A study of nonlinear flight control system designs

Lijun Tian
Iowa State University

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A study of nonlinear flight control system designs

by

Lijun Tian

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Aerospace Engineering

Major Professors: Ping Lu and Bion Pierson

Iowa State University
Ames, Iowa
1999

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Graduate College
Iowa State University

This is to certify that the Doctoral dissertation of

Lijun Tian

has met the dissertation requirements of Iowa State University

Signature was redacted for privacy.

Co-major Professor
Signature was redacted for privacy.

Co-major Professor
Signature was redacted for privacy.

For the Major Program
Signature was redacted for privacy.

For the Graduate College
To my parents and older sister
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOMENCLATURE</td>
<td>xii</td>
</tr>
<tr>
<td>CHAPTER 1 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>CHAPTER 2 AIRCRAFT MODEL</td>
<td>6</td>
</tr>
<tr>
<td>2.1 F18 Aircraft Description</td>
<td>6</td>
</tr>
<tr>
<td>2.2 Engine Dynamics Model</td>
<td>11</td>
</tr>
<tr>
<td>2.3 Aerodynamic Model</td>
<td>15</td>
</tr>
<tr>
<td>2.4 Nonlinear Aircraft Dynamics</td>
<td>17</td>
</tr>
<tr>
<td>2.4.1 Translation Equations of Motion (the force equations)</td>
<td>17</td>
</tr>
<tr>
<td>2.4.2 Rotational Equations of Motion (the moment equations)</td>
<td>18</td>
</tr>
<tr>
<td>2.4.3 The Kinematic Equations</td>
<td>21</td>
</tr>
<tr>
<td>2.4.4 The Navigation Equations (flight path equations)</td>
<td>21</td>
</tr>
<tr>
<td>2.4.5 The Relationship Among $\alpha, \beta$ and True Airspeed $V_T$</td>
<td>22</td>
</tr>
<tr>
<td>2.4.6 Inclusion of Wind</td>
<td>22</td>
</tr>
<tr>
<td>2.5 Longitudinal Aircraft Dynamics</td>
<td>23</td>
</tr>
<tr>
<td>CHAPTER 3 LINEAR ANALYSIS</td>
<td>25</td>
</tr>
<tr>
<td>3.1 Steady-State Trim Condition</td>
<td>25</td>
</tr>
<tr>
<td>3.2 Linearization of the Equations of Motion</td>
<td>29</td>
</tr>
<tr>
<td>3.3 Open-Loop System Characteristics</td>
<td>30</td>
</tr>
<tr>
<td>3.4 The Internal Dynamics of Linearized Systems</td>
<td>33</td>
</tr>
<tr>
<td>3.5 Numerical Solution of the State Equations</td>
<td>34</td>
</tr>
</tbody>
</table>
CHAPTER 4 CONTROL LAW DEVELOPMENT .................................. 35
  4.1 Linear Quadratic Regulator (LQR) Design .......................... 35
  4.2 Dynamics Inversion Control Design ................................. 35
  4.3 Nonlinear Predictive Control Design .............................. 39
  4.4 Approximate Receding-Horizon Control Design ................. 42

CHAPTER 5 CONTROLLER PERFORMANCE FOR HEALTHY AIRCRAFT ........................................... 51
  5.1 LQR Controller Performance ......................................... 51
    5.1.1 Mach = 0.5 case ........................................... 51
    5.1.2 Mach = 0.7 case ........................................... 59
  5.2 Nonlinear Predictive Controller .................................. 66
    5.2.1 Mach = 0.5 case ........................................... 66
    5.2.2 Mach = 0.7 case ........................................... 75
  5.3 Approximate Nonlinear Receding-Horizon Controller ........... 81
    5.3.1 Mach = 0.5 case ........................................... 81
    5.3.2 Mach = 0.7 case ........................................... 87

CHAPTER 6 CONTROLLER PERFORMANCE FOR UNHEALTHY AIRCRAFT ........................................... 91
  6.1 Propulsion Controlled Aircraft (PCA) ............................ 91
  6.2 Simulation Results .................................................. 93

CHAPTER 7 OUTPUT TRACKING PERFORMANCE ....................... 97
  7.1 Introduction .......................................................... 97
  7.2 Output-Tracking Controller ....................................... 99
  7.3 Output Tracking Performance .................................... 101
    7.3.1 Mach = 0.5 case ........................................... 102
    7.3.2 Mach = 0.7 case ........................................... 107
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHAPTER 8 CONCLUSIONS</td>
<td>110</td>
</tr>
<tr>
<td>APPENDIX APPROXIMATE NONLINEAR RECEEDING-HORIZON CONTROL LAW</td>
<td>114</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>125</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>130</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 2.1</td>
<td>Command input limits and sign conventions</td>
<td>10</td>
</tr>
<tr>
<td>Table 2.2</td>
<td>F18 aircraft model dimensions</td>
<td>10</td>
</tr>
<tr>
<td>Table 3.1</td>
<td>Trim data for F18 model at 10,000 ft altitude</td>
<td>28</td>
</tr>
<tr>
<td>Table 5.1</td>
<td>LQR controller gain matrix for Mach number 0.5</td>
<td>52</td>
</tr>
<tr>
<td>Table 5.2</td>
<td>LQR controller gain matrix for Mach number 0.7</td>
<td>60</td>
</tr>
<tr>
<td>Table 5.3</td>
<td>Pole-placement gain matrix for Mach number 0.5</td>
<td>67</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 2.1 Modular structure of the F-18 model ............................... 7
Figure 2.2 F18 aircraft configuration (1) ......................................... 8
Figure 2.3 F18 aircraft configuration (2) ......................................... 9
Figure 2.4 Three view of control surfaces ................................. 10
Figure 2.5 Maximum gross thrust tabular data ............................. 12
Figure 2.6 Power lever angle system configuration .................. 13
Figure 2.7 A schematic of the simple dynamic engine model .......... 14

Figure 3.1 Power curve for the F18 model ............................... 29

Figure 5.1 Mach = 0.5 velocity time history with LQR controller ........ 54
Figure 5.2 Mach = 0.5 angle of attack time history with LQR controller .. 54
Figure 5.3 Mach = 0.5 flight path angle time history with LQR controller . 55
Figure 5.4 Mach = 0.5 pitch rate time history with LQR controller ...... 55
Figure 5.5 Mach = 0.5 throttle setting time history with LQR controller . 56
Figure 5.6 Mach = 0.5 elevator deflection time history with LQR controller 56
Figure 5.7 Mach = 0.5 velocity time history with LQR controller ........ 57
Figure 5.8 Mach = 0.5 angle of attack time history with LQR controller .. 57
Figure 5.9 Mach = 0.5 states time history with LQR controller .......... 58
Figure 5.10 Mach = 0.5 throttle setting time history with LQR controller .. 58
Figure 5.11 Mach = 0.7 velocity time history with LQR controller ........ 61
Figure 5.12 Mach = 0.7 angle of attack time history with LQR controller .. 61
| Figure 5.13 | Mach = 0.7 flight path angle time history with LQR controller | 62 |
| Figure 5.14 | Mach = 0.7 pitch rate time history with LQR controller | 62 |
| Figure 5.15 | Mach = 0.7 throttle setting time history with LQR controller | 63 |
| Figure 5.16 | Mach = 0.7 elevator deflection time history with LQR controller | 63 |
| Figure 5.17 | Mach = 0.7 velocity time history with LQR controller | 64 |
| Figure 5.18 | Mach = 0.7 angle of attack time history with LQR controller | 64 |
| Figure 5.19 | Mach = 0.7 states time history with LQR controller | 65 |
| Figure 5.20 | Mach = 0.7 throttle setting time history with LQR controller | 65 |
| Figure 5.21 | Mach = 0.5 velocity with pole-placement | 69 |
| Figure 5.22 | Mach = 0.5 angle of attack with pole-placement | 69 |
| Figure 5.23 | Mach = 0.5 velocity with predictive controller | 70 |
| Figure 5.24 | Mach = 0.5 angle of attack with predictive controller | 70 |
| Figure 5.25 | Mach = 0.5 flight path angle with predictive controller | 71 |
| Figure 5.26 | Mach = 0.5 pitch rate with predictive controller | 71 |
| Figure 5.27 | Mach = 0.5 throttle setting with predictive controller | 72 |
| Figure 5.28 | Mach = 0.5 elevator deflection with predictive controller | 72 |
| Figure 5.29 | Mach = 0.5 velocity with predictive controller | 73 |
| Figure 5.30 | Mach = 0.5 angle of attack with predictive controller | 73 |
| Figure 5.31 | Mach = 0.5 state variables with predictive controller | 74 |
| Figure 5.32 | Mach = 0.5 throttle setting with predictive controller | 74 |
| Figure 5.33 | Mach = 0.7 velocity with predictive controller | 76 |
| Figure 5.34 | Mach = 0.7 angle of attack with predictive controller | 76 |
| Figure 5.35 | Mach = 0.7 flight path angle with predictive controller | 77 |
| Figure 5.36 | Mach = 0.7 pitch rate with predictive controller | 77 |
| Figure 5.37 | Mach = 0.7 throttle setting with predictive controller | 78 |
| Figure 5.38 | Mach = 0.7 elevator deflection with predictive controller | 78 |
| Figure 5.39 | Mach = 0.7 velocity with predictive controller | 79 |
| Figure 5.40 | Mach = 0.7 angle of attack with predictive controller | 79  |
| Figure 5.41 | Mach = 0.7 state variables with predictive controller | 80  |
| Figure 5.42 | Mach = 0.7 throttle setting with predictive controller | 80  |
| Figure 5.43 | Mach = 0.5 velocity with receding-horizon controller | 83  |
| Figure 5.44 | Mach = 0.5 angle of attack with receding-horizon controller | 83  |
| Figure 5.45 | Mach = 0.5 flight path angle with receding-horizon controller | 84  |
| Figure 5.46 | Mach = 0.5 pitch rate with receding-horizon controller | 84  |
| Figure 5.47 | Mach = 0.5 throttle setting with receding-horizon controller | 85  |
| Figure 5.48 | Mach = 0.5 elevator deflection with receding-horizon controller | 85  |
| Figure 5.49 | Mach = 0.5 velocity with receding-horizon controller | 86  |
| Figure 5.50 | Mach = 0.5 angle of attack with receding-horizon controller | 86  |
| Figure 5.51 | Mach = 0.7 velocity with receding-horizon controller | 88  |
| Figure 5.52 | Mach = 0.7 angle of attack with receding-horizon controller | 88  |
| Figure 5.53 | Mach = 0.7 flight path angle with receding-horizon controller | 89  |
| Figure 5.54 | Mach = 0.7 pitch rate with receding-horizon controller | 89  |
| Figure 5.55 | Mach = 0.7 throttle setting with receding-horizon controller | 90  |
| Figure 5.56 | Mach = 0.7 elevator deflection with receding-horizon controller | 90  |
| Figure 6.1  | Mach = 0.5 state variables with engine-only predictive controller | 95  |
| Figure 6.2  | Mach = 0.5 state variables with engine-only predictive controller | 95  |
| Figure 6.3  | Mach = 0.5 velocity variation with engine-only predictive controller | 96  |
| Figure 6.4  | Mach = 0.5 throttle setting with engine-only predictive controller | 96  |
| Figure 7.1  | Mach = 0.5 climb rate with receding-horizon controller | 103 |
| Figure 7.2  | Mach = 0.5 angle of attack with receding-horizon controller | 103 |
| Figure 7.3  | Mach = 0.5 pitch angle with receding-horizon controller | 104 |
| Figure 7.4  | Mach = 0.5 pitch rate with receding-horizon controller | 104 |
| Figure 7.5  | Mach = 0.5 throttle setting for output tracking control | 105 |
Figure 7.6  Mach = 0.5 elevator deflection for output tracking control  . . . . 105
Figure 7.7  Mach = 0.5 pitch rate for parameter \( h \) influence  . . . . . . . . . . 106
Figure 7.8  Mach = 0.5 flight path angle for parameter \( h \) influence  . . . . . . 106
Figure 7.9  Mach = 0.7 climb rate for output tracking control  . . . . . . . . . . 108
Figure 7.10 Mach = 0.7 flight path angle for output tracking control  . . . . . . 108
Figure 7.11 Mach = 0.7 pitch angle for output tracking control  . . . . . . . . . . 109
Figure 7.12 Mach = 0.7 pitch rate for output tracking control  . . . . . . . . . . 109
NOMENCLATURE

Lower case

\( b \) span of wing
\( c \) mean aero chord
\( c.g \) center of gravity
\( g \) acceleration of gravity
\( m \) airplane mass
\( \bar{q} \) dynamic pressure

Upper case

\( C_D \) coefficient of drag
\( C_{LFT} \) coefficient of lift
\( C_L \) coefficient of rolling moment
\( C_M \) coefficient of pitching moment
\( C_N \) coefficient of yawing moment
\( C_Y \) coefficient of sideforce
\( C_{l\beta} \) dihedral effect derivative (lateral stability)
$C_{ma}$ longitudinal stability derivative

$C_{n\beta}$ weathercock effect derivative (directional stability)

$D_{inl}$ inlet spillage drag, lb

$EPR$ engine thrust ratio

$FG$ gross thrust, lb

$FG_{axial}$ axial gross thrust after thrust vectoring, lb

$FNP$ net propulsive force, lb

$F_{ram}$ ram drag, lb

$F_x, F_y, F_z$ aerodynamic force (including propulsive force) in the $x,y,z$ body axis

HARV high alpha research vehicle

$I_{xx}, I_{yy}, I_{zz}$ moments of inertia about $x, y$ and $z$ axis

$I_{xy}, I_{yz}, I_{xz}$ products of inertia about $x, y$ and $z$ plane

$L, M, N$ roll, pitch and yaw moment w.r.t $x, y, z$ body axis

$L', M', N'$ roll, pitch and yaw moment w.r.t $x$ body axis by propulsive system

$Mach$ Mach number

$MaxAB$ maximum after-burner

$MinAB$ minimum after-burner

$Mil$ power intermediate-rated power

$NPR$ nozzle pressure ratio
\(P, Q, R\) angular velocity in roll, pitch, yaw

\(PCA\) propulsion controlled aircraft

\(PLA\) power lever angle

\(PLA'\) shaped power lever angle

\(S\) wing area

\(T\) engine thrust

\(TVCS\) thrust vectoring control system

\(U, V, W\) velocity component in the \(x, y, z\) body axis

\(U_d, V_d, W_d\) \(x, y, z\) velocity component in the earth fixed frame

\(V_T\) true airspeed

\(W_{dx}, W_{dy}, W_{dz}\) \(x, y, z\) wind component in the earth fixed frame

\(W_x, W_y, W_z\) \(x, y, z\) wind component in the earth fixed frame

\(X, Y, Z\) net applied force in the \(x, y, z\) body axis

\(X_D\) aerodynamic drag force

\(Y_{LIFT}\) aerodynamic lift

\(Z_S\) aerodynamic side force

\(X_{NED}\) position Nord in the NED frame

\(Y_{NED}\) position East in the NED frame

\(Z_{NED}\) altitude
Xₜ  total thrust force along x-body axis, lb
Yₜ  total thrust force along y-body axis, lb
Zₜ  total thrust force along z-body axis, lb

Greek symbols

α  angle of attack
β  sideslip angle
γ  flight path angle
Φ, Θ, Ψ three Euler angles
θ  altitude angle
ϕ  bank angle
ψ  heading angle
λ  eigenvalues
ζ  damping ratio
δₐ  aileron deflection
δ_r  rudder deflection
δ_e  elevator deflection
φₚ  engine installing angle
τ  time constant
ω  frequency
CHAPTER 1 INTRODUCTION

Aircraft flight control systems are traditionally designed based on linearized dynamics and linear control methodologies with the fundamental assumption of small perturbations from equilibrium flight conditions. This assumption neglects high order terms (nonlinearities) in system mathematics expression [1]. While the linear designs have been remarkably successful in the past, increasingly high performance of modern aircraft, usually associated with large flight envelop, high angle of attack, and large angular rates, has invalidated the fundamental assumption of small perturbations of linearization. In these cases the nonlinearity in the aircraft dynamics becomes so prominent that it can no longer be ignored. For most of the cases, linear control will work poorly or the system can become unstable because the system nonlinearities cannot be compensated. In order to respond to these nonlinearities and achieve satisfactory flight performance and quality, a satisfactory nonlinear flight control system must take into account the inherent nonlinearity dictated by the law of physics.

The nonlinear control problem is much more complex than the linear one. During the past decades, significant advances have taken place in the area of nonlinear control. Unfortunately, many of the these developments are scattered in research publications and are understood by a select group of experts. Often, the original ideas and the motivations for pursuing a particular path are lost in a maze of mathematical formalism. So far, there is no universal technique for the analysis of nonlinear control systems because of the varieties of nonlinearities.

Dynamic inversion [2] is a popular method for nonlinear flight control system design.
It is based on input-output feedback linearization techniques of canceling the nonlinearities and replacing the dynamics by desired linear dynamics. The basic idea is to first transform a nonlinear system into a fully or partially linear system, and then use the well-known and powerful linear design techniques to complete the control design. This approach has been used to solve a number of practical nonlinear flight control problems. It applies to important classes of nonlinear systems, i.e. so-called input-state linearizable or minimum-phase systems, typically requires full state measurement. It is worth mentioning that this method is invalid once saturation, dead-zone or modeling uncertainties are present in the system.

There are some situations in which abrupt changes in the system cause significant nonlinear behavior. A case of point is the propulsion-only flight control problem for an aircraft with complete hydraulic failure. Although aircraft control systems are designed with extensive redundancy to ensure a low probability of failure, however, during recent years several aircrafts have experienced major flight control system failures, leaving engine thrust as the only usable control effector. In some of these emergency situations, the engines were used “open-loop” to maintain control of the flight path and bank angle of the airplane. A B-747 aircraft (Boeing Company, Seattle, Washington) lost its entire hydraulic system because of a pressure bulkhead failure [3]. It was flown for almost an hour using throttle control. But the crew were forced to learn by trial and error, and the plane eventually hit a mountain. Perhaps, the best known use of manual throttles-only control occurred in July, 1989, on United Airlines flight 232 [4]. At cruise condition, a DC-10 (McDonnell Douglas Aerospace, Long Beach, CA) suffered an uncontained tail engine failure that caused the loss of all hydraulics. Under extremely difficult circumstances, the crew used wing engine throttles as the controls and was able to crash land at the Sioux City airport, Iowa, and over one-half of the people on-board were saved. A C-5A cargo airplane had a major structural failure that caused loss of all hydraulics to the tail [5, 6]. This airplane was flown for 1/2 hrs with the throttles,
but on a landing attempt, the airplane hit the ground short of runway, broke up, and all aboard were killed in the resulting fire. In the majority of cases surveyed, due to the overload work of manual throttle control, major flight control system failures have resulted in crashes with a total of over 1200 fatalities [7, 8].

The challenge is to design an automatic engine-only thrust control (referred to as the propulsion-controlled aircraft (PCA system) as emergency backup flight control for aircraft when potentially disastrous flight control system failures do occur, and safely land an airplane with severely damaged or inoperative control surfaces. It concerns how an aircraft behaves under the control of only engine thrust. The use of appropriate modulation of engine thrust to stabilize the aircraft may be the only chance of survival for the people on-board. This is particularly true for military airplanes operating in a hostile environment. Many aircraft companies, commercial and military, regard this as an important research topic.

The feasibility and implementation of propulsion controlled system for emergence flight control when the conventional flight control system is inoperative has been established by the propulsion controlled aircraft (PCA) program at the NASA Dryden Flight Research Center. Successful flight experiments have been conducted on F-15, MD-11 and C-17 airplanes using feedback throttle control system. In the flight testing, some notable nonlinear behaviors have also been observed. These include engine dynamics, engine saturation, propulsion and airframe interaction, and strong dynamic cross-coupling. All these nonlinear phenomena are amplified by the fact that the engine has very limited control authority on the attitude of the aircraft. Control laws based on dynamic inversion can easily become invalid because of control (thrust) saturation.

It would appear logical to expect that in these highly nonlinear situations, for both control of healthy high-performance aircraft and impaired aircraft with only differential thrust control, a nonlinear design of the control system may offer better performance. In this thesis, we offer some evidence that nonlinear designs can indeed enhance the
performance of the flight control systems, we shall apply both a recently developed non-linear predictive control approach \[9, 10\] and a newly developed approximate nonlinear receding-horizon control technique \[11\] to nonlinear flight control system design for an F18 aircraft, especially comparison for propulsion controller (PCA) design and show that these methods are effective for an important class of problems in which dynamic inversion encounters difficulty.

The nonlinear predictive control approach is developed based on minimization of local errors between the controlled variables and their desired values. One-step-ahead prediction of \( x(t + h) \) is obtained by expanding each component \( x_i(t + h) \) into an \( i_{th} \) order Taylor series, where \( r_i \) is the relative degree of \( x_i(t) \), and the approximation order \( N \) is always equal to one. It can incorporate nonlinearities in the nonlinear controller design and it is effective, requiring no stringent conditions on the system other than the usual smoothness conditions. Unfortunately, for the output tracking problem of non-minimum phase systems, this method is similar to feedback linearization, and thus is unable to provide satisfactory control for the aircraft.

