The Dynamics of Carbon Sequestration and Measures of Cost-Effectiveness

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Keywords
annualization, carbon sequestration, the study period

Disciplines
Agricultural and Resource Economics | Economics | Natural Resource Economics | Natural Resources Management and Policy

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Abstract

The cost-effectiveness of carbon sequestration alternatives has often been discussed in the economics literature on sequestration. Average or marginal costs and annual carbon supply curves are often used as measures of cost-effectiveness. Sequestration is inherently a temporal process and how time is accounted for in the various measures of cost-effectiveness is critical for appropriate cross-study comparisons. I examine three factors that affect the magnitude of measured cost-effectiveness: the study period, the sequestration path, and the discount rate if discounting is used. The extent to which these factors affect the consistency of cross-study comparisons is empirically illustrated.

Keywords: annualization, carbon sequestration, the study period.

JEL Classification: Q20, Q25
THE DYNAMICS OF CARBON SEQUESTRATION AND MEASURES OF COST-EFFECTIVENESS

Introduction

Carbon sequestration offers a way to mitigate climate change. However, before carbon sequestration is adopted by any policy, it is important to know how costly one method of sequestration is compared to another, and whether sequestration is a cheaper mitigation strategy than emission abatement. This paper examines some critical issues related to cost comparisons of carbon sequestration programs across different studies.

To assess the cost-effectiveness of a carbon sequestration program, we need to know its cost and its carbon sequestration capacity. By dividing its total cost by its total amount of carbon sequestered, we can obtain the average cost of a sequestration program. For the marginal cost, we obtain the cost needed to sequester the last unit of carbon in order to reach a certain carbon target. Given the seemingly common measures of cost-effectiveness of sequestration, cross-study comparisons are often made and the results of different studies are sometimes tabulated together; see Tables 1 and 3 in Sedjo et al. (1995), Table 3 in Stavins (1999), Table 3 in Adams et al. (1999), and Tables 3 and 4 in McCarl and Schneider (2000). Such comparisons, when carefully made, can identify sequestration programs with the lowest cost.

However, a review of the literature suggests that three factors which may significantly affect the consistency of such comparisons have not received adequate attention. The first is the period of study, which may matter whether discounting is used or not. Intuitively, if most carbon sequestration by a program occurs before some time, say 60 years, then the choice of a study period longer than 60 years will reduce the annual average carbon sequestration. Given the same annual cost of the
program, the average cost of sequestration increases as the study period becomes longer. This is especially important given that there is a limit to how long soil or a forest stand will have positive carbon sequestration.\textsuperscript{1}

In the literature, the choice of the study period seems arbitrary. For afforestation programs, Richards, Moulton, and Birdsey (1993) base their study on carbon yield curves over 160 years; Stavins (1999) uses a period of 90 years; Plantinga, Mauldin, and Miller (1999) simulate 60-year programs; Parks and Hardie (1995) limit their study to “the life of a temperate forest”; and van Kooten, Binkley, and Delcourt (1995) employ the optimal harvest age of a forest. For programs seeking to improve forest management, a 30-year study period is employed in Hoen and Solberg (1994), while a 100-year period is used in De Jong, Tipper, and Montoya-Gomez (2000). Adams et al. (1999) examine the costs of sequestration through both afforestation and improvement in forest management. They limit their policy analysis to a 50-year period. Some studies (e.g., McCarl and Schneider 2001 and Pautsch et al. 2001) simulate sequestration using biophysical models over some period of time (e.g., 30 years) and then an annual average carbon sequestration is calculated. While the results of different studies are frequently compared, few studies have discussed the implications of such wide-ranging study periods.

The second factor is the path of carbon sequestration. Although seldom discussed in the cost-of-sequestration literature, the effects of different mitigation paths have been extensively debated in the more general climatic change literature (see Wigley, Richels, and Edmonds 1996 and Ramakrishna 1997). Some have argued that delaying abatement may be costly because there is socioeconomic inertia in the energy system and the process of climate change is difficult to reverse. In addition, if there are marginal costs associated with new emissions, as shown in Tol (1999), then there is a cost of delaying abatement due to discounting.
If earlier carbon sequestration is valued more, then one sequestration program may be preferred over another even if both programs can sequester the same amount of carbon (undiscounted sum) at the same amount of cost over the same period of time. For example, 77 years after afforestation, carbon sequestration (undiscounted sum) is about the same for loblolly pines in the southern plains and ponderosa pines in mountain areas. However, for the former, the majority of sequestration occurs before the first 35 years, while for the latter the majority occurs after the first 35 years. Thus, if early sequestration is preferred, then afforestation with loblolly pines is more valuable than that with ponderosa pines. To take into account the timing of carbon uptake, discounting can be used.

