Estimation and attenuation of reinsurance risk in the crop insurance market

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UMI
Estimation and attenuation of reinsurance risk in the crop insurance market

by

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A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Economics

Major Professor: Dermot J. Hayes

Iowa State University

Ames, Iowa

2000
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CHAPTER 1
INTRODUCTION

Farming is a financially risky undertaking. It is natural, therefore, that the risk-averse producer should seek means to reduce the level of variability in the return on his investment. Recognizing this potential market, a number of private insurance firms in the late nineteenth and early twentieth centuries ventured into the business of writing policies guaranteeing a certain level of revenue per acre of production. However, no firm that wrote multiple-peril crop insurance policies did so for very long (Gardner and Kramer 1986, Kramer 1983). Among the reasons Gardner and Kramer list as causes for these failures of privately provided crop insurance is the insurance firms' failure to spread their risks across the entire range of production areas. In each move into the crop insurance market there was an "inadequate geographical dispersion of risks". This could be restated as "failure to sufficiently diversify."

Among the conditions necessary for a particular risk to be insurable is the absence of "catastrophic" losses (Vaughan and Vaughan 1996). Exclusion of catastrophic losses here means that losses do not occur simultaneously among a high proportion of the insured parties. This condition will be met if losses among the insured risks are independent of one another. If the sources of risk are independent, the insurer can eliminate a great deal of risk by holding a diversified portfolio of the risky liabilities.
Therefore, one might infer that it is the assertion of Gardner and Kramer that the early experiments with market-provided crop insurance would have been less likely to have failed had the insurers had a better-diversified book of business rather than having their policies insuring only farmers in one part of the country. However, a well-diversified portfolio of crop insurance policies does not necessarily mean that the total portfolio risk reaches a level that is acceptable to private insurers. In the crop insurance market, unlike the risks in other insurance markets, pooling does little to mitigate the level of risk (Quiggin 1994).

Miranda and Glauber (1997) examined the effect that the positive correlation of yields has on the risk level of crop insurance. Using a model which generates yields for individual producers, the coefficient of variation (CV) for indemnities is computed for the ten largest crop insurance firms first imposing existing correlation levels, then assuming zero correlation among yields. The authors find that the coefficient of variation ranges between 0.67 and 1.30 for the ten firms. These are between 22 and 49 times the levels of the coefficient of variation for generated indemnities when zero correlation among yields is assumed. On the surface, it does not appear that these high CV levels are due to poor diversification practices as the authors give the CV level for the U.S. total—0.81. This level of variability is much higher than that seen in other lines of insurance which have existed without the level of government participation that has existed in the crop insurance market.

A means of disposing of unacceptable risk must exist lest the profit motive be completely overcome by risk aversion. Such a means is provided to participating
insurers in the Standard Reinsurance Agreement (SRA). The SRA provides for a
transfer of a portion of losses that can occur with widespread yield shortfalls in
exchange for a portion of the gains when premiums are greater than indemnities.
This leaves the Federal Crop Insurance Corporation (FCIC) in its role as reinsurer
with an uncertain level of total outlays. As has been mentioned, the high positive
correlation among yields makes feast or famine returns to the insurer more likely than
they would otherwise be. This risk is magnified when it is passed to the FCIC
because of the non-proportional form of the reinsurance. The SRA yields an
increasing proportion of the firms' profits as positive returns increase and commits
the FCIC to taking responsibility for an increasing proportion of the losses as these
increase. In order for the FCIC to have some notion of its own risk exposure, a model
can be constructed which simulates the behavior of agricultural production.

Objectives

There are three main objectives of this study. The first is to construct the
essentials of the distribution functions which describe the risk exposure to the FCIC
when providing reinsurance services to private firms. The study will consider the
largest sources of risk to the FCIC--Multiple Peril Crop Insurance also known as
Actual Production History insurance (APH), Group Risk Plan (GRP), and Crop
Revenue Coverage (CRC). The crops that will be included in the study are corn,
soybeans, and wheat. These represent the largest of the crops insured by these
programs. Accomplishment of this first part will allow the FCIC to determine how
much should be budgeted annually to reserves to avoid being short of funds for reinsurance obligations for any desired level of confidence. The figure which represents this dollar amount is called the value at risk (VAR). It is merely the dollar value on the horizontal axis that marks the (e.g.) 5% level in the tail of the density function of costs (Jorion 1997).