The newly developed closed-form approximate receding-horizon control laws for the class of continuous-time, affine nonlinear system is based on a multi-step predictive control formulation. In the receding-horizon control strategy, at each time \( t \) and state \( x(t) \), the open-loop solution \( u^* \) for an optimal control problem over a finite horizon \([t, t + T]\) is determined on-line. Then the current control \( u(t) \) is set to equal to \( u^*(t) \). Continuing this process for all \( t \geq 0 \) gives a feedback control since \( u^*(t) \) is dependent on \( x(t) \), so is \( u(t) \). The finite-horizon optimal control problem formulated is as one with a quadratic performance index plus a terminal constraint \( x(t + T) = 0 \). Unfortunately, the heavy computation burden makes the implementation of the receding-horizon control unrealistic. In deriving a closed form approximate receding-horizon control law, the multi-step-ahead prediction of \( x(t + h) \) is obtained by expanding each component \( x_i(t + h) \) into a Taylor series, and the approximation order \( N \) is always greater than one.
The terminal condition $x(t + T) = 0$ must be added to ensure a sufficiently long control horizon achieved and a controllability condition must be satisfied to ensure the existence of the control law. The approximation function leads to a quadratic programming problem (QPP), and the control law can be obtained explicitly. The output-tracking problem can be treated in a similar fashion, and a closed-form tracking control law can be constructed [12].

This thesis is intended to study the feasibility of nonlinear flight control system designs. The study includes the investigation of nonlinear engine dynamics, aerodynamics, flight dynamics, control law development and verification using a high-fidelity six-degree-of-freedom nonlinear model for the F18. Nonlinear predictive control technique and approximate nonlinear receding-horizon control have been applied to the designs of normal flight control system and propulsion control system. In Chapter 2, a realistic nonlinear model of F18 aircraft is described, including all the basic aircraft mathematics models used for nonlinear flight control system design. In Chapter 3, we present some preliminary study by performing linearization analysis. A well-known linear control design approach (Linear Quadratic Regular Technique) is designed based on the linearized system. The newly developed nonlinear predictive control technique and approximate nonlinear receding-horizon control approach are introduced in Chapter 4. The application of the three control approaches for a healthy aircraft and the comparison of the performance of the linear and nonlinear control designs are given in Chapter 5. A unhealthy aircraft flight control problem is studied in Chapter 6 where propulsion control aircraft is the focus. The extension of the approximate receding-horizon control technique to output tracking problem of non-minimum phase systems and the capabilities of these controllers to stabilize nonlinear systems or track the desired output are demonstrated with the F18 model in Chapter 7. Chapter 8 concludes the study.
CHAPTER 2 AIRCRAFT MODEL

This chapter describes a high-fidelity nonlinear F-18 aircraft model, including detailed, full-envelope, nonlinear aerodynamics, fully-developed thrust and first-order engine response data, and the six-degree-of-freedom nonlinear aircraft equations of motion. While this model was primarily developed for the NASA F18 HARV study, the availability of such a model provides a common focus for F18 HARV research in control system design; it is also intended to be useful for a variety of controls system design.

This model is a collection of modules, each performing a specific function. The primary modules are the aircraft actuator and surface command inputs, aircraft mass and geometry modeling, equation of motion, atmospheric model, aerodynamics, and propulsion system. Each major module is described in the following sections. Figure 2.1 shows how the modules would be connected together with user synthesized control laws to form a complete system model.

2.1 F18 Aircraft Description

The F18 aircraft is an all-day, high-performance, supersonic fighter. The complete configuration of the aircraft are shown in Figures 2.2, 2.3. It has a net weight of 36,000 lb and a wing area of 400 ft². A three-view of the aircraft including control surfaces and sign convention is shown in Figure 2.4. The aircraft geometry characteristics are given in Table 2.2. The aircraft primary flight control surfaces consist of horizontal stabilators, conventional ailerons and two vertical rudders.
Figure 2.1 Modular structure of the F-18 model
Figure 2.2  F18 aircraft configuration (1)
Figure 2.3 F18 aircraft configuration (2)
Table 2.1 Command input limits and sign conventions

<table>
<thead>
<tr>
<th>Command name</th>
<th>Symbol</th>
<th>Rate Limiter (deg)</th>
<th>Saturations (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stabilator</td>
<td>$\delta_e$</td>
<td>±40</td>
<td>-24 ~ +10.5</td>
</tr>
<tr>
<td>Aileron</td>
<td>$\delta_a$</td>
<td>±100</td>
<td>-25 ~ +45</td>
</tr>
<tr>
<td>Rudder</td>
<td>$\delta_r$</td>
<td>±82</td>
<td>-30 ~ +30</td>
</tr>
<tr>
<td>Leading edge</td>
<td>$\delta_{lef}$</td>
<td>±15</td>
<td>-3 ~ +33</td>
</tr>
<tr>
<td>Trailing edge</td>
<td>$\delta_{tef}$</td>
<td>±18</td>
<td>-8 ~ +45</td>
</tr>
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</table>

Table 2.2 F18 aircraft model dimensions

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>Mass</td>
<td>36,000 lb</td>
</tr>
<tr>
<td>Wing area</td>
<td>$S$</td>
<td>400 $ft^2$</td>
</tr>
<tr>
<td>Wing span</td>
<td>$b$</td>
<td>37.4 ft</td>
</tr>
<tr>
<td>Mean chord</td>
<td>$\hat{c}$</td>
<td>11.52 ft</td>
</tr>
<tr>
<td>Moments of inertia</td>
<td>$I_{xx}$</td>
<td>22789.08 slug/$ft^2$</td>
</tr>
<tr>
<td></td>
<td>$I_{yy}$</td>
<td>176809.20 slug/$ft^2$</td>
</tr>
<tr>
<td></td>
<td>$I_{zz}$</td>
<td>191743.60 slug/$ft^2$</td>
</tr>
<tr>
<td>Products</td>
<td>$I_{xx}$</td>
<td>-2304.98 slug/$ft^2$</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td></td>
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<td>$\Delta y$</td>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td>$\Delta z$</td>
<td></td>
<td>-0.4460</td>
</tr>
</tbody>
</table>
2.2 Engine Dynamics Model

The F-18 aircraft is powered by two General Electric F404-GE-400 engines [13]. The F404-GE-400 engine is a 16,000-lb thrust class, low bypass, twin spool turbofan with after-burner. It incorporates a three-stage fan and a seven-stage high-pressure compressor, each driven by a single-stage turbine. During flight, power lever angle (PLA) ranges from 23.8° (flight idle) to 130° (full power with after-burner). Intermediate power (full, non-after-burning) occurs at 68° PLA.

Because of the limited control authority of the engine thrust on the attitude of the aircraft, the control commands can easily cause the engines to operate at their limits at its upper-bound when engine is the only flight control effector. For instance, a large velocity perturbation may cause engine to saturate. Also, in low-speed landing approaches, the commanded engine thrust is close to idle. Disturbances caused by initial conditions could result in engine saturation at the lower bound. Being a low bandwidth system, the engines may also experience rate saturation, and this is compounded by the nonlinear aspect of the thrust response.

Due to the execution time constraints, simple first-order engine dynamic model was used

\[
\frac{d \ PLA'}{dt} = \frac{(PLA - PLA')}{\tau}
\]  

(2.1)

where the time constant \( \tau \) is scheduled with respect to the output \( PLA' \), Mach number, and angle of attack. Note that because of these dependence, Eqn. (2.1) is a nonlinear model. The engine gross thrust is computed by performing multidimensional, linear interpolations of tabular data over \( PLA' \), Mach number, altitude and angle of attack. The breakpoints of Mach number and altitude are shown in Fig. 2.5. The real engine
thrust is determined based on several quantities, including gross thrust, ram drag, nozzle pressure ratio and nozzle throat area.

Breakpoints of PLA are flight idle, intermediate-rated power, minimum after-burner and maximum after-burner. An example plot of the gross thrust tabular data values appears in Figure 2.5. The plot symbols represent the table values of maximum gross thrust at different Mach number and altitude conditions for the F404-GE-400 engine.

Figure 2.6 shows the relation between PLA algorithm, controller synthesis and system dynamics. A first-order time delay exists in the PLA system design. At each time instant, the $PLA'$ will be the exact PLA input to aircraft dynamics.

Figure 2.7 represents the overall schematic of the simple dynamic engine model. As
illustrated, input parameters to the simple dynamic engine model are determined from the aircraft simulation flight condition and cockpit PLA position. Simple dynamics are introduced to engine parameters by shaping the PLA command. The values of $F_G, F_{ram}, D_{init}, D_{noz}, NPR,$ and $A8$ are determined by linearly interpolating the tabular data with respect to altitude, Mach number, and shaped power lever angle ($PLA'$).

The Figure 2.7 also illustrates how the simple dynamic engine model interfaces with the thrust vectoring performance model and control laws, giving out the relationship of PLA, Mach, altitude and resultant thrust.

Assume that the left and right propulsion are $T_L$ and $T_R$. The total acting forces and moments produced by propulsive system on the aircraft are

\[
\begin{align*}
F_x' &= (T_L + T_R) \cos \varphi_p \\
F_y' &= 0 \\
F_z' &= (T_L + T_R) \sin \varphi_p \\
L' &= 0 \\
M' &= 0 \\
N' &= (T_L - T_R) \times L
\end{align*}
\]  

(2.2)  

where $\varphi_p$ is engine install angle.
Figure 2.7  A schematic of the simple dynamic engine model
2.3 Aerodynamic Model

This F-18 aircraft features a mid wing configuration with a wing-root leading-edge extension (LEX) that extends from the forward portion of the fuselage and blends into the wing. The aircraft primary flight control surfaces consist of horizontal stabilators which are capable of symmetric movement, conventional ailerons, and two vertical rudders. The individual surface position limits, rate limits, and sign conventions for positive deflection are detailed in Table 2.4.

It has aerodynamic coefficients defined over the entire operational flight envelop of the aircraft by tabulated data. The aerodynamic coefficients are computed by performing multidimensional table lookup and linear interpolation to form nonlinear function generators. The interpolation in general is dependent on the current Mach number, altitude, angle of attack, sideslip angle, angular rates, and control surface deflections. Thus, aerodynamics model consist of a detailed nonlinear model component build up based on look-up tables created from wind-tunnel data with minor adjustments based on flight data. The functional relation is as follows:

\[
\begin{align*}
C_L &= C_{L0}(\alpha) + \Delta C_L(\alpha, \delta_{LEF}) + \Delta C_L(\alpha, \delta_{TEF}) + \Delta C_L(\alpha, \delta_e) + \frac{k}{2} (C_{\alpha}(\alpha) q + C_{\alpha\dot{\alpha}}(\alpha) \dot{\alpha}) \\
C_D &= C_{D0}(\alpha) + \Delta C_D(\alpha, \delta_{LEF}) + \Delta C_D(\alpha, \delta_{TEF}) + \Delta C_D(\alpha, \delta_e) + \Delta C_D(C_L) \\
C_m &= C_{m0}(\alpha) + \Delta C_m(\alpha, \delta_{LEF}) + \Delta C_m(\alpha, \delta_{TEF}) + \Delta C_m(\alpha, \delta_e) + \frac{k}{2} (C_{mq}(\alpha) q + C_{m\dot{\alpha}}(\alpha) \dot{\alpha}) \\
C_Y &= C_{Y0}(\alpha, \beta) + \Delta C_Y(\alpha, \delta_{LEF}) + \Delta C_Y(\alpha, \delta_{TEF}) \\
&+ \Delta C_Y(\alpha, \delta_a) + \Delta C_Y(\alpha, \delta_r) + \frac{k}{2} (C_{Yq}(\alpha) q + C_{Y\dot{\alpha}}(\alpha) \dot{\alpha}) \\
C_t &= C_{t0}(\alpha, \beta) + \Delta C_t(\alpha, \delta_{LEF}) + \Delta C_t(\alpha, \delta_{TEF}) \\
&+ \Delta C_{ts}(\alpha, \beta) + \Delta C_{ts}(\alpha, \beta) + \frac{k}{2} (C_{tq}(\alpha) q + C_{t\dot{\alpha}}(\alpha) \dot{\alpha}) \\
C_n &= C_{n0}(\alpha, \beta) + \Delta C_n(\alpha, \delta_{LEF}) + \Delta C_n(\alpha, \delta_{TEF}) \\
&+ \Delta C_{ns}(\alpha, \beta) + \Delta C_{ns}(\alpha, \beta) + \frac{k}{2} (C_{nq}(\alpha) q + C_{n\dot{\alpha}}(\alpha) \dot{\alpha})
\end{align*}
\]

(2.4)

where \(C_L, C_D, C_m, C_Y, C_t, C_n\) define the coefficient of lift, drag, pitch moment, side force, roll moment and yaw moment. \(\delta_{LEF}, \delta_{TEF}\) are the leading edge and trailing edge. \(\delta_{stab}\)
is the deflection of stabilizer.

The aerodynamic forces (drag force $X_D$, lift $Y_{LFT}$ and side force $Z_S$) on an aircraft are created by its motion relative to the surrounding air. The three most important variables determining the aerodynamic forces and moments are $\alpha$, $\beta$ and $V_T$. The total aerodynamic forces (excluding propulsive forces $T_L$ and $T_R$) are given

$$
\begin{align*}
F_x &= -X_D \cos \alpha \cos \beta + Y_{LFT} \sin \alpha - Z_S \cos \alpha \sin \beta \\
F_y &= -X_D \sin \beta + Z_S \cos \beta \\
F_z &= -X_D \sin \alpha \cos \beta - Y_{LFT} \cos \alpha - Z_S \sin \alpha \sin \beta
\end{align*}
$$

The resultant external forces (excluding propulsive forces $T_L$ and $T_R$) are

$$
\begin{align*}
X &= F_x - mg \sin \Theta \\
Y &= F_y + mg \cos \Theta \sin \Phi \\
Z &= F_z + mg \cos \Theta \cos \Phi
\end{align*}
$$

The total aerodynamic moments are given

$$
\begin{align*}
L &= \frac{1}{2} \rho v^2 s b C_L + F_y \Delta Z - F_z \Delta Y \\
M &= \frac{1}{2} \rho v^2 s c C_M + F_z \Delta X - F_x \Delta Z \\
N &= \frac{1}{2} \rho v^2 s b C_N + F_x \Delta Y - F_y \Delta X
\end{align*}
$$

Thus, we can easily obtain the total external forces and moments acting on aircraft are

$$
\begin{align*}
F_{xt} &= F_x + F_x' \\
F_{yt} &= F_y + F_y' \\
F_{zt} &= F_z + F_z' \\
L_t &= L + L' \\
M_t &= M + M' \\
N_t &= N + N'
\end{align*}
$$
2.4 Nonlinear Aircraft Dynamics

In general, the standard six degrees of freedom (6DOF) equations of motion used for conventional aircraft control design and flight simulation can be obtained by the assumptions of the flat-Earth and the rigid-body aircraft with longitude symmetric plane. These equations of motion are nonlinear, continuous, time-variant, first-order ordinary differential equations with twelve variables. The main variables are:

1. the mass-center velocity components $U, V, W$ in the body-fixed axis

2. the three Euler angles $\Phi, \Theta, \Psi$

3. the angular velocity in the body-fixed axis $P, Q, R$

4. the mass-center position components $X_d, Y_d, Z_d$ in the Earth-fixed frame of reference and $X, Y, Z$ in the NED frame

5. the airspeed $V_T$ and the relative airflow angles $\alpha, \beta$

Usually, we take the coordinate system $F_d(ox'y'z')$ as an Earth-fixed frame of reference and the coordinate system $F_t(oxyz)$ in the body axis as moving coordinate system with the aircraft. The coordinate transfer matrix (rotation matrix) from $F_d$ to $F_t$ is:

\[
T_{td} = \begin{bmatrix}
\cos \Theta \cos \Psi & \cos \Theta \sin \Psi & -\sin \Theta \\
\sin \Phi \sin \Theta \cos \Psi - \cos \Phi \sin \Psi & \sin \Phi \sin \Theta \sin \Psi + \cos \Phi \cos \Psi & \sin \Phi \cos \Psi \\
\cos \Phi \sin \Theta \cos \Psi + \sin \Phi \sin \Psi & \cos \Phi \sin \Theta \sin \Psi - \sin \Phi \cos \Psi & \cos \Phi \cos \Theta
\end{bmatrix}
\]  

(2.10)

2.4.1 Translation Equations of Motion (the force equations)

The basic force equations are:

\[
\begin{cases}
F_x = m(\dot{U} + QW - RV) \\
F_y = m(\dot{V} + RU - PW) \\
F_z = m(\dot{W} + PV - QU)
\end{cases}
\]  

(2.11)
With the aerodynamic forces (including propulsive forces) denoted by \((F_x, F_y, F_z)\), the resultant external forces are:

\[
\begin{align*}
X &= F_x - mg \sin \Theta \\
Y &= F_y + mg \cos \Theta \sin \Phi \\
Z &= F_z + mg \cos \Theta \cos \Phi
\end{align*}
\]  

(2.12)

Thus, we can easily obtain the following translation equations of motion defined in the aircraft body frame:

\[
\begin{align*}
\dot{U} &= RV - QW - g \sin \Theta + F_x/m \\
\dot{V} &= -RU + PW + g \sin \Phi \cos \Theta + F_y/m \\
\dot{W} &= QU - PV + g \cos \Phi \cos \Theta + F_z/m
\end{align*}
\]  

(2.13)

The transformation from the aircraft body-axes to the wind-axes are given as follows:

\[
\begin{align*}
\dot{\Theta} &= [-L + Z_T \cos \alpha - X_T \sin \alpha \\
&+ mg(\sin \Theta \cos \alpha \cos \beta - \cos \Theta \sin \Psi \sin \beta - \cos \Theta \cos \Psi \sin \alpha \cos \beta)]/m \\
\dot{\Psi} &= [D \sin \beta + Y \sin \beta - X_T \cos \alpha \beta + Y_T \cos \beta - Z_T \sin \alpha \beta \\
&- mg(\sin \Theta \cos \alpha \sin \beta + \cos \Theta \sin \Psi \cos \beta - \cos \Theta \cos \Psi \sin \alpha \sin \beta)]/Vm \\
&+ p \sin \alpha - r \cos \alpha
\end{align*}
\]  

(2.14)

2.4.2 Rotational Equations of Motion (the moment equations)

The basic moment equations based on angular momentum \(h\) of the airplane are

\[
\begin{align*}
L &= \dot{h}_x + Qh_z - Rh_y \\
M &= \dot{h}_y + Rh_x - Ph_z \\
N &= \dot{h}_z + Ph_y - Qh_x
\end{align*}
\]  

(2.15)
where $h_x, h_y, h_z$ are the scalar components of $h$.

(1). If the three moments of inertia $I_{xx}, I_{yy}$ and $I_{zz}$ are not be ignored, then the moment equation is

$$
\begin{bmatrix}
I_{xx} & -I_{xy} & -I_{xz} \\
-I_{xy} & I_{yy} & -I_{yz} \\
-I_{xz} & -I_{yz} & I_{zz}
\end{bmatrix}
\begin{bmatrix}
\dot{P} \\
\dot{Q} \\
\dot{R}
\end{bmatrix}
=
\begin{bmatrix}
L \\
M \\
N
\end{bmatrix}
$$

(2.16)

where

$$
\begin{align*}
L &= L' + (I_{zz} - I_{yy})QR + I_{yz}(R^2 - Q^2) + I_{xy}PR - I_{xz}PQ \\
M &= M' + (I_{xx} - I_{zz})RP + I_{xz}(P^2 - R^2) + I_{yx}QP - I_{yx}QR \\
N &= N' + (I_{yy} - I_{xx})PQ + I_{xy}(Q^2 - P^2) - I_{yz}RP + I_{xz}QR
\end{align*}

(2.17)

note that $L, M$ and $N$ are the total resultant moments about the body fixed frame $X$, $Y$ and $Z$ axis, $L', M'$ and $N'$ are moments produced by propulsive system.

The above equations can be transferred into the following form:

$$
\begin{bmatrix}
\dot{P} \\
\dot{Q} \\
\dot{R}
\end{bmatrix}
=
\begin{bmatrix}
I_{xx} & -I_{xy} & -I_{xz} \\
-I_{xy} & I_{yy} & -I_{yz} \\
-I_{xz} & -I_{yz} & I_{zz}
\end{bmatrix}^{-1}
\begin{bmatrix}
L \\
M \\
N
\end{bmatrix}
$$

(2.18)

where the $L, M$ and $N$ are total resultant external moments acting on the aircraft. The variables $P, Q$ and $R$ are pitch, yaw and roll angular velocity defined relative to the body-fixed frame.

