment. First, the vortex should be a single or a misaligned dipole. For this purpose, a large thickness of normal metal layer of the SNS junction is
essential to reduce the magnetic coupling energy between the two superconducting layers. Secondly, the depinning experiment should be made without introducing new vortices by nucleation. This is possible in high temperatures only, in which the pinning force is small. These facts shall be treated again in the final part of this section.

Throughout this section, a vortex will be assumed to be two opposite magnetic monopoles embedded on opposite surfaces within the depth of $\sim \lambda$ of a superconducting layer and connected by magnetic field column with radius of $\sim \lambda$, even though this treatment lacks of mathematical rigour.

In the first part of this section the vortex is introduced in the junction by Field Cooling Process (FCP), that makes it possible to trap a single vortex. In the second part, the vortex will be depinned and moved by the transport current, $I_p$, while avoiding vortex nucleation. From the minimum depinning condition, the elementary pinning force associated with the pinning center is calculated in the third part. Then we investigate the temperature dependence of the pinning force. Finally, we will make remarks on the conditions and limitations of the experiment.

4.3.1. Field cooling process - Experiment

The procedure of the field cooling process is as follows;

1) The sample temperature is raised above 9K in order to eliminate vortices in the junction.
2) A magnetic field, $H_c^\parallel$, is applied perpendicular to the junction surface. 10 mG of the field is comparable to one fluxon threading the junction area. Here the superscript c of $H_c^\parallel$ specifies the use of $H_z$ as n of $I_p^n$.

3) The junction is slowly cooled through $T_{cs}$ down to an operating temperature, say 6.9K, in the presence of the perpendicular field. Fast cooling may not produce reproducible results because there will be thermal gradients in the junction.

4) $H_c^\parallel$ is then decreased to zero.

5) $I_C(\text{H=0})$ is measured, and recorded with $H_c^\parallel$.

6) The above processes are repeated with different $H_c^\parallel$.

This experiment produced a reproducible and well defined step-like $I_C$ vs. $H_c^\parallel$ pattern (Fig. 4.6). This pattern differs from sample to sample, also changes depending on degree of crystallization. For $H_c^\parallel > 0$, the junction showed no vortices for $H_z$ up to 20 mG. At 22 mG of $H_z$, $I_C$ drops to 0.169 mA, a decrease of 4% of $I_{CO}$ (= 0.176 mA). At 27 mG, $I_C$ dropped to 71% of $I_{CO}$ and dropped again to 59% of $I_{CO}$ at 32 mG. We refer the steps as $1^+$, $2^+$ and $3^+$ respectively, where the superscript + denotes the steps of $H_c^\parallel > 0$ region.

Those steps are not symmetric in SNS10 (but almost symmetric in SNS8) with respect to magnetic field reversal, even though full symmetry is expected theoretically for an ideal junction. For $H_c^\parallel < 0$, the step $1^-$, corresponding step to $1^+$, is not clear. In the second step $2^-$, that appears at $H_c^\parallel = -15$ mG, $I_C$ drops to 48% of $I_{CO}$, while $I_C$ increases up to 77% of $I_{CO}$ in the step $3^-$, that appears at $H_c^\parallel = -18$ mG. No explanation was made for this kind of asymmetry.
Fig. 4.6. $I_c$ vs. $H^c_Z$, after the field cooling process
For each step, diffraction patterns, $I_C$ vs. $H_y$ and $I_C$ vs. $H_z$, were taken to see distortions. The more $I_C$ is suppressed, the more significantly distorted the pattern is found to be. For example, $I_C$ vs. $H_z$ of the step 1$^+$ is almost same as the original Fraunhofer pattern, but the diffraction pattern of the step 2$^-$ is remarkably distorted.

Now we bring the theory described in the Chapter 2 to analyze those patterns. For the step 1$^+$, one vortex fitting did not work. Instead, one dipole provides an excellent fit between the theory and the data. The vortex positions were found to be $+(0.06, 0.20)$ (bottom layer) and $-(0.12, 0.28)$ (top layer) in the reduced coordinates, where $+$ ($-$) denotes a plus (minus) pole. The dipole separation is so small that it does not alter the diffraction patterns significantly. Also image vortex effect is negligible since the dipole is close to the center (Fig. 4.7).

