Uncertainty, Risk Aversion and Risk Management for Agricultural Producers

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Disciplines
Agribusiness | Agricultural Economics | Business Administration, Management, and Operations

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Giancarlo Moschini
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David A. Hennessy

July 1999


Abstract

Uncertainty and risk are quintessential features of agricultural production. After a brief overview of the main sources of agricultural risk, we provide an exposition of expected utility theory and of the notion of risk aversion. This is followed by a basic analysis of agricultural production decisions under risk, including some comparative statics results from stylized models. Selected empirical topics are surveyed, with emphasis on risk analyses as they pertain to production decisions at the farm level. Risk management is then discussed, and a synthesis of hedging models is presented. We conclude with a detailed review of agricultural insurance, with emphasis on the moral hazard and adverse selection problems that arise in the context of crop insurance.

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Outline

1. Introduction
   1.1. Uncertainty and risk in agriculture
   1.2. Modeling issues

2. Decision making under uncertainty
   2.1. Preferences over lotteries and the expected utility model
   2.2. Risk aversion
   2.3. Ranking distributions

3. The agricultural producer under uncertainty and risk aversion
   3.1. Modeling price and production uncertainty
   3.2. Static models under risk neutrality
   3.3. Static models under risk aversion
      3.3.1. Introduction of uncertainty
      3.3.2. Marginal changes in environment
      3.3.3. Uncertainty and cost minimization
   3.4. Dynamics and flexibility under uncertainty

4. Selected empirical issues
   4.1. Identifying risk preferences
   4.2. Estimating stochastic structures
   4.3. Joint estimation of preferences and technology
   4.4. Econometric estimation of supply models with risk
   4.5. Risk and equilibrium in supply and production systems
   4.6. Programming models with risk
   4.7. Technology adoption, infrastructure and risk

5. Risk management for agricultural producers
   5.1. Hedging with price contingent contracts
      5.1.1. Forward contracts and futures contracts
      5.1.2. Options on futures
      5.1.3. The time pattern of hedging
      5.1.4. Hedging and production decisions
      5.1.5. The value of hedging to farmers
   5.2. Crop Insurance
      5.2.1. Moral hazard
      5.2.2. Adverse selection
      5.2.3. Further discussion

6. Conclusion
1. Introduction

Because of the complexities of physical and economic systems, the unfolding of most processes that we care about exhibits attributes that cannot be forecast with absolute accuracy. The immediate implication of this uncertainty for economic agents is that many possible outcomes are usually associated with any one chosen action. Thus, decision making under uncertainty is characterized by risk, because typically not all possible consequences are equally desirable. Although uncertainty and risk are ubiquitous, in agriculture they constitute an essential feature of the production environment and arguably warrant a detailed analysis.

Considerable research has been devoted to exploring questions connected with the effects of uncertainty and risk in agriculture, and these efforts have paralleled related developments in the general economics literature. In this chapter we set out to review a number of these studies, especially as they relate to farm-level production decisions. To economize on our coverage of earlier work, and at the risk of not doing justice to some ground-breaking studies, we can refer to Dillon's (1971) survey as a starting point. In addition to providing an exposition of expected utility (EU) theory, which contributed to rooting subsequent studies in modern economic analysis, that survey provides an exhaustive account of previous studies of uncertainty and risk in agricultural economics. Subsequent useful compendia include Anderson, Dillon and Hardaker (1977), who consider a comprehensive set of applications of decision theory to agricultural production under uncertainty, and Newbery and Stiglitz (1981), who not only provide a thorough study of commodity price stabilization issues, but also analyze a number of problems that are relevant to the understanding of risk in agriculture.

The aforementioned contributions have been accompanied and followed by considerable research that is relevant to our pursuit. As we undertake to provide a critical survey of these studies, we are mindful of the subjective bias and unintended oversights that an exercise such as this inevitably entails, a risk heightened in our case by the encompassing nature of the topic and the sheer volume of
the relevant literature. We apologize for errors of omission and commission, and we hope that our review will nonetheless prove useful to the applied researcher.

1.1. Uncertainty and risk in agriculture

Despite the fact that any taxonomy is somewhat arbitrary, it is useful to start by outlining the main sources of uncertainty and risk that are relevant from the point of view of the agricultural producer. First, there is what can be broadly defined as production uncertainty: in agriculture the amount and quality of output that will result from a given bundle of inputs are typically not known with certainty, i.e., the production function is stochastic. This uncertainty is due to the fact that uncontrollable elements, such as weather, play a fundamental role in agricultural production. The effects of these uncontrollable factors are heightened by the fact that time itself plays a particularly important role in agricultural production, because long production lags are dictated by the biological processes that underlie the production of crops and the growth of animals. Although there are parallels in other production activities, it is fair to say that production uncertainty is a quintessential feature of agricultural production.

Price uncertainty is also a standard attribute of farming activities. Because of the biological production lags mentioned above, production decisions have to be made far in advance of realizing the final product, so that the market price for the output is typically not known at the time these decisions have to be made. Price uncertainty, of course, is all the more relevant because of the inherent volatility of agricultural markets. Such volatility may be due to demand fluctuations, which are particularly important when a sizable portion of output is destined for the export market.

Production uncertainty as discussed earlier, however, also contributes to price uncertainty because price needs to adjust to clear the market. In this process some typical features of agricultural markets (a large number of competitive producers, relatively homogeneous output, and inelastic demand) are
responsible for generating considerable price volatility, even for moderate production shocks.

Additional sources of uncertainty are relevant to farming decisions when longer-term economic problems are considered. *Technological uncertainty*, associated with the evolution of production techniques that may make quasi-fixed past investments obsolete, emerges as a marked feature of agricultural production. Clearly, the randomness of new knowledge development affects production technologies in all sectors. What makes it perhaps more relevant to agriculture, however, is the fact that technological innovations here are the product of research and development efforts carried out elsewhere (for instance, by firms supplying inputs to agriculture), such that competitive farmers are captive players in the process. *Policy uncertainty* also plays an important role in agriculture. Again, economic policies have impacts on all sectors through their effects on such things as taxes, interest rates, exchange rates, regulation, provision of public goods, and so on. Yet, because agriculture in many countries is characterized by an intricate system of government interventions, and because the need for changing these policy interventions in recent times has remained strong (witness the recent transformation of key features of the agricultural policy of the United States and the European Union, or the emerging concerns about the environmental impacts of agricultural production), this source of uncertainty creates considerable risk for agricultural investments.

1.2. *Modeling issues*

Two concepts of paramount importance in economic modeling are *optimization* (the rational behavior of economic agents) and *equilibrium* (the balancing of individual claims in a market setting). The application of both of these concepts raises problematic issues when uncertainty is involved. In particular, to apply the powerful apparatus of optimization to individual choices under uncertainty one needs to determine what exactly is being optimized. Although a universally satisfactory answer to this question is far from obvious, the most widely used idea is that agents exposed to uncertainty and
risk maximize expected utility. This paradigm represents the culmination of a research program that dates back to Bernoulli (1738), and rests on some compelling assumptions about individual choice. Most of the applications that we will review rely on the EU model (indeed, often some restricted version of it). Thus, in what follows we will briefly review the EU hypothesis before we proceed with a survey of applications. We should note, however, that despite its normative appeal, the EU framework has recently come under intense scrutiny because of its inability to describe some features of individual behavior under risk, and a number of generalizations of the EU model have been proposed [Machina (1987); Quiggin (1993)].

A modeling strategy that recurs in the applied literature is the distinction between uncertainty and risk attributed to Knight (1921). According to this view, risk arises when the stochastic elements of a decision problem can be characterized in terms of numerical objective probabilities, whereas uncertainty refers to decision settings with random outcomes that lack such objective probabilities. With the widespread acceptance of probabilities as subjective beliefs, Knight’s distinction between risk and uncertainty is virtually meaningless and, like other authors [e.g., Hirshleifer and Riley (1992)], we will ignore it here.¹ Thus, the notions of uncertainty and risk are interchangeable in what follows, although, like Robison and Barry (1987), we tend to use the word uncertainty mostly to describe the environment in which economics decisions are made, and the word risk to characterize the economically relevant implications of uncertainty.

2. Decision making under uncertainty

Economic models of individual choice are necessarily rooted in the assumption of rationality on the part of decision makers. Perhaps the most common and widely understood such model is given by

¹ We should note, however, that in some cases this approach is not totally satisfactory, as illustrated for example by the so-called Ellsberg paradox [Ellsberg (1961)].
the neoclassical theory of consumer choice under uncertainty. The primitive assumption is that there is a preference ordering on commodity bundles that satisfies the consistency requirements of completeness and transitivity. These basic rationality postulates, coupled with the assumption of continuity (a hardly avoidable and basically harmless mathematical simplification), allow consumer choices to be characterized in terms of an ordinal utility function, a construct that enhances the analytical power of the assumptions. Choice under uncertainty could be characterized within this elementary setting, given minor modification of the original assumptions. For example, as in Debreu (1959), the standard preference ordering of neoclassical consumption theory could be applied to state-contingent commodity bundles. The analysis can then proceed without reference to the probability of the various states-of-nature. Whereas such an approach has proven useful for some problems [Arrow (1964); Hirshleifer (1966)], for a number of other cases, including applications typically of interest to agricultural economists, a more specific framework of analysis is desirable. By explicitly recognizing the mutually exclusive nature of alternative random consequences, one can get a powerful representation of decision making under uncertainty. This leads to the so-called EU model of decision under uncertainty, arguably the most important achievement of modern economic analysis of individual behavior. Although there exist a number of lucid expositions of this model [for a textbook treatment, see Mas-Colell, Whinston and Green (1995, chapter 6)], we present (somewhat informally) the main features of EU theory, to set the stage for the review of applications that follows.

2.1 Preferences over lotteries and the expected utility model

Let $A$ represent the set of all possible actions available to decision makers, and let $S$ represent the set of all possible states of nature. The specific action chosen by the agent and the particular state of nature that is realized (with the former choice being made prior to the resolution of uncertainty about the true state of nature) determine the outcomes (consequences) that the agent cares about. In other
words, consequences are random variables as given by the function \( c: S \times A \rightarrow C \), where \( C \) is the set of all possible consequences. For example, \( C \) could be the set of all possible commodity bundles as in standard consumer theory, in which case \( C = \mathbb{N}^n \). Alternatively, as in many applications, it is monetary outcomes that are of interest to the decision makers, in which case one can put \( C = \mathbb{R} \).

Suppose for simplicity that the set \( C \) is finite, and that there are \( N \) possible consequences. Given an objectively known probability for each state of nature, then choosing a particular action will result in a probability distribution (a lottery, a gamble) over outcomes. Formally, one can define a lottery as a probability list \( L = (\ell_1, \ell_2, \ldots, \ell_N) \) such that \( \ell_i \) is the probability (likelihood) that consequence \( c_i \in C \) will arise (of course, \( \ell_i \in [0, 1] \) and \( \sum \ell_i = 1 \)).

In this setting, primitive preferences are represented by a preference relation \( \succeq \) defined over the set of all possible lotteries \( \mathcal{L} \). Assuming that this relation is rational (complete and transitive) and satisfies a specific continuity assumption, then all lotteries can be ranked by a function \( V: L \rightarrow \mathbb{R} \) in the sense that, for any two lotteries \( L \) and \( L' \), we have \( L \succeq L' \iff V(L) \geq V(L') \). Because the underlying assumption is that the decision maker is concerned only with the ultimate consequences, compound lotteries in this setting are always equivalent to the corresponding reduced lottery. Thus, for example, a gamble that gives lottery \( L \) with probability \( \lambda \) and lottery \( L' \) with probability \( (1 - \lambda) \) is equivalent to a simple lottery whose probabilities are given by the mixture \( \lambda L + (1 - \lambda)L'' \). So far, the parallel with standard consumer theory is quite close [in particular, for example, \( V(L) \) is an ordinal function]. To get the EU model, a further assumption is required at this point, namely the "independence axiom" [Samuelson (1952)]. This condition requires that, if we consider the mixture of each of any two lotteries \( L \) and \( L' \) with another lottery \( L'' \), the preference ordering on the two
resulting lotteries is independent of the particular common lottery \( L'' \). That is, for any \( L, L' \) and \( L'' \), and any \( \lambda \in (0, 1) \),

\[
L \succeq L' \Leftrightarrow \lambda L + (1 - \lambda)L'' \succeq \lambda L' + (1 - \lambda)L''.
\]

(2.1)

One may note that an equivalent assumption in the standard choice problem of consumer theory would be very restrictive, which is why it is seldom made in that context. Here, however, the independence assumption is quite natural because of a fundamental feature of decision problems under uncertainty: consequences are mutually exclusive.\(^2\)

The independence axiom, coupled with the other standard rational choice assumptions, has the remarkable implication that there exists a utility function defined over consequences, \( U: C \rightarrow \mathbb{R} \), such that

\[
L \succeq L' \Leftrightarrow \sum_{i=1}^{N} \ell_i U(c_i) \geq \sum_{i=1}^{N} \ell'_i U(c_i),
\]

where again, \( \ell_i \) is the probability that consequence \( c_i \) will attain under \( L \) and \( \ell'_i \) is the probability that consequence \( c_i \) will attain under \( L' \). In other words, with the independence axiom, the utility function over lotteries can always be represented as the mathematical expectation of a utility function defined over consequences, that is \( V(L) = E[U(c)] \) where \( E[\cdot] \) is the mathematical expectation operator. As such, the utility function \( V(L) \) is linear in probabilities. The function \( U(c) \) is usually referred to as the von Neumann-Morgenstern (vNM) utility function.\(^3\) This vNM utility function \( U(c) \) is

\(^2\) Despite its theoretical appeal, the empirical validity of the independence axiom has been questioned, especially in light of the so-called Allais paradox [Allais (1953)].

\(^3\) This convention recognizes these authors' pioneering contribution to the development of the EU model in von Neumann and Morgenstern (1944). But others call \( U(\cdot) \) the Bernoulli utility function, in recognition of Daniel Bernoulli's solution of the St. Petersburg paradox [Bernoulli (1738)], which anticipated some of the features of the EU model.
monotonically increasing and is cardinal in the sense that it is defined up to an increasing linear transformation [that is, if $U(c)$ represents the preference relation $\succeq$, then any $\bar{U}(c) = \alpha + \beta U(c)$, with $\beta > 0$, provides an equivalent representation of this relation]. When the outcomes of interest are described by continuous random variables with joint cumulative distribution function $F(c)$, the EU model implies that $V(F) = \int U(c) \, dF(c)$. In conclusion, in the EU model the problem of selecting the action that induces the most preferred probability distribution reduces to that of maximizing the expected utility of outcomes.

Versions of the EU model more general than the one just discussed are available. Perhaps the most important is the EU model with subjective probability developed by Savage (1954). In this framework one does not assume that the probabilities of various states of the world are objectively given. Rather, the existence of probabilities for the states of nature and of a vNM utility function for the consequences are both implied by a set of axioms. Prominent among these is the "sure-thing" axiom, roughly equivalent to the independence condition discussed earlier. A crucial element for this approach is that probabilities are inherently subjective, an idea pioneered by de Finetti (1931).

2.2. Risk aversion

The EU model allows us to capture in a natural way the notion of risk aversion, which is a fundamental feature of the problem of choice under uncertainty. This notion is made precise when the consequences that matter to the decision maker are monetary outcomes, such that the vNM utility function is defined over wealth, say $U(w)$ where $w \in \mathbb{R}$ is realized wealth. In a very intuitive sense, a decision maker is said to be risk averse if, for every lottery $F(w)$, she will always prefer (at least

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4 Anscombe and Aumann (1963) provide an easier (albeit somewhat different) set-up within which one can derive Savage's subjective EU model.
(weakly) the certain amount \( E[w] \) to the lottery \( F(w) \) itself, i.e., \( U[ \int w \, dF(w)] \geq \int U(w) \, dF(w) \) [Arrow (1965); Pratt (1964)]. But by Jensen’s inequality, this condition is equivalent to \( U(w) \) being concave. Thus, concavity of the vNM utility function provides the fundamental characterization of risk aversion.

In many applied problems it is of interest to quantify risk aversion. For example, when can we say that an agent \( a \) is more risk averse than another agent \( b \)? Given the representation of risk aversion in terms of the concavity of \( U(\cdot) \), then we can say that agent \( a \) is globally more risk averse than agent \( b \) if we can find an increasing concave function \( g(\cdot) \) such that \( U_a = g(U_b) \), where \( U_i \) denotes the utility function of agent \( i \) \((i = a, b)\). An interesting question, in this context, concerns how the degree of risk aversion of a given agent changes with the level of wealth. For this purpose, two measures of risk aversion that have become standard are the Arrow-Pratt coefficient of absolute risk aversion \( A(w) \) and the Arrow-Pratt coefficient of relative risk aversion \( R(w) \) [Arrow (1965); Pratt (1964)]. Because concavity of \( U(w) \) is equivalent to risk aversion, the degree of concavity of \( U(w) \), as captured for example by \( U''(w) \), is a candidate to measure the degree of risk aversion. But because \( U(w) \) is defined only up to an increasing linear transformation, we need to normalize by \( U'(w) > 0 \) to obtain a measure that is unique for a given preference ordering. Thus, the coefficient of absolute risk aversion is defined as \( A(w) = -U''(w) / U'(w) \). As is apparent from its definition, absolute risk aversion is useful for comparing the attitude of an agent towards a given gamble at

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5 Note that \( A(w) \) can also be used to compare the risk aversion of two agents. If \( A_a(w) \) and \( A_b(w) \) are the coefficients derived from the vNM utility functions \( U_a \) and \( U_b \), respectively, then agent \( a \) is more risk averse than agent \( b \) if \( A_a(w) \geq A_b(w) \) for all \( w \). This characterization is equivalent to that given earlier in terms of \( U_a \) being an increasing concave transformation of \( U_b \).
different levels of wealth. It seems natural to postulate that agents will become less averse to a given gamble as their wealth increases. This is the notion of decreasing absolute risk aversion (DARA), i.e., \( A(w) \) is a decreasing function of \( w \) [when \( A(w) \) is merely nonincreasing in \( w \), the notion is labeled nonincreasing absolute risk aversion (NIARA)]. As we shall see, most comparative statics results of optimal choice under uncertainty rely on this condition.

Sometimes, however, it is interesting to inquire about the attitude of risk-averse decision makers towards gambles that are expressed as a fraction of their wealth. This type of risk preference is captured by the coefficient of relative risk aversion \( R(w) = wA(w) \). Unlike the case of absolute risk aversion, there are no compelling \textit{a priori} reasons for any particular behavior of \( R(w) \) with respect to \( w \). An assumption that is sometimes invoked is that of nonincreasing relative risk aversion (NIRRA), implying that an agent should not become more averse to a gamble expressed as a fixed percentage of her wealth as the level of wealth increases.\(^6\)

Of some interest for applied analysis are utility functions for which \( A(w) \) and \( R(w) \) are constant. The constant absolute risk aversion (CARA) utility function is given by \( U(w) = -e^{-\lambda w} \), where \( \lambda \) is the (constant) coefficient of absolute risk aversion. The constant relative risk aversion (CRRA) utility function is given by \( U(w) = (w^{1-\rho})/(1-\rho) \) if \( \rho \neq 1 \), and by \( U(w) = \log(w) \) if \( \rho = 1 \), where \( \rho \) is the (constant) coefficient of relative risk aversion.\(^7\)

\(^6\) Arrow (1965) suggests that the value of \( R(w) \) should hover around 1 and, if anything, should be increasing in \( w \). His arguments are predicated on the requirement that the utility function be bounded, a condition that allows EU to escape a modified St. Petersburg paradox [Menger (1934)]. The relevance of these boundedness arguments for the behavior of \( R(w) \), however, depends on \( U(\cdot) \) being defined on the domain \((0, +\infty)\), a requirement that can be safely dropped in most applications.