Finally, the moment equations of motion are

$$
\begin{align*}
\dot{P} &= A_0RQ + A_1PQ + A_2L + A_3N \\
\dot{Q} &= A_4PR - A_5(P^2 - R^2) + A_6M \\
\dot{R} &= A_7PQ - A_1RQ + A_3L + A_8N
\end{align*}

(2.19)
with the following setting

\[
\begin{align*}
A_0 &= \frac{(l_{yy}-l_{zz})l_{zz}-l_{zz}^2}{l_{xx}l_{zz}-l_{zz}^2} \\
A_1 &= \frac{(l_{yy}-l_{yy}+l_{yy})l_{xx}}{l_{xx}l_{xx}-l_{zz}^2} \\
A_2 &= \frac{l_{xx}}{l_{xx}l_{zz}-l_{zz}^2} \\
A_3 &= \frac{l_{yy}}{l_{xx}l_{zz}-l_{zz}^2} \\
A_4 &= \frac{l_{xx}-l_{xx}}{l_{yy}} \\
A_5 &= \frac{l_{xx}}{l_{yy}} \\
A_6 &= \frac{1}{l_{yy}} \\
A_7 &= \frac{(l_{xx}-l_{yy})l_{xx}+l_{zz}}{l_{xx}l_{xx}-l_{zz}^2} \\
A_8 &= \frac{l_{xx}}{l_{xx}l_{xx}-l_{zz}^2}
\end{align*}
\]

(2.20)

If x-axis, z-axis are in the plane of symmetry xz, y-axis is perpendicular to the xz plane and is a principal axis of inertia, i.e: the products of inertia \( I_{xy} = 0, I_{yz} = 0 \), the simplified equation is

\[
\begin{align*}
l_{xx}\dot{P} - l_{xz}\dot{R} &= L \\
l_{yy}\dot{Q} &= M \\
-l_{xz}\dot{P} + l_{zz}\dot{R} &= N
\end{align*}
\]

(2.21)

where

\[
\begin{align*}
L &= L' + (l_{zz} - l_{yy})QR - l_{xz}PQ \\
M &= M' + (l_{xx} - l_{zz})RP + l_{xz}(P^2 - R^2) \\
N &= N' + (l_{yy} - l_{zz})RP + l_{xz}RQ
\end{align*}
\]

(2.22)

Further, we get

\[
\begin{align*}
\dot{P} &= (l_{yy}L + l_{xy}M)/(l_{xx}l_{yy} - l_{xy}^2) \\
\dot{Q} &= (l_{xx}M + l_{xy}L)/(l_{xx}l_{yy} - l_{xy}^2) \\
\dot{R} &= N/l_{yy}
\end{align*}
\]

(2.23)
2.4.3 The Kinematic Equations

From the basic kinematic eqns:

\[
\begin{align*}
P &= \Phi - \dot{\Phi} \sin \Theta \\
Q &= \dot{\Theta} \cos \Phi + \dot{\Phi} \cos \Theta \sin \Phi \\
R &= \dot{\Psi} \cos \Theta \cos \Psi - \dot{\Theta} \sin \Phi
\end{align*}
\]  

(2.24)

we may derive the following attitude angle equations:

\[
\begin{align*}
\Phi &= P + \tan \Theta (Q \sin \Phi + R \cos \Phi) \\
\Theta &= Q \cos \Phi - R \sin \Phi \\
\Psi &= (Q \sin \Phi + R \cos \Phi) \sec \Theta
\end{align*}
\]  

(2.25)

2.4.4 The Navigation Equations (flight path equations)

Obviously, we have the following relationship between the mass-center position components \(X_d, Y_d, Z_d\) and the mass-center velocity components \(U_d, V_d, W_d\) in the Earth-fixed frame of reference

\[
\begin{bmatrix}
\dot{X}_d \\
\dot{Y}_d \\
\dot{Z}_d
\end{bmatrix}
= T_{dt}(\Psi, \Theta, \Phi)
\begin{bmatrix}
U \\
V \\
W
\end{bmatrix}
\]  

(2.26)

where \(T_{dt} = T_{td}^T\) is the coordinate transfer matrix from \(F_t\) to \(F_d\). The further expansion relative to the above equations by substituting the matrix \(T_{dt}\) from Eqn.(2.10), we can transfer the equations into scalar form

\[
\begin{align*}
\dot{X}_d &= U \cos \Theta \cos \Psi + V (-\cos \Phi \sin \Psi + \sin \Phi \sin \Theta \cos \Psi) + W (\sin \Phi \sin \Psi + \cos \Phi \sin \Theta \cos \Psi) \\
\dot{Y}_d &= U \cos \Theta \sin \Psi + V (\cos \Phi \cos \Psi + \sin \Phi \sin \Theta \sin \Psi) + W (-\sin \Phi \cos \Psi + \cos \Phi \sin \Theta \sin \Psi) \\
\dot{Z}_d &= -U \sin \Theta + V \sin \Phi \cos \Theta + W \cos \Phi \cos \Theta
\end{align*}
\]  

(2.27)
The transformation from the Earth-fixed frame to the NED frame is

\[
\begin{align*}
X_{NED} &= \dot{X}_d \\
Y_{NED} &= \dot{Y}_d \\
Z_{NED} &= -\dot{Z}_d
\end{align*}
\]  

(2.28)

2.4.5 The Relationship Among \(\alpha, \beta\) and True Airspeed \(V_T\)

\[
V_T = \sqrt{V_{Tx}^2 + V_{Ty}^2 + V_{Tz}^2}
\]  

(2.29)

where \(V_{Tx}, V_{Ty}\) and \(V_{Tz}\) are the airspeed components in the body-fixed axis. Thus,

\[
\begin{align*}
\alpha &= \arctan \left( \frac{V_{Tz}}{V_{Tx}} \right) \\
\beta &= \arcsin \left( \frac{V_{Ty}}{V_T} \right)
\end{align*}
\]  

(2.30)

where \(\alpha\) is angle of attack and \(\beta\) is sideslip angle.

2.4.6 Inclusion of Wind

The velocity of the aircraft's c.g with respect to air, \(\vec{V}_T\), and the velocity of the aircraft's c.g with respect to the Earth-fixed frame, \(\vec{V}_d\), are related by

\[
\vec{V}_T = \vec{V}_d - \vec{W}
\]  

(2.31)

where \(\vec{W}\) is wind speed.

The scalar form is:

\[
\begin{align*}
V_{Tx} &= U - W_x \\
V_{Ty} &= V - W_y \\
V_{Tz} &= W - W_z
\end{align*}
\]  

(2.32)

where \(U, V, W\) are the c.g velocity components of the aircraft in the body fixed axis,
$W_x$, $W_y$, $W_z$ are the wind components in the body fixed axis. They can be obtained from the wind components $W_{dx}$, $W_{dy}$, $W_{dz}$ in the Earth-fixed frame:

$$
\begin{bmatrix}
W_x \\
W_y \\
W_z
\end{bmatrix}
= T_{id}(\Psi, \Theta, \Phi)
\begin{bmatrix}
W_{dx} \\
W_{dy} \\
W_{dz}
\end{bmatrix}
$$

(2.33)

In summary, the nonlinear equations of motion (2.13), (2.19), (2.25), (2.27) consist of the general six-degree-of-freedom equations representing the flight dynamics of a rigid aircraft flying in a stationary atmosphere over a flat, non-rotating Earth. These equations are defined on body-fixed axis.

The atmospheric data model is based on tables from the U.S. Standard Atmosphere(1962). This model calculates values for speed of sound, acceleration due to gravity, air density, viscosity, and ambient static pressure and temperature. These values are calculated based on altitude. The tabular data is organized on evenly spaced breakpoints between 0 and 90 km. Linear interpolation is used between table values for altitudes in this range; the extreme values are used for altitudes outside the range.

### 2.5 Longitudinal Aircraft Dynamics

In this section, we will limit our study to longitudinal motion and study the longitudinal aircraft dynamics. The corresponding equations of motion will be given as well.

In longitudinal motion, the aircraft flight is limited to two-dimensions in the vertical plane (symmetric plane $xz$). By setting all lateral parameters ($\beta$, $V$, $P$, $\Psi$, $\Theta$, $\Phi$, $Y$) equal to zero, the original six-degrees-of-freedom equations of motion are reduced to the simplified three-degree-of-freedom equations of motion for longitudinal motion. Equations of motion are still nonlinear, continuous, time-variant, first-order ordinary differ-
ential equations, consist of six nonlinear differential equations with six state variables. The state variables are: the mass-center airspeed \( V \), angle of attack \( \alpha \), pitch rate \( q \) in the body-fixed axis, pitch angle \( \theta \), and the mass-center position coordinates \( d, z \) in an Earth-fixed frame of reference. The equations of motion are defined in the stability axis as follows

\[
\begin{align*}
\dot{V} &= (-D + T \cos \alpha - mg \sin(\theta - \alpha))/m \quad (2.34) \\
\dot{\alpha} &= (-L - T \sin \alpha + mg \cos(\theta - \alpha))/(mV) + q \quad (2.35) \\
\dot{\theta} &= q \quad (2.36) \\
\dot{q} &= (M + T \Delta z)/I_{yy} \quad (2.37) \\
\dot{z} &= V \sin(\theta - \alpha) \quad (2.38) \\
\dot{d} &= V \cos(\theta - \alpha) \quad (2.39)
\end{align*}
\]

where the aerodynamic forces and moment are denoted by \( L, D \) and \( M \). They represent lift, drag and pitch moment, respectively, and are functions of angle of attack, Mach number, altitude, control surface deflections, pitch rate and some other parameters. Through a vertical displacement \( \Delta z \) between the center of the aircraft gravity and the line of thrust, the engine thrust also contributes to the pitch moment. The two available controls are elevator deflection \( \delta_e \) and engine throttle PLA. The complete system equations are Eqns. (2.34–2.39) plus the engine dynamics Eqn. (2.1).
CHAPTER 3 LINEAR ANALYSIS

In this chapter, we will study the characteristics of the linearized aircraft dynamics as the basis for the future nonlinear control system design. A realistic F18 aircraft model is considered consistently through our study for both linear and nonlinear flight control system designs.

The control problem is to design a control law \( u(t) = f(x, t) \) for all \( t \geq 0 \) such that for an arbitrary initial conditions of the system \((2.13) \sim (2.19)\), the system output will track some desired values. Since the linear control problem faces the linear dynamics, there are many methods that can be used to the problem of this class. In this thesis, we choose both linear quadratic regular (LQR) method and pole placement method. However the nonlinear control problem faces the nonlinear dynamics, there are no universal methods can be used so far. Undoubtedly, nonlinear control problem is much more complex than linear one. But, linear problem can give us some insight about nonlinear flight control system design.

3.1 Steady-State Trim Condition

A steady-state trim condition is an equilibrium flight condition where the motion variables in the equations of motion are constant or zero. This requires the solution of a set of nonlinear equations to obtain the trim values of the state and control vectors that satisfy these equations. Since the very complex functional dependence of the aerodynamic data, this can not be done analytically. A numerical algorithm is needed, which
iteratively adjusts the independent variables until some solution criterion is met. The solution will be approximated but can be made arbitrarily close to the exact solution by tightening up the criterion. In some cases, equilibrium points are not unique.

The F18 aircraft is trimmed at the straight-and-level condition for two cases. The first case we consider is the steady-state straight and level flight at an altitude 10,000 feet and Mach number is 0.5 with zero flight path angle. To see the role control play in stabilizing an airplane, we need to fix or to change the equilibrium condition (speed of angle of climb). An adequate control must be powerful enough to produce the whole range of equilibrium states of which the airplane is capable from a performance standpoint to another. So, as comparison, the second case we consider is the steady-state straight and level flight at an altitude 35,000 feet and Mach number equals to 0.7 with zero flight path angle.

Let the system equations (2.34)-(2.38) be

$$\dot{x} = f(x, u)$$  \hfill (3.1)

where $x = (V, \alpha, \theta, q, z)^T$ and $u = (\delta_e, PLA)^T$. $V$ represents the mass-center airspeed, $\alpha$ represents angle of attack, $\theta$ represents pitch angle, $q$ represents pitch rate in the body-fixed axis. And, the two control commands are the elevator deflection angle $\delta_e$ and engine throttle $PLA$.

Supposed that the aircraft flies at straight and level condition with zero flight path angle at the given altitude and Mach number. It satisfies the equilibrium conditions where the right hand sides of Eqns. (2.34–2.38) are zero, i.e: $\Sigma f(x, u) = 0$, and the angle of attack $\alpha$ equals to pitch angle $\theta$, pitch rate $q$ equals to zero. By solving the above equations, the steady-state solutions including airspeed $V$, angle of attack $\alpha$, pitch angle $\theta$ can be obtained. Also, the control commands including engine throttle $PLA$ and elevator deflection angle $\delta_e$ can be derived. We use two methods to derive the trim conditions.
**Nonlinear Solver**

From the general computer build-in solver, algorithm is given by the Gauss-Newton method with a mixed quadratic and cubic line search procedure $F(X) = 0$.

**Simple Iteration**

For some cases, simple subsequent substitution are iteration. Iteration method are adequate to find the trim point. Consider

\[
\begin{align*}
T \cos(\alpha + \varphi_p) - \bar{q}s(C_d + C_d^a \alpha + C_d^\delta \delta_e)\theta &= 0 \\
T \sin(\alpha + \varphi_p) + \bar{q}s(C_y + C_y^a \alpha + C_y^\delta \delta_e)\theta &= 0 \\
-T \Delta z + \bar{q}s b_A(C_m + C_m^a \alpha + C_m^\delta \delta_e) &= 0
\end{align*}
\]

(3.2)

where $T$ is throttle vector, $\varphi_p$ is engine install angle, $\delta_e$ is elevator deflection angle, $\alpha$ is angle of attack, $\theta$ is pitch angle, $\Delta z$ is a vertical displacement between the center of the aircraft gravity and the line of thrust. The aerodynamic lift, drag and pitch moment coefficients are $C_d, C_d^a, C_d^\delta, C_y, C_y^a, C_y^\delta$ and $C_m, C_m^a, C_m^\delta$. $s$ is wing area and $\bar{q}$ is dynamics pressure.

The transformation of the above equations are

\[
\begin{align*}
\alpha &= (mg \cos \Theta - T \sin(\alpha + \varphi_p))/\bar{q}s - C_y/C_y^a \\
T &= (\bar{q}s C_D + mg \sin \Theta)/\cos(\alpha + \varphi_p) \\
\delta_e &= (T \Delta z/\bar{q}s b_A - C_m)/C_m^\delta
\end{align*}
\]

(3.3)  

(3.4)  

(3.5)

Choose the initial guess $\alpha_0, T_0$ and $\delta_{e0}$, we first solve $\alpha_1$ from the eqn (3.3), substitute $\alpha_1$ into eqn (3.4) for $T_1$ and substitute $T_1$ into eqn (3.5) for $\delta_{e1}$. Then repeat the process until convergence is achieved.

According to our experience, the iteration order is a very important factor to make the iteration converge. The current order from $\alpha \rightarrow P \rightarrow \delta_z$ is found to be the most efficient order.
The trim conditions corresponding to altitude 10,000 feet and Mach 0.5 case are

\[ V_{trim} = 551.57 \text{ (ft/sec)}, \alpha_{trim} = 3.39 \text{ (deg)}, \theta_{trim} = \alpha \]  

(3.6)

and control inputs

\[ \delta_{trim} = -0.2413 \text{ (deg)}, PLA_{trim} = 33(\text{deg}) \]  

(3.7)

And, the trim conditions corresponding to altitude 35,000 feet and Mach number 0.7 case are

\[ V_{trim} = 677.65 \text{ (ft/sec)}, \alpha_{trim} = 4.56 \text{ (deg)}, \theta_{trim} = \alpha \]  

(3.8)

and control inputs

\[ \delta_{trim} = -0.7802 \text{ (deg)}, T_{trim} = 42(\text{deg}) \]  

(3.9)

Table 3.1 gives a summary of the F18 trim conditions at the different flight conditions.

From the table, we see that at the same altitude level the engine thrust will increase dramatically with the increase of the speed while keeping the zero flight path angle. For the same value of Mach number, the engine thrust will decrease with the increase of the Mach number. As a result, the angle of attack will correspondingly decrease to trim the aircraft and keep it at the same altitude. The elevator deflection has the opposite trend, with the increase of elevator deflection for the increase of Mach number.

Table 3.1    Trim data for F18 model at 10,000 ft altitude

<table>
<thead>
<tr>
<th>Mach</th>
<th>Airspeed(ft/sec)</th>
<th>Thrust(lb)</th>
<th>Angle of attack(deg)</th>
<th>Elevator(deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>220.63</td>
<td>14151.1986</td>
<td>24.6144</td>
<td>-5.7864</td>
</tr>
<tr>
<td>0.25</td>
<td>275.78</td>
<td>9243.1793</td>
<td>15.2999</td>
<td>-2.4052</td>
</tr>
<tr>
<td>0.30</td>
<td>330.94</td>
<td>6043.3663</td>
<td>10.2297</td>
<td>-1.7254</td>
</tr>
<tr>
<td>0.35</td>
<td>386.07</td>
<td>4157.1417</td>
<td>7.3555</td>
<td>-0.9731</td>
</tr>
<tr>
<td>0.40</td>
<td>441.25</td>
<td>3360.3070</td>
<td>5.5433</td>
<td>-0.5082</td>
</tr>
<tr>
<td>0.45</td>
<td>496.41</td>
<td>3136.1287</td>
<td>4.3008</td>
<td>-0.3912</td>
</tr>
<tr>
<td>0.50</td>
<td>551.57</td>
<td>3122.6774</td>
<td>3.4040</td>
<td>-0.2454</td>
</tr>
<tr>
<td>0.55</td>
<td>606.72</td>
<td>3551.8124</td>
<td>2.7292</td>
<td>-0.0153</td>
</tr>
<tr>
<td>0.60</td>
<td>661.88</td>
<td>4179.4634</td>
<td>2.2313</td>
<td>0.1549</td>
</tr>
<tr>
<td>0.70</td>
<td>772.19</td>
<td>5393.0799</td>
<td>1.5085</td>
<td>0.6396</td>
</tr>
</tbody>
</table>
Figure 3.1 shows the throttle setting plotted against airspeed. There exists the minimum of the power-required curve. If the aircraft is operating on the right side of the power curve, opening the throttle produces an increase in speed, while on the left side of the power curve, opening the throttle produces an increase in altitude, not a increase in speed.

### 3.2 Linearization of the Equations of Motion

As shown in chapter 2, the equations of motion have the nonlinear form

$$\dot{x} = f(x, u)$$

(3.10)

Under the small perturbation assumption from the steady-state condition $x_\epsilon$ and $u_\epsilon$, we may derive a set of linear constant-coefficient state equations. This is done by perturbing the state and control variables from the steady-state condition, and numerically
evaluating the partial derivatives in the Jacobian matrices. The Jacobian matrices may
be determined for any steady-state flight condition.

Expand the nonlinear state equations (2.34)-(2.38) in a Taylor series about the trim
point \((x_e, u_e)\) and keep only the first-order terms. The perturbations in the state and
control vectors will satisfy the following linearized equations

\[
\begin{bmatrix}
\delta x_1 \\
\delta x_2 \\
\vdots \\
\delta x_n
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\
\frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n}
\end{bmatrix}
\begin{bmatrix}
\delta x_1 \\
\delta x_2 \\
\vdots \\
\delta x_n
\end{bmatrix} +
\begin{bmatrix}
\frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_m} \\
\frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_m} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_m}
\end{bmatrix}
\begin{bmatrix}
\delta u_1 \\
\delta u_2 \\
\vdots \\
\delta u_m
\end{bmatrix}
\tag{3.11}
\]

The linearized dynamics about such a trim point \((x_e, u_e)\) are written in a compact
form

\[
\delta \dot{x} = A \delta x + B \delta u 
\tag{3.12}
\]

where \(A = \partial f/\partial x\) and \(B = \partial f/\partial u\) are evaluated at \((u_e, u_e)\), \(\delta x = x - x_e\), and \(\delta u = u - u_e\).

Obviously, for a given flight condition, the coefficient matrices are time-invariant,
and the resulting state equations are linear and time-invariant.

Stability of the linearized system means stability of the original nonlinear system in
a small region around the trim point, and instability of the linearized system indicate
instability of the nonlinear system.

### 3.3 Open-Loop System Characteristics

As mentioned before, two steady-state longitudinal flight cases are studied in this
thesis. The variables involved in the longitudinal equations are airspeed \(v\), angle of
attack \(\alpha\), pitch angle \(\theta\), pitch rate \(q\) and altitude \(h\). The Jacobian A and B matrices are
determined numerically by central differences. For Mach 0.5 and an altitude of 10,000
feet, the A and B are as follows:
It is straightforward to verify that the aircraft is not stable at this condition. In fact, the eigenvalues of system (3.13) are:

\[
\lambda_{1,2} = -0.7107 \pm j1.8449
\]
\[
\lambda_3 = -0.0285
\]
\[
\lambda_4 = 0.0283
\]
\[
\lambda_5 = 0.0000279
\]

where \( \lambda_{1,2} \) represent the short-period heavily damped mode which is stable. \( \lambda_3 \) and \( \lambda_4 \) represent the long-period lightly damped mode called phugoid which is unstable. \( \lambda_5 \) is corresponding to altitude mode which is unstable.

The instability means that if internal or external disturbances are presented in the system, the flight trajectories will diverge from the trim conditions due to the fact that there exist positive eigenvalues located in the right half of the complex plane. Consequently, to maintain the trim condition, feedback control law is needed. Therefore,
we will take the problem of stabilizing the aircraft at this condition to test linear and nonlinear control law designs.