The story of the step 2$^+$ is not so simple, however. One vortex fitting produced an acceptable result, which clearly proved the existence of a single vortex (Fig. 4.8). One can easily infer that the step 2$^+$ may include the vortices of the step 1$^+$. A much better fitting was achieved with 3 vortices, one single vortex at $(0.59, -0.14)$ and one dipole same as that of the step 1$^+$. Since the single vortex is not very close to the center of the junction, the presence of the images play an important role to improve fitting as shown in Fig. 4.9. In addition, the single vortex dominates on the distortion of the diffraction pattern. So there are symmetries to be broken as described in the chapter 2. The symmetry breaking was made from the response to a transport current, $I_p$, or a strong perpendicular field, $H_z$, for
Depinning. As a matter of fact, the presence of the dipole in step 2+ was found after the single vortex was removed out of the junction area. The vortex configurations of the steps are as follows;

1) Step 1+: one dipole at
   \((0.06, 0.20)B, -(0.12, 0.28)T\).

2) Step 2+: one dipole and one single pole at
   \((0.06, 0.20)B, -(0.12, 0.28)T\) and \(-(0.59, -0.14)T\).

In the sense that a dipole is energetically more favored, the single vortex pinning is rather fortuitous.

3) Step 2−: one dipole and one single pole at
   \((0.12, 0.28)T, -(0.06, 0.20)B\) and \((0.42, -0.09)T\).

4) Without image vortices, the single vortex is found to be \((0.57, -0.01)T\) for the step 2− (Fig. 4.12). The curve fitting is very good for small and negative \(H_y\) region, but not very good for positive \(H_y\) region in \(I_C\) vs. \(H_y\). The fitting of \(I_C\) vs. \(H_z\) is not very convincing, even though it surely proves the presence of the single vortex. With a dipole only (step 1+), the curve fitting is same with and without images.

The positions are given in the reduced coordinates and \(T\) and \(B\) stands for the Top and the Bottom layers respectively. The dipole position is not very accurate. It is believed that the inaccuracy of the curve fitting is mainly caused by the inaccurate dipole position. In this determination, the absolute uncertainty of each number is believed not to exceed ±0.05, corresponding to ±1.2 μm, while relative error is smaller than that.

Once trapped in the junction, the vortex did not move at different
Fig. 4.7. Curve fitting for the step 1+ with a small dipole of 
\[+ (0.01, 0.27) \text{ and } -(0.06, 0.19)\] near the center
Fig. 4.8. Single vortex fitting of for the step $2^+$. The vortex is negative at $(0.57, -0.20)$. All images are included.
Fig. 4.9a. Three vortex fitting for the step 2+. A dipole vortex of step 1+ has been added to the single negative vortex at (0.59, -0.14). The dipole has only small effect on the distortion.
Experimental Data

Fitting, Monopole + Dipole

Step 2

\[
\frac{I_c}{I_{co}} (I_{co} = 0.176 \text{ mA})
\]

\[
\frac{H_2}{H_1} (H_1 = 0.365 \text{ G})
\]
Fig. 4.9b. $J_c$ for the same vortex configuration of Fig. 4.9a. Even though the negative current part is dominant, the measured current should be absolute value of the total current.
Fig. 4.10. Single vortex fitting for the step 2-. The vortex is positive one at (0.43, -0.07). The single vortex is mostly responsible for the distortion of the patterns.
Fig. 4.11a. Three vortex fitting for the step 2−. The presence of the dipole clearly improves the fitting. The single vortex is positive one at (0.42, -0.09)
Fig. 4.11b. $J_C$ for the 3 vortex configuration given in Fig. 4.11a
Fig. 4.11c. 3-D version of Fig. 4.11b
Fig. 4.12a. Data fitting of with and without images for the step $2^-$. Without images the single vortex is found to be at $(0.57, -0.10)$
Fig. 4.12b. $J_c$ with three vortices, but without image vortices
temperatures. The vortex was kept for one week without changing $I_C$. The pinning is strong enough to resist external agitation such as thermal excitation, it is believed that the vortex can be kept as long as the temperature is below 7.15K. The stability was confirmed by identical diffraction patterns for 7.0K, 6.9K and 6.8K. Because the step $2^-$ shows a distinguished vortex configuration, we will focus on that step for the vortex depinning experiment, otherwise specified.