\(^7\) Note that, whereas CARA utility can be defined on \((-\infty, +\infty)\), CRRA utility is at most defined on \((0, +\infty)\). CARA and CRRA are special cases of the Hyperbolic Absolute Risk Aversion utility function.
2.3. Ranking distributions

As discussed, the choice problem under uncertainty can be thought of as a choice among distributions (lotteries), with risk-averse agents preferring distributions that are "less risky." But how can we rank distributions according to their riskiness? Earlier contributions tried to provide such ranking based on a univariate measure of variability, such as the variance or standard deviation [for example, the portfolio theory of Markowitz (1952) and Tobin (1958) relied on a mean-standard deviation approach]. But it was soon determined that, for arbitrary distributions, such ranking is always consistent with EU only if the vNM utility function is quadratic. Because of the restrictiveness of this condition, a more general approach has been worked out in what are known as the stochastic dominance conditions [Hadar and Russell (1969); Hanoch and Levy (1969); Rothschild and Stiglitz (1970)].

A distribution $F(w)$ is said to first-order stochastically dominate (FSD) another distribution $G(w)$ if, for every nondecreasing function $U(.)$, we have

$$\int_{-\infty}^{\infty} U(w) dF(w) \geq \int_{-\infty}^{\infty} U(w) dG(w).$$

It can be shown that under FSD one must have $F(w) \leq G(w)$ for all $w$, a condition that provides an operational way of implementing FSD. Thus, this condition captures the idea that more is better in the sense that any agents for which $w$ is a "good" should prefer $F(w)$ to $G(w)$. More to the point of choosing between distributions based on their riskiness, $F(w)$ is said to second-order stochastically dominate (SSD) another distribution $G(w)$ if the condition in (2.3) holds for every increasing and concave function $U(.)$ [such that any risk averter will prefer $F(w)$ to $G(w)$]. It can be shown that in such a case one has
for every \( w \). Thus, (2.4) provides an operational characterization of SSD that can be used to compare distributions. A closely related notion is that of a mean-preserving spread [Rothschild and Stiglitz (1970)], which consists of taking probability mass away from a closed interval and allocating it outside that interval so that the mean of the distribution is unchanged. It turns out that, if a distribution function \( G(.) \) can be obtained from \( F(.) \) by a sequence of such mean-preserving spreads, then \( F(.) \) SSD the distribution \( G(.) \). Thus, when \( F(w) \) and \( G(w) \) have the same mean, the notion of a mean-preserving spread is equivalent to that of second-order stochastic dominance.

One should note that FSD and SSD produce only partial ordering of probability distributions. It is quite possible for any two distributions that neither one stochastically dominates the other, so that we cannot know for sure which one would be preferred by a particular risk-averse agent. Still, stochastic dominance and mean-preserving spreads give a precise characterization of what it means to have an increase in risk, and these conditions have proved to be extremely useful in analyzing the economic impact of changes in risk [Rothschild and Stiglitz (1971)].

When the distributions being compared are restricted to belonging to a particular class, it turns out that the validity of ranking distributions based on their mean and standard deviation can be rescued. In particular, if all distributions being compared differ from one another by a location and scale parameter only [i.e., \( G(w) = F(\mu + \sigma w) \), where \( \mu \) and \( \sigma \) are the location and scale parameters, respectively], then, as Meyer (1987) has shown, the mean-standard deviation ordering of distributions is quite general, in the sense that it is equivalent (for this class of distributions) to second-order stochastic dominance ordering.\(^8\) The location-scale condition is restrictive (for example, it requires

\[ \int_{-\infty}^{w} [F(t) - G(t)] dt \leq 0 \]  

(2.4)

---

\(^8\) As argued by Sinn (1989), there seem to exist earlier statements of this result.
that an increase in variance occurs if and only if a mean-preserving spread occurs). Nonetheless, this condition applies to a number of interesting economic problems by the very definition of the problems themselves (for example, the theory of the competitive firm under price uncertainty) and also has some expositonal value as discussed by Meyer (1987).^9

3. The agricultural producer under uncertainty and risk aversion

The decision environment of agricultural producers is generally multifaceted and complex. Many distinct sources of risk may exist, and many discretionary actions may be available to the decision maker. Decisions and realizations of randomness may occur at several points in time. Further, actions may influence the distributions of yet-to-be realized random variables, while the realizations of random variables may alter the consequences of subsequent actions. To represent such an intricate network of interactions is analytically very difficult, but insights are possible by focusing on simpler stylized models. Thus, in the analysis that follows we start with an exceedingly simple model, and then gradually increase the complexity of the decision environment that we study. But first, an outline of model specifications that have the most relevance to agricultural decision making under uncertainty is in order.

3.1. Modeling price and production uncertainty

As outlined earlier, the main risks that a typical farmer faces are due to the fact that output prices are not known with certainty when production decisions are made and that the production process contains inherent sources of uncertainty (i.e., the relevant technology is stochastic). It is important, therefore, to understand how these fundamental sources of risk affect production decisions.

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^9 In any case, it should be clear that this result does not establish equivalence between EU and a linear mean-variance objective function, a criterion used in many agricultural economics applications.
To capture the essence of price risk for competitive producers, consider the problem of choosing output \( q \) to maximize \( E[U(w_0 + \hat{\pi})] \), where \( w_0 \) is the initial wealth and profit \( \hat{\pi} \) is random due to price uncertainty, that is,

\[
\hat{\pi} = \rho q - C(q, r) - K ;
\]

(3.1)

where \( \rho \) denotes output price, \( C(q, r) \) is the (variable) cost function (conditional on the vector of input prices \( r \)), and \( K \) represents fixed costs.\(^\text{10}\) This is essentially the model considered by Sandmo (1971), among others. Note that, because there is no production uncertainty in this model, the technology of production has been conveniently represented by the cost function \( C(q, r) \) so that the relevant choice problem can be couched as a single-variable unconstrained maximization problem.

When the production function is stochastic, it is clear that a standard cost function cannot represent the production technology [Pope and Chavas (1994)]. Thus, for the pure production uncertainty case, the production problem is best represented as that of choosing the vector of inputs \( x \) to maximize \( E[U(w_0 + \hat{\pi})] \), with random profit given by

\[
\hat{\pi} = p G(x; \tilde{e}) - r x - K ,
\]

(3.2)

where \( G(x; \tilde{e}) \) represents the stochastic production function by which realized output depends on the vector of inputs \( x \) and a vector of random variables \( \tilde{e} \). The latter represents factors that are important for production but are typically outside the complete control of the farmer (examples include weather conditions, pest infestations, and disease outbreaks). It is clear that, in general, the production uncertainty case is more difficult to handle than the pure price risk case. In particular, it is typically necessary to restrict one's attention to the special case where \( \tilde{e} \) is a single random variable. Versions

\(^{10}\) To emphasize and clarify what the source of uncertainty is in any particular model, the overstruck ~ will often be used to denote a random variable.
of this model have been studied by Pope and Kramer (1979) and MacMinn and Holtmann (1983), among others.

Because price and production uncertainty are both relevant to agricultural production, it seems that the relevant model should allow for both sources of risk. Essentially, this entails making price \( p \) a random variable in (3.2). Joint consideration of price and production risk turns out to be rather difficult. Some results can be obtained, however, if the production risk is multiplicative, an assumption that was systematically used by Newbery and Stiglitz (1981), by Innes (1990) and by Innes and Rausser (1989). Specifically, the production function is written as \( \bar{e} H(x) \), where \( \bar{e} \) is a non-negative random variable (without loss of generality, assume \( E[\bar{e}] = 1 \)), and so one chooses input vector \( x \) to maximize \( E[U(w_0 + \bar{\pi})] \) with random profit given by

\[
\bar{\pi} = \bar{p} \bar{e} H(x) - rx - K.
\]  

Obviously, if the analysis is restricted to the consideration of a single random variable \( \bar{e} = \rho \bar{e} \), it is clear that this model is isomorphic to the pure price risk case. In fact, as noted by a number of authors [Pope and Chavas (1994); Lapan and Moschini (1994); O'Donnell and Woodland (1995)], in this case there exists a standard cost function conditional on expected output, say \( C(\bar{q}, r) \) where \( \bar{q} \) is expected output,\(^{11}\) that is dual to the production technology. Hence, the decision problem under joint price and (multiplicative) production risk can also be expressed as a single-variable unconstrained optimization problem because random profit in (3.3) can be equivalently expressed as

\[
\bar{\pi} = \rho \bar{e} \bar{q} - C(\bar{q}, r) - K.
\]  

Before proceeding, we may note some restrictive features of the models just outlined. First, the models are static. There are essentially only two dates: the date at which decisions are made and the

\(^{11}\) Hence, for any given vector \( x \) of inputs, \( \bar{q} = H(x) \).
date at which uncertainty is realized (in particular, all decisions here are made before the resolution of uncertainty). Second, we are considering only one output and, for the time being, we are ignoring the possibility of risk management strategies. Although some of these assumptions will be relaxed later, such simplifications are necessary to get insights into the basic features of the production problem under risk.

In this setting, the basic questions that one may want to ask are: (i) How does the existence of uncertainty affect choice? (ii) Given uncertainty, how does a change in an exogenous variable affect choice? and (iii) To what extent does the existence of uncertainty alter the nature of the optimization problem faced by the decision maker? For three of the basic contexts that we have outlined above (pure price risk, pure production risk with only one random variable, and joint price and production risk with multiplicative production risk), the answers to these questions can be characterized in a unified framework.

3.2. Static models under risk neutrality

Section 2 presented some concepts concerning the effects of riskiness on the expected value of a function. The first- and second-derivatives of a function were found to be key in determining how shifts in a stochastic distribution affect the expected value of a function. Although the structure of risk preferences, as expressed by the utility function, is certainly of consequence in determining the effects of risk on choice, risk-neutral decision makers may also be influenced by risk. Consider an expected profit-maximizing producer who faces a profile of profit opportunities \( z(a, \beta, \varepsilon) \) where \( a \) is a vector of choices (actions) at the discretion of the producer, \( \beta \) is a vector of exogenous parameters, and \( \varepsilon \) is a single random variable that follows the cumulative distribution function \( F(\varepsilon) \). Without loss of generality, let \( \varepsilon \in [0, 1] \). The producer's problem is to
Max \[ a \int_0^1 z(a, \beta, \varepsilon) dF(\varepsilon), \] which yields the vector of first-order conditions \[ \int_0^1 z_a(a, \beta, \varepsilon) dF(\varepsilon) = 0, \] where \( z_a(\cdot) = \partial z(\cdot) / \partial a. \)

Assuming that the choice vector is a singleton, and given concavity of \( z(a, \beta, \varepsilon) \) in \( a \), from the concepts of stochastic dominance discussed earlier it is clear that an FSD shift in \( \varepsilon \) will increase optimal \( a \) if \( z_{ae}(a, \beta, \varepsilon) \geq 0 \) \( \forall \varepsilon \in [0, 1] \), whereas an SSD shift will increase optimal choice if, for all \( \varepsilon \in [0, 1] \), \( z_{ae}(a, \beta, \varepsilon) \geq 0 \) and \( z_{aee}(a, \beta, \varepsilon) \leq 0. \)

A specification of \( z(a, \beta, \varepsilon) \) which is of immediate interest is that of pure price risk as given by (3.1), where \( a = q \) and where the stochastic output price satisfies \( \bar{p} = \beta_1 + (\bar{\varepsilon} - \varepsilon)\beta_2 \) (here \( \bar{\varepsilon} = E[\varepsilon] \)). One may interpret \( \beta_1 + (\bar{\varepsilon} - \varepsilon)\beta_2 \) as a location and scale family of stochastic output price distributions with mean price equal to \( \bar{p} \geq 0 \), and the price variation parameter equal to \( \beta_2 \geq 0 \). Then the first-order condition for expected profit maximization is \( \bar{p} - C_q(q, r) = 0 \), and only the mean of the stochastic price is of relevance in determining optimal choice.

The more general form, where \( \varepsilon \) cannot be separated out in this manner, may arise when production is stochastic. Then, even if \( z_{ae}(\cdot) \geq 0 \), an increase in \( \varepsilon \) does not necessarily imply an increase in optimal \( a \). The stochastic shift in \( \varepsilon \) must be of the FSD dominating type, and an increase in the mean of \( \varepsilon \) is necessary but insufficient for such a shift to occur.

It is also interesting to note that, in this risk-neutral case, an increase in an exogenous variable, say \( \beta_i \), will increase optimal choice if \( z_{a\beta_i}(a, \beta, \varepsilon) \geq 0 \) \( \forall \varepsilon \in [0, 1] \), regardless of the distribution of \( \varepsilon \).
3.3. Static models under risk aversion

Given the payoff \( z(a, \beta, \varepsilon) \), the objective of a risk-averse producer is written as:

\[
\max_a \int_0^1 U[z(a, \beta, \varepsilon)] dF(\varepsilon),
\]

(3.7)

where \( U(.) \) is increasing and concave, profit \( z(.) \) is held to increase in \( \varepsilon \), and the objective function is concave in \( a \), i.e., \( \Delta = E\{ U_{zz}[\cdot] [z_a(.)]^2 + U_{z}[\cdot] z_{aa}(\cdot) \} < 0 \). Aspects of this problem, such as requirements on the nature of the utility function and payoff function and on the nature of the stochastic shift such that \( a \) increases, have been considered in some detail by Meyer and Ormiston (1983, 1985) and Eeckhoudt and Hansen (1992), among others. The first-order condition is

\[
\int_0^1 U_z[z(a, \beta, \varepsilon)] z_a(a, \beta, \varepsilon) dF(\varepsilon) = 0,
\]

(3.8)

with parameterized solution at the value \( a^* = a[F(\varepsilon), \beta] \).

3.3.1. Introduction of uncertainty

To ascertain how uncertainty affects choice for a risk averter, we will follow Krause (1979) and Katz (1981) and compare the solution under uncertainty with the solution when uncertainty is removed by setting the random element equal to its mean (i.e., setting \( \tilde{\varepsilon} = \bar{\varepsilon} \)). When uncertainty is removed, risk preferences are irrelevant, and the optimal choice \( \hat{a} \) satisfies \( z_a(\hat{a}, \beta, \bar{\varepsilon}) = 0 \). When uncertainty exists, on the other hand, then the first-order condition can be expressed as
\[
\text{Cov}[U_z(\cdot), z_d(\cdot)] + E[U_z(\cdot)]E[z_d(\cdot)] = 0.
\]

(3.9)

If \( z_{a_0}(\cdot) \geq 0 \), then the fact that the expectation of the product of two negatively covarying variates is less than the product of the expectations, together with risk aversion, implies that the covariance term must be negative. Because marginal utility is positive, satisfaction of the first-order condition requires that \( E[z_{a_0}(\cdot)] \geq 0 \) when \( z_{a_0}(\cdot) \geq 0 \). We wish to compare \( a^* \), the solution under uncertainty, with \( \hat{a} \). If \( z_{\Delta e}(\cdot) \leq 0 \), then Jensen's inequality implies \( E[z_{a}(\hat{\Delta}, \beta, \overline{\epsilon})] \leq 0 \). But we know that \( E[z_{a}(a^*, \beta, \overline{\epsilon})] \geq 0 \) given \( z_{a}(\cdot) \geq 0 \), and it follows that \( E[z_{a}(a^*, \beta, \overline{\epsilon})] - E[z_{a}(\hat{a}, \beta, \overline{\epsilon})] \geq 0 \). Because the only difference between the two expectations is the evaluation of \( a \), and because \( z_{a}(\cdot) \) is decreasing in \( a \), then \( a^* < \hat{a} \) [Krause (1979)]. The reduction in optimal \( a \) arises for two reasons.

First, even for a risk-neutral producer, the existence of uncertainty reduces input use because it decreases the expected marginal value of the input, \( E[z_{a}(\cdot)] \). Second, risk aversion means that the increase in utility associated with an increase in \( \overline{\epsilon} \) from \( \overline{\epsilon} \) is (in absolute value) lower than the decrease in utility associated with a decrease of the same magnitude in \( \overline{\epsilon} \) from \( \overline{\epsilon} \). Because \( z_{a\Delta e}(\cdot) \geq 0 \) (that is, an increase in \( a \) renders the payoff function more sensitive to the source of risk), the riskaverse producer will reduce sensitivity by decreasing \( a \).

For an expected utility maximizer with payoff (3.1) (i.e., a competitive producer under price uncertainty only), it is clear that \( z_{a\epsilon}(\cdot) = 1 \geq 0 \) and \( z_{a\Delta e}(\cdot) = 0 \leq 0 \), so that the existence of price uncertainty reduces production. For payoff (3.2) (i.e., a competitive producer with stochastic production), \( G_{a\epsilon}(\cdot) \geq 0 \) and \( G_{a\Delta e}(\cdot) \leq 0 \) are sufficient conditions to sign the impact of introducing uncertainty. For a detailed analysis of input choice under stochastic production for risk-averse agents
see Ramaswami (1992), who established requirements on an input-conditioned distribution function for a risk averter to choose less, or more, than an expected profit maximizer. A parallel analysis of equation (3.9) shows that when \( z_{ae}(\cdot) \leq 0 \) and \( z_{ae}(\cdot) \geq 0 \), then risk aversion implies \( a^* \geq \tilde{a} \). The price uncertainty payoff [equation (3.1)] never conforms to \( z_{ae}(\cdot) \leq 0 \), but the production uncertainty model may. Thus, we see that the impact of the existence of uncertainty on optimal choice by a risk averter depends upon second and third cross-derivatives of the payoff function.

### 3.3.2. Marginal changes in environment

We now look at marginal changes in the decision environment, as represented by an increase in \( \beta \).

Intuitively, we know that the conditions required to identify the effects of these marginal changes are likely to be more stringent than those required to sign the effects of introducing uncertainty.