The same problem exists for the Mach 0.7 case. The longitudinal Jacobian matrices for the F18 model at Mach 0.7 and altitude 35,000 feet are

\[
A = \begin{bmatrix}
-2.9153 \times 10^{-4} & -1.8333 \times 10^{-1} & -3.2144 \times 10^{-1} & -1.4055 \times 10^{-2} & 2.8593 \times 10^{-8} \\
-2.5431 \times 10^{-5} & -6.2163 \times 10^{-1} & 0 & 9.9585 \times 10^{-1} & 8.4389 \times 10^{-9} \\
0 & 0 & 0 & 1.0000 \times 10^{0} & 0 \\
-2.2722 \times 10^{-5} & -2.5857 \times 10^{0} & 0 & -2.4547 \times 10^{-1} & -7.0875 \times 10^{-8} \\
0 & -6.7765 \times 10^{2} & 6.7765 \times 10^{2} & 0 & 0
\end{bmatrix}
\]

\(B = \begin{bmatrix}
4.9234 \times 10^{-4} & 1.4869 \times 10^{-2} \\
-9.9348 \times 10^{-8} & -3.6169 \times 10^{-3} \\
0 & 0 \\
-2.5225 \times 10^{-6} & -1.6987 \times 10^{-1} \\
0 & 0
\end{bmatrix}
\]

The open-loop system has the eigenvalues

\[
\begin{align*}
\lambda_{1,2} &= -0.43388 \pm 1.5933i \\
\lambda_{3,4} &= 0.00016768 \pm 0.024998i \\
\lambda_5 &= 0.000032975
\end{align*}
\]

where \(\lambda_{1,2}\) represent the short-period heavily damped mode which is stable. \(\lambda_3\) and \(\lambda_4\) represent the long-period lightly damped mode called phugoid which is unstable. \(\lambda_5\) is corresponding to altitude mode which is unstable. It can be seen again that there exist unstable eigenvalues. It shows that the system is open-loop unstable. This means
that the trajectory will diverge from the trim condition if any initial perturbation exists. Consequently, closed-loop stabilization is needed as well.

3.4 The Internal Dynamics of Linearized Systems

By testing the controllability matrix for Mach = 0.5 case, we know that the original system is controllable, so the application of appropriate control system design technique will stabilize this system. It’s very interesting to know that if we further transfer this system into Jordan form, we get

\[
\begin{bmatrix}
-0.7107 & 1.8449 & 0.0000 & 0.0000 \\
-1.8449 & -0.7107 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0281 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.0283 \\
\end{bmatrix}
\begin{bmatrix}
-0.00007 & -0.04051 \\
0.00018 & 0.11595 \\
-0.01373 & -3.76792 \\
-0.00184 & 3.78688 \\
\end{bmatrix}
\]

(3.19)

In matrix \( \tilde{B} \), the coefficient corresponding to engine throttles \( PLA \) are relatively smaller, especially the first two modes. This means that engine throttle has very little influence on the control of the first two modes in the corresponding equations. Furthermore, if we choose any arbitrary two state variables, for example, \( V \) and \( \alpha \) as the outputs, we will always get at lease one positive zero located in the right half of the complex plane. This indicates that the system is a non-minimum phase system. In other words, the internal dynamics of the system are unstable [14]. While the internal dynamics of a system are stable, the system can be stabilized by simply driving the outputs to zero. Otherwise, if the system internal dynamics are not stable, whatever variables we choose as the outputs, we cannot stabilize the system by only controlling the outputs. This makes control system design more challenging.

In the analysis of nonlinear systems, internal dynamics are also referred to as zero-dynamics [14]. The zero-dynamics are defined to be the internal dynamics of the system
when the system outputs are kept at zero by the control. Zero dynamics do not depend on the choice of control law or the desired trajectories. For linear time-invariant system, the zero dynamics stability is determined by the zeros of the transfer functions. If all the zeros are in the left half of the complex plane, the zero dynamics are stable. Otherwise, the zero dynamics are unstable. The stability of the zero dynamics of the linearized system means stability of the zero dynamics of the corresponding nonlinear system near the trim point. Similarly, instability of zero dynamics of the linearized system signifies instability of zero dynamics of the nonlinear system [14].

3.5 Numerical Solution of the State Equations

To simulate the flight using the aircraft model, we need to solve the nonlinear differential equations that govern the motion. These nonlinear equations of motion depend on the experimental data, and are subjected to arbitrary input signals. Thus, unlike linear ones, nonlinear equations cannot be solved analytically. The powerful mathematical tools like Laplace and Fourier transform do not apply to nonlinear system, and numerical methods must be used to calculate an aircraft trajectory.

The numerical method here means the numerical integration algorithm for solution of ordinary differential equations (ODE). For the continuous state equations, This technique is an initial-value problem. A powerful known algorithm of solving the initial-value problem is Runge-Kutta algorithms. We apply this technique to the F18 model. Nonlinear dynamics are numerically integrated in time to obtain the time histories of state variables. The integration algorithm used here is the fourth-order Runge-Kutta algorithm.
CHAPTER 4  CONTROL LAW DEVELOPMENT

In this chapter, a traditional linear quadratic regulator design (LQR), a conventional nonlinear dynamics inverse technique and the two newly developed nonlinear control methods, a predictive control approach and an approximate nonlinear receding-horizon control technique, are presented. The concepts extended to the output tracking control problem for non-minimum phase system will be discussed in Chapter 7. A closed-form tracking control law is obtained in a similar fashion. The comparisons of controller performance for healthy F-18 aircraft model and unhealthy F-18 aircraft model will be given in the Chapter 5 and 6.

4.1 Linear Quadratic Regulator (LQR) Design

A well-known powerful control system design method for linear, time-invariant system is the LQR approach[1]. Here, we briefly review the procedure for two reasons. First of all, it results in a full-state feedback control law which can be compared with the nonlinear predictive control law and receding-horizontal control law to be introduced later. Secondly, both the nonlinear predictive control law and receding-horizontal control law bear strong similarity with the LQR control law.

To stabilize the linear system (3.12) at the origin, a performance index

\[ J = \frac{1}{2} \int_{0}^{\infty} \left\{ \delta x^T Q \delta x(t) + \delta u^T R \delta u \right\} dt \]  

(4.1)

is minimized , subject to (3.12) and a given initial condition \( \delta x(0) \). The \( Q \) matrix is positive semidefinite, and \( R \) matrix positive definite. Suppose that the system (3.12) is
controllable, the unique optimal control law is then given by

\[ \delta u = -R^{-1}B^TK\delta x \]  \hspace{1cm} (4.2)

where \( K \) is the positive definite solution of the algebraic Riccati equation (ARE)

\[ -KA - A^TK + KBR^{-1}B^T - Q = 0 \]  \hspace{1cm} (4.3)

The controllability guarantees that the ARE has a unique positive solution, thus the control law (4.2) is well defined. Under this control law, the stability of the closed-loop system

\[ \delta x = (A - BR^{-1}B^TK)\delta x \]  \hspace{1cm} (4.4)

is ensured. The applied control to the nonlinear system is then

\[ u = u_{trim} + \delta u \]  \hspace{1cm} (4.5)

### 4.2 Dynamics Inversion Control Design

The first objective of this section is to briefly review the concept and methodology of dynamics inversion, and to yield insights about the tracking control of non-minimum phase systems. Another objective, we want to offer the comparison between dynamics inversion and the methods presented in this thesis.

General nonlinear system has the form

\[ \dot{x} = f(x, u) \]  \hspace{1cm} (4.6)

\[ y = cx \]  \hspace{1cm} (4.7)

where \( x \) is \( n \times 1 \) state vector, \( u \) is \( m \times 1 \) control vector, \( y \) is \( l \times 1 \) output vector, and \( c \) is \( l \times n \) constant matrix. Augmenting the system dynamics with derivatives of approximate control inputs, a transformation of eqns 4.6 and 4.7 can be made into the linear analytic
form

\[
\dot{x} = A(x) + B(x)u
\]  \hspace{1cm} (4.8)

\[
y = C(x)
\]  \hspace{1cm} (4.9)

where \(A(x)\) is \(n \times 1\) vector, \(B(x)\) is \(n \times m\) matrix, \(C(x)\) is \(m \times 1\) vector.

The inverse of dynamics of 4.8 and 4.9 are constructed by differentiating the individual elements of \(y\) a sufficient number of times until a term containing \(u\) appears \([2]\).

Since only \(m\) outputs can be controlled independently with \(m\) inputs, it will be assumed that \(\text{dim}(y) = \text{dim}(u) = m\). The \(k\)th-order differentiation operator is defined \(L^k_A(\cdot)\), such that,

\[
L^k_A(x) = \left[ \frac{\partial}{\partial x} L^{k-1}_A(x) \right] A(x)  \hspace{1cm} (4.10)
\]

\[
L^0_A(x) = x  \hspace{1cm} (4.11)
\]

Using this notation to differentiate the \(i\)th component of \(y\) yield

\[
\begin{align*}
\dot{y}_i & = C_i \dot{x} = C_i A(x) + C_i B(x)u = C_i L^1_A(x) \\
\ddot{y}_i & = C_i \ddot{x} = C_i \left[ \frac{\partial}{\partial x} L^1_A(x) \right] A(x) + C_i \left[ \frac{\partial}{\partial x} L^1_A(x) \right] B(x)u = C_i L^2_A(x) \\
\vdots
\end{align*}
\]

\[
\begin{align*}
y^{(d_i)}_i & = C_i x^{(d_i)} = C_i \left[ \frac{\partial}{\partial x} L^{d_i-1}_A(x) \right] A(x) + C_i \left[ \frac{\partial}{\partial x} L^{d_i-1}_A(x) \right] B(x)u  \hspace{1cm} (4.12)
\end{align*}
\]

where \(d_i\) is the order of the derivative of \(y_i\) necessary to ensure that,

\[
C_i \left[ \frac{\partial}{\partial x} L^{d_i-1}_A(x) \right] B(x) \neq 0  \hspace{1cm} (4.13)
\]

After differentiating the \(m\) elements of \(y\), the output dynamics can be represented as
Using the notation of Singh and Rugh and Freund (1973) let,

\[ A^*_i(x) = C_i[L^d_A(x)] \]  
\[ B^*_i(x) = C_i \left[ \frac{\partial}{\partial x} L^e_A(x) \right] B(x) \]

This allows 4.14 to be written in more compact notation as

\[ y^{(d)} = A^*(x) + B^*(x)u \]

A sufficient condition for the existence of an inverse system model to 4.8 and 4.9 is that \( B^* \) in 4.17 be non-singular. If this is the case, then the inverse system model takes the form,

\[ \dot{x} = [A(x) - B(x)F(x)] + B(x)G(x)v \]  
\[ u = -F(x) + G(x)v \]

where \( v = y^{(d)} \) is the input to the inverse system, \( u \) is its output and

\[ G = [B^*(x)]^{-1} \]  
\[ F = [B^*(x)]^{-1}A^*(x) \]
Applying the control law
\[ u = -F(x) + G(x)u \] (4.22)
to the original system of 4.8 and 4.9 constitute the dynamic inversion control technique.

The dynamics inversion results in unbounded internal dynamics for non-minimum phase nonlinear system. Thus, the control law 4.22 cannot be applied to non-minimum phase nonlinear systems because they cannot be inverted. This is a generation of the linear result that the inverse of the transfer function of a non-minimum phase linear system is unstable. Therefore, for such systems, we cannot expect to design control law to achieve perfect or asymptotic convergent tracking errors. Instead, it is desired to find controllers which lead to small tracking errors for the desired trajectories of interest.

4.3 Nonlinear Predictive Control Design

For the convenience of the reader, a brief review on the nonlinear predictive control design method is given here. For more complete and rigorous derivations and discussions, see Lu [9, 10].

Suppose that the nonlinear dynamic system equations have the form
\[
\begin{align*}
\dot{x}_1 &= f_1(x) \\
\dot{x}_2 &= f_2(x) + B_2(x)u
\end{align*}
\] (4.23) (4.24)

where \( x_1 \in \mathbb{R}^{n_1}, x_2 \in \mathbb{R}^{n_2}, \) and \( n_1 + n_2 = n. \) Here \( x^T = (x_1^T, x_2^T) \in \mathbb{R}^n \) is the state vector of the system. \( u \in U \subset \mathbb{R}^m \) is the control vector, where \( U \) is a compact bounded set in \( \mathbb{R}^m. \) \( B_2(x) \) is continuous and none of its rows are zeros. The function \( f_1 \) is \( C^2, \) and \( f_2 \) is \( C^1. \) Equations (4.23) usually represent the kinematics in the system and Equations (4.24) represent the dynamics. Suppose that a reference trajectory \( s(t) \in \mathbb{R}^n, t \in [t_0, t_f] \) is given. It is assumed that \( s(t) \) satisfies the state equations (4.23) and (4.24) with some reference control \( u^*(t) \in U, \) although \( u^*(t) \) need not be known explicitly. We may
partition the reference trajectory by \( s(t) = (s_1^T(t) \ s_2^T(t))^T \) with \( s_1 \in R^{n_1} \) and \( s_2 \in R^{n_2} \). Suppose that at \( t \in (t_0, t_f) \), \( x(t) \) is known. Consider the system response at \( x(t + h) \), where \( h > 0 \) is a time increment. Expanding \( x_1(t + h) \) in a second-order Taylor series expansion and \( x_2(t + h) \) in a first-order expansion, we have the predicted state at \( t + h \) as a function of the current control \( u(t) \)

\[
x_1(t + h) \approx x_1(t) + h f_1(x) + \frac{h^2}{2} [F_{11} f_1(x) + F_{12} f_2(x) + F_{12} B_2(x) u(t)]
\]

(4.25)

\[
x_2(t + h) \approx x_2(t) + h [f_2(x) + B_2(x) u(t)]
\]

(4.26)

where \( F_{11} = \partial f_1 / \partial x_1 \) and \( F_{12} = \partial f_1 / \partial x_2 \) are the Jacobian matrices of \( f_1(x) \). To find the control \( u(t) \) so that \( x(t) \) tracks \( s(t) \), we define the following performance index of minimization, which is a quadratic function in \( u(t) \)

\[
J = \frac{1}{2} e_1^T(t + h) Q_1 e_1(t + h) + \frac{1}{2} e_2^T(t + h) h^2 Q_2 e_2(t + h) + \frac{1}{2} u^T h^4 R u(t)
\]

(4.27)

where \( e_1(t + h) = x_1(t + h) - s_1(t + h) \) and \( e_2(t + h) = x_2(t + h) - s_2(t + h) \), \( Q_1, Q_2 \) and \( R \) are positive semidefinite square matrices of the appropriate dimensions. The reference states \( s_1(t + h) \) and \( s_2(t + h) \) are further approximated by

\[
s_1(t + h) = s_1(t) + h \dot{s}_1(t) + \frac{h^2}{2} \ddot{s}_1(t)
\]

(4.28)

\[
s_2(t + h) = s_2(t) + h \dot{s}_2(t)
\]

(4.29)

The performance index \( J \) is a quadratic function in \( u \) when \( x_1(t + h) \) and \( x_2(t + h) \) are approximated by equations (4.25) and (4.26). Solving for \( u(t) \) that minimize \( J \) by setting \( \partial J / \partial u = 0 \) yields

\[
u(t) = -W^{-1} \left\{ \frac{1}{2h^2} G^T Q_1 P_1 + \frac{1}{h} B_2^T Q_2 P_2 \right\}
\]

(4.30)

where the following substitutions and expansions have been made:

\[
G = F_{12} B_2(x)
\]

(4.31)
\[ W = \frac{1}{4} G^T Q_1 G + B_2(x)^T Q_2 B_2(x) + R \]  
(4.32)
\[ P_1 = e_1 + h\dot{e}_1 + \frac{h^2}{2} (F_{11}f_1(x) + F_{12}f_2(x) - \ddot{s}_1) \]  
(4.33)
\[ P_2 = e_2 + h (f_2(x) - \ddot{s}_2) \]  
(4.34)

Since the time \( t \) is arbitrarily chosen in the \([t_0, t_f] \), equation (4.30) is a nonlinear, continuous feedback control law. It bears strong similarity with the LQR controller in the way the control law is derived. The weightings \( Q_1, Q_2 \) and \( R \) have the same meaning as in the LQR design. If an element on the main diagonal \( Q_1 \) (or \( Q_2 \)) is nonzero (positive), the corresponding state variable will be controlled to follow its desired value. Typically the performance of the controller is not sensitive to the choices of the weighting values. The parameter \( h \) can be treated as an additional control parameter that can be adjusted to improve the performance of controller. Generally, the smaller value \( h \) has, the faster the system response is, but at larger control effort.

To apply the predictive controller to the longitudinal flight control problem, we let \( x_1 = (\theta, z)^T, x_2 = (V, \alpha, q)^T \), and \( u = (\delta, PLA) \). The control limits are enforced by simple saturators. The reference trajectory \( s(t) \) for stabilization problem is simply the trim value \( x_2 \). So, the dynamic system equations for \( x \in \mathbb{R}^n \) have the form,

\[
\begin{align*}
\dot{x}_1 &= \begin{pmatrix}
\dot{\theta} \\
\dot{h} \\
\dot{x}
\end{pmatrix} = f_1(x) \\
\dot{x}_2 &= \begin{pmatrix}
\dot{\alpha} \\
\dot{V}
\end{pmatrix} = f_2(x) + B(x)u
\end{align*}
\]  
(4.35)

Applying the above method, we can solve a realistic problem.

In the dynamic inversion design [2], the number of the controlled variables (outputs) should not exceed that of the control variables. In the longitudinal control problem
for the F-18, this means that at most two state variables or two functions of the state can be controlled. The overall closed-loop stability then depends on the stability of the uncontrolled internal dynamics, referred to as the zero dynamics [15]. As mentioned in section 3.4, when controlling any two state variables, the zero dynamics of the F-18 at the given trim condition are always unstable (known as non-minimum-phase system). Hence more careful search for appropriate outputs is required before the dynamic inversion approach is applicable. On the other hand, the predictive control method does not have the same restriction since more state variables can be controlled. This gives the controller the possibility to stabilize even a non-minimum-phase system.

4.4 Approximate Receding-Horizon Control Design

In this section, nonlinear feedback control law for stabilization of the class of affine nonlinear systems is derived based on the concept of receding-horizon control. The receding-horizon optimal control problem is approximated by a multi-step predictive control formulation. This approximation leads to a quadratic programming problem (QPP). Under a controllability condition, the unique solution of the QPP exists and gives the closed-form analytical control law which makes on-line implementation of the receding-horizon control strategy possible. The output tracking problem is treated in a similar fashion and a closed-form tracking control law is presented in Chapter 7. The capability of the controllers to stabilize nonlinear system is addressed in Chapter 5 and 6. Output tracking control for non-minimum phase system is demonstrated in Chapter 7.

The concept of receding-horizon control has received considerable attention in recent years. In this strategy, at each time $t$ and state $x(t)$, the open-loop solution $u^*$ for an optimal control problem over a finite horizon $[t, t + T]$ is determined on-line. Then the current control $u(t)$ is set equal to $u^*(t)$. Continuing this process for all $t \geq 0$ gives a feedback control since $u^*(t)$ is dependent on $x(t)$, so is $u(t)$. The receding-horizon
optimal control problem is usually formulated as on with a quadratic performance index plus a terminal constraint \( x(t + T) \) [16].

We consider the nonlinear system

\[
\dot{x} = f(x) + G(x)u
\]  

(4.36)

where \( x \in X \subset \mathbb{R}^n \), and \( u \in \Omega \subset \mathbb{R}^m \). In general \( m \leq n \). The admissible sets \( X \) and \( \Omega \) are compact and \( X \times \Omega \) contains a neighborhood of the origin. Both \( f: X \to \mathbb{R}^n \) and \( G: X \to \mathbb{R}^{n \times m} \) are \( C^2 \), and \( f(0) = 0 \). In the receding-horizon control strategy, the following optimal control problem is solved at each \( t \geq 0 \) and \( x(t) \)

\[
\min_{u} J[x(t), t, u] = \min_{u} \frac{1}{2} \int_{t}^{t+T} [x^T(\tau)Qx(\tau) + u^T(\tau)Ru(\tau)]d\tau
\]  

(4.37)

subject to the state equations (4.36) and

\[
x(t + T) = 0
\]  

(4.38)

for some \( T > 0 \), where \( Q \) is positive semi-definite and \( R \) is positive definite. Denote the optimal control to the above problem by \( u^*(\tau), \tau \in [t, t + T] \). The currently applied control \( u(t) \) is set equal to \( u^*(t) \). This process is repeated for every next \( t \) for stabilization of the system at the origin.

Obviously, this strategy requires, at each time \( t \) and state \( x(t) \), the open-loop solution \( u^* \) for an optimal control problem over a finite horizon \([t, t + T]\) to be determined on-line. For a nonlinear system, this is not realistic because of the on-line computation load, particularly when the terminal constraint \( x(t + T) = 0 \) is imposed. To overcome this difficulty, various extensions of the receding-horizon control concept have been investigated. One of these is to construct closed-form approximate receding-horizon control laws for the class of continuous time, affine nonlinear systems.

In this strategy, we shall approximate the above receding-horizon control problem by the following multi-step-ahead predictive control formulation. Define \( h = T/N \) for some
integer $N \geq n/m$. Let $F(x) = \partial f(x)/\partial x$. Approximate $x(t + h)$ by a first-order Taylor series expansion at $x(t)$

$$
 x(t + h) \approx x(t) + h[f(x) + G(x)u(t)] \\
\triangleq x(t) + \Delta x \\
= [x(t) + hf(x)] + hG(x)u(t) \quad (4.39)
$$

Predict $x(t + 2h)$ by another first-order Taylor series expansion at $x(t + h)$, and then use (4.39) for the first-order expansion of $x(t + h)$ and $f[x(t + h)]$

$$
x(t + 2h) \approx x(t + h) + h \{f[x(t + h)] + G[x(t + h)]u(t + h)\} \\
\approx x(t) + \Delta x + h \{f(x) + F(x) \Delta x + G(x)u(t + h)\} \\
= [x(t) + h(\sum_{i=0}^{1}(1 + hF)^i)f(x)] + h(I + hF)G(x)u(t) + hG(x)u(t + h) \quad (4.40)
$$

where $G[x(t + \delta h)] \approx G[x(t)]$, and later $F[x(t + \delta h)] \approx F[x(t)]$, for $\delta t = h, 2h, \ldots, (N - 1)h$, are used to simplify the expansions and avoid the cross-product terms of $u(t + ih)$ with $u(t + jh)$.