In finding the distribution of reinsurance costs to the FCIC, the second objective will also be met. The details of the Standard Reinsurance Agreement are determined by government policymakers. It is presented to insurers as a "take it or leave it" offer. It is not a product of market forces. As it is not a product of the market, not subject to any sort of bidding process, and does not come about as a product of the reinsurer's profit motive, the value of the SRA cannot be derived by examining the transaction price as if it were a derivative contract traded in the market. The second objective is, therefore, to estimate the fair market value of the SRA—the amount which the reinsurer could obtain for the agreement or the amount the insurers would be willing to pay for it.

The third objective of this study is to determine the extent to which the risk accepted by the reinsurer could be hedged using national area-yield and commodity price contracts. This portion of the study will demonstrate the possible reduction in necessary reserve funds (determined in the first portion) which can be obtained by holding securities that increase in value when events occur that cause claims to increase. For example, holding put options on corn yields will offset some of the payment obligations of the reinsuring party when there is a widespread decline in
corn yields. The fact that the reinsurer holds these put options which increase in value under the above scenario means that less has to be budgeted to reserves. The second part of the study will examine this relationship.

This work will examine crop insurance from the point of view of a reinsurer who pools risk from the business of several insurance companies. The actual situation faced by the FCIC in its role as the provider of reinsurance to crop insurers as described in the Standard Reinsurance Agreement is analyzed. In their paper (discussed below), Miranda and Glauber examined the role the options market could play as a hedge for insurers with different books of business. In their concluding remarks, the case was made that the derivatives market could be used as a substitute to government provision of reinsurance services. While this may very well be the case, the FCIC/RMA has both regulatory and oversight responsibilities as well as service obligations. Authorities may be hesitant to permit (or require) insurers to use the market to rid themselves of systemic risk for fear that hedging activities may lead to speculative activities. Well founded or not, regulators may fear that extreme losses by private insurers in derivatives markets could lead to the withdrawal of crop insurance services from the market.

Therefore, this work examines the possible advantages to the reinsurer of the firm’s participation in the derivatives market. The idea of government participation in the derivatives market as seller and buyer should definitely not be viewed as a prospect without potential adverse affects to current participants in the market. The analysis that follows, however, can be seen a discussion regarding one aspect which
motivates government involvement in the crop insurance market. That aspect of the market is the great variance in returns resulting from high correlation among risks making the portfolio of said risks difficult to diversify. The options market's risk-reducing potential to the FCIC in its role as the provider of reinsurance services is examined and discussed, rather than the potential the market has of replacing the FCIC. If, however, the existence of and the use of yield derivatives substantially reduces the variance of costs to the FCIC, it may be the case that the responsibility of reinsurance could be passed on to a non-governmental firm. While the proceeding investigation will make no more mention on the subject, one could infer that risk reduction by a government reinsurer via use of derivatives as seen in the analysis that follows can also be achieved via their use by a privatized reinsurer.

Organization

The remainder of this work will proceed as follows: The next chapter, entitled "Literature Review," will examine the relevant literature on crop insurance and on the risk-reducing usage of commodity futures and options. In chapter three, "Methods," the procedures used to construct the simulation model are described. The simulation results are then assembled to describe the distribution of costs and are reported in the chapter entitled "Simulation Results." As the secondary inquiry into risk-reducing potential of yield contracts for the reinsurer builds upon the results of the first portion, both the procedures and findings will be presented in a subsection of the results. A summary concludes the work in the chapter entitled "Conclusion".
CHAPTER 2
LITERATURE REVIEW

This chapter reviews the relevant literature on the topic. The first portion examines the work that has been done in the field of crop insurance. Attention then turns to the literature on hedging with commodity derivatives. Finally, the work by Miranda and Glauber which deals with both topics is examined.

Crop Insurance Literature

Federal Crop Insurance has received a significant amount of attention from economists. In general, the literature has concentrated on two subjects. The first is the effect of insurance on the welfare of the insured producer. The second is the effect various pricing and structural schemes can have on the insurer and on the insured particularly in terms of moral hazard and adverse selection results. A survey of this literature follows.