Following Ormiston (1992), we differentiate equation (3.8) partially with respect to \( a \) and \( \beta \) to obtain

\[
\frac{da^*}{d\beta} = \frac{1}{\Delta} \int_0^1 A[z] z_{\beta}(\cdot) U_\sigma(z) z_a(\cdot) dF(e) - \frac{1}{\Delta} \int_0^1 U_\sigma(z) z_{a\beta}(\cdot) dF(e),
\]

where \( A[.] = -U_{z\sigma}[.] / U_{z\sigma}[.] \) is the absolute risk-aversion function defined earlier. Now we can partition the effect of \( \beta \) on \( a \) in three, which we will call (A) the wealth impact, (B) the insurance impact, and (C) the coupling impact [Hennessy (1998)]. The coupling impact is represented by the expression

\[
- \frac{1}{\Delta} \int_0^1 U_\sigma(z) z_{a\beta}(\cdot) dF(e) / \Delta \text{ in (3.10)}
\]

and has the sign of \( z_{a\beta}(\cdot) \) if this term is uniform in sign. If \( \beta \) acts to increase the marginal effect of \( a \) on payoff \( z(\cdot) \), then it will increase the producer’s disposition to use \( a \). For the price uncertainty case of (3.1) with \( \bar{\rho} = \beta_1 + (\bar{\tau} - \bar{\tau}) \beta_2 \), we have

\( z_{a\beta_1}(\cdot) = 1 \). For the production uncertainty case of (3.2), where \( \rho \) is a nonstochastic shift variable,
we have $z_{a\beta}(.) = G_{a}(.) > 0$.

Many agricultural support policies are constructed with the specific intent of having or not having a coupling effect. A price subsidy on an exogenous, institutional output or input quantity is decoupled in the sense that $z_{a\beta}(.) = 0$, whereas with a true price subsidy the actual quantity is coupled. As an illustration, a modification of specification (3.1) is

$z(a, \beta, \xi) = [\beta_{1} + (\xi - \bar{\xi})\beta_{2}] a - C(a, r) - K + \beta_{3} G(a^{0})$, where $G(a^{0})$ is some exogenous institutional reference production level. Here, $z_{a\beta_{1}}(.) \geq 0$, but $z_{a\beta_{2}}(.) = 0$. However, $z_{a\beta_{2}}(\cdot) = \xi - \bar{\xi}$ in this case, and this coupling effect does not have a uniform sign.

Effects (A) and (B) are intertwined in the first term on the right-hand side of (3.10). Let $y(., \xi) = z(., \xi) z^{*}(.)$, so the expression is $Q = \int_{0}^{1} J(., \xi) U_{\xi} [.] z_{a}(.) dF(\xi)/\Delta$. Integrating by parts yields

$$Q = \frac{1}{\Delta} \left[ J(., \xi) \int_{0}^{\xi} U_{\xi} [.] z_{a}(.) dF(\xi) \bigg|_{v=0}^{v=1} - \int_{0}^{\xi} U_{\xi} [.] z_{a}(.) dF(\xi) \frac{dJ(., \xi)}{dv} dv \right]$$

$$= -\frac{1}{\Delta} \int_{0}^{\xi} U_{\xi} [.] z_{a}(.) dF(\xi) \frac{dJ(., \xi)}{dv} dv,$$

(3.11)

where $v$ is used as the dummy variable of integration for the variable $\xi$. To identify effects (A) and (B) note that, if $z_{a\xi}(.) \geq 0$, the first-order condition (3.8) implies that the expression

$\int_{0}^{\xi} U_{\xi} [.] z_{a}(.) dF(\xi)$

is never positive because of the positivity of marginal utility and because $z_{a}(.)$ is negative at low $\xi$ and increases to be positive at high $\xi$. Therefore, given $\Delta < 0$, $Q$ is positive if $dJ(., \xi)/dv \leq 0$. Differentiate to obtain $dJ(., \xi)/dv = z_{a}(.) A_{\xi}[.] z_{q}(.) + A[.] z_{a\xi}(.)$. The first part of this expression may be called the wealth effect (A) because its negativity depends upon the NIARA...
property and the sign of \(z_\beta(.)\) (recall that \(z_e(.) \geq 0\)). All other things equal, if \(\beta\) shifts the
distribution of payoffs rightward \((z_\beta(.) \geq 0)\), as would be the case with a reduction in fixed costs \(K\) in payoff specifications (3.1) or (3.2), and if preferences are NIARA \((A_2[.] \leq 0)\), then \(a\) increases.

When \(\beta = \beta_1 + (\bar{\epsilon} - \bar{e})\beta_2\), then \(z_{\beta_1(.)} \geq 0\) for specification (3.1). Because \(z_{\alpha_2(.)} \geq 0\), both coupling and wealth effects act to increase optimal \(a\), and this is the Sandmo (1971) result that NIARA is sufficient for a shift in mean price to increase production. Notice that because \(z_{e_\beta_1(.)} = 0\), the second part of \(dJ(.)/dv\) may be ignored. Whereas \(\beta_1\) has both wealth and coupling effects, it is easy to describe a wealth effect that does not also involve coupling. Setting

\[
z(a, \beta, \bar{\epsilon}) = [(\beta_1 + (\bar{\epsilon} - \bar{e})\beta_2)] a - C(a, r) - K + \beta_3 G(a_0),
\]

an increase in \(\beta_3\) or a decrease in \(K\) induces an increase in optimal \(a\) under NIARA. Coupling may also occur without wealth effects, although this case is somewhat more difficult to show.

The second part, \(A[.]z_{\beta e(.)}\), is the insurance effect (B). If the favorable exogenous shift acts to stabilize income, that is if \(z_{\beta e(.)} \leq 0\) or \(\beta\) advances less fortunate states of the environment by more than it advances more fortunate states, then optimal \(a\) tends to increase. This would occur in specification (3.2) if \(\beta = p\) and \(G_{pe}(.) \leq 0\). In the case of an insurance contract on the source of uncertainty, say \(M(\beta, \bar{e})\), the payoff is \(pG(a, \bar{e}) - wa - K + M(\beta, \bar{e})\) and the insurance contract decreases risk if \(M_{\beta e(.)} \leq 0\) while \(pG_e(.) + M_e(.) \geq 0\). The similarity of wealth (i.e., risk aversion) and insurance effects has been discussed in detail by Jewitt (1987).

Because of the price uncertainty inherent in agricultural production environments, the effect of an increase in \(\beta_2\) for the specification (3.1), where \(\beta = \beta_1 + (\bar{\epsilon} - \bar{e})\beta_2\), is of particular importance.
From $z \beta_2 = 1$, it can be seen that the $\beta_2$ parameter has a negative insurance effect. It has already been concluded, however, that the coupling effect of $\beta_2$, that is $z \beta_2(\cdot)$, does not have a uniform sign. Thus, although it may be intuitive to expect that an increase in $\beta_2$ would decrease optimal $a$, to determine that requires more work in addition to the NIARA assumption [Batra and Ullah (1974); Ishii (1977)]. Since changing the parameter $\beta_2 \geq 0$ in this setting does not cover the set of all Rothschild and Stiglitz mean-preserving spreads, the above results do not demonstrate that all mean-preserving spreads of price decrease the optimal choice for the model in (3.1). Whereas Méyer and Ormiston (1989), Ormiston (1992), and Gollier (1995), among others, have made advances toward identifying precisely the set of spreads that act to decrease production for NIARA and various conditions on the payoff function, this problem has not yet been completely solved.\footnote{The conclusions drawn thus far are, of course, only relevant for the given context. Noting that peasants in less developed countries often consume a significant fraction of their own production, Finkelstein and Chalfant (1991) concluded that production and consumption decisions cannot be modeled separately for these agents. Their generalization of the Sandmo model suggests that production may plausibly increase under an increase in price uncertainty.}

3.3.3. Uncertainty and cost minimization.

It is well known that profit maximization is predicated upon satisfaction of the cost minimization assumption. Does cost minimization continue to hold under risk, when the objective is expected utility maximization? It turns out that the answer is yes, provided that "cost minimization" is suitably defined. Consider the competitive firm where the input vector $x$ is chosen to maximize $E[U(w_0 + \tilde{Y})]$, where $\tilde{Y} = R(x, \varepsilon) - r x$. Here $R(x, \varepsilon)$ is a revenue profile (that can accommodate both price and/or production uncertainty) and $\varepsilon$ denotes the source of revenue uncertainty. Pope and
Chavas (1994) show that, if the revenue profile satisfies the restriction \( R(x, \bar{e}) = K(\psi(x), \bar{e}) \), where \( \psi(x) \) is (possibly) vector-valued, then the relevant cost function can be written as \( C(q^\psi, r) \), where \( q^\psi \) is the vector of conditioning values corresponding to the function \( \psi(x) \). Hence, technical efficiency is satisfied in the sense that the EU maximizing choice of \( x \) is consistent with the cost minimizing means of obtaining some (vector) level of \( \psi(x) \). The simplest special case arises with multiplicative production risk, when \( R(x, \bar{e}) = H(x) \bar{e} \). As anticipated in section 3.1, in such a case the cost function is written as \( C(q, r) \), where \( q \) is expected output. Thus, the relevant cost function for this special case is rather standard, with the expected output level playing the role of a deterministic output level under certainty. More generally, however, a vector of conditioning values will be needed. For example, if there is no price risk but the production function has the stochastic form suggested by Just and Pope (1978) (to be discussed further in section 4.2), then revenue is written as

\[
R = pM(x) + p [V(x)]^{1/2} \bar{e} \quad \text{with} \quad E[\bar{e}] = 0.
\]

It follows that the EU-consistent cost function here can be written as \( C(\bar{q}, \sigma^2, r) \), where \( \bar{q} \) is a level of expected output [corresponding to the function \( M(x) \)] and \( \sigma^2 \) is a level of output variance [corresponding to the function \( V(x) \)].

That cost minimization always holds for EU maximizers, even when the revenue profile does not satisfy the restriction invoked by Pope and Chavas (1994), is shown by Chambers and Quiggin (1998). Their approach is best illustrated for the production uncertainty case in which the random variable \( \bar{e} \) takes on a finite number (say \( N \)) of values. Given the stochastic production function \( G(x, \bar{e}) \), then realized output for any given realization of the random variable \( (e_i, \text{say}) \) is \( q_i = G(x, e_i) \).

If \( \ell_i \) denotes the probability of \( e_i \) occurring, then the producer's EU problem is:
Now define a cost function $C(q_1, \ldots, q_N, r)$ as

$$C(q_1, q_2, \ldots, q_N, r) = \min_x \{ rx: q_i \leq G(x, e_i), \forall i = 1, 2, \ldots, N \}. \quad (3.13)$$

One may note the formal similarities of $C(q_1, \ldots, q_N, r)$ with a standard multioutput cost function, although the interpretation here is rather different. At any rate, it follows that the producer’s EU maximization problem can be equivalently expressed as

$$\max_{q_1, q_2, \ldots, q_N} \sum_{i=1}^{N} t_i U(pG(x_i, e_i) - rx). \quad (3.14)$$

Thus, it is clear that EU maximizers do minimize costs, in some sense.

3.4. Dynamics and flexibility under uncertainty

A consideration of decision making under risk is not complete without discussion of the interactions between risk and time. Although suppressed in the two dates (one period) models discussed above (i.e., action at time 0 and realization at time 1), the fact is that time and uncertainty are intertwined because information sets become more complete as time passes. To illustrate, we consider a simple extension of the price uncertainty case of model (3.1). Specifically, let $a = (x_1, x_2)$ such that $z(a, \beta, \varepsilon)$ is of form $\varepsilon R(x_1, x_2) - r_1 x_1 - r_2 x_2$ where $\varepsilon$ represents stochastic output price, and assume that $x_1$ is chosen before the realization of $\varepsilon$, whereas $x_2$ is chosen after $\varepsilon$ is observed. Following Hartman (1976), the problem may be posed as
Applying backward induction, the second-stage problem is solved first. The first-order condition is
\[ \epsilon R_{x_2}(x_1, x_2) = r_2, \]
where \( x_1 \) and \( \epsilon \) are now predetermined. Assuming strict concavity of \( R(.) \) in \( x_2 \), the first-order condition is solved to yield \( x_2^* = S(x_1, r_2, \epsilon) \). Given this short-run demand function for \( x_2 \), the producer problem reduces to
\[ \text{Max} \quad \int_0^1 \text{Max} \left[ \epsilon R(x_1, x_2) - r_2 x_2 \right] dF(\epsilon) - r_1 x_1. \]  
(3.15)

Defining \( L(x_1, r_2, \epsilon) = \epsilon R(x_1, S(x_1, r_2, \epsilon)) \), the envelope theorem gives the first-order condition for the first-stage problem (choosing \( x_1 \) as
\[ \int_0^1 L(x_1, r_2, \epsilon) dF(\epsilon) - r_1 = 0. \]  
(3.16)

Now, the Rothschild and Stiglitz mean-preserving spread condition implies that optimum \( x_1 \) increases with such a spread if \( L_{x_1 \epsilon}(x_1, r_2, \epsilon) \geq 0 \). Setting optimum output as \( G^*(x_1, r_2, \epsilon) \), the envelope theorem can be used to show that this is the same as requiring \( G_{x_1 \epsilon}^*(x_1, r_2, \epsilon) \geq 0 \). Further analysis reveals that this condition is equivalent to the requirement that \( \partial[R_{x_1 x_2}(.)/R_{x_2 x_2}(.)]/\partial x_2 \leq 0 \). Thus, when ex-post flexibility exists, the effects of uncertainty depend upon relationships between third derivatives of the production technology. In general, although the impact of a mean-preserving spread in \( \epsilon \) on \( x_2^* \) depends upon the sign of \( \partial^2 S(.)/\partial \epsilon^2 \), the impact on \( x_1 \) is less readily signed
whereas the effect on mean $R(.)$ is yet more difficult to sign. Obviously, the analysis becomes even more involved when decision makers are assumed to be risk averse.

A second set of problems, called real option problems because of structural analogies with financial options, arise from the interactions between time and uncertainty in long-term investment decisions when there are sunk costs or irreversible actions. Consider a decision in 1999 to invest in precision farming education and equipment. At that time it was not yet clear whether the technology was worth adopting. The decision maker may invest early in the hope that the technology will turn out to be profitable. But the investment may turn out to be unprofitable, so there is also an incentive to defer the decision for a year, say, to learn more about the technology in the intervening period. But deferment will mean losing a year of additional profits if the technology turns out to be profitable. Similar sunk cost and information problems may arise in a number of other farm production decisions. Although real option problems such as these can be addressed by rigorous stochastic neoclassical models [e.g., Chavas (1994) or Feinerman, Choi, and Johnson (1990)] or by standard optimal control approaches [Rausser and Hochman (1979)], the more structured contingent claims approach popularized by Dixit and Pindyck (1994) has assumed prominence because it lends itself to empirical and theoretical analysis.

A stylized continuous-time variant of dynamic programming, real option theory connects time and uncertainty by modeling a source of randomness as a stochastic process evolving over time. Some such processes give rise to differential equation relationships between the distribution, time, and the flow of rewards. These relationships can be solved to give a decision-conditioned expected present value, and this expected present value is then optimized over the choice set. The choice set may involve deciding to invest now or to wait, or deciding how much to invest. Marcus and Modest (1984) studied optimal decisions for producers facing price and yield uncertainty and using futures markets, whereas Turvey (1992b) used the approach to study agricultural support policies in Canada.
Purvis et al. (1995) adopted the framework to explain Texas dairy industry technology adoption decisions under cost and regulatory uncertainty, and found that the expected rate of return on the proposed investment might have to be double the threshold identified by a nonstochastic analysis for the decision to be attractive. The approach also provides a simple way of studying adjustment costs. For example, Leahy (1993) studied shutdown and startup costs for a competitive firm facing random prices.

4. Selected empirical issues

Our cursory review thus far has privileged analytical methods and theoretical analyses. But considerable empirical research in agricultural economics has been done to test, quantify, and otherwise put to use a number of features of risk models. In this section we will look, in some detail, at a number of contributions that have had a primarily empirical bent.

4.1. Identifying risk preferences

In an early empirical study of agricultural decision making under risk, Lin, Dean and Moore (1974) elicited preferences over hypothetical lotteries from managers of six large Californian farms. Using quadratic programming methods, they estimated the mean-variance frontier available to the farmer. They then compared the farm plans suggested by the elicited preference structure with plans suggested by the expected profit maximization rule, with plans suggested by lexicographic preference structures, and with the actual implemented plans. They found that, although no stylized preference structure was clearly a superior fit, for each of the six farms the EU framework performed at least as well as the other paradigms. For Nepalese rice farmers, Hamal and Anderson (1982) also used hypothetical lotteries and found evidence in support of DARA. The analysis was less conclusive concerning the slope of relative risk aversion.
Dillon and Scandizzo (1978) modified the approach of Lin, Dean and Moore (1974) by eliciting preferences from a relatively large number of subsistence farmers and sharecroppers in northeastern Brazil. Risk attitudes were imputed from choices between hypothetical lotteries that realistically reflected the farm payoffs faced by these decision makers. Unlike the study by Lin, Dean and Moore, however, the hypothetical decisions were not validated through comparison with actual decisions. The lotteries posed were of two types; those in which the family subsistence requirement was covered but surplus income was at risk, and those in which the subsistence requirement was also at risk. Hypothetical returns were adjusted until certainty equivalence between lottery comparisons was established. The replies were then fitted to three decision criteria: mean-standard deviation, mean-variance, and CARA-expected utility objective functions. As expected, both farmers and sharecroppers tended to be more risk averse when subsistence income was at risk. Surprisingly, smallholders tended to be more risk averse than sharecroppers. Dillon and Scandizzo (1978) found less clear evidence about the impact of socioeconomic factors on risk attitudes. Perhaps the most interesting indication was that, even within seemingly homogeneous groups, a wide dispersion of risk preferences appeared to exist.

Taking an econometric approach, Moscardi and de Janvry (1977) estimated a Cobb-Douglas production function for corn with data from small Mexican subsistence farms. Using a safety-first framework, they imputed a measure of risk aversion from the divergence between actual fertilizing decisions and optimal decisions under risk neutrality. They found evidence of considerable risk aversion, and they also suggested that risk attitudes might be functions of socioeconomic variables (such as family size and age of operator) that may evolve over time. Brink and McCarl (1978) also estimated risk attitudes as a residual that rationalizes observed choices relative to "optimal" ones as predicted by a mathematical programming model (relying on a linear mean-standard deviation objective function). Thirty-eight mid-western crop producers at a Purdue University decision analysis
workshop listed their resources and identified their preferred crop acreage allocation plan. The risk parameter giving a plan deemed closest to the announced plan was assumed to represent risk preferences. The analysis concluded that risk aversion seemed to be low. Measuring risk essentially as a residual, however, is an obvious limitation of these studies (because such a procedure ignores other potential reasons for observed decisions to depart from the model’s optimal decisions.)

Because of the limitations of inferring risk from observed production decisions, and because hypothetical payout surveys can give unstable results, Binswanger (1980) made real payments to peasant farmers in India. Outcomes were determined by tossing a dice, and the amount at risk varied from 0.5 rupees to 500 rupees (negative payout states were not considered). The 500 rupees payout amounted to about 2.3 percent of average household wealth, and corresponded in magnitude to substantial fertilization investments. (It was believed that some households were constrained by capital resources from fertilizing adequately.) Preliminary tests found that individuals tended to treat money gifted to them on the day of the experiment for the purpose of participating in the experiment as if it were their own. Preliminary results also suggested that once lotteries for low gambles had primed individuals to making lottery decisions about real money, then a hypothetical 500 rupees game appeared to give results that were statistically similar to a real 500 rupees game. To conserve financial resources, the hypothetical 500 rupees game was used thereafter.

