Optimal trade policy, time consistency and uncertainty in an oligopsonistic world market

Jean-Philippe Gervais
Iowa State University

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Optimal trade policy, time consistency and uncertainty
in an oligopsonistic world market

by

Jean-Philippe Gervais

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Economics

Major Professor: Harvey E. Lapan

Iowa State University

Ames, Iowa

1999

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Graduate College
Iowa State University

This is to certify that the Doctoral dissertation of
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has met the thesis requirements of Iowa State University

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Major Professor

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For the Major Program

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For the Graduate College
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I would like to express my most sincere gratitude to my major professor Dr. Harvey E. Lapan for guiding my research efforts. Completing this dissertation under his supervision has been an enriching and enjoyable experience. His knowledge of economic theory, advising skills and teaching ability have been particularly helpful. I would like to thank Dr. GianCarlo Moschini for serving on my committee and being so supportive during my four years at Iowa State. He significantly improved my understanding of economic theory through the classes that he taught. I also wish to express my deep appreciation to Dr. John Schroeter, Dr. David Hennessy and Dr. Hal Stern for serving on my dissertation committee.

Special thanks go to Dr. C. Phillip Baumel for generously funding my studies during my four years at Iowa State University. His valuable experience and expertise in the economics of agricultural markets allowed me to study a number of challenging research topics. I also acknowledge the financial support of the Iowa Farm Bureau Federation through their fellowship program.

Finally, the completion of this dissertation would not have been possible without the love and support of my parents. They have taught me how hard work and determination would lead me to achieve any endeavor I choose to pursue. I am particularly thankful for their understanding and encouragement during my four-year voyage away from home.
This dissertation investigates the strategic behavior between countries that have purchasing power on the world market for a certain good. Tariffs and quotas are not equivalent protection instruments under non-cooperative behavior. Importers are better off if they choose their trade instrument cooperatively. If production decisions occur before consumption decisions, the \textit{ex-ante} optimal policy is not time consistent because the \textit{ex-post} elasticity of the residual foreign export supply curve is lower than the \textit{ex-ante} elasticity. However, we show that the importers’ inability to irrevocably commit to their trade instrument may be welfare superior to the precommitment solution.

We also derive the equilibrium production tax and quota if production decisions occur before consumption decisions and importers are not able to irrevocably commit to their \textit{ex-ante} trade policy. Production instruments are not equivalent under non-cooperative behavior. Provided trade is restricted with an import quota, the equilibrium production policy is to restrict domestic production below the competitive level. If trade is restricted with an import tariff, the equilibrium production policy may be to subsidize production. We derive conditions under which the ability of each importer to control domestic production increases welfare.

In the next part of the dissertation, we endogenize the decision of two policy active importers to either commit to their import quota or keep the flexibility to revise their \textit{ex-ante} quota once production decisions are made. Production in both importing countries is subject to an asymmetric random shock. Both policy active importers prefer not to commit to their trade policy given a sufficiently high degree of variance in production uncertainty. Under
certain conditions, there exists an equilibrium where one country commits to its \textit{ex-ante} quota while the other keeps the flexibility to revise the level of its quota after uncertainty is resolved.

Finally, we examine the endogenous choice of trade instrument among two policy active importers. Production and consumption decisions are carried out simultaneously and the producers' cost function is subject to an asymmetric random shock. In the case where random disturbances are zero, the equilibrium of the game entails both importers choosing to use an import quota. Under production risk, there exists an equilibrium where one country uses an import quota while the other uses a tariff.
CHAPTER 1. GENERAL INTRODUCTION

1.1 – Introduction

In today's economic world, most countries face considerable pressure from trading partners to eliminate trade barriers. Some nations successfully resist adhering to the concept of free trade. Their actions can be motivated by political pressure coming from domestic agents or by the attainment of some non-economic objectives. However, countries may also be preoccupied by the exercise of their market power. In agricultural markets, the existence of state trading enterprises (STEs) may be an indication that some countries actually think they possess market power for a certain good.

The exercise of that market power differs greatly across countries and commodities. In this thesis, we look at importers who can influence the world price at which they buy a certain good due to their economic size. This is referred to as the large country assumption in the economic literature. We assume that countries behave non-cooperatively when deciding how to use their respective market power. This setting has been neglected in the literature as researchers' efforts have mainly concentrated on the behavior of a single policy active importer.

Pursuing an active commercial policy has potentially dynamic implications, which have often been ignored in research. The timing at which commercial policy is used if production decisions and consumption decisions are separated in time can influence greatly the impacts of commercial policies. Agricultural markets provide a good example of this situation, since there exists a lag between planting decision and harvest and consumption decisions. Similarly, there may exist a lag between investment decisions of firms and labor
hiring decisions because capital takes time to accumulate. Policy active countries may have
the incentive to change their policies between the two stages. This has potentially enormous
impacts on the ranking of trade policies and important welfare considerations. The purpose
of this thesis is to study the impact of production lags on policy active importers' welfare
under various assumptions.

1.2 – Thesis Organization

Chapter 2 investigates the strategic behavior between countries that have purchasing
power on the world market for a certain good. It stresses the potential welfare superiority of
the 'no-commitment' equilibrium compared to the precommitment solution from the
importers' perspective. The next chapter explores the policy active importers' incentives and
the welfare implications of using both production policies and trade polices if importers are
not able to irrevocably commit themselves to their ex-ante trade policy. We derive
conditions under which the ability of each importer to control domestic production increases
welfare. The ensuing chapter allows two policy active importers to endogenously decide
whether to commit or not commit to their trade policy before production decisions are made
if each importer's domestic production is subject to a country-specific random shock. The
introduction of risk in production provides an option value for importers not to commit to
their trade policy. The next chapter endogenizes the type of trade instrument used by
importers. Non-cooperative behavior in our model implies a non-equivalence of trade
instruments. We emphasize the factors leading importers to prefer a certain type of
instrument to another. The last chapter finally provides general conclusions and discusses
future areas of research.
CHAPTER 2. TRADE POLICY AND TIME CONSISTENCY IN AN OLIGOPSONISTIC WORLD MARKET

2.1 - Introduction

Much has been written on the theory of optimal tariffs (see Corden (1994) for a detailed survey). However, as pointed out by Grant and Quiggin (1997), strategic issues raised by a market structure where there is more than one country with market power for a good have not been addressed formally. This issue of market structure in trade policy is somewhat related to the study of oligopoly theory.

The contribution of this chapter to the literature is twofold. First, it investigates the strategic behavior between countries that have purchasing power on the world market. This strategic game between policy active importers has been introduced first by Bergstrom (1982) and later by Karp and Newbery (1991, 1992).\textsuperscript{1} We formalize the non-equivalence of tariffs and quotas given the structure of the world market and the non-cooperative behavior among importers. Importers set their trade instrument given their belief about the type of instrument the other importers will use. When the strategy space is restricted to the use of a tariff, the Nash equilibrium entails lower tariffs for each country than in the situation when they collude and act as a single monopoly importer. If the strategy space is restricted to quotas, the non-cooperative solution also implies that a too large quantity is imported in each country. Each country would be better off by colluding and importing a smaller quantity.

\textsuperscript{1} These papers analyze the strategic behavior between importers of a depletable resource. There is a significant difference between the optimal tariff for an ordinary good compared to an exhaustible resource such as oil. Oil is available in a fixed amount and, if costless to extract, its supply will be inelastic. However, in a trade context, exports are not inelastic, \textit{i.e.} there is a role for demand.
The main contribution of this chapter analyses time consistency issues. As pointed out by Staiger (1995), time consistency problems and rules versus discretion issues have occupied a major place in the macroeconomics and public finance literature, but less so in the international trade field. With a sufficient degree of discretion, an optimal trade policy is bound to lack credibility because it is almost surely time inconsistent. Most of the time consistency issues addressed in the economic literature emphasize the inferiority of the no commitment solution.

We assume there is a lag between production and consumption, and that all countries can change their policy between the two stages. For example, this setting applies to agricultural markets with spring planting and fall harvest. In the case of tariffs, the *ex-post* (given production decisions) tariff will be higher than the *ex-ante* (before production decisions are made) tariff because the residual export supply curve elasticity faced by each country is lower *ex-post*. With perfect foresight, foreign producers fully anticipate the time consistent tariff and decrease their production accordingly. Therefore, the lower *ex-post* elasticity of the residual foreign export supply curve may be welfare increasing for the policy active importers compared to the *ex-ante* situation because of its off-setting welfare effect with respect to the trade instrument competition. The same argument also applies to the strategic quota game.

This chapter is organized as follows. First, it provides a review of the literature on time consistency of trade policy. The theoretical model is set out in the second section to

---

2 For example, the same issue would arise if there was a lag between capital/investment decisions and labor decisions.
address optimal trade policy and time consistency issues. Next, we develop a numerical example to illustrate the various results. The last section provides concluding remarks.

2.2 - Review of Literature

According to the well-known theory of Johnson (1954), the optimal tariff for a large country equals the reciprocal of the foreign export elasticity of supply. Lapan (1988) points out that, in the case where production decisions are made before consumption and trade decisions, and that the government can readjust its tariff between the two stages, the standard optimal tariff will not be time consistent. From an ex-post perspective, i.e. once production decisions are made, the foreign export supply elasticity is lower than the ex-ante elasticity. Therefore, policy makers have an incentive to set ex-post tariffs at a higher level than they would if they could precommit to the ex-ante tariff.

The foreign and domestic producers, knowing that the ex-ante tariff is not time consistent will adjust their production accordingly (i.e. foreign production will be lower than if the large country could precommit to the ex-ante optimal tariff) and both countries will be worse off. The importance of the timing assumption is immediate once it is recalled that a tariff can always be decomposed into a production subsidy and a consumption tax on the importable good.

Maskin and Newbery (1990) model the behavior of a large importer of oil unable to commit to future tariffs. Time consistency models of exhaustible resources point out that scarcity can be artificially induced by the exercise of market power. Suppliers not only have to make their extraction (and hence their production) decision according to current prices, but also by comparing anticipated future prices, which will depend on future levels of a tariff. In
their two period model, if the importer places sufficient weight on second period consumption of oil, and can revise costlessly the tariff set in the first period, the welfare level in the time consistent tariff equilibrium may be less than the free-trade welfare level.

Karp and Newbery (1991,1992) build a continuous time model where oligopsonistic importers choose a time path of tariffs to maximize their domestic welfare. They show that the open loop strategy is not time consistent. They rely on numerical methods to illustrate the welfare inferiority of the closed-loop solution compared to the open-loop solution. However, they simplify the model so that there is only one large importer. In that case, there is disadvantageous market power for the importer. In their 1991 paper, they illustrate the differences between the time consistent tariff and the importers' welfare under different sequential games between policy active importers and competitive exporters.

Karp and Perloff (1995) consider the impacts of government commitment on output and investment subsidies in a strategic trade model à la Brander-Spencer. Output policies based on static models are not altered for a dynamic model. However, investment decisions depend on future as well as current government policies. If precommitment is not feasible, investment policy is of a limited strategic use. Table 2.1 summarizes the principal papers on time consistency of trade polices.

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3 A closed-loop solution refers to an equilibrium where players can observe and respond to their opponents' actions at the end of each period. The open-loop solution is a function of calendar time alone.

4 Other interesting issues of time consistency in international trade have been addressed in the literature. Staiger and Tabellini (1987, 1989) considered the credibility issue arising from the use of tariffs as a redistributive tool. Torneill (1991), Brainard (1994) and Wright (1995) explain that future tariff removal is time inconsistent if protection was either granted to provide incentives to firms to reduce their costs or to a declining industry. Karp and Paul (1998) analyze the impact of tariff commitment on the ability to affect the reallocation of labor in a dying industry. All of these papers address redistributive or second-best issues.
Table 2.1 Summary of the literature review on time consistency of trade policies

<table>
<thead>
<tr>
<th>Paper</th>
<th>Domestic market structure</th>
<th>World market structure</th>
<th>Instruments</th>
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<tbody>
<tr>
<td>Lapan (1988)</td>
<td>Competitive</td>
<td>Monopolistic</td>
<td>Equivalence of tariffs and quotas</td>
<td>Domestic country and foreigners lose from the inability to commit by the large country.</td>
</tr>
<tr>
<td>Maskin and Newbery</td>
<td>Competitive</td>
<td>Monopolistic</td>
<td>Equivalence of tariffs and quotas</td>
<td>No commitment implies a welfare loss for the large importer.</td>
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<td>(1990)</td>
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<tr>
<td>Karp and Newbery</td>
<td>Competitive</td>
<td>Oligopolistic</td>
<td>Tariffs</td>
<td>No commitment implies lower welfare for single large importer.</td>
</tr>
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<td>(1991)</td>
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<td>Karp and Newbery</td>
<td>Competitive</td>
<td>Oligopolistic</td>
<td>Tariffs</td>
<td>Welfare comparisons of sequential games between importers and exporters.</td>
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<td>(1992)</td>
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<tr>
<td>Karp and Perloff</td>
<td>Monopolistic</td>
<td>Competitive</td>
<td>Output and investment subsidies</td>
<td>Lack of commitment makes investment policy unattractive but has no effect on output policy.</td>
</tr>
<tr>
<td>(1995)</td>
<td></td>
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</table>

2.3 – Precommitment and Optimal Trade Policies

Consider a partial equilibrium model. Suppose there are $N$ importing nations with purchasing power on the world market for a certain good. Their inverse domestic demand is denoted by $p_i(d_i)$ where $d_i$ and $p_i$ are the domestic demand and domestic price respectively. Denote the world price and foreign exports from the rest of the world by $\bar{p}$ and $\bar{X}$. 
respectively. The aggregate foreign export supply is defined by: \( \bar{X}(\bar{p}^*, \bar{p}) = \bar{Q}(\bar{p}^*) - \bar{D}(\bar{p}) \)

where \( \bar{Q}(\bar{p}^*) \) and \( \bar{D}(\bar{p}) \) are the foreign supply and demand respectively. The superscript \( e \) is used to denote producers' price expectations when production decisions are made. This notation is introduced to model the time consistency issue later in the chapter. Foreign supply depends on the producers' price expectations and hence on their expectation of the trade policies, whereas foreign demand depends on the realized world price. This structure gives rise to a foreign export supply curve: \( \bar{X}(\bar{p}, \bar{p}^*) \).

In this section we assume decisions are made in the following sequence:

1. Importing nations irrevocably commit to their trade policy (tariffs or quotas).
2. Production decisions are made.
3. Consumption and trade decisions are carried out.

Under the above sequence of actions, \( \bar{p}^* = \bar{p} \), and thus the foreign export supply curve is:

\[
\phi(\bar{p}) = \bar{X}(\bar{p}^*, \bar{p}) \bigg|_{\bar{p}^* = \bar{p}} = \bar{Q}(\bar{p}) - \bar{D}(\bar{p})
\]

Denote by \( \phi' \) the slope of the \textit{ex-ante} foreign export supply curve so that \( \phi' = \bar{Q}' - \bar{D}' \).

2.3.1 Tariff competition

If \( \tau_i \) is the \textit{ad-valorem} tariff imposed by country \( i \) on imports, we have the following arbitrage condition between the domestic and world price: \( p_i = \bar{p}(1 + \tau_i) \). If \( q_i \) is the quantity produced in country \( i \), imports are defined by \( m_i(\bar{p}_i^*, p_i) = d_i(p_i) - q_i(\bar{p}_i^*) \), where again the superscript \( e \) denotes producers' expectation. From an \textit{ex-ante} perspective, \( \bar{p}_i^* = p_i \), and the slope of the import demand is: \( m'_i(p_i) = d'_i - q'_i \). World market equilibrium
implies \( \phi(\bar{p}) = \sum_j m_j(\bar{p}(1+\tau_j)) \). For further reference, totally differentiate this equilibrium condition to obtain:

\[
\frac{\partial \phi(\bar{p})}{\partial \tau_i} = \frac{m_i \bar{p}}{\phi' - \sum_j (1+\tau_j)m_j'} < 0
\] (2.1)

Sufficient conditions for (2.1) to be negative are that the foreign export supply be positively sloped and that the import demand be negatively sloped. The welfare of foreign exporters is increasing with the world price and thus decreasing in every tariff. The objective function of the government in country \( i \) is to maximize domestic welfare defined as the sum of consumer surplus\(^5\), producer surplus and tariff revenue. Domestic welfare of country \( i \) is:

\[
W_i = \int_0^q p_i(y_i)dy_i - p_i d_i + p_i q_i - \int_0^q c'(z_i)dz_i + \tau_i \bar{p} m_i
\] (2.2)

such that \( d_i = q_i - m_i \), and \( p_i(q_i + m_i) = \bar{p}(1+\tau_i) \). Rewrite (2.2) as:

\[
W_i = \int_0^{q_i - m_i} p_i(y_i)dy_i - c(q_i) - \bar{p} m_i
\] (2.3)

To solve for the optimal ex-ante tariff, totally differentiate equation (2.3):

\[
dW_i = [p_i(d_i) - c'(q_i)]dq_i + [p_i(d_i) - \bar{p}]dm_i - m_i d\bar{p}
\] (2.4)

The expression in the first bracket on the right hand-side of (2.4) is equal to zero absent government domestic policy since we assume perfect competition in production. The world price is determined according to the ex-ante residual foreign export supply:

\[
\phi(\bar{p}) - \sum_{j=x} m_j(\bar{p}(1+\tau_j)) - m_i = 0
\] (2.5)

\(^5\)Consumer surplus is an exact measure of consumer welfare if demand is derived from a quasi-linear utility function.
Differentiating the behavioral equation in (2.5), given the other countries' tariff choice, yields:

\[
\phi' - \sum_{j \neq i} (1 + \tau_j) m'_j \frac{dp}{d \tau} - dm_i = 0
\]  

(2.6)

The optimal tariff is found by setting (2.4) to zero and substituting (2.6) in (2.4) for \( dm_i \).

Finally, divide both sides of (2.4) by \( d \tau_i \) to get:

\[
\frac{\partial W_i}{\partial \tau_i} = \left( \tau_i \bar{p} \left[ \phi' - \sum_{j \neq i} (1 + \tau_j) m'_j \right] - m_i \right) \frac{\partial \bar{p}}{\partial \tau_i} = 0
\]

(2.7)

Assuming that the welfare function in (2.3) is strictly concave in its own tariff everywhere, the second order condition for a maximum is satisfied. Equation (2.7) implicitly yields the reaction function of country \( i \) as a function of its belief about other countries' tariff, \( \tau_i = R(\tau_1, \ldots, \tau_{i-1}, \tau_{i+1}, \ldots, \tau_N) \). The Nash equilibrium is the set of tariffs \( (\tau^*_1, \ldots, \tau^*_i, \ldots, \tau^*_N) \) such that \( \tau^*_i = R(\tau^*_1, \ldots, \tau^*_N) \) and \( \tau^*_j = R(\tau^*_1, \ldots, \tau^*_N) \), \( \forall i, j \). After some manipulations, the \textit{ex-ante} (or precommitment) tariff in elasticity form is:

\[
\tau^*_i = \frac{\alpha_i}{\xi^p + \sum_{j \neq i} \alpha_j \eta^p_j}
\]

(2.8)

where \( \xi^p = \phi' \bar{p}/\phi \) is the foreign export supply elasticity with precommitment, \( \alpha_i \) is country \( i \)'s share of world imports, and \( \eta^p_j = -m'_j \bar{p}/m_j \) is the import demand elasticity of country \( j \).

Therefore, the denominator in (2.8) represents the elasticity of the residual foreign export supply curve. If all countries are symmetric, their share of world imports is equal to \( 1/N \). As \( N \to \infty \), the optimal policy becomes free trade.
Consider the case where the $N$ countries maximize their joint welfare. Intuitively, countries will internalize the adverse effect on world price given by the change of other countries' imports with respect to their own tariff. The joint welfare of all importers is:

$$ W = \sum_{i=1}^{N} q_i \int_0^{y_i} p_j(y_j) \, dy_j - \sum_{i=1}^{N} c(q_i) - \sum_{i=1}^{N} \bar{p} m_i \quad (2.9) $$

Differentiating (2.9) with respect to $\bar{p}$, $q_i$ and $m_i$ yields after some manipulations:

$$ dW = \left[ p_i (d_i) - c'(q_i) \right] dq_i + (p_i (d_i) - \bar{p}) dm_i - \sum_j m_j \, d\bar{p} \quad (2.10) $$

Since the expression $\left[ p_i (d_i) - c'(q_i) \right]$ equals zero, (2.10) can be rewritten as:

$$ dW = \tau_i \bar{p} dm_i - \sum_j m_j \, d\bar{p} \quad (2.11) $$

Differentiating the behavioral equation (2.5) yields $\phi' \bar{p} = dm_i$. Use this to substitute in (2.11) for $dm_i$, set (2.11) to zero to find a maximum, and divide both sides by $d\tau_i$ to yield:

$$ \frac{\partial W}{\partial \tau_i} = \left[ \tau_i \bar{p} \phi' - \sum_j m_j \right] \frac{\partial \bar{p}}{\partial \tau_i} = 0 \quad (2.12) $$

Solving (2.12) yields $\tau_i^* = 1/\varepsilon^p \forall j$, the optimal collusive tariff for importer $i$. It is proportional to the inverse of the foreign export supply elasticity, which is the standard result in the optimal tariff literature. Evaluate (2.12) at $\tau_i = \tau_i^*$ to obtain:

$$ \left. \frac{\partial W}{\partial \tau_i} \right|_{\tau_i = \tau_i^*} = \sum_{j=1}^{N} \left[ \tau_i \bar{p} m_j' (1 + \tau_j) - m_j \right] \frac{\partial \bar{p}}{\partial \tau_i} > 0 \quad (2.13) $$

---

6 Obviously, the choice of instrument does not matter for the collusive and full information case. It can be readily shown that quotas and tariffs are equivalent instruments under collusion. Therefore, differentiating equation (2.5), we treat imports $m_{p,j} \neq i$, as given.
Equation (2.13) provides the following ranking between the precommitment tariff and collusive tariff: \( \tau^p < \tau^c \). Countries would be better off acting cooperatively; thus maximizing their joint welfare. In the non-cooperative setting, when a country decreases its tariff, it fails to consider the reduction in other countries’ welfare that is caused by the ensuing increase in the world price.

2.3.2 Quota competition

We will now restrict the strategy space of the \( N \) importers to a quota. We set \( \tau_j = 0 \) \( \forall j \) and use imports as the choice variable. The domestic price is determined by:
\[
m_i = d_i(p_i) - q_i(p_i^c).
\]
In the \textit{ex-ante} situation, when production decisions are carried out simultaneously with consumption, \( p_i = p_i^c \). The domestic price is then \( p_i(m_i) \). If the choice variable is imports, the \textit{ex-ante} residual foreign export supply faced by country \( i \) is defined by:
\[
\phi(p) - \sum_{j \neq i} m_j - m_i = 0
\]  
(2.14)

Auctioning the quota licenses raises government revenue \( (p_i - \bar{p})m_i \). Therefore, country \( i \)'s welfare is still defined by (2.3). Differentiating the residual export supply curve in (2.14) given the other countries’ quota gives:
\[
\phi' \Delta \bar{p} - \Delta m_i = 0
\]  
(2.15)

Using (2.15) to substitute for \( \Delta \bar{p} \) in (2.4), we get:
\[
\frac{\partial \Delta W}{\partial m_i} = (p_i - \bar{p}) - \frac{m_i}{\phi'} = 0
\]  
(2.16)
We assume the welfare function to be concave everywhere in its own imports. Equation (2.16) yields the reaction function of country $i$. The intersection of every countries’ reaction function gives the Nash equilibrium precommitment quota $m^p_i$. Define $\theta^p_i$ as the tariff equivalent measure of the difference between the domestic price and the world price given $m^p_i$. From (2.16), the precommitment tariff equivalent in elasticity form is:

$$\frac{p_i - \bar{p}}{\bar{p}} = \theta^p_i = \frac{m_i}{\bar{p} \phi^i} = \frac{\alpha_i}{\xi^p}$$

(2.17)

As we mentioned before, in the case of collusion, the optimal policy is independent of the instrument used. The optimal collusive quota is $m^*_{c}$ and the collusion tariff equivalent of the import quota in elasticity form is: $\theta^*_{c} = \tau^*_{c} = 1/\xi^p$. It can readily be shown that the collusion quota is lower than the ex-ante Nash equilibrium quota. Under the strategic quota game, when a country increases its quantity imported, it fails to consider the reduction in other countries’ welfare that is caused by the ensuing increase in the world price.

**Proposition 2.1:** Assume $N$ symmetric policy active importers. The ex-ante Nash equilibrium import quota induces a higher price differential between the domestic price and the world price than the optimal ex-ante tariff. Moreover, the importer’s welfare associated with the ex-ante quota will be higher than the welfare associated with the precommitment tariff.

**Proof:** Evaluating (2.7) at $\tau_i = \theta^p_i$ yields:

$$\frac{\partial W_i}{\partial \tau_i, \tau_i = \theta^p_i} = -\theta^p_i \bar{p} \left( \sum_{j \neq i} m'_j (1 + \tau_j) \right) \left( \frac{\partial \bar{p}}{\partial \tau_i} \right) < 0$$

(2.18)
From (2.1) and because the import demand is negatively sloped, the expression in (2.18) provides the following ranking: \( \tau_i^p < \theta_i^p \), given symmetry among policy active importers. Because tariffs and quotas are equivalent under collusive behavior, we have the following rankings: \( \tau_i^p < \theta_i^p < \tau_i^* = \theta_i^* \). Domestic welfare of each importer under both Nash equilibria is lower than in the collusive equilibrium. Since the ex-ante quota is closer to the collusive solution than the ex-ante tariff, it must bring a higher welfare level in equilibrium. Q.E.D.

The intuition behind proposition 2.1 is simple because it is tantamount to the standard oligopoly theory. By using a tariff as their trade instrument, each country faces a more elastic residual foreign export supply at the tariff set by the other countries than in the monopsony case. Moreover, the residual foreign export supply curve will be even more elastic than under the strategic quota game. When countries use a quota, the Nash equilibrium will induce a higher welfare than tariffs because countries increasing their imports by one (differential) unit cause an increase in the world price of \( \ddot{p}'(\ddot{X}) \). Tariffs do not have an equivalent effect on the world price, since imports of other countries also vary following a change in one importer's tariff.

The results in proposition 2.1 contrast with the bilateral monopoly case (two-good, two-country retaliation world). The use of a quota in our model does not eliminate trade as in the Rodriguez's model (1974). In the quota-retaliation framework, each country cannot enforce a favorable terms of trade shift. Although both wish to achieve the same level of trade restriction on a certain good, they have different preferred levels of trade in the other
good. In our model the foreign exporters are passive. Therefore, competing importers are able to induce a terms of trade shift, albeit not optimal, because they fail to take into account (or do not care about) the consequence of their trade policy on the other policy active importers’ welfare. The same basic story applies to the bilateral monopoly with tariffs as strategic variables. In that situation, it is possible to find an equilibrium where both countries are worse off than under free trade. This is ruled out in our setting since every country gains by imposing a tariff, and importer $i$ gains when importer $j$ imposes a tariff.

2.4 - Time Consistency and Optimal Trade Policies

This section derives the time consistent trade instrument when there is a lag between production and consumption decisions. The timing of events is of great importance. We follow the hypothesis made in Lapan (1988). First, each country announces its tariff given its own belief about the tariff choice of other countries. Then, production decisions are made according to price expectations of domestic and foreign producers. Before consumption decisions (and trade decisions) are made, each government can costlessly revise the level of its trade instrument set at the beginning of the game. Finally, consumption decisions are made and trade between countries is carried out.