The discount rate is the third factor to be examined. Unlike some existing studies (to be discussed in the next paragraph), the interest here is not merely in how the results of a particular study are sensitive to different discount rates. I examine how the discount rate interacts with sequestration paths and the study period to affect the measured cost-effectiveness of sequestration programs. The advantage of discounting is that it can reflect preferences for early carbon reduction and can focus on some summary measures (e.g., annualized numbers) without being too concerned about the paths of sequestration. However, discounting also brings its own complications. The first complication is that a different discount rate may favor a different sequestration path. Moreover, such effect is not necessarily monotone in the sense that a higher discount rate always favors one path while a lower discount rate always favors another.

The second complication of discounting is that the implications of different discount rates vary with the study period and the carbon sequestration paths concerned. Given a fixed study period and specific sequestration paths, the relationship between the discount rate and the marginal cost of sequestration has
been found to be increasing (e.g., Richards, Moulton, and Birdsey 1993, and Newell and Stavins 2000), decreasing (Adams et al. 1999), or indeterminate (Hoen and Solberg 1994). Later it will be shown that the relationship actually depends on the choice of the study period, the relevant sequestration paths, as well as how marginal cost is calculated.

In this paper I examine the effects of the three factors mentioned above. Utilizing empirical results from the existent literature, I illustrate the likely magnitudes of these effects. The following section describes a simple modeling framework. In what remains, I analyze the effects of the three factors in turn. The last section gives concluding comments.

**A Simple Model Specification**

Consider a program with a funding of $\bar{M}$, which selectively enrolls fields from a total of $N$ agricultural or forest fields to sequester carbon for a period of time, $T$. Throughout this paper, $T$ is referred to as the study period. Each piece of land is enrolled for some carbon sequestration practices during this period of time. The size of field $n$ is denoted as $\bar{A}_n$, where $n$ is the index of a field. Carbon sequestered at time $t$ by field $n$ is denoted as $x_n(t) \geq 0$. Denote the unit cost of enrolling land from field $n$ as $p_n(t)$, which is the profit foregone and/or establishment expenditures due to the adoption of carbon sequestration practices. The benefit from sequestration, denoted as $b_n(t)$ for each unit of carbon sequestered, is derived from mitigating the adverse effects of global warming. It is reasonable to assume that the same amount of carbon sequestration provides the same amount of benefit across all fields, that is, $b_n(t) = b(t)$, for any $n$.

The policymaker’s problem is to choose $a_n$ for each field to maximize the present discounted value (PDV) of the benefit from sequestration over the whole
study period, i.e.,

\[
\begin{align*}
    \text{max} & \quad \int_0^T e^{-rt} \left( b(t) \sum_{n=1}^N a_n x_n(t) \right) dt \\
    \text{s.t.} & \quad \int_0^T e^{-rt} \left( \sum_{n=1}^N p_n(t) a_n \right) dt = \bar{M}, \\
    & \quad 0 \leq a_n \leq \bar{A}_n,
\end{align*}
\]

where \( r \) is the discount rate for policymakers.\(^2\) It is important to note here that \( a_n \) does not change with time. Because of the easy reversibility of carbon sequestration, it may be critical for a policy to enroll a farmer for a relatively long period of time. Otherwise, carbon can be released back to the atmosphere shortly after being sequestered, reducing or undoing its benefits.

The solutions to (1) are as follows:

\[
\begin{align*}
    a_n^* = \bar{A}_n & \quad \text{if } \bar{b}_n > \lambda^* \bar{p}_n; \\
    a_n^* = 0 & \quad \text{if } \bar{b}_n < \lambda^* \bar{p}_n; \\
    0 < a_n^* < \bar{A}_n, & \quad \text{if } \bar{b}_n = \lambda^* \bar{p}_n;
\end{align*}
\]

where \( \lambda^* \) is the multiplier for the budget constraint (1b), and

\[
\bar{p}_n \equiv \int_0^T e^{-rt} p_n(t) dt, \quad \text{and} \quad \bar{b}_n \equiv \int_0^T e^{-rt} b(t) x_n(t) dt.
\]