Capturing risk attitudes by the coefficient of partial risk aversion, it was found that subjects tended to become more risk averse as the size of the gamble increased.\(^{13}\) Compared with the hypothetical scenario interviewing method, the imputed risk aversion coefficient was less dispersed when real money was involved. This would suggest that the interviewees may have had difficulty taking the interviews as seriously as they would real-world decisions. On the effects of

\[^{13}\text{The coefficient of partial risk aversion is defined as } \frac{-\pi}{U''(\tilde{\pi} + w_0)} U'(\tilde{\pi} + w_0), \text{ where } w_0 \text{ is initial wealth and } \tilde{\pi} \text{ is profit.}\]

30
socioeconomic characteristics, Binswanger (1980) found that wealthier, better educated, and more progressive farmers tended to be less risk-averse, as did those who had off-farm salaries. Prior luck in the game also tended to reduce the degree of risk aversion (only the luck regressor, however, had consistently high $t$ statistics across all gamble sizes). Overall, Binswanger interpreted the results as being supportive of the hypothesis that it is resource- and infrastructural constraints, such as access to information and credit, that induce caution among peasants rather than the hypothesis of innate conservatism.

In a different analysis of these Indian data, Binswanger (1981) considered the foundations of the EU framework and concluded that decision makers did not integrate possible outcomes from a gamble with pre-existing income, but rather treated them separately in their decision calculus. This conclusion is somewhat at variance with Binswanger's (1980) conclusion from pretest analysis that subjects treated gifted money as their own. The separation of gamble money from pre-existing wealth lends some support to Kahneman and Tversky's (1979) prospect theory approach to decision making. Failure of income integration has serious implications for modeling decisions, but had generally been ignored in the empirical literature. Binswanger also used inferences drawn from safety-first type models to identify inconsistencies with the data, and he concluded that the decision makers did not appear to act in a safety-first manner. Finally, Binswanger identified evidence in the data to support both DARA and decreasing relative risk aversion (DRRA) preferences.

Surveying work on risk preferences and risk management to that time (including work by Binswanger already cited), Young (1979) and Hazell (1982) raised concerns about all approaches. The direct elicitation (interview) method is reliable only to the extent that it captures the preference structure that would be used in real decisions, and evidence suggested that it might not do so.
Experimental approaches might be too expensive to implement in developed countries.\textsuperscript{14} Approaches based on observed supply and input demand behavior impute risk as the residual component explaining discrepancies between expected profit-maximizing solutions and actual decisions. But discrepancies may be due to other effects, such as imperfect information and heterogeneous resource endowments. To the extent that such research had identified determinants of risk preferences, Young concluded that farmers in developing countries appeared to be more risk averse than those in developed countries, and he observed that this conclusion is consistent with DARA. But because the studies considered did not explicitly control for the availability and use of risk management institutions, which tend to be more widely available in developed countries, developed-country farmers may appear to be less risk averse than they actually are.

Returning to the task of econometrically estimating risk structures, Antle (1987) expressed the optimality conditions for EU maximizing choices in terms of a given individual’s absolute risk aversion and downside risk aversion coefficients.\textsuperscript{15} The Generalized Method of Moments (GMM) procedure was then applied to identify means, variances, and covariances of risk preference parameters based on data from the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT) pertaining to one of the six Indian villages (Aurepalle) that had been considered by Binswanger (1980, 1981). Antle (1987) found a mean Arrow-Pratt index similar to that reported in Binswanger (1980). Dissatisfied that this approach required some, if only minimal, assumptions concerning the technology available, Antle (1989) developed a method that did not involve joint estimation with technology. Antle’s view was that it would be better to estimate risk preference.

\textsuperscript{14} Binswanger estimated that, were he to run his experiments in the United States, it would have cost $150,000 (circa 1978) rather than $2,500.

\textsuperscript{15} This downside risk aversion coefficient is defined as $\frac{U''''(\cdot)}{U'(\cdot)}$. Note that $U''''(\cdot) > 0$ is necessary for DARA.
structures separately from technology rather than jointly. His concerns about a joint estimation arose mainly from problems involving the data required for the estimation of technology, and the discontent with alternative econometric approaches to joint estimation. The econometric methods applied again involved GMM estimation on data from the ICRISAT India village study. The means of the Arrow-Pratt and downside risk aversion indices were, as expected, similar to those estimated earlier.

Among other econometric estimations of risk attitudes, Myers (1989) assumed CRRA and joint lognormality of the distributions of output price and producer consumption, and developed a reduced-form rational expectations approach to testing for the aggregate level of relative risk aversion for U.S. producers who store crops. Annual data over the period 1945 to 1983 suggest a coefficient of relative risk aversion between 1.5 and 4.5 for corn and wheat storers, but the estimates for soybeans are implausible. Exploiting technical attributes of CRRA and of constant partial relative risk aversion (CPRRA), Pope (1988) developed implications for optimal choices by individuals expressing such preferences. In Pope and Just (1991), these implications, together with implications for choice under CARA preferences, were tested on state-level Idaho potato acreage data. CARA and CPRRA hypotheses were rejected, but CRRA was not. Chavas and Holt (1990), studying U.S.-level corn and soybean acreage allocation decisions, also used the tests proposed by Pope (1988) and rejected both CRRA and CPRRA. Testing for the impact of wealth, proxied by an index of proprietor equity, on allocation decisions, they found evidence to reject CARA in favor of DARA.

4.2. Estimating stochastic structures

As mentioned earlier, production risk is an essential feature of agriculture, and estimation of such stochastic production structures has obvious immediate interest for farm management as well as to

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16 This means that $-\pi U''(\pi + w_0)/U'(\pi + w_0)$ is invariant to changes in $\pi$ for the level of $w_0$ in question.
address agricultural policy issues. For example, production uncertainty has implications for the implementation of crop insurance. Also, environmental externalities such as water contamination and ecosystem destruction may sometimes be traced back to the use of such agricultural inputs as nitrogen and pesticides; production uncertainty, together with risk aversion, may increase application of these inputs. Existing statistical procedures for studying relationships between stochastic distributions have tended to emphasize stochastically ordered comparisons, such as first- and second-degree dominance, between elements in a set of distributions. But economists, especially agricultural economists, are often interested in conditional relationships. To reconstruct nonparametric stochastic relationships between crop yield and input use would often require volumes of data beyond that usually available to analysts. Further, as the literature on the impacts of stochastic shifts on decisions has shown, the necessary and the sufficient conditions for a stochastic shift to have a determinate impact on the decisions of a meaningful class of decision makers are generally not among the simpler types of stochastic shifts.

The complexity of the decision environment is substantially reduced if one can treat technology as being nonrandom. If one is primarily concerned with price uncertainty, then it might be convenient to assume deterministic production. Thus, one can estimate the distribution of the realized random element without regard to the choices made. In other cases, however, it is not possible to simplify the decision environment in this way. Although random yield—the consequence of interactions between choices and random weather variables—can be measured, it would be more difficult to measure and aggregate in a meaningful manner the various dimensions of weather. In such a case, it is more convenient to estimate the input-conditioned distribution of yield. Although they do not lend themselves to estimating or testing for general production function relations, existing stochastic ordering methods can be useful in testing for the nature of and impacts of exogenous stochastic shifts in, say, the distribution of output price, and for studying discrete decisions such as the adoption of a
new technology.

Although studies applying stochastic dominance methods to agricultural problems are numerous [e.g., Williams et al. (1993)], most of these studies compare point estimates of the distributions and do not consider sampling errors. Tolley and Pope (1988) developed a nonparametric permutation test to discern whether a second-order dominance relationship exists. More recently, Anderson (1996) used the nonparametric Pearson goodness-of-fit test on Canadian income distribution data over the years 1973 to 1989 to investigate, with levels of statistical confidence, whether first-, second-, and third-order stochastic dominance shifts occurred as time elapsed.

For input-conditioned output distributions, Just and Pope (1978) accounted for heteroskedasticity by developing a method of estimating a two-moment stochastic production function by three-stage nonlinear least squares techniques. The function is of the form

\[ q = M(x) + [V(x)]^{1/2} \bar{\varepsilon}, \]  

(4.1)

where \( q \) is output, \( E[\bar{\varepsilon}] = 0, \) \( \text{Var}[\bar{\varepsilon}] = 1, \) and \( x \) is a vector of input choices. The functions \( M(x) \) and \( V(x) \) determine the conditional mean and variance of \( q \), respectively, and can be chosen to be sufficiently flexible to meet the needs of the analysis. Just and Pope (1979) applied their method to Day's (1965) corn and oats yield-fertilization data set, and found the results generally, but not totally, supportive of the hypothesis that an increase in fertilization increases the variance of output. Their readily estimable approach has proven to be popular in applied analyses. For example Traxler et al. (1995) used the approach in a study of the yield attributes of different wheat varieties in the Yaqui Valley (Mexico), and found that whereas earlier varietal research appeared to emphasize increasing mean yield, later research appeared biased toward reducing yield variance.

Suggesting that mean and variance may not be sufficient statistics to describe stochastic production, Antle and Goodger (1984) established a method for estimating an arbitrarily large number
of input-conditioned moments. Applying their approach to large-scale California milk production, they rejected the statistical hypothesis that input-conditioned mean and variance are sufficient statistics. An interesting simulation finding was that a CARA decision maker facing the estimated technology substantially increased dairy rations relative to a risk-neutral decision maker. This suggests that the marginal risk premium in Ramaswami (1992) may be negative on occasion.

Nelson and Preckel (1989) identified the need for a flexible approach to estimating parametric yield distributions when accommodating skewness is important. Gallagher (1987), among others, has observed negative skewness for crop yields. The Just-Pope approach is insufficiently flexible, whereas the Antle-Goodger method, which is nonparametric, may be inefficient. Finding inspiration in Day's (1965) suggestion that the beta distribution would likely fit most yield distributions quite well, Nelson and Preckel conditioned beta distribution parameters on input choices. The output density function is then

\[
f(q|x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{(q - q^{\text{min}})^{\alpha-1}(q^{\text{max}} - q)^{\beta-1}}{(q^{\text{max}} - q^{\text{min}})^{\alpha+\beta-1}},\]

where \(\Gamma(.)\) is the gamma function, output \(q\) is supported on the interval \([q^{\text{min}}, q^{\text{max}}]\), and the distribution parameters are conditional on inputs, i.e., \(\alpha = \alpha(x)\) and \(\beta = \beta(x)\). For field-level corn yields in five Iowa counties over the period 1961 to 1970, Nelson and Preckel set \(y^{\text{min}} = 0\), and let both \(\alpha(x)\) and \(\beta(x)\) be Cobb-Douglas functions of nitrogen, phosphorus, potassium, field slope, and soil clay content. Using a two-stage maximum likelihood method, they found that the marginal effects of nitrogen, phosphorus, and potassium on skewness, variance, and even mean were mixed in sign.

The maximum likelihood approach to estimating parameterized conditional densities has proven to be quite popular. A gamma distribution relationship between applied nitrogen levels and late spring
soil nitrate levels has been used in Babcock and Blackmer (1992) to study the effects of information concerning spring soil nitrate levels on subsequent side-dressing and on expected profit; a beta distribution has been applied by Babcock and Hennessy (1996) to study input use in the presence of crop insurance. A different line of inquiry has sought to model the nonnormality of crop yield distributions by estimating transformations of the normal distribution. Taylor (1990) employed a hyperbolic trigonometric transformation to deviations from a linear yield trend estimation on corn, soybean, and wheat crops. Moss and Shonkwiler (1993) and Ramírez (1997) have extended this approach to accommodate stochastic yield trends and multivariate distributions, respectively. But the presumption that yields are not normally distributed has been called into question by Just and Weninger (1999), who criticize a number of features of statistical analyses implemented by previous studies and conclude that the empirical evidence against normality is weak.

Stochastic production has implications for the estimation of dual representations of production technologies. For example, as discussed in section 2.3.3, when the production function is affected by multiplicative risk and producers maximize expected utility the relevant cost function is $C(\tilde{q}, r)$, where $\tilde{q}$ is expected output. When the stochastic production function is written more generally as $G(x, \varepsilon)$, the relevant cost function still has the structure $C(\tilde{q}, r)$ if producers are risk neutral (they maximize expected profits).\(^\text{17}\) Pope and Just (1996) call such a function the "ex-ante cost function," and convincingly argue that a number of previous studies have resulted in inconsistent estimates of technological parameters because they have estimated a standard cost function $C(q, r)$ (conditional on realized output $q$) when in fact they should have been estimating $C(\tilde{q}, r)$. Estimation of the ex-ante cost function $C(\tilde{q}, r)$ is problematic, on the other hand, because it is conditional on expected output

\(^{17}\) Of course, in such a case the parameters of the cost function $C(\tilde{q}, r)$ may include parameters of the distribution of the random variable $\varepsilon$. 37
$\bar{q}$, which is not observable. The solution proposed by Pope and Just (1996) entails estimating $\bar{q}$ jointly with the structure of the ex-ante cost function. The specific procedure that they suggest fails to achieve consistent estimation of technological parameters because it does not address the nonlinear errors-in-variables problem that typically arises in this context [Moschini, (1999)]. But by exploiting the full implications of expected profit maximization, Moschini (1999) shows that it is possible to effectively remove the errors-in-variables problem and obtain consistent estimation of the ex-ante cost function parameters.

4.3. Joint estimation of preferences and technology

Most research studies considered thus far have sought to identify risk preferences without estimating the source of randomness, or they have sought to estimate the source of randomness without simultaneously estimating the risk preference structure. Those papers that have simultaneously identified risk preferences and the source of randomness [e.g., Moscardi and de Janvry (1977) or Antle (1987)] have treated either one or both components in a rather elementary manner. Separating the estimation of the two structures is econometrically inefficient to the extent that a joint estimation imposes cross-estimation restrictions and accommodates error correlations. Using a Just-Pope technology with Cobb-Douglas mean and variance functions together with a CARA risk preference structure, cross-equation restrictions and a nonlinear three-stage least squares estimator, Love and Buccola (1991) applied a joint estimation for Iowa corn and soybean production. The data pertained to three of the five counties studied by Nelson and Preckel (1989). Love and Buccola found considerable variation in the estimated coefficient of risk aversion across the three Iowa counties under consideration. Concerning technology, they contrasted their results with a straightforward Just-Pope estimation and with the Nelson and Preckel analysis to find that each estimated similar technology structures.
The Love and Buccola approach is restrictive in the sense that CARA was imposed. Chavas and Holt (1996) developed a joint estimation method that is able to test for CARA or DARA. Applying their estimator to corn and soybean acreage allocation in the United States, and on a data set much the same as that used in their 1990 work, they assumed that the production technology was a quadratic function of allocated acres and that the utility function is

\[ u(\pi_t, t) = \int_L^\infty \exp(\alpha_0 + \alpha_1 \pi + \alpha_2 \pi^2 + \alpha_3 t) \, dz \],

where \( \alpha \) are parameters to be estimated, \( \pi \) is profit in year \( t \), and \( z \) is a dummy variable of integration. Their analysis found strong statistical evidence for the presence of downside risk aversion and for rejecting CARA in favor of DARA.

Although the approach by Chavas and Holt does generalize the representation of risk preferences, the assumed technology was not flexible in the Just-Pope sense. Further, their specification can say little about the impact of relative risk aversion. Using Saha's (1993) expo-power utility specification,

\[ U[\pi] = -\exp(-\beta \pi^\alpha) \]

where \( \alpha \) and \( \beta \) are parameters to be estimated, Saha, Shumway and Talpaz (1994) assumed a Just-Pope technology and jointly estimated the system using maximum likelihood methods. Data were for fifteen Kansas wheat farms over the four years 1979 to 1982, and there were two aggregated input indices in the stochastic technology (a capital index and a materials index). The results supported the hypotheses of DARA and increasing relative risk aversion (IRRA). Also, the materials index was found to be risk decreasing, so risk-averse agents may have a tendency to use more fertilizer and pesticides than risk-neutral agents.

Before leaving the issue of risk estimation, a comment is warranted about subsequent use of the estimates. There may be a tendency on the part of modelers engaged in policy simulation to use without qualification risk preference structures that were identified in previous research. Newbery and Stiglitz (1981, p. 73) have shown that caution is warranted in accommodating the particular
circumstances of the simulation exercise. One must ensure that the chosen risk preference structure is consistent with reasonable levels of risk premia for the problem at hand. The set of coefficients of absolute risk aversion that give reasonable risk premia vary from problem to problem.

4.4. Econometric estimation of supply models with risk

One of the most widely agreed upon results from the theory of the firm under price uncertainty is that risk affects the optimal output level. Normally, the risk-averse producer is expected to produce less than the risk-neutral producer, ceteris paribus, and the risk-averse producer will adjust output to changing risk conditions (e.g., decrease production as risk increases). Econometric studies of agricultural supply decisions have for a long time tried to accommodate these features of the theory of the firm. There are essentially two reasons for wanting to do so: first, to find out whether the theory is relevant, i.e., to "test" whether there is risk response in agricultural decisions; second, assuming that the theory is correct and risk aversion is important, accounting for risk response may improve the performance of econometric models for forecasting and/or policy evaluation, including welfare measurement related to risk bearing.

To pursue these two objectives, a prototypical model is to write supply decisions at time \( t \) as

\[
y_t = \beta_0 + x_t' \beta_1 + \beta_2 \mu_t + \beta_3 \sigma_t^2 + e_t, \tag{4.3}
\]

where \( y \) denotes supply, \( \mu \) denotes the (subjective) conditional expectation of price, \( \sigma^2 \) denotes the (subjective) conditional variance of price, \( x \) represents the vector of all other variables affecting decisions, \( e \) is a random term, \( t \) indexes observations, and \( (\beta_0, \beta_1, \beta_2, \beta_3) \) are parameters to be estimated (\( \beta_1 \) is a vector). Clearly, this formulation simplifies theory to the bone by choosing a particular functional form and, more important, by postulating that mean and variance can adequately
capture the risk facing producers. Whereas more sophisticated models may be desirable, from an econometric point of view equation (4.3) is already quite demanding. In particular, the subjective moments of the price distribution $\mu_t$ and $\sigma^2_t$ are unobserved, and thus to implement equation (4.3) it is necessary to specify how these expectations are formed.

