ACCOUNTING FOR ULTRASONIC SIGNAL ATTENUATION THROUGH MODEL PARAMETER INTERPOLATION

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INTRODUCTION

In ultrasonic NDE of materials, deconvolution techniques are widely used to improve time/space resolution, minimize spectral coloring, and compensate for different experimental settings, e.g., transducer variations, pulser-receiver energy/damping settings, etc. The reference signal that is used for deconvolution is typically obtained as the front (or back) surface echo from a suitable sample under conditions identical to those used in acquiring the signal to be processed (deconvolved). When the signal to be processed is acquired from an attenuating medium, the effect of signal attenuation should be appropriately accounted for in the deconvolution technique. If the signal arises from a localized inhomogeneity as in the case of flaw scattered signals, this is easily accomplished by suitably modifying the reference signal; for instance, in the Wiener filter based deconvolution technique [1], the frequency dependent attenuation corresponding to the flaw location is determined and incorporated into the reference signal spectrum. When the inhomogeneities are distributed throughout the material as in the case of grain backscattered signals, the correction for attenuation should vary along the depth of the material. A suitable deconvolution technique for incorporating such correction is based on the Kalman filter [2, 3]. In this technique, the reference signal and the signal to be processed are modeled respectively as the impulse response of a system and the system output. The input to the system is the deconvolved signal that has to be estimated. The Kalman filter algorithm processes the data sequentially and its formulation allows the system parameters to change at each step. This property can be taken advantage of in providing varying amounts of correction for attenuation along the depth of the material.

In this paper, we investigate the use of a model parameter interpolation method to provide suitable correction for attenuation. System models (AR or ARMA) are first built for the front and back surface echos obtained from a suitable sample. The parameters of these models are then interpolated to obtain models corresponding to intermediate depths. The impulse responses of the interpolated models represent the reference signals corrected for attenuation. The effectiveness of this approach is evaluated using experimentally obtained signals from copper samples of different thicknesses (1/4″, 1/2″, 3/4″ and 1″).

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MODELING THE REFERENCE SIGNAL

In the Kalman filter based deconvolution technique, the signal to be processed, e.g., grain backscattered signal, is modeled as follows:

$$\begin{aligned} z(k) &= u(k) * r(k) + v(k) \\ &= y(k) + v(k), \end{aligned} \tag{1}$$

where k denotes the sample index, " $*$ " denotes deconvolution, $z(k)$ is the measured signal to be processed, $v(k)$ is the measurement noise, $r(k)$ is the reference signal, and $u(k)$ is the deconvolved signal to be estimated. If we regard $r(k)$ as the impulse response of a system and $u(k)$ as the system input, the measured signal $z(k)$ is just the system output $y(k)$ corrupted by the additive noise $v(k)$. Using state-space notation, (1) can be expressed as follows:

$$\mathbf{x}(k+1) = \mathbf{F}_k \mathbf{x}(k) + \mathbf{G}_k u(k) \tag{2}$$

$$z(k) = \mathbf{H}_k \mathbf{x}(k) + v(k), \tag{3}$$

where $\mathbf{x}(k)$ is the $(N \times 1)$ system state vector, $\mathbf{F}_k$ is the $(N \times N)$ state transition matrix, $\mathbf{G}_k$ is the $(N \times 1)$ input matrix, and $\mathbf{H}_k$ is the $(1 \times N)$ measurement matrix. The matrices $\mathbf{F}_k$, $\mathbf{G}_k$, and $\mathbf{H}_k$ which describe the system are chosen such that the system impulse response approximates the reference signal $r(k)$.

Two of the popular system models are the ARMA (Auto-Regressive Moving Average) and the AR (Auto-Regressive) models. The difference equation relating the input and output of an N-th order ARMA model is given by

$$\begin{aligned} y(n) &+ \alpha_{1,k} y(n-1) + \alpha_{2,k} y(n-2) + \cdots + \alpha_{N,k} y(n-N) \\ &= \beta_{1,k} u(n-1) + \beta_{2,k} u(n-2) + \cdots + \beta_{N,k} u(n-N), \end{aligned} \tag{4}$$

where n is the sample index and ($\alpha_{i,k}$, $\beta_{i,k}$: $i = 1, 2, \ldots, N$) represent the system parameters. These parameters are chosen to minimize the average squared error between $r(n)$ and the system impulse response, i.e., $y(n)$ when $u(n)$ is the unit sample sequence. This is accomplished using a nonlinear least squares optimization technique, viz., Levenberg-Marquardt method [4]. In Z-transform notation, the system function of the ARMA model in (4) is represented by

$$H_k(Z) = \frac{\beta_{1,k} Z^{-1} + \beta_{2,k} Z^{-2} + \cdots + \beta_{N,k} Z^{-N}}{1 + \alpha_{1,k} Z^{-1} + \alpha_{2,k} Z^{-2} + \cdots + \alpha_{N,k} Z^{-N}}. \tag{5}$$

