Monopoly Power in Domestic Production, Smuggling, and the Non-Equivalence Between Tariffs and Quotas

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Abstract
During the last thirty years, one of the most popular research topics in international trade has been the non-equivalence among policy instruments such as specific and ad valorem import tariffs, voluntary export restraints and import quotas. The non-equivalence principle was shown to hold under revenue/rent seeking behavior (Vousden, 1990), under uncertainty (Young and Anderson, 1982), and in the presence of retaliation (Melvin, 1986; Syropoulos, 1994). Furthermore, it has been shown that different policy instruments have different effects on the stability of world prices (Zwart and Blandford, 1989) in addition to having different effects on the quality/composition of imports (Falvey, 1979; Das and Donnefeld, 1987). Perhaps the best known case of non-equivalence is the one described by Bhagwati (1965, 1969) where domestic production is controlled by a monopolist. For a given volume of imports, an import tariff generates a lower domestic price and less deadweight loss than an import quota. Casual empirical evidence from developing and developed countries alike indicates that highly distorted prices, resulting from trade and domestic taxes, provide consumers and firms the necessary incentives to engage in various types of illegal activities usually referred to as smuggling. In spite of the prevalence of this by-product of government intervention, it is often ignored for policy analysis purposes. In this paper, we revisit Bhagwati’s non-equivalence when domestic production is controlled by a monopolist and allow smuggling activities to take place when the differential between the domestic price and the world price is high enough.

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MONOPOLY POWER IN DOMESTIC PRODUCTION, SMUGGLING, AND THE NON-EQUIVALENCE BETWEEN TARIFFS AND QUOTAS

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MONOPOLY POWER IN DOMESTIC PRODUCTION, SMUGGLING, AND THE NON-EQUIVALENCE BETWEEN TARIFFS AND QUOTAS

I. INTRODUCTION

During the last thirty years, one of the most popular research topics in international trade has been the non-equivalence among policy instruments such as specific and *ad valorem* import tariffs, voluntary export restraints and import quotas. The non-equivalence principle was shown to hold under revenue/rent seeking behavior (Vousden, 1990), under uncertainty (Young and Anderson, 1982), and in the presence of retaliation (Melvin, 1986; Syropoulos, 1994). Furthermore, it has been shown that different policy instruments have different effects on the stability of world prices (Zwart and Blandford, 1989) in addition to having different effects on the quality/composition of imports (Falvey, 1979; Das and Donnefeld, 1987). Perhaps the best known case of non-equivalence is the one described by Bhagwati (1965, 1969) where domestic production is controlled by a monopolist. For a given volume of imports, an import tariff generates a lower domestic price and less deadweight loss than an import quota.

Casual empirical evidence from developing and developed countries alike indicates that highly distorted prices, resulting from trade and domestic taxes, provide consumers and firms the necessary incentives to engage in various types of illegal activities usually referred to as smuggling. In spite of the prevalence of this by-product of government intervention, it is often ignored for policy analysis purposes. In this paper, we revisit Bhagwati's non-equivalence when domestic production is controlled by a monopolist and allow smuggling activities to take place when the differential between the domestic price and the world price is high enough.

The paper is organized as follows: Bhagwati's non-equivalence and recent extensions are discussed in the next section. The model is introduced in the third section. At first, a quota and its import-equivalent specific and *ad valorem* tariffs are compared in terms of their effects on domestic output and welfare without smuggling. We show that the specific tariff dominates the *ad valorem* tariff which dominates the quota. Smuggling is then introduced in the model and its effects on domestic production and welfare are assessed under an *ad valorem tariff* and then under a quota. A comparison of these policy instruments reveals that, in the presence of smuggling, the dominance of the *ad valorem* tariff over the quota no longer
holds. We also show that under a quota, smuggling can increase welfare. Finally, we extend the ranking of policy instruments under smuggling to include the specific tariff. Our results suggest that the latter remains the best instrument.

II. BHAGWATI'S NON-EQUIVALENCE AND EXTENSIONS

Professor Bhagwati has compared tariffs and quotas when domestic production is controlled by a single firm under various assumptions regarding the behaviors of foreign suppliers and import quota holders. The two panels of Figure 1 do not reproduce all of Bhagwati's (1969) scenarios but present the main argument behind the non-equivalence. In the left panel, \( \omega \) is the world price/foreign export supply schedule consistent with the large country assumption, \( \omega(1+r) \) is the tariff-distorted foreign export supply schedule, \( R \) is the import quota, and \( D \) is the domestic demand curve. To ensure that the subsequent tariff-quota comparison involves import-equivalent policy instruments, \( R \) is set equal to the level of foreign exports under the \textit{ad valorem} tariff. The right panel illustrates three residual demand curves, defined as the difference between domestic demand and the policy-distorted foreign export supply schedules. The domestic monopolist's marginal cost curve is labelled \( MC \). Under free trade, the domestic firm faces the residual demand curve \( D-DR''-D \) which is more elastic than the tariff-distorted residual demand curve \( Dr-Dr'-D \). Under the quota, the foreign export supply schedule is given by \( \omega \) when the quota is not binding and is perfectly inelastic when exports equal \( R \). This vertical segment of the foreign export supply schedule makes the quota-distorted residual demand curve \( (DR-DR'-DR''-D) \) less elastic except at domestic prices low enough to make the quota non-binding. \( MR_\tau \) and \( MR_\omega \) are the marginal revenue curves associated with the tariff and quota-distorted residual demand curves. The profit maximizing monopolist chooses the quantity that equalizes its marginal revenue to its marginal cost. Thus, the optimal quantity produced under the \textit{ad valorem} tariff is \( Q_\tau \) which exceeds the quantity \( Q_R \) produced under the import-equivalent quota. The tariff equilibrium is characterized by a lower domestic price than the quota equilibrium \( (P_\tau < P_R) \), which implies the dominance of the tariff over the quota. This dominance is more obvious when the foreign export supply curve is infinitely elastic, as under the small country assumption. In that case, the segment \( Dr-Dr' \) of the tariff-distorted residual demand curve is perfectly elastic and acts as a price ceiling.
Other authors have revisited Bhagwati’s non-equivalence. Sweeney, Tower, and Willett (1977) assume that the government uses a tariff or quota to attain a targeted imports/domestic production ratio. When the policy instrument is perceived as exogenous by the monopolist, they show that a quota leads to a higher domestic price and hence lower welfare than a tariff. However, the equivalence between the two instruments holds when the rule used by the government to set the policy instrument is taken into account in the decision-making process of the monopolist. Rotemberg and Saloner (1989) have replaced the assumption of a monopolist firm by a cartel of firms and found that the cartel’s incentives to deter deviations from collusive behavior will bring about a lower price under a quota than under a tariff.

Kaempfer, Marks, and Willett (1988) relax the small country assumption and compare the welfare implications of tariffs and quotas generating equivalent producer rents for a domestic monopolist. They find that quotas induce higher domestic prices, higher volumes (and price) of imports and hence less overproduction than tariffs at high levels of producer rents. Consequently they conclude that quotas are more (less) efficient than tariffs at high (low) levels of protection. They also show that higher world prices and higher levels of imports are sufficient conditions for foreign countries to prefer the quota. Finally, Kaempfer, McClure and Willett (1989) explores the non-equivalence issue by holding constant the domestic price in the tariff/quota comparison. Their small country political economy setting assumes that the government provides protection for a domestic monopolist selecting the policy instrument that meets the domestic price target at the lowest welfare cost. They show that for low tariffs, price-equivalent quotas may not exist while at high levels of protection, the ranking of tariffs and quotas depends upon the domestic monopolist’s market share under free trade. Quotas dominate tariffs when, at the free trade equilibrium, the monopolist has a market share of less than fifty percent.

III. THE MODEL

We adopt the modeling strategy of many previous studies and use a partial equilibrium model based on the assumptions of a homogenous product, endogenous terms of trade and monopoly in domestic production. The model is presented in two parts. First, the behavior of the monopolist is analyzed without smuggling. It is shown that neither a tariff nor a quota is an optimal policy under these circumstances and
that the specific tariff is a better policy instrument than the *ad valorem* tariff and that both tariffs dominate the quota. In the second part, the behavior of the smugglers is described and the effect of smuggling on the monopolist’s behavior is investigated.

(i) *The monopolist’s behavior without smuggling.* In this case, imports can come only from legal channels such that legal imports \( L \) equals total imports \( I \). Domestic price (or inverse demand), as shown in equation (1), depends upon domestic consumption, which is the sum of imports \( I \) and domestic production \( Q \) \((D=I+Q)\). The endogenous world price \( P^w \) is an increasing function of the volume of imports as shown by equation (2). Domestic production is done by a single firm whose cost of production \( C \) depends on its output level \( Q \) as shown by equation (3).

\[
P^d = \theta(I + Q); \quad \theta = (d\theta/dD) < 0, \quad D = (I + Q) \tag{1}
\]

\[
P^w = \omega(I); \quad \omega = (d\omega/dI) > 0 \tag{2}
\]

\[
C = C(Q); \quad C = (dC/dQ) > 0; \quad C'' = (d^2C/dQ^2) \geq 0 \tag{3}
\]

The monopolist is assumed to be a profit maximizing firm. Under free trade, arbitrage implies that world and domestic prices will be equalized. In the presence of import tariffs (which may, for now, include both *ad valorem* tariffs (at rate \( \tau \)) and *specific* tariffs (\( t \)), the arbitrage condition implies:

\[
\theta(I + Q) - (1 + \tau)\omega(I) - \tau \leq 0; \quad I\{\theta(I + Q) - (1 + \tau)\omega(I) - \tau\} = 0 \tag{4}
\]

where (4) holds with equality when imports occur. From (4), assuming \( I > 0 \), defines:

\[
I(Q,\tau,t) s.t.: \quad \theta(Q + I(Q,\tau,t)) = (1 + \tau)\omega(I(Q,\tau,t)) + t; \tag{5}
\]

\[
I_Q = (\theta(Q,\tau,t)/\theta Q) = (\theta/((1 + \tau)\omega - \theta)); \quad \{1 + I_Q\} = [(1 + \tau)\omega'/((1 + \tau)\omega - \theta)] \in (0,1)
\]

where a subscript denotes partial differentiation. If trade is instead restricted by import quotas (of magnitude \( R \)), the arbitrage condition implies:

\[
\theta(I + Q) - \omega(I) \leq 0; \quad I \leq R; \quad (R - I)\{\theta(I + Q) - \omega(I)\} = 0 \tag{6}
\]
Assuming the quota is binding, then (by definition) \( I_Q = 0 \).