Intuitively, \( \lambda^* \) is the shadow cost of funding. The variables \( \bar{p}_n \) and \( \bar{b}_n \) are, respectively, the PDV of cost per acre and the PDV of benefit per acre from field \( n \) for carbon sequestration over the study period. Whether to enroll field \( n \) depends on the relative magnitude of \( \lambda^* \bar{p}_n \) and \( \bar{b}_n \). If the benefit of carbon sequestration is constant over time, that is, \( b(t) = b \) for any \( t \), then \( \bar{b}_n = b \bar{x}_n \) with

\[
\bar{x}_n \equiv \int_0^T e^{-rt} x_n(t) dt.
\]
Equations (2) are modified as follows:

\[
    a_n^* = \bar{A}_n \quad \text{if} \quad b > \lambda^* \bar{p}_n / \bar{x}_n; \quad (5a)
\]

\[
    a_n^* = 0 \quad \text{if} \quad b < \lambda^* \bar{p}_n / \bar{x}_n; \quad (5b)
\]

\[
    0 < a_n^* < \bar{A}_n, \quad \text{if} \quad b = \lambda^* \bar{p}_n / \bar{x}_n. \quad (5c)
\]

We use \( \hat{n} \) to specifically denote the marginal field such that \( b = \lambda^* \bar{p}_{\hat{n}} / \bar{x}_{\hat{n}} \). The optimal number of acres to be enrolled from field \( \hat{n} \) is determined as follows:

\[
    a_{\hat{n}}^* = (\bar{M} - \sum_{\{i: a_i^* = \bar{A}_i\}} \bar{p}_i a_i^*) / \bar{p}_{\hat{n}} \quad \text{if} \quad b = \lambda^* \bar{p}_{\hat{n}} / \bar{x}_{\hat{n}}, \quad (6)
\]

where \( \{i: a_i^* = \bar{A}_i\} \) is the set of fields which are completely enrolled in the program.

From its definition (4), we can view \( \bar{x}_n \) as the PDV of sequestered carbon. It may seem unusual to use the PDV of a physical good, carbon, because PDVs are in general calculated for monetary values. In fact, in the model setup (1), only monetary values are discounted. Carbon discounting results because benefit provided by future sequestration is discounted. In the literature, carbon discounting such as that in (4), or its variation such as annualized carbon (more on this later), has been adopted by many studies, including Richards, Moulton, and Birdsey (1993), Hoen and Solberg (1994), Adams et al. (1999), Plantinga, Mauldin, and Miller (1999), Stavins (1999), and Newell and Stavins (2000).

In (5), \( \lambda^* \bar{p}_n \) is the cost per acre to enroll land from field \( n \), which equals the direct cost per acre multiplied by the shadow cost of funding. Then, \( \lambda^* \bar{p}_n / \bar{x}_n \) is the average cost of sequestration from field \( n \) over the study period. Note that the average is based on PDVs, not on the undiscounted accumulation. For a program as defined by (1), the marginal cost of enrolling land is the cost of enrolling land from the marginal field, that is, \( \lambda^* \bar{p}_{\hat{n}} \). The marginal cost per unit of carbon sequestration for the program is \( \lambda^* \bar{p}_{\hat{n}} / \bar{x}_{\hat{n}} \).
Conditions in (5) basically require that the optimal program starts enrolling fields with the lowest (direct) cost, and then moves to those with higher (direct) cost. This goes on until available funding is exhausted. Thus, $\lambda^*$ is not really relevant in the decision on which field to enroll. Moreover, the shadow cost of funding is rarely considered in empirical studies. Accordingly, $\lambda^*$ will be disregarded for the rest of the analysis and the direct cost of a sequestration program will be referred to simply as the cost of the program.

For later reference, I define the annualized cost per acre and annualized carbon sequestration per acre for field $n$.

**Definition 1** The annualized cost of enrolling an acre of land from field $n$ is a fixed annual cost, denoted as $\overline{p}_n$, such that $\int_0^T e^{-rt} \overline{p}_n \, dt = \overline{p}_n$. Or,

$$\overline{p}_n = \frac{\overline{p}_n}{\int_0^T e^{-rt} \, dt}, \quad (7)$$

where $\int_0^T e^{-rt} \, dt$ is the annualizing factor. Similarly, the annualized carbon sequestration by an acre of land from field $n$ is a fixed annual number, denoted as $\overline{x}_n$, such that