Because there is a lag between production and consumption, and all countries can change their tariff after production decisions are made, the time consistent tariff is higher than the precommitment tariff. This is because the \textit{ex-post} residual foreign export supply elasticity faced by country $i$ is lower than the \textit{ex-ante} elasticity. The lack of commitment by importers may be collectively beneficial since the \textit{ex-post} tariff is bounded below by the \textit{ex-ante} tariff. With the perfect foresight assumption, foreign producers fully anticipate the tariff.
change after production decisions are made. Since the anticipated time consistent tariff is larger, the lower world price causes a contraction in production. Therefore, the lack of commitment can increase domestic welfare by offsetting the policy competition welfare effect. However, if foreign supply is very elastic, thus making the residual ex-post export supply elasticity lower than the ex-ante elasticity, the potential gain of not committing to the ex-ante tariff can vanish.

3.4.1 Tariff competition

Formally, the slope of the ex-post foreign export supply curve is:

\[ \bar{X}_p = \phi' - \bar{D'} = -\bar{D'} \].

Clearly, given output levels, the slope of the foreign export supply is smaller ex-post than ex-ante if \( \bar{D'} > 0 \). Restricting the strategy space to the use of a tariff, the following arbitrage condition between the world price and domestic price must hold if imports in country \( i \) are positive: \( p_i = \bar{p}(1 + \tau_i) \). From an ex-post perspective, i.e. when production decisions are made: \( \partial m_i / \partial p_i = m'_i + q'_i \). The welfare function of the government, once production decisions are made is still defined as in (2.3). The ex-post residual foreign export supply is:

\[ \bar{X}(\bar{p}, \bar{p}^*) - \sum_{j \neq i} m_j(\bar{p}(1 + \tau_j), \bar{p}^*(1 + \tau_j)) - m_i = 0 \] (2.19)

Totally differentiate (2.19) given the tariff choice of other countries and predetermined output levels. Assume perfect foresight so that producers correctly anticipate the tariffs set by governments ex-post, and thus \( \tau_i = \tau^*_i, \bar{p} = \bar{p}^* \) and \( p_i = p^*_i \).

\[ \left[ \bar{X} - \sum_{j \neq i} (1 + \tau_j) \frac{\partial m_j}{\partial \bar{p}_j} \right] \partial \bar{p} - \partial m_i = 0 \] (2.20)
Set equation (2.4) to zero, substitute (2.20) in (2.4) for \(dm\), and divide both sides by \(d\tau\). After some manipulations, you get the ex-post tariff reaction function implicitly defined by:

\[
\frac{\partial W_i}{\partial \tau_i} = \left( r_i \bar{p} \left[ \bar{X}_p - \sum_{j \neq i} \left( 1 + \tau_i \right) \frac{\partial m_j}{\partial \tau_j} \right] - m_i \right) \frac{\partial \bar{p}}{\partial \tau_i} = 0
\] (2.21)

The tariff reaction function for country \(i\) is: \(\tau_i^\ast = R(\tau_1, \ldots, \tau_{i-1}, \tau_{i+1}, \ldots, \tau_N, \bar{q}, \bar{O})\). Define the ex-post elasticities: 

\[
\xi_i = \frac{(\bar{p} \bar{X}_p)}{\bar{X}_p} = \bar{\mu}(\bar{p} \phi' / \phi)
\]

where \(\bar{\mu} = \bar{D}' / (\bar{O}' - \bar{D}') \in [0, 1]\) and 

\[
\eta_i = \frac{(p_j (\partial m_j / \partial \tau_j) / m_j)}{m_j} = -\mu_j (p_j m_j / m_j)
\]

where \(\mu_j = (-d_j' / (d_j' - d_j)) \in [0, 1]\). Imposing a subgame perfect equilibrium, the time consistent tariff in elasticity form is:

\[
\tau_i^\ast = \frac{\alpha_i}{\xi_i + \sum_{j \neq i} \alpha_j \eta_j^i}.
\]

**Proposition 2.2:** Assuming symmetry among the policy active importers, the time consistent tariff is higher than the ex-ante tariff. Moreover, the inability to precommit to the ex-ante tariff may result in a higher welfare ex-post for all importing countries.

**Proof:** Evaluating (2.21) at the precommitment solution \(\tau_i^p\), we get:

\[
\frac{\partial W_i}{\partial \tau_i} \bigg|_{\tau_i = \tau_i^p} = \tau_i^p \bar{p} \left( -\bar{O}' - \sum_{j \neq i} (1 + \tau_i^p) q_j^i \right) \frac{\partial \bar{p}}{\partial \tau_i} > 0
\] (2.22)

Assuming \((\partial W_i / \partial \tau_i)\) is decreasing in the (symmetric) tariff vector, (2.22) implies that the ex-post tariff will be higher than the ex-ante tariff, \(\tau_i^p < \tau_i^\ast\). From (2.13), we have: \(\tau_i^p < \tau_i^\ast\).
Domestic welfare must be lower under both the *ex-ante* and *ex-post* equilibrium tariffs than under the collusive tariff since the *ex-ante* collusive tariff yields the global optimum. Since the domestic welfare function is continuous and monotonic over the interval \([r^p, r^*]\), a sufficient condition for the time consistent tariff to be welfare superior to the *ex-ante* tariff is to fall within the previous interval. However, a Nash equilibrium resulting in an *ex-post* tariff higher than the collusive *ex-ante* tariff has an indeterminate effect on domestic welfare compared to the precommitment tariff. QED.

To compare the time consistent tariff to the optimal collusive tariff, evaluate (2.21) at \(\tau_i^*\):

\[
\frac{\partial W_i}{\partial \tau_i} \bigg|_{\tau_i = \tau_i^*} = \tau_i^* \bar{P} \phi'(\bar{\mu} - \alpha, + \sum_{j \neq i} (-\mu_j m_j'/\phi')(1 + \tau_i^*)) \frac{\partial \Phi}{\partial \tau_i}
\]

From (2.23), with symmetric importers, we have that \(\tau_i^e < \tau_i^*\) if \(\bar{\mu} - \alpha + (N - 1)(-\mu m'/m)(1 + \tau^*) > 0\). This can not occur for \(N = 1\). For \(N > 1\), the likelihood that \(\tau_i^e < \tau_i^*\) increases as: (i) \(\bar{\mu}\) increases; (ii) \(\alpha\) decreases; and (iii) the price responsiveness of the import demand in importing nations increases compared to the price responsiveness of the foreign export supply.

The intuition behind proposition 2.2 is the following. The *ex-ante* Nash equilibrium entails tariffs set too low compared to the collusive equilibrium because countries perceive a more elastic residual foreign export supply than they would if they acted cooperatively. The lower *ex-post* foreign supply elasticity partly offsets that fact. When the supply in exporting nations is not very elastic (\(\bar{\mu}\) large), the failure to precommit is not very costly, and hence
the welfare of importing nations will be improved by this failure to precommit. Similarly, when the number of nations is large (low $\alpha$), or demand is very elastic in importing nations, the tariff competition among these nations is severe and hence the increase in tariff due to the inability to precommit is welfare improving. However, if foreign production is very elastic (low $\mu$), then welfare can be lower $ex-post$ since the equilibrium $ex-post$ will be far apart from the $ex-ante$ equilibrium.

2.4.2 Quota competition

The time consistent quota is derived in a similar way to the time consistent tariff. Imposing a quota on imports is equivalent to setting the domestic price. In this case, since $m_i = d_i(p_i) - q_i(p_i^*)$, the domestic price is $p_i(m_i, p_i^*)$. The residual foreign export supply curve faced by country $i$ is $X(p, p^*) - \sum_{j \neq i} m_j - m_i = 0$. Differentiate the latter expression to get: $X(p, p^*) - dm_i = 0$. Perfect foresight implies $p_i^* = p_i$ and $p^* = \bar{p}$. Setting equation (2.4) to zero after appropriate substitutions implicitly yields the $ex-post$ quota reaction function $m_i^e = g(m_1, \ldots, m_{i-1}, m_{i+1}, \ldots, m_N)$. Imposing a subgame perfect equilibrium, the time consistent tariff equivalent quota in elasticity form can be written as:

$$\frac{\partial W_i}{\partial m_i} = - \frac{m_i}{X_p} + (p_i - \bar{p}) = 0 \Rightarrow \theta_i^e = \frac{p_i - \bar{p}}{\bar{p}} \equiv \frac{\alpha_i}{\xi}$$

(2.24)

Proposition 2.3: Assuming symmetry among the policy active importers, the time consistent quota is lower than the precommitment quota. It induces a higher price differential between the domestic price and the world price $ex-post$ than in the $ex-ante$ case. Moreover, the
inability to precommit to the \textit{ex-ante} quota may be collectively welfare improving for all importing countries.

\textbf{Proof:} Evaluating (2.24) at the precommitment solution \(m_i^p\), we get:

\[
\frac{\partial W}{\partial m_i\bigg|_{m_i=m_i^p}} = p_i - \bar{p} - \frac{m_i^p}{\phi - \bar{Q}'} = \frac{(p_i - \bar{p})\bar{Q}'}{D'} < 0
\]  

Equation (2.25) gives the following ranking between the \textit{ex-ante} Nash equilibrium quota and the time consistent \textit{ex-post} quota, \(m_i^c < m_i^p\). Similar to the case of tariffs, the incapability to precommit to a quota can be welfare improving since we have proved the following rankings:

\(m_i^p > m_i^*\) and \(m_i^c > m_i^*\); and that the \textit{ex-ante} collusive quota yields the global optimum. The world market structure in our model implies that the quantity imported in each country is too large \textit{ex-ante}. Since \textit{ex-post}, the residual foreign export supply is less elastic, each country will import a smaller quantity. QED.

To compare the time consistent quota to the optimal collusive quota, evaluate (2.24) at \(\theta_i^*\):

\[
\frac{\partial W}{\partial m_i\bigg|_{m_i=m_i^c}} = \frac{\theta_i^* \bar{p}}{\bar{\mu}} (\bar{\mu} - \alpha_i)
\]  

The inability to commit to the \textit{ex-ante} import quota is collectively beneficial if \(\bar{\mu} \geq 1/2\) since (2.25) and (2.26) yield: \(m_i^p > m_i^c > m_i^*\)
**Proposition 2.4:** Assuming symmetric policy active importers who cannot precommit to a trade policy: (i) the time consistent tariff leads to lower equilibrium tariff than the time consistent equilibrium quotas \( \tau^c < \theta^c \); (ii) the time consistent quota equilibrium yields higher welfare than the tariff equilibrium if \( N\bar{\mu} > 1 \); (iii) the time consistent tariff equilibrium yields higher welfare than the quota equilibrium if \( N\bar{\mu} + N\mu(N - 1)(m'/\phi')(1 + \tau^*) < 1 \); and (iv) the ranking between the two instruments is indeterminate if \( N\bar{\mu} + N\mu(N - 1)(m'/\phi')(1 + \tau^*) > 1 > N\bar{\mu} \).

**Proof:** Evaluating (2.21) at \( \theta^c \): \( \frac{\partial W}{\partial \tau_i} \bigg|_{\tau_i=\theta^c} = -\theta^c \frac{\partial P}{\partial \tau_i} \left( \sum m_j \frac{\partial m_j}{\partial P_j} (1 + \tau_j) \right) \frac{\partial P}{\partial \tau_i} < 0 \) implies \( \tau^c < \theta^c \).

From proposition 2.3, if \( \bar{\mu} > \alpha \), then \( \theta^c < \tau^*_i \) and combining with the previous statement yields \( \tau^c < \theta^c < \tau^*_i \); this proves the second claim. In case where \( N\bar{\mu} + N\mu(N - 1)(m'/\phi')(1 + \tau^*) < 1 \), then we have \( \tau^c > \tau^*_i \) from (2.23); combining with the first statement yields \( \theta^c > \tau^c > \tau^*_i \), proving the third claim. Finally, under (iv) we have: \( \theta^c > \tau^*_i > \tau^c \) and the two equilibria cannot be ranked. QED.

**2.5 - A Numerical Example**

This section tries to illustrate the welfare implications of the precommitment and time consistent trade instrument discussed in propositions 2.1 to 2.4. Suppose domestic preferences are represented by a quasi-linear utility function: \( U(w_i, x_i) = w_i + \frac{a}{b} x_i - \frac{x_i^2}{2b} \).
where $w_i$ is a numéraire good. These preferences implies the domestic demand \( d_i \) for good $x_i$ is: $d_i = a - bp_i$, where $a$ and $b$ are positive constants and $p_i$ is the domestic price. Domestic producers of the importable good in country $i$ have the following cost function:

\[
c(q_i) = \frac{q_i^2}{2g} + \frac{c}{g}q_i.
\]

Competitive domestic markets imply the domestic supply function is:

\[
q_i = -c + gp_i,
\]

where $g$ is a positive constant. Assume for simplicity the available policy is a specific tariff\(^7\), so that $p_i = \bar{p} + t_i$. The import demand function of country $i$ is then:

\[
m_i = a + c - (b + g)(\bar{p} + t_i).
\]

The welfare function for country $i$ is the sum of consumer surplus, producer surplus and tariff revenue:

\[
W_i = \int_0^{q_i^*} \left( \frac{a-y_i}{b} \right) dy_i - c(q_i) - \bar{p}m_i = \frac{a}{b}d_i - \frac{d_i^2}{2b} - q_i^2 + \frac{c}{g}q_i - \bar{p}m_i
\]

such that $d_i = q_i + m_i$. The export supply curve of the rest of the world is:

\[
\bar{X}(\bar{p}, \bar{p}^*) = \bar{Q}(\bar{p}^*) - \bar{D}(\bar{p}) = (\delta - \alpha) + \beta \bar{p} + \gamma \bar{p}^*
\]

At the ex-ante level $\bar{p}^* = \bar{p}$, so the ex-ante foreign export supply curve is: $\phi(\bar{p}) = (\delta - \alpha) + (\beta + \gamma)\bar{p}$. World equilibrium implies

\[
\sum_i m_i = \phi(\bar{p}). \quad \text{Solving for } \bar{p} \text{ yields: } \bar{p} = \frac{N(a+c) - (\delta - \alpha) - (b + g)(\sum_i t_i)}{(\beta + \gamma) + N(b+g)}. \quad \text{The first order condition of the strategic tariff game can be rewritten as } t_i(\partial m_i/\partial t_i) = m_i(\partial \bar{p}/\partial t_i).
\]

Therefore, the reaction function for country $i$ is implicitly defined by:

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\(^7\)In our model, proposition 2.1 to 2.4 still hold if the strategy space is restricted to specific tariffs instead of ad-valorem tariffs. However, specific tariffs and ad-valorem tariffs are not equivalent instruments. The use of a specific tariff is uniquely to facilitate the computation of the numerical example in this section.
\[-t_i = \frac{(a+c)}{[(\beta+\gamma)+(N(b+g))] + (b+g)\left[\frac{N(a+c)-(\delta-\alpha)-(b+g)\sum t_j}{((\beta+\gamma) + N(b+g))^2}\right]} \quad (2.28)\]

Imposing symmetry among the importing countries, the intersection of the reaction functions solves for the ex-ante Nash equilibrium. To simplify the derivation of further results, rescale the following parameters: \(\alpha' = \alpha/N\), \(\delta' = \delta/N\), \(\beta' = \beta/N\) and \(\gamma' = \gamma/N\). Moreover, define \(\lambda = (\beta' + \gamma')/(b+g)\) and \(A = [(a-c)\lambda - (\alpha' - \delta')]/(b+g)\). The precommitment optimal tariff is:

\[t_i^p = \frac{A}{\lambda(2+\lambda)+(N-1)(1+\lambda)^2} \quad (2.29)\]

The optimal collusive tariff is given by maximizing the sum of the \(N\) countries' welfare. This gives a set of \(N\) first order conditions of the type: \(t_i \sum m_j/\partial t_i = \bar{x}(\partial \bar{p}/\partial t_i)\). Because of the symmetry between the countries, \(t_i = t_j\). The solution is:

\[t_i^* = \frac{A}{\lambda(2+\lambda)} \quad (2.30)\]

We have expressed the precommitment tariff as a deviation from the collusive tariff. Both tariffs have the same numerator and only differ in their denominator. It is readily seen that if \(N > 1\), \(t_i^p < t_i^*\), because \((N-1)(1+\lambda)^2 > 0\). The latter expression illustrates the theoretical result of equation (2.13). The collusive tariff is higher than the non-cooperative precommitment tariff, provided \(N > 1\).

The time consistent tariff is found by maximizing:

\[W_i = \int_0^{\phi_i} \left(\frac{a-\gamma}{b}\phi_i - c(q_i)^* - \bar{p}m_i\right) = \frac{a}{b}d_i - \frac{d_i}{2b} \left(\frac{q_i^*}{2g}\right)^2 + \frac{c}{g}q_i^* - \bar{p}m_i \quad (2.31)\]
where \( q_i^* \) represents domestic production given the trade policy expectation of producers in country \( i \). From the government’s perspective, production is fixed. In this case,

\[
\bar{p} = \frac{N\alpha + \alpha - \bar{Q}^* - \sum_j b t_j - \sum_j q_j^*}{N b + \beta}
\]

and \( m_i = a - b t_i - q_i^* - b \bar{p} \). The first order condition is:

\[
t_i (\partial m_i / \partial t_i) = m_i (\partial \bar{p} / \partial t_i)
\]

Imposing perfect foresight implies, \( t_i^* = \bar{t}_i \) and \( q_i^* = c + g p_i \). The implicit reaction function of country \( i \) is:

\[
(\bar{t}_i + \alpha - \bar{Q}^*) \quad \bar{m}_i = (N - 1) b \beta + \gamma
\]

For further reference, define the following two parameters \( \mu = b / (b + g) \in [0,1] \) and \( \bar{\mu} = \beta / (\beta + \gamma) \in [0,1] \). The parameters \( \mu \) and \( \bar{\mu} \) can be interpreted as the relative domestic and foreign demand responsiveness respectively. Using the symmetry across countries, the optimal time consistent tariff is:

\[
t_i^* = \frac{A}{\lambda (2 + \lambda) + (1 + \lambda)(\lambda (N \bar{\mu} - 1) + (N - 1) \mu)}
\]  

(2.32)

Using equations (2.29), (2.30) and (2.32), we can illustrate the point made in proposition 2. Because we used linear domestic and foreign demand schedules and quadratic cost functions, it should be clear that welfare of country \( i \) is a quadratic function. The welfare function reaches a maximum at \( t_i^* \) and is symmetric around that point. With those properties, we show that the inability to precommit to the ex-ante tariff increases welfare for an importer if the following inequality holds: \( t_i^* < t_i^* + (t_i^* - t_i^*) \). The first inequality is ensured by proposition 2. The second inequality holds if:

\[
\Delta_i = [\lambda (2 + \lambda) + 2(N - 1)(1 + \lambda)^2][\lambda (N \bar{\mu} - 1) + (N - 1) \mu + (N - 1) \lambda (2 + \lambda)(1 + \lambda) > 0
\]

The inability to commit to the specific tariff is collectively beneficial for the policy active importers if the inequality above is satisfied. Clearly, if \( N = 1 \), \( \Delta_i < 0 \) implying
precommitment is desirable. For \( N > 1 \), a sufficient condition for \( \Delta' \) to be greater than zero is for \( \bar{\mu} \geq 1/N \) because then \( t_i^e < t_i^c < t_i^e \). The only remaining question is what if \( t_i^e > t_i^c \).

Since \( \Delta' \) is increasing in \( N \), the likelihood that the inability to precommit is beneficial increases with \( N \). Fixing \( N = 2 \) and \( \lambda = 1 \), \( \Delta' \) becomes: \( 22 \bar{\mu} + 11 \mu - 5 > 0 \). Thus, for not very high relative foreign and domestic demand responsiveness, the collective inability to commit to the *ex-ante* specific tariff increases welfare for policy active importers.

In the case of a strategic quota game, \( m_i \) is the choice variable. Since \( d_i = q_i + m_i \), in the precommitment case, domestic market equilibrium implies: \( p_i = (a - c - m_i) / (b + g) \).

Solving for the inverse *ex-ante* foreign export supply gives: \( \bar{p}(\bar{X}) = (\bar{X} + (\alpha - \delta)) / (\beta + \gamma) \).

Maximizing domestic welfare in (2.27), the first order condition is: \( \dot{p}_i - \bar{p} = \bar{p}' m_i \), where \( \bar{p}' \) is the derivative of the inverse foreign export supply with respect to imports in country \( i \).

If the policy active importers can precommit to their policy, the quota reaction function of country \( i \) is then implicitly defined by: \( \frac{a - c - m_i}{b + g} - \frac{m_i}{\beta + \gamma} = \sum_j m_j + (\alpha - \delta) / \beta + \gamma \).

Imposing symmetry between the countries allows one to solve for the optimal Nash equilibrium quota: \( m_i^* = \frac{(a + c)(\beta + \gamma) - (\alpha - \delta)(b + g)}{(N + 1)(b + g) + (\beta + \gamma)} \). The tariff equivalent is given by:

\[ \theta_i^p = p_i - \bar{p} \]

With various simple manipulations, one finds:

\[ \theta_i^p = \frac{A}{\lambda(2 + \lambda) + (N - 1)\lambda(1 + \lambda)} \]  \hspace{1cm} (2.33)

Note that equations (2.29) and (2.33) give the same optimal tariff when \( N = 1 \). This shows the equivalency result between quotas and tariffs in case of a single large country.
However, following proposition 1, if $N > 1$, $t_i^r < \theta_i^r$ since $(N-1)((1+\lambda)^2 - \lambda(1+\lambda)) = (N-1)(1+\lambda) > 0$; and quotas are welfare superior to tariffs. In the collusive case, the choice of tariff or quota is irrelevant and thus, the tariff equivalent $\theta_i^*$ is equal to $t_i^*$ in equation (2.30).

Finally, the time consistent quota is derived by assuming production is fixed. The price in importing countries is: $p_i = (a - m_i - q_i^*)/b$. Similarly, the foreign inverse export supply curve is: $\bar{p}(\bar{X}, \bar{Q}^*) = (\alpha + \bar{X} - \bar{Q}^*)/\beta$, and the world price is determined such that $\bar{X} = \sum_j m_j$. The first order condition implicitly defines the quota reaction function of country $i$: $\beta(\alpha - m_i - q_i^*) - b(\alpha + \bar{X} - \bar{Q}) = bm_i$. Imposing perfect foresight and symmetry among the $N$ countries gives: $m_i^c = \frac{\beta(a + c)(\beta + \gamma) - (\alpha - \delta)(b + g)}{\beta(\beta + \gamma) + (N+1)(\gamma + \beta)(b + g) - N\gamma(b + g)}$. The time consistent tariff equivalent is:

$$\theta_i^c = \frac{A}{\lambda(2+\lambda) + (\mu N - 1)(1+\lambda)\lambda}$$  (2.34)

Again, because we have expressed equations (2.33) and (2.34) in terms of deviation from the optimal collusive tariff equivalent, we can easily illustrate proposition 2.3. The importers' collective inability to precommit to their trade policy will be welfare improving if the following inequality holds: $\theta_i^p < \theta_i^c < \theta_i^* + (\theta_i^* - \theta_i^p)$. The first inequality in the latter expression always holds for $\bar{\mu} < 1$. The second inequality holds if:

$$\Delta^\theta = [2N(1+\lambda) - \lambda](N\bar{\mu} - 1) + (2 + \lambda)(N - 1) > 0$$  (2.35)
If you set $N = 1$, then $\Delta^\theta < 0$, and the inability to precommit to the *ex-ante* quota hurts the large country [Lapan (1988)]. For $N \neq 1$, note that (2.35) is independent of the parameter $\mu$. This means that domestic demand and supply do not play a role in the *ex-post* versus *ex-ante* welfare analysis in the strategic quota game. Further note that $\Delta^\theta$ is an increasing function of $\overline{\mu}$. The inability to precommit to the *ex-ante* quota is more likely to be welfare improving the higher the foreign exporters relative demand responsiveness is. A sufficient condition for the inequality to hold is that: $\overline{\mu} \geq 1 / N$, since it implies: $\theta_i^* < \theta_i^e$. The sign of $\partial \Delta^\theta / \partial N$ is ambiguous. The likelihood that the inability to precommit is beneficial increases with $N$ ( $\Delta^\theta$ is increasing in $N$) as long as $\overline{\mu} \geq \lambda / (4 + 3 \lambda)$.

Assuming commitment is not feasible, (2.32) and (2.34) can be used to compare tariffs and quotas. Clearly if $\theta_i^e < t_i^* \ [\overline{\mu} N > 1]$, then quotas dominate tariffs; comparably, if $t_i^e > t_i^*$ then tariffs dominate quotas (for $(N\overline{\mu} - 1) + (N - 1)\mu < 0$). Thus the ambiguity arises only for $\theta_i^e > t_i^* > t_i^e$. Using the same reasoning as earlier, tariffs will dominate quotas if and only if $\theta_i^e - t_i^* > t_i^* - t_i^e$. Hence, tariffs are superior to quotas if and only if $\Delta^\theta < 0$ where:

$$\Delta^\theta = \left((N\overline{\mu} - 1)(1 + \lambda)\right)\left((N - 1)\mu + \lambda(N\overline{\mu} - 1)\right) + (2 + \lambda)\left[\lambda(N\overline{\mu} - 1) + \frac{(N - 1)\mu}{2}\right] < 0$$

Finally, in some extreme cases, an importing country can end up worse off than if it had no market power at all on the world market. In the case of tariffs, the condition is:

$$2(N - 1)\mu + \lambda N\overline{\mu}(2 + \lambda) + \lambda^2(N\overline{\mu} - 1) < 0$$

(2.36)
The inequality in (2.36) implies the policy active importers would collectively be better off if they had precommitted to pursue free trade. In the quota game, market power is collectively disadvantageous if: 
\[ \lambda \mu N (2 + \lambda) + \lambda^2 (\mu N - 1) < 0. \]

2.6 - Conclusion

We have shown that tariffs and quotas are not equivalent protection instruments in a strategic setting where importing countries have purchasing power on the world market over a certain good. Each policy active importer would be better off by colluding and setting their trade instrument cooperatively. If production lags are present, the *ex-ante* optimal policy is not time consistent because the *ex-post* elasticity of the residual foreign export supply curve is lower than the *ex-ante* elasticity. However, we have shown that the importers' inability to precommit to their trade instrument may be welfare superior to the precommitment solution. The negative welfare implication of non-cooperative behavior may be balanced off by the welfare effect of the *ex-post* elasticity. Given the structure of the world market, these conclusions extend to any dynamic framework in which some inputs are committed before trade decisions are made. The political reasons why the government does not, or cannot, precommit itself to a predetermined policy is not the focus of the paper. However, it could very well be an interesting exercise to explain the precommitment incapability of the importers.

The next chapter extends the model by adding another stage to the time consistency game. Suppose that the timing of economic decisions is modified as follows: First, governments of the policy active countries announce a tax/subsidy they will pay domestic producers as well as the tariff rate. Next, domestic and foreign production is made according
to producers' price expectations. Assuming precommitment is not feasible, each policy active government can revise the tariff rate or the production tax/subsidy. Finally, consumption decisions and trade are carried out. Lapan (1988) has shown that, with a single large country, the optimal production policy is a tax on domestic production of the importable. The purpose is to signal foreign producers to increase their production. If it were possible, a government would want to assure foreign producers they will not exploit the lower *ex-post* elasticity once production is made. The appropriate questions to answer are the following. Do policy active importers have any incentive to use a production tax and/or subsidy? Would the production policy result in a higher or lower welfare for those nations? Is an agreement to limit domestic production policies beneficial?
CHAPTER 3. DOMESTIC PRODUCTION AND
TIME CONSISTENT TRADE POLICIES

3.1 - Introduction

Governments have a multitude of policy instruments available to implement an even larger variety of objectives. More often than not, those instruments do not yield equivalent results. It is well known that the optimal policy for a large country which can influence the price at which it imports a good is to set a tariff (or tariff equivalent quota) equal to the inverse of the foreign export supply elasticity [Johnson (1953)]. Governments can rely on commercial policy to increase social welfare by improving its terms of trade at the expense of increased domestic deadweight loss. Casual observation of commercial policy reveals that countries usually use a large combination of instruments. We frequently observe a trade policy co-existing with some form of domestic production policy in a given sector of the economy. However, the terms of trade argument in protection does not offer any rationale to use domestic production policies in conjunction with trade policy in a static framework.