The firm's profit \( \pi^M \) is given by equation (7), while the monopolist's profit-maximizing first order condition is given by (8):

\[
\pi^M = \theta (1 + Q) Q - C(Q) \\
(\partial \pi^M / \partial Q) = J(Q, t, I_Q, I) = \theta Q (1 + I_Q) - C'' = 0
\]

Note that the specific tariff rate, \( t \), enters the FOC indirectly only through its impact on imports \( I \), whereas the ad valorem tariff rate enters both indirectly (through \( I \)) and directly (through \( I_Q \)). Also note that under a (binding) tariff: \( \{1 + I_Q\} < 1 \), whereas under a (binding) quota \( \{1 + I_Q\} = 1 \). Equation (8) determines the monopolist's output in terms of the trade policy instruments: \( Q(t, \tau, R) \). We assume:

ASSUMPTION I: With tariffs, the profit-maximizing output level of the monopolist is an increasing function of the (specific or ad valorem) tariff rate: \( (\partial Q^* / \partial t)^{>0} \), \( (\partial Q^* / \partial I)^{>0} \). If quotas are used, the optimal output level is a decreasing function of the quota level: \( (\partial Q^* / \partial \tau)^{<0} \).

Discussion: As is well-known, the SOC does not guarantee how changes in tariffs (or imports) shift the marginal revenue curve. The assumptions we make are comparable to the assumption of strategic substitutes in the strategic trade policy literature. Mathematically, the SOC require: \( (J_Q + J_I I_Q)^{<0} \). Since, from (5), \( [(\partial Q(t, I, \tau) / \partial \tau)]^{<0} \), the assumption that domestic monopoly output increases with the specific tariff implies \( J_I^t < 0 \). This, in turn, implies that \( J_Q < 0 \) for the SOC to hold. Similarly, for quotas, the assumption \( (\partial Q^* / \partial \tau)^{<0} \) implies \( J_I^t < 0 \).

Under the monopoly solution, domestic price exceeds marginal cost, which is inefficient from a welfare standpoint. A second source of inefficiency arises under free trade, since domestic price equals world price (rather than the marginal cost of imports). One trade policy instrument cannot remove both sources of inefficiency and hence cannot be by itself a first-best policy instrument. A combination of policy instruments could support the first best solution by restricting imports to obtain the optimal world price and by subsidizing imports enough to exogenize the domestic price in the eyes of the monopolist. Tariffs and quotas, if used individually, can be used to extract rents from trade partners but cannot address the issue of
market power in the domestic market. Since these policy instruments generate different residual demand curves, domestic price and hence welfare are dependent upon the choice of commercial policy instrument. The following proposition compares the effects of quotas, specific and ad valorem tariffs on domestic production when smuggling is not feasible. It complements Bhagwati's (1969) comparison between an ad valorem tariff and a quota by including a specific tariff.

**PROPOSITION 1:** Without smuggling, if a domestic monopolist produces the import-competing good, domestic production is higher under a specific tariff than under an import-equivalent ad valorem tariff, and domestic production is higher under an ad valorem tariff than under the import-equivalent quota.

**PROOF:** Ceteris paribus, output is higher the more elastic is the residual demand curve. As is well-known, under monopoly the import-equivalent quota results in lower domestic output than either tariff since the residual demand curve is less elastic, and hence marginal revenue is lower under the quota than under the tariff. To compare the two types of tariffs, it is apparent from (5) and (8) that, given \( Q, I \), marginal revenue is a decreasing function of \( \tau \). Thus, raising the specific tariff and lowering the ad valorem tariff in order to hold imports constant will increase domestic output. From the FOC, we have:

\[
J_Q dQ + J_I dI + J_\tau d\tau = 0.
\]

Under Assumption I, \( J_Q < 0 \), and \( J_\tau = \theta Q I \), thus, increasing \( I \) and decreasing \( \tau \), such that imports do not change, will lead to higher domestic output. QED

This proposition is illustrated in Figures 1 and 2. The right panel of each figure shows the residual demand curves and their corresponding marginal revenue schedules. The second part of the proposition is illustrated in Figure 1 which has been described in the previous section. Nevertheless, it might be useful to recall that \( D \tau - DR \) and \( DR - DR'' \) are the residual demand curves when "import-equivalent" ad valorem tariff \( \tau \) and quota \( R \) are imposed. The more elastic tariff-distorted residual demand induces the monopolist to produce more than under an import-equivalent quota. The comparison between import-equivalent specific and ad valorem tariffs is pictured in Figure 2. It confirms that production is higher under the specific tariff than under the ad valorem tariff, a result specific to the large country assumption.
The welfare level generated by the commercial policy instruments can be measured by adding up consumer surplus, the monopolist's profit and the government revenue generated by the policy instrument. After some simplifications, it can be shown that the level of welfare can be represented by:

\[ W = \int_0^{Q^*} \left[ \theta(D) dD - C(Q) - \omega(I) I \right] \]  

(9)

Since tariffs and quotas alter both domestic production and imports, total differentiation of equation (9) with respect to \( Q \) and \( I \) shows how welfare varies with these instruments:

\[ dW = [\theta - C'] dQ + [\theta - (\omega + I\omega')] dI \]  

(10)

As usual, welfare maximization entails the equality among domestic price, the domestic marginal cost of production, and the marginal import cost. With monopoly, two policy instruments are needed to generate this equilibrium. Given the second-best nature of the tariffs and the quota, two issues need to be addressed: i) the magnitude of the second-best tariff and ii) the welfare ranking of the second-best tariffs and quota.

**PROPOSITION 2:** Under the conditions of Assumption 1: (i) the second-best tariff rate exceeds the reciprocal of the foreign export supply elasticity; and (ii) the specific tariff dominates the second-best ad valorem tariff which in turn dominates the optimal second-best quota.  

**PROOF:** Consider the second-best quota. Introducing a quota at the free-trade level (discontinuously) lowers domestic output and thus lowers domestic welfare; thus, the optimal quota may be "infinite". The condition for a locally optimal quota (\( R \)) is given by:

\[ \frac{dW}{dR} = (\theta - C') \frac{dQ}{dR} + \left[ \theta - (\omega + I\omega') \right] = 0 \rightarrow \left[ \theta - (\omega + I\omega') \right] = -(\theta - C') \frac{dQ}{dR} > 0 \]  

(11)

since \( \theta > C' \) due to monopoly behavior and \( (dQ/dR) < 0 \) by assumption. Thus, the (locally) optimal quota is chosen so that domestic price exceeds the marginal cost of imports (a smaller quota is used to encourage domestic production). If the globally optimal quota is "infinite" (free trade), then either tariff will improve upon the free trade solution (as shown below). If the globally optimal quota is interior, then from
Proposition 1: A tariff which yields the same level of imports will result in higher domestic output. Thus, from (10), it follows that, given imports, the increase in domestic output raises welfare.

The comparison of specific and ad valorem tariffs is similar. The ad valorem tariff solves:

\[
\frac{dW}{d\tau} = (\theta - C') \cdot \frac{dQ}{d\tau} + \{\theta - (\omega + 1)\} \frac{dI}{d\tau} = 0 \rightarrow \{\theta - (\omega + 1)\} = - (\theta - C') \left\{ \frac{dQ}{d\tau} \right\}
\]

By assumption I, \((dQ/d\tau) > 0\) which implies \((dI/d\tau) < 0\). Hence, under domestic monopoly, the second-best tariff implies \(\theta - (\omega + 1) > 0 \rightarrow \tau^* > (1/\omega)/\omega\).

Finally, from Proposition I, the specific tariff that results in the same level of imports as the optimal ad valorem tariff yields higher domestic output and hence higher welfare. Thus, the optimal specific tariff must dominate the optimal ad valorem tariff. QED

Intuitively, since the monopoly position of the local firm yields too little output at the traditional optimal tariff, it is desirable to stimulate production with a higher tariff.

(ii) The monopolist's behavior and smuggling. We assume there are a large number of smugglers who behave competitively. The domestic price depends on domestic production \(Q\) and total imports \(I\), where \(I\) is the sum of aggregate legal imports \(L\) and aggregate illegal imports \(S = \sum S_i\). Generally, the profits \(\pi_i\) of smuggler \(i\), as shown below, depend on the domestic price, \(P^d = \theta(Q + L + S)\), the probability that smuggled imports will be seized at the border, \(1 - \lambda\), the cost to obtain the smuggled goods \(\alpha P^w\) (\(P^w = \omega(L + S)\) is the world price and \(\alpha\) a terms of trade parameter specific to illegal imports) and its other costs \(\chi(s_i, S)\). The latter are assumed to depend on the individual and aggregate volumes smuggled \(s_i\) and \(S\). \(\chi(s_i, S)\) is assumed to be increasing in \(S\) to introduce a congestion effect that makes the individual smuggler use more expensive routes to maintain the probability of not getting caught at \(\lambda\):

\[
\pi_i = (\lambda \theta - \alpha \omega) s_i - \chi(s_i, S); \quad \lambda \geq \lambda
\]

The parameter \(\alpha\) is expected to be greater or equal to one in most situations, which implies illegal importers can not get a better price on imports than legal importers. However, it is conceivable that people escaping commercial taxes may be able to avoid domestic taxes in the exporting countries so that \(\alpha < 1\) cannot be
ruled out completely. The free entry zero profit condition determines the individual smuggler's volume and the number of smugglers. For simplicity we assume that the smuggler's cost is linear with respect to its own volume of smuggling and that all smugglers are identical. A smuggler's profit can then be written as:

$$\pi_i = ([A - a_i] - \eta(S))s_i$$  \hspace{1cm} (14)

where $\eta(S)$ is the marginal and average private cost of smuggling. An interior solution $(S>0)$ requires that $\eta(S)/\lambda = (\theta - \delta\omega)$, where $\delta=\alpha/\lambda$. Smuggling will not occur if $\eta(S)/\lambda \geq (\theta - \delta\omega)$. Another corner solution could occur when legal imports are more expensive than illegal imports (legal imports cease). This rarely observed case is not without interest since Bhagwati and Hansen (1973) have shown that a smuggling-only equilibrium can dominate a no-smuggling equilibrium if the terms of trade gain is sufficiently large.