Repeating this process to predict $x(t + 3h)$ and $x(t + 4h)$, we have

$$
x(t + 3h) \approx \left[ x(t) + h[\sum_{i=0}^{2}(1 + hF)^i]f(x) \right] \\
+ h(I + hF)^2G(x)u(t) + h(I + hF)G(x)u(t + h) + hG(x)u(t + 2h) \quad (4.41)
$$

and

$$
x(t + 4h) \approx \left[ x(t) + h[\sum_{i=0}^{3}(1 + hF)^i]f(x) \right] \\
+ h(I + hF)^3G(x)u(t) + h(I + hF)^2G(x)u(t + h) + h(I + hF)G(x)u(t + 2h) \\
+ hG(x)u(t + 3h) \quad (4.42)
$$

Hence, we obtain the general expression for $x(t + kh), 1 \leq k \leq N$

$$
x(t + kh) \approx \left[ x(t) + h[\sum_{i=0}^{k-1}(1 + hF)^i]f \right] + \left\{ \sum_{i=0}^{k-1}(1 + hF)^iGu[t + (k - 1 - i)h] \right\} \quad (4.43)
$$
where \( f, F, \) and \( G \) are all evaluated at \( x(t) \). Let \( L(\tau) = x^T(t + \tau)Qx(t + \tau) + u^T(t + \tau)Ru(t + \tau) \) for \( \tau \in [0, T] \). The integral in (4.37) may be approximated by the standard trapezoid formula

\[
J \approx J_t = \frac{T}{2N} [\frac{1}{2} L(0) + L(h) + \ldots + L((N - 1)h) + \frac{1}{2} L(Nh)]
\]

Next, replace \( x(t + kh), 1 \leq k \leq N, \) by (4.43) in the expression of \( L(kh) = x^T(t + kh)Qx(t + kh) + u^T(t + kh)Ru(t + kh) \). Then \( J \) in (4.44) becomes a quadratic function of an \((mN)\)-dimensional vector \( v = \text{col} \{ u(t), u(t + h), \ldots, u(t + (N - 1)h) \} \), assuming that \( x(t) \) is known (note that the optimal \( u(t + Nh) \) can be shown to be trivially zero from the following formulation, thus is excluded from \( v \)). With this substitution, we can rewrite (4.44) in the conventional quadratic form

\[
\tilde{J} = \frac{1}{2}v^T H(x)v + g^T(x)v + q(x)
\]

where \( H(x) \in R^{mN \times mN} \) is positive definite from the definition of (4.44). The terminal constraint (4.38), \( x(t + T) = 0 \), can also be expressed as a constraint on \( v \) with \( x(t + T) = x(t + Nh) \) approximated by (4.43)

\[
A^T(x)v = b(x)
\]
where

\[
A^T = [(I + hF)^{N-1}G, \ldots, (I + hF)G, G], \quad b = -\frac{1}{h}x - \left[\sum_{i=0}^{N-1} (I + hF)^i\right]f \quad (4.47)
\]

Now the receding-horizon optimal control problem is reduced to the problem of minimizing \(\hat{J}\) with respect to \(v\) subject to (4.46), which is a quadratic programming problem (QPP). If the linear system \(\dot{x} = F(x(t))x + G(x(t))u\) is controllable, then \(A^T \in R^{n \times mN}\) is of full rank, i.e., \(\rho(A^T) = n\) (note that \(mN \geq n\) by the choice of \(N\)) and inverse of matrix \(A\) exists. Thus, the closed-form optimal solution exists as follows

\[
v = -\left[H^{-1} - H^{-1}A(A^TH^{-1}A)^{-1}A^TH^{-1}\right]g + \left[H^{-1}A(A^TH^{-1}A)^{-1}\right]b \quad (4.48)
\]

An analytical, nonlinear feedback control law for \(u(t)\), denoted by \(\bar{u}(t; x, N)\) hereafter to signify its dependence on the state and the value of \(N\), is then defined by the first \(m\) equations in (4.48). The sufficient conditions for the existence of \(v\) are studied in the paper [11].

The approximation order \(N\) is an important influence factor on the performance of the approximate receding-horizon control law. The larger \(N\) (thus the smaller \(h\) is), the better an approximation \(\bar{u}(t; x, N)\) is to the original receding-horizon control, therefore the closer the performance comparison will be. Generally, for \(N \geq n/m\) where \(n\) is the total number of states and \(m\) is the total number of controls, the relative lower order \(N\) is enough to stabilize the general nonlinear systems. The stabilizing property of the approximate receding-horizon control law is easily established for linear systems. For nonlinear systems, however, a theoretical proof of stability seems quite difficult. However, numerical simulations provide the demonstrations for the promising capability of the control law in stabilizing nonlinear system.

Since \(L(Nh) = x^T(t + Nh)Qx(t + Nh) + u^T(t + Nh)Ru(t + Nh)\) is defined, the final state constraint condition \(x(t + T) = 0\) can make the state variable \(x(t + Nh)\) equal to zero. Also, from \(\frac{\partial L}{\partial u}(t + Nh) = 0\), we may find that control variable \(u(t + Nh)\) equal
to zero. Thus, \( L(Nh) \) term is excluded from the approximate performance index \( J_i \).

Hence, by setting \( J = \tilde{J} \), \( H \) matrix always has the following specific form, i.e:

\[
H_{N \times N} = \begin{bmatrix}
    H_{11} & H_{12} & \ldots & H_{1(N-1)} & 0 \\
    H_{21} & H_{22} & \ldots & H_{2(N-1)} & 0 \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    H_{(N-1)1} & H_{(N-1)2} & \ldots & 0 & 0 \\
    0 & 0 & \ldots & 0 & H_{NN}
\end{bmatrix}
\] (4.49)

This gives us systematic way to derive explicit formulas for closed-loop control law for different \( N \) value. Furthermore, based on careful observation of the above equations, we match the coefficient of states and controls in \( J \) to that in \( \tilde{J} \), it is easy to obtain explicit expression for \( H(x), g(x) \) and \( q(x) \) so that we can obtain explicit mathematical expression for close-loop control law \( v \).

For the convenience, here we provide the formulas for \( N = 1, \ N = 2 \). For the larger \( N \) value (\( N = 3, \ N = 4 \) and \( N = 5 \)), the derivation will be given in Appendix.

For \( N = 1 \)

\[
J_1 = \frac{k}{2}[\frac{1}{4}L(0)] = \frac{k}{4}L(0) \\
\tilde{J}_1 = \frac{k}{2}u^T(t)H_{11}u(t) + g_{1}^T(x)u(t) + q(x)
\] (4.50)

\[
\begin{cases}
    H_{11} = \frac{k}{2}R \\
    g_{1}^T(x) = zeros(1, m) \\
    q(x) = \frac{k}{4}x^T(t)Qx(t)
\end{cases}
\] (4.51)
For $N = 2$

\[
J_2 = \frac{1}{2} \left[ \frac{1}{2} L(0) + L(h) \right] = \frac{1}{4} L(0) + \frac{1}{2} L(h)
\]

\[
\tilde{J}_2 = \frac{1}{2} \left[ \begin{array}{c} u(t) \\ u(t + h) \end{array} \right]^T \left[ \begin{array}{cc} H_{11} & 0 \\ 0 & H_{22} \end{array} \right] \left[ \begin{array}{c} u(t) \\ u(t + h) \end{array} \right] + \left[ \begin{array}{c} g_1 \\ g_2 \end{array} \right]^T \left[ \begin{array}{c} u(t) \\ u(t + h) \end{array} \right] + q(x)
\]

\[
(4.52)
\]

\[
\Rightarrow
\]

\[
\begin{align*}
H_{11} &= \frac{1}{2} R + h \{hG(x)\}^T Q \{hG(x)\} \\
H_{22} &= h R \\
H_{12} &= \text{zeros}(m, m) \\
H_{21} &= H_{12} \\
g_1^T(x) &= h \{x(t) + hf(x)\}^T Q \{hG(x)\} \\
g_2^T(x) &= \text{zero}(1, m) \\
q(x) &= \frac{h}{4} x^T(t)Q x(t) + \frac{h}{2} \{x(t) + hf(x)\}^T Q \{x(t) + hf(x)\}
\end{align*}
\]

\[
(4.53)
\]

For $N = 3$

\[
J_3 = \frac{1}{2} \left[ \frac{1}{2} L(0) + L(h) + L(2h) \right] = \frac{1}{4} L(0) + \frac{1}{2} L(h) + \frac{1}{2} L(2h)
\]

\[
\tilde{J}_3 = \frac{1}{2} \left[ \begin{array}{c} u(t) \\ u(t + h) \\ u(t + 2h) \end{array} \right]^T \left[ \begin{array}{ccc} H_{11} & H_{12} & 0 \\ H_{21} & H_{22} & 0 \\ 0 & 0 & H_{33} \end{array} \right] \left[ \begin{array}{c} u(t) \\ u(t + h) \\ u(t + 2h) \end{array} \right] + \left[ \begin{array}{c} g_1 \\ g_2 \\ g_3 \end{array} \right]^T \left[ \begin{array}{c} u(t) \\ u(t + h) \\ u(t + 2h) \end{array} \right] + q(x)
\]

\[
(4.54)
\]

\[
\Rightarrow
\]
\[
\begin{align*}
H_{11} &= \frac{1}{2} R + h \{hG(x)\}^T Q \{hG(x)\} + h \{h(I + hF)G(x)\}^T Q \{h(I + hF)G(x)\} \\
H_{22} &= hR + h \{hG(x)\}^T Q \{hG(x)\} \\
H_{33} &= hR \\
H_{12} &= h \{h(I + hF)G(x)\}^T Q \{hG(x)\} \\
H_{21} &= H_{12} \\
H_{13} &= H_{31} = H_{23} = H_{32} = \text{zeros}(m, m) \\
\end{align*}
\]

\[
\begin{align*}
g_1^T(x) &= h [z(t) + h f(x)]^T Q \{hG(x)\} + h \left\{z(t) + h \left[\sum_{i=0}^{1} (I + hF)^i \right] f(x) \right\}^T Q \{h(I + hF)G(x)\} \\
g_2^T(x) &= h \left\{z(t) + h \left[\sum_{i=0}^{1} (I + hF)^i \right] f(x) \right\}^T Q \{hG(x)\} \\
g_3^T(x) &= \text{zeros}(1, m) \\
q(x) &= \frac{1}{2} x^T(t) Q x(t) + \frac{1}{2} [z(t) + h f(x)]^T Q [z(t) + h f(x)] \\
   &\quad + \frac{1}{2} \left\{z(t) + h \left[\sum_{i=0}^{1} (I + hF)^i \right] f(x) \right\}^T Q \left\{z(t) + h \left[\sum_{i=0}^{1} (I + hF)^i \right] f(x) \right\}
\end{align*}
\]

The above method works for \( f(0) = 0 \) case. In order to extend this method to \( f(0) \neq 0 \) case, we need to perform coordinate transformation.

For the nonlinear system

\[
\dot{x}(t)_{nx1} = f(x)_{nx1} + G(x)_{nxm} u(t)_{mx1}
\]

where \( x(0) = x_0, \dot{x}(0) = 0 \), and satisfies

\[
f(x_0) + g(x_0) u_0 = 0
\]

\[
\implies (x_0, u_0, t = 0) \text{ is equilibrium points}
\]

Redefine
\[ \begin{align*}
V &= u - u_0 \\
Z &= x - x_0
\end{align*} \] (4.58)

\[ \begin{align*}
\dot{z} &= \dot{x} = f(x) + G(x)u \\
&= f(z + x_0) + G(z + x_0)(v + u_0) \\
&= f(z + x_0) + G(z + x_0)u_0 + G(z + x_0)v
\end{align*} \] (4.59)

The newly transformed system is

\[ \dot{z} = f_{\text{new}}(z) + G_{\text{new}}(z)v \] (4.60)

where \( z(0) = 0, f_{\text{new}}(z = 0) = f(x_0) + G(x_0)u_0 = 0 \). Obviously, receding-horizon control method is applicable for this system. The corresponding transformation relation between \((z, v)\) and \((x, u)\) are

\[ \begin{align*}
z &= 0 & \Rightarrow & & x &= x_0 \\
v &= 0 & \Rightarrow & & u &= u_0
\end{align*} \] (4.61)

It is interesting to note that in the continuous-time, nonlinear predictive control methodology [9, 10], the one-step-ahead prediction of \( x(t + h) \) is obtained by expanding each component \( x_i(t + h) \) into an \( r_i \)-th order Taylor series, where \( r_i \) is the relative degree of \( x_i(t) \), and \( N \) is always equal to one. Another major difference is that no terminal constraint \( x(t + T) = 0 \), hence no controllability condition is explicitly required in the predictive control approach [10, 11]. As a consequence, neither of the approximate receding-horizon control and the predictive control can replace the other. They complement each other in that when one is not applicable, the other may work well in a particular problem.
CHAPTER 5 CONTROLLER PERFORMANCE FOR HEALTHY AIRCRAFT

The performance of the nonlinear predictive controller, the approximated nonlinear receding-horizon controller and the LQR controller for the longitudinal motion of a healthy F18 aircraft are compared in this chapter. The healthy aircraft in the following refers to the aircraft with normal horizontal stabilators (elevator), ailerons, rudders and throttle control, as opposed to engine-only flight control where only the throttle is the available control.

In this chapter, the simulations are performed based on the two flight conditions of the F18 aircraft in longitudinal motion to demonstrate the application of the controllers and to illustrate that non-intuitive results may be achieved when the system nonlinearity is appropriately taken into account. First of all, we consider the case that F18 aircraft flying at Mach = 0.5 and altitude = 10,000 feet; Secondly, we consider the case that F18 aircraft flying at Mach = 0.7 and altitude = 35,000 feet.

5.1 LQR Controller Performance

5.1.1 Mach = 0.5 case

A LQR controller (4.2) based on the linearized dynamics is designed where weighting matrices \( Q = \text{diag}\{1, 1, 1, 1, 1\} \) and \( R = \text{diag}\{0.01, 100\} \). The application of LQR controller gives the following close-loop linearized system.
The closed-loop system has the eigenvalues

\[ \lambda_{1,2} = -0.908332 \pm j2.446431 \]
\[ \lambda_{3,4} = -1.766698 \pm j0.9239787 \]
\[ \lambda_5 = -0.386554 \]  

The eigenvalues are located in the left half of the complex plane, showing that the system is closed-loop stable despite the open-loop instability. This means that the flight will return to the trim conditions if perturbed slightly.

For the designed LQR controller, the corresponding gain matrix \( K \) is listed in Table 5.1. By properly adjusting the weighting matrices, satisfactory pole positions can be achieved. This can be proved by calculating the eigenvalues of the closed-loop system equation. The control law has the following form

\[ \delta u = -K\delta x \]  

The control \( u = u_{trim} + \delta u \) is applied to the original nonlinear dynamic model for the F-18 aircraft. The simulation results for applying this control to both the linearized

<table>
<thead>
<tr>
<th>Table 5.1</th>
<th>LQR controller gain matrix for Mach number 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airspeed</td>
<td>Angle of Attack</td>
</tr>
<tr>
<td>4.3847e+02</td>
<td>4.6202e+03</td>
</tr>
<tr>
<td>2.7308e-01</td>
<td>1.4192e+02</td>
</tr>
</tbody>
</table>
model and the complete nonlinear model are shown in Figures 5.1 - 5.6. Initial perturbations off the trim condition are created at -5 ft/sec in velocity, +5 deg in angle of attack and 0.1 rad/sec in pitch rate.

Velocity initially decreases and then returns to the trim value quickly. Corresponding to the decrease of velocity, angle of attack initially increases a little bit and then vibrates to restore to its trim value eventually. Flight path angle restoring to zero value in about 7 seconds shows that the aircraft reaches its steady state with angle of attack equals to pitch angle. pitch rate approaches to its trim value in about 5 seconds.

Observations from the variations of the state variable time histories show that LQR technique is well designed to stabilize the linearized systems by providing good transient response performance. For nonlinear systems, even though LQR approach has the capability to stabilize them in the small region around the equilibrium points, the nonlinear system response for LQR technique is not as good as the linear one since LQR design neglects nonlinearity inside the physical systems when doing the controller design.

The plots for controls PLA and $\delta_e$ are given in Figures 5.5 and 5.6. Time delay exists between the PLA and shaped PLA due to the first order engine dynamics model (2.1). Figures 5.7 and 5.8 gives the time histories with the three different initial perturbations, showing the ability of the controller to stabilize this aircraft. The results show that the LQR control scheme performs well in stabilizing the aircraft even for the relative large initial errors in both the linearized model and the complete nonlinear model. In fact, tests indicate that the maximum size of perturbations accommodated by the controls within control limits, called stability region, under the linear control law is about ±25 ft/sec in velocity, ±8 deg in angle of attack and 0.2 rad/sec in pitch rate. Perturbations beyond this range will cause instability. Figures 5.9 and 5.10 give the time histories corresponding to the maximum region of stability. So the LQR type of technique for linear controller design is applicable to stabilize the linearized system, and the original nonlinear system when the aircraft has operational normal flight control system.
Figure 5.1  Mach = 0.5 velocity time history with LQR controller

Figure 5.2  Mach = 0.5 angle of attack time history with LQR controller
Figure 5.3  Mach = 0.5 flight path angle time history with LQR controller

Figure 5.4  Mach = 0.5 pitch rate time history with LQR controller
Figure 5.5  Mach = 0.5 throttle setting time history with LQR controller

Figure 5.6  Mach = 0.5 elevator deflection time history with LQR controller
Figure 5.7  Mach = 0.5 velocity time history with LQR controller

Figure 5.8  Mach = 0.5 angle of attack time history with LQR controller
Figure 5.9  Mach = 0.5 states time history with LQR controller

Figure 5.10  Mach = 0.5 throttle setting time history with LQR controller
5.1.2 Mach = 0.7 case

We now consider another straight and level flight condition at Mach number 0.7 and altitude 35,000 feet. Calculation shows that dynamic pressure of this case is reduced to 45 percent of the dynamic pressure at Mach number 0.5 and altitude 10,000 feet case. The increase of dynamic pressure due to the increase of velocity is not enough to compensate the decrease of dynamic pressure due to the increase of altitude. Thus, with the increasing of the altitude, the aerodynamic control effectiveness decreases and the engine performance decreases. We will follow a similar procedure as in Mach = 0.5 case to study the F-18 flight control performance.

A LQR controller based on the linearized dynamics is obtained by solving the algebra matrix Riccati equation. The controller gain matrix $K$ is obtained in Table 5.2. Substituting the controller into the linearized system equation, we obtain the closed-loop system under the LQR control law

$$A - BK = \begin{bmatrix}
-2.5767E - 1 & 3.0631E + 1 & -8.9735E + 1 & -4.6329E + 0 & -5.5910E - 2 \\
3.1155E - 4 & -2.1801E - 1 & -5.3515E - 1 & 9.2869E - 1 & -5.7116E - 4 \\
0 & 0 & 0 & 1.0000E + 0 & 0 \\
1.9533E - 2 & 2.2403E + 1 & -3.3100E + 1 & -4.3882E + 0 & -3.5303E - 2 \\
0 & -6.7765E + 2 & 6.7765E + 2 & 0 & 0
\end{bmatrix}$$

The closed-loop system has the eigenvalues

$$\begin{align*}
\lambda_{1,2} &= -0.82300 \pm 2.1653i \\
\lambda_{3,4} &= -1.4677e \pm 0.69690i \\
\lambda_5 &= -0.28242
\end{align*}$$

The eigenvalues show that the system is closed-loop stable despite the open-loop instability. This means that the linearized system has the ability to restore to its equi-
Table 5.2 LQR controller gain matrix for Mach number 0.7

<table>
<thead>
<tr>
<th>Airspeed</th>
<th>Angle of Attack</th>
<th>Pitch Angle</th>
<th>Pitch Angular Rate</th>
<th>Altitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1362e+02</td>
<td>-2.2504e+04</td>
<td>2.1670e+04</td>
<td>-1.8827e+02</td>
<td>1.6934e+01</td>
</tr>
<tr>
<td>1.6545e-01</td>
<td>2.2083e+02</td>
<td>-2.9233e+02</td>
<td>-3.6524e-01</td>
<td>-3.1166e-01</td>
</tr>
</tbody>
</table>

The control $u = u_{trim} + \delta u$ is applied to the nonlinear dynamic model for F18. The simulation results for applying this control to both the linearized model and the complete nonlinear model are shown from Figures 5.11 - 5.16. Perturbations are initialized at -5 ft/sec in velocity, +5 deg in angle of attack and 0.05 rad/sec in pitch rate.

The variations of the time histories of state variables are shown in Figures 5.11 to 5.14. The controls $PLA$ and $\delta_e$ are plotted in Figures 5.15 and 5.16. The capability of the LQR control scheme to stabilize the F18 model are also shown in Figure 5.17, 5.18 by offering three different initial perturbations. The simulation results show the controller perform well to stabilize the F18 aircraft.

As expected, the transient response corresponding to the nonlinear model performs relatively large overshoot and oscillatory, showing the nonlinear model has worse response performance than the linear one. This is remarkable because of the neglection of the system nonlinearity inside the LQR controller design.