$$\overline{x}_n = \frac{\overline{x}_n}{\int_0^T e^{-rt} \, dt}. \quad (8)$$

The notation “$\overline{\cdot}$” above $\overline{p}_n$ and $\overline{x}_n$ is used to mean average, or more specifically in (7) and (8), annualization over the study period. Then for the program as defined by (1), the marginal cost of carbon sequestration based on PDVs is

$$\overline{MC} = \frac{\overline{p}_n}{\overline{x}_n} = \frac{\overline{p}_n}{\overline{x}_n}, \quad (9)$$

where “$\overline{\cdot}$” above $MC$ is used to emphasize the fact that PDVs of cost and carbon are used. We can interpret $\overline{MC}$ in two slightly different ways. The middle term in (9) indicates that it is the extra cost (in PDV) to sequester an extra ton of carbon (in PDV) over the whole study period, while the last term in (9) indicates it is the
extra annual cost (in annualized value) to sequester an extra ton of carbon annually (also in annualized value).

Denoting as \( \bar{C} \) the total amount of carbon sequestered by the program, we get

\[
\bar{C} = \int_{0}^{T} e^{-rt} \left( \sum_{\{i: a_i = A_i\}} a_i^* x_i(t) \right) dt + \int_{0}^{T} e^{-rt} a_n^* x_n(t) dt. \tag{10}
\]

By varying funding \( \bar{M} \) for the program, we can derive different pairs of \((\bar{C}, \bar{MC})\). Plotting these pairs on a two-dimensional plain, we obtain the carbon supply curve from these \(N\) fields.

From (9) and (10), it is clear that two otherwise identical programs may have quite different measured marginal cost of sequestration if different \( r \) or \( T \) or \( x_n(t) \) is used in the analyses of the programs. While recognizing \( p_n(t) \) is also an important factor, only the first three factors will be discussed. In fact, the focus here is on how the measured marginal cost of sequestration varies with the changes in the three factors for given \( p_n(t) \). First, I examine the effects of \( x_n(t) \).

**The Effects of Carbon Paths**

To avoid confusion, I first define the (undiscounted) annual average carbon sequestration in field \( n \) over the study period \( T \) as

\[
\bar{x}_n \equiv \left( \int_{0}^{T} x_n(t) dt \right) / T. \tag{11}
\]

Many studies base the marginal cost of carbon sequestration on \( \bar{x}_n \), for example, Moulton and Richards (1990), Dudek and LeBanc (1990), Adams et al. (1993), Dixon et al. (1994), Parks and Hardie (1995), De Jong, Tipper, and Montoya-Gomez (2000), Pautsch et al. (2001), and McCarl and Schneider (2001). When carbon sequestration is linear, that is, \( x_n(t) = \bar{x}_n, \forall t \), then from (4) and (8) we know \( \bar{x}_n \geq \bar{x}_m \) if and only if \( \bar{x}_n \geq \bar{x}_m \). In such cases, paths of sequestration do not affect the choice between \( x_n(t) \) and \( x_m(t) \).
However, the time path of carbon sequestration is usually non-linear. For example, carbon sequestration by afforestation is largely determined by the accumulation of biomass which is known to follow an S-shaped curve in general: slow at the beginning, faster in the midterm, and then slow again in the long run. The duration of each stage could range from a few years to more than a hundred years, depending on the timber species. As shown by Richards, Moulton, and Birdsey (1993), the paths for loblolly pine, ponderosa pine, and black walnut are all S-shaped and quite different from each other. A similar process also exists for carbon sequestration by switching from conventional tillage to conservation tillage (Lal et al. 1998).

When linearity is not satisfied, two fields may have quite different annualized carbon even if the undiscounted annual averages of carbon are the same and the same $r$ and $T$ are used. In other words, $\bar{x}_n - \bar{x}_m$ may be large in absolute value even if $\bar{x}_n - \bar{x}_m$ is zero. The disparity between $\bar{x}_n - \bar{x}_m$ and $\bar{x}_n - \bar{x}_m$ is affected by the curvature of $x_n(t)$ and $x_m(t)$ in the carbon-time plane (e.g., see Figures 1(a) and 1(b)). If we view carbon sequestration $x_n(t)$ as the weight attached to the corresponding time point $t$, then $x_n(t)$, upon appropriate normalization, can be viewed as the probability density function of $t$. This view of $x_n(t)$ enables utilization well-known results in the literature on finance and risk/uncertainty.

Before invoking any result, I present the definitions of two concepts.