One notable critique of the optimal tariff argument resides in determining the appropriate timing to use such a policy. This chapter assumes that production and consumption decisions are made sequentially and that the level of trade policy can be revised between the two stages.\(^8\) This setting obviously applies to agricultural markets with spring planting and fall harvest. It also naturally applies to any setting where firms’ input decisions

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\(^8\) International agreements generally put a cap on the level of protection. However trade agreements generally do not prevent any country from changing their policy through time as long as it respects the established ceilings in the level of protection. Of course, if the cap on the level of protection is lower than the optimal trade policy, the time inconsistency problem does not arise.
are chosen sequentially. The production tax/subsidy and consumption tax components of a tariff combined with the existence of production lags create an incentive to use a production policy as a partial commitment mechanism when countries cannot credibly commit to their trade policy. The key assumption is to allow importers to revise their trade policy once production decisions are made. We propose to extend the model of chapter two to allow $N$ policy active importers to capture the terms of trade gain with a production policy and a trade instrument. The market structure implies a non-equivalence of the different trade instruments along with a similar non-equivalence among production policies.

The purpose of this chapter is twofold. First, we want to examine if large importers have any private incentives to use a production policy along with a trade instrument. Second, we want to investigate the welfare implications of using a production policy. The first section of this chapter provides a brief review of the literature on domestic production policies and trade. The next section introduces the theoretical model to derive the equilibrium production and trade policies. A welfare analysis of the various equilibria follows. The last section provides concluding remarks.

3.2 - Review of Literature

Lapan (1988) has shown that, with a single large importer and the existence of production lags, the time consistent *ex-post* tariff is Pareto inferior to the precommitment *ex-ante* tariff. Once production decisions are made, the foreign export supply elasticity faced by a large importer is lower than in the *ex-ante* case. This gives an incentive to the large

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9 The crucial assumption is that importing countries face a different residual foreign export supply when they revise their trade policy. Our argument could be extended to a model where firms make their investment decisions before hiring labor.
country to change its tariff announced at the beginning of the game if it did not irrevocably commit itself not to do so. Given complete information, for the \textit{ex-post} tariff to be time consistent, it requires that producers' price expectations are correct. Therefore, foreign producers, anticipating a higher tariff \textit{ex-post} than \textit{ex-ante}, will decrease their production and both the importer and foreigners' welfare is lower under the time consistent solution. However, if the large importer can control production at the beginning of the game, the optimal production policy is to tax (or use a production equivalent quota) the domestic production of importables. The purpose is to signal foreign producers to increase their production. The resulting equilibrium makes every country better off compared to the case where the large country is not able to control domestic production.

We showed in chapter 2 that if \( N \) policy active importers behave non-cooperatively \((N > 1)\), the time consistent \textit{ex-post} equilibrium may be welfare superior compared to the \textit{ex-ante} precommitment equilibrium. The welfare effect of a lower \textit{ex-post} foreign export supply elasticity due to predetermined output levels, is balanced by the perceived higher residual foreign export supply elasticity due to non-cooperative behavior. This chapter examines the impact of production policies under the no-commitment situation.\(^{10}\)

We propose to study two different production instruments, namely a production quota and a production tax. A production quota is equivalent to a situation in which the government chooses the level of domestic production and thus is managing domestic supply. Supply management is a much debated topic in agriculture. Most of the discussion on the

\(^{10}\) Again, because of the production tax/subsidy and consumption tax component of a tariff, the existence of a lag between production and consumption gives an incentive to use a production policy before production decisions are made. However, if irrevocably precommitting to the \textit{ex-ante} trade policy were feasible, the incentive to use a production policy would not be present.
economics of production quotas has focused on the analysis of producers' welfare.\textsuperscript{11} In a trade context, Moschini and Meiike (1991) explored the economic consequences of converting import quotas to import tariffs in case domestic production is under a supply management program.\textsuperscript{12}

Vercammen and Schmitz (1992) have looked at the transfer of rents following import concessions of a small country using a supply management program. Producers, in specific circumstances, prefer to offer import concessions than abandon supply management because supply management programs generally result in large rent transfers. De Gorter and Meiike (1989) have analyzed the relative efficiency of four policy options\textsuperscript{13} available to the European Community (EC) in terms of their efficiency at achieving the highest producer welfare. Their empirical model of the EC wheat sector confirms that a dual system of production quota and two-price plan is superior at achieving producer income goals.

In almost all cases, production quotas and taxes are either exogenous to the model or exist to implement some non-economic objective(s) [Bhagwati's (1971) terminology]. In this paper, it should be clear that production quotas are used to capture the welfare gains associated with the terms of trade argument. However, given the non-cooperative nature of the model, the welfare effect of the equilibrium production policies from the importers' perspective is \textit{a priori} indeterminate. The apparent policy makers' preference of production

\textsuperscript{11} Among others, Moschini (1984) has shown that part of the production quota value is due to the reduction in output price uncertainty in case supply management programs reduce the output price variability. Hennessy (1997) investigates the economic impact of altered price risk following the introduction of supply management.

\textsuperscript{12} They show that if the objective of the government is to replace a quota with a tariff that reflects the current difference between the domestic price and the world price, the price to domestic producers is left unchanged. On the other hand, if the objective is to replace the import quota with a tariff that leaves imports unchanged, the producers' domestic price would decline considerably.

\textsuperscript{13} The four policy alternatives considered were: a production quota, a production tax, a price reduction and a two price plan. The two-price plan proposed to maintain price supports for the portion of the production sold domestically, accompanied by a lower price for the grain exported on the world market.
quotas over production taxes in the real world is not well documented. While it is clear that supply management programs may be driven by political economy motives, we wish to rank the production tax and production quota on the basis of a social welfare maximizing goal.

There are numerous examples of the co-existence of trade and production policies. The Common Agricultural Policy (CAP) of the EC provides many relevant illustrations. It is generally accepted that the CAP has the potential to affect the level of world market prices [Swimbank and Tanner (1996)] based on the European common market size. The CAP is also a major source of disagreements between the EC and other large countries like the U.S. and Australia. Swimbank and Tanner provide numerous case studies where the production side of the domestic agricultural output markets is distorted while being the object of protectionist measures on the world market. For example, production quotas are used in the dairy sector and sugar industry to restrict producers’ supply. On the other hand, the livestock sector of the EC is generally subsidized at a very high level.

A similar pattern of trade and domestic policies has emerged in the steel industry over the years [Hogan (1983, 1991)]. In the late 1970’s and early 1980’s, the European Economic Community (EEC), a major exporter of steel to the U.S., called for mandatory reductions in production schedules of steel in every members’ domestic market. This resulted in a dramatic reduction of the production capacity in the EEC. Similar policies have been implemented in the U.S. and Japan. Officials came to realize that trade policies were not sufficient to maintain a relatively high world price for exports. This was done at a time when exports to the U.S. from the EEC and Japan were subject to Voluntary Restraint Agreements (VRAs).
In light of the previous discussion, there are two important issues to address. First, we need to determine if policy active importers have any private incentive to use a production policy (either a production tax/subsidy or production quota) given production lags. The mere existence of production lags is not sufficient however for the *ex-ante* trade policy not to be time consistent. The important assumption is that countries have the opportunity to revise their trade instrument level between production and consumption decisions. Second, the welfare consequences of such actions need to be evaluated with respect to the no production policy case. Assuming policy active importers behave non-cooperatively and do not irrevocably commit themselves to their *ex-ante* trade policy, the opportunity to use a production policy before production decisions are made, may decrease importers’ welfare compared to a no production policy situation.

### 3.3 – Domestic Production Policies and Time Consistent Import Quota

Consider a partial equilibrium model. There are $N$ importing countries with purchasing power on the world market for a certain good. Exporting nations behave competitively and pursue free trade. Domestic demand in each importing country is $d(p_i)$ where $p_i$ is the domestic consumer price. Domestic production is denoted by $q_i$. Suppose that the timing of economic decisions is as follows:

1. Policy active importers announce production policy level and trade policy level.
2. Domestic and foreign production decisions are made.
3. Importing nations can revise their trade policy and production policy levels costlessly.
4. Consumption and trade decisions are made.
Note that once output is determined at stage 2, there is no incentive to change the production policy level in a welfare maximizing framework in the following stage. Revising the domestic production policy is uniquely a redistributive issue. A change in the production policy level has no real effect if all agents are alike or marginal utility of income is constant.

The time consistent game is solved backwards. At the last stage of the game, the production levels of foreigners and importers are known. The domestic welfare function is the sum of consumer surplus, producer surplus, and government revenue. All tax and auction revenues are rebated to consumers in a lump-sum manner. The welfare function of importer $i$ is:

$$W_i = \int_0^{q_i} [p_i(y_i)dy_i - c(q_i) - \bar{p}m_i]$$

The objective of each importer is to maximize (3.1) such that

$$\bar{X} = \bar{O}(\bar{p}^*) - \bar{D}(\bar{p}) = \sum_j m_j$$

and

$$d_i = q_i + m_i,$$

where $\bar{X}$, $\bar{O}$ and $\bar{D}$ are the foreign exports, foreign supply (production) and foreign demand respectively and $m$, represents imports of country $i$. We work with a linear model and thus $\bar{D}'' = d_i'' = 0$. Equilibrium on the world market gives raise to an inverse foreign export supply, $\bar{p}(\bar{X}, \bar{O})$. Differentiate the welfare function in (3.1) with respect to $q_i$, $m_i$ and $\bar{p}$ to get:

$$dW_i = [p_i - c'(q_i)]dq_i + [p_i - \bar{p}]dm_i - m_i d\bar{p}$$

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14 In case of a production or import quota, we assume that each importer auctions off the quota licenses.
3.3.1 Framework

The strategy space of the policy active importers is restricted to the use of import quotas. From an ex-post perspective, production is fixed and so \( dq_i = 0 \) in (3.2). Differentiate the residual foreign export supply holding other importers' quota fixed to get:

\[
- \overline{D}' \cdot \overline{p} - dm_i = 0 \tag{3.3}
\]

where \( \overline{D}' \) is the slope of the foreign demand curve. Substitute (3.3) into (3.2) for \( \overline{dp} \) and divide both sides of (3.2) by \( dm_i \) to get country \( i \)'s first order condition:

\[
J'(m_i, q_i, \overline{p}(\overline{Q}, \overline{X})) = \partial W_i/\partial m_i = (p_i - \overline{p}) + m_i/\overline{D}' = 0 \tag{3.4}
\]

Solving (3.4) gives country \( i \)'s reaction function: \( m_i = f(m_{-i}, q_i, \overline{Q}) \). It is a function of the predetermined production levels \( q_i \) and \( \overline{Q} \) and other importers' quota \( m_{-i} \). For the second order condition of the maximization problem to hold, \( J''_{m_i} \) is assumed to be negative.

We need to solve for the response of country \( i \)'s imports following a change in the production policy. From the equilibrium condition on each domestic market, we have ex-post \( dp_i = (dq_i + dm_i)/d_i' \). Similarly, differentiating the equilibrium condition on the world market yields: \( \overline{dp} = (d\overline{Q} - d\overline{X})/\overline{D}' \). Totally differentiate (3.4) to get:

\[
\overline{D}' \left( \frac{dq_i + dm_i}{d_i'} \right) + (d\overline{X} - d\overline{Q}) + dm_i = 0 \tag{3.5}
\]

Define \( \alpha_i \equiv d_i'/\overline{D}' > 0 \) and \( \beta_i \equiv \alpha_i/(1 + \alpha_i) \in [0,1] \). Rewrite (3.5) as:

\[
\frac{dq_i}{\alpha_i} + (d\overline{X} - d\overline{Q}) + \frac{dm_i}{\beta_i} = 0 \tag{3.6}
\]

Summing (3.6) over all \( N \) importers yields:
\[ d\bar{X} = \frac{-\sum_{j} (1-\beta_j) dq_j + \sum_{j} \beta_j d\bar{O}}{1+\sum_{j} \beta_j} \quad (3.7) \]

At the beginning of the game, policy active importers non-cooperatively set their production policy. We need a closure rule to determine \( \bar{O} \). From an ex-ante perspective, we require that predetermined output levels be consistent with what producers would have chosen had they correctly forecast prices. Assuming perfect foresight, we have \( \bar{p}^* = \bar{p} \) and thus \( \bar{O}(\bar{p}^*) = \bar{S}(\bar{p}) \) with \( \bar{S}' > 0, \bar{S}'' = 0 \). The world price is determined accordingly by:

\[ \bar{X} = \bar{S}(\bar{p}) - \bar{O}(\bar{p}) \]

Totally differentiating the previous identity gives:

\[ d\bar{p} = d\bar{X}/(\bar{S}' - \bar{D}') \]

Similarly, differentiating the foreigners' supply yields:

\[ d\bar{O} = \bar{S}' d\bar{p} \]

Therefore, using the two previous equations, we can write

\[ d\bar{O} = (1-\bar{\mu})d\bar{X}, \]

where \( \bar{\mu} = -\bar{D}'/(\bar{S}' - \bar{D}') \) is the relative foreign demand responsiveness. Substitute into (3.7) for \( d\bar{O} \) to get:

\[ d\bar{X} = \frac{-\sum_{j} (1-\beta_j) dq_j}{1+\bar{\mu} \sum_{j} \beta_j} \quad (3.8) \]

Now, substituting (3.8) into (3.6), we get:

\[ dm_i = -(1-\beta_i) dq_i + \bar{\mu} \beta_i \frac{\sum_{j} (1-\beta_j) dq_j}{1+\bar{\mu} \sum_{j} \beta_j} \quad (3.9) \]

Equations (3.1) through (3.9) hold regardless of whether the production policy in the first stage is a quota or tax, given that importers are using an import quota to restrict trade in the second stage.
3.3.2 Production quota competition

First, we solve for the equilibrium production quota. Taking as given the other importers' production quota, we have \( \frac{\partial q_j}{\partial q_i} = 0, \forall j \neq i \). From (3.9), we have the following results:

\[
\frac{\partial m_i}{\partial q_i} \bigg|_{q_*} = \frac{- (1 - \beta_i) (1 + \mu \sum_{j \neq i} \beta_j)}{1 + \mu \sum_j \beta_j} < 0
\]

(3.10)

\[
\frac{\partial m_j}{\partial q_i} \bigg|_{q_*} = \frac{\mu \beta_j (1 - \beta_i)}{1 + \mu \sum_j \beta_j} > 0
\]

(3.11)

Equations (3.10) and (3.11) are intuitive. Given the production quota of the other importers, if country \( i \) increases (decreases) its domestic production quota, it lowers (raises) its imports. The decrease (increase) in its imports has a negative (positive) impact on the world price, and thus increases (decreases) other importers' quota, ceteris paribus.

In the first stage of the game, policy active importers set their production quota non-cooperatively. To get the first order condition of country \( i \), use (3.4), (3.8) and (3.10) to substitute into (3.2):

\[
\frac{\partial W_i}{\partial q_i} = p_i - c'(q_i) - (p_i - \bar{p})(1 - \beta_i) \left[ \frac{1 - \mu + \mu \sum_{j \neq i} \beta_j}{1 + \mu \sum_j \beta_j} \right] = 0
\]

(3.12)

Solving simultaneously the system of \( N \) equations in (3.12), we get the Subgame Perfect Nash (SPN) equilibrium production quota, \( q_i^* \) along with the specific production tax equivalent: \( \omega_i^* = p_i - c(q_i^*) \). Let \( \theta_i = p_i - \bar{p} \) be the import tariff equivalent of the import quota associated with a production quota. The SPN equilibrium yields:
Proposition 3.1: Assume that symmetric policy active importers can not precommit to their *ex-ante* import quota, but they can directly control their domestic production using a production quota. Under linear demand and supply schedules, the equilibrium production tax equivalent is positive. Moreover, the production tax equivalent is lower than the equilibrium tariff equivalent of the import quota, *i.e.* $0 < \omega_i^m < \theta_i^a$.

Proof: The claim $\omega_i^m > 0$ is easily verified since $0 < \beta_j < 1, \forall j$ and $0 \leq \bar{\mu} \leq 1$. To prove that $\omega_i^m < \theta_i^a$, we need to show that $(1 - \beta_i) \frac{1 - \bar{\mu} + \bar{\mu} \sum_j \beta_j}{1 + \bar{\mu} \sum_j \beta_j} < 1$. This condition is satisfied because we can rearrange the last inequality as: $-\beta_i - \bar{\mu} \beta_i \left(1 + \sum_j \beta_j\right) < 0$. Q.E.D.

Proposition 3.1 states that the production tax/subsidy is lower than the consumption tax. Remember that a quota (or tariff) can always be decomposed into a production subsidy and consumption tax. Consequently, since $\omega_i^m < \theta_i^a$, the overall impact of the trade and production policies in equilibrium is to subsidize production at a lesser rate than the implicit tax on consumption. Imposing symmetry, from (3.13) we have that:

$$\frac{\omega}{\theta} = (1 - \beta) \left\{ \frac{\bar{\mu}(1 + \beta)}{1 + N\bar{\mu}\beta} \right\}$$ (3.14)
The impact of the variable $N$ in (3.14) is positive: $\partial(\omega/\Theta)/\partial N > 0$. This implies that the consumption tax component of the quota increases (decreases) relatively to the production tax/subsidy as the number of symmetric importers decreases (increases). Before explaining in details the intuition behind proposition 3.1, we solve for the production tax equilibrium.

3.3.3 Production tax competition

Suppose all countries use a specific tax on production, $\sigma$, instead of a production quota. As a country varies its tax rate, it has an effect on other countries' production and thus unlike before: $\partial q_j/\partial q_i\big|_{\sigma_i} \neq 0$. Perfect competition in production on each domestic market determines $q_j$, such that: $p_j - \sigma_j = c'(q_j)$. Inverting the previous equation yields the domestic supply rule $q_j = s_j(p_j - \sigma_j)$. Differentiate the latter equation to get:

$$dq_j = s'_j(dp_j - d\sigma_j).$$

From before, equilibrium on the domestic market implies:

$$dp_j = (dq_j + dm_j)/d'_j.$$

Substitute the latter term into the former to get:

$$dq_j = \left(1 - \mu_j\right)\left(d'_j d\sigma_j - dm_j\right) \quad (3.15)$$

where $\mu_j = -d'_j/(s'_j - d'_j)$ is the relative domestic demand responsiveness. Using $d\bar{Q} = (1 - \bar{\mu})d\bar{X}$ and substituting (3.15) into (3.6) for $dq_i$, we get:

$$dm_j = -\left[\frac{\alpha_j(1 - \mu_j)D'_j}{\alpha_j + \mu_j}\right]d\sigma_j - \frac{\bar{\mu} \alpha_j}{\mu_j + \alpha_j} d\bar{X} \quad (3.16)$$

Summing (3.16) over all importing countries yields:
Define $\gamma_j = \alpha_j / (\alpha_j + \mu_j) \in [0,1]$. Substitute (3.17) into (3.16) to get:

$$dX = \sum_i \frac{\alpha_i \mu_i}{\mu_i + \alpha_i} = -\sum_i \left[ \frac{\alpha_i (1 - \mu_i) \bar{D}'}{\mu_i + \alpha_i} \right] d\sigma_i$$  \hspace{1cm} (3.17)

We can evaluate the marginal impact of the production tax on each importers' import quota holding $\sigma_j, \forall j \neq i$ fixed:

$$\frac{\partial m_i}{\partial \sigma_i} = -\gamma_i (1 - \mu_i) \bar{D} d\sigma_i + \bar{D}' \mu_i \frac{\sum_j \gamma_j (1 - \mu_j) d\sigma_j}{1 + \mu \sum_j \gamma_j}$$  \hspace{1cm} (3.18)

Equations (3.19) and (3.20) have a reasonable intuitive interpretation. Given other importers' production tax, an increase (decrease) in country $i$'s production tax lowers (raises) its domestic production and increases (decreases) its imports. This causes the world price to increase (decrease) and thus lowers (raises) the other importers' quota. Moreover, from (3.15) and (3.17) we have the following results: $\partial X / \partial \sigma_i = -[\bar{D}' \gamma_i (1 - \mu_i)] / [1 + \mu \sum_j \gamma_j] > 0$ and $\partial q_i / \partial \sigma_i = (1 - \mu_i) [d_i' - \partial m_i / \partial \sigma_i] < 0$. Hence, an increase (decrease) in the production tax increases (decrease) foreign exports.

We have all the necessary information to find the equilibrium production tax. Substitute $d \rho = dX / (\bar{S}' - \bar{D}')$ into (3.2) and divide both sides of the equation by $d\sigma_i$ to get:
Using (3.4), (3.19) and substituting for \( \frac{\partial X}{\partial \sigma_i} \) into (3.21), we can rewrite the first order condition in (3.21) as:

\[
\frac{dW_i}{\partial \sigma_i} = \sigma_i \frac{dq_i}{\partial \sigma_i} + (p_i - \bar{p}) \left[ \frac{\partial m_i}{\partial \sigma_i} - \mu \frac{\partial X}{\partial \sigma_i} \right] = 0
\]

(3.21)

Substituting for \( \frac{\partial q_i}{\partial \sigma_i} \) into (3.22), we solve the system of \( N \) equations simultaneously.

The SPN equilibrium production tax under an import quota, \( \sigma^m_i \) is:

\[
\sigma^m_i = \frac{\theta^\sigma \left[ \gamma_i \left( 1 - \mu \right) + \bar{\mu} \sum_{j \neq i} \gamma_j \right]}{\alpha_i \left( 1 + \bar{\mu} \sum_j \gamma_j \right) + \gamma_i \left( 1 - \mu \right) \left( 1 + \bar{\mu} \sum_j \gamma_j \right)}
\]

(3.23)

**Proposition 3.2:** Assume that symmetric policy active importers can not precommit to their import quota before domestic production decisions are made. However, they are able to control domestic production using a production tax. Under linear demand and supply schedules: (i) The non-cooperative equilibrium production tax is positive but lower than the import quota tariff equivalent, i.e. \( 0 < \sigma^m_i < \theta^\sigma \); (ii) The implicit tax resulting from a production quota is less than the non-cooperative production tax under an import quota, i.e. \( \omega^m_i < \sigma^m_i \).

**Proof:** It is readily seen from (3.23) that \( \sigma^m_i \) is positive. Also, \( \sigma^m_i < \theta^\sigma \) if the term between brackets on the right hand-side of (3.23) is less than one. This inequality holds if:
The term \( \gamma, -\alpha, -\gamma, (1-\mu_i) \) can be rewritten as: 
\[ -\alpha_i^2/(\mu_i + \alpha_i) < 0; \]
and thus the above inequality is rewritten as:
\[ -\alpha^2(1-\mu)/(\mu_i + \alpha_i) - \mu \sum_{j \neq i} \alpha_j^2 \gamma_j - \mu \gamma_i < 0; \] proving claim (i). From symmetry, rewrite (3.23) as:
\[
\frac{\sigma}{\theta} = \frac{\gamma((1-\mu) + \mu(N-1)\gamma)}{\alpha(1+\mu N\gamma) + \gamma(1-\mu)[1+\mu(N-1)]} = (1-\beta)\left[\frac{(1-\mu) + (N-1)\mu \gamma}{(1+\beta \mu) + (N-1)\mu \gamma}\right]
\] (3.24)

Given \( \theta \), using (3.14) and (3.23), we have that \( \omega^m \leq \sigma^m \) as \( \gamma \geq \beta \). We know that \( \gamma \geq \beta \) since \( \mu \leq 1 \) and therefore \( \omega^m < \sigma^m \), provided \( \mu < 1 \). Q.E.D.

Proposition 3.2 states that the equilibrium production tax/subsidy is lower than the consumption tax component of the quota. From (3.24), it is relatively easy to see that:
\[ \partial(\sigma/\theta)/\partial N > 0. \]
Consequently, as the number of symmetric policy active importers increases (decreases), the production tax/subsidy level increases (decreases) relatively to the consumption tax component of the quota. Moreover, from (3.14) and (3.24), the equilibrium production tax and production quota tax equivalent are equal as \( N \) tends to infinity.

The economic intuition behind the results of proposition 1 and 2 are fairly straightforward. Evaluate the first order condition in (3.21) at \( \sigma_i = 0 \):
\[
\frac{dW_i}{\partial \sigma_i} \bigg|_{\sigma_i = 0} = (p_i - \bar{p})(1-\mu)\frac{\partial m_i}{\partial \sigma_i} - \mu \sum_{j \neq i} \frac{\partial m_j}{\partial \sigma_i} > 0
\] (3.25)

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15 The first order conditions in (3.12) and (3.21) have a similar interpretation. They both dictate that the optimal policy is to restrict domestic production below the competitive level. The two instruments however yield different values for the production tax and production tax equivalent quota.
Equation (3.25) can be decomposed into two separate effects to gauge the incentives of importers to restrict domestic production. The first term between brackets on the right hand-side of (3.25) is the ex-post welfare incentive [Lapan (1988)]. Each importer has an incentive to tax production in order to encourage foreigners’ production as long as foreigners’ supply is positively sloped ($\bar{\mu} < 1$). In case the foreigners’ supply is perfectly inelastic ($S' = 0 \Rightarrow 1 - \bar{\mu} = 0$) and $N$ equals one, the optimal production tax is zero. This is expected since the ex-post and ex-ante residual foreign export supply are identical when $\bar{\mu} = 1$.

In case $N$ is greater than one, the optimal production tax is positive as $\frac{\partial M_j}{\partial \sigma_i} < 0$ even if $S' = 0$ (perfectly inelastic supply). This incentive to tax production comes from the second term between brackets on the right hand-side of (3.23). This term is similar to the standard strategic trade argument à la Brander-Spencer. An importer gains by using a production tax in order to shift rents away from other importers. Importer $i$ has an incentive to tax its own production because of the positive impact on its imports. This causes an increase in the world price and, ceteris paribus, decreases other importers’ trade quota. This provides a welfare gain to importer $i$ as the decrease in other policy active countries’ imports lowers the world price. The two distinct effects reinforce each other.

Propositions 3.1 and 3.2 answered the first question of the introduction. We showed that importers have a private incentive to use a production policy (either a production tax or a

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16 The strategic trade literature generally focuses on the transfer of rents from foreign firms to domestic firms and on the ability of the domestic firms to export more. However, it should be clear that the qualitative results of our model could be replicated in terms of an export subsidy and/or production subsidy if we had modeled the behavior of $N$ policy active exporters facing a passive importer.
production quota). However, the two instruments are not equivalent in equilibrium. Section 3.5 examines at the welfare implications of the two domestic production policies.