Skees and Reed (1986) examined the premium structure of the Federal Crop Insurance program based on individual producers’ Actual Production History (APH). Their main concern was the effect the premium-determination process has on the level of adverse selection. When higher-risk producers are charged the same premium as lower-risk producers, the policy will tend to attract those with higher levels of risk and drive away those who have less to gain from buying the insurance. If rates are then adjusted (raised) to reflect the average, those with less variance in their production have even greater disincentive to purchase insurance.
The "theoretical premium" is equal to the expected loss in bushels times commodity price. Assuming a normal distribution governs crop yields (so that mean and variance are all that are needed to find expected loss), Skees and Reed demonstrate that unless the coefficient of variation (CV) is the same for all producers, the theoretical premium will differ among producers. If standard deviation is held constant, it is shown that the theoretical premium falls as expected yield rises.

The authors note, however, that while Federal Crop Insurance rates had recently begun to be adjusted for different APH yields (previously they had been based on the average for the area where the producer was located), APH average yields are not the same as the mathematically expected value for yields. The fact that the FCIC did not adjust APH yields for any trend, "means that farmers with positive yield trends are not able to purchase as much protection as is implied because APH yield is a biased estimate of expected yield," since producers are allowed to purchase protection on, at most, 75% of their APH yield.

The authors also raised concerns about individual rates based on the mean of a small sample of annual yields for the individual producer. This means that individuals with the same average APH values will pay the same premium even though they may have very different variation in yields. If equal APH averages do not imply the same CV across producers, this causes an adverse selection problem since farmers with very inconsistent yields will have greater incentive to purchase insurance than will those with production that is more reliable. Of course, if the CV is similar for farms with equal expected yields, this would tend to support the practice
of basing premiums on average APH yields.

Using farm level corn and soybean data, they first test for the existence of a positive trend in yields and whether the trend is the same for all producers within a state (data for Illinois and Kentucky farms are used). Positive trends are found. The hypothesis of equal trends among farms within a region is not rejected. Adjusting the data for productivity changes, another regression is then run for each region with standard deviation as the dependent variable and average (or expected) yield as the independent. The “hypothesis that standard deviation is independent of expected yield cannot be rejected....” When CV is similarly tested, evidence is found that this tends to decline as expected yield increases. A test for normality of farm yields generates uncertain results with normality being rejected for a substantial minority of the producers. Skees and Reed conclude that, while it would be preferable to include the CV directly into the premium function, if only average APH yield is used, there is evidence to support premium reductions as yields increase. The measure of APH yield should, however, be adjusted for changes in productivity to obtain a more realistic estimate of expected yield.

This problem was further investigated later by Goodwin (1994). In his study, Goodwin questions the assertion that there is a consistently strong relationship between average yields and the CV. Even if it is true that the CV tends to decrease as average yield increases, use of this tendency for rating policies will not likely eliminate the adverse selection problem because there will certainly exist high-yield, high variance operations which have more incentive to purchase insurance. There
will also be low-yield, low-variance producers whose expected loss (or theoretical premium) is much less than the premium calculated based on the low-yield, high-CV assumption.

Goodwin investigates the relation between average and standard deviation using a much larger data set of Kansas farms which are not all insurance purchasers (Skees and Reed use data from Federal Crop Insurance purchasers). Running separate regressions for each of eight crops (four commodities each divided into dryland and irrigated land), six are found to have significant relation between yield and standard deviation. In some cases the relation is positive. In others, it is negative. For all eight crops, the regression equation has very low explanatory power.

The data is then tested for the existence of adverse selection. The decision to purchase or not to purchase crop insurance is known for a portion of Goodwin’s data set. CV was found to be higher for those who participated in FCI but the null hypothesis of equal CV could be rejected for only two of the four crops. Next, expected losses are calculated for participants and non-participants and are compared to estimated FCI premiums. For both groups, the average of expected losses are greater than estimated FCI premiums for all four crops. In three of the four cases, average expected losses of participants were greater than the average losses for non-participants with two of the three being significant. He concludes by advocating the direct use of the CV or of other observable factors which might be good indicators of variability for premium rate setting.
As a solution to adverse selection and moral hazard problems, there was frequently discussed a policy in which indemnities would be based on a large area where a farm exists. Because there would be little moral hazard problem, there would be no need for a high deductible as there is under MPCI based on individual APH yields.