**Definition 2** Let $F^i(y)$ and $F^j(y)$ be two cumulative distribution functions (cdf’s) of a random variable $y \in [\underline{y}, \overline{y}]$. (i) $F^i(y)$ first-order stochastically dominates (FOSD) $F^j(y)$ if

$$\int_{\underline{y}}^{\overline{y}} \phi(y)F^i(y) \geq \int_{\underline{y}}^{\overline{y}} \phi(y)dF^j(y)$$

(12)

for any non-decreasing function $\phi(y)$. (ii) $F^i(y)$ second-order stochastically dominates (SOSD) $F^j(y)$ if (12) holds for any non-decreasing concave function $\phi(y)$. 

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We can show that $F^i(y)$ FOSD $F^j(y)$ if $F^i(y) \leq F^j(y)$, for any $y \in [y, \bar{y}]$.

Also, given the same mean, if $Y$ has larger variability under $F^i(y)$ than under $F^j(y)$ then $F^i(y)$ SOSD $F^j(y)$. Loosely speaking, FOSD compares the means (levels) of two distributions while SOSD, in addition, compares their spread over the domain of a random variable. Suppose the two cdf’s are the distributions of random net returns associated with two investment projects. If $F^i(y)$ FOSD $F^j(y)$, that is, higher returns are more likely to occur under $F^i(y)$ than under $F^j(y)$, then the former will be preferred over the latter by any investor who values higher returns more. If $F^i(y)$ SOSD $F^j(y)$, then the former will be preferred over the latter by any risk-averse investor who values higher returns more because net returns from the former tend to be less variable and/or higher in all states of nature.

In our context, we can artificially construct cdf’s as follows. Define

$$F^i(t) = \frac{\int_0^t x_i(s)ds}{\int_0^T x_i(s)ds}, \quad i = 1, 2, ..., N. \quad (13)$$

The term $F^i(t)$ is actually a cdf because it satisfies all the conditions required for a cdf. However, $F^i(t)$ is artificial because $t$ is not really a random variable, and different probability densities are only artificially attached to different values of $t$. Based on $F^i(t)$, I next provide two propositions which compare $\bar{x}_n - \bar{x}_m$ and $\bar{x}_n - \bar{x}_m$.

**Proposition 1** Given $\bar{x}_n = \bar{x}_m$, if $F^m(t)$ first-order stochastically dominates $F^n(t)$ then $\bar{x}_n \geq \bar{x}_m$ for any $r > 0$.

Proofs are given in the appendix. Intuitively, the probability density artificially given to each point of time is determined by the rate of carbon sequestration. The first-order stochastic dominance by $F^m(t)$ over $F^n(t)$ means that proportionally less carbon is accumulated earlier under path $x_m(t)$ than under path $x_n(t)$. Given that earlier sequestration is valued more (for $r > 0$) in calculating the PDV of a stream
of carbon sequestration, the PDV is greater from \( x_n(t) \) than from \( x_m(t) \); that is, \( \bar{x}_n \geq \bar{x}_m \), or equivalently, \( \overline{\Pi}_n \geq \overline{\Pi}_m \).

Figure 1(a) gives one example of Proposition 1. Two paths are shown: one constant and the other monotone decreasing over time. The area under each path is the carbon accumulated over time through the path. Given that the undiscounted annual averages of carbon sequestration under the two paths are equal, it is clear that at each moment of time but the last moment \( T \), carbon accumulated under \( x_n(t) \) is more than that under \( x_m(t) \); that is, \( F^m(t) \leq F^n(t) \) for any \( 0 \leq t < T \).
Then, according to Proposition 1, we have \( \overline{x}_n \geq \overline{x}_m \).

Figure 1(b) displays a similar situation with non-monotone paths. The two pines have about the same (undiscounted) annual average carbon sequestration from year 0 to year 77, which is about 2.15 tons/year/acre. However, over the same period and at a 2 percent discount rate, the PDVs of carbon sequestration for loblolly and ponderosa pines are \( \bar{x}_n = 102.96 \) and \( \bar{x}_m = 72.463 \) tons/acre, respectively (which correspond to annualized carbon: \( \overline{\Pi}_n = 2.62 \) and \( \overline{\Pi}_m = 1.84 \) tons/year/acre, respectively). The difference between \( \overline{\Pi}_n - \overline{\Pi}_m \) and \( \bar{x}_n - \bar{x}_m \) is about 0.78 tons/year/acre, which accounts for 36 percent of the (undiscounted) annual average carbon sequestration.
Figure 1 (a). Comparison of carbon paths

Figure 1(b). The paths of loblolly and ponderosa pines
In situations where the accumulated carbon under neither path is higher than under the other path for all time points, as shown in Figure 2, Proposition 1 does not apply. It is not immediately obvious whether the annualized carbon under path $x_n(t)$ is more than that under $x_m(t)$, even if $\bar{x}_n = \bar{x}_m$. However, with more information on $F^m(t)$ and $F^n(t)$, that is, on $x_m(t)$ and $x_n(t)$, we can identify a condition under which $\bar{x}_n \geq \bar{x}_m$.