IV. SMUGGLING AND THE AD VALOREM TARIFF

For a given tariff rate, the monopolist's output decision influences the composition of imports (between legal and illegal). In general, for high levels of domestic output, the gap between world and domestic prices will be small, and smuggling will be nonexistent. However, as the domestic firm reduces its output level, the profitability of smuggling increases. It is possible, for some tariff rates, that at low domestic output levels all imports will be diverted to illegal channels. Since, as shown below, the residual demand curve facing the monopolist is unaffected by smuggling, as long as legal imports and smuggling coexist, the smuggling affects the monopolist's behavior only when legal imports cease.

(i) Deriving the residual demand curve under smuggling: The tariff-distorted residual demand curve pictured in Figure 1 is characterized by a kink. The introduction of smuggling is tantamount to adding another source of competition which adds another kink to the residual demand curve. To identify the conditions supporting the three different kind of equilibria $(S=0, L>0; S,L>0; S>0, L=0)$, it is necessary to locate the kinks on the residual demand curve facing the monopolist. To derive the residual demand curve, recall the arbitrage condition for importers:

$$\theta(I + Q) \leq (1 + \tau)\omega(I) \quad \text{for legal imports, with equality if any legal imports occur;} \hspace{1cm} (15)$$

$$\theta(I + Q) \leq \delta\omega(I) + (\eta(S)/\lambda); \quad S \in [0,I] \quad \text{with equality if } S>0. \hspace{1cm} (16)$$
If some legal imports enter the country, then (15) holds with equality. Define $I'(Q, \tau)$ such that:

\[ I'(Q, \tau) \text{ s.t. } \theta(Q + I'(Q, \tau)) = (1 + \tau)\omega(I'(Q, \tau)), \]

where:

\[
\frac{\theta'}{\theta} = \frac{\theta - \omega}{(1 + \tau)\omega - \theta} < 0; \quad \frac{\theta'}{\tau} = \frac{-\omega}{(1 + \tau)\omega - \theta} < 0
\]  

(17)

$I'(Q, \tau)$ represents the level of imports, as a function of domestic output, assuming some legal imports enter the country. The monopolist's residual demand curve, assuming legal imports occur, is $P'(Q, \tau)$:

\[ P'(Q, \tau) = \theta(Q, I'(Q, \tau)); \text{ where: } \frac{\partial P'}{\partial Q} = \theta'(1 + \{I'/\partial Q\}) = \theta'(1 + \kappa)^{-1}; \]

\[
\frac{\partial P'}{\partial \tau} = (\theta\kappa/(1 + \tau)(1 + \kappa)) > 0; \quad \kappa = (-\theta/(1 + \tau)\omega) > 0;
\]

If the disparity between world and domestic prices is small, no smuggling will occur. Thus, define $Q_0(\tau)$, $I_0(\tau)$ as the threshold level of output (and hence imports) such that:

\[ P'(Q_0, \tau) - \delta\omega(I'(Q_0, \tau)) - (\eta(0)/\lambda) = 0; \quad (1 + \tau - \delta)\omega(I_0) - (\eta(0)/\lambda) = 0; \]

where: $I'(Q_0(\tau), \tau) = I_0(\tau)$

(18)

It is readily shown that \( \{P'(Q, \tau) - \delta\omega(I'(Q, \tau))\} \) is a decreasing function of $Q$, and an increasing function of $\tau$; hence, it follows that for $Q > Q_0(\tau)$ ($I < I_0(\tau)$) no smuggling will occur, whereas for $Q < Q_0(\tau)$ ($I > I_0(\tau)$) smuggling will occur. Assuming some legal imports occur, the level of smuggling is given by:

\[ S = 0 \text{ if } Q \geq Q_0(\tau) \rightarrow I \leq I_0(\tau); \quad \text{where: } (dQ_0/d\tau) > 0 \text{ and } (dI_0/d\tau) < 0 \]

(19)

Further, define: $P_0(\tau) = P'(Q_0(\tau), \tau)$; thus, $P_0$ represents the domestic price such that no smuggling (just) occurs. Since: $P_0(\tau) = \delta\omega(I_0(\tau)) + (\eta(0)/\lambda)$, and $(dI_0/d\tau) < 0$, it follows that: $(dP_0/d\tau) < 0$, a result we use later. This result states that the increase in output necessary to deter smuggling following a rise in the tariff rate induces a fall in the domestic price. Smuggling is profitable only if: $P'(Q, \tau) > P_0(\tau)$.

As domestic output shrinks, imports increase - as does the differential between domestic and world prices; hence, smuggling also increases. If the tariff is high enough, then for sufficiently low output (high import) levels smuggled imports may entirely replace legal imports. Denote $I'(Q)$ as the level of imports and $P'(Q)$ as the corresponding domestic residual demand when all imports are smuggled:
\[
I^*(Q) \quad \text{s.t.} \quad \theta(Q + I^*) = \delta \omega(I^*) + \left( \eta(I^*) / \lambda \right), \quad P^*(Q) = \theta(Q + I^*);
\]
\[
\left( \frac{dI^*}{dQ} \right) = \left( \frac{-\mu}{(1 + \mu)} \right), \quad \left( \frac{dP^*}{dQ} \right) = \left( \frac{\theta}{(1 + \mu)} \right), \quad \mu = \left( -\theta / (\delta \omega' + (\eta / \lambda)) \right) \geq 0
\] (20)

Legal imports will cease if \( P^*(Q, \tau) \geq P^*(Q) \), i.e., if \( \left( 1 + \tau - \delta \right) \omega(I) - \left( \eta(I) / \lambda \right) \geq 0 \). Define:
\[
\phi(I, \tau) = \left( 1 + \tau - \delta \right) \omega(I) - \left( \eta(I) / \lambda \right) \geq 0
\] (21)

where the strict inequality implies legal imports are unprofitable \( (I^*(Q) > I^*(Q, \tau)) \) in this domain. Since \( I \) depends inversely on \( Q \), and since \( \phi(I_0(\tau), \tau) < 0 \), an all smuggling equilibrium can occur only if:
\[
\left[ \left( 1 + \tau - \delta \right) \omega' - \left( \eta' / \lambda \right) \right] > 0
\] (22)

Finally let \( Q_o(\tau), I_o(\tau), P_o(\tau) \) be the output, import and price schedules such that legal imports just vanish:
\[
\left( 1 + \tau - \delta \right) \omega(I_o(\tau)) - \left( \eta(I_o(\tau)) / \lambda \right) = 0; \quad P_o(\tau) = \theta(Q_o(\tau) + I_o(\tau)) = \delta \omega(I_o(\tau)) + \left( \eta(I_o(\tau)) / \lambda \right)
\] (23)

It is readily shown that \( \left( dQ_o(\tau) / d\tau \right) > 0 > \left( dI_o(\tau) / d\tau \right) \) if \( \left( 1 + \tau - \delta \right) \omega'(I) - \left( \eta'(I) / \lambda \right) > 0 \). Hence, the domestic price at which legal imports cease is a decreasing function of the tariff rate \( (dP_o(\tau) / d\tau < 0) \) since the higher tariffs make smuggling more attractive. We shall use this result shortly.

To summarize, the legal imports-only, coexistence, and smuggling-only equilibria occur when:

- \( Q \geq Q_o(\tau) \rightarrow P^*(Q, \tau) \leq P_o(\tau) \) and \( S = 0 \)
- \( Q \in (Q_o(\tau), Q_o(\tau)) \rightarrow P^*(Q, \tau) \in (P_o(\tau), P_o(\tau)) \) and \( S \in (0, I^*(Q, \tau)) \)
- \( Q \leq Q_o(\tau) \rightarrow P^*(Q, \tau) \geq P^*(\tau) \) and \( S = I^*(Q) \)

Actual imports are given by: \( I^*(Q, \tau) = \text{Max} \left[ I^*(Q, \tau), I^*(Q) \right] \), and the actual domestic price is given by:
\[
P^*(Q, \tau) = \text{Min} \left[ P^*(Q, \tau), P^*(Q) \right].
\]

Note that the residual demand curve is more elastic under the smuggling-only scenario than under a scenario allowing only for legal imports if: \( \left( 1 + \tau - \delta \right) \omega'(I) - \left( \eta'(I) / \lambda \right) > 0 \), which is precisely the condition that is required to allow a smuggling-only outcome. Figure 3 illustrates the residual demand curve facing the monopolist, assuming the tariff rate is such that all three cases can occur.

The flatter segment (A'B") of the residual demand curve corresponds to the smuggling-only case while coexistence of legal and illegal imports can only occur along the B"C' segment. The legal-only and no-import scenario segments are respectively delimited by C'D" and D"E. Note that the slope of B"D" is equal
to the slope of $D\tau - D\tau'$ in Figure 2 and that an increase in the tariff pushes $B''$ to the right, thus extending the elastic segment and confirming an important property of $Q(t)^{11}$.