Halcrow (1949) is credited by others as being the first to suggest an area-yield scheme. The idea was investigated again by Miranda in a 1991 article. Miranda began by assuming that individual yields are of the form

\[ y_i = \mu_i + \beta_i \cdot (y - \mu) + \varepsilon, \]

where variables with subscripts are particular to individual producers and those without are those associated with the area. The \( y \)'s are random yield variables, the \( \mu \)'s are the associated yield means, \( \varepsilon \) is a stochastic variable with mean zero, and \( \beta \) is equal to the covariance of individual and area yield divided by the variance of the area yield. Assuming that indemnities, \( n \), are paid when area yield is less than a certain "critical yield," it is shown that risk reduction for the individual is determined by his individual value for \( \beta \). It is not necessarily true, however, that area-yield insurance will be risk reducing for all producers. Risk reduction increases with correlation between the individual and area yields and increases as variance of individual yields increases. Specifically, the value of risk-reduction is equal to

\[ \sigma^2 \cdot \left[ \frac{\beta_i}{\beta} - 1 \right] \text{ where } \beta = \frac{\sigma_i^2}{2 \cdot \text{Cov}(y, n)}. \]
Variance of the indemnity is denoted by $\sigma_i^2$. If the producer is able to choose any coverage level (a scaling factor for premiums and indemnities), the risk-minimizing coverage level is equal to

$$
\frac{\beta_i}{2\beta_i}.
$$

In addition, it is demonstrated that this optimal coverage level will be greater than one for some producers.

Miranda then examined the level of protection offered by a policy in which indemnities and premiums were based on the yield of all producers in the area. He proceeded by decomposing an individual farm's total production risk into systemic and nonsystemic components. The systemic portion represents factors such as area temperature and rainfall which affect area producers in a common manner. The nonsystemic portion consists of those factors such as one's own production practices which influence only the individual producer. Miranda found that area-yield insurance with low deductible, which protects an individual against only the systemic risk, provides a greater level of protection against total risk than does 75% coverage of APH yield. This assumes that producers may choose any positive level of coverage under the area-yield scheme.

Mahul (1999) examined the optimal design of an area-yield policy in the form of a utility maximization problem. As in Miranda (1991) the difference in individual yield from its mean is equal to the sum of a multiple ($\beta$) times the difference in area yield and its mean and a stochastic term with mean zero and uncorrelated with
yields. A general structure for the policy is specified: indemnities are a function of area yields and premiums depend on expected indemnities. It is shown that when an indemnity is paid, the amount is linear with respect to area yield. Assuming the farmer's beta is positive, the indemnity will equal this beta times the shortfall in yield (or zero if the area yield is greater than the critical yield). This means that for the producer with a beta greater than one, the optimal policy will indemnify by more than one bushel for each bushel reduction in yields below the critical yield. It is also shown that the critical yield is equal to the maximum area yield if the policy is actuarially fair and is less than the maximum if the premium is greater than expected indemnities. If the insurance purchaser is constrained to a certain value in the coverage parameter, under constant absolute risk aversion the optimal critical yield will increase as the coverage constraint is lowered.

The idea of area-yield insurance was implemented in the 1994 crop year. Skees, Black, and Barnett (1997) describe the methods used to determine rates for the policy that was named the Group Risk Plan (GRP). The area currently used for GRP is the individual county in which the operation exists. Limits are placed on the range of coverage levels purchasers may choose. The authors emphasize that area-yield insurance can only serve to reduce risk for the individual where a significant portion of total risk is systemic in nature.

There have recently become available policies which concentrate on producer revenues rather than on yields. Currently existing policies that guarantee revenue levels are Crop Revenue Coverage (CRC), Revenue Assurance (RA), and Income
Protection (IP) with CRC currently obtaining the largest premium revenues of the three.

Potential benefits of a revenue insurance scheme are examined in Hennessy, Babcock, and Hayes (1997). It is first demonstrated that insurance which guarantees a fixed level of revenue is less costly to the provider than any separate guarantees of price and quantity which achieves a like revenue. It is then shown that insuring the revenue of a group of crops ("portfolio" revenue insurance) is less costly than insuring the revenues of each individual crop.

Implications for government cost and producer welfare are then tested using a simulation where the subject is a producer of corn and soybeans who exhibits CARA utility. Two levels of risk aversion are examined, as are four different insurance schemes. The four scenarios are the 1990 farm program, no insurance, farm-level revenue insurance, and county-level revenue insurance. Within the latter two scenarios, both crop-specific and portfolio insurance are tested each at 75% of expected revenue guarantee and at the 100% level. Yields and prices are random variables. Acreage is permitted to be allocated between the two crops to maximize expected utility.