**Proposition 2** Given $\bar{x}_n = \bar{x}_m$, if $F^m(t)$ second-order stochastically dominates $F^n(t)$, then $\bar{x}_n \geq \bar{x}_m$ for $r > 0$.

Graphically, the second-order stochastic dominance of $F^m(t)$ over $F^n(t)$ implies that carbon uptake spreads out more evenly over time and/or occurs earlier along path $x_n(t)$ than along path $x_m(t)$ (see Figure 2). In either case, proportionately more carbon is accumulated earlier under path $x_n(t)$. Because of discounting, the
value of carbon sequestration decreases at an exponential rate, $e^{-rt}$, and so the annualized carbon is higher for a carbon path with relatively more early carbon sequestration. Thus, we have $\bar{x}_n \geq \bar{x}_m$.

As a numerical example, let

$$x_n(t) = \frac{100}{\sqrt{2\pi} \times 25 \times .6826} \times \exp\left[-\frac{(t - 25)^2}{2 \times 25^2}\right]; \quad (14a)$$

$$x_m(t) = \frac{100}{\sqrt{2\pi} \times 10 \times 0.9876} \times \exp\left[-\frac{(t - 25)^2}{2 \times 10^2}\right]. \quad (14b)$$

Consider a period of 50 years, i.e., $0 \leq t \leq T = 50$. The plots of the two paths from time 0 to time $T$ are shown in Figure 2. The (undiscounted) annual average carbon sequestration over these $T$ years is the same for both paths: $\bar{x}_n = \bar{x}_m = 2$. However, as shown by the figure, carbon path $x_n(t)$ is flatter. According to (13), $F^m(t) = \int_0^t x_m(s)ds/100$ and $F^n(t) = \int_0^t x_n(s)ds/100$. The artificial random variable $t$ has the same mean under these cdf’s but a larger variance under $F^n(t)$ than under $F^m(t)$. In fact, $F^m(t)$ second order stochastically dominates $F^n(t)$. By Proposition 2, we know $\bar{x}_n \geq \bar{x}_m$. This is consistent with our numerical results: at $r = 5\%$, $\bar{x}_n = 1.94$ and $\bar{x}_m = 1.75$; or at $r = 10\%$, $\bar{x}_n = 1.84$ and $\bar{x}_m = 1.29$.

**The Effects of Study Period**

In the examples of the loblolly pine and ponderosa pine as shown in Figure 1(b), if $T < 76$, then the (undiscounted) annual average carbon sequestered by loblolly pine will be greater than that by ponderosa pine. If $T > 77$, the reverse is true. This simple example illustrates how summary measures of carbon sequestration may produce different results when different study periods are used.

The following analysis illustrates how much the results may differ empirically. Using a study period of 90 years, Stavins (1999) establishes carbon supply curves through afforestation for 36 counties in the U.S. Mississippi delta states (Arkansas, Louisiana, and Mississippi) and then calibrates his model for the whole United
States in order to conduct cross-study comparisons. While he tries to take into account different discount rates used in different studies, the implication of different study periods used in different studies is in general not examined.

Based on the marginal costs of land conversion in Table 2 of Stavins (1999) and carbon yield curves in Richards, Moulton, and Birdsey. (1993), I construct carbon supply curves for the study area in Stavins (1999) using different study periods, \( T \). The results for loblolly pine are shown in Figure 3. To illustrate the size of the difference in marginal costs for different \( T \), more details are given in Table 1 for the case where 25 million tons of annualized carbon is sequestered for the given area. The table suggests that the implication of different choices of \( T \) is significant in this case.