4.3.2. Vortex depinning by the transport current, $I_P$

In the above, the vortex configuration of the step $2^-$ was found to be a dipole plus a single pole. The dipole is so strongly pinned that $I_P$ could not depin without vortex nucleation. Instead, the single pole could be moved by $I_P$. The experimental procedure is:

1) The sample is cooled with $H_2 = -16 \text{ mG}$ to get the step $2^-$ (field cooling process). This step is confirmed by $I_C(0)$ measurement.

2) Apply $I^d_P$ through the bottom superconducting layer.

3) Decrease $I^d_P$ to zero. Here $d$ of $I^d_P$ specifies the use of $I_P$ to be vortex depinning as $n$ of $I_P$.

4) Measure $I_C(0)$, and recorded it with $I^d_P$.

5) Repeat the processes with increased $I^d_P$.

The result of above experiment is another well defined step like pattern as shown in Fig. 4.13a. In the step $2^-$, positive $I^d_P$ caused step like increase of $I_C$, while negative $I^d_P$ caused step like decrease. Geometrically $I^d_P > 0$ pushed the single vortex toward the edge ($x = 1$).
Fig. 4.13a. Step-like pattern of $I_c$ vs. $I_p^0$ for various temperatures.
Each step corresponds to different vortex configuration.
Fig. 4.13b. $I_c$ vs. $I_p^d$ for 6.9K. Each step corresponds the given pinning site.
to increase $I_C$. Similarly, $I_p^d < 0$ pulled the single vortex into the center ($x = 0$) of the junction, decreasing $I_C$.

For each step of $I_C$ vs. $I_p^d$, the diffraction patterns, $I_C$ vs. $H_y$ and $I_C$ vs. $H_z$, were prepared for curve fitting to find the vortex configuration. This work clearly proved vortex depinning by $I_p^d$ as discontinuous vortex jump from one place to the another place. The single vortex did not move by positive $I_p^d < 12$ mA (or $I_p^d < 20$ mA for negative direction). Detailed depinning story is given below.

$I_p^d = 12$ mA at 6.9K caused the vortex by move a small, but clearly discernible, distance of 0.2 from $A = (0.42, -0.09)$ to $B = (0.44, -0.09)$, and increased $I_C$ from 0.093 mA to 0.098 mA. This pinning center is strong enough to resist up to 19 mA of $I_p^d$. For $I_p^d = 19.5$ mA, it jumps outward by 0.37 from $B = (0.44, -0.09)$ to $E = (0.81, -0.09)$. The single vortex has disappeared by $I_p^d = 25$ mA, and it is believed that it was kicked out of the junction area (F). Now the diffraction pattern is identical to that of the step $1^+$ with opposite polarity. This, in fact, is how the presence of a dipole was found in the step $2^-$. Note that the vortex nucleation starts at $I_p^{d1} = 36.5$ mA. Up to this current $I_C$ was not changed, meaning no dipole depinning.

Here the letters A,B,E and F are individual pinning sites. The site F is expected to be somewhere outside the junction (top layer extends to $x > 1$), but the possibility of vortex-antivortex annihilation is not excluded. In either case, the Josephson junction can not see it.

In the negative $I_p$, depinning behavior is different from the positive $I_p$ case just described. By $I_p^d = -22$ mA, the vortex jumps toward the center of the junction by $= 0.1$ from $A = (0.42, -0.09)$ to G =
(0.33, -0.09) to decrease $I_C$ from 0.093 mA to 0.074 mA. At -30 mA it jumps almost to the center of $H = (-0.02, -0.10)$, giving almost zero $I_C(0)$. After this move, next motion is not clear. Higher $I_{P}^{d}$ may move it further, but the nucleation of vortices makes the determination of the single vortex very difficult.

So far, there looks to be only small number of pinning centers. Once the vortex is depinned by $I_{P}$, it overpasses all pinning centers between A and G, if any, and so on. Even though depinning does not tell what the moving path is, the path is strongly believed to be linear and parallel to the x axis as shown in the depinning diagram of Fig. 4.14.