Under all scenarios the expected utility-maximizing allocation of acreage between the two crops is an equal division. Farm-level revenue insurance with a 75% revenue guarantee results in mild producer welfare reduction in terms of certainty equivalents and great reductions in government expenses compared to the results under the 1990 farm program. With a 100% guarantee, government costs are slightly
higher than they are under the 1990 program but producer welfare is considerably greater. Compared to the "no program" alternative, each dollar of government expenditure under the 1990 program increases farmer welfare by less than one dollar. The revenue insurance policies in all cases increase welfare by more than government expenditures.

Prior to this, Turvey and Amanor-Boadu (1989) discussed the pricing of such a policy. In order to rate revenue insurance premiums, it is important to have an understanding of the distribution of revenue. Testing various crop prices, yields, and revenues for normality produces conflicting results. The authors suggest that revenues may be either approximately normally distributed or it may be the case that revenue distributions may be positively skewed. They therefore offer two different models as providing bounds on the actuarially fair premium. The first is based on a model developed by Botts and Boles in 1957 which assumes that revenues are distributed normally. The second is based on the Black-Scholes option pricing model which, for this use, must assume crop revenues are distributed lognormally.

Using county-level yield and price data, the crop allocation necessary to achieve given levels of expected income and the associated standard deviations which result are calculated under the two different distribution assumptions. Per-acre premiums are then calculated for the various expected income levels and six different coverage levels. The crop insurance model and the Black-Scholes model yield significantly different premiums. Premiums are considerably higher with the normal distribution of revenue. This makes intuitive sense as it would seem that low revenue
results would be much less likely if they are governed by a positively skewed lognormal distribution.

Stokes, Nayda, and English (1997) criticize the use of the Black-Scholes formula for rating revenue insurance in part for the assumption of lognormal revenue. In addition, “[t]he assumption of the existence of a riskless hedge portfolio, long in gross revenues and put options written against the revenue is tenuous at best. This is because the Black-Scholes model would require the producer to continuously update this portfolio (by buying and selling farm gross revenue) . . . to maintain a riskless position.” Since this cannot be done, “...the no-arbitrage condition required...is violated.” They develop a model in which premiums and maximum coverage are determined by a moving average of individual production. The model is used together with county-level data to generate actuarially fair premium rates for revenue insurance. This model results in premiums inversely related to expected gross revenue in contrast to results obtained by Turvey and Amanor-Boadu.

**Hedging Literature**

The abundant literature on the use of futures contracts by producers to hedge risk began with McKinnon’s work in 1967. Using variance of income as a measure of risk, McKinnon showed the position a farmer should take in the futures market to minimize risk for a given expected production level and given futures prices. In his analysis he assumed normally distributed production levels and prices. The results indicate that the optimal forward sale increases as output variance increases and
varies inversely with the variance of price. Unless either the correlation between output and price is nonnegative or the variance of output is zero (i.e. output is a deterministic variable), the optimal hedge will be less than the amount of the expected output. In addition, as the correlation between price and output becomes more negative, the amount of the optimal hedge decreases.

A large number of articles followed McKinnon's work putting more structure on the problem. Collins (1997) categorizes the types of studies which make up the hedging literature. In the main, these can be characterized by two avenues of inquiry. One of these is the risk-minimizing area of investigation. Studies that fall into this category target hedge ratios which minimize the variance of net returns. Collins cites Lence and Hayes (1994) as an example.

In the Lence and Hayes article, it is noted in that work that minimum variance hedge (MVH) ratios are generally computed as if there is no uncertainty with regard to parameters. Given that data sets used in these computations are sample data, there must be some level of uncertainty about any parameter estimate. If the hedging activity is undertaken by a risk-averse individual, then uncertainty about the components of the variance-minimizing hedge ratio should itself affect the ratio.

The authors first review the derivation of the MVH ratio under the assumption that parameters are known with certainty. Emphasized is the fact that these parameters generally are not known with certainty but researchers usually proceed as though they were. The result is the standard one that the MVH ratio is equal to the covariance of spot and futures prices divided by the variance of futures prices.
The problem of uncertainty in parameters (the functional form of the pdf of prices being assumed known) is attacked using Bayesian tools. After mathematically defining the objective of finding the MVH ratio given the additional uncertainties, MVH is analyzed for an agent desiring to hedge a cash position in soybeans. The ratio is calculated for various prior estimates of spot-futures correlation coefficients using point estimates for all other variables. Three different levels of confidence are considered for the prior knowledge. As the prior estimate of correlation increases, the MVH ratio increases as might be expected. Confidence level in the prior seems to have the potential of having dramatic effect on the MVH ratio, lower confidence causing the MVH to be considerably lower than it would be under typical point estimation.