![Figure 3. Supply curves, loblolly pine, (r=.05), different \( T \)](image)

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<th>30</th>
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</tr>
<tr>
<td>Differences (compared to ( T=60 ))</td>
<td>-40%</td>
<td>0</td>
<td>26%</td>
<td>36%</td>
</tr>
</tbody>
</table>

Table 1. Costs to sequester 25 million tons of annualized carbon (for loblolly pine, \( r=.05 \), and \( T=30, 60, 90, \) and 160 years)
Figure 3 suggests that as $T$ increases, the carbon supply curve for loblolly pine moves leftward. However, as illustrated by Figure 4, the pattern shown in Figure 3 does not generalize. As $T$ increases, the carbon supply curve for ponderosa pine actually shifts rightward. Moreover, the response of carbon supply curves to changes in the study period is not necessarily linear. Although not shown, for ponderosa pine at $r = 0$, as $T$ increases the carbon supply curve first shifts rightward and then leftward. Given such irregular shifts of the carbon supply curve with the change of $T$, any cross-study comparison is made more challenging if studies base their results on different study periods.

![Graph showing supply curves for ponderosa pine](image)

**Figure 4. Supply curves, ponderosa pine, ($r=.05$), different $T$**

The lack of a specific pattern in the shifts of the carbon supply curve in response to the change of $T$ can be explained by the definition of $\bar{x}_n$ in (8). As $T$ increases both the denominator, i.e., the annualizing factor ($\int_0^T e^{-rt}dt$), and the numerator, i.e., the PDV of carbon, increase, given non-negative sequestration.
However, depending on the path of sequestration, their rates of change may differ and the difference may vary over time. Thus, as $T$ increases annualized carbon may change non-monotonely. By the definition of marginal cost (9), this in turn implies that, even given the same annualized marginal cost per acre of afforestation, marginal cost of carbon sequestration may change non-monotonely with monotone changes in $T$.

**The Effects of Discount Rate**

As mentioned earlier, here we are not merely interested in the sensitivity-type analyses. Instead, we want to know how the effects of discount rate are affected by different $T$ and different carbon sequestration paths. Table 2 shows annualized carbon sequestration on an acre of afforestation with two different pines. As $r$ increases, for ponderosa pine the annualized carbon decreases for all four study periods, while for loblolly pine the relationship varies with $T$. Thus, the relationship between the discount rate and the annualized carbon sequestration varies with the study period and the specific carbon paths (or tree species).

<table>
<thead>
<tr>
<th>(a) Loblolly pine</th>
<th>(b) Ponderosa pine</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>T=30</strong></td>
<td><strong>T=60</strong></td>
</tr>
<tr>
<td>$r=0$</td>
<td>3.562</td>
</tr>
<tr>
<td>$r=.02$</td>
<td>3.485</td>
</tr>
<tr>
<td>$r=.05$</td>
<td>3.333</td>
</tr>
<tr>
<td>$r=.15$</td>
<td>2.704</td>
</tr>
</tbody>
</table>

The various relationships between $r$ and annualized carbon again can be explained by the definition of annualized carbon in (8). As $r$ increases, say by $\Delta r > 0$, both the integrand in the annualizing factor ($\int_0^T e^{-rt}dt$) and the integrand in the PDV of carbon ($\int_0^T e^{-rt}x_n(t)dt$) decrease proportionately to $e^{-\Delta r^* t}$ times their original values. Since $e^{-\Delta r^* t}$ decreases with $t$, the integrands associated with a
larger \( t \) reduce more than those associated with a smaller \( t \). Thus, depending on how the integrands in them change over time, the PDV and the annualizing factor may change at quite different rates when \( r \) increases. This causes the annualizing factor to change in various patterns as \( r \) increases. Since the integrand in the PDV is affected by sequestration paths and the study period \( T \), the relationship between \( r \) and the annualized carbon is also affected by these two factors.

Even given the same study period, discounting may lead to inconsistent comparisons. In order to compare the annualized carbon sequestration along two different paths \( x_n(t) \) and \( x_m(t) \), we can assess the sign of

\[
D(r) \equiv \tilde{x}_n - \tilde{x}_m = \int_0^T e^{-rt} [x_n(t) - x_m(t)] \, dt.
\]

From the definition of \( \tilde{x}_n \) in (8), we know \( \tilde{x}_n - \tilde{x}_m \geq 0 \) if and only if \( D(r) \geq 0 \). If we treat \( x_n(t) - x_m(t) \) as the net cash flow of a project, then \( D(r) \) is the net present discounted value of this project. The discount rate \( r \) such that \( D(r) = 0 \) is called the internal rate of return (ROR) of a project. The well-known disadvantage of ROR is that it is not unique. This same disadvantage is manifested in the PDV of a project in the form of non-monotonicity of \( D(r) \) in \( r \). Saak and Hennessy (2001) and Oechmke (2000) identify conditions where \( D(r) \) is monotone in \( r \).