The system matrices corresponding to the ARMA model in (4) are realized in the controllable canonical form as follows:

$$\mathbf{F}_k = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ -\alpha_{N,k} & -\alpha_{N-1,k} & \cdots & -\alpha_{1,k} \end{pmatrix}, \quad \mathbf{G}_k = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix},$$

$$\mathbf{H}_k = (\ \beta_{N,K} \quad \beta_{N-1,k} \quad \cdots \quad \beta_{1,k} \ ). \tag{6}$$

In the case of an AR system model, (4), (5) and (6) are modified so that $\beta_{1,k} = 1$ and $\beta_{i,k} = 0$ for $i = 2, 3, \ldots, N$.

PARAMETER INTERPOLATION

Suppose ($\alpha_{i,0}$, $\beta_{i,0}$: $i = 1, 2, \ldots, N$) and ($\alpha_{i,L}$, $\beta_{i,L}$: $i = 1, 2, \ldots, N$) represent the parameters of the systems obtained respectively using the front and back surface echos from a suitable sample as reference signals. The system model parameters at any intermediate depth k is then obtained by interpolation of these parameters. For example, the interpolation corresponding to $\beta_{i,k}$ is given by the following equation:

$$\beta_{i,k} = \beta_{i,0} + f(\frac{k}{L})\,(\beta_{i,L} - \beta_{i,0})\,, \tag{7}$$

where $f(\cdot)$ is used to control the type of interpolation, e.g., $f(k/L) = k/L$ corresponds to linear interpolation and $f(k/L) = \sqrt{k/L}$ corresponds to a nonlinear interpolation.

If (7) is used to interpolate $\alpha_{i,k}$'s, the stability of the resulting system cannot be guaranteed. To overcome this problem, we first convert the $\alpha_{i,k}$'s into an equivalent set of parameters $\gamma_{i,k}$'s called the PARCOR (Partial Correlation) coefficients. These coefficients are interpolated using (7) and the resulting values are converted back to $\alpha_{i,k}$'s. Such an interpolated system will always be stable if the systems corresponding to $k = 0$ and $k = L$ are stable. The procedure for converting $\alpha_{i,k}$'s to $\gamma_{i,k}$'s and vice versa is described below [5]. Let $\{a_m(i),\ i = 1, 2, \ldots, m,\ a_m(0) = 1\}$ denote the coefficients of the denominator polynomial of an m-th order system function. The conversion of $\alpha_{i,k}$'s to $\gamma_{i,k}$'s proceeds as follows. First, set

$$a_N(i) = \alpha_{i,k} \qquad i = 1, 2, \ldots, N. \tag{8}$$

Next, for $m = N, N-1, \ldots, 1$, compute

$$K_m = a_m(m) \tag{9}$$

and

$$a_{m-1}(i) = \frac{a_m(i) - K_m a_m(m-i)}{1 - K_m^2} \qquad i = 1, 2, \ldots, m-1. \tag{10}$$

Then the PARCOR coefficients $\gamma_{i,k}$ are given by

$$\gamma_{i,k} = K_i \qquad i = 1, 2, \ldots, N. \tag{11}$$

Conversion of $\gamma_{i,k}$'s to $\alpha_{i,k}$'s is done as follows. First, set

$$K_i = \gamma_{i,k} \qquad i = 1, 2, \ldots, N. \tag{12}$$

Next, for $m = 1, 2, \ldots, N$, compute

$$a_m(m) = K_m \tag{13}$$

and

$$a_m(i) = a_{m-1}(i) + K_m a_{m-1}(m-i) \qquad i = 1, 2, \ldots, m-1. \tag{14}$$

Then $\alpha_{i,k}$'s can be determined as

$$\alpha_{i,k} = a_N(i) \qquad i = 1, 2, \ldots, N. \tag{15}$$

EXPERIMENTAL RESULTS

The effectiveness of the model parameter interpolation method was verified using experimentally obtained signals from copper samples of different thicknesses (1/4″, 1/2″, 3/4″ and 1″). Front and back surface echos obtained from the 1″ thick sample were used to build 14-th order ARMA models and 20-th order AR models. The parameters of these models were interpolated to obtain the system models and their impulse responses at depths of 1/4″, 1/2″ and 3/4″. These signals were then compared with experimentally obtained back surface echos from the 1/4″, 1/2″ and 3/4″ thick copper samples.