3.4 - Domestic Production Policies and Time Consistent Import Tariff

In this section, we turn our attention to the case where importers use a specific import tariff to restrict trade. The time consistent game is solved backwards. At the last stage of the game, production levels are predetermined. The objective of country $i$ is to maximize the sum of consumer surplus, producer surplus and government revenue:

$$W_i = \int_0^{\bar{Y}_i} p_i(Y_i) dY_i - c(q_i) - \bar{p} m_i$$

such that $\bar{X} = Q'(\bar{p}) - D(\bar{p})$ and $p_j = \bar{p} + \tau_j, \forall j$. Therefore, the world price is a function of the type: $\bar{p} = \phi(Q, \bar{X})$. Totally differentiate the previous function to get:

$$d\bar{p} = -(d\bar{X} - d\bar{Q})/\bar{D}'$$

Remember that from an ex-post perspective, $Q$ and $q_j, \forall j$ are predetermined. Totally differentiate (3.26) to get:

$$dW_i = [p_i - \bar{p}] dm_i - m_i d\bar{p}$$

3.4.1 Framework

The trade equilibrium condition implies that: $\bar{Q}(\bar{p}'') - D(\bar{p}) = \sum_j (d_j(p_j) - q_j(p_j'))$.

Holding the tariffs of the other importers and output levels constant, differentiate the ex-post residual foreign export supply to get:

$$(-\bar{D}' - \sum_{j=1}^{\bar{Y}_i} d'_j) d\bar{p} - dm_i = 0$$

(3.28)

Substitute (3.26) into (3.25) for $dm_i$ to get the first order condition of country $i$: 
\[ J'(\tau_1, \ldots, \tau_N, q_1, \ldots, q_N, \bar{Q}) = \frac{\partial W_i}{\partial \tau_i} = (p_i - \bar{p}) \left( \sum_{j=1}^{n_i} d_j' + \bar{D}' \right) + m_i = 0 \] (3.29)

We assume linear foreigners and importers' demand and supply. Hence, all second derivatives of the demand and supply schedules are zero \( (i.e. d''_j = \bar{D}'' = s''_j = \bar{S}'' = 0) \). For notational purpose, define \( \kappa = \sum_j d_j' < 0 \). For the second order condition to be satisfied, we assume \( J'_\tau < 0 \). The domestic price \( p_i \) is determined according to the equilibrium condition on each domestic market: \( d_i(p_i) = m_i + q_i \). Totally differentiate the previous equation to get:
\[ dp_i = (dm_i + dq_i)/d_i'. \] Totally differentiate the first order condition in (3.29):
\[ \left[ \kappa + \bar{D}' - d'_i \right] \left[ \frac{dq_i + dm_i}{d_i'} + d\bar{X} - d\bar{Q} \right] + \frac{dm_i}{\bar{D}'} = 0 \] (3.30)

Using the previous definition for \( \alpha_i \), we have: \( \kappa + \bar{D}' - d'_i = \bar{D}'(\sum_{j=1}^{n_i} \alpha_j + 1) \).

Collecting terms in (3.30), we can rewrite that equation as:
\[ \left[ 1 + \sum_{j=1}^{n_i} \alpha_j \right] \left[ \frac{dq_i + dm_i}{\alpha_i} + d\bar{X} - d\bar{Q} \right] + dm_i = 0 \] (3.31)

We need a closure rule to determine \( \bar{Q} \). With perfect foresight, producers correctly anticipate the resulting world and domestic prices after governments have moved. This expectation framework is consistent with our game of complete information. Therefore, \( \bar{p}^* = \bar{p} \), and thus \( \bar{Q}(\bar{p}^*) = \bar{S}(\bar{p}) \). The \textit{ex-ante} response of foreigners' production following a price change is: \( d\bar{Q} = \bar{S}' dp \). Substituting for \( dp \) into the last equation yields:
\[ d\bar{Q} = (1 - \bar{p}) d\bar{X} \]. Define \( \psi = 1 + \sum_j \alpha_j = 1 + \kappa/\bar{D}' > 0 \) and rewrite (3.31) as:
\[ dm_i = -\left[ \frac{\psi - \alpha_i}{\psi} \right] dq_i - \alpha_i \mu \left[ \frac{\psi - \alpha_i}{\psi} \right] d\bar{X} \] (3.32)
Summing (3.32) over all importers yields:

\[
d\bar{X} = -\left[ \sum_j (1 - \alpha_j / \psi) dq_j \right] \left[ \frac{1 + \bar{m}(\psi - 1) - (\bar{m} / \psi)(\sum_j \alpha_j^2)}{1 + \bar{m}(\psi - 1) - (\bar{m} / \psi)(\sum_j \alpha_j^2)} \right] \quad (3.33)
\]

Substituting (3.33) into (3.32) for \( d\bar{X} \) yields country \( i \)'s imports as a function of every importer's production level:

\[
dm_i = -(1 - \alpha_i / \psi) dq_i + \left[ \frac{\bar{m} \alpha_i (1 - \alpha_i / \psi)}{R} \right] \left( \sum_j (1 - \alpha_j / \psi) dq_j \right) \quad (3.34)
\]

where \( R = 1 - \bar{m} + (\bar{m} / \psi)(\psi^2 - \sum_j \alpha_j^2) > 0 \). Equations (3.28) through (3.34) hold regardless of the production policy employed (quota or tax) in the first stage of the game given that the strategy space of the policy active importers is restricted to a specific import tariff in the second stage.

### 3.4.2 Production quota competition

This section finds the optimal production quota of the policy active importers. Holding the other importers' production quota fixed, we have: \( \partial q_j / \partial q_i = 0, \forall j \neq i \). From (3.34), the responses of import quantities to a change in \( q_i \) are:

\[
\left. \frac{\partial m_i}{\partial q_i} \right|_{q_j} = -(1 - \alpha_i / \psi) \left( R - \bar{m} \alpha_i (1 - \alpha_i / \psi) \right) < 0 \quad (3.35)
\]

\[
\left. \frac{\partial m_j}{\partial q_i} \right|_{q_j} = \bar{m} \alpha_j (1 - \alpha_j / \psi) (1 - \alpha_i / \psi) \frac{R}{R} > 0 \quad (3.36)
\]

The expression in (3.35) is negative because \( R - \bar{m} \alpha_i (1 - \alpha_i / \psi) > 0 \). The intuition behind (3.35) and (3.36) is similar to the import quota game of section 3.3. However, the
response of country $i$'s imports following a change in any production quota is different under an import tariff than under an import quota, i.e. $\frac{\partial m_j}{\partial q_i} |_{\text{tariff}} \neq \frac{\partial m_j}{\partial q_i} |_{\text{quota}}, \forall i, j$.

Differentiate the welfare function in (3.26) to get:

$$dW_i = [p_i - c'(q_i)]dq_i + \tau(dm_i - m_i d\bar{p}) = 0 \quad (3.37)$$

Using the first order condition for the import tariff in (3.29) and since, before production decisions are made $d\bar{p} = d\bar{x}/(S' - D')$, we obtain the production quota first order condition of country $i$:

$$\frac{\partial W_i}{\partial q_i} = [p_i - c'(q_i)] - \frac{\tau_i (\psi - \alpha_i)}{R \psi} \left[ (1 - \bar{\mu}) - \bar{\mu} \sum_{j \neq i} \alpha_j^2 / \psi \right] = 0 \quad (3.38)$$

Solving simultaneously the system of $N$ first order conditions in (3.38) yields the SPN equilibrium production quota, $q_i^*$, with the specific tax equivalent $\omega_i^*$:

$$\omega_i^* = \frac{\tau_i (\psi - \alpha_i)}{R \psi} \left[ (1 - \bar{\mu}) - \bar{\mu} \sum_{j \neq i} \alpha_j^2 / \psi \right] \quad (3.39)$$

**Proposition 3.3:** Assume policy active importers can not precommit to their import specific tariff before domestic production decisions are made. However, importers are able to control domestic production using a production quota. Under linear demand and supply schedules, the equilibrium production tax equivalent is either positive or negative but lower than the import tariff, i.e. $0 \leq \omega_i^* < \tau_i^a$.

**Proof:** Define $q_i^0$ such that $p_i - c'(q_i^0) = 0$; hence $q_i^0$ is defined as the competitive production level absent any domestic distortions. Evaluate (3.38) at $q_i = q_i^0, \forall i$:
\[
\frac{\partial W_i}{\partial q_i} \bigg|_{q_i = q_i^*} = -\frac{\tau_i (\psi - \alpha_i)}{R \psi} \left[ (1 - \overline{\mu}) - \overline{\mu} \sum_{j=1}^{\infty} \alpha_j^2 / \psi \right] \tag{3.40}
\]

From (3.40), \( \omega_i > 0 \) as \( (1 - \overline{\mu}) - \overline{\mu} \sum_{j=1}^{\infty} \alpha_j^2 / \psi > 0 \). Clearly, the likelihood that the production tax is negative is increasing with \( \overline{\mu} \) and \( \alpha \). Rewrite (3.39) as:

\[
\omega_i = \frac{1 - \mu - \mu \sum_{j=1}^{\infty} \alpha_j^2 / \psi}{1 - \mu - \mu \sum_{j=1}^{\infty} \alpha_j^2 + \mu / \psi (\psi^2 - \alpha_i^2)} \tag{3.41}
\]

Since both terms between brackets on the right hand-side of (3.29) are less than one, it follows that: \( \omega_i / \tau_i < 1 \). Q.E.D.

The intuition behind the equilibrium production quota under an import tariff is similar to the one under an import quota, albeit there is one important term missing from (3.25). Rewrite (3.40) as:

\[
\frac{\partial W_i}{\partial q_i} \bigg|_{q_i^* = q_i^*} = \tau_i \left[ (1 - \overline{\mu}) \frac{\partial m_i}{\partial q_i} - \overline{\mu} \sum_{j=1}^{\infty} \alpha_j \frac{\partial m_i}{\partial q_i} - \overline{\mu} (\psi - \alpha_i) \sum_{j=1}^{\infty} \frac{\partial m_i}{\partial q_i} \right] > 0 \tag{3.42}
\]

The first term between bracket on the right hand-side of (3.42) is the \textit{ex-post} welfare incentive. A policy active importer has an incentive to restrict its own production of the importable in order to signal to exporters to increase their production. In case of monopsony power on the world market \( (N = 1) \), the optimal production policy is to tax production as long as \( \overline{\mu} < 1 \). In case of a perfectly inelastic supply \( (\overline{S} = 0, \overline{\mu} = 1) \), the \textit{ex-ante} foreign export supply is identical to the \textit{ex-post} foreign export supply. The incentive to tax its own production to encourage foreigners' production disappears. The third term between brackets in (3.42) is the strategic trade argument. In the event where \( N > 1 \), a policy active importer
can restrict its own production to encourage other importers to lower their imports. *Ceteris paribus*, this lowers the world price and importer \( i \) gains from this restriction in its own production. This effect reinforces the *ex-post* welfare effect.

The second term works in a different direction from the two previous effects, *i.e.* \( \bar{\mu} \sum_{j \neq i} \alpha_j \partial m_i / \partial q_i > 0 \). In the import quota game, this term did not appear because given the other importers' quota, country \( i \) does not have any incentive to change its own import quota in equilibrium. In the import tariff game, provided \( \bar{\mu} \) is sufficiently high, committing to decrease imports may actually raise country \( i \)'s welfare. But lowering its imports, given the tariff of other importers causes a decrease in the world price and increases other countries imports. Therefore a policy active importer does not get the full benefit of lowering its imports. It prefers to subsidize its own production to effectively precommit to lower imports.

Assuming symmetry among importers, we can rewrite (3.41) as:

\[
\frac{\omega}{\tau} = \left[ \frac{1 + (N - 1)\alpha}{1 + N\alpha} \right] \left[ \frac{(1 - \bar{\mu})(1 + N\alpha) - \bar{\mu}(N - 1)\alpha^2}{(1 - \bar{\mu})(1 + N\alpha) - \bar{\mu}(N - 1)\alpha^2 + \bar{\mu}(1 + N\alpha)^2 - \alpha^2} \right] \tag{3.43}
\]

From (3.43), a change in the number of policy active importers \( (N) \) has an indeterminate impact on the ratio \( \omega/\tau \).

### 3.4.3 Production tax competition

Suppose a country is using a specific tax on production, \( \alpha_i \) instead of a production quota. Substitute (3.15) into (3.32) for \( dq_i \) to get:

\[
dm_j = \left[ \frac{(\psi - \alpha_j)(1 - \mu_j)d\sigma_j + \bar{\mu}\alpha_j(\psi - \alpha_j)d\bar{X}}{(1 - \mu_j)\alpha_j + \mu_j\psi} \right] \tag{3.44}
\]
Summing (3.44) over all importing countries yields:

\[ d\overline{X} \left[ 1 + \sum_j \frac{\alpha_j \mu_j (\psi - \alpha_j)}{(1 - \mu_j) \alpha_j + \mu_y \psi} \right] = -\sum_j \left[ \frac{d_j (1 - \mu_j) (\psi - \alpha_j) d\sigma_j}{(1 - \mu_j) \alpha_j + \mu_y \psi} \right] \]  

(3.45)

Since \( \gamma_j = \alpha_j / (\alpha_j + \mu_y) \), we can define \( V_j = \frac{\gamma_j (\psi - \alpha_j)}{(1 - \mu_j) \gamma_j + (1 - \gamma_j) \psi} > 0 \). Substitute (3.45) into (3.44) for \( d\overline{X} \) to get:

\[ dm_i = -\overline{D}'(1 - \mu_i) V_i d\sigma_i + \overline{D}' \mu V_i \sum_j \left[ \frac{(1 - \mu_j) V_j d\sigma_j}{(1 + \mu \sum_j V_j)} \right] \]  

(3.46)

The marginal impact of the production subsidy on imports of each country is derived from (3.46) holding \( \sigma_j, \forall j \neq i \) fixed:

\[ \frac{\partial m_i}{\partial \sigma_i_{\mid \sigma_j}} = -\frac{\overline{D}'(1 - \mu_i) V_i (1 + \mu \sum_j V_j)}{(1 + \mu \sum_j V_j)} > 0 \]  

(3.47)

\[ \frac{\partial m_j}{\partial \sigma_i_{\mid \sigma_j}} = \frac{\overline{D}' \mu V_j V_i}{(1 + \mu \sum_j V_j)} < 0, \forall j \neq i \]  

(3.48)

Equations (3.47) and (3.48) have a reasonably intuitive interpretation. Given other importers’ production tax, an increase (decrease) in country \( i \)'s production tax lowers (raises) its domestic production and increases (decreases) its imports. This causes the world price to increase (decrease) and thus, lowers (raises) the other importers’ quota, \textit{ceteris paribus}. We now have all the necessary information to find the optimal production subsidy. Substituting \( d\overline{\sigma} = d\overline{X}/(\overline{S}' - \overline{D}') \) into (3.27) and using (3.29), we get the first order condition of importer \( i \):

\[ \frac{\partial \overline{W}}{\partial \sigma_i} = \sigma_i \frac{\partial q_i}{\partial \sigma_i} + (p_i - \overline{p}) \left[ \frac{\partial m_i}{\partial \sigma_i} - \overline{\mu} (\psi - \alpha_i) \right] = 0 \]  

(3.49)
Using (3.47) and substituting for \( \frac{\partial \bar{X}}{\partial \sigma_i} \) and \( \frac{\partial q_j}{\partial \sigma_i} \) into (3.49), we can rewrite the first order condition as:

\[
\frac{dW_i}{\partial \sigma_i} = \frac{(1 - \mu_i)D'}{(1 + \bar{\mu}\sum_j V_j)} \left\{ \sigma_i \left[ \alpha_i (1 + \mu_i \sum_j V_j) + (1 - \mu_i) \psi_i (1 + \bar{\mu}) \sum_{j \neq i} V_j \right] - \tau_i \psi_i [1 + \bar{\mu} \sum_{j \neq i} V_j - \bar{\mu} (\psi_i - \alpha_i)] \right\} = 0
\]  

(3.50)

Solving the system of \( N \) equations simultaneously yields the Subgame Perfect Nash (SPN) equilibrium production tax under an import tariff, \( \sigma_i^* \):

\[
\sigma_i^* = \tau_i^* \frac{V_i}{\psi_i - \alpha_i} \left[ \frac{1 + \bar{\mu} \sum_{j \neq i} V_j - \bar{\mu} (\psi_i - \alpha_i)}{\alpha_i (1 + \mu_i \sum_j V_j) + \psi_i (1 - \mu_i) (1 + \bar{\mu} \sum_{j \neq i} V_j)} \right]
\]  

(3.51)

**Proposition 3.4:** Assume that symmetric policy active importers can not precommit to their import specific tariff before domestic production decisions are made. However, they are able to control domestic production using a production tax. Under linear demand and supply schedules: (i) The equilibrium production tax can be either positive or negative, but is smaller than the time consistent import tariff, i.e. \( 0 < \sigma_i^* < \tau_i^* \); (ii) The production tax is higher than the tax equivalent production quota in equilibrium, \( \omega_i^* < \sigma_i^* \).

**Proof:** See Appendix A.

The intuition behind proposition 3.4 is identical to the intuition behind proposition 3.3. Figure 3.1 summarizes the incentives for policy active importers to control domestic production for both the import quota and import tariff game.
Figure 3.1 Decomposition of importers’ incentives to control domestic production in the import quota and import tariff games.

Each importer has an incentive to tax domestic production to encourage foreigners to produce more if $\tilde{S}' > 0$. There exists also an incentive to tax domestic production to reduce other countries’ imports. These two effects reinforce each other. A third effect induces importers to subsidize production. If $\bar{\mu}$ is sufficiently high, it may be desirable for a policy active importer to lower its imports from an ex-post perspective. However, an importer would not want to use tariffs for this purpose because a decrease in imports caused by an increase in the tariff of importer $i$ induces a decrease in the world price and therefore
encourages other countries to import more. This is not desirable from country i’s perspective. Therefore, a production subsidy is preferable to an increase in tariff to lower imports.

3.5 - Welfare Implications of Production Policies

We have demonstrated that policy active importers have incentives to use domestic production instruments if they can not commit to their trade policy before production decisions are made. Whether, in a symmetric equilibrium, the importers actually gain from the flexibility to use production policies is however left to determine. This section intends to answer the question using the collusive production policy as a benchmark to compare welfare levels under the SPN equilibrium policy and the no production policy case.

First, note that there is equivalence between the production tax and production quota either with collusive behavior or when \( N = 1 \). We assume non-cooperative behavior among importers in the latter stage of the game when setting their trade instrument. However, as a reference tool, we define collusive welfare, as the sum of producer surplus, consumer surplus and government revenue of the \( N \) importers. Since in a symmetric equilibrium all importers get the same welfare, this is an appropriate measure to determine the optimal collusive production policy. First, assume that the strategy space of the trade policy is restricted to import quotas. The collusive welfare function is:

\[
W = \sum_{i} \left[ \int_{0}^{y_i^*} p(y_i)dy_i - c(q_i) - pm_i \right]
\]  

(3.52)

Differentiating (3.52) with respect to \( q_i \) holding other importers production quota fixed, we have:
\[
\frac{dW}{\partial q_i} = [p_i - c'(q_i)] + \sum_j \left( (p_j - \overline{p}) \frac{dm_j}{\partial q_i} \right) - \sum_j m_j \frac{\partial \overline{p}}{\partial q_i} \tag{3.53}
\]

3.5.1 Import quota game

In a symmetric equilibrium, \( p_j = p_i, \forall i, j \). Using \( d\overline{X} = (\overline{S'} - \overline{D'})d\overline{p} \) and the ex-post first order condition for the import quota maximization problem in (3.4) yields the first order condition of country \( i \):

\[
\frac{\partial W}{\partial q_i} = \omega_i^{*m} + \theta \left[ \frac{\overline{S'} + (N-1)\overline{D'}}{\overline{S'} - \overline{D'}} \right] \frac{\partial \overline{X}}{\partial q_i} = 0 \tag{3.54}
\]

where the superscripts \(^{*m}\) indicate the equilibrium under a collusive production instrument and non-cooperative import quota. Evaluate (3.54) at \( q_i^0 \) such that \( p_i - c'(q_i^0) = 0 \) to get:

\[
\frac{\partial W}{\partial q_i} \bigg|_{q_i=q_i^0} > 0 \text{ as } \overline{S'} + (N-1)\overline{D'} \leq 0 \text{ since } \frac{\partial \overline{X}}{\partial q_i} < 0. \text{ Therefore, } \omega_i^{*m} > 0 \text{ as } N\overline{\mu} \leq 1.
\]

**Proposition 3.5:** Assume symmetric policy active importers who are restricted to use an import quota for their trade instrument. Under linear demand and supply schedules: (i) The optimal collusive production tax is always lower than the SPN equilibrium production tax and the equilibrium production tax equivalent quota; (ii) Welfare under both the equilibrium production tax and production quota is lower than under the time consistent solution without any production policy if \( N\overline{\mu} > 1 \). In case where \( N\overline{\mu} < 1 \), the welfare effect is ambiguous.

**Proof:** Evaluate (3.12) at the collusive solution \( q_i = q_i^{*m} \) to get:
The signing of (3.55) comes from (3.8) and (3.11). We have: \( q^* < q^*_m \) and since \( c'' > 0 \), it implies \( \omega_i^m > \omega_i^*_m \). From proposition 3.2, we have: \( \sigma_i^m > \omega_i^m > \omega_i^*_m \), proving claim (i). The proof for the second part of proposition 3.5 comes from the definition of collusive behavior and the second order condition \( \frac{\partial^2 W}{\partial q_i^2} < 0 \). The collusive production quota solution brings the highest welfare level attainable for every symmetric importer, given the trade policy rule. Therefore, if the optimal collusive production policy yields a negative production tax (subsidy), the inability (or an irrevocable commitment not) to control domestic production in the first stage must be welfare superior as the non-cooperative equilibrium entails a positive tax on domestic production. From (3.54), this happens if \( N\mu > 1 \). In case both the optimal collusive and non-cooperative solutions entail a positive tax (\( \sigma_i^m > \omega_i^m > \omega_i^*_m > 0 \)), the welfare impact of controlling production is indeterminate. Q.E.D.

Under the assumption of linear demand and supply schedules, from (3.4), the reaction function of country \( i \) has the form: \( m_i = k_0 + k_1 \overline{Q} + k_2 q_i + \sum_{j \neq i} k_{3j} m_j \), where \( k_0, k_1, k_2 \) and \( k_{3j} \) are constants. Solving for \( m_i \) implies that the imports of each country are linear in \( \overline{Q} \) and \( q_i \). \( \forall j \). Differentiating the welfare function in (3.1), we have: 

\[
p_i (q_i + m_i) - c'(q_i) - \overline{p}(\overline{Q}, \sum_j m_j) (\partial m_i / \partial q_i) - \overline{p}' \sum_j (\partial m_j / \partial q_i). 
\]

The inverse domestic demand, marginal cost and inverse export supply function are linear in their arguments. Hence, the second derivative of the welfare function with respect to \( q_i \) is a constant and thus...
the welfare function is quadratic in $q_i$. Since the production tax is linear in domestic production, the welfare function is also quadratic in the production tax $\sigma$. This property of the welfare function is important to gauge the welfare implications of the equilibrium policies.

Figure 3.2 illustrates the different welfare implications of the production quota and production tax under an import quota. Since the production tax equivalent quota in equilibrium is positive, the condition for the production quota to be welfare superior to the no production policy case is: $2\omega^m - \sigma^m > 0$. Using (3.13) and (3.54), this condition can be written as: $\mu < \frac{1 + \alpha}{2N - 1 + (3N - 2)\alpha}$. The solid lines in figure 3.2 trace out the previous equation as a function of $\alpha$. For a given value of $N$, if $\mu$ is located above (below) the solid line, the ability to control domestic production lowers (improves) welfare from the policy active importers' perspective. As $N$ increases, the area under the curve decreases and therefore, the likelihood that a production quota is welfare improving is decreasing with $N$.

In case of a production tax, the inequality $2\omega^m - \sigma^m > 0$ implies that the ability to tax domestic production at the beginning of the game is welfare improving. Using (3.23) and (3.54), the solution for this inequality is given in Appendix B. The dotted lines in Figure 3.2 illustrate the different values of $\mu$ as a function of $\alpha$ for which the importer's welfare is equal under a production quota and no production policy. In case $\mu$ lies above (below) the dotted line for any given value of $N$, the ability to tax domestic production is welfare inferior (superior) to the situation where importers are collectively committed not to use any production policy.
Figure 3.2 Welfare implications of the SPN equilibrium production tax and production quota under an import quota

Note that, in appendix B, the parameter $\gamma$ is function of $\mu$. Therefore, the results in figure 1 are not invariant to the relative demand responsiveness of importers for the import quota/production tax game. The dotted lines in figure 1 are traced out fixing the value of $\mu$ at 0.1. Simulations showed that increasing $\mu$ only reduces the distance between the solid and dotted lines for a given $N$. In other words, as the relative domestic demand responsiveness increases (decreases) the likelihood that a production tax increases (decreases) welfare is increasing (decreasing). The potential effect of the parameter $\mu$ on importer's welfare seems very small in the import quota game.
Figure 3.2 is conformed to the theoretical results of section 3.3. In case importers cannot commit to their ex-ante import quota, the ability to impact domestic production is welfare improving only if \( \bar{\mu} \) is sufficiently small. In other words, the ex-ante residual foreign export supply must be sufficiently different from the ex-post supply. Naturally, because \( \sigma^m > \omega^m > \omega^* \), using a production quota is always welfare superior to a production tax. Therefore, for a given \( N \), the solid line associated with the import quota/production quota game always lies above the respective dotted line of the import quota/production tax case.

3.5.3 Import tariff game

Similarly to the previous section, we can investigate the welfare implications of the production tax and quota under an import tariff. Non-cooperative behavior once production decisions are made implies: \( \tau \bar{D}'(1 + \sum_{j \neq i} \alpha_j) + m_i = 0 \). Dropping all subscripts because of symmetry among the policy active importers, we can substitute the previous expression into (3.53) to yield the first order condition of importer \( i \):

\[
\frac{\partial W}{\partial q_i} = \omega^{*i} + \tau \left[ (1 - N\bar{\mu}) - N\bar{\mu}(N - 1)\alpha \right] \frac{\partial \bar{X}}{\partial q_i} = 0
\]  

(3.56)

where \( \omega^{*i} \) denotes the equilibrium collusive production policy with an import tariff. Since \( \partial \bar{X}/\partial q_i < 0 \), evaluating (3.56) at \( q_i = q_i^0 \) yields: \( \partial W/\partial q_i|_{q_i=q_i^0} < 0 \) as

\[
1 - N\bar{\mu} - N\bar{\mu}(N - 1)\alpha > 0. \quad \text{Therefore } \sigma^{*i} = \omega^{*i} > 0 \text{ as } 1 - N\bar{\mu} - N\bar{\mu}(N - 1)\alpha > 0.
\]
Proposition 3.6: Assume symmetric policy active importers who can revise their import tariff once production decisions are made. Under linear demand and supply schedules: (i) The optimal collusive production tax is always lower than the SPN equilibrium production tax and the SPN equilibrium production tax equivalent quota, \( \sigma_i^* > \omega_i^* > \omega_i^{**} \); (ii) If \((1 - \overline{\mu})(1 + N\alpha) - (N - 1)\overline{\mu}\alpha^2 < 0\), the ability to control the level of domestic production is welfare improving compared to a commitment not to use any production policies; (iii) If \((1 - \overline{\mu})(1 + N\alpha) - (N - 1)\overline{\mu}\alpha^2 > 0 > 1 - N\overline{\mu} - N\overline{\mu}(N - 1)\alpha\), the ability to control domestic production is welfare decreasing; (iv) If \(1 - N\overline{\mu} - N\overline{\mu}(N - 1)\alpha > 0\), the welfare ranking between the SPN equilibrium production quota and the no-production policy is ambiguous.