(ii) Monopolist Output Choice. The monopolist's profit function is given by:

$$\pi(Q, \tau) = P^a(Q, \tau)Q - C(Q) \text{ where: } P^a = P^a(Q) \text{ for } Q \leq Q_1(\tau); \quad P^a = P_1(Q, \tau) \text{ for } Q \geq Q_1(\tau) \quad (24)$$

From (24), it is apparent that the residual demand curve is continuous, but not differentiable at $Q_1(\tau)$. This kink, represented by $B''$ in Figure 3, brings about the following marginal profit expressions:

$$(\partial \pi / \partial Q) = \theta (Q + I') + Q \cdot \theta' (1 + \mu)^{-1} - C'(Q) \text{ when: } Q < Q_1(\tau) \quad (25)$$

$$(\partial \pi / \partial Q) = \theta (Q + I'(Q, \tau)) + Q \cdot \theta' (1 + \kappa)^{-1} - C'(Q) \text{ when } Q > Q_1(\tau) \quad (26)$$

Because the derivative is discontinuous at $Q_1(\tau)$, the possibility of multiple local optima arises. However, for $\lim_{Q \to q_1(\tau)} (\partial \pi / \partial Q) > \lim_{Q \to q_1(\tau)} (\partial \pi / \partial Q)$, assuming $\pi(Q)$ is concave in each domain, there is a unique optimum. Define $Q_1'(\tau)$, $Q^*$ such that:

$$Q^\ast \text{ solves: } (\partial \pi / \partial Q) = \theta (Q + I') + Q \cdot \theta' (1 + \mu)^{-1} - C'(Q) = 0$$

$$Q_1'(\tau) \text{ solves: } (\partial \pi / \partial Q) = \theta (Q + I'(Q, \tau)) + Q \cdot \theta' (1 + \kappa)^{-1} - C'(Q) = 0 \quad (27)$$

$Q^*$ is the monopolist's optimal output if no legal imports can occur, whereas $Q_1'(\tau)$ is optimal output when smuggling is not feasible. Thus, the true optimal solution for the monopolist, denoted $Q^*(\tau)$, is given by:

$$Q^*(\tau) = Q^* \text{ if } \left(\theta (Q + I') + Q \cdot \theta' (1 + \mu)^{-1} - C'(Q)\right)_{Q_1(\tau)} < 0$$

$$Q^*(\tau) = Q_1(\tau) \text{ if } \left(\theta (Q + I') + Q \cdot \theta' (1 + \mu)^{-1} - C'(Q)\right)_{Q_1(\tau)} > 0$$

$$Q^*(\tau) = Q_1'(\tau) > Q_1(\tau) \text{ if } \left(\theta (Q + I') + Q \cdot \theta' (1 + \kappa)^{-1} - C'(Q)\right)_{Q_1(\tau)} > 0 \quad (28)$$

For small $\tau$, the divergence between world and domestic prices will be small, and the monopolist's optimal choice will result in no smuggling. As the tariff rate increases, the demand for the monopolist's output increases; while the higher tariff need not increase marginal revenue, we assume it does (a condition guaranteed with linear demands and marginal cost). Thus, using Assumption I:
PROPOSITION 3: Consider the monopoly solution with both the threat of legal and illegal imports.

Further, assume that, with legal imports, the monopoly solution is such that output and domestic price increase with the tariff rate. Then there exist tariff rates \( \tau_0 < \tau_1 < \tau_2 \) such that

\[
\begin{align*}
\text{for } \tau \leq \tau_0, & \quad Q'(\tau) = Q'(\tau), \quad S^* = 0, \quad I'(\tau) = I'(Q'(\tau), \tau); \quad \frac{dP^*}{d\tau} > 0, \quad \frac{dQ^*}{d\tau} > 0 > \frac{dI^*}{d\tau} \\
\text{for } \tau \in (\tau_0, \tau_1), & \quad Q'(\tau) = Q'(\tau), \quad I'(\tau) = I'(Q'(\tau), \tau); \quad S^*(\tau) \in (0, I'(\tau)) \quad \frac{dP^*}{d\tau} > 0, \quad \frac{dS^*}{d\tau} > 0, \quad \frac{dQ^*}{d\tau} > 0 > \frac{dI^*}{d\tau} > 0 \\
\text{for } \tau \geq \tau_2, & \quad Q'(\tau) = Q'(\tau), \quad S^*(\tau) = I'(\tau) = I'(Q'(\tau)); \quad \frac{dP^*}{d\tau} = \frac{dQ^*}{d\tau} = \frac{dS^*}{d\tau} = \frac{dI^*}{d\tau} = 0
\end{align*}
\]

PROOF: Define \( P_1(\tau) = \delta(Q'(\tau) + I'(Q'(\tau), \tau)) \) as the domestic price associated with the optimal monopoly output when legal imports occur. For \( \tau = 0 \), \( P_1(\tau) < P_0(\tau) \), where \( P_0(\tau) \) denotes the domestic price at which illegal imports will just occur. As proven earlier, \( (dP_1/d\tau) > 0 > (dP_0(\tau)/d\tau) \). Hence, there exists a \( \tau_0 \) such that \( P_1(\tau) < P_0(\tau) \) as \( \tau \geq \tau_0 \). For \( \tau \leq \tau_0 \), \( S = 0 \), whereas \( S > 0 \) for \( \tau > \tau_0 \). That proves the first part of the proposition.

As previously shown, as \( \tau \) increases, \( P_1(\tau) \) increases, whereas \( P_1(\tau) \) (the price at which legal imports cease) decreases. Thus, define \( \tau_1 \) such that \( P_1(\tau_1) = P_1(\tau_1) \). For \( P_1(\tau) \in (P_0(\tau), P_1(\tau)) \) (i.e., \( \tau \in (\tau_0, \tau_1) \)) legal and smuggled imports coexist, proving the second part of the proposition.

For \( \tau > \tau_1 \), \( P_1(\tau) > P_1(\tau) \) and legal imports cease. Define \( \tau_2 \) such that \( Q_1(\tau_2) = Q' \); recalling that \( Q_1(\tau) \) is an increasing function of \( \tau \), it must be that \( Q_1(\tau_2) < Q_1(\tau) \) for \( \tau < \tau_2 \). Thus, for \( \tau \in (\tau_1, \tau_2) \),

\[
\lim_{\theta \to Q'(\tau)} (\partial \pi / \partial Q) > 0 > \lim_{\theta \to Q'(\tau)} (\partial \pi / \partial Q)
\]

implying the optimal solution in this domain is \( Q'(\tau) = Q_1(\tau) \), i.e., at the kink where the smuggling-distorted segment of the residual demand merges with the tariff-distorted segment. Finally, for \( \tau > \tau_2 \), \( Q_1(\tau) > Q_1(\tau) \), and \( (\partial \pi / \partial Q)|_{Q'} = 0 \), implying \( Q'(\tau) = Q_1(\tau) \) for \( \tau \geq \tau_2 \). QED

Figure 4 shows the impact of tariffs on output, domestic price, total and smuggled imports and, in doing so, it illustrates several interesting phenomena. First, for \( \tau \in (\tau_0, \tau_1) \), while domestic output and total imports are not affected by smuggling, legal imports decline and smuggled imports increase with the tariff.
rate. If smuggled imports entail a higher social cost than legal imports, this has implications for the second-
best tariff. Secondly, note that for $\tau > \tau_1$, legal imports cease, and for $\tau \in (\tau_1, \tau_2)$ domestic price is actually
a decreasing function of the tariff rate due to the discontinuity in the marginal revenue curve, which results
from the fact that the foreign supply of legal imports is less elastic than that of smuggled imports. The
combination of decreases in smuggled imports and output increases in the $(\tau_1, \tau_2)$ domain suggests that the
higher corner solution might be the best tariff rate in that domain. Finally, note that for $\tau > \tau_2$, changes in
the tariff have no further effect. In this domain, where the tariff-distorted schedule of legal imports is
everywhere above the illegal imports schedule, the solution entails what Fishelson and Hillman (1979) calls
"water in the tariff" or redundant protection and it is identical to a ban on legal imports.

(iii) Smuggling and welfare under an ad valorem tariff. For simplicity, we assume $\lambda = \delta = 1$, and
that smugglers earn no profits. Under these conditions $\eta(S)S$ represents the excess social cost of smuggled,
over legal, imports. Within the context of a partial equilibrium approach, the welfare function can be
written as, after some simplification:

$$W = \int_{x=0}^{Q+1} \theta(x)dx - C(Q) - \omega(I) \cdot 1 - \eta(S) \cdot S$$

which has the usual interpretation, except that the last term which represents the excess social cost
associated with smuggling. Consider the optimal choice of an ad valorem tariff:

$$\frac{\partial W}{\partial \tau} = (\theta - C')(\partial Q/\partial \tau) + (\theta - \omega - \omega')(\partial I/\partial \tau) - (\eta + \eta')(\partial S/\partial \tau)$$

(30)

The first two terms of equation (30) are the ones from which the familiar optimal tariff argument is derived,
and the last term represents the change in social smuggling costs due to the change in the tariff rate.

Let $\tau^*$ denote the second-best tariff consistent with equation (30) when smuggling is not possible,
and $\bar{\tau}$ denote the second-best tariff in the presence of smuggling. Since $\theta > C'$ and $\theta = \omega$ at $\tau = 0$, it
follows that $\tau^* > 0$ provided $(\partial Q/\partial \tau) > 0 > (\partial I/\partial \tau)$. 
PROPOSITION 4: First, if \( \tau^* \leq \tau_0 \), then \( \tau = \tau^* \); i.e., if the no-smuggling second-best tariff will not result in smuggling when the smuggling technology is introduced, the threat of smuggling will not affect the optimal equilibrium. Second, if \( \tau^* \in (\tau_0, \tau_1) \) then: \( \tau_0 \leq \tau < \tau^* \); furthermore, the introduction of smuggling activities lowers welfare.

PROOF: The first claim is trivial since the behavioral rules for output and total imports are unaffected by smuggling, as long as legal and illegal imports coexist. The second claim follows directly because, in that domain, the behavioral rules for output and total imports are not affected by the smuggling, but the amount of smuggling is an increasing function of the tariff. Hence, \( \frac{\partial W}{\partial \tau} \bigg|_{\tau} = -\left(\eta + S\eta'\right)\left(\frac{\partial S}{\partial \tau}\right) < 0 \), implying the result. The reduction in welfare follows since the domestic output and import levels chosen with smuggling (and the second-best tariff) were feasible without smuggling and since smuggling imposes a social cost beyond legal imports. Thus, welfare must be reduced by the smuggling. QED

The above proposition goes beyond Bhagwati and Srinivasan's (1973, p.385) result under the assumption of a perfectly competitive domestic sector as they could not provide an unambiguous ranking of the second-best tariffs except under the assumption of constant-elasticity offers.

Due to the kinks in the residual demand curve, corner solutions are likely. By continuity, the solution at the kink is influenced by the solutions on either side. The following lemma looks at the size of the second-best tariff when smuggling is possible but does not occur.

LEMMA 1: The threat of smuggling may reduce the second-best optimal tariff, even though the resulting equilibrium results in no smuggling.

PROOF: Note that the welfare function is not differentiable at \( \tau_0 \); from (30) \( \lim_{r \to r_0^+} \left(\frac{\partial W}{\partial \tau}\right) > \lim_{r \to r_0^-} \left(\frac{\partial W}{\partial \tau}\right) \) since \( \eta(0) > 0 \) and \( \lim_{r \to r_0^-} \left(\frac{\partial S}{\partial \tau}\right) > 0 \). Thus for some parameter values, the second-best optimal tariff when smuggling is not feasible will exceed \( \tau_0 \) but will equal \( \tau_0 \) when smuggling is a threat. Hence, even though no smuggling occurs, the threat of smuggling is sufficient to induce the setting of a lower tariff. QED

The welfare analysis of ad valorem tariffs up to now has been limited to tariff rates low enough so that illegal imports did not completely replace legal imports (i.e., \( \tau < \tau_1 \)). However, we noted earlier that
output is increasing and smuggling decreasing in the \( (\tau_1, \tau_2) \) domain and that tariff rate increases beyond \( \tau_2 \) have no effect. The welfare implications of these results are captured in the following proposition.