Further test the stability region under this linear control law by increasing the values of initial perturbation within the control limits, the results show that the LQR control scheme performs quite well in stabilizing the aircraft even for the relative large initial errors in the linearized model. For the nonlinear model, the size of the stability is about $\pm 15$ ft/sec in velocity, $\pm 5$ deg in angle of attack and 0.1 rad/sec in pitch rate. Perturbations beyond this range will cause either the system unstable. The stability region greatly decrease comparing to the first case due to the limit of control authority. Figure 5.19, 5.20 give the time histories corresponding to the maximum region of stability.
Figure 5.11  Mach = 0.7 velocity time history with LQR controller

Figure 5.12  Mach = 0.7 angle of attack time history with LQR controller
Figure 5.13  Mach = 0.7 flight path angle time history with LQR controller

Figure 5.14  Mach = 0.7 pitch rate time history with LQR controller
Figure 5.15  Mach = 0.7 throttle setting time history with LQR controller

Figure 5.16  Mach = 0.7 elevator deflection time history with LQR controller
Figure 5.17  Mach = 0.7 velocity time history with LQR controller

Figure 5.18  Mach = 0.7 angle of attack time history with LQR controller
Figure 5.19  Mach = 0.7 states time history with LQR controller

Figure 5.20  Mach = 0.7 throttle setting time history with LQR controller
5.2 Nonlinear Predictive Controller

Now we apply the nonlinear predictive control method to stabilize the F-18 aircraft. As mentioned before, nonlinear control methods are much more complex than linear ones since they take the nonlinearity of the system dynamics into account. Precisely because of this reason, though, a nonlinear controller can offer better performance than a linear controller, or stabilize the system in a larger region.

The nonlinear predictive control law follows directly from Eq. (4.30). The controller parameters are chosen to be $Q_1 = \text{diag}(1, 0)$, $Q_2 = \text{diag}(1, 1, 1)$, $R = 0$, and $h = 1 \text{ sec}$.

5.2.1 Mach = 0.5 case

In order to examine the closed-loop stability theoretically, we linearize the closed-loop system around the trim points of the aircraft. The closed-loop stability under predictive controller can be verified by examining the eigenvalues of the linearized closed-loop system dynamics which in this case are

$$\begin{align*}
\lambda_{1,2} &= -0.85704 \pm j1.36572 \\
\lambda_3 &= -0.50017 \\
\lambda_4 &= -0.07252 \\
\lambda_5 &= -0.0027
\end{align*}$$

(5.6)

All the eigenvalues of the closed-loop system corresponding to the desired nonlinear control law are located in the left half of the complex plane. The closed-loop system is stable at the trim point.

Because the system is controllable, the poles of the above closed-loop system with the complex poles appearing in conjugate pair can be placed by arbitrary pole-placement algorithm based on the linearized system model. The control law has the form $\bar{u} = k_{\text{mxn}} \bar{x}$. Due to the fact that the system has different zeros, the gain matrix $k$ is not
Table 5.3  Pole-placement gain matrix for Mach number 0.5

<table>
<thead>
<tr>
<th>Airspeed</th>
<th>Angle of Attack</th>
<th>Pitch Angle</th>
<th>Pitch Angular Rate</th>
<th>Altitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5288e-02</td>
<td>-7.0140e+00</td>
<td>7.7595e+00</td>
<td>-1.1721e+00</td>
<td>4.8277e-03</td>
</tr>
<tr>
<td>2.1758e-01</td>
<td>-2.1609e+01</td>
<td>1.4455e+01</td>
<td>5.3602e+00</td>
<td>1.3906e-02</td>
</tr>
</tbody>
</table>

unique and the different $k_{mn}$ will result in different response despite that the closed-loop poles are the same. The gains corresponding to a better response are listed in Table 5.3.

The calculation results show that it takes much longer time for pole-placement method to stabilize this system, roughly about five thousands second, although it eventually drives the system back to be the trim condition. Furthermore, we apply the control $u = u_{trim} + \ddot{u}$ to the original nonlinear dynamic model for the F-18 aircraft. The simulation results for applying both pole-placement method and nonlinear predictive method to the complete nonlinear model are shown in Figure 5.21 - 5.22. Initial perturbations off the trim condition are created at -5 ft/sec in velocity, +5 deg in angle of attack and 0.1 rad/sec in pitch rate.

Obviously, for the given closed-loop poles, the nonlinear predictive controller can stabilize the system within about ten seconds. However, the pole-placement method has no ability to stabilize this system within the same time. Meanwhile, the system response under the pole-placement method is very slow with high overshot. The velocity response is much sluggish comparing to the other state response.

Now, the same initial perturbations used for the LQR controller are added to the system dynamics under the predictive controller to demonstrate the controller performance. Figures 5.23 to 5.26 show the variations of time histories of state variables. The time histories of the $PLA$ command and actual response $PLA'$, and the stabilator deflection are given in Figure 5.27 and Figure 5.28. Figures 5.29 and 5.30 give the time histories with the different initial perturbations. Also plotted in these figures are the
corresponding variations of states and controls under the LQR control law.

The control parameter $h$ is an important factor for the system response. The controller gain is proportional to $1/h$. For smaller $h$ value, the system response is faster with high peak value; for larger $h$ value, the system response is slower and it takes more time to reach its steady state. For applications, we may treat the choice of the value $h$ as a control parameter. On the other hand, the choices of the weighting matrix $Q$ and $R$ are not critical to the system response for this particular problem. Theoretically, larger $R$ values require less control effort and smaller $R$ values require more control effort. Generally, it is good to choose the unit matrix for matrices $Q$ and $R$.

Simulation results provided above show that the nonlinear system for the given perturbations under the nonlinear control law is stable. Figures 5.31 and 5.32 show the region of stability that the nonlinear predictive controller can accommodate is found to be about the same as that of the LQR controller, which is rather remarkable for the LQR controller, given its simple linear form. But clearly, the performance of the nonlinear predictive controller is much better than that of the LQR controller. The response is generally much less oscillatory, faster, and has smaller overshoot.

It should be noted that the dynamic inversion method also leads to nonlinear feedback control laws for the two controls ($\delta_e, PLA$). But in this case if any two of the five state variables ($V, \alpha, \theta, q, z$) of the F-18 are used as the controlled outputs for the control law design, the system is always non-minimum-phase. This can be verified by examining the transmission zeros of the transfer matrix of the linearized open-loop dynamics in any given combination, at least one of the transmission zeros lies in the right-half of the complex plane. By Ref [14], the zero dynamics of the nonlinear system coincide with that of the linearized system. Hence the aircraft cannot be stabilized using any two state variables as the outputs and the dynamic inversion control laws at this trim condition will not work as well.
Figure 5.21  Mach = 0.5 velocity with pole-placement

Figure 5.22  Mach = 0.5 angle of attack with pole-placement
Figure 5.23  Mach = 0.5 velocity with predictive controller

Figure 5.24  Mach = 0.5 angle of attack with predictive controller
Figure 5.25  Mach = 0.5 flight path angle with predictive controller

Figure 5.26  Mach = 0.5 pitch rate with predictive controller
Figure 5.27  Mach = 0.5 throttle setting with predictive controller

Figure 5.28  Mach = 0.5 elevator deflection with predictive controller
Figure 5.29  Mach = 0.5 velocity with predictive controller

Figure 5.30  Mach = 0.5 angle of attack with predictive controller
Figure 5.31  Mach = 0.5 state variables with predictive controller

Figure 5.32  Mach = 0.5 throttle setting with predictive controller
5.2.2 Mach = 0.7 case

The ability of the nonlinear predictive controller to stabilize the F-18 aircraft can be further tested by considering the case where Mach = 0.7 and altitude = 35,000 feet. As noted before, this is a more challenging flight condition because it involves a lower dynamic pressure and less available engine thrust.

The initial perturbations are the same as those used for the LQR controller. Simulation results of state variables are given in Figures 5.33 - 5.36. Figure 5.37 contains the time history of the PLA command and actual response $PLA'$, and Figure 5.38 gives the stabilator deflection time history. Again, the comparison to the LQR controller is provided in these figures.

The simulation results for this case bear the same performance as that of Mach 0.5 case. Figures 5.39 and 5.40 give the time histories under the different initial perturbations. The maximum size of perturbations, also called stability region defined in the previous section for the nonlinear controller is found to be about the same as that of the LQR controller at the same condition. The size is smaller than that in the case of Mach 0.5 because of less aerodynamic pressure and decreased engine thrust as a result of the increased altitude.

Figures 5.41 and 5.42 show the variations of the time histories of velocity and angle of attack with initial perturbation of -5 ft/sec in velocity, +5 deg in angle of attack and 2.86 deg/sec in pitch rate corresponding to the different $h$ values. The controller performance can be improved by adjusting the value of $h$ to achieve satisfactory response quality.

From the simulation results, it is clear that while both LQR and nonlinear predictive control law stabilize the aircraft in about the same region, the performance of the predictive control law is superior to the LQR control law. This is because the design of predictive control law incorporates the system nonlinearity.
Figure 5.33  Mach = 0.7 velocity with predictive controller

Figure 5.34  Mach = 0.7 angle of attack with predictive controller
Figure 5.35  Mach = 0.7 flight path angle with predictive controller

Figure 5.36  Mach = 0.7 pitch rate with predictive controller
Figure 5.37  Mach = 0.7 throttle setting with predictive controller

Figure 5.38  Mach = 0.7 elevator deflection with predictive controller
Figure 5.39  Mach = 0.7 velocity with predictive controller

Figure 5.40  Mach = 0.7 angle of attack with predictive controller
Figure 5.41  Mach = 0.7 state variables with predictive controller

Figure 5.42  Mach = 0.7 throttle setting with predictive controller
5.3 Approximate Nonlinear Receding-Horizon Controller

The performance of the LQR controller and nonlinear predictive controller has been studied in the previous section. In this section, we will apply the approximate nonlinear receding-horizon controller to the given F18 aircraft and compare the controller performance.

The approximated nonlinear receding-horizon control law follows directly from Eq. (4.48). The controller parameter $N$ is chosen equal to 3 and the step size $h$ is chosen to be 1.

5.3.1 Mach = 0.5 case

The closed-loop stability under the nonlinear control law can be verified by examining eigenvalues of the linearized closed-loop dynamics which are

$$
\begin{align*}
\lambda_{1,2} &= -1.35704 \pm j1.36572 \\
\lambda_3 &= -0.80017 \\
\lambda_4 &= -0.7452 \\
\lambda_5 &= -0.5647
\end{align*}
$$

The same initial perturbations used for the LQR controller and nonlinear predictive controller are added to the F18 model to demonstrate the performance of the approximate receding-horizon controller.

The variations of time histories of states are given in Figures 5.43 - 5.46. Figure 5.47 presents the time histories of the power level angle $PL_A$ command and actual response $PL_A'$, and Figure 5.48 gives the stabilator deflection time history. Figures 5.49 and 5.50 give the time response corresponding to three different initial perturbation in velocity and angle of attack, showing the capability of the approximate receding-horizon controller to stabilize the F18 model. Also, the variations of state and control under the LQR control law and the predictive control are plotted in these figures.
It is very interesting to note that the time response under the approximate receding-horizon control law is smooth, faster, less overshoot and oscillation. It further shows that the approximate receding-horizon controller offers the better response performance than both of the LQR controller and the predictive controller while providing good stability performance. Also, by initializing the same values of perturbations, stabilizing the F18 aircraft requires less control authority than that of the other two controllers. This is rather remarkable and is what we expect.

While the performance of the approximate receding-horizon control law may depend on the value of $N$, the closed-loop stability does not necessarily require a large $N$ to achieve. In fact, for our study, $N$ chosen to be 3 is enough for controller to offer satisfactory system performance. Very important tests also show that the stability region of the approximate receding-horizon depends on both step size and the choice approximation order. Generally, the larger $N$ (thus the smaller $h$ is), the better an approximation $\tilde{u}(t; x, N)$ is to the original receding-horizon control, therefore the closer the performance comparison will be. The higher order $N$ is, the smaller value $h$ is, and relatively better response performance can be expected. On the other hand, the step size is another important factor for the system stability. The satisfied flight quality can be achieved based on the appropriate adjustment of the step size.

The approximate receding-horizon controller offers comparable, or slightly better performance when compared with the predictive control law. It is very reasonable since the predictive control is just one-step time ahead predictive method. However, the approximate receding-horizon control is multi-step time ahead predictive method with the satisfactory of the performance index. There are a lot of complex work inside the controller design. The minimum of the performance index plus the enforced the terminal constraint condition limit unexpected increasing of the state variables. As the function of the states, generally, the unexpected increasing of control variables are also limited.
Figure 5.43  Mach = 0.5 velocity with receding-horizon controller

Figure 5.44  Mach = 0.5 angle of attack with receding-horizon controller
Figure 5.45  Mach = 0.5 flight path angle with receding-horizon controller

Figure 5.46  Mach = 0.5 pitch rate with receding-horizon controller
Figure 5.47  Mach = 0.5 throttle setting with receding-horizon controller

Figure 5.48  Mach = 0.5 elevator deflection with receding-horizon controller
Figure 5.49 Mach = 0.5 velocity with receding-horizon controller

Figure 5.50 Mach = 0.5 angle of attack with receding-horizon controller
5.3.2 Mach = 0.7 case

For Mach number is 0.7 and altitude is 35,000 feet case, the initial perturbations are chosen with +5 deg in angle of attack, +2.87 deg/sec in pitch rate. The receding-horizon controller with order 3 applied to the F18 model to demonstrate the controller performance.

Figures 5.51 - 5.54 show the variations of time histories of velocity, angle of attack, pitch angle, pitch rate and flight path angle. Figure 5.55 contains the time history of the power level angle $PLA$ command and actual response $PLA'$, and Figure 5.56 gives the stabilator deflection.

It is noticed from the Figures that for the same size of perturbation in velocity, angle of attack and pitch rate, the response curve corresponding to the approximate receding-horizon controller has less overshoot and oscillation than that of the LQR controller and the predictive controller. The pitch rate response curve share the same characteristics. Once again, the simulation results shows that compared to the other two controllers, the approximate receding-horizon controller performs well to stabilize the F18 aircraft. Obviously, for normal flight condition, nonlinear control system approach has advantages over the linear one.

The approximate receding-horizon control law still work well for this case. But with the smaller sizes of the initial perturbation than that of the Mach number 0.5 and altitude 10,000 feet case, stabilizing the F18 aircraft requires large control effort. This can be seen in Figure 5.55. Since the engine authority is limited a lot, the engine throttle upper-bound decreases due to the increase of Mach number and altitude. The control surface can easily reach its limits and cause the engine saturation at this point even under small initial perturbation. Elevator response also become worse with large oscillation compared to Mach 0.5 and altitude 10,000 feet case. This is the reason why the stability region for this case is remarkably reduced.
Figure 5.51  Mach = 0.7 velocity with receding-horizon controller

Figure 5.52  Mach = 0.7 angle of attack with receding-horizon controller
Figure 5.53 Mach = 0.7 flight path angle with receding-horizon controller

Figure 5.54 Mach = 0.7 pitch rate with receding-horizon controller
Figure 5.55  Mach = 0.7 throttle setting with receding-horizon controller

Figure 5.56  Mach = 0.7 elevator deflection with receding-horizon controller
CHAPTER 6 CONTROLLER PERFORMANCE FOR UNHEALTHY AIRCRAFT

In the preceding section, we have demonstrated the performance of the LQR controller, pole-placement algorithm, the nonlinear predictive controller and the approximate receding-horizon controller in normal, less challenging, flight conditions. In this section, we investigate engine-only flight control for the F-18. We assume that the F-18 is flying with the stabilators locked in the trimmed positions. The only control available is the throttle $PLA$.

6.1 Propulsion Controlled Aircraft (PCA)

The concept of using only engine thrust as emergency backup flight control for aircraft, is known as the propulsion controlled aircraft (PCA). A number of nonlinear behaviors have been observed during PCA flight tests. Compared to the conventional flight control surfaces, the engines respond to commands slowly and have limited control effectiveness. Hence, the ability of the system promptly respond to flight condition changes is limited. Consequently, many nonlinear effects, which are easily accommodated by a conventional flight control system, become significant issues in the design of an effective controller when the engines are used as the only control effectors.

Because of the loss of the primary attitude control effector (stabilator) in this case, and the fact that the engine has rather limited control authority on any state other than the airspeed, nonlinearities in the system which would be well accommodated by
the normal flight control system thus not influential to the performance now become prominent factors. Indeed, despite that a stabilizing LQR engine-only control law can still be designed for the linearized F-18 dynamics Eq. (3.12), simulations show that the stability region of the closed-loop system with the nonlinear F-18 dynamics for Mach number is 0.5 and altitude 10,000 feet case is extremely small because of the strong nonlinearity. The aircraft becomes unstable even for very small perturbations in the state away from the trim condition. This means that the system has no ability to restore to its trim condition. In other words, the linear engine-only controller would practically fail to stabilize the aircraft in the event when the stabilator becomes inoperative at the trim condition considered.

In fact, we may design the linear propulsion controller based on the fact that there is only one control available. For example, an LQR PCA controller gives the following linearized closed-loop system matrix

\[
A - BK = \begin{bmatrix}
-2.1649E - 5 & -1.0076E + 0 & 0 & 9.9086E - 1 & -4.1579E - 8 \\
0 & 0 & 0 & 1.0000E + 0 & 0 \\
-1.7297E - 4 & -3.5226E + 0 & 0 & -4.1341E - 1 & -2.4675E - 7 \\
0 & -5.5157E + 2 & 5.5157E + 1 & 0 & 0
\end{bmatrix}
\] (6.1)

The closed-loop system eigenvalues with one controller are

\[
\begin{align*}
\lambda_{1,2} &= -0.710716 \pm j1.84489 \\
\lambda_{3,4} &= -0.026165 \pm j0.34489 \\
\lambda_5 &= -0.00507586752
\end{align*}
\] (6.2)

It is seen that the linearized system is stable. But even with small initial perturbations of \(\delta V = -1 \text{ ft/sec}, \delta \alpha = 1 \text{ deg} \) and \(\delta q = 0.01 \text{ deg/sec}\), simulating with nonlinear
F18 model shows that the aircraft becomes unstable. So the linear propulsion controller doesn’t work well.

On the other hand, the nonlinear predictive controller designed for the engine is still capable of stabilizing the aircraft. The controller is still designed by following the procedure in Section 4.3. this time only for the PLA. It should be noted that better performance could be achieved if the PLA controller parameters are readjusted for the engine-only case. But we deliberately used the same parameters to emulate the realistic situation in which it would not be possible to readjust the engine controller parameters in time should a complete failure of the stabilator occur in flight. Under this nonlinear control law, the linearized closed-loop dynamics at the trim point have the poles

\[
\begin{align*}
\lambda_{1,2} &= -0.703879 \pm j1.84753 \\
\lambda_3 &= -0.49209 \\
\lambda_4 &= -0.0217 \\
\lambda_5 &= -0.0007338
\end{align*}
\] (6.3)

Note that the pair of the complex poles are very close to those of the open-loop dynamics in Eq. (3.15), which represents the so-called short-period mode in flight mechanics. This is because this mode primarily reflects rapid changes in angle of attack $\alpha$ and pitch angle $\theta$, and is almost uncontrollable by engine only. Thus any state-feedback control law for the throttle can barely change them.

### 6.2 Simulation Results

Figure 6.1 shows the time histories of the state variables with the same initial perturbations of $\delta V = -5$ ft/sec, $\delta \alpha = 5$ deg and $\delta q = 2.87$ deg/sec to the F-18. Figure 6.2 presents the time history comparison of angle of attack between normal flight control and PCA system control with the same initial perturbations. The normal flight control system exhibits the better response performance than the PCA system.
Figure 6.3 shows the variation of velocity time history. Velocity time response is pretty good. It restores to its trim value in about six seconds. Figure 6.4 illustrates variations of the commanded PLA and response PLA'. The same capability exists for the PLA command. To avoid the control throttle saturation, the appropriate weighting matrix should be chosen here. For our case, a large control weighting matrix R is chosen to be 100.

It is clear that the aircraft remains stabilized at the trim point, but the aircraft response, particularly in the pitch, is much more sluggish as compared to the response of the healthy aircraft. This comes as no surprise, given the loss of the use of the primary pitch control effector (stabilator). However in situations like this the foremost concern is not the performance, but stabilization of the aircraft with the only remaining control - the engines. The nonlinear predictive controller is able to accomplish this objective. The stability region in this case is about the same size as that of the healthy aircraft under the two controls $\delta_e$ and PLA. This is quite impressive, given that now the stabilator is inoperative and the linear controller cannot stabilize the aircraft.

It is worth to mentioning that the approximate receding-horizon control method shows the difficulty of the F18 aircraft control in propulsion-controlled aircraft (PCA) system. When the throttle is the only available control, the $\hat{B}$ matrix in Jordan form is reduced to contain just the first column of the original $\hat{B}$ matrix in Eqn. (3.19). We see the coefficient corresponding to the first mode is very small. This means that this mode is less influenced by the control. Although at this time the system controllability matrix is still full rank, the determinant of the matrix is pretty small. Therefore, the controllability condition is not satisfied. The receding-horizon control law does not exist due to the singularity. A valuable understanding is that the PCA control problem is really challenging, predictive control approach shows its ability to control both the healthy aircraft and the unhealthy aircraft.
Figure 6.1  Mach = 0.5 state variables with engine-only predictive controller

Figure 6.2  Mach = 0.5 state variables with engine-only predictive controller
Figure 6.3 Mach = 0.5 velocity variation with engine-only predictive controller

Figure 6.4 Mach = 0.5 throttle setting with engine-only predictive controller
CHAPTER 7 OUTPUT TRACKING PERFORMANCE

In this chapter, approximate receding-horizon control technique developed in Chapter 4 will be extended to the output-tracking problem. A closed-form output tracking control law can be obtained in a similar fashion. The desired outputs may be chosen as constant or function of time. The capability of the controller to stabilize nonlinear system and realize the output tracking control for non-minimum phase system is demonstrated by application of the control law to the F18 model.