Proof: Evaluate (3.42) at the collusive solution \( q_i = q_i^{**} \) to get:

\[
\frac{\partial W_i}{\partial q_i} = -\tau \frac{(\psi - \alpha)}{R\psi} \left[ (N - 1)\overline{\mu} + \left( N - 1 \right)\overline{\mu}(N - 1)\alpha^2 \right] < 0
\]

(3.57)

From (3.57), we have: \( q_i^{**} < q_i^{*} \) and since \( c'' > 0 \), it implies \( \omega_i^* > \omega_i^{**} \). Furthermore, from proposition 3.4, we have: \( \sigma_i^* > \omega_i^* \). The proofs of claims (ii) through (iv) come from the definition of collusive behavior. The collusive solution brings the highest welfare level attainable for all symmetric importers. From (3.51), \( (1 - \overline{\mu})(1 + N\alpha) - (N - 1)\overline{\mu}\alpha^2 < 0 \) implies the SPN equilibrium production tax equivalent is negative. The ability to control the level of domestic production is welfare improving since, from claim (i), we have \( 0 > \omega_i^* > \omega_i^{**} \); proving claim (ii). From (3.51) and (3.56), the inequality \( (1 - \overline{\mu})(1 + N\alpha) - (N - 1)\overline{\mu}\alpha^2 > 0 > 1 - N\overline{\mu} - N\overline{\mu}(N - 1)\alpha \) implies \( \omega_i^{**} < 0 < \omega_i^* \). Thus,
welfare is higher if we can prevent importers to control domestic output, proving claim (iii). Finally, under the conditions of the fourth claim, (3.56) implies: $0 < \omega^* < \omega^r$ and hence the welfare ranking is ambiguous. Q.E.D.

Figure 3.3 illustrates the welfare implications of a production quota given the importers are restricted to use an import tariff for their trade instrument. As stated in proposition 3.6, if the non-cooperative production policy is to choose output above the competitive level, the ability to control production is welfare improving from the importers' perspective as compared to an irrevocable commitment not to control domestic production. Using (3.39), this condition is satisfied if: $\bar{\mu} > \varphi(\alpha, N) = \psi / ((N - 1)\alpha + \psi)$.

For a given value of $N$ and $\alpha$, if $\bar{\mu}$ is located above the dotted curve in figure 3.3, controlling production is welfare improving. It is easy to see that $\varphi_N < 0$. Hence, an increase (decrease) in the number of policy active importers increases (decreases) the likelihood that the equilibrium production tax is negative.

In case the equilibrium production quota entails a positive tax, the following inequality insures that the ability to tax production is welfare improving: $2\omega^* - \omega^m > 0$. Using (3.43) and (3.56), this condition yields: $\bar{\mu} < \psi / ((N - 1)\alpha(2\psi N - \alpha) + \psi(2N - 1))$. For any given $N$, a value of $\bar{\mu}$ located below the solid line in figure 3.3 assures that the production policy is welfare improving from the importers' perspective compared to the laissez-faire situation in domestic production.
Figure 3.3 Welfare implications of the SPN equilibrium production quota under an import tariff

**Proposition 3.7:** Assume symmetric policy active importers who can revise their import tariff once production decisions are made. Under linear demand and supply schedules: (i) If $1 - \mu - \mu(N-1)(\alpha - V) < 0$, the ability to tax domestic production is welfare improving compared to a commitment not to use any production policies; (ii) If $1 - \mu - \mu(N-1)(\alpha - V) > 0 > 1 - N\mu - N\mu(N-1)\alpha$, the ability to tax domestic production is welfare decreasing; (iii) If $1 - N\mu - N\mu(N-1)\alpha > 0$, the welfare ranking between the SPN equilibrium production tax and the no-production policy is ambiguous.
Proof. From proposition 3.6, we have: $x_i^* > \omega_i^* > \sigma_i^*$. The collusive solution brings the highest welfare level attainable for all symmetric importers. From (3.51), the SPN equilibrium production tax is negative if: 

$$1 - \overline{\mu} - \overline{\mu}(N-1)(\alpha - V) < 0.$$ 

The ability to tax domestic production must be welfare superior to the no-production policy equilibrium since $0 > \sigma_i^* > \omega_i^*$; proving claim (i). From (3.56) and (3.51), if 

$$1 - \overline{\mu} - \overline{\mu}(N-1)(\alpha - V) > 0 > 1 - N\overline{\mu} - N\overline{\mu}(N-1)\alpha,$$

we have: $\omega_i^{**} < 0 < \sigma_i^*$; and importers are collectively worse off in the production tax equilibrium than under an agreement to ban domestic production taxes. Finally, under the conditions of the third claim, (3.56) implies: 

$$0 < \omega_i^{**} < \sigma_i^*;$$

and hence the welfare ranking is ambiguous. Q.E.D.

Figure 3.4 presents the values of $\overline{\mu}$ for which the production tax is welfare improving with respect to the no production policy scenario. First, from proposition 3.7(i), 

$$1 + \overline{\mu}(N-1)V - \overline{\mu}(\psi - \alpha) < 0$$

implies the equilibrium production tax is negative (subsidy) and the ability to tax production is welfare improving. The parameter $\mu$ influences the welfare implications through its impact on $V$. Unlike the import quota/production tax case, changes in $\mu$ have a greater impact on the welfare implications in the import tariff game. We fix the value of $\mu$ at 0.5. In case $N = 2$, there are no values of $\alpha$ lower than 1 that yields a negative production tax (subsidy). As the value of $\alpha$ moves over one, there exists values of $\overline{\mu}$ that guarantees the production tax will be negative. In case the parameter $\overline{\mu}$ lies above the dotted line associated with $N = 2$, the ability to tax production in the first stage of the game is welfare improving compared to the no production policy case.
Since $\mathcal{N}/\partial \mu < 0$, the likelihood that the production tax is negative is increasing with $\mu$. In other words, an increase in the elasticity of the domestic supply will shift down the dotted curve. In case $N = 4$, there are no values of $\alpha$ that yields a production subsidy independently of the value of $\bar{\mu}$ if $\mu = 0.5$. Moreover, an increase in $N$, has an indeterminate impact on the likelihood of the SPN equilibrium tax of being negative.

In case the equilibrium production tax is positive, the condition $2\omega^{*r} - \sigma^r > 0$ assures that the ability to tax production is welfare improving. Appendix B solves the previous
equation for $\mu$ as a function $\alpha, N$ and $\mu$. Fixing $\mu$ at 0.5, any value of $\mu$ below the solid lines in figure 3.4 for a given $N$ implies policy active importers are collectively better off using a production tax compared to an agreement to ban domestic production policies.

### 3.6 - Conclusion

In this chapter, we assumed that production and consumption decisions are not carried out simultaneously. Moreover, the importers are assumed not to irrevocably commit themselves to their *ex-ante* trade policy. They can revise the level of their trade instrument once production decisions are made. We have derived the equilibrium domestic production policies and trade policies for policy active importers who can influence the price at which they buy a good on the world market. Because of the non-cooperative behavior among importers, price and quantity instruments available to importers are not equivalent.

If the trade strategy space of importers is restricted to use an import quota, the equilibrium production policy is to restrict domestic production below the competitive level. The production quota instrument is welfare superior from the importers' perspective to the production tax. In case of an import tariff, the equilibrium production policy may be to subsidize production. We derive conditions under which the ability of each importers to intervene on its domestic market increases welfare. Using numerical simulations, we relate those conditions to restrictions on the supply and demand elasticities of the model. Generally speaking, importers are likely to gain from using a production policy if the *ex-post* and *ex-ante* residual foreign export supply curve are either very different or very similar from one another.
This paper rationalizes the argument for a large importer to use both production and trade policies to increase welfare in case there is a lag between production and consumption decisions and they cannot commit to their trade policy. If trade agreements do not prevent importers from using production policies, the numerical simulations have shown that there exists a possibility for importers to enter in a prisoner's dilemma. Given the opportunity to set a production tax or control production directly, importers will do so, even though in equilibrium, it is likely to be welfare inferior to an agreement to ban production policies.
CHAPTER 4. PRODUCTION UNCERTAINTY AND ENDOGENOUS PRECOMMITMENT IN TRADE POLICY

4.1 - Introduction

The topic of quantity or price leadership versus simultaneous play in the standard oligopoly literature has been studied in detail. With one notable exception [Syropoulos (1995)], country specific leadership in trade policy has never fully been explored. While a few studies have looked at the implication of firms' timing of moves within a strategic trade framework, the oligopoly theory suggests a natural extension to the time consistency game introduced in chapter two. One explanation for the lack of research in trade policy leadership is the potential absence of any argument to explain a heterogeneous timing of moves in trade policy among governments.

Uncertainty may create different timing incentives in trade policy for governments depending on the information set of firms/consumers and policy makers. Suppose there exists a lag between production and consumption decisions and that each policy active importer can choose to adopt a policy/regime that makes revision costly. Given the commitment decision of other countries, a policy active importer may either prefer to precommit to its trade policy or use the flexibility to revise its policy after production decisions are made if additional information is revealed.

The production lag assumption is particularly relevant in agricultural markets. There may, for example, exist uncertainty in production due to weather patterns. Governments may not have complete information on weather conditions at the time they set their trade policy if they move before firms. In a setting, where there exists a lag between firms’ investment
decisions and labor hiring decisions, uncertainty in technology could create the same incentives discussed above if additional information on the technology shock is revealed after capital decisions were made.

The purpose of this chapter is to answer two questions. First, we wish to endogenize the precommitment decision of two policy active importers given that they are allowed either to commit to their trade policy before production decisions are made and uncertainty resolved or choose to revise their trade policy once production decisions are made and additional information revealed. This question seems most relevant since chapter 2 assumed exogenous commitment decisions. Second, we wish to analyze whether two importers can enter into a prisoner’s dilemma. Given the non-cooperative nature of our model, importers may choose a Pareto dominated outcome in equilibrium if we endogenize their precommitment decisions.

Uncertainty is introduced to model asymmetries among the strategic importers. In contrast, the previous chapters mainly relied on the assumption of symmetry among importers to derive some of the results. The strategy space of importers is restricted to the physical quantity of imports. Domestic production in each importing country is subject to a random shock. In this case, there is an option value from retaining flexibility by setting the optimal policy once production decisions (and outcomes) are made if additional information is available. The value of revising the trade policy must be weighted against the strategic value of committing to a policy at the beginning of the game. The randomness in domestic production allows for the possibility of asymmetric equilibria if production risk is specific to each importer.
The remainder of the chapter is organized as follows. The next section provides a review of the literature on the endogenous timing of moves in oligopoly theory. We introduce the theoretical framework in the following section. We study a monopsony market and measure the welfare trade-off between precommitting to a trade policy before production decisions are made versus keeping the flexibility to revise its trade policy once uncertainty is resolved and production decisions are made. The next section analyzes the import quota endogenous commitment game for two policy importers under uncertainty. Finally, concluding remarks are presented along with possible extensions.

4.2 - Review of Literature

Dowrik (1986), and Hamilton and Slutsky (1990) have analyzed the endogenous timing of moves in a duopoly model without uncertainty. Hamilton and Slutsky considered two specific types of game. Under their first assumption, the first stage of the game consists of two firms announcing the time at which they choose an action and are committed to this choice. In the second stage, a firm selects its action knowing when the other will make her decision. This is called the observable delay extended game. If the two firms have downward sloping reaction functions (as in a quantity setting game with output being strategic substitutes), the unique equilibrium is a simultaneous move equilibrium. In the case of a price setting game with both firms' reaction function sloping upward, the extended game has multiple equilibria, where each firm prefers to be the follower.

In their second extended game, leadership means committing to a particular action whether or not the rival attempts to lead or follow. Thus, the leadership action must be chosen without observing the timing of the opponent's move. In this game with action
commitment, both simultaneous play and each waiting and the other playing its Stackelberg leader choice in the first period are equilibria in the quantity game.

Spencer and Brander (1991) considered a strategic duopoly setting in which uncertainty creates an option value from retaining flexibility by delaying output decisions until after market demand uncertainty is resolved. The value of flexibility is weighted against the strategic value of precommitment. They also characterize the profit trade-off between flexibility and capital stock precommitment for an incumbent confronted with the possibility of entry by a firm with uncertain costs.

Syropoulos (1994) has endogenized trade policies in a bilateral monopoly model (two-good, two-country world). In the first stage, both countries declare simultaneously whether they wish to move early or late. A declaration of the type \([\text{Early}(E), E]\) or \([\text{Late}(L), L]\) implies that both players set their trade instrument simultaneously at the first or second stage. A declaration \([E, L]\) or \([L, E]\) implies that one county becomes the leader while the other is the follower. If the strategy space is restricted to a quota, moving simultaneously will cause the asymptotic elimination of trade [Rodriguez’s result (1974)]. If countries play a leader-follower game, the equilibrium is welfare superior to a simultaneous move game; as a follower, the individual country can exercise its market power in trade.

In Syropoulos’ tariff game, the Nash equilibrium entails a positive trade flow if the tariff reaction function are negatively sloped everywhere (tariffs are strategic substitutes throughout the strategy space). Sequential play arises if both reaction functions are increasing in the neighborhood of the Cournot Nash equilibrium. As an extension, the second stage of the game is modified so that governments choose the type of instruments and the level of protection. If the first stage of the game results in a simultaneous move, tariffs
are superior to quotas by an iterated dominance argument. In a sequential play, it is shown that: (i) the policy leader will always prefer to intervene with tariffs; and (ii) the follower is indifferent between using tariffs and quotas.

4.3 - Precommitment and Production Uncertainty under Monopsony Power

The purpose of this section is to examine the impact of uncertainty on the precommitment decision of a policy active importer if there exists a lag between production and consumption decisions. We change the world market structure of the previous chapters to consider the case of a single policy active importer. This diversion has one primary purpose. It makes the strategic trade-off between commitment and uncertainty more apparent since strategic interactions among importers are assumed away. We come back to an oligopsonistic market structure in the next section. The critical assumption is that producers and government do not have the same information set when making their decisions. Assume the following timing of events:

1. Government chooses its trade policy
2. Random disturbance in production is observed
3. Production decisions are made
4. Government can revise its trade policy
5. Consumption and trade decisions are made.

Producers' cost function in the importing country is subject to a linear random shock. Foreigners' supply schedule is known with certainty. Producers in both countries are
assumed to know the realization of the shock.\textsuperscript{17} In other words, firms make their production decisions after having observed the random disturbance. Hence, firms and the policy maker do not have the same information \textit{ex-ante}. There exists a lag between production and consumption decisions. If the government irrevocably precommits in the first period to its \textit{ex-ante} trade policy, stage 4 disappears. However, if it did not irrevocably precommit to its \textit{ex-ante} trade policy, the government can observe the random shock and revise its \textit{ex-ante} trade policy.

The uncertainty creates an option value for the government of not committing to its \textit{ex-ante} trade policy and revising its policy after it has observed the random shock. Lapan (1988) has shown that without uncertainty, a government prefers to commit to its \textit{ex-ante} trade policy since precommitment Pareto dominates the time consistent solution without commitment. However, the introduction of a random shock may reverse this ranking since more information is revealed to the policy maker once production decisions are made.

We use a stylized model to assess the impact of uncertainty on the value of precommitment. We work with a partial equilibrium framework, and abstain from consumer risk attitude considerations.\textsuperscript{18} In the case of monopsony power, the choice of trade instrument of the policy active importer is irrelevant in a certain world. However, under a random production shock, the equivalence between an import quota and import tariff breaks

\textsuperscript{17} It may have been more appealing in the monopsony game to introduce the source of uncertainty into foreigners' cost function as the large country may have less information about the production schedule of foreigners than about his own production schedule. However, the qualitative results are invariant with respect to the country specific source of uncertainty in this game. We introduce uncertainty into the importer's domestic production only for internal consistency throughout the chapter.

\textsuperscript{18} In a model without uncertainty, qualitative results are similar using either a partial equilibrium or general equilibrium framework as long as the import good is a normal good. However, with uncertainty, using a partial equilibrium framework is equivalent to assuming that agents in the model are risk neutral.
down. We restrict our analysis to the import quota game since it is the instrument modeled in the oligopsonistic world market structure in the next section.

Denote the world price and imports by $\bar{p}$ and $m$, respectively. Importer's welfare is the sum of consumer surplus, producer surplus and the auctioned quota licenses, which can be rewritten as:

$$W = \int_0^d p(y)dy - C(q, \varepsilon) - \bar{p}m$$

(4.1)

where $p(d)$ and $C(q, \varepsilon)$ are the inverse demand of domestic consumers and the cost function of domestic producers respectively. The importer's objective is to maximize (4.1) such that $Q(p) - D(p) = m$ and $d = q + m$, where $\bar{X} = \bar{Q} - \bar{D}$, $\bar{Q} = \delta + \gamma \bar{p}$ and $\bar{D} = \alpha - \beta \bar{p}$ are exports, foreign supply and foreign demand respectively. Foreigners are assumed to pursue a free trade policy.

For simplicity, assume the marginal cost of producers is linear in the random shock. The cost function is quadratic in output: $C(q, \varepsilon) = q^2/2g + cq/g - q\varepsilon/g$. Therefore, we have: $C_q = -q/g < 0$ and $C_{qe} = -1/g < 0$. Domestic producers behave competitively and the aggregate domestic supply is: $q = -c + gp + \varepsilon$. Consequently, a positive realization of the random shock $\varepsilon$ decreases total cost and marginal cost of domestic producers. Assume the distribution of $\varepsilon$ has mean zero and variance $\sigma^2$. In the precommitment case, the importer maximizes expected welfare in (4.1). The first order condition is:

$$\partial E[W]/\partial m = E[(p - \bar{p}) - m/(\beta + \gamma)] = 0$$

(4.2)

since $p = C_q$ absent any distortion on the domestic market. The world price is determined according to the equilibrium on the world market: $\bar{p} = [(\alpha - \delta) + m]/(\beta + \gamma)$. Preferences of
domestic consumers are quasi-linear: \( U(w, x) = w + \frac{a}{b} x - \frac{x^2}{2b} \), where \( w \) is a numéraire good.

Hence, the domestic demand of good \( x \) is: \( d = a - bp \).

The domestic price is determined from the equilibrium condition on the domestic market: \( p = \frac{(a + c) - m - \varepsilon}{b + g} \). After appropriate substitutions, the first order condition in (4.2) becomes:

\[
\frac{\partial E[W]}{\partial m} = (a + c)(\beta + \gamma) - (\alpha - \delta)(b + g) - m[2(b + g) + (\beta + \gamma)] = 0
\]  

(4.3)

Defining \( \lambda = (\beta + \gamma) /[2(b + g)] \), \( \alpha' = \alpha/2 \) and \( \delta' = \delta/2 \), we solve (4.3) to get the optimal precommitment import quota:

\[
m^e = \frac{(a + c)\lambda - (\alpha' - \delta')}{1 + \lambda}
\]  

(4.4)

Assume the government does not irrevocably commit itself to its \textit{ex-ante} trade policy. In other words, after production decisions are made and the random shock observed, it can revise its import quota. The government maximizes (4.1) subject to the equilibrium conditions on the world market and domestic market: \( \bar{Q} - \alpha + \beta \bar{p} = m \) and \( a - bp - q = m \) respectively. Production levels are predetermined. The first order condition is:

\[
\frac{\partial W}{\partial m} = (a - q - m) / b - (\alpha - \bar{Q} + m) / \beta - m / \beta = 0
\]  

(4.5)

The production levels \( \bar{Q} \) and \( q \) in (4.5) depend on the price expectations of producers. Because producers know that the government did not irrevocably commit itself to its \textit{ex-ante} import quota in the first stage of the game, producers adjust their production accordingly. Therefore, with perfect foresight, \( \bar{p}^* = \bar{p} \) and \( p^* = p \) yield the time consistent solution. Equation (4.5) can be rewritten as:
\[(a-c-s)(\beta + \gamma) - (a - \delta)(b + g)\]  
\[= m[\beta(b + g + \beta + \gamma) + (b + g)(\beta + \gamma)] \quad (4.6)\]

Solving (4.6) yields the optimal time consistent quota without commitment:

\[m^* = \frac{[(a+c-s)\lambda - (a'-s')]}{(1+\lambda)+(1-\bar{\mu})/2\bar{\mu}} \quad (4.7)\]

where \(\bar{\mu} = \beta/(\beta + \gamma)\) is the relative foreign demand responsiveness. The time consistent quota and precommitment quota in (4.4) and (4.7) have to be compared in terms of their expected welfare. Substituting for \(p\), \(C(q,\varepsilon)\) and \(\bar{p}\) in (4.1) yields the following welfare function:

\[W = K + \frac{1}{(b + g)} \left[ \frac{Am}{\lambda} \left( \frac{1+\lambda}{2\lambda} \right) m^2 + \frac{\mu}{2(1-\mu)} \varepsilon^2 \right] - \frac{\varepsilon m}{1-\mu} \left( a + c\bar{\mu} \right) \quad (4.8)\]

where \(\lambda \equiv (a+c)\lambda - (a'-\delta')\), \(K = a^2/2b + c^2/2g - (a+c)/2(b+g)\) and \(\mu = b/(b+g)\).

Denote by \(E[W^p]\) and \(E[W^n]\) the expected welfare under precommitment and no commitment respectively. Using (4.4) and (4.7) to substitute back into (4.8), we have:

\[E[W^p] = K + \frac{1}{2(b+g)} \left( \frac{A^2}{\lambda(1+\lambda)} + \frac{\mu\sigma^2}{(1-\mu)} \right) \quad (4.9)\]

\[E[W^n] = K + \frac{(1+\bar{\mu})}{2(b+g)} \left[ \frac{4\bar{\mu}A^2}{\lambda[1+(1+2\lambda)\bar{\mu}]^2} + \frac{4\bar{\mu}\lambda\sigma^2}{[1+(1+2\lambda)\bar{\mu}]^2} + \frac{\mu\sigma^2}{(1+\bar{\mu})(1-\mu)} \right] \quad (4.10)\]

From (4.9) and (4.10), we have \(E[W^p] > E[W^n]\) as:

\[\frac{\sigma^2}{A^2} > f(\bar{\mu},\lambda) \equiv \frac{(\bar{\mu} - 1)^2}{16(1+\lambda)\bar{\mu}^2[1+\lambda\bar{\mu}]} \quad (4.11)\]

From (4.11), it is straightforward to see that \(f_{\bar{\mu}} < 0\). As the parameter \(\bar{\mu}\) increases (decreases), the expected welfare gain from precommitting decreases (increases) relatively to
the no-commitment solution. This is to be expected since a high (low) value of $\bar{\mu}$ implies that the precommitment and no-commitment residual foreign export supply curves are similar (different).

Note that in the full information case (i.e. when government observes the random shock before moving in the first stage of the game), the expected welfare is given by:

$$E[W^p]_{FI} = K + \frac{1}{2(b+g)} \left\{ \frac{A^2}{\lambda(1+\lambda)} + \frac{\mu\sigma^2}{(1-\mu)} + \frac{\lambda\sigma^2}{(1+\lambda)} \right\}$$

(4.12)

Hence, we have $E[W^p]_{FI} > E[W^p]$. Expected welfare is an increasing function of the amount of information available. We have $E[W^p]_{FI} > E[W^n]$, and thus the precommitment equilibrium is preferred to the no-commitment solution under full information. Introduction of uncertainty is not sufficient to reverse the ranking between the precommitment and no-commitment solution. Agents' information sets have to differ along the game tree.

To illustrate the implications of (4.11), assume the production shock is distributed uniformly on the interval $[-A\theta, A\theta]$. The mean of the shock is zero and its variance is $\sigma^2 = \theta^2 A^2/3$. We have $E[W^p] \geq E[W^n]$ as:

$$\theta \geq \left[ \frac{3(\bar{\mu}-1)^2}{16(1+\lambda)\bar{\mu}^2[1+\lambda\bar{\mu}]} \right]^{1/2}$$

(4.13)

Figure 4.1 plots the value of $\theta$ with respect to the parameter $\bar{\mu}$ for which the equality in (4.13) strictly holds given a value of $\lambda$. Any value of $\theta$ located below (above) the solid line for a given value of $\lambda$ and $\bar{\mu}$ implies that the expected welfare associated with precommitment is higher (lower) than the expected welfare of not committing.
Figure 4.1 Expected welfare with precommitment and no-commitment in terms of the production disturbance variance and the relative foreign demand responsiveness.

4.4 — Endogenous Precommitment and Quota Competition without Uncertainty

In chapter two, we have shown that if \( N \) policy active importers behave non-cooperatively \((N > 1)\), the time consistent \textit{ex-post} equilibrium may be welfare superior to the \textit{ex-ante} precommitment equilibrium. The intuition is that the welfare effect of a lower \textit{ex-post} foreign export supply elasticity is balanced off by a higher foreign export supply elasticity due to the instrument competition among importers. In this section, we examine the impact of sequential play among importers.
Consider a partial equilibrium model. There are 2 policy active importers with purchasing power on the world market for a certain good. We restrict the strategy space of importers to import quotas. Consider the following four-stage game. In stage one, each country decides whether to commit to its trade policy or whether to retain flexibility to make its trade policy decision after production decisions are known. The decision by each importing country is publicly observed. For now, we assume there is no uncertainty and full information. In the next stage, if either country has decided to precommit, it then chooses its trade policy. In stage three, foreign and domestic producers make their production decisions based on price expectations. If both countries have committed to their trade policy, then nothing can be changed in the following stage. If only one country has precommitted to its policy, then the other country chooses its trade policy given the import quota of the leader. If neither country precommitted to its policy in stage one, then both choose their trade policy simultaneously. Finally, consumption and trade decisions are carried out.

Domestic demand in each importing country is \( d_i(p_i) \) where \( p_i \) is the domestic consumer price. Domestic production is denoted by \( q_i \). Importer's welfare is the sum of consumer surplus, producer surplus and the auctioned quota licenses. The welfare function is:

\[
W_i = \int_{0}^{d_i} p_i(y_i) dy_i - c(q_i) - \bar{p}m_i
\]  

(4.14)

The objective of each importer is to maximize (4.14) such that \( \bar{X} = \bar{O}(\bar{p}^*) - \bar{D}(\bar{p}) = \sum_j m_j \) and \( d_i = q_i + m_i \), where \( \bar{X}, \bar{O} \) and \( \bar{D} \) are the foreign exports, foreign supply and foreign demand respectively and \( m_i \) represents imports of country \( i \). We
work with a linear model and thus $D'' = d'' = 0$. Equilibrium on the world market gives rise to an inverse foreign export supply, $\tilde{p}(\tilde{X}, \tilde{Q})$. Differentiate the welfare function in (4.14) with respect to $q_t$, $m_i$, and $\tilde{p}$ to get:

$$dW_i = [p_i(d_j) - c'(q_j)]dq_i + (p_t - \tilde{p})dm_i - m_i d\tilde{p}$$

(4.15)

The term $[p_i(d_j) - c'(q_j)]$ is equal to zero absent any domestic distortions in production. Each importer has two options. It must decide whether to precommit to a trade policy level before production and consumption decisions are made or whether to wait and delay its trade policy after production decisions are known.