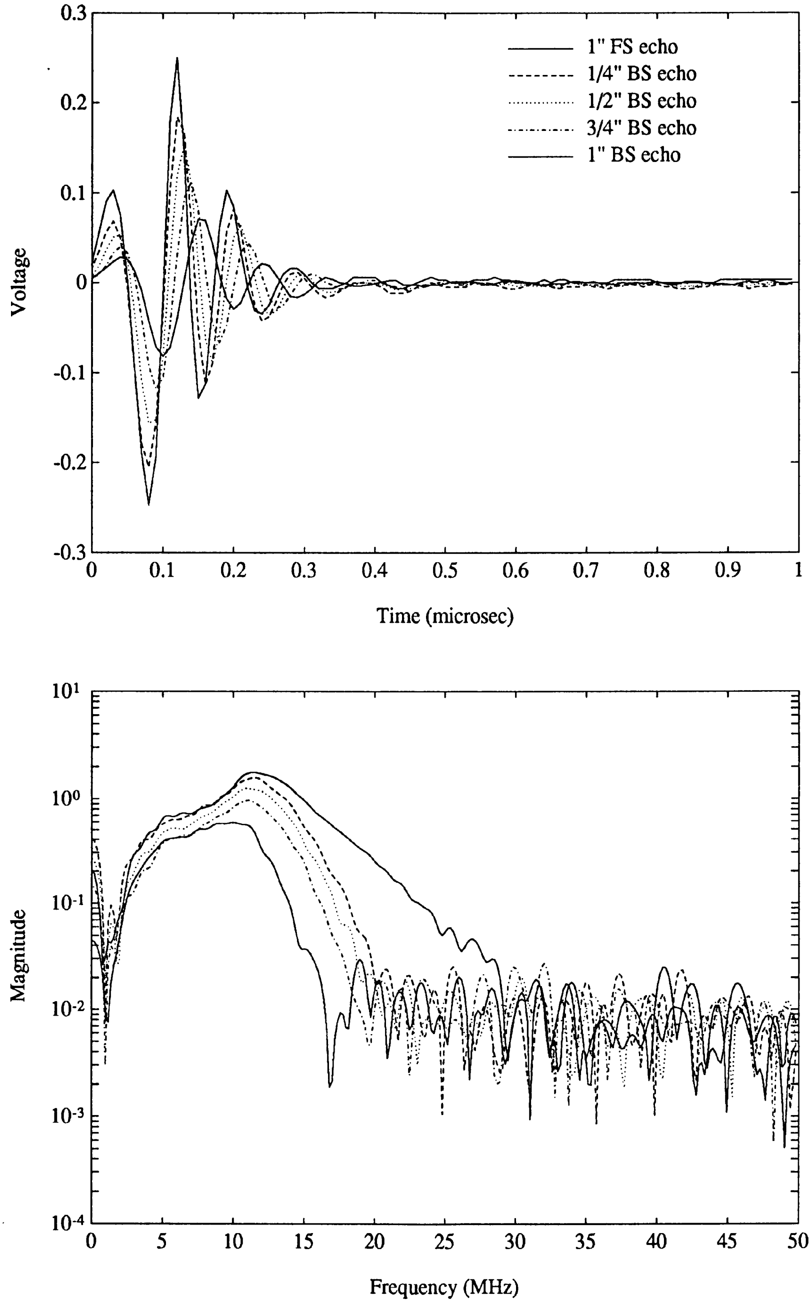

Figure 1. Measured signals from copper samples of different thicknesses

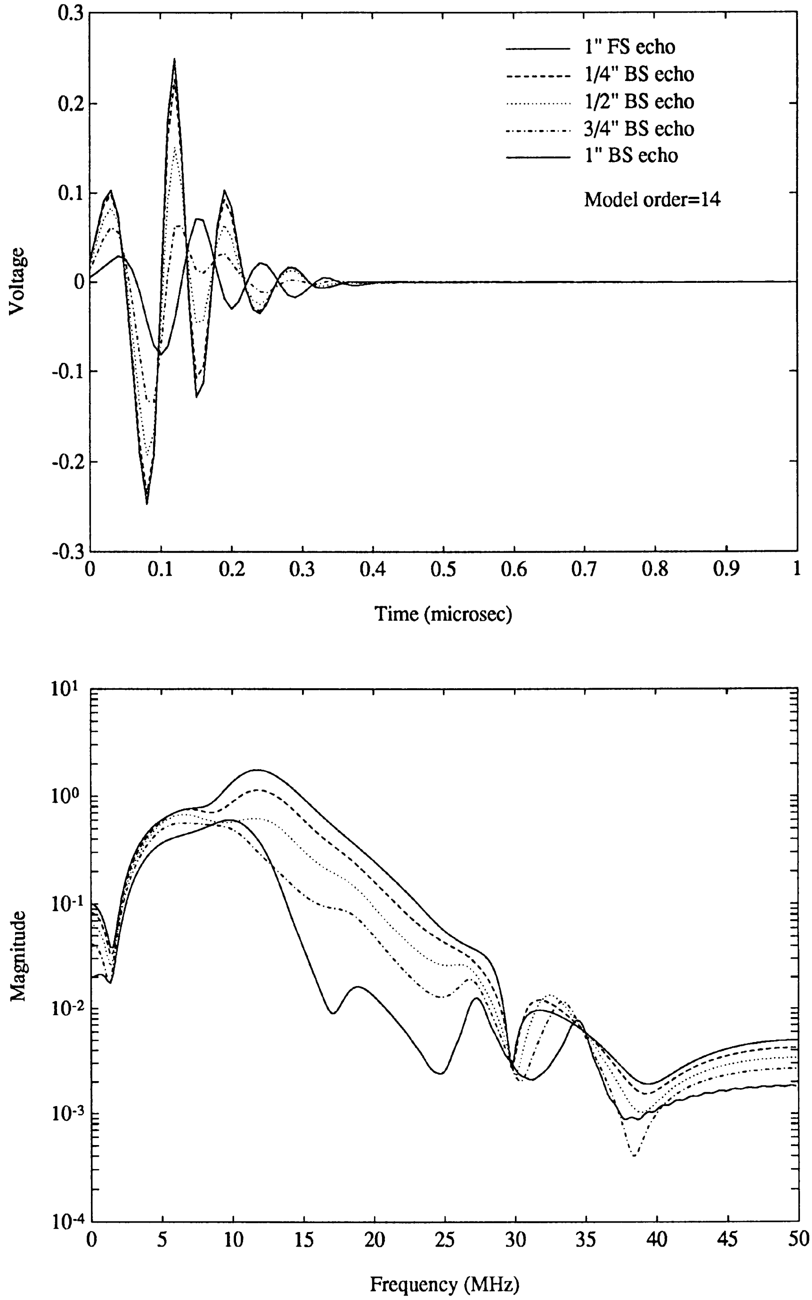

Figure 2. Interpolated signals using 14-th order ARMA models

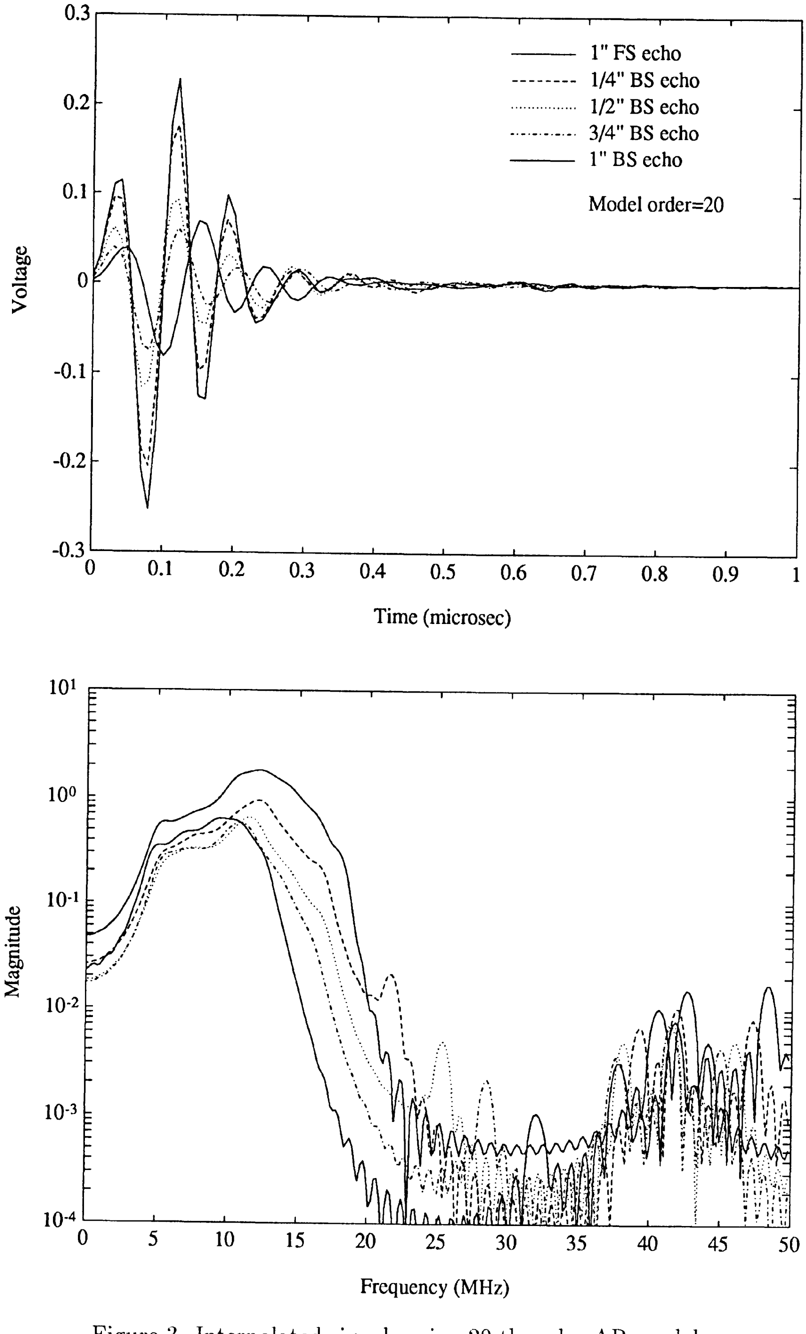

Figure 3. Interpolated signals using 20-th order AR models

Table 1. Comparison of performances of model parameter interpolation method and Wiener filter method

Signal	ARMA model		AR model		Wiener filter		
	Linear	Nonlinear	Linear	Nonlinear	Q=1%	Q=0.1%	Q=0.01%
1/4″ BS	8.48	8.16	5.28	7.43	5.57	6.41	6.63
1/2″ BS	4.93	6.58	4.62	7.70	2.78	3.99	4.20
3/4″ BS	4.07	4.46	5.86	5.70	4.10	5.77	6.02

In obtaining the front and back surface echos, a 15MHz focused transducer (radius: 0.25″, focal length: 3.5″) was used and was adjusted to focus respectively on the front and back surfaces of the samples. The back surface echos obtained from the samples were normalized by accounting for