The specification of expectations for the first moment is a familiar problem in econometric estimation. Solutions that have been proposed range from naive expectations models (where $\mu_t = p_{t-1}$), to adaptive expectations (where $\mu_t$ is a geometrically weighted average of all past prices), to rational expectations (where $\mu_t$ is the mathematical expectation arrived at from an internally consistent model of price formation, for example). A review of price expectations formation for price levels is outside the scope of this chapter, but we note that, not surprisingly, parallel issues arise in the context of modeling variance. Behrman (1968) allowed for price risk to affect crop supply in a developing country by measuring $\sigma^2_t$ as a three-year moving average (but around the unconditional mean of price). Similar ad hoc procedures have been very common in other studies, although often with the improvement of a weighted (as opposed to simple) average of squared deviations from the conditional (as opposed to unconditional) expectation of the price level [e.g., Lin (1977); Traill (1978); Hurt and Garcia (1982); Sengupta and Sfeir (1982); Brorsen, Chavas and Grant (1987); Chavas and Holt (1990, 1996)]. A more ambitious and coherent framework was proposed by Just (1974, 1976), whereby first and second moments of price are modeled to the same degree of flexibility by extending Nerlove's (1958) notion of adaptive expectations to the variance of price. This procedure has been used in other studies, including Pope and Just (1991), Antonovitz and Green (1990), and Aradhyula and Holt (1990). More recently, advances have been made by modeling the time-varying variance within the autoregressive conditional-heteroskedasticity (ARCH) framework of

The empirical evidence suggests that risk variables are often significant in explaining agricultural production decisions. The early work by Just (1974), as well as some other studies, has suggested that the size of this supply response to risk may be quite large, but the quantitative dimension of this risk response is more difficult to assess because results are typically not reported in a standardized manner. For example, an interesting question in the context of supply response concerns the size of the likely output contraction due to risk. As model (4.3) suggests, an approximate estimate of this output reduction (in percentage terms) is simply given by the elasticity of supply with respect to the price variance $\sigma_t^2$, but this basic statistic often is not reported. As a yardstick, however, we note that for broiler production Aradhyula and Holt (1990) found a long-run price variance elasticity of -0.03, whereas for sow farrowing, the comparable long-run elasticity estimated by Holt and Moschini (1992) was -0.13.

Although such estimates may suggest a fairly sizeable production response to the presence of risk, caution is in order for several reasons. First, as is often the case in applied economic modeling, these empirical results are drawn from models that are based on individual behavior but that are estimated with aggregate data without explicit consideration of aggregation conditions. Second, insofar as producers use appropriate risk management procedures (see section 5), the conditional variance typically used may not be measuring the relevant risk. Finally, estimating response to conditional variance is inherently difficult. To illustrate this last point, consider the adaptive expectation approach that specifies the (subjective) conditional mean and the conditional variance as follows:

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18 For example, a producer facing price risk and using futures contracts optimally to hedge risk would be exposed only to residual basis risk, and conceivably that is what the variance terms should measure.
where usually $\lambda \in (0,1)$ and $\phi \in (0,1)$. These parameterizations are appealing because they make the unobservable variable a function of past realizations (which are, at least in principle, observable) in a very parsimonious way. It is known that the assumption of adaptive expectations for the mean of price is rather restrictive, and it turns out that such an assumption for the variance is even more restrictive.

By definition, if $\mu_t$ denotes the agent's conditional expectation of price, then a price-generating equation consistent with the agent's beliefs is $p_t = \mu_t + u_t$, where $u_t$ is a random term with a zero conditional mean. Hence, an equivalent way of saying that the producer's expected price is formed adaptively as in equation (4.4) is to say that the producer believes that price is generated by

$$p_t = p_{t-1} - \lambda u_{t-1} + u_t$$

(4.6)

with $E[u_t|P_{t-1}] = 0$, where $P_{t-1}$ denotes the entire price history up to period $t-1$. Thus, adaptive expectation for the conditional mean of price is equivalent to assuming that the agent believes that price changes follow an invertible first-order moving-average process, a rather restrictive condition.\(^{19}\)

Given that equation (4.6) is the relevant price model, the adaptive expectation model for the variance of equation (4.5) can be rewritten as

\[^{19}\text{See, for example, Pesaran (1987, p. 19).}\]

43
\[ \sigma_t^2 = \phi \sigma_{t-1}^2 + (1 - \phi) u_{t-1}^2. \] (4.7)

Note that for the model to be internally consistent the agent must believe that the random terms \( u_t \) are drawn from a distribution with mean zero and variance \( \sigma_t^2 \). But, as is apparent from (4.7), for most types of distributions (including the normal), \( \sigma_t^2 \) is bound to converge to zero as time passes. Indeed, equation (4.7) shows that the adaptive expectation model for conditional price variance is a special case of Bollerslev's (1986) generalized ARCH (GARCH) model, specifically what Engle and Bollerslev (1986) called the "integrated" GARCH model. For this model, \( \sigma_t^2 \to 0 \) almost surely for most common distributions [Nelson (1990)].20 The fact that these models imply that \( \sigma_t^2 \to 0 \) leads to the somewhat paradoxical situation of modeling response to risk with models that entail that risk is transitory. As Geweke (1986, p. 59) stated, "... the integrated GARCH model is not typical of anything we see in economic time series."

These undesirable modeling features are avoided if the conditional price variance is modeled by a regular GARCH model, such as the GARCH(1,1) model:

\[ \sigma_t^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \alpha_2 u_{t-1}^2, \] (4.8)

where \( \alpha_0 > 0 \) bounds the conditional variance away from zero (and thus precludes \( \sigma_t^2 \to 0 \)), and \( \alpha_1 + \alpha_2 < 1 \) ensures stationarity of the conditional variance process. This class of models, popular in finance studies, has been applied to agricultural supply models by Aradhyula and Holt (1989, 1990),

\[ \sum_k \alpha_k = 1. \]

20 Similar problems also apply to other more ad hoc parameterizations, such as that used by Chavas and Holt (1990), where \( \sigma_t^2 = \sum_k \alpha_k u_{t-k}^2 \) and \( \alpha_k \) are predetermined constants satisfying \( \sum_k \alpha_k = 1 \).
Holt and Moschini (1992), Holt (1993), and others. Whereas this approach offers a coherent framework for modeling production response to risk, the GARCH model makes explicit the relation between conditional and unconditional variance and brings to the fore an important feature of the problem at hand. Namely, models such as (4.3) can identify response to variance only if the latter is time-varying. If, on the other hand, producers perceive variance to be relatively constant, then no response to risk can be estimated. For example, in the logic of the model (4.8), a constant variance would imply that $\alpha_1 = \alpha_2 = 0$, such that the conditional variance is the same as the unconditional variance ($\alpha_0$, in such a case), and the term $\beta_3 \alpha_0$ in equation (4.3) would then be absorbed by the intercept.

We conclude this section with two observations. First, the assumption that producers perceive a constant conditional variance may not be a bad approximation. Most economic time series do seem to display ARCH properties, but the ability to forecast squared errors is usually very limited even in these models [Pagan and Schwert (1990)], and this is particularly true for the planning horizons typical of agricultural production decisions [Holt and Moschini (1992)]. Thus, in such cases conditional variance does not do much better than unconditional variance for the purpose of measuring the relevant risk; hence, identifying and estimating risk response may be too ambitious an undertaking. But second, the fact that we may have trouble identifying risk response does not mean that production adjustments to risk are not present. Indeed, virtually any supply model that has been estimated without a risk term is consistent with a potentially large risk response insofar as the relevant risk is an unconditional variance that is captured by the intercept.

\[\text{\textsuperscript{21} A related point is that, unlike typical finance applications, agricultural supply models with risk are usually estimated with a small sample of observations.}\]
4.5. Risk and equilibrium in supply and production systems

The models that we have just reviewed introduce a risk variable as a single equation supply model. As mentioned earlier, representing risk in terms of a single variable (say, price variance) may be justified as an approximation to the more general EU model and will be an admissible procedure only under certain restrictive conditions (for example, normality and CARA). Whereas consideration of higher moments has been advocated [Antle and Goodger (1984)], it is arguable that such ambitions may be frustrated in most empirical applications. The single equation nature of these supply models, on the other hand, can only be a partial representation of the more complete production and supply system that may represent the agricultural producer's decision problem. Thus, generalizing risk response models to systems of equations may be desirable, and it has been pursued by Coyle (1992), Chavas and Holt (1990, 1996), and Saha, Shumway and Talpaz (1994), among others. Consideration of such complete supply systems is common in applied work under assumptions of certainty or risk neutrality, thanks partly to the popularization of flexible functional forms for dual representations of technology (such as profit and cost functions), which greatly simplify the derivation of coherent systems of output supply and input demand equations. Extension of this "dual" approach under risk has been explored by Coyle (1992), but because his set-up relies on a linear mean-variance objective function (which, as discussed earlier, is consistent with EU only under restrictive assumptions), it is unclear whether this dual approach is better than the corresponding "primal" approach.

The system approach typically can accommodate such integrability conditions as symmetry, homogeneity, and curvature (say, convexity in prices of the profit function). Interest in these restrictions can arise for at least two reasons. First, this set of testable restrictions may be used to validate the theoretical framework. Second, if testing the theory is not an objective, then maintaining these restrictions may be useful in improving the feasibility/efficiency of estimation, as well as improving the usefulness of empirical results for policy and welfare analysis. If one wanted to
consider the integrability conditions for EU maximizing producers, what would such conditions look like? Pope (1980) pursued this question and showed that the simple symmetry and reciprocity conditions that hold under certainty need not hold under uncertainty. But, as in any optimization problem, some symmetry conditions must exist, and for the case of a producer who maximizes expected utility under price uncertainty, these conditions were characterized by Pope (1980), Chavas and Pope (1985), and Paris (1989). In general the relevant symmetry conditions will involve wealth effects (and thus will depend on risk attitudes). Restrictions on preferences, however, can reduce the symmetry and reciprocity conditions of the risk-averse case to those of the certainty case. That will happen, for example, if the utility function is of the CARA type [Pope (1980)]. Alternatively, restrictions on the technology can also reduce the symmetry and reciprocity conditions of the risk-averse case to those of the certainty case. Specifically, if the production function is homothetic, then input demands satisfy the symmetry conditions that hold under certainty; and if the production function is linearly homogeneous, then the corresponding reciprocity conditions also hold [Dalal (1990)].

A fundamental restriction of output supply and input demand functions under certainty is that of homogeneity of degree zero in prices. Thus, for example, if all input and output prices are scaled by a constant (for instance, a change of units of measurement from dollars to cents), then all real decisions are unaffected, i.e., there is no money illusion. In general the homogeneity property does not seem to hold under price uncertainty, as noted by Pope (1978) and Chavas and Pope (1985), unless restrictions are placed on preferences. Because a proportional change in all input and output prices induces a corresponding change in profit, the decisions of a producer with CARA preferences are affected by such a proportional change. On the other hand, if the producer holds CRR
preferences, then decisions are not affected by such a proportional change in all prices.\textsuperscript{22}

Spelling out such homogeneity conditions is quite useful, and indeed Pope (1988) used homogeneity to derive tests for the structure of risk preferences. But because homogeneity of degree zero of choice functions in prices is typically associated with the absence of money illusion, the conclusion that homogeneity need not hold under uncertainty may seem somewhat puzzling. One way to look at the problem is to recognize that the absolute risk-aversion coefficient is not unit-free; thus, for example, it is meaningless to postulate a particular numerical value for \( \lambda \) independent of the units of measurement of prices. If doubling of all prices were associated with halving of \( \lambda \), for example, then even under CARA choices would not be affected by such a change. There is, however, a more fundamental way of looking at the homogeneity property. The crucial element here is to recognize that the vNM utility function of money, say \( U(\pi) \), is best interpreted as an indirect utility function of consumer demand, such that \( \pi \) creates utility because it is used to purchase consumption goods. Thus, \( U(\pi) = V(p^C, \pi) \) where \( V(p^C, \pi) \) is the agent's indirect utility function, and \( p^C \) denotes the price vector of consumption goods. In analyses of risk models, the vector \( p^C \) is subsumed in the functional \( U(\cdot) \) under the presumption that these prices are held constant. Because \( V(p^C, \pi) \) is homogeneous of degree zero in \( p^C \) and \( \pi \), it follows that, when consumption prices are explicitly considered, the vNM utility function is homogeneous of degree zero in all prices (i.e., consumption prices, output prices, and input prices). Thus, homogeneity (i.e., lack of money illusion) must hold even under uncertainty, when this property is stated in this extended sense.

\textsuperscript{22} For example, if output and input prices are scaled by a constant \( k > 0 \), then profit changes from \( \pi \) to \( k\pi \). If utility is CARA, then \( -\exp(\lambda \pi) \neq -\exp(-k\lambda \pi) \), because scaling prices by \( k \) is equivalent to changing the constant coefficient of risk aversion. On the other hand, if utility is CRRA, say \( U = \log(\pi) \), then scaling profit by \( k \) clearly has no effect on choices.
Storage opportunities introduce dynamics and require a more careful accounting for equilibrium issues as well as for expectation formation when modeling supply. In particular, because negative storage is impossible, nonlinearities are inherent in the equilibrium problem. Using U.S. soybean market data over the period 1960 to 1988, Miranda and Glauber (1993) develop an equilibrium rational expectations model that explicitly represents the behavior of producers, consumers, and storers (both private and public). They find evidence to suggest that both acres supplied and storage activities respond negatively to increased price risk. The storage result suggests that risk management institutions may facilitate efficiency by reducing impediments to intertemporal transactions.

4.6. Programming models with risk

In a number of agricultural economics applications, especially those with a normative focus, risk has been considered within suitably parameterized programming models that can readily be solved (and simulated) by appropriate computational methods. The classical quadratic programming problem of Freund (1956) maximizes a weighted linear summation of mean and variance subject to resource constraints:

$$\begin{align*}
\max_x \mu(x) - \frac{1}{2} \lambda V(x) \\
s.t. \quad G(x) \leq 0,
\end{align*}$$

where \(\mu(x)\) and \(V(x)\) are mean and variance of returns as a function of choices, \(G(x) \leq 0\) is a vector of equality and inequality constraints, and \(\lambda\) measures the magnitude of risk aversion. Sharpe (1963), among others, refined the approach into a convenient and economically meaningful single-index model for portfolio choice. Applications of the method in agricultural economics include Lin, Dean and Moore (1974) and Collins and Barry (1986); both of which consider land allocation decisions. Because solving quadratic programming problems was, at one time, computationally difficult, Hazell (1971) linearized the model by replacing variance of reward with the mean of total absolute deviations.
(MOTAD) in the objective function. Hazell's MOTAD model has been extended in several ways by Tauer (1983), among others, and the general method has been used widely in economic analyses of agricultural and environmental issues [Teague, Bernardo and Mapp (1995)]. Risk considerations can also be introduced as a constraint, and many such programming problems go under the general rubric of safety-first optimization as studied by Pyle and Turnovsky (1970) and Bigman (1996).²³

Given the strong relationship between time and uncertainty, risk has a natural role in dynamic optimization problems. The analytical problems associated with identifying the time path of optimal choices often requires numerical solutions for such problems. This is particularly true in agricultural and resource economics, where the necessity to accommodate such technical realities as resource carry-over may preclude stylized approaches such as the real options framework discussed previously. Stochastic dynamic programming is a discrete-time variant of optimal control methods and is robust to accommodating the technical details of the rather specific problems that arise in agricultural and natural resource economics. A standard such problem is:

\[
\text{Max } \sum_{t=0}^{T} \beta^t E_0[\pi(x_t,y_t)] \quad \text{s.t. } y_t = f(y_{t-1}, x_{t-1}, \epsilon_t),
\]

\[
y_0 \text{ given},
\]

where \(T\) may be finite or infinite, \(\beta\) is the per-period discount factor, and \(\pi(x_t,y_t)\) is the per-period reward. The goal is to choose, at time 0, a contingently optimal sequence, \(x_0\) through \(x_T\), to maximize the objective function. But the problem is not deterministic because randomness, through the sequence \(\epsilon_t\), enters the carry-over equation, \(y_t = f(y_{t-1}, x_{t-1}, \epsilon_t)\). This means that a re-optimization is required at each point in the time sequence. To initialize the problem, it is necessary

²³ Note that safety-first approaches to risk modeling may be difficult to reconcile with the EU framework.
that \( y_0 \) be known. For analytical convenience, Markov chain properties are usually assumed for the stochastic elements of the model. Many variants of the above problem can be constructed. For example, time could modify the per time period reward function or the carry-over function. Applications of the approach include capital investment decisions [Burt (1965)] and range stocking rate and productivity enhancement decisions [Karp and Popé (1984)].

4.7. Technology adoption, infrastructure and risk

A class of production decisions where risk is thought to play an important role is that of new technology adoption. Early work in this area, reviewed by Feder, Just and Zilberman (1985), analyzed the relationships among risk, farm size, and technology adoption, as well as the role of information, human capital, and various market constraints affecting labor, credit, and land tenure. More recent studies that consider the possible impact of risk on adoption include Antle and Crissman (1990) and Pitt and Sumodiningrat (1991). The availability of irrigation has been shown to be an important risk factor for technology adoption. It both increases average productivity and reduces variability of output, and often involves community or government actions (thus emphasizing how risk management opportunities may often depend upon local institutional factors). For references to the impacts of risk and irrigation on technology adoption, with special regard to the adoption of high-yielding but flood-susceptible rice in Bangladesh, see Azám (1996), Bera and Kelley (1990), and other research cited therein. This line of research suggests that technologies are often best introduced in packages rather than as stand-alone innovations. Other work on structure includes Rosenzweig and Binswanger (1993), who studied the structural impacts of weather risk in developing countries, and Barrett (1996), who considered the effects of price risk on farm structure and productivity. In the context of hybrid maize adoption, Smale, Just and Leathers (1994) argue that it is very difficult to disentangle the importance of competing explanations for technology adoption, and suggest that
previous studies may have overstated the importance of risk aversion.

The introduction of a new technology often requires a substantial capital investment, and so the functioning of credit markets plays a crucial role. For collateral-poor farmers in rural communities of the less developed world, credit is often unattainable through formal channels. For example, Udry (1994) finds that in four northern Nigeria villages more than 95 percent of borrowed funds were obtained from neighbors or relatives. One of the reasons for the importance of informal lending channels is the limited means by which formal credit providers can obtain relevant information concerning the riskiness of projects. As discussed in Ray (1998), less formal sources (such as the landlord, a local grain trader, or the village moneylender) are in a better position to judge risks and to provide credit. But, perhaps due to high default risk or to the systemic nature of risk when all borrowers are from the same village, interest rates are often very high. Bottomley (1975) developed a simple model that relates equilibrium rates to default risk. It has been suggested that moneylender market power may also affect rates but, from a survey of the literature, Ray (1998) concludes that local moneylending markets tend to be quite competitive. However, as Bottomley (1975) pointed out, the true interest rate may often be difficult to ascertain because loans are often tied in with other business dealings such as labor, land lease, and product marketing agreements.

Faced with production and price risks, poorly performing credit markets would seem to imply inadequate investments, perhaps especially in risk-reducing technologies. On the other hand, the limited liability nature of credit may create incentives for borrowers to engage in riskier projects that are also less productive on the average, compared with the projects that would have been chosen if the credit line were not available. Basu (1992) studies the effect of limited liability and project substitution on the structure of land lease contracts.

5. Risk management for agricultural producers
The purpose of risk management is to control the possible adverse consequences of uncertainty that may arise from production decisions. Because of this inherently normative goal, stating the obvious might yet be useful: risk management activities in general do not seek to increase profits per se but rather involve shifting profits from more favorable states of nature to less favorable ones, thus increasing the expected well-being of a risk-averse individual. It should also be clear that production and risk management activities are inherently linked. Most business decisions concerning production have risk implications, and the desirability of most risk management choices can only be stated meaningfully with reference to a specific production context. As for the risk implications of production decisions, a useful classification of inputs can be made following Ehrlich and Becker (1972), who identified "self-insurance" and "self-protection" activities. Self-insurance arises when a decision alters the magnitude of a loss given that the loss occurs. Self-protection takes place when a decision alters the probability that a loss will occur. Of course, agricultural inputs may have both self-insurance and self-protection attributes; for instance, fertilizer may reduce both the probability and conditional magnitude of a crop nutrient deficiency, and livestock buildings can operate in the same way upon weather-related losses. Ehrlich and Becker (1972) use this classification to show that input choices modify the demand for market insurance. Expenditures on market insurance and self-insurance substitute for each other, whereas expenditures on self-protection could actually increase the demand for market insurance.