7.1 Introduction

Generally, the primary tasks of a control system can be divided into two categories: stabilization and tracking. In stabilization problems, a control system, called a stabilizer is to be designed so that the states of the closed-loop system will be stabilized around an equilibrium point. This has been discussed in the previous chapters. In tracking control problems, discussed in this chapter, the design objective is to construct a controller, called a tracker, so that the system output tracks a given time-varying trajectory. Problems such as making an aircraft fly along a specified path or at special tracking control tasks may be very typical situations.

The output tracking problem is a very important topic in control system design. Normally, tracking problems are more difficult to solve than stabilization problems, because in tracking problems the controller should not only keep the whole state stabilized but also drive the system output toward the desired output.
As mentioned in Chapter 3, the linearized F18 aircraft system is a non-minimum phase system, i.e: the system internal dynamics are unstable. Therefore, the nonlinear system is also a non-minimum phase system. The control of non-minimum phase systems is a topic of active current research since the controller must bear the capability to redefine the output function so that the resulting zero-dynamics is stable. The control law derived based on the dynamic inversion cannot be applied to this system [14]. On the other hand, for output tracking problems, the nonlinear predictive control law becomes a special case of the dynamics inversion control law [9, 10], thus is not applicable either to non-minimum phase system. This is verified by numerical simulation.

Design of output tracking controllers for non-minimum phase nonlinear system is highly challenging. Stable inversion is a newly designed control strategy to solve the output tracking problem for a class of non-minimum phase nonlinear system with smooth dynamics and affine in control input [17]. In this strategy, given any smooth reference output trajectory $y_d$ with compact support, find a bounded control input $u_d$ and a bounded state trajectory $x_d$ such that $u_d \to 0$ and $x_d \to 0$ as $t \to \pm \infty$ and their image by the input/output map of the control system is exactly $y_d$. The pair $(x_d, u_d)$ is the stable inverse solution for a given reference output $y_d$. The stable inverse pair can be obtained by solving the two-point boundary value problem.

Stable inversion based design requires to know the prescribed output tracking trajectory. For a large number of problems of interest associated in aerospace area, the sufficient smooth function of reference output trajectory $y_d$ cannot be known beforehand. Thus, approximate receding-horizon control technique provide a promising design method for output tracking control of the non-minimum phase system. It doesn’t impose restrict limit on the desired trajectory. The control problem can be realized by simply solving initial value problems. Numerical simulation results have proved that the multi-step predictive method, i.e: the nonlinear receding-horizon control technique can efficiently implement the desired output tracking control for a class of non-minimum
phase nonlinear system.

In the next section, we will address the principle of the method. The tracking performance analysis will be demonstrated in the section 7.3.

### 7.2 Output-Tracking Controller

In the output tracking problem, suppose that, in addition to the nonlinear state equation \( (4.36) \), the output of the system is defined by the nonlinear relationship

\[
y = c(x)
\]  

where \( c : R^n \rightarrow R^m \) is sufficiently differentiable. The desired output is specified by a given function of \( C^\infty, y_d(t) \in R^m \) for \( t \geq 0 \). The following receding-horizon problem may be set up for providing the output-tracking control

\[
\min_u J[x(t), t, u] = \min_u \frac{1}{2} \int_t^{t+T} [e^T(\tau)Qe(\tau) + u^T(\tau)Ru(\tau)] d\tau
\]  

subject to the state equations \( (7.2) \) and

\[
e(t + T) = 0
\]  

where \( e(t) = y(t) - y_d(t) \). The current control \( u(t) \) is then set equal to \( u^*(t) \), the current value of the optimal open-loop control of the above problem.

For nonlinear system, the major difficulty for implementation of the receding-horizon control strategy lies in obtaining the optimal control \( u^* \) on-line, particularly when the state constraint \( e(t + T) = 0 \) is imposed because of heavy on-line computation demands. The approximate receding-horizon control methodology can overcome the above difficulty.

A similar approximation approach can be taken to obtain a closed-loop control law for the above receding-horizon optimal control problem [11]. For \( h = T/N \), the output
$y(t + kh)$ is approximated by the first-order Taylor series expansion

$$y(t + kh) \approx y(t) + C[x(t)]x(t + kh) - x(t), 1 \leq k \leq N$$ (7.4)

where $C = \partial c(x)/\partial x$. The previously obtained expression (7.2) is then used for $x(t + kh)$. The desired output $yd(t + kh)$ is predicted similarly by recursive first-order Taylor series expansions

$$yd(t + h) \approx yd(t) + hyd(t)$$ (7.5)

$$yd(t + 2h) \approx yd(t + h) + hyd(t + h)$$

$$\approx yd(t + h) + h[\dot{yd}(t) + h\ddot{yd}(t)]$$ (7.7)

$$\approx yd(t) + hyd(t) + h[\dot{y} - d(t) + \ddot{yd}(t)]$$ (7.8)

where another first-order expansion $\dot{yd}(t) \approx \dot{yd}(t) + h\ddot{yd}(t)$ and (7.5) are used in arriving at (7.8). Continuing this process, we have for $yd(t + kh)$

$$yd(t + kh) \approx yd(t) + h \left[ \sum_{i=0}^{k-1} (l + hp)pyd(t) \right]$$ (7.9)

where $p = d/dt$ is the differentiation operator. Combining the predictions of $y(t + kh)$ and $yd(t + kh)$, we obtain the prediction of the tracking error

$$e(t + kh) \approx y(t + kh) - yd(t + kh) \approx e(t) + h \left\{ \sum_{i=0}^{k-1} [C(I + hF')f] \right\} + h \left\{ \sum_{i=0}^{k-1} [C(I + hF')G[u(t + (k - 1 - i)h) - (1 + hp)pyd(t)] \right\}$$ (7.10)

Now with the cost function (7.2) approximated by the trapezoidal or Simpson’s rule, it can be written as a quadratic function of $v = col(u(t), u(t + h), \ldots, u[t + (N - 1)h]$, similar to Eqn. (4.45)

$$\bar{J}_{out} = \frac{1}{2}v^TH_0(x)v + g_0^T(x)v + g_0(e, x, yd)$$ (7.11)

The constraint (7.3) is then expressed as $e(t + Nh) = 0$ which leads to

$$M^T(x)v = d(e, x, yd)$$ (7.12)
where

\[ M^T = C \left[ (I + hF)^{-1}G, \ldots, (I + hF)^{N-1}G \right] \] (7.13)

\[ d = -\frac{1}{r} e - \left[ \sum_{i=0}^{N-1} C(I + hF)^i f - (1 + hp)^i p y_d(t) \right] \] (7.14)

Following a similar argument as in Section 4.4, if the linear system \( \dot{\xi} = F(x(t))\xi + G(x(t))u \) is controllable, then the matrix M is full rank, the inverse of M exists and the QPP has one solution given by

\[ v = -\left[ H_0^{-1} - H_0^{-1} M(M^T H_0^{-1} M)^{-1} A^T H_0^{-1} \right] g_0 + \left[ H_0^{-1} M(M^T H_0^{-1} M)^{-1} \right] d \] (7.15)

The first \( m \) equations in (7.15) give a closed-loop output-tracking control law \( \bar{u}_{out}(t; x, N) \). The sufficient conditions for the existence of \( v \), thus the existence of \( \bar{u}_{out}(t; x, N) \) can be proved by further theoretical derivation [11].

### 7.3 Output Tracking Performance

In applying the above output tracking controller to F-18 aircraft in longitudinal motion, we need to make the standard assumption that the number of outputs is equal or less than that of the controllers.

We choose climb rate and angle of attack as the outputs, where climb rate is defined to be: \( \dot{h} = V \sin (\theta - \alpha) \), \( V \) is velocity, \( \theta \) is pitch angle and \( \alpha \) is angle of attack. System analysis demonstrates that the corresponding nonlinear system is a non-minimum phase system. In fact, the F-18 dynamic are always the non-minimum phase no matter which two state variables we choose as the outputs. This definitely precludes the application of both dynamics inversion and predictive control techniques. We expect that the approximate receding-horizon control applied to the F18 model can compensate this shortcoming. Assume that healthy aircraft with the throttle and elevator as controls. We design the F18 model to track constant climb rate commands, and at the same time, keep velocity, angle of attack and pitch rate maintaining at the trim values.
7.3.1 Mach = 0.5 case

First of all, let's consider F18 aircraft flying at the trim condition of Mach number 0.5 and altitude 10,000 feet. The desired output tracking performance are designed separately based on +10 ft/sec, +20 ft/sec and +30 ft/sec constant climb rate commands with velocity, angle of attack and pitch rate keeping at the trim condition values. Figure 7.1 shows that the F18 model respond well to three different climb rate commands. The approximate receding-horizon control law provides good output-tracking performance to stabilize the aircraft and to realize the output tracking control.

The variations of the state variables are exhibited in Figures 7.2 - 7.4. The trajectories are smooth and stable. While the output exactly follow the constant climb rate, all the state variables except for pitch angle $\theta$, are kept at the trim values. To maintain the constant climb rate, pitch angle $\theta$ is re-trimmed at a new trim value. The overall closed-loop system is clearly stable to track the desired trajectory, despite that the system is non-minimum phase.

It is all the interesting to see that while system satisfies the desired tracking performance, all the control variables are within their limits, no saturation happens. The plots of control variable time histories are shown in Figure 7.5 and Figure 7.6. After 5 seconds, the control variables approach the new constant value corresponding to the desired climb rate.

Figures 7.7 and 7.8 show the influence of control parameter $h$ with the different approximation order, $h$ varys from 0.8, 1.0 to 1.3. The larger value the $h$ is, the slower time response; the smaller $h$ value is, the faster the system response is.

The simulation results show that the approximate receding-horizon control methodology can perform good output tracking performance for the non-minimum phase system even for the case where both dynamics inversion and predictive control method show their difficulties to realize the system output tracking control.
Figure 7.1  Mach = 0.5 climb rate with receding-horizon controller

Figure 7.2  Mach = 0.5 angle of attack with receding-horizon controller
Figure 7.3 Mach = 0.5 pitch angle with receding-horizon controller

Figure 7.4 Mach = 0.5 pitch rate with receding-horizon controller
Figure 7.5  Mach = 0.5 throttle setting for output tracking control

Figure 7.6  Mach = 0.5 elevator deflection for output tracking control
Figure 7.7  Mach = 0.5 pitch rate for parameter h influence

Figure 7.8  Mach = 0.5 flight path angle for parameter h influence
7.3.2 Mach = 0.7 case

Secondly, let's consider the F18 aircraft flying at a trim condition of Mach number of 0.7 and an altitude of 35,000 ft. The given climb rate commands are +5 ft/sec, +10 ft/sec constant climb rate with zero perturbations in the states. While keeping the desired climb rate, we maintain angle of attack at its trim value. As mentioned before, this is pretty challengeable case compared to Mach 0.5 and altitude 10,000 feet case due to the decrease of the total dynamics pressure and the decrease of the aerodynamics efficiency and control authority.

The simulation results are presented in Figure 7.9 to Figure 7.12. The results bear a strong similarity as in the case of Mach 0.5 and altitude 10,000 feet. We see that the two desired tracking values are exactly satisfied. To match the constant climb rate and the same trim angle of attack, the flight path angle will be exactly constant, but maintain at a new value. To maintain +10 ft/sec constant climb rate, the overshoot of response trajectory for pitch rate is relatively larger than that of Mach 0.5 and altitude 10,000 feet case. From the simulation results, we see the capability of the approximate nonlinear receding-horizon control method when applied to output-tracking control problem for non-minimum phase system.

The performance of the approximate receding-horizon control law depend on the order of approximation performed. In fact, the closed-loop stability does not necessarily require a larger N to achieve. For the cases demonstrated here, the values of N equal to 3. Actually, test shows that N equals to 3 or 4 will yield satisfactory system response.

From the simulation results based on the two flight conditions for the F18 aircraft, we confirm that the approximate receding-horizon control approach works well in output-tracking problem. While the other two control methods are invalid for this kind of control problem, the receding-horizon control method shows its advantage.
Figure 7.9  Mach = 0.7 climb rate for output tracking control

Figure 7.10  Mach = 0.7 flight path angle for output tracking control
Figure 7.11  Mach = 0.7 pitch angle for output tracking control

Figure 7.12  Mach = 0.7 pitch rate for output tracking control
CHAPTER 8 CONCLUSIONS

Linear or nonlinear, that is a question one would ask when it comes to controller design for the inherently nonlinear system of an airplane. The traditional approach has been linear, perhaps dictated historically by the limited capability of avionics and availability of only linear control theory. But its success over the history of aviation is no coincidence. As the F-18 application demonstrated in Chapter 5, a linear controller is stabilizing the aircraft, even compared with a nonlinear design, in the normal flight scenarios. But the performance of the linear design degrades significantly when the operation point is not very close to the trim point. Also the limitations of linear designs become obvious in more challenging situations such as high-performance flight with high angle of attack and large angular rates or unconventional emergency engine-only flight control applications illustrated in Chapter 6.

The major objective of this thesis is to address nonlinear flight control system design for both normal aircraft flight control where the control surfaces are the primary effectors, and unconventional emergency flight control by engines only. By offering the comparison of nonlinear flight control system design and linear flight control system design, we successfully proved the feasibility of nonlinear flight control system designs.

Our first contribution was focusing on the normal flight control system design. We spanned our study among the nonlinear engine dynamics, aerodynamics, flight dynamics, control law development and verification using a high-fidelity six-degree-of-freedom nonlinear model for the F18 aircraft. Based on the original nonlinear models, a LQR controller and two nonlinear controllers, i.e., a predictive controller and a approximate
receding-horizon controller, were designed. The successful applications of the controllers to the F18 aircraft demonstrated the capability of the nonlinear predictive control method and nonlinear approximate receding-horizon control method for controlling nonlinear systems. Our study shows that, even though both the linear controller and nonlinear controllers have the ability to stabilize the aircraft, the nonlinear controllers exhibit better response performance. Also, while the LQR and dynamic inversion methods are not applicable, continuous-time predictive control approach and approximate receding-horizon control approach introduced in this thesis have become effective methods in control system design. System nonlinearities can be accommodated by the nonlinear flight control systems to improve system performance and increase system stability.

Another valuable contribution of this thesis was that we successfully developed a nonlinear predictive control technique for flight control design of a propulsion-controlled aircraft (PCA) system. This problem has long posed a serious challenge to controller design. When the primary control surface, the stabilator, is inoperative, linear control methods cannot stabilize the aircraft. However, the nonlinear predictive control strategy is effective in the case of propulsion controlled aircraft. Simulation results exhibit better performance for the nonlinear controllers than for the linear one, and clearly show the capability of stabilizing unhealthy aircraft. These results also show that the crippled aircraft can be controlled to achieve flight conditions that are infeasible according to the linearized model. From our study, a very important conclusion can be drawn that when nonlinearities dominate in system dynamics, linear control methods work poorly or do not work at all, whereas a nonlinear flight control system can potentially accomplish the control objectives beyond the extent linear controllers can ever reach.

The last contribution of this thesis was that we accomplished the challenging design of output tracking control for non-minimum phase systems. Output tracking control for non-minimum phase systems is an active research topic in recent years. In this thesis, we extended the concept of the approximate receding-horizon control method to the
output tracking problem for non-minimum phase systems. A closed-form control law was developed. Application to the F18 model, which is a non-minimum phase system, demonstrates the approximate receding-horizon control method effectively realized output tracking control for this non-minimum phase system where both dynamic inversion and predictive control are not applicable.

As expected, our study shows that nonlinearities in aircraft dynamics are a prominent consideration in the design of high-performance conventional flight control systems. An appropriately designed nonlinear predictive controller and an approximate receding-horizon controller will respond better to system nonlinearities and hence provide more effective control in even large perturbation or the stringent situation of propulsion-only control system and even output tracking control for non-minimum phase system. This highlights the importance of appropriately incorporating nonlinearities in controller design. The comparison of the performance with that of linear flight controllers provides some insights into when nonlinear controllers may improve performance.

The work presented in this thesis is just the first step toward thoroughly understanding and utilizing nonlinear predictive controllers and approximate receding-horizon controllers for various flight control systems. There are many related issues need to be deeply studied in the future. These issues may include:

- Since the approximate receding-horizon control strategy is based on the multi-step predictive control formulation, the errors must exist due to the approximation. Finding the exact solution and analyzing the errors may be the very important research topic in the future;

- Theoretically proving the possibilities of both the predictive control method and the approximate receding-horizon control method for stabilizing normal flight control systems;

- Extending the predictive control approach and the approximate receding-horizon
control approach to various normal nonlinear flight control systems to test controller performance and explore the possibility of the various applications;

- Applying the approximate receding-horizon control approach to the controllable propulsion only control system to explore the capability of the controller in stabilizing unhealthy aircrafts;

- Numerically constructing the approximate receding-horizon controller is time-consuming for nonlinear systems. A more efficient numerical algorithm will greatly reduce the execution time with the development of modern high performance computer;

- Theoretically proving the possibility of the approximate receding-horizon control approach in accomplishing the output tracking control for both minimum phase and non-minimum phase system;

- Real-time on-board implementation of both the predictive control method and the approximate receding-horizon control method may be very challenging issue to explore;

- Real-time on-board implementing of output tracking controllers for non-minimum phase system using the approximate receding-horizon control method is an interesting topic to study;

The hope is that with the advances in nonlinear control methodology and avionics, more capable and higher-performance flight control system designs will become feasible.
The higher-order explicit formulas for approximate nonlinear receding-horizon control laws are given in this Appendix.

As known from Chapter 4, an analytical, nonlinear feedback control law $v$ is defined by the first $m$ equations in 4.48 as follows.

For $N = 1$

$$v = \begin{bmatrix} u(t) \end{bmatrix}_{m \times 1}$$  \hspace{1cm} (A.1)

For $N = 2$

$$v = \begin{bmatrix} u(t) \\ u(t + h) \end{bmatrix}_{2m \times 1}$$  \hspace{1cm} (A.2)

For $N = 3$

$$v = \begin{bmatrix} u(t) \\ u(t + h) \\ u(t + 2h) \end{bmatrix}_{3m \times 1}$$  \hspace{1cm} (A.3)

For $N = 4$

$$v = \begin{bmatrix} u(t) \\ u(t + h) \\ u(t + 2h) \\ u(t + 3h) \end{bmatrix}_{4m \times 1}$$  \hspace{1cm} (A.4)
For $N = n$

$$v = \begin{bmatrix} u(t) \\ u(t+h) \\ u(t+2h) \\ \vdots \\ u[t + (n - 1)h] \end{bmatrix}_{n \times 1}$$

(A.5)

Since the general expression for state $x(t + kh), 1 \leq k \leq N$

$$x(t + kh) \approx \left[ x(t) + h \left( \sum_{i=0}^{k-1} (I + hF)^i \right) f \right] + \left\{ \sum_{i=0}^{k-1} (I + hF)^i G u[t + (k - 1 - i)h] \right\}$$

(A.6)

where $f, F,$ and $G$ are all evaluated at $x(t)$.