4.4.1 Simultaneous play

It is useful to restate the solutions of the precommitment game and the time consistency game of chapter two. The domestic price is determined by the equilibrium on the domestic market: $m_i = d_i(p_i) - q_i(p_i^*)$, where $p_i^*$ is used to determine producers' expectations of the trade policy. In the precommitment game, $p_i = p_i^*$ and $\tilde{p} = \tilde{p}^*$. Hence, $\tilde{Q} = S(\tilde{p})$ and $q_j = s_j(p_j)$. The ex-ante residual foreign export supply faced by country $i$ is defined by: $\tilde{S}(\tilde{p}) - D(\tilde{p}) - m_i - m_i = 0$. Differentiating the residual export supply curve given the other importer's quota gives: $(\tilde{S}' - D')d\tilde{p} - dm_i = 0$. Use the latter equation to substitute into (4.15) for $d\tilde{p}$. We get the implicit reaction function of country $i$:

$$\partial W_i / \partial m_i = (p_i - \tilde{p}) - m_i / (\tilde{S}' - D') = 0$$

(4.16)

Solving (4.16) yields: $m_i = f(m_j)$ for $i = 1, 2$ and $j \neq i$. The intersection of both countries' reaction function gives the Nash equilibrium precommitment quota, denoted
The first superscript letter indicates whether country 1 is assumed to have precommitted \((p)\) or not \((n)\) to its trade policy. The second superscript letter indicates country 2’s timing of move. With precommitment and linear supply and demand schedules, the slope of each country’s reaction function is derived from (4.16):

\[
\frac{\partial m_j^i}{\partial m_i} = \frac{d_j^i - s_j^i}{S^i - D^i - 2(d_j^i - s_j^i)} < 0 \quad i, j = 1, 2; i \neq j
\]  

(4.17)

Turning to the no-commitment game, assume that both countries can revise costlessly the quota set at the beginning of the game after production decisions are made. Due to some exogenous factor, we assume that they can not (or do not want to) precommit to their ex-ante trade policy. The residual foreign export supply curve faced by country \(i\) once production decisions are made is: \(Q - D(p) - m_j - m_i = 0\). Differentiate the latter expression given the quota of country \(j\) to get: \(-D'dp - dm_i = 0\). The first order condition implicitly defines the reaction function of country \(i\):

\[
\frac{\partial W_i}{\partial m_i} = p_i - \bar{p} + m_i / D' = 0
\]  

(4.18)

Solving (4.18) yields the no commitment quota reaction function \(m_i = g(m_j, \bar{O}, q_i)\), \(i = 1, 2, j \neq i\). To derive the time consistent equilibrium, we impose perfect foresight such that \(\bar{p}' = \bar{p}\) and \(p_i^* = p_i\). The subgame perfect equilibrium (SPN) of the no-commitment game is denoted by: \(m_1^{\text{nn}}\) and \(m_2^{\text{nn}}\).

We proved in proposition 2.3 that \(m_1^{\text{nn}} < m_1^{pp}\). The welfare ranking between the two equilibria is however ambiguous. Generally speaking, a large foreign demand elasticity and/or a small foreign supply elasticity increases the likelihood that the inability to
precommit to the *ex-ante* import quota is welfare improving from both policy active importers' perspective.

4.4.2 Sequential play

In the previous section, we assumed that both policy importers move simultaneously. In this section, we assume that due to some exogenous factor, one country moves before production decisions are made and irrevocably commits to its import quota while the other has the opportunity to revise its import quota once production decisions are made. Without loss of generality, assume country 2 does not precommit in the first stage and is allowed to revise its quota after production decisions are made. This makes country 2 the follower and country 1 the leader. Conversely, assume country 1 irrevocably precommits to its import quota at the beginning of the game. We solve the game backwards. At the last stage of the game, production levels of foreigners and importers are known along with the import quota of country 1.

Country 2 maximizes (4.14) such that \( \overline{Q} - D(p) = m_1 + m_2 \). Differentiate the *ex-post* residual export supply faced by country 2 taking as given country 1's quota and foreigners' output: \( \overline{D}' d\overline{p} - dm_2 = 0 \). Substitute for \( d\overline{p} \) in (4.15) to get:

\[
\frac{\partial W_z}{\partial m_2} = (p_2 - \overline{p}) + m_2 / \overline{D}' = 0 \tag{4.19}
\]

Equation (4.19) implicitly defines the reaction function of country 2, \( g_2(m_1, \overline{Q}, q_2) \). Totally differentiate (4.19) to get:

\[\text{Note that our game is not a typical Stackelberg leader of game due to the difference in the residual export supply faced by the leader and follower. Hence, it is not obvious that a country will prefer to lead than being a follower in this quantity game.}\]
\[(1/d'_1 + 2/D')dm_2 + (2/D')dm_1 - (1/D')d\overline{Q} + (1/d'_2)dq_2 = 0 \quad (4.20)\]

Holding \(\overline{Q}\) and \(q_2\) constants, we have: \(\partial m_2/\partial m_1|_{\overline{Q}, q_2} = -d'_2/(D' + 2d'_2) = -\alpha_2/(1 + 2\alpha_2)\). However, at the time consistent equilibrium, we have: \(d\overline{Q} = \overline{S}'d\overline{p}\) and \(dq_2 = \overline{s}'d\overline{p}_2\). Therefore, the effective\(^{20}\) slope of the ex-post reaction function of country 2 is:

\[
\frac{\partial m_2}{\partial m_1} = \frac{-\overline{\mu}\alpha_2}{\mu_2 + (1 + \overline{\mu})\alpha_2} = \frac{-\alpha_2}{1 + 2\alpha_2 + 1/\overline{\mu}((\mu_2 - \overline{\mu}) + (1 - \overline{\mu})\alpha_2)} < 0 \quad (4.21)
\]

where \(\alpha = d'_1/D'\), \(\overline{\mu} = -D'/(\overline{S}' - D')\) and \(\mu = -d'/s_1 - d'_1\). The residual export supply faced by country 1 is: \(\overline{S}(\overline{p}) - \overline{D}(\overline{p}) - g_2(m_1, \overline{Q}, q_2) - m_1 = 0\). Differentiate to obtain:

\[(\overline{S}' - D')d\overline{p} - g_2'dm_1 - dm_1 = 0\]. Using (4.21), substitute the previous equation into (4.15) for \(d\overline{p}\) to get:

\[
\frac{\partial \overline{W}_1}{\partial m_1} = (p_1 - \overline{p}) - \frac{m_1}{(\overline{S}' - D')}
\left[\frac{\alpha_2 + \mu_2}{(1 + \overline{\mu})\alpha_2 + \mu_2}\right] = 0 \quad (4.22)
\]

Solving (4.22) yields for the equilibrium import quota of country 1, denoted \(m_1^{eq}\).

Plugging back the optimal quota of country 1 in country 2's reaction function yields:

\(m_1^{eq} = g_2(m_1^{eq})\). For further reference, evaluate the first order condition of both importers in (4.19) and (4.22) at \(m_1 = m_1^{eq}\):

\[
\frac{\partial \overline{W}_1}{\partial m_1}_{m_1=m_1^{eq}} > 0; \quad \frac{\partial \overline{W}_2}{\partial m_2}_{m_1=m_1^{eq}} = 0 \quad (4.23)
\]

\(^{20}\) In other words, (4.21) is the slope of country 2's reaction function from country 1's perspective since it chooses \(m_1\) when production levels are not predetermined. Hence, country 1 recognizes the impact of its policy on production levels of the other importer and foreigners.
Proposition 4.1: Assume two symmetric policy active importers and no uncertainty in domestic production. In the Subgame Perfect Nash (SPN) equilibrium: (i) Country 1’s precommitment to its import quota if country 2 does not commit yields larger equilibrium imports in country 1 than if both importers had moved simultaneously before or after production decisions were made. Moreover, country 2’s imports in equilibrium are lower than the equilibrium quota under both simultaneous precommitment and no-commitment.

From symmetry we have: $m_1^{pn} > m_1^{pp} > m_1^{mn} > m_1^{pp}$ and $m_2^{np} > m_2^{pp} > m_2^{mn} > m_2^{pp}$.

Proof: From (4.23), the ability to precommit to its trade policy shifts country 1’s reaction function to the right with respect to its ex-post reaction function. The equilibrium will be along country 2’s ex-post reaction function. It implies $m_1^{pn} > m_1^{mn}$ and $m_2^{pn} < m_2^{mn}$ since both reaction function are negatively sloped. From symmetry, we also have: $m_1^{np} < m_1^{mn}$ and $m_2^{np} < m_2^{mn}$. Evaluate the first order conditions in (4.19) and (4.22) at the solution $m_1^{pp}$:

$$\frac{\partial W_1}{\partial m_1} \bigg|_{m_1=m_1^{pp}} = -\frac{m_1^{pp}}{S' - D'} \left[ \frac{\mu \alpha_2}{(1 + \mu) \alpha_2 + \mu_2} \right] > 0$$

(4.24)

$$\frac{\partial W_2}{\partial m_2} \bigg|_{m_1=m_1^{pp}} = \frac{m_2^{pp}(1 - \mu)}{D'} < 0$$

(4.25)

Equations (4.24) and (4.25) indicate that in case country 1 precommits to its import quota while country 2 does not, the reaction function of country 1 is shifted outward and country 2’s reaction function is shifted downward with respect to the simultaneous precommitment reaction functions. Since reaction functions are negatively sloped, we have: $m_1^{pn} > m_1^{pp}$ and $m_2^{pn} < m_2^{pp}$. Q.E.D.
Since the residual foreign export supply elasticity is higher *ex-ante* than *ex-post*, country 1 has an incentive to import more if it precommits to its quota before production decisions are made. Moreover, since country 1 moves before importer 2 and that imports are strategic substitutes, it has an incentive to increase its imports in order to decrease the imports of country 2. The two effects reinforce each other.

**Proposition 4.2:** Assume two symmetric policy active importers and no uncertainty in domestic production. In the first stage of the game, each importer must decide whether to commit to its *ex-ante* import quota or keep the flexibility to revise its quota once production decisions are made. The Subgame Perfect Nash (SPN) equilibrium of this game entails precommitment by both countries.

**Proof:** Consider the case where country 1 commits and country 2 does not precommit. Country 1 chooses its preferred point on the *ex-post* reaction function of country 2. Thus, the SPN equilibrium under simultaneous no-commitment is feasible for the leader. Since the simultaneous no-commitment equilibrium differs from the equilibrium under which country 1 precommits, it must be true that country 1 does strictly better as a leader (\(W_i^{pn} > W_i^{pm}\)). Now consider the case where country 1 does not commit and country 2 precommits. From proposition 4.1, we have that \(m_1^{pp} < m_1^{pp}\) and \(m_2^{pp} > m_2^{pp}\). Let \(\Delta[a,b]\) denote the change in welfare between equilibrium \(a\) and equilibrium \(b\). Therefore, the welfare comparison between \(W_i^{pp}\) and \(W_i^{np}\) is:

\[
\Delta[W_i^{pp}, W_i^{np}] = \Delta[W_i^{pp}, W_i(R_i^p(m_2^{np}), m_2^{np})] + \Delta[W_i(R_i^p(m_2^{np}), m_2^{np}), W_i^{np}]
\] (4.26)
The first term between brackets on the right hand-side of (4.26) describes a move up the reaction function of country 1. Totally differentiate (4.14) to get
\[ \frac{\partial W_1}{\partial m_2} = -m_1/(S' - D') < 0 \] since, as we move along country 1’s reaction function,
\[ p_1 - \bar{p} - m_1/(S' - D') = 0. \]
Hence, since \( m_2^{\text{pp}} > m_2^{\text{pp}} \), the first term (4.26) is negative. The second term on the right hand-side of (4.26) implies \( m_2 \) is held constant as we decrease \( m_1 \) and so \( \frac{\partial W_1}{\partial m_1}|_{m_2} = (p_1 - \bar{p}) - m_1/(S' - D') > 0 \) from the second order condition. Since both terms in (4.26) are negative, it must be that \([W_1^{pp}, W_1^{\text{pp}}]\) is negative; hence \( W_1^{pp} > W_1^{\text{pp}} \).
From symmetry, we have that precommitting is a dominant strategy for both importers and thus the SPN equilibrium entails precommitment for both countries. Q.E.D.

Figure 4.2 clarifies proposition 4.2. Point \( A \) is the simultaneous precommitment solution, as the intersection of the \textit{ex-ante} reaction function (solid lines) yields the SPN equilibrium of this game. Following (4.24) and (4.25), both reaction functions shift appropriately to yield the SPN equilibrium of the \([n, p]\) game at the intersection of the dotted reaction functions (point \( C \)). Since the movement from \( A \) to \( C \) entails a move along country 1’s reaction function (\( A \) to \( B \)), plus an horizontal movement holding \( m_2 \) constant (\( B \) to \( C \)), country 1 is located on a higher iso-welfare curve in the \([n, p]\) than in the \([p, p]\) equilibrium. Therefore, we have: \( W_1^{pp} - W_1^{\text{pp}} > 0 \).
4.5 - Endogenous Precommitment under Production Uncertainty

In this section, we use a stylized model to introduce uncertainty. Suppose domestic preferences of a representative consumer in country $i$ are represented by a quasi-linear utility function: $U(w_i, x_i) = w_i + \frac{a}{b} x_i - \frac{x_i^2}{2b}$, where $w_i$ is a numéraire good. These preferences imply the domestic demand ($d_i$) for good $x_i$ is: $d_i = a - b p_i$, where $a$ and $b$ are positive constants and $p_i$ is the domestic price in country $i$. Domestic producers of the importable good in country $i$ have the following cost function: $c(q_i, \epsilon_i) = \frac{q_i^2}{2g} + \frac{c q_i}{g} - \frac{\epsilon_i q_i}{g}$, where $\epsilon_i$ is a country specific random shock with mean zero and variance $\sigma_i^2$. A positive realization of $\epsilon_i$ decreases the
marginal cost of producers in country \( i \). For example, the randomness may be due to unknown weather conditions at the time each government sets their import quota level.

Competitive domestic markets imply the domestic supply function is: \( q_i = -c + g p_i + \varepsilon_i \), where \( g \) is a positive constant.

The welfare function for country \( i \) is the sum of consumer surplus, producer surplus and government revenue:

\[
W_i = \int_0^{q_i + m_i} \left( \frac{a - y_i}{b} \right) dy_i - c(q_i) - \bar{p}m_i = \frac{a}{b} d_i - \frac{d_i^2}{2b} - \frac{q_i^2}{2g} + \frac{c}{g} q_i \varepsilon_i q_i - \bar{p}m_i
\]

Each importer \( i \) maximizes (4.27) such that \( d_i = q_i + m_i \) and \( m_1 + m_2 = \bar{X} = \bar{O}(p^*) - \bar{D}(\bar{p}) \),

where \( \bar{O}(p^*) = \delta + \bar{p}p^* \) is the foreign supply and \( \bar{D}(\bar{p}) = \alpha - \beta \bar{p} \) is the foreign demand.

4.5.1 Simultaneous Play

First, we solve the simultaneous precommitment game. In this case, importers move before production decisions are made; hence they do not observe the random shock. They maximize expected domestic welfare in (4.27) given the other importer's quota. The first order condition of importer \( i \) is: \( E[p_i - \bar{p} - \bar{p}'m_i] = 0 \), where \( \bar{p}' \) is the derivative of the inverse ex-ante foreign export supply with respect to imports in country \( i \). The quota reaction function of country \( i \) is implicitly defined by:

\[
E \left[ \frac{a + c - m_i - \varepsilon_i}{b + g} - \frac{m_i}{\beta + \gamma} - \frac{m_2 + (\alpha - \delta)}{\beta + \gamma} \right] = 0, \quad i = 1, 2
\]
Denote the equilibrium import quotas of the simultaneous precommitment game by $(m_{1}^{pp}, m_{2}^{pp})$. Since the random production shock in both countries has mean zero, solving (4.28) yields:

$$m_{i}^{pp} = \frac{2A}{2(1+\lambda)+1}, \quad i = 1, 2$$  \hspace{1cm} (4.29)

where $\overline{\mu} = \beta/\gamma$, $\lambda = (\beta + \gamma)/2(b + g)$, $A = [(a+c)\lambda - (\alpha' - \delta')]$, $\alpha' = \alpha/2$ and $\delta' = \delta/2$. The parameter $\overline{\mu}$ can be interpreted as the relative slope of the foreign demand with respect to the slope of the ex-ante foreign export supply.

Assume that both policy active importers can not commit to their ex-ante import quota. They can revise the quota set at the beginning of the game once production decisions are made and that uncertainty has been resolved. Obviously each importer's best response function will depend upon the realization of the production shocks, $\varepsilon_1$ and $\varepsilon_2$. Both importers maximize (4.27) taking as given the other importer’s quota. The first order condition of importer $i$ is:

$$p_i(m_i, q_i) - \overline{p}(X, \overline{D}) + m_i/\overline{D}' = 0.$$  \hspace{1cm} (4.27)

Predetermined production levels are a function of price expectations of producers. Firms correctly anticipate the revision of the import quotas. Hence, with perfect foresight, $p_i^* = p_i$ and $\overline{p}^* = \overline{p}$. Substitute for the world price $\overline{p}$ and the domestic price $p_i$ in the first order condition of country $i$. The implicit ex-post quota reaction function is:

$$\beta(\beta + \gamma)(a + c - m_i - \varepsilon_i) - (b + g)\beta(\alpha - \delta + m_i + m_2) = (b + g)(\beta + \gamma)m_i$$  \hspace{1cm} (4.30)

Solving (4.30) yields the SPN equilibrium of the ex-post simultaneous quota game. Denote this solution by $(m_{1}^{m}, m_{2}^{m})$, where:
Without loss of generality, assume country 2 does not precommit to its import quota while country 1 commits to its import quota before production decisions are made. This strategic game is solved backwards. Country 2 chooses its import quota once production quantities are known and treats the import quota of country 1 as given. The reaction function of country 2 is given by equation (4.30). Rearranging some terms, we have:

\[
m_2 = \frac{2\mu (A - \lambda \epsilon_i)}{1 + 2\mu(1 + \lambda)} - \frac{2\mu^2 \lambda (\epsilon_i - \epsilon_j)}{[1 + 2\mu(1 + \lambda)][1 + 2\lambda \mu]}, \quad i, j = 1, 2, i \neq j
\]  

(4.31)

4.5.2 Sequential play

Country 1 maximizes expected welfare since it does not observe the random disturbances. However, it has full information on the distribution characteristics of the shocks. The first order condition is: 

\[
E\left\{ p_i - \bar{p} - \frac{m_1}{S_i - D_i}[1 + \frac{\partial m_2}{\partial m_1}] \right\} = 0.
\]

Solving the previous equation yields:

\[
m_1^{\text{en}} = \frac{(1 + 2\lambda \mu) A}{\mu \lambda + (1 + 2\mu \lambda)(1 + \lambda)}
\]  

(4.33)

Plugging back (4.33) into (4.32) to substitute for \( m_2 \) gives the equilibrium quota of country 2 in the sequential game:

\[
m_2^{\text{en}} = \frac{\mu A [2\lambda (1 + 2\mu \lambda) + 4\mu \lambda + 1]}{[1 + 2\mu(1 + \lambda)][\mu \lambda + (1 + 2\mu \lambda)(1 + \lambda)]} - \frac{2\mu \lambda \epsilon_2}{[1 + 2\mu(1 + \lambda)]}
\]  

(4.34)
As a benchmark to compare welfare levels between the different subgame perfect equilibria, we need to rewrite the welfare function in (4.27) in terms of import quotas. In the first stage of the game, each country simultaneously decides to irrevocably precommit to its import quota or keep the flexibility to revise its quota once production decisions are made and the random production shocks observed. The decision to precommit at the beginning of the game is made by comparing the expected welfare levels associated with the various equilibria. Substituting for the cost function, the world price and domestic price in (4.27), the expected welfare function is:

\[ E[W_i] = K + \frac{1}{(b + g)} \left[ \frac{Am_i}{\lambda} - \frac{(1 + \lambda)}{2\lambda} m_i^2 - \frac{m_i}{2\lambda} \right] + \varepsilon_i m_i + \varepsilon_i \left( a + \frac{c \mu}{(1 - \mu)} \right) + \frac{\mu}{(1 - \mu)} \varepsilon_i^2 \]  

We restrict the parameter \( \lambda \) to equal \( \frac{1}{2} \). It implies that the slope of the import demand is equal in absolute value to the slope of the foreign export supply. However, the particular slopes of the demand and supply function may differ between importers and exporters. This assumption does add a little more structure to the model, but does not change any of the qualitative results. It is introduced to simplify the solutions of the various equilibria. Moreover, assume the two random shocks are identically and independently distributed (i.i.d.). Hence the covariance \( (\sigma_{12}) \) between the two shocks is zero.

Substituting (4.29) into (4.35) for the precommitment SPN import quota, the expected welfare is:

\[ E[W_i^{SPN}] = K + \frac{1}{2(b + g)} \left[ \frac{3A^2}{4} + \frac{\mu}{(1 - \mu)} \sigma_i^2 \right] \]
In case country 1 precommits to its import quota while country 2 does not, using (4.33) and (4.34), the expected welfare of country 1 and 2 are respectively:

\[
E[W_{1m}] = K + \frac{2A^2(1+\mu)(1+\bar{\mu})(3/4 + \bar{\mu})}{(b + g)(3 + 4\mu)^2(1 + 2\mu)} + \frac{\mu}{2(b + g)(1 - \mu)} \sigma_1^2
\]  

(4.37)

\[
E[W_{2m}] = K + \frac{2\mu A^2(2 + 3\mu)^2(2 + \bar{\mu})}{(b + g)(3 + 4\mu)^2(1 + 2\mu)^2} + \frac{\bar{\mu}(2 + \mu)\sigma_2^2}{2(b + g)(1 + 2\mu)^2} + \frac{\mu\sigma_1^2}{2(b + g)(1 - \mu)}
\]  

(4.38)

Finally, the expected welfare of country 1 in case both importers do not commit to their ex-ante import quota is:

\[
E[W_{1m}^m] = K + \frac{1}{(b + g)} \left[ \frac{2\mu A^3(2 + \bar{\mu})}{(1 + 3\mu)^2} + \frac{\bar{\mu}\sigma_1^2[4\bar{\mu}^3 + 12\bar{\mu}^2 + 9\bar{\mu} + 2]}{2(1 + 3\mu)^2(1 + \bar{\mu})^2} \right.

\left. + \frac{(2 + \bar{\mu})\mu^3\sigma_2^2}{2(1 + 3\mu)^2(1 + \mu)^2} + \frac{\mu\sigma_1^2}{2(1 - \mu)} \right]
\]  

(4.39)

Since the only source of asymmetry among the two importers is due to the specific random shock in production, equations (4.37) through (4.39) allow to solve for \(E[W_{2m}]\), \(E[W_{1m}^m]\) and \(E[W_{2m}^m]\) respectively by appropriately replacing \(\sigma_1^2\) with \(\sigma_2^2\) and vice-versa.

4.5.3 Equilibria of the game

Recall that in the first stage of the game, both importers decide non-cooperatively whether or not to irrevocably commit to their ex-ante import quota. In order to build a payoff matrix for this subgame, we need to compare the expected welfare levels associated with all possible commitment decisions of importers.

In order to rank the expected welfare levels, we introduce a few more assumptions. Assume \(\epsilon\) is uniformly distributed with mean zero on the interval: \([-\theta, \theta, \theta, \theta]\). The
variance of \( \varepsilon \) is then: \( \sigma_i^2 = \theta^2 \tilde{A}^2 / 3 \). To facilitate the treatment of asymmetry in risk, suppose that \( \theta_1 = \theta \) and \( \theta_2 = \phi \theta_1 = \phi \theta \), with \( \phi > 0 \). Consequently, we have: \( \sigma_1^2 = \theta^2 \tilde{A}^2 / 3 \) and \( \sigma_2^2 = \phi^2 \theta^2 \tilde{A}^2 / 3 \).

As we have seen in proposition 2.3, without uncertainty, the welfare levels between simultaneous precommitment and no-commitment can not be ranked a priori. Clearly, uncertainty creates an additional incentive not to commit and keep the flexibility to revise its \textit{ex-ante} import quota since additional information can be revealed to policy makers. Using (4.36) and (4.39), we have that \( E[W_{1}^{mn}] \geq E[W_{1}^{pp}] \) as:

\[
\begin{align*}
\theta^2 & \geq \frac{3(1 + \overline{\mu})^2 (3 - 14 \overline{\mu} + 11 \overline{\mu}^2)}{[4 \overline{\mu}(3 + 4 \overline{\mu})(2 + 9 \overline{\mu} + 12 \overline{\mu}^2 + 4 \overline{\mu}^3 + \phi^2 \overline{\mu}^2 (2 + \overline{\mu})]}
\end{align*}
\]

(Note that the polynomial \( 3 - 14 \overline{\mu} + 11 \overline{\mu}^2 \) between brackets in (4.40) can be rewritten as \( 11(\overline{\mu} - 1)(\overline{\mu} - 3/11) \). Therefore, if \( \overline{\mu} > 3/11 \), the expected welfare of both importers do not commit to their quota is larger than the expected welfare under simultaneous precommitment. Under this condition, the ranking is independent of the variance of the random shocks since the right hand-side of (4.40) is negative. Similarly, for country 2, we have \( E[W_{2}^{mn}] \geq E[W_{2}^{pp}] \) as:

\[
\begin{align*}
\theta^2 & \geq \frac{3(1 + \overline{\mu})^2 (3 - 14 \overline{\mu} + 11 \overline{\mu}^2)}{[4 \overline{\mu}(3 + 4 \overline{\mu})(\phi^2 (2 + 9 \overline{\mu} + 12 \overline{\mu}^2 + 4 \overline{\mu}^3) + \overline{\mu}^2 (2 + \overline{\mu})]}
\end{align*}
\]

As outlined in the introduction, we want to endogenize the sequence of moves by policy active importers and emphasize the values of \( \theta \) and \( \overline{\mu} \) leading to sequential play among countries.
Proposition 4.2: Consider two policy active importers. They can either precommit to their \textit{ex-ante} import quota before production decisions are made or keep the flexibility to revise their import quota after production decisions are made and the random shocks observed. If the random shocks $\epsilon_1$ and $\epsilon_2$ are distributed uniformly on the interval $[-\theta A, \theta A]$ and $[-\phi\theta A, \phi\theta A]$ respectively with $0 < \phi < 1$:

(i) The SPN equilibrium of the game entails precommitment from both importers if:

$$\theta < \left[ \frac{3(1+2\mu)^2 - 4(2+3\mu^2)}{4(2+\mu)(2+3\mu^2)} \right]^{1/2} \quad (4.42)$$

(ii) under the condition of (4.42),

$$\theta > \left[ \frac{3(1+2\mu)^2(3-14\mu+11\mu^2)}{4\mu(3+4\mu)\left[\phi^2(2+9\mu+12\mu^2+4\mu^3)+\mu^2(2+\mu)\right]} \right]^{1/2} \quad (4.43)$$

implies that both importers enter into a prisoners' dilemma in the equilibrium of the game.

Proof: Without uncertainty ($\theta = 0$), we know precommitting is a dominant strategy for both importers. Using (4.36) and (4.38), we have that: $E[W_{1}^{pp}] > E[W_{1}^{mp}]$ if (4.42) holds.

Precommitting is importer 1's best response given country 2 has precommitted to its quota. Since $0 < \phi < 1$, it must be the case that $E[W_{2}^{pp}] > E[W_{2}^{pp}]$ as well. Hence, the best response of country 2 is to precommit to its \textit{ex-ante} import quota given importer 1 has precommitted; proving claim (i). The inequality in (4.43) comes from (4.36) and (4.39). It implies that $E[W_{2}^{mm}] > E[W_{2}^{pp}]$. Since $0 < \phi < 1$, it must be the case that $E[W_{1}^{mm}] > E[W_{1}^{pp}]$ as well. Both
countries would be better off colluding and agree not to commit to their *ex-ante* import quota, which proves claim (ii). Q.E.D.

Figure 4.3 illustrates the implications of proposition 4.2. The dotted line indicates the values of \( \theta \) for which (4.42) holds with strict equality as a function of the foreigners relative demand responsiveness parameter (\( \bar{\mu} \)).