Abstracting from self-insurance and self-protection effects of production choices, farmers usually have access to a number of other tools that have a more direct risk management role. These include contractual arrangements (e.g., forward sales, insurance contracts) as well as the possibility of

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24 In a comprehensive review of literature on crop yield variability determination, Roumasset et al. (1989) conclude that nitrogen tends to increase variability. For technology adoption, Antle and Crissman (1990) suggest that variability tends to increase initially but decrease again after more is learned about the innovation.
diversifying their portfolio by purchasing assets with payoffs correlated with the returns on production activities. Risk management decisions are obviously constrained by the given institutional and market environments, i.e., what tools and programs are actually available to the farmer. Thus, the possible incompleteness of risk markets and the imperfections of capital markets are bound to be crucial to risk management. As will be discussed in this section, existing risk markets, such as contingent price markets and crop insurance, typically do not allow producers to eliminate all risk (for given production choices, it may be impossible to take market positions such that the resulting total payoff is invariant to the state of nature.) Whereas this may suggest scope for welfare-increasing government intervention, it also indicates that farmers just may have to bear some residual risk.

In what follows we analyze in some detail contractual relationships that a producer may enter into in order to manage price and quantity risk. In particular, we emphasize price-contingent contracts (forward, futures and options) and crop insurance contracts. Whereas the analysis hopefully will clarify the role of various risk-management tools, we should emphasize that the results of most of the models analyzed below do not translate into direct risk management recommendations. For example, given the endogeneity of many of the risks faced by producers, a discussion of risk management that takes production decisions as given is to some extent artificial, although it may be analytically useful. More generally, one should keep in mind that farmers ultimately likely care about their consumption, itself the result of an intertemporal decision. Risky production and risky prices of course imply a

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25 When capital markets are imperfect, internal funding can be very important for production decisions. For this reason, Froot, Scharfstein, and Stein (1993) argue that one of the main purposes of hedging in a business is to manage cash flow so that profitable investment opportunities that arise might be pursued, so that risk management has clear dynamic implications. The time sequence of cash flows may also be important under the risk of business failure, as discussed by Foster and Rausser (1991).

26 From a welfare point of view, farmers may not be the main losers from market incompleteness. Myers (1988) showed empirically that the incompleteness may benefit producers when food demand is inelastic and may benefit consumers under other circumstances. Lapan and Moschini (1996) in a partial equilibrium framework, and Innes and Rausser (1989) and Innes (1990) in a general equilibrium framework, identified roles for second-best policy interventions when some risk markets are missing.
risky farm income, but such income uncertainty may not necessarily translate into consumption risk because borrowing/saving opportunities, as well as income from other assets and/or activities (diversification), may be used to smooth consumption over time. It is nonetheless instructive to consider certain aspects of risk management in stylized models.

5.1. Hedging with price contingent contracts

"Hedging" here refers to the purchasing of assets for the purpose of insuring one's wealth against unwanted changes. As discussed earlier, output price is one of the most important sources of risk for agricultural producers. Several instruments are available to farmers of developed countries to "hedge" this price risk, notably forward contracts and price contingent contracts traded on organized futures exchanges.

5.1.1. Forward contracts and futures contracts

The biological lags that characterize agricultural production mean that inputs have to be committed to production far in advance of harvest output being realized, at a time when output price is not known with certainty. The simplest instrument often available to farmers to deal with this price risk is a "forward contract." With such a contract a farmer and a buyer of the agricultural output agree on terms of delivery (including price) of the output in advance of its realization. For example, a farmer and a buyer can agree that a certain amount of corn will be delivered at a given time during the harvest season at the local elevator for a certain price. It is readily apparent that conditions exist under which such a contract can completely eliminate price risk. To illustrate, let \( q \) = output quantity produced, \( h \) = output quantity sold by means of a forward contract, \( p \) = the output price at the end of the production period, \( f_0 \) = the forward price quoted at the beginning of the period, and \( \pi \) = the profit at the end of the period. Then the random end-of-period profit of the firm that uses forward
contracts is

\[ \hat{\pi} = \tilde{p} q - C(q) + (f_0 - \tilde{p}) h, \quad (5.1) \]

where \( C(q) \) is a strictly convex cost function (which subsumes the effects of input prices).\(^{27}\) If the farmer's utility function of profit is written \( U(\pi), \) where \( U''(.) < 0 < U'(.), \) the first-order conditions for an optimal interior solution of an EU maximizer require

\[
E[U'(\hat{\pi})(\tilde{p} - C'(q))] = 0, \quad (5.2)
\]
\[
E[U'(\hat{\pi})(f_0 - \tilde{p})] = 0, \quad (5.3)
\]
from which it is apparent that optimal output \( q^* \) must satisfy \( C'(q^*) = f_0. \) This is the "separation" result derived by Danthine (1978), Holthausen (1979), and Feder, Just and Schmitz (1980). Optimal output depends exclusively on the forward price, which is known with certainty when inputs are committed to production, and hence the production activity is riskless.

The importance of the separation result lies in the fact that the agent's beliefs about the distribution of cash and futures prices, and her degree of risk aversion, are inconsequential for the purpose of making production decisions. The agent's beliefs and her risk attitudes, however, may affect the quantity of output that is sold forward. In particular, from (5.3) it follows that

\[
h^* \gtrless q^* \quad \text{as} \quad E[\tilde{p}] \lesssim f_0. \quad (5.4)
\]

Thus, for example, a producer who believes that the forward price is biased downward (i.e., \( E[\tilde{p}] > f_0 \)) has two ways of acting to take advantage of her information (i.e., "speculating"): she

\(^{27}\) Input prices are implicitly compounded to the end of the period using the (constant) market interest rate, so that all monetary variables in (5.1) are commensurable.
could produce more than under an unbiased forward price, while holding constant the amount sold forward; or she could decrease the amount sold forward, while holding output at the level that is optimal when the forward price is unbiased. Either action results in some uncommitted output being available at harvest time that will fetch the (risky) market price. But speculating by varying output has decreasing returns [because \( C''(q) > 0 \) by assumption], whereas speculating by varying the amount sold forward has constant returns. Hence, speculation here takes place exclusively by varying the amount sold forward. Similarly, changes in risk aversion, and in the riskiness of the price distribution, in this setting affect forward sales but not production decisions.

An extension of the results just discussed considers futures contracts instead of forward contracts. A futures contract is, essentially, a standardized forward contract that is traded on an organized exchange, such as the Chicago Board of Trade or the Chicago Mercantile Exchange [Williams (1986)]. A futures contract typically calls for delivery of a given quantity (say, 5,000 bushels) of a certain grade of a commodity (say, No. 2 yellow corn) at a specified delivery time (say, December of a given year) at a specified location (say, a point on the Mississippi River). Because of these features, the futures price may not be exactly suited to hedge the risk of a given producer. On the other hand, futures markets are quite liquid and hedging by using futures is readily possible for all producers, even when a local buyer offering a forward contract is not available. Using futures contracts, a producer can lock in on a price for future delivery; the problem, of course, is that this precise futures price may not be what the producer needs. Such discrepancies may be due to any one of the three main attributes of an economic good: form, time, and space. Because of that, the

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28 For example, the commodity grown by the producer may be of a different kind (or a different grade) than that traded on the exchange; or, the producer may realize the output at a different time than the delivery time of the contract; or, the producer may realize the output at a different location than that called for in the futures contract. Grade differences may be handled by pre-specified premiums or discounts over the futures price; differences in the type of commodity lead to the problem of "cross-hedging" of Anderson and Danthine (1981); see DiPietre and Hayenga (1982) for an application. The
local cash price that is relevant for the producer is not the one that is quoted on the futures market, although usually it is highly correlated with it. In addition, one should note that futures entail lumpiness (only 5,000 bu. at a time for most grains, for example) as well as transactions costs. Thus, relative to a forward contract, a futures contract is an imperfect (although possibly effective) risk-reduction instrument, i.e., the producer that uses futures contracts retains "basis risk." 29

To illustrate hedging under basis risk, let us modify the notation of the previous section by letting \( f_0 \) = futures price quoted at the beginning of the period, \( f_t \) = futures price at maturity of the futures contract, and \( h \) = amount of commodity sold in the futures market. As before, \( p \) represents the cash price at harvest time, and thus basis risk means that, typically, \( p \neq f_t \). In general, it is difficult to fully characterize the production and hedging decisions under basis risk. Some results may be obtained, however, by restricting the relationship between cash and futures prices to be linear, as in Benninga, Eldor and Zilcha (1983):

\[
\bar{p} = \alpha + \beta \bar{f} + \theta ,
\]

where \( \alpha \) and \( \beta \) are known constants, and \( \theta \) is a zero-mean random term that is independent of the futures price. 30 The end-of-period profit of the producer can then be represented as

\[
\pi = (\alpha + \beta f_0 + \theta) q - C(q) + (f_0 - f_t) (h - \beta q).
\]

imperfect time hedging problem was explicitly addressed by Batlin (1983).

29 Basis in this context refers to the difference, at the date of sale, between the (local) cash price and futures price.

30 Actually, whereas independence is sufficient for our purposes, the slightly weaker assumption that \( \bar{f} \) is conditionally independent of \( \theta \) is both necessary and sufficient [Lence (1995)]. Of course, for some distributions (such as the multivariate normal) these two notions of independence are equivalent. Indeed, if \((\bar{p}, \bar{f})\) are jointly normally distributed, then the linear basis representation in (5.5) follows.
Now, if the futures price is unbiased (i.e., if $E[\hat{f}] = f_0$), it is apparent that, for any given output $q$, the optimal futures hedge is $h^* = \beta q$. Additional results for this basis risk case are presented in Lapan, Moschini and Hanson (1991). Because in this case random profit reduces to $\bar{\pi} = (\alpha + \beta f_0 + \bar{\theta})q - C(q)$, the effective (hedged) price, $\alpha + \beta f_0 + \bar{\theta}$, is still random. Hence, under risk aversion, production takes place at a point at which marginal cost is lower than the expected price (given optimal hedging), i.e., $C'(q^*) < (\alpha + \beta f_0)$, indicating that a portion of price risk due to the basis cannot be hedged away. Because there is some residual uncertainty concerning the local cash price, the degree of risk aversion also influences optimal output. Specifically, the output level $q^*$ is inversely related to the degree of risk aversion, as in earlier results of models of the competitive firm under price risk [Baron (1970); Sandmo (1971)]. Also, a ceteris paribus increase in nondiversifiable basis uncertainty (a mean-preserving spread of $\bar{\theta}$) will in general decrease the optimal output level, a sufficient condition being that preferences satisfy DARA [Ishii (1977)].

It is important to realize that with basis risk, even in its special formulation of equation (5.5), the separation result, discussed earlier for the case of forward contracts, does not apply. Because hedging does not eliminate basis risk, if the agent believes that the futures price is biased then her choice will involve the possibility of investing in two risky assets (production of output and trading in futures). Thus, if the agent believes that the futures price is biased, her optimal speculative response will entail changes in both these risky assets. For the special case of CARA preferences and of a linear basis as in (5.5), however, one can still prove a separation result between production and

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31 Hence, the optimal futures hedge ratio $h^*/q$ is equal to $\beta = \text{Cov}(\bar{p}, \hat{f})/\text{Var}(\hat{f})$, the coefficient of the theoretical regression of cash price on futures price, a result that has been used in countless empirical applications.
hedging (speculative) decisions. Specifically, in such a case the optimal output level \( q^* \) does not depend on the parameters of the producer's subjective distribution of futures prices [Lapan, Moschini and Hanson (1991)], although it does depend on the agent's degree of risk aversion and on the parameters \( \alpha \) and \( \beta \), which define the expectation of the cash price (conditional on the futures price).

The results just outlined pertain to a static problem and, more crucially, pertain to a competitive producer who faces only price risk. For most commodities, however, the hedging problem needs to consider the fact that farmers typically are exposed to both price and production uncertainty. An early attempt at allowing both price and production risk was that of McKinnon (1967), who considered the hedging problem of minimizing the variance of profit for a given planned output level. Because of the complications generated by the joint presence of price and production risk, efforts to extend McKinnon's risk-minimization analysis to EU maximization often have relied on the assumption that producers maximize an objective function increasing in the mean and decreasing in the variance of revenue/profit. This approach was followed by Rolfo (1980), Newbery and Stiglitz (1981, chapter 13), and Anderson and Danthine (1983), among others. In these studies it is shown that the correlation between the random production and random price is crucial for determining the optimal hedging strategy. Because demand considerations suggest the correlation is typically negative, a "natural" hedge is already built into the price system and the optimal strategy is to hedge an amount lower than expected output.

Such a mean-variance approach usually is justified on the grounds that it is exact for a CARA utility function if wealth/profit is normally distributed. But profit typically is not normally distributed when output is uncertain because it entails the product of two random variables [Newbery (1988)]. Indeed, the need to analyze our hedging problem in a general framework is clearly illustrated by noting that, under production uncertainty, the optimal hedge in general is less than expected output even when output and price are independent [Losq (1982)], a result that cannot be established by
mean-variance analysis. Of course, the difficulty is that it is not possible to establish useful general hedging results that hold for arbitrary concave utility functions and arbitrarily jointly distributed random prices and quantities. If one assumes a CARA utility function, however, an exact solution to the hedging problem under production uncertainty may be possible, as illustrated by Bray (1981), Newbery (1988), and Karp (1988).

A model that captures the essence of a typical farmer's planting hedge was presented in Lapan and Moschini (1994), who consider futures hedging for a competitive producer who faces both production (yield) and price risk and whose only available hedging instrument is a futures contract (with basis risk). Following Newbery and Stiglitz (1981), stochastic output is represented in terms of a production function with multiplicative risk, i.e., \( \tilde{Q} = \tilde{y}q(x) \), where \( x \) denotes the vector of inputs, \( \tilde{y} \) is a random variable with mean \( \bar{y} \), and \( \tilde{Q} \) is random output. As noted earlier, with multiplicative production risk, input choices can still be represented by a standard cost function, say \( C(q) \) where \( q \) denotes the scale of production.\(^{32}\) With input prices assumed constant (they are typically known at the time production and hedging decisions are made) and subsumed in the function \( C(.) \), realized total profits are\(^{33}\)

\[
\tilde{\pi} = \tilde{p} \tilde{y}q - C(q) + (f_0 - \tilde{f})h. \quad (5.7)
\]

Thus, the producer knows \( f_0 \) when \( q \) and \( h \) are chosen, but the realizations of the random variables \( \{\tilde{f}, \tilde{p}, \tilde{y}\} \) are not known. The difference between \( \tilde{f} \) and \( \tilde{p} \) reflects basis risk.

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\(^{32}\) Thus, for any level of inputs, \( q = \tilde{q}(x) \). In this setting, \( q \) aggregates planted acreage and other inputs, and \( \tilde{y} \) reflects random yield.

\(^{33}\) Of course, simultaneous use of crop insurance contracts (discussed later) would alter the nature of this problem.
Within this context, and assuming that producers maximize a CARA utility, and that the three random variables \( \{ \tilde{f}, \tilde{p}, \tilde{y} \} \) are jointly normally distributed, Lapan and Moschini (1994) derive and discuss the exact analytic solution to the optimal hedging problem. In particular, they show that the optimal futures hedge satisfies

\[
q^* = \frac{f_0 - \tilde{f}}{\lambda s_{11}} + q \left[ \frac{\bar{y} s_{12}}{s_{11}} + \frac{\bar{p} s_{13}}{s_{11}} \right].
\]

Here \( s_{ij} \) are the elements of the matrix \( S = [\lambda qB + V^{-1}]^{-1} \), where \( \lambda \) is the coefficient of absolute risk aversion, \( V \) is the variance-covariance matrix of the three random variables, and \( B \) is an accounting matrix of zeros and ones. Hence, an important result here is that the optimal hedge does depend on the degree of risk aversion, even when the futures price is perceived as unbiased. This insight was not present in earlier mean-variance models of hedging under production uncertainty [e.g., Rolfo (1980); Newbery and Stiglitz (1981)]. For likely parameter values, this risk preference effect may be important and the optimal hedge may differ substantially from the mean-variance one. Furthermore, the optimal hedge under yield uncertainty depends on the conditional forecast of the harvest price \( (\bar{p}) \) and of the yield term \( (\bar{y}) \), even when the futures price is perceived as unbiased. Thus, in addition to precluding the separation result, production uncertainty also entails that the optimal hedge is inherently time-varying because conditional forecasts will be revised as harvest approaches.

The empirical application reported by Lapan and Moschini (1994), based on a generalization of Myers and Thompson's (1989) hedge ratio estimation procedure, showed that the optimal hedge is considerably less than the full hedge, and that the amount sold forward declines as risk aversion increases. Of course, CARA, joint normality, and multiplicative production risk are rather restrictive assumptions, but nonetheless this model is useful because it can relax the straitjacket of the mean-
variance framework and provide insights into the EU-maximizing optimal hedge. Although analytical results based on more general assumptions are difficult to obtain, empirically it is easy to consider alternative risk preference structures and stochastic distributions. For example, Lapan and Moschini (1994) solve numerically for the optimal hedge for CRRA preferences and log-normally distributed \( \{\tilde{f}, \tilde{p}, \tilde{y}\} \), and find that the conclusions obtained under CARA and normality are reasonably robust.\(^{34}\)

5.1.2. Options on futures

Among the instruments traded on commodity exchanges, futures contracts arguably have the most direct relevance to risk management for farmers. With the introduction of options on futures for many commodities in the 1980s, however, the possibility of trading put and call options has attracted considerable attention.\(^{35}\) The use of options as hedging devices when the producer faces only price (and basis) risk (but not production risk) was considered by Lapan, Moschini and Hanson (1991). They emphasize that the inclusion of commodity options in a decision maker's portfolio leads to a violation of the two main conditions for a mean-variance representation of expected utility: (i) options truncate the probability distribution of price (so that the argument of the utility function, profit or wealth, is not normally distributed even if the random price is normal), and (ii) the use of options generally means that the argument of utility is not monotonic in the random attributes. The model essentially entails adding another hedging instrument (options) to the payoff in equation (5.7). A

\(^{34}\) Whereas the discussion here has emphasized price-contingent contracts, some yield futures have recently been introduced by the Chicago Board of Trade. Clearly, such contracts are potentially useful for farmers (provided enough liquidity exists). A mean-variance analysis of the hedging problem with both price and yield futures is presented by Vukina, Li, and Holthausen (1996).