transmission and reflection coefficients and phase inversion. The front and back surface echo signals obtained from the copper samples and their frequency spectra are shown in Figure 1. Figure 2 shows the interpolated signals and their frequency spectra using 14-th order ARMA models; Figure 3 shows the interpolated signals and their frequency spectra using 20-th order AR models. Table 1 compares the performances of the model parameter interpolation method using both AR and ARMA models with the performance of Wiener filter based method. The performance measure is the signal-to-noise ratio (SNR) in dB computed using the actual (measured) back surface echo and the difference between the actual and interpolated signals. The nonlinear weighting function used here is $f(x) = \sqrt{x}$. In the Wiener filter based method, the spectrum of the attenuated signal $F_k(\omega)$ is computed as

$$F_k(\omega) = F_0(\omega)e^{-\delta(\omega)k}. \tag{16}$$

An estimate of the frequency dependent attenuation $\delta(\omega)$ in (16) is obtained as follows:

$$e^{-\delta(\omega)L} = \frac{F_L(\omega)\tilde{F}_0(\omega)}{|F_0(\omega)|^2 + Q|F_0(\omega)|^2_{\max}}, \tag{17}$$

where $F_0(\omega)$ and $F_L(\omega)$ respectively denote the spectra of front and back surface echos, "~" denotes complex conjugate, and Q is a desensitizing factor that avoids division by zero. The performance of the Wiener filter method was computed for different values of Q (1%, 0.1%, 0.01%) as shown in Table 1.

CONCLUSIONS

We have investigated a model parameter interpolation method to provide varying amounts of correction for ultrasonic attenuation along the depth of a medium. The method is especially suited for use with a Kalman filter based deconvolution technique. Both ARMA and AR system models with nonlinear interpolation yield reasonably good results.

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