![Figure 4.3 Combinations of production risk and relative foreign demand responsiveness producing the various equilibria](image)

Figure 4.3 Combinations of production risk and relative foreign demand responsiveness producing the various equilibria
We fixed the value of $\phi$ at 0.5 in figure 4.3. Any combination of $\theta$ and $\bar{\mu}$ located below the dotted line yields a SPN equilibrium with simultaneous precommitment (regions 1, 4 and 6). Any value of $\theta$ and $\bar{\mu}$ located above the solid bold line indicates that $E[W_2^{ep}] > E[W_2^{pp}]$. Therefore, any combination of $\theta$ and $\bar{\mu}$ in regions 4 and 6 implies both countries enter into a prisoner’s dilemma in equilibrium.

**Proposition 4.3:** Consider two policy active importers. They can either precommit to their *ex-ante* import quota before production decisions are made or keep the flexibility to revise their import quota after production decisions are made and the random shocks observed. The random shocks $\epsilon_1$ and $\epsilon_2$ are distributed uniformly on the interval $[-\theta A, \theta A]$ and $[-\phi \theta A, \phi \theta A]$ respectively with $0 < \phi < 1$. The SPN equilibrium entails country 2 committing to its *ex-ante* import quota and country 1 revising its import quota once production decisions are made if:

$$
\frac{3(1+2\mu)^2 - 4(2+3\mu)^2}{4(2+\mu)\mu} < \theta
$$

$$
\frac{6(1+\mu)(3+4\mu)(1+2\mu)(2+3\mu)(2+4\mu)}{\mu(3+4\mu)(1+2\mu)(\phi^2(2+9\mu+12\mu^2+4\mu^3)+\bar{\mu}^4(2+\mu)}}
$$

(4.44)

**Proof:** The first inequality in (4.44) comes from proposition 4.2; hence we have $E[W_2^{ep}] > E[W_2^{pp}]$. Using (4.37) and (4.39), we have that $E[W_2^{ep}] > E[W_2^{sp}]$ if the second inequality holds. Combining the two rankings, country 2’s best response to importer 1 not committing is to precommit to its import quota. Also country 1’s best response to country 2 precommitting is not to commit to its import quota. Q.E.D.
Any combination of \( \theta \) and \( \bar{\mu} \) located above the dotted line in figure 4.3 implies \( E[W_{1}^{\text{np}}] > E[W_{1}^{\text{sp}}] \). Any pair \((\theta, \bar{\mu})\) located below the thin solid line in figure 4.3 implies \( E[W_{2}^{\text{np}}] > E[W_{2}^{\text{sp}}] \). Therefore, regions 2 and 3 indicate the values of \( \theta \) and \( \bar{\mu} \) yielding an asymmetric sequence of move between importers. Since country 1 is facing a higher degree of production risk than country 2, it has an incentive keep the flexibility to revise its quota in order to observe the production shocks in both countries. However, a low value of \( \bar{\mu} \) provides incentives to country 2 to precommit since the \textit{ex-post} and \textit{ex-ante} residual foreign export supply elasticity are different. As increases \( \bar{\mu} \), the relative \textit{ex-post} welfare incentive of precommitting disappears.

**Proposition 4.4:** Consider two policy active importers. They can either precommit to their \textit{ex-ante} import quota before production decisions are made or keep the flexibility to revise their import quota after production decisions are made and the random shocks observed. The random shocks \( a_1 \) and \( a_2 \) are distributed uniformly on the interval \([-\Theta A, \Theta A]\) and \([-\phi \Theta A, \phi \Theta A]\) respectively, with \( 0 < \phi < 1 \). The SPN equilibrium of the game entails both countries not committing to their \textit{ex-ante} import quota if:

\[
\theta > \left[ \frac{6(1 + \bar{\mu})^2(3 + 4\bar{\mu})^2(1 + 2\bar{\mu})(2 + \bar{\mu})\bar{\mu} - (1 + 3\bar{\mu})^2(1 + \bar{\mu})(3 + 7\bar{\mu} + 4\bar{\mu}^2)}{\bar{\mu}(3 + 4\bar{\mu})^2(1 + 2\bar{\mu})(\phi^2(2 + 9\bar{\mu} + 12\bar{\mu}^2 + 4\bar{\mu}^3) + \bar{\mu}^2(2 + \bar{\mu}))} \right]^{1/2}
\]  

(4.45)

**Proof:** The inequality in (4.45) implies that \( E[W_{2}^{\text{nm}}] > E[W_{2}^{\text{np}}] \) from proposition 4.3. Since \( \phi < 1 \), \( E[W_{2}^{\text{nm}}] > E[W_{2}^{\text{np}}] \) implies that \( E[W_{1}^{\text{nm}}] > E[W_{1}^{\text{np}}] \). The best response of country 2
given that country 1 does not commit is to not commit to its import quota since
\[ E[W_2^{mp}] > E[W_2^{np}] \]. A similar argument applies to country 1. Q.E.D.

Any pair \((\theta, \bar{\mu})\) located above the thin solid line in figure 4.3 implies
\[ E[W_2^{mp}] > E[W_2^{np}] \]. Therefore, regions 5 and 6 indicate the values for which \(\theta\) and \(\bar{\mu}\) yields a simultaneous no commitment decision between importers. Note that, from propositions 4.2 and 4.4, there exist two different SPN equilibria in region 6. Hence, there exist values for the parameters \((\theta, \bar{\mu})\) and \((\theta, \bar{\mu})\) for which the SPN equilibrium of the endogenous precommitment game is not unique. However, in this region, the simultaneous precommitment equilibrium is dominated by the simultaneous no-commitment equilibrium.

4.6 – Conclusion

This chapter investigates endogenous leadership in import quotas among two policy importers if production decisions occur before consumption decisions. Each policy active importers can either precommit to its trade policy at the beginning of the game or keep the flexibility to revise its trade policy once production decisions are made. Producers' cost function in both importing countries is subject to a specific random disturbance. Firms make their production decisions under complete information. However, a government may commit to a trade policy before observing the random disturbance.

Committing to its trade policy before production decisions are made may give a strategic leadership welfare advantage. However, the potential precommitment advantage must be weighted against the benefit of not irrevocably committing to its \textit{ex-ante} trade policy...
if additional information is revealed once firms have moved. We consider asymmetries in risk. There exists an equilibrium where one importer prefer not to commit and observed the random shocks while the other country precommits to its \textit{ex-ante} import quota before production decisions are made. Both importers prefer to revise their import quota after production decisions are made given a sufficiently high degree of variance in both production random disturbances.
CHAPTER 5. ENDOGENOUS CHOICE OF TRADE INSTRUMENT UNDER PRODUCTION UNCERTAINTY

5.1 - Introduction

There exists a large body of literature on the (non) equivalence of trade instruments. A wide variety of situations have been advanced to explore the topic. More often than not, uncertainty, various market structures and different optimizing criteria have been used to examine the question. What is most surprising is that researchers have often neglected to endogenize the type of instrument used by policy makers. In other words, the strategy space of governments is usually specified exogenously. Some research efforts on endogenous type of protection have originated from the literature on the political economy of trade protection. It has been somewhat successful (empirically and theoretically) at explaining why certain types of protection are preferred over others.

We have shown in chapter 2 that import tariffs and import quotas are not equivalent instruments in an oligopsonistic world market where countries attempt to capture a terms of trade gain. The purpose of this chapter is to endogenize the countries’ choice of trade instrument. We abstract from any time consistency issue introduced earlier. Therefore, we assume that production and consumption decisions are made simultaneously.

We build a two-stage game. In the first stage, policy active importers simultaneously choose the type of instrument they will be using in the second stage. Given the observed type of instrument the other importer has committed to use, each country chooses the level of its trade policy. This type of game can be rationalized as a repeated international trade game.
where countries argue whether to use tariffs or quotas and then annually decide the level of trade protection non-cooperatively.

We have seen that import quotas are Pareto superior to import tariffs in the case where both importers are symmetric. However, it is generally recognized that tariffs are preferred to import quotas under uncertainty. Therefore, we introduce some uncertainty in the production schedules of each importer through a random disturbance in producers’ cost function. The production uncertainty may reverse the ranking between import tariffs and import quotas in our model. We proceed to endogenize the type of trade instrument used by each importer under risk.

This chapter is organized as follows. The first section presents a review of the literature on the endogenous choice of instruments and on the non-equivalence of trade instruments under uncertainty. The following section solves the equilibrium of the game where importers choose the type of trade policy under certainty. In the next section, we introduce uncertainty in the producers’ cost function in both importing countries. Given asymmetry in production risk, the equilibrium of the endogenous type of protection game may involve importers choosing different instruments. The last section presents concluding remarks.

5.2 - Review of Literature

Cassing and Hillman (1985) consider the endogenous choice between a tariff and a quota in the presence of a domestic monopoly. The choice between the two instruments is made by a policy-maker who maximizes a political support function by trading off the gains to the beneficiary of protection against the penalty inflicted to the losers from protection. If
we assume that the trade policy’s revenue does not enter the policy maker’s calculations, producer rents are higher under a tariff than a quota at any given domestic price. Therefore, governments prefer to intervene with a tariff. Although the foundations of the above model are ad-hoc and based on some non-economic objective, it still provides some insights on how interests groups within an economy can shape up a government’s trade policies.

In a bilateral monopoly framework, Grant and Quiggin (1997) have endogenized the type of tariff used by a large country. They assume that each country has an export supply function subject to a random shock. In equilibrium, the optimal tariff rule is given by the common inverse elasticity rule. However, given the structure of export supply functions, the optimal type of tariff in equilibrium vary from a specific tariff (semi-log linear schedules), to an ad-valorem tariff (constant elasticity schedules) or to a quadratic tariff (linear export schedules). Their model can explain the prevalence of ad-valorem trade taxes in the real world. In their view, it is evidence that export supplies are believed to have a constant price elasticity. Since all countries observe their own shock but not the other countries' shock, the equilibrium of the game is an equilibrium in beliefs. It may not be the case that the export supply schedules literally exhibit constant price elasticity. It may just be that the perception of policy makers is shaped by numerous econometric studies which have used a log-linear specification to estimate export supply schedules.

Singh and Vives (1984) and Cheng (1985) were the first to look at price setting and quantity setting firms in an oligopolistic industry. They note judiciously that those mixed

\footnote{See Vousden (1990) for a normative analysis of the non-equivalence between tariffs and quotas under monopoly in production. For a quota and a tariff yielding the same domestic output, the quota involves a higher price and lower consumption of the good than the tariff. However, for a quota and a tariff yielding the same domestic price, output under a tariff is higher than under a quota; and thus producer rents are higher with a tariff.}
oligopolies are likely to exist in reality. In the case of linear demand and cost functions, they showed that, for each firm, choosing the quantity (price) strategy is superior to the price (quantity) strategy if the goods are substitutes (complements). Under more general assumptions, Sato (1996) proved that mixed duopoly equilibrium prices do not necessarily range within Cournot and Bertrand prices. In the Cournot-Bertrand mixed market, the equilibrium prices for the price-setting firm are higher than for the quantity-setting firm.

The non-equivalence of tariffs and quotas with uncertainty has been extensively discussed in the literature. Dasgupta and Stiglitz (1977) have compared specific tariffs and quotas under uncertainty for a small country constrained to raise a fixed expected tariff revenue. They use expected domestic surplus as their welfare measure. The sources of uncertainty are the foreign price and domestic supply and demand intercepts. Their finding is that tariffs are unambiguously superior to quota. Their analysis is limited by the fact that they used linear demand and supply functions. Young (1979) challenged that result. He showed that the ranking between the instruments can be reversed if the tariff rate has to be large compared to the degree of uncertainty in the world price, domestic demand and supply.

Pelcovits (1977) has compared tariffs and quotas when the two instruments are constrained to yield a fixed level of expected imports. Specific tariffs are superior to quotas in case of random world price. Whether an ad-valorem tariff is superior to a quota is unclear. Under an ad-valorem tariff, the world price movements are reflected in magnified movements in the domestic price. However, Pelcovits has shown that if the demand curve for imports is linear, the ad-valorem tariff is superior to the quota provided that the tariff rate is not too high. Young (1979) has demonstrated that a specific tariff is a superior instrument to a quota given a ceiling on expected imports for a small or large country.
All of the above papers have not explicitly considered risk preferences in their modeling. One notable exception is found in Young and Anderson (1982). The expected surplus measure used in the previous papers is a valid welfare criterion only if marginal utility of income is constant. If the uncertainty arises from abroad and the non-economic constraint is on the average level of imports, quotas will be superior to tariffs if the representative consumer is sufficiently risk averse. Tariffs lead to arbitrage of imports across states of the world. For example, if the world price is high, it restricts imports and imposes a cost on the economy additional to the one associated with the higher world price. If the world price is low, it encourages imports and the country benefits from the positive income effect of a lower world price. But this arbitrage also implies that real income fluctuations are greater under the tariff. If individuals are risk averse, the fluctuations in income reduce the attractiveness of the tariff compared to the quota.

However, all this literature generally suffers from one major flaw. It fails to identify why there is protection in the first place. It has been the usual practice to compare tariffs and quotas under the expected imports criterion or some other non-economic objective. The results should not be interpreted as offering an argument for protection per se. Lapan and Choi (1988) provide the ranking of tariffs and quotas under an import-induced externality for a small country facing foreign price uncertainty and domestic production disturbances. In case the external damage function is linear in imports and the indirect utility function is linear in income, their model reduces to the standard expected welfare analysis described above. In the more general case, quotas are more likely to dominate tariffs when the price elasticity of demand for imports or the elasticity of the marginal external damage function is large.
It should be clear that the use of trade policy in our model is to equate marginal cost (or domestic distortion cost of the policy) and marginal benefit (terms of trade gain) from trade. That is, if a country is large enough to affect the price of the good it buys, it can usually gain by restricting trade below the free trade level. When there is more than one country with purchasing power over a certain good on the world market and there is no uncertainty, in equilibrium, the use of a quota to exercise market power brings a higher domestic welfare than tariffs. This ranking may be reversed under uncertainty.

5.3 — Endogenous Choice of Instrument without Uncertainty

Assume there are two large countries on the world market with purchasing power over the same good ($N = 2$). Production and consumption decisions are made simultaneously by all agents in each country. We assume away the existence of production lags in this chapter. Moreover, let us consider the case of symmetry between the two countries and no uncertainty for the time being. These two assumptions will be relaxed later. The game of endogenous type of protection is as follows. In the first stage, the two countries choose either to use an import quota or an import tariff to restrict trade. Given the observed choice of the other country's type of instrument, each government simultaneously chooses the level of protection in the second stage. This type of game is solved backwards. Proposition 2.1 showed that, given country $j$ responds to country $i$’s policy with the same type of instrument, quotas are preferred to tariffs, i.e. $W_i^{qm} > W_i^{mr}$ for $i = 1, 2$. $W_i^{ab}$ denotes the welfare of country $i$ if country 1 uses instrument $a$ while importer 2 uses instrument $b$, with $a, b = m$ or $r$. Since we extensively refer to the previous notation, table 5.1 summarizes the notation used in this chapter.
For simplicity, we work with a value function defined in terms of imports. Suppose consumers in each importing nation have quasi-linear preferences. We can define the function $V(m) = \arg\max_{\omega} \omega + U(q_i + m) - c(q_i)$, where $m$, $q$, and $\omega$ denote imports, domestic production and consumption of an aggregate export good in country $i$ respectively. By the envelope theorem, we have: $V'(m_i) = p_i$. The quota optimization problem for country $i$, given country $j$ uses an import quota, is:

$$\text{Max } W_i(m_1, m_2) = V_i(m_1) - \bar{p}(m_1 + m_2)m_i$$  \hfill (5.1)

where the inverse foreign export supply $\bar{p}(m_1 + m_2)$ is defined by the world trade equilibrium condition: $m_1 + m_2 = \bar{S}(\bar{p}) - \bar{D}(\bar{p})$.

The first order condition of (5.1) is:

$$V_i' - \bar{p} - m_i\bar{p}' = 0, \quad i = 1, 2$$  \hfill (5.2)
if country \( i \) believes \( j \) uses a quota. Solving simultaneously (5.2) for both importers yields the Subgame Perfect Nash (SPN) equilibrium \((m_1^{em}, m_2^{em})\). For the second order condition to hold we have that: \( V''_i - 2\bar{p}'_i - m_i\bar{p}''_i < 0 \). Assume that import quotas are strategic substitutes: 
\[-\bar{p}' - m_i\bar{p}'' < 0.\]
Therefore, along the reaction function of country \( i \), we have:

\[
\frac{\partial m_i}{\partial m_i} \bigg|_{RF} = \frac{\bar{p}' + m_i\bar{p}''}{V''_i - 2\bar{p}'_i - m_i\bar{p}''_i} < 0, \quad i = 1, 2; \quad i \neq j
\]  

(5.3)

Suppose that both countries are using a specific tariff. Country \( i \) maximizes (5.1) such that \( p_i = \bar{p} + \tau_i \) and \( V'_i = \bar{p}(m_1 + m_2) + \tau_i \), where \( \tau_i \) is a specific tariff on imports in country \( i \). Holding country \( j \)'s tariff constant, the first order condition is:

\[
\frac{\partial W_i}{\partial m_i} = V'_i - \bar{p} - m_i\bar{p}' - m_i\bar{p}' \left( \frac{\partial m_i}{\partial m_j} \bigg|_{\tau_j} \right) = 0, \quad i = 1, 2; \quad i \neq j
\]  

(5.4)

From the envelope theorem we have: \( \tau_j = \kappa_j(m_1, m_2) = V'_j - \bar{p} \). Totally differentiate the previous equation to get: \( d\tau_j = (V'_j - \bar{p}') dm_j - \bar{p}' dm_i \). Setting \( d\tau_j = 0 \) yields:

\[
\frac{\partial m_j}{\partial m_i} \bigg|_{\tau_j} = \frac{\bar{p}'}{V'_j - \bar{p}'} < 0
\]  

(5.5)

Substituting (5.5) into (5.4), if country \( i \) believes \( j \) is using a tariff, we have:

\[
V'_i - \bar{p} - m_i\bar{p}' \left( \frac{V''_i}{V'_j - \bar{p}'} \right) = 0, \quad i = 1, 2; \quad i \neq j
\]  

(5.6)

Solving simultaneously the first order condition in (5.6) for both importers yields the SPN equilibrium \((m_1^{\pi}, m_2^{\pi})\). Evaluate (5.6) at the solution \((m_1^{em}, m_2^{em})\) to get:
Equation (5.7) implies that \((m_1^*, m_2^*) > (m_1^m, m_2^m)\). The last inequality only reaffirms the results of proposition 2.1. The next step is to find the equilibrium policies associated with the mixed tariff and quota competition games.

Suppose, without loss of generality, that country 1 is using a quota and that country 2 is using a tariff. From country 1's perspective, \(\tau_2\) is held fixed when it is optimizing. Conversely, from country 2's perspective, imports of country 1 are fixed when optimizing its objective function. The equilibrium quota and tariff solve the set of first order conditions:

\[
V_1' - \bar{p} - m_1 \bar{p}' \left( \frac{V_2''}{V_2' - \bar{p}'} \right) = 0, \quad \text{if country 1 believes 2 uses a tariff} \tag{5.8}
\]

\[
V_2' - \bar{p} - m_2 \bar{p}' = 0, \quad \text{if country 2 believes country 1 uses a quota} \tag{5.9}
\]

Solving simultaneously (5.8) and (5.9) yields the SPN equilibrium \((m_1^m, m_2^m)\). Evaluating (5.8) and (5.9) at \((m_1^m, m_2^m)\) gives:

\[
\left. \frac{\partial W_1}{\partial m_1} \right|_{(m_1^m, m_2^m)} = -m_1 \bar{p}' \frac{\bar{p}'}{V_1' - \bar{p}'} > 0 \tag{5.10}
\]

\[
\left. \frac{\partial W_2}{\partial m_2} \right|_{(m_1^m, m_2^m)} = 0 \tag{5.11}
\]
**Proposition 5.1:** Consider two symmetric policy active importers. Suppose country 1 restricts trade using an import quota while the second importer restricts trade using an import tariff. In equilibrium, we have that: (i) $m_1^{nr} > m_1^{mm}$ and $m_2^{nr} < m_2^{mm}$; and (ii) $W_2^{nr} < W_2^{mm}$.

**Proof:** From equation (5.11), the equilibrium will be along country 2's reaction function. Equation (5.10) indicates that the reaction function of country 1 is shifted outward. Since we move down importer 2's reaction function, we have: $m_1^{nr} > m_1^{mm}$ and $m_2^{nr} < m_2^{mm}$, proving claim (i). Since the equilibrium is located along country 2's reaction function and that country 1's reaction function is shifted outward, the equilibrium implies a lower welfare for country 2. Totally differentiate (5.1) to get:

$$dW_2|_{RF} = \left(V_2' - \bar{p} - m_2 \bar{p}' \right) dm_2 - m_2 \bar{p}' dm_1$$

(5.12)

Since we are moving along country 2's reaction function, $V_2' - \bar{p} - m_2 \bar{p}' = 0$ from (5.9) and $\frac{\partial W_2}{\partial m_1}|_{RF} < 0$. Hence, we have: $W_2^{nr} < W_2^{mm}$. Note that from symmetry, $W_2^{nr} < W_2^{mm}$ implies $W_1^{mm} < W_1^{mm}$. Q.E.D.

Without additional assumptions, we are not able to say anything about $W_1^{nr}$ and $W_1^{mm}$.

Differentiate (5.1) to get:

$$dW_1 = (V_1' - \bar{p} - m_1 \bar{p}') dm_1 - m_1 \bar{p}' dm_2$$

(5.13)

Using (5.8), we can rewrite (5.13) as:

$$dW_1 = -m_1 \bar{p}' [\theta dm_1 + dm_2]$$

(5.14)
where $\theta^r = \bar{P}'/(\bar{P}' - V''') > 0$ and can be seen as the conjectural variation parameter. The term between brackets in (5.14) has an undetermined sign since $dm_1 > 0$ and $dm_2 < 0$ in the mixed $[m, r]$ game compared to the $[m, m]$ game.

Figure 5.1 explains the intuition behind the results of proposition 5.1. The two solid lines represent the reaction function of both countries given that each believes the other uses an import quota. Point $A$ represents the equilibrium of the $[m, m]$ game. The equilibrium of the mixed $[m, r]$ game is located to the right of $A$ along 2’s reaction function. It is easy to see the equilibrium yields a lower welfare for country 2. In the event the equilibrium is on the segment between points $A$ and $C$, the equilibrium of the $[m, r]$ game results in a higher welfare for country 1 than in the import quota game $[m, m]$. However, nothing prevents the equilibrium to be located to the right of point $C$; thus yielding a lower welfare level from country 1’s perspective.

Given country $i$’s choice of a trade policy level, country $j$ will be indifferent between a tariff and a quota. That is a country’s choice of a tariff or quota strategy does not change its own reaction function but that of its rival. When country 1 optimizes, it faces a more elastic residual foreign export supply holding $\tau_2$ fixed than when $m_2$ is held fixed. A change in $m_1$ also affects $m_2$ in the quota-tariff game. Therefore, country 1 imposes a higher quota when country 2 uses a tariff than when it is using a quota.
Without loss of generality, suppose that country 1 believes country 2 is using a tariff and that country 2 believes country 1 is using a quota. The first order conditions of this problem are given in (5.8) and (5.9). Evaluate the set of first order conditions at the solution $(m_1^*, m_2^*)$:

\[
\frac{\partial W_1}{\partial m_1} \bigg|_{(m_1^*, m_2^*)} = 0 \tag{5.15}
\]

\[
\frac{\partial W_2}{\partial m_2} \bigg|_{(m_1^*, m_2^*)} = \frac{m_2(\bar{p}')}{{V'_1} - \bar{p}'} < 0 \tag{5.16}
\]
**Proposition 5.2**: Consider two symmetric policy active importers. Country 1 restricts trade using an import quota while the second importer restricts trade using an import tariff. In equilibrium, we have that: (i) \( m^q_1 < m^q_2 \) and \( m^q_2 > m^q_1 \); and (ii) \( W^{mr}_1 > W^{mr}_2 \).

**Proof**: From (5.15) and (5.16), the equilibrium of the mixed \([m, \tau]\) game is along country 1's reaction function. Since we are moving down country 1's reaction function, we have that: \( m^q_1 < m^q_2 \) and \( m^q_2 > m^q_1 \), proving claim (i). Since we are moving down country 1's reaction function, importer 1 get a higher welfare level in the mixed \([m, \tau]\) game than in the \([\tau, \tau]\) game, \( W^{mr}_1 > W^{mr}_2 \). From (5.13), since \( V_1' - \bar{p} - m_2 \bar{p}' = 0 \), we have \( dW_1^{mr} = -m_2 \bar{p}' dm_2 > 0 \) since \( dm_2 > 0 \). Hence, \( W^{mr}_1 > W^{mr}_2 \). Note that, from symmetry, we have: \( W^{mr}_2 > W^{mr}_1 \). Q.E.D.

Figure 5.2 represents the equilibria of the mixed \([m, \tau]\) game and import tariff game \([\tau, \tau]\). Point A represents the equilibrium \((m^q_1, m^q_2)\) while point C represents the potential location of the equilibrium quantity \((m^{mr}_1, m^{mr}_2)\). Point B locates the equilibrium of the \([m, m]\) game. Point C yields a higher welfare than at point B from country 1's perspective. However, it is not possible to rank unambiguously the welfare level of country 2 at point C with respect to B without further assumptions.
Combining the results of propositions 5.1 and 5.2, we have that: $R_1^{\text{mm}} < W_{1}^{\text{mm}}$ and $W_{1}^{\text{mr}} > W_{1}^{\text{rr}}$. Hence, using an import quota is a dominant strategy for country 1. From symmetry, it follows that using an import quota is a dominant strategy for country 2 as well. Suppose we allow both importers in the first stage of the game to simultaneously announce the trade instrument that they are committing to use in a second stage. The pair $(m_1^{\text{mm}}, m_2^{\text{mm}}) \rightarrow (W_1^{\text{mm}}, W_2^{\text{mm}})$ is a Subgame Perfect Nash (SPN) equilibrium in dominant strategy of this game. In the next section, we introduce uncertainty to allow importers to choose a different type of instrument in equilibrium.
5.4 - Production Uncertainty and the Endogenous Choice of Trade Instrument

5.4.1 Framework

The purpose of section 5.4 is to introduce uncertainty into the endogenous type of protection game. We introducing heterogeneous random disturbances into domestic production schedules of each importer using a stylized model.

The sequence of events is the following. Two policy active importers simultaneously choose between using an import quota or an import tariff to restrict trade. However, both importers’ domestic production is subject to a random disturbance which is not observed at the time importers commit themselves to a type of trade policy. Next, the decision of both importers is publicly observed. Given the irrevocable commitment of each importer to a trade instrument, both countries choose the level of their trade policy. Finally, both production shocks are publicly revealed and production, consumption and trade decisions are carried out.

Preferences and technology are identical in both countries. Domestic preferences are represented by a quasi-linear utility function: 
\[ U(w_i, x_i) = w_i + \frac{a}{b} x_i - \frac{x_i^2}{2b} \]
where \( w_i \) is a numéraire good. These preferences imply the domestic demand \( d_i \) for the import good \( x_i \) is:
\[ d_i = a - bp_i \]
where \( a \) and \( b \) are positive constants and \( p_i \) is the domestic price.