**PROPOSITION 5**: *In the presence of smuggling, it cannot be socially optimal to choose a tariff rate in the \( (\tau_1, \tau_2) \) domain.*

**PROOF**: In this domain, domestic price is a decreasing function of the tariff rate; hence: \( S = I \), \( [(\partial Q/\partial \tau) + (\partial S/\partial \tau)] > 0 > (\partial S/\partial \tau) \). Furthermore, the arbitrage condition states that \( \theta = \omega + \eta \). Hence, \( (\partial W/\partial \tau) \big|_{\tau \in (\tau_1, \tau_2)} = (\theta - C')(\partial Q/\partial \tau) - S(\omega' + \eta')(\partial S/\partial \tau) > 0 \). \( \text{QED} \)

Intuitively, when all imports are smuggled, the effective price paid for imports is not the world price, but the domestic price. Since increasing the tariff results in higher output and lower domestic price (i.e., improved terms of trade), welfare must be an increasing function of the tariff rate when \( \tau \in (\tau_1, \tau_2) \).

**V. SMUGGLING AND THE QUOTA**

(i) **Smuggling and domestic production under a quota.** Under domestic monopoly, Professor Bhagwati (1969) has shown that the range of \{output, import\} options available under a quota are different from, and inferior to, those available with a tariff when smuggling is not feasible. Implementing a quota at the free trade level of imports leads to higher domestic prices and lower welfare (the gap between marginal cost and demand price widens). If the quota is equal to the import level under a tariff, domestic output will be lower with the quota, implying lower utility (because of the discrepancy between price and marginal cost).

The question is how smuggling changes that conclusion. We have shown that, under a tariff, smuggling lowers welfare\(^{14}\). This result occurs because the presence, or threat, of smuggling does not change the monopolist's output unless legal imports are eliminated. We demonstrate in this section that these results are reversed under a quota, and that the threat of smuggling can be beneficial. More concretely, we show that the threat of smuggling raises the contestability of the domestic market and can induce an increase in domestic output even though smuggling does not actually occur.

Let \( R \) denote the quota limit, and assume the same smuggling technology as in the previous section. Define the schedule of outputs \( Q_0(R) \), and of prices, \( P_0(R) \) as follows:
For each \( R \), \( Q_0(R) \) is the level of domestic production such that it is just (not) profitable to smuggle any imports into the economy, and \( P_0(R) \) is the corresponding domestic price. Given \( R \), there will be no smuggling if \( Q \geq Q_0(R) \), whereas smuggling will occur if \( Q < Q_0(R) \). Thus, there will be a kink in the monopolist’s residual demand at \( Q_0(R) \). At lower output levels \( (Q < Q_0(R)) \) demand will be more elastic because of the competition from smugglers, while at higher output levels demand is less elastic because of the usual insulating effect of the quota. The domestic price when output exceeds \( Q_0(R) \) is:

\[
\text{when } Q > Q_0(R), \quad S = 0: \quad P^d_n(Q, R) = \theta(Q + R); \quad \text{and } \frac{\partial P^d_n}{\partial Q} = \theta(Q + R) \tag{33}
\]

On the other hand, when output falls below \( Q_0(R) \) smuggling becomes profitable. The level of smuggling \( (S(Q, R)) \) and the corresponding domestic price are determined by:

\[
\text{when } Q < Q_0(R), \quad S > 0: \quad P^d_n(Q, R) = \theta(Q + R + S(Q, R)); \quad \frac{\partial P^d_n}{\partial Q} = \theta(1 + S_Q) = \theta(1 + \mu)^{-1} \tag{35}
\]

where, as defined earlier in the paper, \( \mu = \left(-\theta/\left[\delta\omega^1(\eta/\lambda)\right]\right) \).

Note that, unlike the case of the ad valorem tariff, smuggled imports do not squeeze out legal imports, and hence the demand curve appears more elastic to the monopolist\(^5\). The (formula for the) slope of the residual demand curve under smuggling is the same as for the residual demand curve under an ad valorem tariff when all imports are smuggled since the monopolist responds to the elasticity of the smuggling schedule only when legal imports do not change. For the tariff case, this requires that there be no legal imports, whereas this need not be the case under a quota.

Equations (33) and (35) reveal that the residual (inverse) demand curve facing the monopolist is continuous, but not differentiable at \( Q_0(R) \). Following the same methodology as for the ad valorem tariff, the profits on either side of the kink can be expressed as:

\[
\pi(Q, R) = P^d_n(Q + R + S(Q, R)Q - C(Q); \quad (\partial\pi/\partial Q) = P^d_n + \theta\nu(1 + \mu)^{-1} - C'(Q) \quad \text{when } Q < Q_0(R) \tag{34}
\]

\[
\pi(Q, R) = P^d_n(Q + R)Q - C(Q); \quad (\partial\pi/\partial Q) = P^d_n + \theta\nu - C'(Q) \quad \text{when } Q > Q_0(R) \tag{35}
\]
It follows that: \[ \lim_{Q \to Q_0(R)} \frac{\partial Q}{\partial Q} = (\theta + Q \theta (1 + \mu)^{-1} - C(Q)) \to \theta + Q \theta - C(Q) = \lim_{Q \to Q_0(R)} \frac{\partial Q}{\partial Q} \] since \((1 + \mu)^{-1} \epsilon (0,1)\). Assuming the profit function is concave in each domain, this implies the global concavity of the profit function. Define \(Q_n(R)\) as the profit-maximizing output level when smuggling is not feasible and \(Q_s(R)\) as the monopoly output level when only smuggling can occur. Thus, \(Q_n(R), Q_s(R)\) solve:

\[ Q_n(R) \text{ solves: } (\partial Q/\partial Q) = P_n^d(Q + R) + Q \theta - C(Q) = 0 \]
\[ Q_s(R) \text{ solves: } (\partial Q/\partial Q) = P_s^d(Q + R + S) + Q \theta (1 + \mu)^{-1} - C(Q) = 0 \]

Clearly \(Q_s(R) > Q_n(R)\). Finally, define \(Q^*(R)\) as the optimal solution for the monopolist. Then:

**PROPOSITION 6:** The monopolist's optimal decision rule is given by the following output schedules:

\[ Q^*(R) = Q_n(R) \text{ if } Q_n(R) > Q_s(R) \]
\[ Q^*(R) = Q_s(R) \text{ if } Q_s(R) > Q_n(R) > Q_s(R) \]
\[ Q^*(R) = Q_s(R) \text{ if } Q_s(R) < Q_n(R) \]

**PROOF:** If \(Q_n(R) > Q_s(R)\), \(\left[ (\theta + Q \theta - C(Q)) \right]_{Q(R)} > 0 \to \left[ (\theta + Q \theta (1 + \mu)^{-1} - C(Q)) \right]_{Q(R)} > 0 \), since \(\mu > 0\) and \(S(Q_0(R), R) = 0\). Hence, the optimal solution \(Q^*(R) > Q_0(R) \to Q^*(R) = Q_n(R)\).

Similarly, if \(Q_s(R) < Q_0(R)\), \(\left[ (\theta + Q \theta (1 + \mu)^{-1} - C(Q)) \right]_{Q_0(R)} < 0 \to \left[ (\theta + Q \theta - C(Q)) \right]_{Q_0(R)} < 0 \), and hence the optimal solution \(Q^*(R) < Q_0(R) \to Q^*(R) = Q_s(R)\).

Finally, if \(Q_s(R) > Q_0(R) > Q_n(R)\), \(\left[ (\theta + Q \theta (1 + \mu)^{-1} - C(Q)) \right]_{Q_0(R)} > 0 \to \left[ (\theta + Q \theta - C(Q)) \right]_{Q_0(R)} > 0 \), implying the maximum occurs at \(Q^*(R) = Q_0(R)\). QED

The discontinuity in the marginal revenue schedule raises the likelihood the firm will choose an output level that "just" deters entry by smugglers. Without further assumptions, it is possible that some types of solutions (e.g., an equilibrium with smuggling) would not emerge. To prevent this, we assume:

**Assumption II:** \(Q_s(R) < Q_0(R)\) when \(R = 0\).

**Assumption III:** \(Q_n(R) > Q_0(R)\) at \(R = I^*_f\) (\(I^*_f\) denotes the free trade level of imports).
Assumption II implies that smuggling costs are low enough to make smuggling profitable under the autarkic monopoly solution ($R=0$), whereas Assumption III implies that smuggling is not profitable when the quota is set at the free trade level of imports (i.e., the price increase from imposing a quota at the free trade level of imports is less than smuggling costs). If this latter assumption does not hold, then smuggling will occur for all quota levels less than, or equal to, the free trade level of imports.\textsuperscript{17} In what follows, we rule out this possibility to consider equilibria in which the threat of smuggling affects the monopolist's behavior. The following two propositions show that there are unique values of $R$ that divide the solution space.

**PROPOSITION 7:** The assumptions of linear demand, world export supply, marginal cost and smuggling functions suffice to imply the $Q_n(R)$ and the $Q_s(R)$ curves intersect the $Q_0(R)$ curve (at most) once, and will cut the $Q_0(R)$ curve from below\textsuperscript{18}.

**PROOF:** Suppose there exists $R_0$ such that $Q_n(R_0) = Q_0(R_0)$. From (31), $(\partial Q_0/\partial R) < -1$ and domestic price falls as $R$ decreases. Thus, as we move along the $Q_0(R)$ locus:

$$\left(d\left(P_n + Q\theta - C'(Q)\right)/dR\right) = (\theta + Q\theta')(1 + (\partial Q_0(R)/\partial R)) + (\theta - C''(\partial Q_0(R)/\partial R)) > 0$$

under the usual assumption that $(\theta + Q\theta') < 0$. Given concavity, this in turns implies: $Q_n(R) > Q_0(R)$ as $R > R_0$.