When the study period varies, the dividing discount rate such that \( D(r) = 0 \) also varies. In the example of the loblolly pine and ponderosa pine, at \( T = 160 \), the dividing discount rate is about 0.0153, that is, the PDV of carbon sequestration by loblolly pine is larger if \( r > 0.0153 \). At \( T = 90 \), the dividing discount rate is about 0.0074, which is smaller than that at \( T = 160 \). This is because carbon sequestration by loblolly pine almost tapers off to zero for \( T > 90 \), while sequestration by ponderosa pine is still significant (see Figure 1(b)). Thus, for a study period longer than 90 years, a higher discount rate is needed in order for the PDV of carbon sequestration by loblolly pine to remain larger.
Discussion

Summary measures like the annualized marginal cost of sequestration and annualized carbon supply curves are intended to normalize the time dimension of carbon sequestration. In the literature, few have explored how such normalization may affect the measured cost-effectiveness of sequestration programs. In this paper, I examine the effects of three factors: the sequestration path, the study period, and the discount rate. It is empirically demonstrated that differences in any one of these factors may lead to significant disparities in measured cost-effectiveness for otherwise identical sequestration programs.

The results suggest that, in cross-study comparisons of sequestration programs, researchers should be careful about the underlying parameters, for example, the study period and the species of trees concerned or, in general, the carbon yield curves of the underlying sequestration processes. In addition, results from different scenarios may be helpful. While some studies include sensitivity analysis with respect to the discount rate, few have analyzed how the presented results may vary with the change of study period or alternative carbon yield curves. Sensitivity analysis with respect to the study period can also be plotted by adding a time dimension to the plot of carbon supply curves.
Endnotes

1In cases where trees are harvested and kept as long-lasting wood, or used as bio-fuel, such limit may not exist.

2Some studies use the cost minimization problem dual to (1) where the objective is to minimize the cost of sequestration given a carbon target. The results from the dual and primal models are similar.

3The regions suitable for loblolly pine and ponderosa pine may have marginal costs of afforestation different from those in Stavins (1999). However, when $p_n(t) = p_n$ for all $0 < t \leq T$, as is the case in Stavins, the annualized costs of afforestation per acre stay the same for each field when $T$ and $r$ change. Thus how the supply curve shifts depends on how annualized carbon per acre changes with $T$ and $r$.

4From (9), we know, given the same annualized cost of afforestation, marginal cost of sequestration is determined by annualized carbon sequestration. This is the case if $p_n(t) = p_n$ for all $0 < t \leq T$. In general, as $r$ varies annualized cost per acre of afforestation will also change. However, given the focus here on the effects of the dynamics of sequestration, only changes in the annualized carbon will be discussed.
Appendix

Proof for Proposition 1

Note that $\phi(t) = -e^{-rt}$ is an increasing function for any $r > 0$. Thus, if $F^m(y)$ FOSD $F^n(y)$, then by (12) we have

$$\int_0^T (-e^{-rt})dF^m(t) \geq \int_0^T (-e^{-rt})dF^n(t), \quad \forall \; r > 0. \quad \text{(A-1)}$$

Plugging in (13) and rearranging, we obtain

$$\int_0^T \frac{e^{-rt}x_m(t)}{\int_0^T x_m(s)ds}dt \leq \int_0^T \frac{e^{-rt}x_n(t)}{\int_0^T x_n(s)ds}dt, \quad \forall \; r > 0.$$

Given $\bar{x}_n = \bar{x}_m$ and (11), we have

$$\int_0^T e^{-rt}x_m(t)dt \leq \int_0^T e^{-rt}x_n(t)dt, \quad \forall \; r > 0,$$

i.e., $\bar{x}_n \geq \bar{x}_m$. The proposition follows because $\bar{x}_n \geq \bar{x}_m$ if and only if $\bar{x}_n \geq \bar{x}_m$.

Proof for Proposition 2

Since $\phi(t) = -e^{-rt}$ is non-decreasing and concave for any $r > 0$, (A-1) still holds if $F^m(y)$ SOSD $F^n(y)$. The rest of the proof follows in a way similar to the proof for Proposition 1.
References


139-165.