Domestic producers of the importable in country \( i \) have the following cost function:
\[ c(q_i, \varepsilon_i) = \frac{q_i^2}{2g} + \frac{cq_i}{g} - \frac{\varepsilon_i q_i}{g} \]
where \( \varepsilon_i \) is a random shock with mean zero and variance \( \sigma_i^2 \). A positive realization of \( \varepsilon_i \) decreases the marginal cost and total cost of producers in country \( i \).
Profit maximization under full information implies: \( q_i = -c + gp_i + \varepsilon_i \), where \( g \) is a positive constant.

Foreigners are passive and follow a free trade policy. Foreign demand and supply are respectively: \( \overline{D} = \alpha - \beta \overline{p} \) and \( \overline{Q} = \delta + \gamma \overline{p} \). The world price is determined according to the equilibrium condition in trade: \( \overline{p} = \frac{(\alpha - \delta) + m_z + m_i}{(\beta + \gamma)} \). The domestic price is determined according to the equilibrium on each domestic market: \( p_i = \frac{(a + c - \varepsilon_i) - m_i}{(b + g)} \). Welfare of importer \( i \) is defined as the sum of consumer surplus, producer surplus and government revenue:

\[
W_i = \int_0^{q_i} \left( \frac{a - q_i}{b} \right) dq_i - c(q_i) - \overline{p}m_i = \frac{a}{b} d_i - \frac{d_i^2}{2b} - \frac{q_i^2}{2g} + \frac{c}{g} q_i + \frac{\varepsilon_i q_i}{g} - \overline{p}m_i \quad (5.17)
\]

5.4.2 Import quota

We restrict the strategy space of both importers to an import quota. Both policy active importers maximize expected welfare in (5.17) given the other importer's quota. The first order condition is: \( E[p_i - \overline{p} - m_i \overline{p}'] = 0 \). Let \( \lambda = (\beta + \gamma) / (2(b + g)) \). Solving the first order condition simultaneously yields the equilibrium import quota of both importers:

\[
m_i^{em} = \frac{2A}{2(1 + \lambda) + 1} \quad (5.18)
\]

where \( A \equiv (a + c)\lambda - (\alpha' - \delta') \), \( \alpha' = \alpha / 2 \) and \( \delta' = \delta / 2 \).
5.4.3 Import tariff

Assume both importers use a specific tariff $\tau_i$. There is an arbitrage condition between the world price and domestic price if imports are positive in equilibrium: $p_i = \bar{p} + \tau_i$. Imports in country $i$ are defined by:

$$m_i = (a - c) - (b + g)(\bar{p} + \tau_i) - \varepsilon_i$$

(5.19)

The world price is determined according to the equilibrium condition $m_1 + m_2 = \bar{X}$, which implies:

$$\bar{p} = \frac{2(a - c) + (\alpha - \delta) - (b + g)(\tau_1 + \tau_2) - (\varepsilon_1 + \varepsilon_2)}{(\beta + \gamma) + 2(b + g)}.$$

Both importers maximize expected domestic welfare given the other importer's tariff. The first order condition is: $E\left[\frac{\partial m_i}{\partial \tau_i} - m_i \frac{\partial \bar{p}}{\partial \tau_i}\right] = 0$. With appropriate substitutions, the first order condition implicitly defines the reaction function of both importers:

$$E[m_i - \tau_i (b + g)(1 + 2\lambda)] = 0$$

(5.20)

Solving simultaneously (5.20) for both importers yields the Nash equilibrium tariff. The equilibrium import quantities in each country are:

$$m_i^* = \frac{A(1 + 2\lambda)}{(1 + 4\lambda + 2\lambda^2)} + \frac{\varepsilon_j - (1 + 2\lambda)\varepsilon_i}{2(1 + \lambda)}$$

(5.21)

Note that $\frac{\partial m_i^*}{\partial \varepsilon_j} > 0$ and $\frac{\partial m_i^*}{\partial \varepsilon_i} < 0$. A decrease in country $j$'s marginal cost ($\uparrow \varepsilon_j$) decreases country $j$'s imports and lowers the world price ceteris paribus. This provokes an increase in imports of country $i$. For $\varepsilon_j = \varepsilon_i = 0$, we have that $m_i^* > m_i^{**}$ for $i = 1, 2$ since from (5.18) and (5.21), we have that:

$$m_i^* = m_i^{**} + \left[\frac{A}{(1 + 4\lambda + 2\lambda^2)(2(1 + \lambda) + 1)} + \frac{\varepsilon_j - (1 + 2\lambda)\varepsilon_i}{2(1 + \lambda)}\right]$$

(5.22)
5.4.4 Mixed instruments

Without loss of generality, assume country 1 is using a specific tariff while country 2’s instrument is a quota. We have the following arbitrage condition between country 1’s domestic price and the world price: \( p_1 = \tilde{p} + \tau_1 \). Equilibrium on the world market implies:

\[
\tilde{p} = \frac{(a + c) - (\delta - \alpha) - (b + g)\tau_1 + m_2}{\beta + \gamma + b + g}.
\]

The first order condition for country 1’s maximization problem is:

\[
E \left[ t_1 \left( \frac{\partial m_1}{\partial t_1} \right) - m_1 \left( \frac{\partial \tilde{p}}{\partial t_1} \right) \right] = 0.
\]

It implicitly defines the tariff reaction function of country 1 as a function of country 2’s import quota:

\[
((b + g)^2 - (\beta + \gamma + b + g)^2)\tau_1 = -(a - c)(\beta + \gamma) - (b + g)(\delta - \alpha) + (b + g)m_2 \quad (5.23)
\]

Rearranging terms in (5.23) we have:

\[
\tau_1 = \frac{A}{2\lambda(b + g)(1 + \lambda)} - \frac{m_2}{4\lambda(b + g)(1 + \lambda)} \quad (5.24)
\]

The first order condition of country 2 is:

\[
E \left[ p_2 - \tilde{p} - m_2 / (S' - D') \right] = 0.
\]

The previous equation can be solved to get the quota reaction function of country 2 as a function of country 1’s tariff:

\[
m_2 = \frac{2[(a - c)\lambda - (\alpha' - \delta')] + (b + g)\tau_1}{(2\lambda + 3)} \quad (5.25)
\]

Using (5.24) and (5.25), we can solve the Nash equilibrium of the \([\tau, m]\) game:

\[
m_2^m = \frac{2A[1 + 4\lambda(1 + \lambda)]}{4\lambda(1 + \lambda)(2\lambda + 3) + 1} \quad (5.26)
\]

\[
\tau_1^m = \frac{4\lambda(1 + \lambda)}{4\lambda(1 + \lambda)(2\lambda + 3) + 1} \Rightarrow m_1^m = \frac{16\lambda(1 + \lambda)}{[4\lambda(1 + \lambda)(2\lambda + 3) + 1](1 + 2\lambda)} - \frac{2\lambda e_1}{(1 + 2\lambda)} \quad (5.27)
\]
5.4.5 Equilibria of the game

Recall that in the first stage of the game, both policy active importers simultaneously announce the trade instrument they commit to use in the second stage. In order to compare the expected welfare levels associated with each equilibrium, we rewrite the welfare function in terms of imports. This is done by substituting the producers’ cost function, demand schedules and foreign supply into (5.17).

\[ E[W] = K + \frac{1}{2(b+g)} \left( \frac{2Am_i}{\lambda} - m_i \right)^2 - \frac{m_i m_j}{\lambda} - 2\varepsilon_i m_i + \frac{\mu}{(1-\mu)} \varepsilon_i^2 + 2\varepsilon_i \left( a + \frac{c\mu}{(1-\mu)} \right) \]

(5.28)

where \( K = a^2/2b + c^2/2g - (a+c)^2/2(b+g) \).

Assume that the slope of the countries’ import demand and the slope of the foreign export supply are identical. Hence, \( \lambda = 1/2 \). Rewriting (5.28) gives:

\[ E[W] = K + \frac{1}{(b+g)} \left[ \frac{Am_i}{2} - \frac{3m_i^2}{2} - m_i m_j - \varepsilon_i m_i + \frac{\mu}{(1-\mu)} \varepsilon_i^2 + 2\varepsilon_i \left( a + \frac{c\mu}{(1-\mu)} \right) \right] \]

(5.29)

Under the restriction imposed on the parameter \( \lambda \), we can rewrite (5.18), (5.21), (5.26) and (5.27) as:

\[ m_1^{m_1} = \frac{3A}{13} - \frac{\varepsilon_1}{2}, \quad m_1^{m_2} = \frac{4A}{13}, \quad m_1^{m_3} = \frac{A}{4}, \quad m_1^{nr} = \frac{2A}{7} + \frac{(\varepsilon_2 - 2\varepsilon_1)}{2} \]

(5.30)

\[ m_2^{m_1} = \frac{3A}{13} - \frac{\varepsilon_2}{2}, \quad m_2^{m_2} = \frac{4A}{13}, \quad m_2^{m_3} = \frac{A}{4}, \quad m_2^{nr} = \frac{2A}{7} + \frac{(\varepsilon_1 - 2\varepsilon_2)}{2} \]

(5.31)

Assume the two random shocks are jointly distributed and drawn from a bivariate normal distribution with density function:
From the properties of a bivariate normal distribution, we have that the covariance between the random variables is: \( \sigma_{12} = \rho \sigma_1 \sigma_2 \). Assume also that the standard error of \( \varepsilon_2 \) can be written as a function of the standard error of \( \varepsilon_1 \). Hence, if \( \sigma_2 = \phi \sigma_1 \), we have: \( \sigma_{12} = \phi^2 \sigma_1^2 \) and \( \sigma_{12} = \rho \phi \sigma_1^2 \). Using these assumptions, the expected welfare levels of the import tariff game is:

\[
W_{1}^{w} = K + \frac{1}{2(b + g)} \left[ \frac{16 A^2}{49} + \frac{4 \sigma_1^2}{18} + \frac{\phi^2 \sigma_1^2}{18} - \frac{4 \rho \phi \sigma_1^2}{18} + \frac{\mu \sigma_1^2}{(1 - \mu)} \right] \tag{5.33}
\]

\[
W_{2}^{w} = K + \frac{1}{2(b + g)} \left[ \frac{16 A^2}{49} + \frac{4 \phi^2 \sigma_1^2}{18} + \frac{\sigma_1^2}{18} - \frac{4 \rho \phi \sigma_1^2}{18} + \frac{\mu \sigma_1^2}{(1 - \mu)} \right] \tag{5.34}
\]

Using (5.30) and (5.31) to substitute back into (5.29), we can compute the expected welfare levels under the various equilibria:

\[
W_{1}^{w} = K + \frac{1}{(b + g)} \left[ \frac{3 A^2}{8} + \frac{\mu \sigma_1^2}{2(1 - \mu)} \right] \tag{5.35}
\]

\[
W_{1}^{w} = K + \frac{1}{(b + g)} \left[ \frac{64 A^2}{169} + \frac{\mu \sigma_1^2}{2(1 - \mu)} \right] \tag{5.36}
\]

\[
W_{1}^{w} = K + \frac{1}{(b + g)} \left[ \frac{54 A^2}{169} + \frac{\sigma_1^2}{8} + \frac{\mu \sigma_1^2}{2(1 - \mu)} \right] \tag{5.37}
\]

\[
W_{2}^{w} = K + \frac{1}{(b + g)} \left[ \frac{64 A^2}{169} + \frac{\mu \sigma_1^2}{2(1 - \mu)} \right] \tag{5.38}
\]

\[
W_{2}^{w} = K + \frac{1}{(b + g)} \left[ \frac{54 A^2}{169} + \frac{\phi^2 \sigma_1^2}{8} + \frac{\mu \sigma_1^2}{2(1 - \mu)} \right] \tag{5.39}
\]
Consider two policy active importers who simultaneously commit either to use an import tariff or an import quota to restrict trade in the first stage of the game. Their trade instrument commitment is publicly observed. Each importer faces a random production disturbance in his own country and does not observe either shock before making its trade instrument commitment. In the second stage, both importers simultaneously set their trade policy level given their commitment at the beginning of the game. Finally, production disturbances are observed and production, consumption and trade are carried out. Assume country 2 faces a higher level of production risk than country 1. Therefore, \( \phi > 1 \).

**Proposition 5.3:** Assume the random shocks are distributed according to (5.32) and \( \phi > 1 \):

The endogenous type of instrument game is characterized by: (i) The SPN equilibrium of the game entails both importers using an import quota if: \( \sigma_1^2 / A^2 < 0.444 / \phi^2 \); (ii) Both importers enter into a prisoner’s dilemma in the SPN equilibrium of the game if: \( 0.872 / (4 + \phi^2 - 4 \rho \phi) < \sigma_1^2 / A^2 < 0.444 / \phi^2 \).

**Proof:** The inequality \( \sigma_1^2 / A^2 < 0.444 / \phi^2 \) implies that \( W_2^{mm} > W_2^{mt} \) from (5.35) and (5.39). Since \( \phi > 1 \), it must be the case that \( W_1^{mm} > W_1^{mt} \). Therefore, given country 1 uses a quota, country 2’s best response is also to use an import quota in the first stage of the game. Similarly, since \( W_1^{mm} > W_1^{mt} \), country 1’s best response to country 2 using a quota is to choose an import quota in the first stage of the game; proving claim (i). Using (5.33) and (5.35), the inequality \( 0.872 / (4 + \phi^2 - 4 \rho \phi) < \sigma_1^2 / A^2 < 0.444 / \phi^2 \) implies that \( W_1^{tr} > W_1^{mt} \). Since \( \phi > 1 \), it
must also be the case that $W_{2r} > W_{2mm}$. Since $\sigma_1^2 / A^2 < 0.444 / \phi^2$, the pair of actions $(m_{1mm}, m_{2mm})$ is still a SPN equilibrium of the game. However, both importers would be better off in terms of expected welfare if they collude and use import tariffs since $W_{1r} > W_{1mm}$ and $W_{2r} > W_{2mm}$. Q.E.D.

Proposition 5.3 states that, given a sufficiently low level of variance in the production shocks, quotas are preferred to import tariffs for both countries. As $\rho$ increases (decreases), the likelihood that both importers use import quotas in equilibrium increases (decreases). The expected welfare under the $[r, r]$ game is decreasing with $\rho$. Intuitively, given that the marginal cost in country $i$ is small (large) due to a large (small) random disturbance, country $i$ prefers country $j$ to have a high (low) marginal cost since it will raise (lower) country $j$'s imports and beneficiate (hurt) country $i$ through a higher (lower) world price.

**Proposition 5.4:** If the random shocks are distributed according to (5.32) and $\phi > 1$, the SPN equilibrium of the game entails country 1 using an import quota and country 2 using an import tariff if: $0.444 / \phi^2 = f(\phi) < \sigma_1^2 / A^2 < \bar{f}(\phi, \rho) = 0.939 / (4 + \phi^2 - 4 \rho \phi)$

*Proof:* From claim (i) in proposition 5.3, $f(\phi) < \sigma_1^2 / A^2$ implies that $W_{2mr} > W_{2mm}$. From (5.33) and (5.36), $\sigma_1^2 / A^2 < \bar{f}(\phi, \rho)$ implies $W_{1mr} > W_{1rr}$. Hence, using a quota is country 1's best response given country 2 uses a tariff. Given that country 1 is using an import quota, $W_{2mr} > W_{2mm}$ implies country 2's best response is to use an import tariff. Q.E.D.
In the case where \( \sigma_1^2 / A^2 \) is in the interval of \([f, \overline{f}]\), country 1, knowing country 2's best response to an import quota is to use an import tariff, has an incentive to use an import quota since it can restrict country 2's imports further by committing to use an import quota instead of a tariff. Country 2's best response is to arbitrage imports across the different state of the world given a relatively high variance in its own domestic production. Note that 

\[
f_\neq < 0 \quad \text{and} \quad \overline{f}_\neq > 0 \quad \text{as} \quad \phi > 2\rho.
\]

Hence, an increase in country 2's production risk unequivocally increases the likelihood of an asymmetric equilibrium in the trade instrument if \( \phi < 2\rho \). Note that the interval in \([f, \overline{f}]\) may very well be empty, preventing the equilibrium of the game to be asymmetric.

**Proposition 5.5:** If the random shocks are distributed according to (5.32) and \( \phi > 1 \), a SPN equilibrium of the game entails both importer choosing to use an import tariff if:

\[
\sigma_1^2 / A^2 > \frac{0.939}{(4 + \phi^2 - 4\rho\phi)}.
\]

**Proof:** From proposition 5.4, \( \sigma_1^2 / A^2 > \frac{0.939}{(4 + \phi^2 - 4\rho\phi)} \) implies that \( W_1^{\ast} > W_2^{\ast} \).

Therefore, using an import tariff is country 1's best response to country 2 using an import tariff. Since \( \phi > 1 \), using a tariff must be country 2's best response to country choosing a tariff in the first stage of the game. Q.E.D.
Note that the equilibrium of the game may not be unique since the condition \( \sigma_i^2 / A^2 > 0.939 / (4 + \phi^2 - 4\rho\phi) \) does not prevent \( \sigma_i^2 / A^2 \) from being less than \( 0.444 / \phi^2 \) as well. However, from proposition 5.3, the equilibrium where both countries use an import quota will be Pareto dominated by the other SPN equilibrium where both countries use tariffs.

5.5 - Conclusion

We proved that a sufficiently high variance in production risk creates incentives for both importers to use import tariffs. This is rather intuitive since tariffs allow for arbitrage across the states of the world. The ability to profit from this arbitrage opportunity must be weighted against the higher residual foreign export supply elasticity using tariff rather than quotas. There exists also a range of values for production risk parameters that entails an equilibrium where both importers choose a different instrument. The importer facing the largest production risk uses an import tariff to arbitrage imports across states of the world while the other importer chooses to restrict imports with a quota.
CHAPTER 6. GENERAL CONCLUSION

6.1 – Summary

This dissertation includes four three chapters dealing with time consistency of trade policies. First, we investigate the strategic behavior between countries that have purchasing power on the world market for a certain good. Tariffs and quotas are not equivalent protection instruments in this oligopsonistic market. Policy active importers would be better off by colluding and setting their trade instrument cooperatively. If production decisions occur before consumption decisions, the ex-ante optimal policy is not time consistent because the ex-post elasticity of the residual foreign export supply curve is lower than the ex-ante elasticity. However, we show that the importers’ inability to irrevocably commit to their trade instrument may be welfare superior to the precommitment solution. The negative welfare implication of non-cooperative behavior may be balanced off by the welfare effect of the ex-post elasticity.

The following chapter explores the policy active importers’ incentives and the welfare implications of using production policies and trade polices if production decisions occur before consumption decisions. The existence of production lags rationalizes the argument for a large country to use both production and trade polices in order to increase its welfare if it can not irrevocably commit to its ex-ante trade policy. Production instruments are not equivalent under non-cooperative behavior. Provided trade is restricted with an import quota, the equilibrium production policy is to restrict domestic production below the competitive level. If trade is restricted with an import tariff, the equilibrium production
policy may be to subsidize production. We derive conditions under which the ability of each importer to control domestic production increases welfare.

Chapter four analyzes the endogenous decision of two policy active importers to either commit to their import quota or keep the flexibility to revise their *ex-ante* quota once production decisions are made. We assume that production in both importing countries is subject to an asymmetric random shock. Committing to its import quota before production decisions are made may provide a strategic leadership welfare gain. However, not irrevocably committing to its *ex-ante* quota gives an importer the flexibility to revise its import quota once uncertainty on the domestic market is resolved. Both policy active importers prefer not to commit to their trade policy given a sufficiently high degree of variance in production uncertainty, ceteris paribus. Under certain conditions, there exists an equilibrium where one country commits to its *ex-ante* quota while the other keeps the flexibility to revise the level of its policy after uncertainty is resolved.

The final chapter examines the endogenous choice of trade instrument among two policy active importers. Production and consumption decisions are carried out simultaneously; thus we assume away the existence of production lags. The producers’ cost function is subject to a linear random shock. In the first stage of the game, each government commits to either use an import quota or an import tariff to reduce trade in the second stage of the game in an attempt to capture a potential terms of trade gain. In the case where random disturbances are null, the equilibrium of the game entails both importers choosing to use an import quota. If there is asymmetry in production risk, there exists a range of values for the uncertainty parameters yielding an equilibrium where one country uses an import quota while the other country uses an import tariff.
6.2 – Recommendations for Future Research

We derived the equilibrium production policies under the assumption that importers are not able to irrevocably commit themselves to their import quota. If importers fail to ban the use of production policies, the ability to control domestic production may hurt them. Given that possibility, what are the conditions for which importers would prefer to commit to their \textit{ex-ante} trade policy (and thus eliminate the need for production policies)? The ranking between the precommitment solution and the time consistent solution under production policies is not obvious as it involves comparing the two distorted foreign rate of transformation with a distorted domestic rate of production and an undistorted one.

The models in chapter four and five are obviously highly stylized and some of the assumptions on the structure of preferences and technology should be relaxed in order to get more robust results. The impact of risk preferences on the results could be analyzed using a general equilibrium framework. However, it is not clear how much would be gained from doing so. Import quotas generally induce less variation in consumers’ income than tariffs. Hence, within our framework where quotas are preferred to tariffs under certainty, introducing risk averse consumers may potentially reinforce this preference.

Other types of instrument could be analyzed within our framework. Minimum Access Commitments (MACs) are increasingly popular in agricultural markets. Under this type of protection, an importer commits itself to let a predetermined quantity of imports enter its domestic market. This commitment imposes a lower bound on imports and thus differs from import quotas. Can policy active importers use MACs to capture a terms of trade gain if they are used in conjunction with other instruments? What are the strategic differences from the importers’ perspective if MACs are used compared to an import quota game?
APPENDIX A. PROOF OF PROPOSITION 3.4

Proof of proposition 3.4(i): Using (3.51), dropping all subscripts because of the symmetry assumption, the SPN equilibrium production tax is lower than the tariff if:

\[
\frac{V[1 + \bar{\mu}(N-1)\bar{V} - \bar{\mu}(\psi - \alpha)]}{\alpha(1 + \bar{\mu}N\bar{V}) + V(1 - \mu)(1 + \bar{\mu}(N-1)\bar{V})} < 1 \quad (A.1)
\]

The inequality (A.1) is satisfied if:

\[
\frac{\bar{\mu}(N-1)^2 V^2 - \sqrt{\bar{\mu}(\psi - \alpha)}}{\alpha(1 + \bar{\mu}N\bar{V}) + V(1 - \mu)(1 + \bar{\mu}(N-1)\bar{V})} < 1 - \frac{V}{\alpha(1 + \bar{\mu}N\bar{V}) + V(1 - \mu)(1 + \bar{\mu}(N-1)\bar{V})}
\]

The above inequality can be rewritten as:

\[
[(2N-1)\alpha - (N-1)\mu V]\alpha + \alpha - \mu V + \bar{\mu}V > 0 \quad (A.2)
\]

The inequality in (A.2) is respected if \( \alpha - \mu V > 0 \). Using the definition of \( V \), it is readily seen that the latter inequality holds. Therefore, the inequality in (A.2) is respected, and thus \( \sigma_i^* < \tau_i^* \). Q.E.D.

Proof of proposition 3.4(ii): Evaluate the first order condition in (3.49) at the equilibrium production quota under an import tariff \( (\omega_i^*) \):

\[
\left.\frac{\partial W_i}{\partial \sigma_i}\right|_{\sigma_i = \sigma_i^*} = \frac{\partial q_i}{\partial \sigma_i} \tau_i \left[ \left( \frac{\partial m_i}{\partial \sigma_i} \right|_{\sigma_i^*} - \frac{\partial m_i}{\partial q_i} \right] - \bar{\mu}(\psi - \alpha) \left( \frac{\partial \bar{X}}{\partial \sigma_i} \right|_{\sigma_i^*} - \frac{\partial \bar{X}}{\partial q_i} \right|_{\sigma_i^*} \quad (A.3)
\]

From (3.34), we have: \( \frac{\partial m_i}{\partial \sigma_i} = \frac{\partial m_i}{\partial q_i} \frac{\partial q_i}{\partial \sigma_i} + \sum_{j \neq i} \frac{\partial m_i}{\partial q_j} \frac{\partial q_j}{\partial \sigma_i} \). Therefore, the first term between parenthesis on the right hand-side of (A.3) can be rewritten as: \( \sum_{j \neq i} \frac{\partial m_i}{\partial q_j} \frac{\partial q_j}{\partial \sigma_i} \frac{\partial q_i}{\partial \sigma_i} < 0 \).
Similarly, from (3.33), we have that the second term between parenthesis on the right hand-side of (A.3) can be rewritten as: 
\[ \sum_{j=n} \frac{\partial X}{\partial q_j} \frac{\partial q_j}{\partial \sigma_i} > 0. \]
Therefore, 
\[ \left. \frac{\partial W}{\partial \sigma_i} \right|_{\sigma_i = \omega^*_i} > 0, \]
implying that \( \sigma_i^* > \omega^*_i \). Q.E.D.
The inequality $2\omega^m - \sigma^m > 0$ implies that the ability to tax domestic production at the beginning of the game is welfare improving. The solution to this inequality yields:

$$2N\gamma(1-\mu) + (N-1)\gamma\mu + \gamma^2\alpha(N-1) + 2\alpha N + \gamma^2 - (1+\alpha)\gamma \pm \sqrt{\kappa}$$

$$\bar{\mu} < \frac{2N\gamma(1-\mu) + (N-1)\gamma\mu + \gamma^2\alpha(N-1) + 2\alpha N + \gamma^2 - (1+\alpha)\gamma \pm \sqrt{\kappa}}{2\gamma N(\alpha(1+\gamma) + 2\gamma(1-\mu) - 2N(\gamma + \alpha) - \alpha \gamma)}$$

where:

$$\kappa = -4\gamma N(2\alpha + \gamma - \alpha \gamma - 2\gamma\mu)(\alpha + 2\gamma + \alpha \gamma - 2\gamma\mu - 2\alpha N - 2\gamma N - \alpha \gamma N + 2\lambda^2)$$

$$\lambda = \gamma(1+\alpha) - \gamma^2 + \alpha \gamma^2 + 2\gamma^2 \mu - 2\alpha N - 2\gamma N - \alpha \gamma^2 N + 2\gamma \mu N(1-\gamma)$$

The solution of the inequality $2\omega^* - \sigma^* > 0$ is:

$$\bar{\mu} < \frac{c_1 + c_2 \pm \sqrt{a^2 - 4(-2(\psi - \alpha)\theta + \psi\psi')b}}{2(D_1 + D_2)}$$

where:

$$a = -2H(\psi - \alpha) - 2a^2 \theta N(N - 1)^2 + 2N\psi\theta(1 - \alpha) + 2\alpha N^2\psi\theta - \alpha^2 NV - \psi V(1 - \psi) + \psi V$$

$$b = -2HN(1 - \alpha) - 2a^2 HN^2 + 2HN\psi(1 - \alpha) + 2\alpha HN^2\psi - \alpha^2 KNV + K\psi V(1 - \psi)$$

$$c_1 = 2H(\psi - \alpha) + 2aN\theta(1 - \alpha) + 2a^2 N^2\theta - 2N\psi\theta + 2aN\psi\theta$$

$$c_2 = -2aN^2\psi\theta + a^2 NV + \psi V - K\psi V - \psi^2 V$$

$$D_1 = -2aN H(1 - \alpha) - 2a^2 HN^2 + 2HN\psi(1 - \alpha) + 2\alpha HN^2\psi - \alpha^2 KNV,$$

$$D_2 = K\psi V(\psi - 1), \quad \theta = \alpha + (1 - \mu)\psi$$

$$K = (N - 1)V - (\psi - \alpha) \quad \text{and} \quad H = NV\alpha + (N - 1)V^2(1 - \mu).$$
BIBLIOGRAPHY