Depinning at 6.8K, at which $I_{P1}^{d}$ = 43 mA, shows different major pinning sites from those of 6.9K. For $I_{P}^{d} < 0$, depinning was not possible without vortex nucleation. To the right side of A ($x > x_{A}$), new pinning centers of C = (0.53, -0.09) and D = (0.59, -0.09) are developed (Fig. 4.14). First depinning (A $\rightarrow$ B) occurred at $I_{P}^{d} = 22$ mA so that $I_C$ was enhanced from 0.150 mA to 0.157 mA. Second depinning (B $\rightarrow$ C) occurred by $I_{P}^{d} = 28.5$ mA, followed by third depinning (C $\rightarrow$ D) by $I_{P}^{d} = 36$ mA. Finally, $I_{P}^{d} = 38$ mA moved the single vortex out of the junction without stopping by the sites of E.

At higher temperature of 7.0K, the depinning behavior is simpler than lower temperature cases. The first depinning (A $\rightarrow$ B) seemed to occur at $I_{P}^{d} = 6$ mA, but the site B is not stable any more at this temperature so that the vortex easily returned back to A by small external agitations such as spike current or thermal excitation. $I_{P}^{d} = 11.5$ mA surely pushed the vortex out of the junction (site F) without stop by C,D or E, meaning no stable pinning centers between $x = x_{A}$ and x.
= 1 along y = -0.1 line. For \( x < x_A \) region, two pinning centers, G and H, were still stable to allow depinning to G at \( I_p^G = -12.5 \) mA and to H at -19.5 mA. At \( I_p^d = -26.5 \) mA the single vortex was depinned from the site H and removed from the junction (\( y < -1 \)), leaving the dipole only around the middle of the junction.

At 7.1K, at which no quantitative analysis were made, new pinning centers for \( x < x_A \) were observed between G and H, but no pinning centers for \( x > x_A \). This appearance of new site is not because of development of new site, but because of weakened pinning force of the site G at higher temperature, allowing the vortex to stop at new pinning center.

After initial depinning (forward), tracing backward is particularly of interest. Unfortunately, this was not done much in the present experiment. For most cases, certainly not all, backward depinning did not follow forward depinning pattern. So more pinning centers were found, even though they are less stable. No quantitative work was undertaken.

Finally, depinning by applied perpendicular field was demonstrated. Here a current equivalent to \( I_p \) is induced by the applied field rather than an external current source. The magnetic field inside the junction is given by Eq. (2.13) shall interact with the magnetic charge. Outside the junction (outside surface of the superconducting layer), the field is so weak that the interaction can be ignored. For the step 2- with pinning site A, \( H_z \) of same direction as \( H_2^S < 0 \) pushes outward if the vortex is in the top layer (plus pole), while inward if it is in the bottom layer (minus pole). Experimental results show that the vortex stops at the site B by \( H_2^S = 1.6 \) gauss, at the site E by 2.0 gauss and
Fig. 4.14. Stable pinning sites for the single vortex. The dipole is too strongly pinned to be depinned by $I_p^d < I_{p1}^d$. The open circle is a negative vortex and closed circles are positive vortices.
to the site F(outside) by at = 2.2 gauss at 6.9K, proving the vortex in
the top layer (plus pole).

A summary of the depinning behavior is as follows:

1) A linear path of vortex motion was observed. From this, the
size of a pinning center is believed to be much smaller than
depinning distances. This means the pinning potential is well
localized within $\lambda$. Once the vortex is depinned, it moves
through superconducting region until it encounters a stronger
pinning center.

2) By depinning forward and backward sequentially, we can find many
pinning centers. It may be very interesting to compare this
pinning center map with real grain boundary map taken by SEM.
This was not done in this experiment.

3) The pinning potential is shown to be asymmetric. For the
pinning site B the minimum pinning forces in $+x$ and $-x$ directions
differ by factor of 2.

4) Pinning potential topography changes as a function of
temperature. As a result, (less stable) pinning centers may
disappear at higher temperatures, while new pinning centers can
be developed at lower temperatures, resulting different
depinning patterns and different depinning forces. As expected,
the pinning force is greater at lower temperatures than that of
higher temperatures. Temperature dependence of pinning force
will be discussed again.

5) The detailed curve fittings are given in the Appendix.
4.3.3. Elementary pinning force

Depinning force is essentially the interaction of the vortex with current. There are two different ways to discuss the Lorentz force.