Next, define $R_1$ be such that $Q_n(R_1) = Q_0(R_1)$. Thus, as we move along the $Q_0(R)$ locus:

$$\frac{d\left(P_n + Q(1+\mu)^{-1} - C'(Q)\right)}{dR} = \left(\theta + Q(1+\mu)^{-1}\theta'\right)\left(1 + \left(\partial Q_0(R)/\partial R\right)\right) + \theta(1+\mu)^{-1} - C''\left(\partial Q_0(R)/\partial R\right) - Q\theta^2(1+\mu)^{-2}\left(\partial\mu/\partial R\right)$$

Under the linearity assumption, $(\partial\mu/\partial R)$ is zero while the remaining terms are positive. Assuming concavity of the profit function, this implies $Q_n(R) > Q_0(R)$ as $R > R_1$. Finally, note that since

$$\left[\theta + Q\theta - C\right]_{Q=Q_0(R)} < 0,$$

this implies $Q_n(R_1) < Q_0(R_1)$, and hence $R_1 < R_0$. \textbf{QED}

**PROPOSITION 8:** Under assumptions (II) and (III) there exists $R_o > R_1 > 0$ such that:

- for $R \geq R_0$, $Q^*(R) = Q_n(R)$;
- for $R \in (R_0, R_1)$, $Q^*(R) = Q_0(R)$;
- for $R < R_1$, $Q^*(R) = Q_s(R)$;
PROOF: It follows from propositions 6 and 7. For $R \geq R_0$, the $Q_0(R)$ locus lies above the $Q_0(R)$ locus, implying the monopoly solution is not affected by the potential smugglers in this domain. For $R < R_1$, 

$$0 > \lim_{Q \to Q_0(R)} (\partial \pi / \partial Q) > \lim_{Q \to Q_0(R)} (\partial \pi / \partial Q)$$

implying the optimal value of $Q$ is less than $Q_0(R)$. Thus, $Q^*(R) = Q_0(R)$ in that domain. Finally, for $R \in (R_1, R_0)$, 

$$\lim_{Q \to Q_0(R)} (\partial \pi / \partial Q) > 0 > \lim_{Q \to Q_0(R)} (\partial \pi / \partial Q),$$

implying $Q^*(R) = Q_0(R)$.

QED

Figure 5 illustrates how the monopoly solution changes with the quota level. The pertinent segments of all three output schedules are dashed. For very restrictive quota levels ($R < R_1$), $Q_0(R)$ is the relevant schedule and decreases in $R$ give rise to more modest increases in domestic output than in the intermediate quota range ($R_0, R_4$) within which the $Q_0(R)$ schedule is relevant. For quota sizes below $R_1$, the monopolist has to contend with the pressure exerted by the smugglers while at quota sizes above $R_1$, the monopolist is himself putting pressure on the smugglers to keep them away from the market. Figure 5 shows that smuggling need not occur in order to have an effect on the monopolist's behavior. This is captured by the following proposition.

PROPOSITION 9: For $R \in (R_1, R_0)$, the threat of smuggling increases domestic output, even though no smuggling occurs. For $R < R_1$, smuggling will occur, and domestic output will be higher than if no smuggling were feasible.

PROOF: It follows directly from the previous propositions.

This proposition establishes the contrasting impact of smuggling under different policy instruments. Under a tariff, smuggling has no impact on domestic output as long as legal and illegal imports coexist. Only when all imports are smuggled can smuggling activities lead to a higher domestic output. Thus, smuggling affects domestic production only at very high tariff rates. By contrast, under a quota in the range $(0, R_1)$, legal and illegal imports always coexist and output is always increased. However, the greatest impact is observed when no smuggling actually occurs, that is at higher/less restrictive quota levels. The threat of any smuggling acts as a deterrent on the monopolist's behavior.
(ii) Smuggling and welfare under a quota. The welfare criterion is the same as that used under a tariff. Naturally, there are no tariff revenues per se, but there are revenues derived from selling the quotas licenses. The welfare function as defined in equation (29) can be used to determine the effect of smuggling on domestic welfare in the presence of a quota.

**PROPOSITION 10:** For any $R \in (R_1, R_0)$, the threat of smuggling raises domestic welfare. Furthermore, if the second-best quota, in the absence of a smuggling threat, is $R^* \in (R_1, R_0)$, then the threat of smuggling must raise domestic welfare.

**PROOF:** For any quota in this region, no smuggling occurs, so the only impact, as compared to the case in which smuggling is not feasible, is to increase domestic output. Since $(\partial W/\partial Q) = (\theta - C') > 0$, it follows that the higher output due to the threat of smuggling is beneficial. The proof of the second part of the proposition follows directly from the first part, since if the government chooses the same quota, no smuggling will occur, but domestic output increases. QED

The difference between the welfare implications of smuggling under the quota and under the tariff is worth discussing. Since decreasing $R$ in this "no-smuggling" domain reduces imports, raises domestic output and reduces the domestic price, it is very likely that the optimal quota in this region is $R_1$. The second-best quota (if interior) solves:

$$(\partial W/\partial R) = (\theta - C')(\partial Q/\partial R) + (\theta - \omega - 1\omega') - (1\omega' + S\eta')(\partial S/\partial R) = 0 \quad (37)$$

Note that, when smuggling occurs ($R < R_1$) it may be worthwhile to expand the quota to reduce smuggling even if the marginal cost of legal imports exceeds the domestic price. In the "no-smuggling" domain, $(\partial Q_0(R)/\partial R) < -1$ and the welfare effects of an increase in the quota are given by:

$$(\partial W/\partial R)|_{R \in (R_1, R_0)} = (\theta - C')(\partial Q/\partial R) + (\theta - \omega - 1\omega') < (\theta - C'(-1) + (\theta - \omega - 1\omega') = (C' - \omega - 1\omega')$$

Thus, raising the quota will lower welfare in this domain if the marginal cost of imports exceeds the domestic firm's marginal cost. Though an interior solution cannot be ruled out a priori, it seems logical to consider the more restrictive corner solution ($R = R_1$) as highly probable.
VI. SMUGGLING AND THE AD VALOREM TARIFF-QUOTA COMPARISON

When different policy instruments are compared, a variable is often chosen to remain constant to “anchor” the comparison. One variable often used for this purpose is the volume of imports. The presence of smuggling complicates matters because legal and total imports may differ, and hence it is not immediately apparent which should be used to compare the two policies. In this section we show that, because of the threat of smuggling, there exists a unique critical level of legal imports such that, if legal imports are the same under both instruments, the output under the quota will be higher (lower) than under the tariff as long as the level of legal imports is lower (higher) than the critical level. This result can then be used to (partially) rank the quota and the ad valorem tariff when smuggling is feasible.

Since legal imports will be held constant in the subsequent tariff/quota comparison, it is useful to express output under the tariff as a function of the (resulting) legal import level to establish a matching relation with the appropriate quota level. Before doing so, we state the following:

PROPOSITION 11: With smuggling, a quota that bans legal imports weakly dominates a tariff that eliminates legal trade.

PROOF: As shown in Figure 4, any tariff $\tau \geq \tau_1$ leads to no legal imports. We have also shown that welfare under the tariff is strictly increasing in $\tau$ in the interval $(\tau_1, \tau_2)$, and hence the local maximum occurs at $\tau_2$. However, this (or any higher) tariff rate results in the same equilibrium that would pertain with an absolute ban on (legal) imports. Thus, welfare under a quota of $R = 0$ is the same as that under a tariff of $\tau \geq \tau_2$, and therefore dominates any tariff in the interval $(\tau_1, \tau_2)$. QED

This is very different than the usual result according to which a prohibitive tariff dominates a quota because of the remaining threat of imports under the tariff. In this case, if smuggling can occur, these imports under a tariff are not beneficial since the price paid for the imports (including smuggling costs) exceeds domestic production costs.

In the remaining part of this section, we rely heavily on the results upon which Figures 4 and 5 are based, that is equations (25), (26), (36) and the propositions describing the behavior of the monopolist.
under the *ad valorem* tariff and the quota. We focus attention on cases where legal imports occur and for simplicity we assume that all functions are linear and that $\delta = \lambda = 1$. We define:

$$\hat{\tau} = \left( \frac{\eta'}{\omega'} \right); \quad \text{thus: } \kappa \geq \mu \text{ as } \tau \leq \hat{\tau}$$

where $\hat{\tau}$ is the tariff rate at which the slopes of the legal import and smuggled import schedules are equal. To facilitate the comparison of tariffs and quotas, we use the notation $\{Q^*(\tau), L^*(\tau), S^*(\tau)\}$ to denote the solution for output, legal imports and smuggled imports under a tariff. Due to the previous proposition, we can restrict attention to the domain $\tau \in (0, \tau_1)$. In this domain, legal imports are a monotonically declining function of the tariff rate, and are zero for $\tau \geq \tau_1$. Thus, for $\tau \in (0, \tau_1)$, we can invert the relationship between legal imports and $\tau$:

$$L^* = \psi(\tau) \rightarrow \tau = \psi^{-1}(L^*) = \beta(L^*); \quad L^* \in [0, I_{\beta}] \rightarrow \tau \in [0, \tau_{1}]; \quad \left( d\tau / dL^* \right) = \beta' < 0$$

where $L^*$ denotes legal imports under the tariff and $I_{\beta}$ stands for the free trade level of imports.

Using legal imports under the tariff, let $Q^*(L^*) = Q^*(\tau L^*)$, $S^*(L^*) = S^*(\tau L^*)$ denote output, and smuggling, under the tariff as a function of legal imports. Recalling the definition of $\tau_0$ as the largest tariff level in the no-smuggling domain, define $L^*_0$ such that $L^*_0 = \psi(\tau_0)$; hence, $S^*(L^*_0) = 0$ for $L^*_0 \geq L^*_0$, $S^*(L^*_0) > 0$ for $L^*_0 < L^*_0$.

Similarly, let $\{Q^R(R), S^R(R)\}$ denote the domestic output and smuggling levels under a quota system, with $R$ denoting the quota on legal imports. Recall that $R_0$ represents the largest quota level such that the smuggling constraint just affects the monopolist’s behavior and that $R_1$ is the smallest quota that deters smuggling. In order to prove the main propositions regarding the ranking of the *ad valorem* tariff and the quota, two minor lemmas need to be proven. The first lemma establishes the magnitude of $L^*_0$ relative to $R_0$. Because $L^*_0$ cannot be unambiguously ranked relative to $R_1$, the second lemma compares the level of domestic output under both instruments for the two possible cases.