The second avenue identified by Collins pursues an optimal hedge by maximizing a producer's expected utility function which contains returns and a measure of risk as arguments. The position undertaken by agents in these models is generally given by an equation with a hedging component and a speculative component. Lapan and Moschini (1994) analyzed the problem from the point of view of the producer who faces risk from both price and output (as did McKinnon) and, in addition, faces basis risk. Lapan and Moschini initially make assumptions regarding risk attitudes (Constant Absolute Risk Aversion) and the distribution of the random variables (bivariate normal) in order to obtain a solution in the form of an equation which has a speculative component based on perceived bias in futures prices and a hedging component. Optimal futures positions are estimated using soybean
production for regions of Iowa assuming various levels of risk aversion and different harvest dates. Results depend somewhat on the level of risk aversion and rather less on the harvest date. The robustness of these results is tested by use of a Monte Carlo simulation using different distributional and utility specifications (lognormality and Constant Relative Risk Aversion). The results hold up fairly well under the different specifications.

Collins himself departs from these two areas of emphasis which have investigated what agents should do and pursues a model which explains observed behavior in participation or lack of participation in commodity derivatives markets. He writes that a positive model of hedging should be able to simulate four different observed behaviors. First, some agents do not hedge. Others hedge completely. Those who hedge incompletely will increase their hedge if the volatility of the spot price increases. Agents who use an incomplete hedge will also increase their hedge when their debt increase.

While acknowledging that most of the literature in hedging has not had a positive emphasis but rather has sought to solve optimization problems, he evaluates various models for their ability to explain the observed behavior of various agents with regard to hedging in futures markets. Models which derive the risk-minimizing hedge may be appropriate descriptions of what an arbitrageur does when he is long in the spot market and short the futures but do not simulate behavior of farmers or intermediate users of commodities who are usually either partially hedged or do not use the futures market at all. Utility-maximizing models also fall short of the
conditions set by Collins. It is possible to obtain a no-hedge solution if by chance the hedging component happens to exactly equal the speculative component. These also emulate the behavior of the agent who hedges completely if the agent is modeled with infinite aversion toward risk. These conditions, however, are not intuitively appealing.

Collins uses a two-period model where the objective is to maximize expected wealth in the second period subject to the likelihood that wealth falls below a certain level, \( d \), is no larger than a certain probability, \( \alpha \). If \( d = \alpha = 0 \), then if the worst possible commodity price will not drive total assets below liabilities, the producer has no reason to sell his crop forward. (It is assumed that expected spot prices are greater than futures prices). If this is not the case, the producer will hedge "such that [he] will just avert bankruptcy" under the worst-case scenario. Thus, a producer will generally not hedge in the futures market if the expected spot price is greater than the futures price. The exception occurs for the producer who carries a high level of debt. If the level of debt is high enough, cash prices could conceivably be sufficiently low for debt to exceed equity in the operation at the end of the crop year. The model is also applicable to the arbitrageur who maintains a complete hedge in any transaction since "[a] true arbitrage transaction requires no equity," and any loss could therefore reduce total equity to a negative figure.

The development of contracts on official NASS yield estimates for certain crops and areas has led to another area of study within the field. These contracts can
be used as imperfect substitutes for crop insurance for the producer. Vukina, Li, and Holthausen (1996) examined the way these might be used in conjunction with commodity price contracts to hedge risk. A mean-variance utility function is assumed which allows a solution to be obtained for optimal price and yield futures positions. The authors show the version of these equations that minimize the variance of profit as in McKinnon (1967). The derived solution for the utility-maximizing positions in the price and futures markets contain four terms each: a term for price hedging, a term for yield hedging, a speculative component for price, and a speculative component for yield. The risk minimizing positions is shown to be equal to the utility-maximizing positions without the speculative components. With the given structure, the authors next show the conditions under which the existence of yield contracts will significantly reduce variance of profit when only price futures exist. In short, the yield upon which the yield contract is based should not have too high a variance. In general, however, not all risk can be eliminated even assuming no price or yield basis risk.