The transport electric current interacts with a vortex. The Lorentz force, \( \mathbf{f}_p \), caused by local current density \( J_p \) is given by

\[
\mathbf{f}_p = J_p \times \mathbf{A} \quad (4.10)
\]

Let \( J_p(x) = J_0 \cdot f(x) \), where \( J_0 = I_p/(\pi d_s) \), uniform current density, and \( f(x) \) is the fraction for non-uniform current density. We assume \( J_p \) is uniform in thickness. Then \( \mathbf{f}_p = \mathbf{f}_p x \), the elementary pinning force per unit length of the vortex, so

\[
\mathbf{f}_p = J_p \phi_0/c \\
= (I_p \phi_0/(\pi d_s)) f(x) \quad (4.11)
\]

The elementary pinning force, total force, is

\[
f_p = d_s \cdot \mathbf{f}_p \\
= (I_p \phi_0/(\pi d_s)) f(x) \quad (4.12)
\]

In the other scheme, a pole of a vortex is regarded as a magnetic charge of \( q_m = \phi_0/(4\pi) \). Maxwell's equation \( \nabla \times \mathbf{B} = (4\pi/c)J_p \) is integrated over the closed path surrounding the local transport current. If the path is not close to the edge, perpendicular component of \( \mathbf{B} \) would be ignored, so

\[
(2\pi/c) I_p f(x) \]

Force acting on the charge is

\[
f_p = 2q_m B \\
= (I_p \phi_0/(\pi d_s)) f(x) \quad (4.14)
\]
where the factor 2 comes from two poles on each side of the current
carrying superconductor. The two forces of Eq. (4.12) and Eq. (4.14)
are identical even though they are derived from different point of view.
We extend this idea.

External field generates surface shielding current, which interacts
with a vortex. So $I_p$ through the bottom layer can exert Lorentz force
on the vortex in the top layer. The magnetic field due to $I_p$ on the
bottom surface of the top layer is not calculated accurately. If we
ignore the presence of the top layer, the field may be given
approximately by Eq. (4.1) with $f(x) = 1$. If we assume full ground
plane, the field is $(4\pi\alpha/cW)I_p$ with $\alpha \approx 0.9$ [69]. From the linear path
of the vortex motion, we assume that $J_p$ is approximately along $y$
direction. So Eq. (4.13) may be a good approximation. In this
approximation, the force is

$$f_p = q_mB$$

$$= (1/2)(\Phi_0/cW)I_p$$

$$= (1/2)(4.5 \times 10^{-6})I_p \text{ (dyne/Amp) (Practical unit)} \quad (4.15)$$

B field on the top surface of the top layer is very weak at the vortex
position if the vortex is not near the edge of $y = 1$. So the
contribution from the outer pole is ignored.

With Eq. (4.14), we get the pinning force. For depinning from the
site A to the site B, minimum depinning current is 12 mA, so

$$f_p(A\rightarrow B) = (1/2)(4.5 \times 10^{-6})(12 \times 10^{-3})$$

$$= 2.7 \times 10^{-8} \text{ (dyne)}$$

Average force per unit length of the vortex is
\( \tilde{f}_p(A\to B) = \frac{f_p}{d_g} \)
\[ = 4.5 \times 10^{-4} \text{ (dyne/cm)} \]

This is the maximum pinning force associated with the pinning center \( A = (0.42, -0.09) \). For other pinning sites, \( f_p(B\to E) = 4.4 \times 10^{-8} \) dyne and \( f_p(E\to F) = 5.2 \times 10^{-8} \) dyne.

Different pinning center shows different pinning force. In addition, the pinning force is not isotropic. For depinning \( A \to G \), \( \tilde{f}_{p1} = -22 \) mA, so
\[ f_p(A\to G) = 4.9 \times 10^{-8} \text{ (dyne)}, \text{ or} \]
\[ \tilde{f}_p(A\to G) = 8.2 \times 10^{-4} \text{ (dyne/cm)} \]

The external field generates a parallel field according to Eq. (2.13). In reduced coordinates, \( B = 2H_z(x\hat{x} - y\hat{y}) \). The depinning motion from \( A \to B \) is caused by a field of \( H_z \approx -1.6 \) gauss, while \( B \to E \) by \( H_z = -2.0 \) gauss, and \( E \to F \) by \( H_z \approx -2.2 \) gauss. Since the vortex is at \( (0.42, -0.09) \), ignoring \( y \) component, we get \( f_p(A\to B) = q_mB = 2.2 \times 10^{-8} \) dyne (or \( 3.7 \times 10^{-4} \) dyne/cm). Comparing the pinning forces from two different experiments gives the ratio \( \left( \frac{f_p(I_p)}{f_p(H_2)} \right) \) of 1.2 for \( A \to B \), 1.5 for \( B \to E \) and 0.83 for \( E \to F \). Noting approximations in calculating the forces, the two measurements are rather in good agreement.