\(^{35}\) A "put" conveys to the buyer the right to sell the underlying futures contract at a given price (the "strike price"). This right can be exercised over a certain period of time (the life of the option), and for this right the buyer must pay a "premium" (the price of the option) to the seller (the underwriter). Similarly, a "call" conveys to the buyer the right to sell the underlying futures at the strike price during the life of the option. See Cox and Rubinstein (1985) for more details.
basic modeling issue here is that, given the presence of futures, one of these basic types of options is redundant (for example, a put can always be constructed using a futures and a call). Hence, for modeling purposes attention can be limited to any two of the three types of assets (futures, puts, and calls). Equivalently, as emphasized by Lapan, Moschini and Hanson (1991), one can consider futures and a combination of puts and calls such as straddles. The use of futures and straddles is fully equivalent to allowing the use of futures and calls (or puts), but it has the analytical advantage of illuminating the interpretation of a number of results because the payoff of a straddle is essentially orthogonal to the payoff of a futures contract.

Lapan, Moschini and Hanson (1991) show that, when the futures price is unbiased (from the producers' own point of view), then options are redundant hedging instruments. The key insight here is that, unlike futures contracts, options allow the construction of payoffs that are nonlinear in the realized futures price. But when futures prices and options premiums are perceived as unbiased (such that the only reason to trade these instruments is to hedge the risky cash position), the relevant payoff of the producer is linear in the futures price. Hence, the optimal hedging strategy involves using only futures contracts, which provide a payoff that is linear in the price of interest (the option payoff is uncorrelated with the risk that remains after the optimal futures hedge). If futures prices and/or options premiums are perceived as biased, however, then there is a speculative motive to trade futures and options, and options are typically used along with futures.

In this context it is clear that a hedging role for options is likely when there is a nonlinear relation between profit and the futures prices, such as the presence of nonlinear basis risk or the presence of production uncertainty together with price uncertainty. The latter situation is obviously of great interest to farmers, and has been analyzed by Moschini and Lapan (1995). They study the problem of

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A (short) straddle can be constructed by selling one call and one put at the same strike price (or, because of the redundancy just mentioned, it can be constructed by buying one futures and selling two calls).
a farmer with end-of-period profit given by

\[ \tilde{\pi} = p\tilde{y}q - C(q) + (f_0 - \tilde{f})h + (r - |f - k|)z, \]  \hspace{1cm} (5.9) \]

where \( z \) is a short straddle with strike price \( k \) and premium \( r \) (note that the payoff of the straddle depends on the absolute value of the difference between realized futures price and strike price). The producer knows \( f_0 \), \( k \), and \( r \) when \( q, h, \) and \( z \) are chosen, but the realizations of the random variables \( \{\tilde{f}, \tilde{p}, \tilde{y}\} \) are not known. Under the assumption of CARA and normality, Moschini and Lapan (1995) provide analytic solutions for the optimal use of futures and straddles. If futures and options prices are perceived as unbiased, then the optimal hedging strategy entails selling futures and buying straddles. Of course, because of the simultaneous presence of price and production uncertainty, the optimal use of the hedging instruments depends on the agent's degree of risk aversion, and in general the optimal hedge is less than the full hedge. For example, for a representative soybean producer with a local relative risk aversion of \( R = 2 \), and after translating optimal levels of futures and straddles into futures and puts, the optimal hedge is to sell futures in an amount of about 63 percent of the expected output and to buy puts in an amount of about 15 percent of expected output.

If the producer perceives the futures and straddle prices as being biased, then there is a speculative motive to trade these assets. An interesting result here is that, if the agent perceives only the options price to be biased, then only the straddle position is affected, whereas if only the futures price is perceived as biased, both futures and options positions will be affected.\(^{37}\) This result is reminiscent of the speculative hedging role of options illustrated by Lapan, Moschini and Hanson (1991, 1993), and in particular, cannot be obtained by using the special mean-variance framework.

\(^{37}\) Thus, options are useful to provide insurance against the risk of speculating on the futures price because the nonlinearity of their payoffs can compensate for the speculation outcome of extreme price realizations. But futures are not useful to hedge the speculative risk induced by the optimal option use under biased option prices.
5.1.3. The time pattern of hedging

The discussion so far has dealt with a simple version of the hedging problem, a one-period (two-dates) model. At the beginning of the period, when the risky cash position is incurred (say, when corn is planted or when feeder cattle are bought and placed in the feedlot), the farmer hedges by trading futures and other derivatives (options). At the end of the period, when the cash position is liquidated (because the crop is harvested or the cattle are sold), the financial positions are closed. But what if the farmer were free to adjust the futures hedge after it is established and before it is closed? Two questions are relevant here. Does the possibility of revising the optimal hedge affect the initial hedging decision? And, if it is optimal to revise the hedge over time, how is the hedge revised?

These problems have been addressed, in different contexts, by (among others) Anderson and Danthine (1983), Karp (1988), and Myers and Hanson (1996). It turns out that the answer to these questions depends crucially, among other things, on whether the producer believes that futures prices are biased or unbiased, and on whether or not there is production uncertainty in the model.

Because our focus is on risk reduction (hedging), suppose that futures prices are unbiased. Also, consider first the pure price and basis risk case (no production risk), and suppose that there are \( T \) periods, with the initial hedge being taken at \( t = 0 \), and the last hedge being lifted at \( t = T \), and that the terminal profit of the producer is

\[
\bar{\pi}_T = \bar{p}_T q + \sum_{t=1}^{T} (1+i)^{T-t} (\bar{f}_t - \bar{f}_{t-1})h_{t-1} - C(q),
\]

(5.10)

where \( i \) is the per-period interest rate. If the producer maximized the EU of terminal profit, \( E[U(\bar{\pi}_T)] \), then the optimal hedging problem (for any given level of output \( q \)) can be solved by backward induction. Suppose first that \( i = 0 \) and that the linear basis assumption made earlier is rewritten as
\[ \hat{\rho}_T = \alpha + \beta \hat{f}_T + \theta_T. \] (5.11)

Then, it is easily shown that the optimal hedge is to sell an amount \( h_t^* = \beta q \) for all \( t = 0, \ldots, T - 1 \).

Thus, if futures prices are unbiased, the static optimal hedge solution gives the optimal hedging strategy at any time based upon the conditional moments available at that time. In particular, the myopic hedging rule (i.e., the hedge that does not take into account that later revisions in the hedge positions are possible) is the same as the optimal dynamic hedging strategy [Karp (1988)].

Because profits/losses of the futures position are marked to market in equation (5.10), if the interest rate is positive then the optimal futures hedge at time \( t \) should be adjusted by a factor of \((1 + r)^{T-t}\). This gives a first, albeit trivial, reason for the pure hedge to change as time \( t \) moves from 0 to \( T \), as the amount sold forward will increase over time because of this pure discounting effect. As harvest approaches, the agent may revise her expectations about futures (and therefore cash) prices at \( T \). However, there would be no need to adjust the futures position through the growing season due to these changed price expectations, provided the farmer continued to believe that the futures price was unbiased. A second reason to revise the hedge position arises if the moments of the distribution of cash and futures prices (for time \( T \)) change over time (as a result of new information), in which case the optimal hedge will be revised as time progresses from \( t \) to \( T \), as illustrated by Myers and Hanson (1996). Furthermore, in that situation the ability to revise the futures hedge does affect the initial (at time \( t = 0 \)) hedge position, so that myopic and optimal dynamic hedges differ.

As illustrated by Anderson and Danthine (1983), Karp (1988), and Lapan and Moschini (1994), production uncertainty gives yet another fundamental reason for the optimal hedge to change over time. Because production uncertainty implies that the futures market cannot provide a perfect hedge, the hedge itself depends on the agent's forecast of realized cash price (realized futures price) and
realized yield, even when the futures price is unbiased [recall equation (5.8)]. Clearly, changes in
expectations of realized yields (and hence output) will lead to revisions in the futures position. Even
if yield forecasts do not change, however, changes in the futures price (and therefore in the expected
cash price) will lead to changes in the optimal hedge if the realizations of yields and price are
correlated.

A somewhat different dynamic hedging problem arises when the production setting allows for
some inputs to be chosen after the uncertainty is resolved, as in the ex-post flexibility models of
Hartman (1976) and Epstein (1978). This hedging problem has been studied by Moschini and Lapan
(1992), who emphasize that in this model the ex-ante profit of the firm is nonlinear (convex) in the
risky price (hence, once again, the mean-variance framework is unlikely to be very useful unless one
is willing to assume that the utility function is quadratic). They derive a special case of the separation
result for this instance of production flexibility (without basis and production risk, of course), which
attains when the shadow price of the quasi-fixed input (the input that is chosen ex-ante) is linear in the
output price. This linearity means that the incremental risk due to changes in the quasi-fixed inputs
can be fully hedged using futures (because the payoff of the futures position is also linear in price).
The nonlinearity of profit in the risky price, however, means that all income risk cannot be hedged
via futures for the case of production flexibility, and thus there is a pure hedging role for options,
over and above that of futures.

5.1.4. Hedging and production decisions

The hedging review so far has emphasized the optimal use of hedging instruments conditional on a
given output or a given expected output. An important but distinct question concerns how the
availability of these hedging opportunities affects the firms' choice of output. As mentioned earlier,
in the special case where basis risk and production risk are absent, the availability of futures contracts
allows a separation between production and hedging (speculative) decisions. Specifically, the futures price determines the optimal output level, irrespective of the subjective beliefs of the producer, and any difference between the agent's price expectations and the prevailing futures price only affects the hedging/speculative position. Even in this simple case, however, whether the hedging opportunity increases output depends crucially on whether the futures price is biased or not. If the futures price is perceived as unbiased, then the availability of futures hedging induces the risk-averse firm to expand output.

When we relax the restrictive assumptions that lead to the separation result, and allow for basis and production risk (in addition to futures price risk), in general the planned output of the risk-averse firm will depend on both the futures price and price expectations. The question of how hedging affects the choice of planned output, therefore, is only meaningful in the context of unbiased prices, but even in this context it turns out that general propositions are not possible. Some insights, however, are provided by Moschini and Lapan (1995) for the case of jointly normally distributed random variables and CARA preferences. In particular, they show that if the level of risk aversion is small or if the orthogonal production risk is sufficiently small and the futures price is unbiased, then the availability of futures hedging induces the risk-averse firm to produce a larger output level. Essentially, the ability to hedge effectively changes (increases) the risk-adjusted price the firm perceives for its output. Similarly, it is shown that, if the degree of risk aversion or the level of pure production risk is not too large and futures and option prices are unbiased, then the availability of options (in addition to futures) also causes the firm to increase output.

5.1.5. The value of hedging to farmers

Whereas the foregoing cursory review suggests a potentially important role for futures and option contracts to manage farmers' risk, empirical surveys often find that use of such contracts by farmers
is limited.\textsuperscript{38} Many explanations for this situation have been offered. From a purely economic point of view, it is clear that existing futures markets do not complete the set of markets in the Arrow-Debreu sense, and thus futures are unlikely to provide a full hedge in a number of production situations. For example, as discussed earlier, consideration of basis and other risks may substantially affect (typically reduce) the optimal futures hedge. Furthermore, even abstracting from basis and other risks, one may note that the time horizon of existing futures is limited (i.e., the most distant delivery date for agricultural futures is often little beyond one year). Thus, producers who hedge optimally their one-period risk are still exposed to some intertemporal price risk even after accounting for "rollover" hedging [Gardner (1989); Lapan and Moschini (1996)].

From a more practical viewpoint, certain costs of hedging that are typically neglected in the analysis, such as brokerage fees, initial deposit, and the requirement to mark to market, may deter hedging activities. Lence (1996) argues that such costs may make the net benefits of hedging almost negligible and may help explain why many farmers do not hedge. Also, limited use of futures by farmers may, to a certain extent, result from mistrust and lack of proper education on the working of such instruments, an observation that suggests a clear scope for extension activities. But one should also keep in mind that the futures markets may be indirectly quite important for agricultural risk management even when many farmers do not use futures contracts directly. For example, futures may be routinely used by country elevators to hedge the risk of storing grain, and these elevators may in turn offer forward contracts to farmers.

\textsuperscript{38} But a recent survey [U.S. General Accounting Office (1999)] finds that use of risk-management tools by farmers is actually fairly common in the United States. In 1996, 42 percent of the United States' two millions farmers used one or more risk management tool, and use of risk management strategies was even more frequent for larger farms. For example, among farmers with annual sales greater than $100,000, 55 percent used forward contracts and 32 percent engaged in hedging with futures and/or options (52 percent of these farmers also purchased crop insurance, a risk management tool discussed below).
5.2. Crop insurance

Given the susceptibility of crop yields to weather fluctuations, there is obviously a latent demand for crop insurance. Although crop insurance markets have existed for a long time in some parts of the world (e.g., the United States, Canada, and Sweden), their existence has depended crucially on government support, and these governments often have seen fit to subsidize or even run crop insurance markets. Unsubsidized private insurance markets for agricultural risks have been confined mostly to single-peril insurance contracts. Wright and Hewitt (1990) express the belief that private agricultural insurance markets may fail because the costs of maintaining these markets imply unacceptably low average payouts relative to premiums. Furthermore, they suggest that the perceived demand for crop insurance may be overstated because farmers can use diversification and savings to cushion the impact of a poor harvest on consumption. Although Wright and Hewitt’s conjectures are solidly motivated, little has been done to verify the claims empirically. It seems clear, however, that unsubsidized agricultural insurance may not be attractive to farmers because it may be too costly. In particular, the costs of private insurance contracts arise, in part, from information problems that are inherent in these insurance contracts, and it is to these problems that we now turn.

Almost invariably crop insurance markets that have benefited from government intervention, especially for multiple-peril contracts, have been either unexpectedly costly to maintain or unattractive to producers, or both. Consider, for example, the case of the U.S. Federal Crop Insurance Corporation (FCIC), which subsidizes insurance for U.S. crop growers. Below is a table of acreage participation rates and loss ratios for some of the major grain and oilseed crops over the ten-year period 1987 to 1996. The loss ratio is the ratio of indemnities to premium payments, and does not include premium subsidies. When one notes that loss ratios of no more than 0.7 are deemed necessary for unsubsidized insurance to be viable given the administrative costs of running it [Wright

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39 In addition to subsidizing premiums, the FCIC also absorbs the administrative costs.
(1993)], it is clear that the acreage premia would have to be raised substantially for the program to be self-sustaining. Even so, despite heavy government involvement, the subsidized programs are insufficiently generous to attract even a majority of acres planted to these crops. Indeed, the reported participation rates are artificially high because in 1989 and some subsequent years producers had to sign up to be eligible in the event of ad hoc relief, and in 1995 producers had to sign up in order to be eligible for very attractive target price programs. Knight and Coble (1997) provide a detailed overview of the multiple-peril crop insurance environment since 1980. Given that a good insurance policy should attract decision makers who are willing to lose money on average in order to have a less variable income, it is obvious that the FCIC programs have left much to be desired.

Table 5.1. FCIC Coverage and Payouts 1987—1996.

<table>
<thead>
<tr>
<th>Crop</th>
<th>U.S. Acres Planted (Millions)</th>
<th>Acres that are FCIC Insured (Percent)*</th>
<th>Loss Ratio*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td>71.0</td>
<td>46.8</td>
<td>1.53</td>
</tr>
<tr>
<td>Corn</td>
<td>73.6</td>
<td>38.3</td>
<td>1.22</td>
</tr>
<tr>
<td>Soybeans</td>
<td>59.9</td>
<td>35.3</td>
<td>1.06</td>
</tr>
<tr>
<td>Sorghum</td>
<td>11.4</td>
<td>37.9</td>
<td>1.37</td>
</tr>
<tr>
<td>Barley</td>
<td>8.9</td>
<td>44.0</td>
<td>1.44</td>
</tr>
<tr>
<td>Rice</td>
<td>2.9</td>
<td>29.5</td>
<td>2.42</td>
</tr>
</tbody>
</table>

* Averages reported are the annual numbers averaged over 10 years.


Not the least of the problems that arise in crop insurance markets is the existence of a strong political interest in their perceived success. Although the political aspects of these markets are many
and varied, the following provides a flavor. Just as in the United States, government involvement in
Canadian crop insurance markets has been both extensive and of questionable success. One of the
precursors to crop insurance in Canada was the 1939 (federal) Prairie Farm Assistance Act. In the
words of the Minister of Agriculture at the time, and referring to a long-standing federal policy of
encouraging the settlement of the Prairie provinces, the act "... is intended to take care of people
who were put on land that they should never have been put on. That is our reason for being in this at
all, and it is our reason for paying two-thirds or three-quarters of the costs out of the treasury of
Canada (Standing Committee on Agriculture and Colonization)." Sigurdson and Sin (1994) provide a
description of the political history of Canadian crop insurance policy, and Gardner (1994) gives an
overview of the United States crop insurance policy in relation to other agricultural policies.

In the United States, one of the more important political aspects of crop insurance is the
unwillingness of the federal government to ignore the pleas for monetary disaster assistance when a
crop failure is widespread. Given that farm-level crop failures tend to be strongly positively
correlated, this undermines the incentive to purchase crop insurance. Disaster assistance is an
example of one economic problem—moral hazard—that afflicts crop insurance markets.

When considering a risk, insurance companies may observe certain parameters of the decision
environment such as geographic location, soil type, and yield history. They may also observe certain
actions such as input use. It is often infeasible to observe all relevant facts, however, and even if
observable it may be impossible to write an insurance contract based upon these observations. When
it is impossible or excessively costly to write a contract based upon relevant actions, then moral
hazard problems may arise. Similarly, when contracts based upon relevant environmental parameters
are infeasible, then adverse selection problems may arise. In the remainder of this section, we
delineate the nature of the two major economic incentive problems that impede well-functioning crop
insurance contracts, and we discuss possible remedies to these problems.
5.2.1. Moral hazard

A risk-neutral insurer who is contemplating the business of a risk-averse producer will seek to specify a contract payout schedule, net of premium, such that a profit is made on the average and also that the producer finds the contract to be sufficiently attractive to sign. Using a standard principal-agent model, as in Chambers (1989), let $R$ be gross revenue and let $I(R)$ be the net contract payoff schedule (premium minus payout), with $C[I(R)]$ as the cost of administering that payoff schedule. Then, assuming symmetric information, i.e., that the insurer can contract upon observable input choices, the insurer’s problem is

$$\text{Max}_{x, I(R)} \int_a^b \left( I(R) - C[I(R)] \right) dF(R|x) \quad \text{s.t.} \quad \int_a^b U[R - I(R) - rx] dF(R|x) \geq \bar{u}, \quad (5.12)$$

where $R$ is supported on $[a, b]$, $\bar{u}$ is the minimum level of expected utility that must be maintained to entice the producer to insure, $F(R|x)$ is the revenue distribution function conditional on the input vector $x$, and $r$ is the input price vector.

Standard analysis, due to Borch (1962), yields the requirement that $I(R)$ satisfy the point-wise condition

$$\frac{1 - C[I(R)]}{U_\pi[\pi]} = \mu, \quad (5.13)$$

where $\mu$ is the Lagrange multiplier for the EU constraint in problem (5.12). Now, if the insurer’s cost is invariant to the nature of the schedule, then optimality requires $U_\pi[\pi]$ to be constant, and so for risk-averse producers $I(R)$ must be such that $R - I(R) - rx$ is constant. This is the classical risk-sharing result, namely that the risk-neutral insurer should accept all risk from the risk-averse producer. Under general conditions, this result continues to hold if the insurer is risk averse but
contracts upon a large number of independent risks. Because the insurer here assumes all the
risk, and given the participation constraint, then \( I(R) = R - rx - U^{-1}[\bar{u}] \), and the optimal \( x \) is that
which maximizes the producer’s expected profit.  