**Lemma 2**: $L^*_0 < R_0$

**Proof**: Under a quota of $R_0$, the smuggling (just) does not affect the monopolist behavior as shown by Figure 5. As argued by Bhagwati (1969), output under a tariff that yields the same (legal) imports will be
higher when smuggling can be ignored: \( Q'(R_0) > Q^t(R_0) \); but this implies:

\[
\left( \Theta(Q'(R_0) + R_0) - \omega(R_0) - \eta(0) \right) < 0;
\]

hence, the associated tariff \( (\tau^\theta = \beta(R_0)) \) must be lower than \( \tau_0 \), implying \( L^* < R_0 \).

QED

**LEMMA 3:** Let \( R < \text{Min}\left[ R_1, L^*_0 \right] \). Then, in this domain either the output under the quota is always higher, or the output levels cross only once, i.e.,

\[
Q^k(R) > Q^t(R) \quad \forall R \leq \text{Min}\left[ L^*_0, R_1 \right] \quad \text{or:}
\]

\[
\exists R' \in (0, \text{Min}\left[ L^*_0, R_1 \right]) \quad \text{s.t.} \quad Q^k(R) > Q^t(R) \quad \text{as} \quad R \leq R'.
\]

**PROOF:** Note that: (i) the output level under a prohibitive quota is the same as that under a tariff when \( \tau = \tau_2 \); (ii) for \( \tau \in (\tau_1, \tau_2) \), legal imports under the tariff are zero, but output is increasing in \( \tau \); (iii) output under the quota is continuous in the quota limit. Hence, for small \( \varepsilon \),

\[
Q^k(\varepsilon) > Q^t(\tau_1 - \gamma) \quad \text{where} \quad L^*(\tau_1 - \gamma) = \varepsilon.
\]

Thus, \( Q^k(R) > Q^t(R) \) for \( R \) near zero.

Next, assume there exists some \( R' \in (0, \text{Min}\left[ L^*_0, R_1 \right]) \) s.t. \( Q^k(R') = Q^t(R') \). Smuggling occurs in this domain and domestic output, imports, and price move according to the schedules to the right of \( \tau_0 \) in Figure 4 (under the tariff) and to the left of \( R_0 \) in Figure 5. By smuggling arbitrage, at the same levels of output and legal imports, smuggled imports must be the same. Since price, output and total imports are the same, equations (25), (26) and (36) tell us that: \( \kappa = \mu \rightarrow \tau = \hat{\tau} \). Further, for \( R < R' \), \( \tau > \hat{\tau} \rightarrow \kappa < \mu \).

Thus, a comparison of (26) and (36) shows that the output level under the tariff will be lower (and smuggling will be higher). Conversely, for \( R > R' \rightarrow \tau < \hat{\tau} \), and hence output will be higher under the tariff.

QED

The information contained in lemmas 2 and 3 provide the foundation for the comparison of output under the two regimes as expressed in the following proposition.

**PROPOSITION 12**\(^{10} \): If \( L^*_0 < R_1 \), \( \exists R' < L^*_0 \) s.t. \( Q^k(R) \geq Q^t(R) \) as \( R \leq R' \). If \( L^*_0 > R_1 \), then \( Q^k(R) > Q^t(R) \) as \( R \leq L^*_0 \).
PROOF: For both cases, first note that: \( Q^R(R) < Q^*(R) \) \( \forall R \geq R_0 \) since smuggling is irrelevant in this domain. Second, recall that \( (\partial Q^R / \partial R) < 1 \) for \( R \in (R_1, R_0) \), whereas for all levels of legal imports \( (\partial Q^*/\partial R) \in (-1,0) \). Thus, if \( Q^*(R_i) \geq Q^R(R_i) \rightarrow Q^*(R) > Q^R(R) \) \( \forall R > R_i \)

Next, assume \( L_0^R < R_1 \). At \( R = L_0^R \), \( S^R = 0 \), whereas \( S^R > 0 \) (by definition of \( R_i \)). For smuggling to be profitable under the quota, it must be that: \( Q^R(R_i) < Q^*(R_i) \). The first part of the proposition follows after applying the second part of the previous lemma and the results in the previous paragraph.

For \( L_0^R > R_1 \), consider that, by definition, at \( L_0^R \), \( \theta(R_i + L_0^R) = \{\omega(R) + \eta(0)\} \); similarly, for \( R \in (R_i, R_0) \), \( \theta(Q^R(R) + R) = \{\omega(R) + \eta(0)\} \). Thus for \( R = L_0^R \) (\( L_0^R \in [R_1, R_0] \)) we must have \( Q^R(L_0^R) = Q^*(L_0^R) \). Further, since \( Q^*(R_i) > Q^R(R_i) \) and \( (\partial Q^R / \partial R) < (\partial Q^*/\partial R) < 0 \) for \( R \in (R_1, R_0) \), this implies output levels cross only once in this domain. Applying the previous lemma implies the output levels will not cross in \( R < R_1 \) and hence: \( Q^R(R) > Q^*(R) \) as \( R \leq L_0^R \). \( \text{QED} \)

COROLLARY: If \( L_0^R = R_1 \rightarrow Q^R(R) < Q^*(R) \) as \( R \geq R_1 \).

PROOF: It follows immediately from the previous proposition.

Figures 6 and 7 illustrate the results of the previous proposition, and hence show the comparison of output levels (and implicitly, smuggling levels) under the two cases. Finally, the above proposition can be used to establish the welfare comparison. For the range of tariffs and quotas of interest \( (\theta - \omega - \eta)S = 0 \); hence, differentiating the welfare function and using that result in the following proposition yields

\[
\frac{dW}{dS} = (\theta - C)dQ + (\theta - \omega - (L + S)\omega')d(L + S) - (\eta + S\eta')dS = (\theta - C)dQ + (\theta - \omega - (L + S)\omega')dL - ((L + S)\omega' + S\eta')dS
\]

PROPOSITION 13: If legal import levels are the same under the two policy instruments, the instrument which induces the larger level of domestic output will also induce the higher level of welfare.

PROOF: The proof is immediate since \( \theta > C' \) and the policy with higher output level must, by "smuggling arbitrage", result in no higher smuggling levels. However \( S \) can be zero for both cases. \( \text{QED} \)
The above proposition and Figures 6 and 7 lead to more concrete welfare results which are presented in the next four propositions.

**PROPOSITION 14**: Suppose \( L_0^* < R_1 \), so that output under the tariff is larger (smaller) than output under the quota if \( \tau \) is less (greater) than \( \hat{\tau} \). Let \( \tau^* \) denote the second-best tariff, \( R^* \) the second-best quota, and \( \hat{L} = L^*(\hat{\tau}) \), the legal import level at which the two policies yield the same solution. Then:

(i) maximized welfare is under the quota if \( \tau^* > \hat{\tau} \) \( \left( L^*(\tau^*) < \hat{L} \right) \); (ii) maximized welfare is higher under the tariff if \( R^* > \hat{L} \). Finally, if \( \tau^* < \hat{\tau} \) and \( R^* < \hat{\tau} \), the two policies cannot generally be ranked.

**PROOF**: The proof follows from the previous proposition. (i) If \( \tau^* > \hat{\tau} \), then by choosing the quota level such that \( R = L^*(\tau^*) \), output will be higher under the quota and hence welfare will be higher. This implies that the optimal quota must dominate the tariff. (ii) Similarly, if \( R^* > \hat{L} \), then by choosing \( \tau = \beta(R^*) < \hat{\tau} \), output will be higher (smuggling lower) under the tariff, implying higher welfare. Thus, the optimal tariff must dominate the quota. (iii) Finally, if \( \tau^* < \hat{\tau} \) and \( R^* < \hat{\tau} \), no conclusion can be reached since neither policy strictly dominates the other. Thus, while welfare under the quota may be less than welfare under the tariff at \( R = L^*(\tau^*) \), decreases in \( R \) may raise welfare under the quota. QED

A similar proposition holds if \( L_0^* > R_1 \).

**PROPOSITION 15**: Suppose \( L_0^* > R_1 \), so \( Q^R(R) \geq Q^\tau(R) \) as \( R \leq L_0^* \). Then, (i) maximized welfare will be higher under the quota if \( \tau^* > \tau_0 \), implying \( L^*(\tau^*) < L_0^* \). However, (ii) maximized welfare will be higher under the tariff if \( R^* > L_0^* \). Finally, if \( \tau^* < \tau_0 \) and \( R^* < L_0^* \), the two policy instruments cannot automatically be ranked.

**PROOF**: Identical to the proof of proposition 14.

Finally, consider whether it is possible to compare the level of the second-best instruments. The impact of a change in legal imports on welfare (via either a quota or a tariff) can be measured by:

\[
\frac{dW}{dL} = (\theta - C)\frac{dQ}{dL} + (\theta - \omega - (L + S)\omega') - \left( (L + S)\omega' + S\eta' \right)\frac{dS}{dL}
\]

(39)

In general, it is not possible to compare \( \frac{dQ}{dL} \) and \( \frac{dS}{dL} \) under a quota to their value under a tariff. However, if the tariff solution leads to legal imports in the region \( (R_t, R_0) \), it is clear that, in this
domain \( \frac{dQ^R}{dL} < -1 < \frac{dQ^*}{dL} < 0 \). Hence, letting \( \tau^*, L^*(\tau^*) \) denote the optimal solution under a tariff, and evaluating \( \frac{dW}{dL} \) for a quota at that point yields:

**PROPOSITION 16:** If the second-best tariff leads to legal imports in the domain \( (R_0, R_1) \), then the second best quota will be more restrictive than the tariff; i.e., \( R^* < L^*(\tau^*) \).

**PROOF:** The result follows from evaluating equation (30), the optimal policy rule for the tariff, and using the fact \( \frac{dQ^R}{dL} < -1 < \frac{dQ^*}{dL} < 0 \). QED

If the optimal tariff leads to a solution in which smuggling occurs, it is less easy to compare the levels of the optimal policy instruments. However, it can be shown that:

\[
\left( \frac{dQ^R}{dL} \right) < \left( \frac{dQ^*}{dL} \right) < 0, \quad \text{and} \quad 0 > \left( \frac{dS^R}{dL} \right) > \left( \frac{dS^*}{dL} \right) \quad \text{for: } \tau \geq \hat{\tau}. \tag{40}
\]

Thus:

**PROPOSITION 17:** If the second-best tariff \( \tau^* \geq \hat{\tau} \), then the second-best quota will yield lower legal imports than the second-best tariff.