It may be useful to estimate the size of vortex-image interaction. Using the reduced coordinates, \( x \to (W/2)x \) and \( y \to (W/2)y \), and with \( d_{\text{eff}} = 1200 \) nm, we get from Eq. (4.3)
\[ f_m = (2.0 \times 10^{-9} \text{ dyne}) \cdot f(x_0, y_0) \]
(4.16)
where \( f(x_0, y_0) = \frac{\Delta p(x-x_0)}{|x-x_0|} \). Numerical calculation gives \( f(0.42, -0.1) = 0.478\hat{x} - 0.059\hat{y}, f(0.81, -0.1) = 2.47\hat{x} - 0.007\hat{y} \), but \( f(0.95, -0.1) = 9.95\hat{x} - 0.004\hat{y} \) and \( f(0.99, -0.1) = 50.0\hat{x} - 0.004\hat{y} \).
Ignoring the y component, $f_m = 1 \cdot 10^{-9}$ dyne for the vortex $(0.42, -0.09)$, which corresponds to at most $0.25$ mA of depinning current. For weak pinning materials this force may not be ignorable.

### 4.3.4. Temperature dependence of $f_p$

Although the pinning force is not accurately calculated, it is strictly proportional to the depinning current, whatever the local current density is. Minimum depinning currents were measured for various temperatures and various jumps. The temperature dependence of $(I_p^{(A-B)})^{2/3}$ is shown in Fig. 4.15. From the linearity of the graph, we infer as

$$f_p = (1 - t)^{3/2} \quad (4.17)$$

since $f_p \propto I_p^d$, where temperature independent distribution of $I_p$ is assumed. The reduced temperature $t = T/T_C$ with $T_C = 7.3K$ was used. The line does not extrapolates to $T_C$. This is ascribed to the fact that a stable pinning potential well does not form until same temperature below $T_C$. At higher temperatures the well is too shallow to catch a vortex. One must note that $I_p^{d(B-E)}$ line runs higher than $I_p^{d(A-B)}$. The exponent of $(3/2)$ in $f_p$ was cited by J. Galland and H. Lee [70], and L. Allen and J. Claassen [71].

Temperature dependence of $f_p$ may be closely related to the pinning mechanism, that is little known. D. Finnemore suggested a simple derivation of Eq. (4.17) [72]. If pinning potential is a triangular well with radius of $\xi$, the coherence length, pinning force is the slope of the well. The energy difference between with vortex and without
Fig. 4.15. Temperature dependence of $+I_p^d$ (open circles) and $-I_p^d$ (closed circles). By $+I_p$ the single vortex moves from A to B, while by $-I_p$ from A to G.
vortex is equalized with the work done by depinning, as

\[ f_p \cdot \xi_s = \frac{H_c^2}{8\pi} (\pi \xi_s)^2 d_s \]  

(4.18)

Since \(H_c \sim (1 - t^2)\) and \(\xi_s \sim (1 - t)^{-1/2}\), the above equation reduces to

\[ f_p \propto (1 - t^2)^{1/2}(1 - t) \]

\[ = (1 - t)^{3/2} \]  

(4.17)

near \(T_c\).

4.3.5. Limitations of the experiment

Although the single vortex pinning, depinning and locating were successfully carried out, the SNS system shows some limitations and disadvantages for the performance of the experiment, as listed below.

1) Maximum vortex depinning current must be smaller than the minimum vortex nucleation current, \(I_{d1}\), otherwise depinning is messed up by nucleation. Since \(I_d \gg I_{d1}\) below 6.7K, there is a window between \(I_d^{1}\) and \(I_{d1}\) above 6.7K, where depinning experiment is available as illustrated in Fig. 4.16. This kind of window was observed in granular Al film by M. Fang [73].

2) The superconductivity of the S layer is somewhat weakened by the presence of the N layer (proximity effect), resulting smaller pinning force than that of an isolated S layer.

3) N layer must be thick to induce misalignment of a dipole vortex.