This set-up is drastically changed, and moral hazard problems arise, when the insurer contracts on
a risk-averse producer whose inputs are unobservable (i.e., there is asymmetric information). This is
because the insurer has but one instrument, the payoff schedule, to address two goals. To be
attractive a contract must mitigate the uncertainty facing insurers, but to make a profit the contract
must ensure that producers do not take advantage of the limited control over insurance payoffs that
arise from the insurer’s inability to observe input use. The insurer’s problem when inputs are not
observable, but the stochastic technology \( F(R|x) \) is known, can be stated as

\[
\begin{align*}
\text{Max} & \quad \int_a^b \{ I(R) - C[I(R)] \} dF(R|x) \quad \text{s.t.} \quad \int_a^b U[R - I(R) - rx] dF(R|x) \geq \bar{u}, \\
\hat{x} & = \arg\max_{a} \int_a^b U[R - I(R) - rx] dF(R|x).
\end{align*}
\]

(5.14)

The additional incentive compatibility constraint ensures that the rational insurer endogenizes the input
consequences of the payoff schedule posed. For both problems (5.12) and (5.14), in general the
participation constraint is binding and the producer achieves utility level \( \bar{u} \). Under moral hazard,
however, it is not optimal for the risk-neutral principal to assume all risk. Some residual risk must
be borne by the (risk-averse) producer and hence, to achieve a given \( \bar{u} \), the expected payouts to the
producer have to be larger than under symmetric information. Chambers (1989) discusses the welfare

\[\text{40}\] Unfortunately, risks across crop production units usually tend to be more systematic than
idiosyncratic in nature.

\[\text{41}\] In the trivial case where inputs are unobservable but the producer is risk neutral, this expected
profit-maximizing result may also be achieved by setting the schedule \( I(R) \) equal to a constant. In this
way, the producer faces all the consequences of the actions taken. But then, of course, the insurance
company serves no purpose and will never be able to cover any administrative costs.

75
loss associated with the incentive constraint as well as the possibility that it might cause crop insurance markets to fail.

The implications of the moral hazard problem are not as clear-cut as intuition might suggest. Being relieved of some of the consequences of low input use, the producer may reduce input intensity. On the other hand, as previously shown, if input use is risk increasing then a high-risk environment may cause the producer to use fewer inputs than a lower-risk environment. Thus the existence of insurance may, in mitigating risk, encourage input use. That is, risk sharing and moral hazard effects may oppose each other.

To model econometrically the moral hazard problem, the crop producer contemplating whether to insure may be viewed as having to make two decisions: whether or not to insure, and the choice of input vector. In one of the first econometric analyses of the effects of crop insurance, Horowitz and Lichtenberg (1993) assumed that the decision to insure affects input use but not the other way around. Modeling the insurance decision by Probit analysis and modeling input choice as a linear regression on the insurance decision, among other regressors, they studied corn production decisions in ten Corn Belt states and concluded that the decision to insure increased significantly the use of nitrogen and pesticides. These results are somewhat surprising, so other researchers sought to confirm the conclusions on different data sets and using other methodologies. Smith and Goodwin (1996) estimated a simultaneous equations model of input use and crop insurance purchases for Kansas dryland wheat farmers, and concluded that insurance and input decisions are likely simultaneously determined. Further, their results suggest that insurance reduces the use of agricultural chemicals. Estimating an input-conditioned beta distribution for farm-level Iowa corn production, Babcock and Hennessy (1996) simulated optimal input use under different types and levels of insurance for risk-averse producers and also concluded that insurance would likely decrease input use. Although more empirical investigations are warranted, it would appear that risk sharing through crop insurance
reduces input use.

The moral hazard problem was also studied in the West African Sahel region, which is at risk to drought. Following on work by Hazell (1992), among others, Sakurai and Reardon (1997) identified quite strong potential demand for area-level rainfall insurance. Their analysis also raises the concern that moral hazard arising from food aid could undermine the viability of such contracts.

In identifying two types of risk, production risk and land value risk arising from soil depletion, Innes and Ardila (1994) suggest an intertemporal environmental aspect to the incentive problem. For fragile land, a contract tailored to insure against production risk may exacerbate land value deterioration, and so one might not be able to ignore dynamic aspects of moral hazard. This is especially true if the operator does not own the land. Dynamic issues also arise in work by Coble et al. (1997) who find evidence that input reduction by insured producers occurs mainly when a crop loss is most likely, thus exacerbating the magnitude of the loss. Further empirical analysis is required to decompose the magnitudes of these impacts.

Moral hazard problems may not be confined to input intensity issues. If output is difficult to verify, then false yields may be reported. Such illegal acts raise questions concerning contract design, the structure of legal sanctions, and the nature of detection technologies. Hyde and Vercammen (1997) argue that, whereas it is difficult to motivate the structure of insurance contracts actually offered (i.e., the attributes of monotonicity, convexity, deductibility, and co-insurance) as a response to moral hazard on input use alone, actual contracts can plausibly be an optimal response to moral hazard on both input use and yield verification together.

5.2.2. Adverse selection

When, unlike the producer, the insurer is not completely informed about the nature of the risk being insured, then the insurer faces the problem of adverse selection. Ignoring input choices, let a risk-
neutral insurer have categorized three production units owned by different operators and of equal size (say, one acre without loss of generality), A, B, and C, into the same risk cohort. From the information available to it, say common average yield ($\bar{y}$), the insurer can observe no difference among these three acres. In fact, the associated yield distributions differ; suppose all acres realize two outcomes, each with the same probability $1/2$, but the realizations for acre A are \{\bar{y} - 10, \bar{y} + 10\}, those for B are \{\bar{y} - 20, \bar{y} + 20\}, and those for C are \{\bar{y} - 30, \bar{y} + 30\}. With unit price, if the insurance payout equaled $\text{Max}[\bar{y} - y, 0]$, then the expected payouts for acres A, B, and C would be 5, 10, and 15, respectively. In such a case, assuming full participation, the actuarially fair premium for a contract covering all three risks would be 10/acre. However, if the acre A producer is insufficiently risk averse, then she may conclude that the loss ratio for acre A, at $5/10 = 1/2$, is too low and may not insure the acre. If the insurer continues to charge 10/acre when covering only acres B and C, then an average loss of 2½/acre is incurred. On the other hand, if the premium is raised to 12½/acre so that a loss is avoided, then acre B may not be insured. Thus, the market may unravel in stages.

Avoiding adverse selection may require the successful crop insurance program to identify, acquire, and skillfully use data that discriminate among different risks. Although perhaps costly to implement, such data management procedures may be crucial because, unless rates are perceived as being acceptable, the market may collapse. The phenomenon of unravelling suggests that identifying a sufficiently large number of relatively homogeneous risks is a prerequisite for a successful contract. Useful discriminators would appear to include mean yield. Skees and Reed (1986) and Just and Calvin (1993) have found evidence suggesting that yield variance may decrease with increased mean yield, and so, even if the trigger insurance yield increases with mean yield, rates should probably be lower for more productive acres. Goodwin (1994), studying Kansas crops (1981—90), finds the
relationship between yield variability and mean yield to be tenuous and suggests that farm yield histories be used to calculate yield variability rather than impute variability from historical mean yield. He also concludes that other factors, such as enterprise size, could be informative in setting premium rates.

The degree of homogeneity required to sustain the contract depends upon, among other things, the degree of risk aversion expressed by producers. The more risk averse the producers, the more tolerant they will be of actuarially unfair rates. In an investigation of adverse selection in contracts on corn production, Goodwin (1993) studied county-level data for the ninety-nine Iowa counties over the period 1985 to 1990 and found the elasticities of acreage insured to expected payoff to be in the range of 0.3–0.7. At a farm level, these elasticities may be higher. Further, he found that counties where the risk of payout is low are quite sensitive to the premium charged, so that an across-the-state (of Iowa) premium increase might not make corn yield insurance more profitable because substantial cancellations by the better risk prospects may occur. He concluded that the best approach to loss ratio reduction may involve fine-tuning the rate setting at the county or farm level.

Adverse selection may be either spatial or temporal in nature. The problem type discussed thus far may be categorized as being spatial in the sense that the factors differentiating risks occur at a given point in time. An alternative form of adverse selection, identified by Luo, Skees and Marchant (1994), may arise when attributes of a given risk vary temporally. Coble et al. (1996) consider the case of adverse selection in crop insurance contracts for Kansas dryland wheat farmers over the years 1987 to 1990. Pre-season rainfall was used as an indicator for intertemporal adverse selection whereby an unseasonably low (high)-level of rainfall occurring before contract signing would entice

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42 If the producer is better informed about the temporal evolution of risk, then adverse selection may occur. However, as discussed in Knight and Coble (1997), the insurer may be just as informed about the temporal risk as the producer, but may be either unable or unwilling to adjust rates. In such a situation, the problem is not one of adverse selection.
marginal risks into (out of) signing, thus increasing the loss ratio if rates do not reflect the implications of the water deficit prevailing at signing. Although finding some evidence of adverse selection, they did not identify any of an intertemporal nature.

There are, of course, many factors other than adverse selection that determine the decision for, and the magnitude of, crop insurance participation. To understand adverse selection it is necessary to isolate its impact by accounting for other determinants of participation. In addition to the aforementioned research, econometric analyses of the determinants of insurance participation have been conducted by Gardner and Kramer (1986), Just and Calvin (1990), and Smith and Baquet (1996), among others. Although the conclusions are somewhat mixed, an overview of results suggests that participation tends to increase with farm size. This may be because of the negative correlation between farm size and the importance of off-farm income, or because of increased borrowing. Also, enterprise specialization tends to increase participation, presumably because of increased risk exposure. Further, and suggestive of adverse selection, higher yield variability land is more likely to be insured. However, estimates by Coble et al. (1996) infer that this is true even if rates account for the increased riskiness.

5.2.3. Further discussion

Though conceptually distinct, the differences between the moral hazard and adverse selection problems often disappear in practice. Noting that both moral hazard and adverse selection are problems of information asymmetry. Quiggin, Karagiannis and Stanton (1993) posed the situation in which a wheat and corn producer contemplating crop insurance has one acre of good land and one acre of bad land. Given the decision to insure wheat but not corn, the planting of wheat on poor quality land might be viewed as moral hazard. However, given the decision to plant poor land to wheat, the decision to insure wheat only may be viewed as adverse selection. Thus, it should be no
surprise that the potential remedies to each problem are similar.

Due to the informational nature of the main barriers to successful crop insurance markets, the obvious solution is, where feasible, to acquire and use as much information as marginal cost and profit considerations allow. To improve performance by reducing adverse selection, the FCIC changed its approach to rate-setting in 1985 to accommodate additional information. Subsequent contracts changed the determination of the insurable yield from an average of past yields observed in a locality to an average of past yields observed on the farm in question. Even so, sensible a reform, however, may give rise to incentive problems. As pointed out by Vercammen and van Kooten (1994), producers might manipulate input use in a cyclical manner to build up insurable yield levels before cashing in (in a probabilistic sense) by reducing input use for a few years.

On the other hand, area yield insurance [Halcrow (1949); Miranda (1991); Mahul (1999)], where indemnities are based upon the average yield of a suitably wide area (say, a county), eliminates the moral hazard problem and may reduce or eliminate adverse selection. In addition, just as futures markets permit hedge ratios in excess of one, a producer may take out an arbitrary level of area yield insurance coverage without giving rise to concerns about increased moral hazard. Area yield insurance rates are likely to be lower than farm-specific rates because an area yield index will usually be less variable than yield on a given farm. However, because farm-specific risks are not insured, producers may continue to be subjected to some (possibly substantial) production risk.

Revenue insurance is a recurrently popular concept because it directly addresses the real problems facing producers, namely income variability. A further possible advantage is that, in combining price and yield insurance, the approach may mitigate somewhat the incidence of moral hazard and adverse selection. Miranda and Glauber (1991), as well as Babcock and Hennessy (1996), conducted simulation analyses for U.S. crop production, and Turvey (1992a, 1992b) studied the costs and benefits of such a program in Canada. The potential for revenue insurance arises from the fact that,
even together, price contingent markets (for a fixed quantity) and yield contingent markets (for a fixed price) are not likely to fully stabilize income. Hennessy, Babcock and Hayes (1997) have shown that this targeting attribute of revenue insurance means that it can increase the welfare impact of a given expenditure on income support relative to various alternatives of price and yield support.

Compulsory insurance has often been proposed to eliminate the political need for continual ex-post interventions. If adverse selection is a major problem in competitive insurance markets, however, then compulsory insurance is unlikely to gain the political support necessary for a long-term solution. More effective re-insurance on the part of crop insurers may facilitate the reduction of market rates, and thus reduce adverse selection, because systemic risk is pervasive in the insurance of crop risks and so pooling is largely ineffective for the insurer [Miranda and Glauber (1997); Duncan and Myers (1997)]. Given the diminishing importance of agriculture in developed economies, the introduction of crop loss risks into a well-diversified portfolio of risks would reduce the high level of systematic risk in crop insurance markets, and so may reduce the risk premia required by crop insurers. But crop insurance differs in many ways from other forms of insurance, and it may prove difficult to entice reinsurers into accepting these contracts. If a permanent solution exists that is politically more acceptable than a laissez-faire market approach, it may involve a package of reforms that is balanced to mitigate the incentive impacts but incurs low budgetary costs. Such a package should also take care not to undermine existing or potentially viable risk markets. Finally, the policy mix must be flexible because the technology and organization of crop production may undergo fundamental changes in the coming years.

6. Conclusion

It is abundantly clear that considerations of uncertainty and risk cannot be escaped when addressing most agricultural economics problems. The demands imposed on economic analysis are complex and
wide-ranging, with issues that extend from the pure theory of rational behavior to the practicality of developing risk-management advice. The economics profession at large, and its agricultural economics subset, has responded to this challenge with a wealth of contributions. In this chapter we have emphasized theoretical and applied analyses as they pertain to production decisions at the farm level. The EU model provides the most common approach to characterizing rational decisions under risk, and it has been the framework of choice for most applied work in agricultural economics. Whereas our review has provided only a nutshell exposition of the framework’s main features, the careful student will dig deeper into its axiomatic underpinning as a crucial step to appreciating what modeling decisions under risk means. More generally, we can note that a satisfactory model of decision making under risk requires assuming an extended notion of rationality. Agents need to know the entire distribution of risky variables, and need to take into account how this randomness affects the distribution of outcomes over alternative courses of action. Thus, the decision maker’s problem is inherently more difficult under uncertainty than under certainty.

Because the notion of rational behavior under risk arguably requires agents to solve a complex problem, it is perhaps useful to distinguish between whether our models are meant to provide a *positive theory* (aiming to describe how agents actually make decisions under risk) or a *normative theory* (the purpose of which is to prescribe a rational course of action for the particular risky situation). This distinction is admittedly somewhat artificial, and most models are suitable to either interpretation. Yet being more explicit about whether one’s analysis is pursuing a positive or normative exercise is possibly quite important in applied contexts such as those covered in this chapter. Much agricultural risk-management work is meant as a normative activity, and this may have implications for the choice of models. For instance, the EU model has been criticized, on positive grounds, for failing to describe accurately how agents actually behave under risk in some situations; such a critique, of course, says nothing about the suitability of the EU model for normative
(prescriptive) purposes.

Models of decision making under risk bring to the forefront the fact that decisions will be affected in a crucial way by the agent's preferences, i.e., her attitudes towards risk. Consequently, it is quite important to quantify the degree of agricultural producers' risk aversion, and a number of studies have endeavored to do just that. The conclusions may be summarized as follows: within the EU framework, producers typically display some aversion to risk, and risk preferences probably conform to DARA. But evidence on the magnitude of risk aversion is less conclusive and falls short of providing useful parameters that are critical for normative statements (whether in terms of risk management advice to farmers or in terms of suggesting desirable government policies).

Considerations of risk aversion also raise concerns about a very common attribute of applied studies that have a positive orientation. Namely, whereas theoretical models are meant for individual decision making, empirical models are often implemented with aggregate data. The danger of ignoring the implicit aggregation problem is obviously a general concern that applies to economic models of certainty as well. But the fact that risk attitudes play an important role in models with risk, and given that such preferences are inherently an individual attribute, suggests that agents heterogeneity is bound to be more important when risk matters. It seems that more can and should be done to tackle aggregation considerations in a satisfactory manner.

The complexities of the decision maker's problem under risk raise additional issues for the applied researcher. Agents' beliefs about the characteristics of uncertainty are obviously crucial in this context: The EU model, by relying on the notion of subjective probabilities, neatly solves the theoretical modeling question. But the applied researcher may need to model explicitly how the agent makes probability assessments (i.e., to model her expectations). Whereas the rational expectation hypothesis provides perhaps the most ambitious answer to this question, it is informationally very demanding when (as is typically the case in risky situations) the entire distribution of the random
variables matters. This raises the question of whether rational expectations are legitimate from a theoretical point of view, but also implies that empirical models that wish to implement rational expectations can be computationally quite demanding, even for the simplest model under risk. Indeed, many empirical models reviewed in this chapter appear somewhat oversimplified. The modus operandi seems to be to allow theoretical modeling to be as sophisticated as desired but to keep empirical models as simple as possible. Such oversimplifications naturally beg the question of the relationship of empirical models to the theoretical constructs that are used to interpret results, and raise some concerns about what exactly we can learn from this body of empirical studies.

Notwithstanding the remaining criticisms and concerns that one may have, the studies surveyed in this chapter have addressed an important set of problems. Uncertainty and risk are essential features of many agricultural activities, and have important consequences for the agents involved and for society at large. Although welfare and policy considerations related to risk are discussed elsewhere in this Handbook, we should note that the economic implications of the existence of risk and uncertainty are related to the particular institutional setting in which agents operate. Insofar as the set of relevant markets is not complete, then this market failure has the potential of adversely affecting resource allocation, as well as resulting in less than optimal allocation of risk-bearing. Indeed, the incompleteness of risk markets for agricultural producers has often been cited as a motivation for agricultural policies in many developed countries. But arguably neither existing markets nor government policies have solved the farmers' risk exposure problems, and risk continues to have the potential of adversely affecting farmers' welfare, as well as carrying implications for the long-run organization of agricultural production and for the structure of resource ownership in the agricultural sector.
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References


de Finetti, B., 1931, Sul significato soggettivo della probabilità, _Fundamenta Mathematicae_ 17, 298-29.


Duncan, J. and R.J. Myers, 1997, _Crop insurance under catastrophic risk_. Staff Paper 97-10, Department of Agricultural Economics, Michigan State University.


Goodwin, B.K., 1994, Premium rate determination in the Federal crop insurance program: What do
averages have to say about risk?, *Journal of Agricultural and Resource Economics* 19: 382-95.


Standing Committee on Agriculture and Colonization, 1947, Minutes of the proceedings and evidence, (Canadian House of Commons, Ottawa, Canada) June 10.


(Giannini Foundation, California Agricultural Experiment Station, Berkeley).