**PROOF:** The proof follows from equation (39), and noting that welfare is increasing in output, decreasing in smuggling and that \( \left( \frac{dQ^R}{dL} \right) - \left( \frac{dQ^*}{dL} \right) < 0, \quad \left( \frac{dS^R}{dL} \right) - \left( \frac{dS^*}{dL} \right) > 0 \). QED

**THE DOMINANCE OF THE SPECIFIC TARIFF WITH SMUGGLING**

The contrast between the specific tariff and the two other instruments can best be described by going back to Figures 1 to 3. First note that the schedule \( (1+\tau)\omega \) is steeper than \( \omega + \tau \), a fact that explains the superiority of the specific tariff over the other instruments at low enough tariff rates for smuggling to have no effect (i.e., \( t \leq \eta(0) \)). For \( t > \eta(0) \) and \( \eta' > 0 \), the residual demand curve facing the monopolist is such that the first imports entering will be smuggled. As the domestic price increases, there will be a level at which the marginal cost of legal and illegal imports is equal. Let \( P_L \) and \( Q_L \) be the price and domestic output at which this happens. For \( Q > Q_L \), the arbitrage is driven by the cost of illegal imports such that the monopolist's behavioral rule is: \( J^* = \theta + Q \theta(1+\mu)^{-1} - C' = 0 \) while for \( Q < Q_L \), the arbitrage is driven by the specific tariff which implies that \( J^* = \theta + Q \theta(\omega/(\omega-\theta)) - C' = 0 \). At \( Q_L \), \( J^* > 0 > J^* \) and it can be shown that this kink moves to the left as the tariff rate increases.
(i.e., $(\frac{\partial Q^*_e}{\partial \lambda}) > 0$). Thus there will be a range of tariffs supporting legal imports-only equilibria \((t \leq t_0 = \eta(0))\) and a set of higher tariffs allowing the coexistence of legal and illegal imports \((t_0 < t < t_1)\).

Under both sets of tariffs, the monopolist's behavior is consistent with \(J^-\). There is also a set of yet higher tariffs prohibiting legal imports \((t_1 < t < t_2)\) for which the monopolist's output is \(O_L\). Tariffs above \(t_2\) also prohibit legal imports but provide redundant protection and generate the same equilibrium as under \(ad \ text{valorem}\) tariff rates \(t \geq t_2\) or under a quota \(R=0\).

**PROPOSITION 18:** For \(t < t_2\), the specific tariff will result in higher domestic production and hence higher welfare than the \(ad \ text{valorem}\) tariff and quota generating the same level of legal imports.

**PROOF:** From proposition 1 we know that for legal imports-only equilibria, the specific tariff dominates the other instruments. To show why this is also true for \(t_0 < t < t_2\), note that in Figure 3 and from our analysis for the \(ad \ text{valorem}\) tariff and the quota, the behavior captured by \(J^-\) is observed along the most elastic segment of the residual demand curve, that is at tariff rates/prices high enough to eliminate legal imports or for a quota restrictive enough to make smuggling profitable. With a specific tariff sufficiently high to have a segment of the residual demand along which legal and illegal imports would coexist, the smuggling rule \((J^-)\) is followed at prices low enough to prevent legal imports from entering. At prices high enough for legal and illegal imports to coexist, the residual demand has yet a flatter slope which triggers the \(J^-\) rule. Thus the marginal revenue curve of the monopolist under the specific tariff when legal and illegal imports coexist must lie above its counterparts under the quota and the \(ad \ text{valorem}\) tariff. It follows that domestic output under the specific tariff will always be at least as large as that under an \(ad \ text{valorem}\) tariff or a quota when illegal and legal imports coexist. Finally, when legal imports are zero, we know that for \(t_1 < t < t_2\), \((\frac{\partial Q^*_e}{\partial \lambda}) < 0\). This and proposition 13 imply that output and welfare are higher at the lowest possible prohibitive tariff level \(t_1\) than at any other level above that. Therefore smuggling does not alter the strong dominance of the specific tariff over the other instruments. QED
VII. CONCLUSION

We have shown that Professor Bhagwati’s classic result about the non-equivalence between tariffs and quotas when a monopolist operates in the domestic market is not robust when smuggling takes place. Hence, it is possible for a quota to dominate an import-equivalent *ad valorem* tariff, especially if the tariff rate of the latter is high. Our results appear most pertinent in the wake of the recent tariffication of very restrictive import quotas. In some cases, like Canadian agricultural products marketed under supply management programs, the new tariffs exceed 300% and are certainly prohibitive. Minimum access commitments accompany the new tariffs to insure that exporters are not completely driven out. These minimum access commitments act as binding quotas and thus reduce the welfare losses that otherwise would accompany the switch to tariffs high enough to provide redundant protection and to attract smugglers.

Future research should address enforcement and evasion issues and explore the implications of different cost structures for smugglers. Even though we have not analyzed these problems, it can be conjectured from our results that laissez-faire or little enforcement might be the best course of action in some cases.
REFERENCES


Footnotes

1 For simplicity, we assume throughout the paper that the monopolist cannot export the good.

2 Pushing the argument to the logical extreme, the residual demand curve can be made to look perfectly elastic by using an *ad valorem* tariff of (-1) and a specific tariff.

3 Under the small country assumption, the specific and *ad valorem* tariffs are equivalent.

4 It is insightful to rewrite the smuggler's profit as: \( \pi_s = (\theta - \omega) s - [(1 - \lambda) \omega + (\alpha - 1) \omega] s - \chi(s, S) \). The term \((\theta - \omega)\) is the gross per unit profit margin of the smuggler, \((1 - \lambda) \omega\) is the average cost associated with the risk of seizure, \((\alpha - 1) \omega\) is the average terms of trade deterioration cost, and \(\chi(s, S)\) captures the total cost of the resources used in smuggling. The presence of S in the smuggler's cost function creates an externality that makes the marginal social cost of smuggling higher than its private counterpart.

5 Due to the hypothesized congestion effect, the marginal social cost of smuggling, \((\eta + \eta' S)\), exceeds the marginal private cost of smuggling \(\eta\). The difference between the marginal social and private smuggling costs is irrelevant in the positive analysis, but matters in the subsequent welfare analysis.

6 Naturally, the welfare of society is affected by the smuggling, but the monopolist's behavior is not, provided legal imports enter the country.

7 If \(\left(1 + \tau - \delta \omega(I) - (\eta(I)/\lambda)\right) < 0\), it will never be profitable to smuggle all imports (hence, \(Q_0(\tau) = 0\)); thus, if \(\omega(I)\) and \(\eta(I)\) are linear, a necessary condition for all imports to be smuggled is: \(\tau > \hat{\tau} = (\delta - 1) + (\eta/\lambda \omega)\). If \(\tau > \hat{\tau}\) and \(\left(1 + \tau - \delta \omega(0) - (\eta(0)/\lambda)\right) > 0\), there will be no legal imports \(Q_0(\tau) = \infty\).

8 Some of these intervals may be degenerate; i.e., \(Q_0(\tau) \to \infty\) as \(\tau \to \infty\); and \(Q_0(\tau) \to 0\) as \(\tau \to 0\).

9 For some level of domestic output, imports - legal or smuggled - will be reduced to zero.

10 It is also possible to use Figure 3 to show that \(Q_0(\tau)\) is increasing in \(\tau\). An increase in the tariff rate moves \(B^n\) down and \(D^n\) up thus pushing \(C^n\) north east of its original location.

11 Again, if \(\left(1 + \tau - \delta \omega(I) - (\eta(I)/\lambda)\right) < 0\), \(Q_0(\tau) = 0\), so an all smuggling equilibrium cannot occur.
We assume that, in the presence of smuggling, this local optimum is the global optimum. As discussed below, welfare is increasing in $\tau$ for $\tau \in (\tau_1, \tau_2)$.

Formally, we have not been shown that the infinite tariff "smuggling-only" case is inferior to the no-smuggling (possible) second-best optimal tariff. However, it seems very unlikely that it would be a more desirable outcome under most plausible circumstances.

Under a quota, the threat of smuggling makes the demand curve appear more elastic than it would if smuggling were not feasible. Further, the demand curve under the quota and smuggling can appear more elastic than it would under an ad valorem tariff.

An alternative to assumption (III) is that, at $R = I_R$, $Q_\delta(R) > Q_0(R) > Q_n(R)$. The main point of either assumption is that the monopolist's maximizing behavior does not result in smuggling for all $R$.

Naturally, a quota set above the free trade level may also be binding, but the case is not interesting since there is no welfare justification for imposing a quota at that level.

The linear restriction is sufficient, not necessary and the conclusion probably holds for other functional structures, though it cannot be proven.

It is readily shown that $L_0 < R_0 \iff \tau < \hat{\tau}$ and $L_0 \in (R, R_0) \iff \tau_0 > \hat{\tau}$, where $\hat{\tau} = (\eta/\omega')$.

With legal imports we have shown $(\partial Q^*/\partial \tau) > 0, (\partial S^*/\partial \tau) \geq 0, (\partial A^*/\partial \tau) < 0, (\partial \alpha^*/\partial \tau) > 0 \to$ $(\partial [Q^* + S^* + L^*]/\partial \tau) < 0$. Thus, as legal imports increase (the tariff decreases), output (and perhaps smuggling) decrease, but total consumption rises, hence: $(\partial Q^*/\partial \tau) \in (-1,0)$.

At $L_0^*$ (i.e. at $\tau = \tau_0$) the smuggling function is not differentiable; i.e., as $L \to (L_0^*)^-$, $(\partial S/\partial L) < 0$, and as $L \to (L_0^*)^+$, $(\partial S/\partial L) = 0$, increasing the likelihood of a solution at $L_0^*$. Similarly, under a quota, neither the output rule nor the smuggling rule is differentiable at $R_1$, raising the likelihood of a solution there.

This is a sufficient, not necessary, condition and the inequality will hold elsewhere.
Figure 1. Ad Valorem Tariff ($\tau$) vs Quota (R)
Figure 2. Ad Valorem Tariff ($\tau$) vs Specific Tariff ($T$)
Figure 4 Monopoly Output, Imports and Smuggling with Ad Valorem Tariff
Figure 4 Monopoly Output, Imports and Smuggling with Quota
Figure 6 Output under Tariff vs. Quota

Figure 7 Output under Tariff vs. Quota