4) The result is highly dependent upon the preparatory condition of the sample. The sample should not be kept in room temperature
Fig. 4.16. Temperature dependence of vortex depinning current, $I_{p1}^d$, and vortex nucleation current, $I_{p1}^n$. There is a window of shaded region for the vortex depinning experiment.
long time, otherwise the operating temperature may be far below
the window mentioned above mainly because of small $I_C$.

5) Pinning forces measured in this experiment are in $\pm x$ directions
only. No other component of the forces were measured. By
applying $I_p$ in $x$ direction as well as $y$ direction, pinning force
can be studied in all directions in principle.

6) $I_C$'s in $+$ and $-z$ directions are not same when the junction
contains vortices. Double data points in Fig. 4.11a are two
$I_C$'s in $+$ and $-z$ directions. The most serious case occurred
after $I_p^0$ was applied for vortex nucleation at 6.5K. $I_{C+} = 0.42$
mA and $I_{C-} = 0.38$ mA, while $I_{CO} = 0.80$ mA, giving $\Delta I_C/I_{CO} = 5\%$.
Also $\Delta I_C$ oscillates in $H_\gamma$. The source of $\Delta I_C$ has not been
explained.

4.4. Future Work and Application

We just started playing with vortex in a SNS junction. There are
many other subjects to be studied. Here a closely related topic and its
possible application to electronics are presented.

4.4.1. Easier way of reading vortex position

A single vortex originally trapped at (0.42, -0.09) was depinned
and moved by $I_p$ along the straight line of $y = -0.1$, corresponding to
the Lorentz force direction. This characteristic response of the vortex
to the current brings an easier way of the vortex position.
Fig. 4.17. Experimental voltage vs. Josephson currents of the junction. Each curve corresponds to different single vortex position. The curves well fit to RSJ model (Eq. 3.1) with same $R_n$ and different $I_c$'s.
For every possible pinning positions, there is only one $I_c(0)$ as the vortex moves along the x axis. So under the assumption that the vortex motion is confined in a specific path, say the x axis, we can determine the vortex position by measurement of $I_c(0)$, without preparing the diffraction patterns (Fig. 4.17).

Instead of $I_c(0)$, we can use the corresponding voltage across the junction with $I_J > I_c$. The RSJ model applies to the SNS junction with vortices as well. Since the presence of the vortex reduces $I_c$, the increase of the voltage for $I_J (> I_c)$ is according to the vortex position.

4.4.2. Confined vortex and application

To use the above idea, the vortex needs to be confined in a specific path. We know that the favorable pinning sites are grain boundaries, defects or voids, for which the pair potential is low. So confining a vortex in a desired region is reduced to making the pair potential of the region lower than other regions.

If a normal metal layer of narrow strip is deposited on the top layer as illustrated in Fig. 4.18a, the proximity effect suppresses the order parameter of the strip region. The region then will be the most favorite place of vortex trapping by the field cooling process. Also, the vortex can be depinned by smaller $I_c$ within the region, meanwhile strong $I_p$ is necessary to get out of the region, resulting vortex confinement.

Thinner part of the superconducting layer can provide weaker order parameter. Dry etching technique (Fig. 4.18b) may work for the purpose,
(a) Extra N layer is deposited on the S layer.

(b) A narrow part of S layer is ditched.

Fig. 4.18. Fabrication of weak superconducting region
but expensive. Scratching can leave ill-defined boundaries of strong pinning centers. Or, shadowed evaporation may be adopted. For all methods there are technical difficulties to be overcome.

If the sample is prepared by evaporation of a strip, the strip is supposed to run from near the edge to the center along the x axis. By the field cooling process a vortex is mostly likely trapped in the strip region. Vortex depinning within the region becomes much easier than outside. So application of appropriate $I_p$ can provide a controlled vortex motion. The critical current, $I_c(H=0)$, is $\approx I_{c0}$ with the vortex near the edge, while zero with the vortex at the center. Similarly, the voltage across the junction for $I_J > I_c$ is very small with a vortex near the edge, while it is large with the vortex at the center (Fig. 4.17). This distinctive two voltage states provide an application as a digital signal of 0 and 1. It can also be used as a constant digital memory because the vortex is very stable without $I_p$ or thermal excitation.