Let $L(\tau) = x^T(t + \tau)Qx(t + \tau) + u^T(t + \tau)Ru(t + \tau)$ for $\tau \in [0, T]$. The integral in (4.37) may be approximated by the standard trapezoidal formula

$$J \approx J_t = \frac{T}{2N} \left[ \frac{1}{2} L(0) + L(h) + \ldots + L((N - 1)h) + \frac{1}{2} L(Nh) \right]$$

(A.7)

Substitution of $x(t + kh)$ in $L(kn)$ gives us the following expansion

$$L(0) = x^T(t)Qx(t) + u^T(t)Ru(t)$$

$$L(h) = x^T(t + h)Qx(t + h) + u^T(t + h)Ru(t + h)$$

(A.8)
\[ L(2h) = x^T(t + 2h)Qx(t + 2h) + u^T(t + 2h)Ru(t + 2h) \]
\[ = \left[ x(t) + h \sum_{i=0}^{1} (I + hF)^i f(x) \right]^T Q \left[ x(t) + h \sum_{i=0}^{1} (I + hF)^i f(x) \right] \]
\[ + 2 \left[ x(t) + h \sum_{i=0}^{1} (I + hF)^i f(x) \right]^T Q \{ h(I + hF)G(x) \} u(t) \]
\[ + 2 \left[ x(t) + h \sum_{i=0}^{1} (I + hF)^i f(x) \right]^T Q \{ hG(x) \} u(t + h) \]
\[ + 2u^T(t) \{ h(I + hF)G(x) \}^T Q \{ hG(x) \} u(t + h) \]
\[ + u^T(t) \{ h(I + hF)G(x) \}^T Q \{ hG(x) \} u(t) \]
\[ + u^T(t + h) \{ hG(x) \}^T Q \{ hG(x) \} u(t + h) \]
\[ + u^T(t + 2h)Ru(t + 2h) \]

\[ L(3h) = x^T(t + 3h)Qx(t + 3h) + u^T(t + 3h)Ru(t + 3h) \]
\[ = \left[ x(t) + h \sum_{i=0}^{2} (I + hF)^i f(x) \right]^T Q \left[ x(t) + h \sum_{i=0}^{2} (I + hF)^i f(x) \right] \]
\[ + 2 \left[ x(t) + h \sum_{i=0}^{2} (I + hF)^i f(x) \right]^T Q \{ h(I + hF)^2G(x) \} u(t) \]
\[ + 2 \left[ x(t) + h \sum_{i=0}^{2} (I + hF)^i f(x) \right]^T Q \{ h(I + hF)G(x) \} u(t + h) \]
\[ + 2 \left[ x(t) + h \sum_{i=0}^{2} (I + hF)^i f(x) \right]^T Q \{ hG(x) \} u(t + 2h) \]
\[ + 2u^T(t) \{ h(I + hF)^2G(x) \}^T Q \{ h(I + hF)G(x) \} u(t + h) \]
\[ + 2u^T(t) \{ h(I + hF)^2G(x) \}^T Q \{ hG(x) \} u(t + 2h) \]
\[ + 2u^T(t + h) \{ h(I + hF)G(x) \}^T Q \{ hG(x) \} u(t + 2h) \]
\[ + u^T(t) \{ h(I + hF)^2G(x) \}^T Q \{ h(I + hF)G(x) \} u(t) \]
\[ + u^T(t + h) \{ h(I + hF)G(x) \}^T Q \{ h(I + hF)G(x) \} u(t + h) \]
\[ + u^T(t + 2h) \{ hG(x) \}^T Q \{ hG(x) \} u(t + 2h) \]
\[ + u^T(t + 3h)Ru(t + 3h) \]
\[ L(4h) = x^T(t + 4h)Qz(t + 4h) + u^T(t + 4h)Ru(t + 4h) \]
\[ \begin{aligned}
&= \left[ x(t) + h\left[ \sum_{i=0}^{3} (I + hF)^i \right] f(x) \right]^T Q \left[ x(t) + h\left[ \sum_{i=0}^{3} (I + hF)^i \right] f(x) \right] \\
&\quad + 2 \left[ x(t) + h\left[ \sum_{i=0}^{3} (I + hF)^i \right] f(x) \right]^T Q \left\{ h(I + hF)^3G(x) \right\} u(t) \\
&\quad + 2 \left[ x(t) + h\left[ \sum_{i=0}^{3} (I + hF)^i \right] f(x) \right]^T Q \left\{ h(I + hF)^2G(x) \right\} u(t + h) \\
&\quad + 2 \left[ x(t) + h\left[ \sum_{i=0}^{3} (I + hF)^i \right] f(x) \right]^T Q \left\{ h(I + hF)G(x) \right\} u(t + 2h) \\
&\quad + 2 \left[ x(t) + h\left[ \sum_{i=0}^{3} (I + hF)^i \right] f(x) \right]^T Q \left\{ hG(x) \right\} u(t + 3h) \\
&\quad + 2u^T(t) \left\{ h(I + hF)^3G(x) \right\}^T Q \left\{ h(I + hF)^2G(x) \right\} u(t + h) \\
&\quad + 2u^T(t) \left\{ h(I + hF)^3G(x) \right\}^T Q \left\{ h(I + hF)G(x) \right\} u(t + 2h) \\
&\quad + 2u^T(t) \left\{ h(I + hF)^3G(x) \right\}^T Q \left\{ hG(x) \right\} u(t + 3h) \\
&\quad + 2u^T(t + h) \left\{ h(I + hF)^2G(x) \right\}^T Q \left\{ h(I + hF)G(x) \right\} u(t + 2h) \\
&\quad + 2u^T(t + h) \left\{ h(I + hF)^2G(x) \right\}^T Q \left\{ hG(x) \right\} u(t + 3h) \\
&\quad + u^T(t + 2h) \left\{ h(I + hF)G(x) \right\}^T Q \left\{ h(I + hF)G(x) \right\} u(t + 3h) \\
&\quad + u^T(t + 3h) \left\{ hG(x) \right\}^T Q \left\{ hG(x) \right\} u(t + 3h) \\
&\quad + u^T(t + 4h)Ru(t + 4h)
\end{aligned} \] (A.10)

Hence, we obtain the general expression for \( L(kh), 1 \leq k \leq N \)

\[ L(kh) = x^T(t + kh)Qz(t + kh) + u^T(t + kh)Ru(t + kh) \]
\[ \begin{aligned}
&= \left[ x(t) + h\left[ \sum_{i=0}^{k-1} (I + hF)^i \right] f(x) \right]^T Q \left[ x(t) + h\left[ \sum_{i=0}^{k-1} (I + hF)^i \right] f(x) \right] \\
&\quad + 2 \left[ x(t) + h\left[ \sum_{i=0}^{k-1} (I + hF)^i \right] f(x) \right]^T Q \left\{ \sum_{i=0}^{k-1} h(I + hF)^iG(x)u[t + (k - 1 - i)h] \right\} \\
&\quad + u^T(t + kh)Ru(t + kh)
\end{aligned} \] (A.11)
Since $L(Nh) = x^T(t + Nh)Qx(t + Nh) + u^T(t + Nh)Ru(t + Nh)$ is defined, the final state constraint condition $x(t + T) = 0$ can make the state variable $x(t + Nh)$ equal to zero. Also, from $\frac{\partial J}{\partial u}(t + Nh) = 0$, we may find that control variable $u(t + Nh)$ equal to zero. Thus, $L(Nh)$ term is excluded from the approximate performance index $J_i$. Hence, by setting $J = \tilde{J}$, H matrix always has the following specific form.

$$H_{N \times N} = \begin{bmatrix}
H_{11} & H_{12} & \cdots & H_{1(N-1)} & 0 \\
H_{21} & H_{22} & \cdots & H_{2(N-1)} & 0 \\
\vdots & \vdots & \ddots & \vdots & 0 \\
H_{(N-1)1} & H_{(N-1)2} & \cdots & 0 \\
0 & 0 & \cdots & 0 & H_{NN}
\end{bmatrix} \quad (A.12)$$

This further simplifies the explicit formulas for the closed-loop control law for different $N$ values. Furthermore, it's easy for us to obtain explicit expressions for $H(x)$, $g(x)$ and $q(x)$ so that we can get an explicit mathematical expression for the closed-loop control $v$.

For $N = 1$

$$J_1 = \frac{h^2}{2} L(0) = \frac{h}{4} L(0) \quad (A.13)$$

$$\tilde{J}_1 = \frac{1}{2} u^T(t)H_{11}u(t) + g_1^T(x)u(t) + q(x)$$

$$\Rightarrow$$

$$\begin{align*}
H_{11} &= \frac{h}{2} R \\
g_1^T(x) &= zeros(1, m) \quad (A.14) \\
q(x) &= \frac{h}{4} x^T(t)Qx(t)
\end{align*}$$
For $N = 2$

\[ J_2 = \frac{h}{2} \left[ \frac{1}{2} L(0) + L(h) \right] = \frac{h}{4} L(0) + \frac{h}{2} L(h) \]

\[ \tilde{J}_2 = \frac{1}{2} \begin{bmatrix} u(t) \\ u(t + h) \end{bmatrix}^T \begin{bmatrix} H_{11} & H_{12} \\ 0 & H_{22} \end{bmatrix} \begin{bmatrix} u(t) \\ u(t + h) \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}^T \begin{bmatrix} u(t) \\ u(t + h) \end{bmatrix} + q(x) \]  
(A.15)

implies

\[ \begin{align*}
H_{11} &= \frac{h}{2} R + h \{ hG(x) \}^T Q \{ hG(x) \} \\
H_{22} &= h R \\
H_{12} &= \text{zeros}(m, m) \\
H_{21} &= H_{12} \\
g_1^T(x) &= h \{ x(t) + hf(x) \}^T Q \{ hG(x) \} \\
g_2^T(x) &= \text{zero}(1, m) \\
q(x) &= \frac{h}{4} x^T(t) Q x(t) + \frac{h}{2} \{ x(t) + hf(x) \}^T Q \{ x(t) + hf(x) \} 
\]  
(A.16)

For $N = 3$

\[ J_3 = \frac{h}{2} \left[ \frac{1}{2} L(0) + L(h) + L(2h) \right] = \frac{h}{4} L(0) + \frac{h}{2} L(h) + \frac{h}{2} L(2h) \]

\[ \tilde{J}_3 = \frac{1}{2} \begin{bmatrix} u(t) \\ u(t + h) \\ u(t + 2h) \end{bmatrix}^T \begin{bmatrix} H_{11} & H_{12} & 0 \\ H_{21} & H_{22} & 0 \\ 0 & 0 & H_{33} \end{bmatrix} \begin{bmatrix} u(t) \\ u(t + h) \\ u(t + 2h) \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix}^T \begin{bmatrix} u(t) \\ u(t + h) \\ u(t + 2h) \end{bmatrix} + q(x) \]  
(A.17)

implies
\[
\begin{align*}
H_{11} &= \frac{1}{2}R + h\{hG(x)\}^TQ\{hG(x)\} + h\{h(I + hF)G(x)\}^TQ\{h(I + hF)G(x)\} \\
H_{22} &= hR + h\{hG(x)\}^TQ\{hG(x)\} \\
H_{33} &= hR \\
H_{12} &= h\{h(I + hF)G(x)\}^TQ\{hG(x)\} \\
H_{21} &= H_{12} \\
H_{13} &= H_{31} = H_{23} = H_{32} = \text{zeros}(m,m) \\
g_1^T(x) &= h[x(t) + hf(x)]^TQ\{hG(x)\} + h\left\{x(t) + \sum_{i=0}^{1}(I + hF)^i\right\}f(x)^TQ\{h(I + hF)G(x)\} \\
g_2^T(x) &= h\left\{x(t) + h\left[\sum_{i=0}^{1}(I + hF)^i\right]f(x)\right\}^TQ\{hG(x)\} \\
g_3^T(x) &= \text{zero}(1,m) \\
q(x) &= \frac{h}{4}x^T(t)Qx(t) + \frac{h}{2}[x(t) + hf(x)]^TQ[x(t) + hf(x)] \\
&\quad+ \frac{h}{2}\left[x(t) + h\left[\sum_{i=0}^{1}(I + hF)^i\right]f(x)\right]^TQ\left[x(t) + h\left[\sum_{i=0}^{1}(I + hF)^i\right]f(x)\right] \\
\end{align*}
\]

(A.18)

For \(N = 4\)

\[
J_4 = \frac{h}{2}\left[L(0) + L(h) + L(2h) + L(3h)\right] = \frac{h}{2}L(0) + \frac{h}{2}L(h) + \frac{h}{2}L(2h) + \frac{h}{2}L(3h) \\
\]

\[
\tilde{J}_4 = \frac{1}{2} \left[
\begin{array}{c}
  u(t) \\
  u(t+h) \\
  u(t+2h) \\
  u(t+3h)
\end{array}
\right]^T \left[
\begin{array}{cccc}
  H_{11} & H_{12} & H_{13} & 0 \\
  H_{21} & H_{22} & H_{23} & 0 \\
  H_{31} & H_{32} & H_{33} & 0 \\
  0 & 0 & 0 & H_{44}
\end{array}
\right] \left[
\begin{array}{c}
  u(t) \\
  u(t+h) \\
  u(t+2h) \\
  u(t+3h)
\end{array}
\right]
\]

(A.19)

\[
\left[
\begin{array}{c}
  g_1 \\
  g_2 \\
  g_3 \\
  g_4
\end{array}
\right]^T + \left[
\begin{array}{c}
  u(t) \\
  u(t+h) \\
  u(t+2h) \\
  u(t+3h)
\end{array}
\right]q(x)
\]
\[
\begin{align*}
H_{11} &= \frac{1}{2} R + h \{ hG(x) \}^T Q \{ hG(x) \} + h \{ h(I + hF)G(x) \}^T Q \{ h(I + hF)G(x) \} \\
&\quad + h \{ h(I + hF)^2G(x) \}^T Q \{ h(I + hF)^2G(x) \} \\
H_{12} &= hR + h \{ hG(x) \}^T Q \{ hG(x) \} + h \{ h(I + hF)G(x) \}^T Q \{ h(I + hF)G(x) \} \\
H_{13} &= h \{ h(I + hF)^2G(x) \}^T Q \{ hG(x) \} \\
H_{14} &= hR \\
H_{21} &= H_{12} \\
H_{22} &= hR + h \{ hG(x) \}^T Q \{ hG(x) \} + h \{ h(I + hF)G(x) \}^T Q \{ h(I + hF)G(x) \} \\
H_{23} &= h \{ h(I + hF)G(x) \}^T Q \{ hG(x) \} \\
H_{31} &= H_{13} \\
H_{32} &= H_{23} \\
H_{33} &= hR + h \{ hG(x) \}^T Q \{ hG(x) \} + h \{ h(I + hF)G(x) \}^T Q \{ h(I + hF)G(x) \} \\
H_{34} &= H_{14} = H_{24} = H_{42} = H_{34} = H_{43} = 0
\end{align*}
\]

\[
\begin{align*}
g_1^T(x) &= h \{ x(t) + hf(x) \}^T Q \{ hG(x) \} + h \left[ x(t) + h \sum_{i=0}^{1} (I + hF)^i f(x) \right]^T Q \{ h(I + hF)G(x) \} \\
&\quad + h \left[ x(t) + h \sum_{i=0}^{2} (I + hF)^i f(x) \right]^T Q \{ h(I + hF)^2G(x) \} \\
g_2^T(x) &= h \left[ x(t) + h \sum_{i=0}^{1} (I + hF)^i f(x) \right]^T Q \{ hG(x) \} + \\
&\quad h \left[ x(t) + h \sum_{i=0}^{2} (I + hF)^i f(x) \right]^T Q \{ h(I + hF)G(x) \} \\
g_3^T(x) &= h \left[ x(t) + h \sum_{i=0}^{2} (I + hF)^i f(x) \right]^T Q \{ hG(x) \} \\
g_4^T(x) &= \text{zero}(1, m) \\
q(x) &= \frac{1}{4} x^T(t) Q x(t) + \frac{1}{2} \{ x(t) + hf(x) \}^T Q \{ x(t) + hf(x) \} + \\
&\quad \frac{1}{2} \left[ x(t) + h \sum_{i=0}^{1} (I + hF)^i f(x) \right]^T Q \left[ x(t) + h \sum_{i=0}^{1} (I + hF)^i f(x) \right] + \\
&\quad \frac{1}{2} \left[ x(t) + h \sum_{i=0}^{2} (I + hF)^i f(x) \right]^T Q \left[ x(t) + h \sum_{i=0}^{2} (I + hF)^i f(x) \right]
\end{align*}
\]
For $N = 5$

\[
J_5 = \frac{h}{2} \left[ \frac{1}{2} L(0) + L(h) + L(2h) + L(3h) + L(4h) \right] \\
= \frac{h}{4} L(0) + \frac{h}{2} L(h) + \frac{h}{2} L(2h) + \frac{h}{2} L(3h) + \frac{h}{2} L(4h)
\]

\[
\dot{J}_5 = \frac{1}{2} \begin{bmatrix}
  u(t) & u(t+h) & u(t+2h) & u(t+3h) & u(t+4h) \\
  u(t+h) & H_{11} & H_{12} & H_{13} & H_{14} & 0 \\
  u(t+2h) & H_{21} & H_{22} & H_{23} & H_{24} & 0 \\
  u(t+3h) & H_{31} & H_{32} & H_{33} & H_{34} & 0 \\
  u(t+4h) & H_{41} & H_{42} & H_{43} & H_{44} & 0 \\
  0 & 0 & 0 & 0 & 0 & H_{55}
\end{bmatrix} \begin{bmatrix}
  u(t) \\
  u(t+h) \\
  u(t+2h) \\
  u(t+3h) \\
  u(t+4h)
\end{bmatrix}
\]

\[
\begin{bmatrix}
  g_1 \\
  g_2 \\
  g_3 \\
  g_4 \\
  g_5
\end{bmatrix} \begin{bmatrix}
  u(t) \\
  u(t+h) \\
  u(t+2h) \\
  u(t+3h) \\
  u(t+4h)
\end{bmatrix} + x + q(x)
\]

\[
\Rightarrow \begin{cases}
H_{11} = \frac{h}{2} R + h \{hG(x)\}^T Q \{hG(x)\} + \\
\quad h \{h(I + hF)G(x)\}^T Q \{h(I + hF)G(x)\} + \\
\quad h \{h(I + hF)^2G(x)\}^T Q \{h(I + hF)^2G(x)\} + \\
\quad h \{h(I + hF)^3G(x)\}^T Q \{h(I + hF)^3G(x)\} \\
H_{22} = hR + h \{hG(x)\}^T Q \{hG(x)\} + \\
\quad h \{h(I + hF)G(x)\}^T Q \{h(I + hF)G(x)\} + \\
\quad h \{h(I + hF)^2G(x)\}^T Q \{h(I + hF)^2G(x)\} \\
H_{33} = hR + h \{hG(x)\}^T Q \{hG(x)\} + \\
\quad h \{h(I + hF)G(x)\}^T Q \{h(I + hF)G(x)\} \\
H_{44} = hR + h \{hG(x)\}^T Q \{hG(x)\} \\
H_{55} = hR
\end{cases}
\]
\[
\begin{align*}
H_{12} &= H_{21} = h \{h(I + hF)G(z) \}^T Q \{hG(z)\} + \\
&\quad h \{h(I + hF)^2G(z) \}^T Q \{h(I + hF)G(z)\} + \\
&\quad h \{h(I + hF)^3G(z) \}^T Q \{h(I + hF)^2G(z)\} \\
H_{13} &= H_{31} = h \{h(I + hF)^2G(z) \}^T Q \{hG(z)\} + h \{h(I + hF)^3G(z) \}^T Q \{h(I + hF)G(z)\} \\
H_{14} &= H_{41} = h \{h(I + hF)^3G(z) \}^T Q \{hG(z)\} \\
H_{23} &= H_{32} = h \{h(I + hF)G(z) \}^T Q \{hG(z)\} + h \{h(I + hF)^2G(z) \}^T Q \{h(I + hF)G(z)\} \\
H_{24} &= H_{42} = h \{h(I + hF)^2G(z) \}^T Q \{hG(z)\} \\
H_{34} &= H_{43} = h \{h(I + hF)G(z) \}^T Q \{hG(z)\} \\
H_{15} &= H_{51} = H_{25} = H_{52} = 0 \\
H_{35} &= H_{53} = H_{45} = H_{54} = 0
\end{align*}
\] (A.24)

\[
\begin{align*}
g_1^T(x) &= h \{x(t) + h f(x) \}^T Q \{hG(z)\} + \\
&\quad h \left[ x(t) + h \sum_{i=0}^{1} (I + hF)^i f(x) \right]^T Q \{h(I + hF)G(z)\} + \\
&\quad h \left[ x(t) + h \sum_{i=0}^{2} (I + hF)^i f(x) \right]^T Q \{h(I + hF)^2G(z)\} + \\
&\quad h \left[ x(t) + h \sum_{i=0}^{3} (I + hF)^i f(x) \right]^T Q \{h(I + hF)^3G(z)\} \\
g_2^T(x) &= h \left[ x(t) + h \sum_{i=0}^{1} (I + hF)^i f(x) \right]^T Q \{hG(z)\} + \\
&\quad h \left[ x(t) + h \sum_{i=0}^{2} (I + hF)^i f(x) \right]^T Q \{h(I + hF)G(z)\} + \\
&\quad h \left[ x(t) + h \sum_{i=0}^{3} (I + hF)^i f(x) \right]^T Q \{h(I + hF)^2G(z)\} \\
g_3^T(x) &= h \left[ x(t) + h \sum_{i=0}^{2} (I + hF)^i f(x) \right]^T Q \{hG(z)\} + \\
&\quad h \left[ x(t) + h \sum_{i=0}^{3} (I + hF)^i f(x) \right]^T Q \{h(I + hF)G(z)\} \\
g_4^T(x) &= h \left[ x(t) + h \sum_{i=0}^{3} (I + hF)^i f(x) \right]^T Q \{hG(z)\} \\
g_5^T(x) &= \text{zero}(1, m)
\end{align*}
\] (A.25)
\[ q(x) = \frac{h}{4} x^T(t) Q x(t) + \frac{h}{2} \{ x(t) + hf(x) \}^T Q \{ x(t) + hf(x) \} + \]
\[ \frac{h}{2} \left[ x(t) + h \left[ \sum_{i=0}^{1} (I + hF)^i \right] f(x) \right]^T Q \left[ x(t) + h \left[ \sum_{i=0}^{1} (I + hF)^i \right] f(x) \right] + \]
\[ \frac{h}{2} \left[ x(t) + h \left[ \sum_{i=0}^{2} (I + hF)^i \right] f(x) \right]^T Q \left[ x(t) + h \left[ \sum_{i=0}^{2} (I + hF)^i \right] f(x) \right] + \]
\[ \frac{h}{2} \left[ x(t) + h \left[ \sum_{i=0}^{3} (I + hF)^i \right] f(x) \right]^T Q \left[ x(t) + h \left[ \sum_{i=0}^{3} (I + hF)^i \right] f(x) \right] \quad \text{(A.26)} \]

Thus, having the expressions of matrix \( H \), \( g \) and \( q \), we are ready to calculate the closed-form control law.
REFERENCES


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