So far, we have examined electrical properties with Pb(Bi)-Ag(Al)-Pb(Bi) SNS junction. This junction produces very small voltage of ~2 nanovolt for $I_J = 2I_c$ across the junction at 6.9K, requiring the SQUID detection. A high impedance SNS junction is desired for more convenient experiment. This junction must be of lower pinning superconductor with high stability. One of the example is a SINS type junction, in which I layer provides most of the impedance and N layer gives thick barrier with small impedance. A Nb based SNS junction with Si barrier [74,75] is a good candidate for the experiment with convenience.
5. CONCLUSIONS

In this research the Abrikosov vortex motion in a SNS Josephson junction has been studied. The theory has been developed with magnetic monopole charge approximation of a vortex and improved by including the image vortices. The diffraction pattern distortion has been used to locate the vortex inside the junction.

A single vortex was trapped by the field cooling process. The presence of a vortex severely changed the critical current of the junction, $I_c$, as well as the diffraction pattern, $I_c$ vs. $H$. The vortex was moved in the influence of the Lorentz force by the transport current, $I_p$ and the perpendicular field. By moving the vortex back and forth, many pinning centers could be found. Distortion of diffraction patterns were successfully used to find the vortex configuration.

Particularly for the single vortex case, the critical current of the junction varies from zero to $I_{c0}$ as the vortex moves from the center to the edge of the junction. As long as the vortex is constrained in the linear path, it can be located by the zero field critical current, not by the full diffraction patterns. Alternatively, the output voltage across the junction for the Josephson current greater than $I_{c0}$ can also be used. This discrete change of critical current and output voltage can be used for an electronic digital device.

From the minimum depinning transport current, the elementary pinning force associated with a particular pinning center was measured. The single vortex initially pinned at the site $A = (0.42, -0.09)$ in
units of W/2 by the field cooling process. The pinning force of the site is calculated from the minimum depinning current to move the vortex to the nearest pinning center. The force was found to be $2.7 \times 10^{-8}$ dyne (or $4.5 \times 10^{-4}$ dyne/cm) at $T/T_C = 0.95$. The force was asymmetric and different from one pinning site to another. From the experiment the temperature dependence of the pinning force was suggested to be $(1 - T/T_C)^{3/2}$ near $T_C$.

There are two main ingredients of successful vortex depinning experiment. First, the N layer of the Josephson junction must be thick to reduce the magnetic field energy of a vortex such that a vortex pair is easily misaligned. Secondly, the vortex depinning experiment must be carried out at higher temperatures where the depinning currents are smaller than the vortex nucleation current. There was a window for the experiment above 6.7K.
6. APPENDIX

Here, the detailed experimental facts such as curve fitting of the single vortex depinning for the step $2^-$ are presented. There are three poles inside the junction, a dipole of $+(0.01, 0.27)$ and $-(0.06, 0.19)$ and a single positive pole. The dipole was too strongly pinned to be moved by the depinning current, $I_d$, which is supposed to be smaller than the minimum vortex nucleation current, $I_{pl}$. The single positive vortex initially pinned at the pinning center $A = (0.42, -0.09)$ could be depinned by the depinning current at the temperatures above $6.7 \, ^\circ K$. It also was moved to several other pinning centers by successive depinnings.
Fig. 6.1. Pinning sites found by the single vortex depinning for the step 2—
Fig. 6.2. Experimental $I_c$ vs. $H_y$ (and $H_z$) without vortex pinning
Fig. 6.3a. Stability of the vortices. The three diffraction patterns, $I_c$ vs. $H_y$, at different temperatures are identical, meaning the same vortex configuration.
Fig. 6.3b. Stability of the vortices. $I_c$ vs. $H_z$ case
Fig. 6.4a. Curve fitting for the vortex configuration A
Fig. 6.4b. $J_c$ for A
Fig. 6.4c. 3-D graph of $J_c$. 
Fig. 6.5a. Vortex configuration B
Fig. 6.6. Vortex configuration C
Fig. 6.7a. Vortex configuration D
Fig. 6.8a. Pinning site E
Fig. 6.8b. $J_c$ for E
Fig. 6.8c. 3-D graph of $J_c$
Fig. 6.9a. Diffraction pattern with the dipole only. The single pole was removed from the junction.
Fig. 6.9b. $J_c$ with the dipole
Fig. 6.9c. 3-D graph of $J_c$
Fig. 6.10a. Pinning site G
Fig. 6.10b. $J_c$ for $G$
Fig. 6.11a. Pinning site of H
Fig. 6.11b. $J_c$ for $H$
Fig. 6.11c. 3-D graph of $J_c$. 

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