Synthesis

In a 1997 article, Miranda and Glauber demonstrate the potential benefit to the crop insurance industry of a market for area-yield contracts. Private insurers take advantage of the law of large numbers insuring many parties against unlikely independent events but, in the case of agricultural production, "[t]he lack of stochastic independence among individual yields defeats insurer efforts to pool crop loss risk"
among farms, causing crop insurers to bear substantially higher risk per unit of premium than other property liability and business insurers.” Therefore, without reinsurance crop insurance would have to charge premiums that would drive a great number of purchasers from the market.

Miranda and Glauber use CV of indemnities as a measure of risk. County-level and farm-level data are detrended using a quadratic regression equation of state-level yields. A regression equation for each farm is estimated using adjusted county yield as the independent variable. A simulation is then run by selecting a crop year at random and then generating farm yields using the regression equations and random draws for the error terms. Indemnities are then calculated and totaled for ten insurance companies. Risk levels range from 0.67 to 1.30. These levels are compared to CV levels of 0.05 to 0.15 in other lines of insurance. The authors estimate that crop insurers take on between 22 and 50 times the risk they would if yields were independent.

Miranda and Glauber also investigate the possibility of using area-yield contracts as tools for reinsuring crop insurance policies first assuming that such contracts are available only for the entire U.S. production area and then for every individual state. A simulation is run assuming optimal hedging under each of these two scenarios and under a third scenario assuming each insurer reinsures with the FCIC’s current Standard Reinsurance Agreement placing all policies in the
Commercial Fund. FCIC reinsurance significantly reduces risk to the insurers (CV ranges from 0.26 to 0.35). Availability of national yield contracts further reduces variability for nine of the ten largest crop insurers (CV ranges from 0.20 to 0.42). The existence of yield option contracts for all producing states reduces the CV of indemnities for all ten insurers to levels seen in other lines of the insurance industry which exist without government involvement (0.07-0.16).

\[1\] Policies in the Commercial Fund leave the insurer with the largest amount of premium revenues and with the greatest responsibility for indemnities.
CHAPTER 3
METHODS

This chapter will cover the methods used in constructing the simulation. It begins with a summary of the path which leads to FCIC returns on reinsurance activities. From here the discussion turns to the details starting with the probability distribution used to simulate county yields. This entails selection of the distribution family and the estimation of parameters. An overview of the crop insurance policies demonstrates that most indemnities result from shortfalls in farm yields and it is therefore necessary to simulate yields at the farm level in addition to those of the counties. This portion also incorporates calibration of the model such that expected loss ratios conform to assumptions. The paragraphs which follow motivate the importance of correlation levels among the sources of risk. The means of incorporating correlation within the model is then explained. Obtaining from indemnities the FCIC’s financial obligations to insurers necessitates an examination of the Standard Reinsurance Agreement. The implementation of the particulars of the SRA concludes the chapter.

The Simulation Model

The first objective of this study is to determine the value at risk of the FCIC’s reinsurance obligations. It is achieved by a Monte Carlo simulation in which draws for yields are made and indemnities computed based on these draws. The simulation is run on a spreadsheet with the use of a software add-in by the Palisade Corporation
called @Risk. This software permits the user to designate probability distributions and the relevant parameters according to which random draws will be made. The method used here is to first make a draw from a uniform distribution with limits zero and one. This draw can be viewed as representing a probability. A draw for a yield value is made by using this probability value in the inverse of the yield distribution function. After yields are determined for an iteration, indemnities are computed based on the particulars of the various policies. Using the specifications elaborated in the Standard Reinsurance Agreement, the FCIC’s share of revenues and expenses are then tallied.

**County Yields**

**Accounting for Changes in Productivity**

The first step is to assign a distribution function for county yields. This requires the use of data on county-level production. A difficulty in determining the probability distribution function for yields lies in the fact that the function almost certainly changes each year. Production levels for a given year can be viewed as the result of a draw from a probability distribution. However, since production possibilities change each year, the draw for next year's production level must be made from a different distribution function. The annual county yields recorded by the National Agricultural Statistics Service are therefore generated each from a different distribution and are not immediately useful for estimating the 1997 distribution function. Productivity changes over the years to an extent such that a typical yield in
one particular year would be quite unlikely in another year. In using historical data, one must take into account the fact that technological and other changes have altered the production possibilities.

With this in mind, an effort is made to adjust the data for changes in productivity. This adjustment can be made by identifying the trend in yields. As the desire is merely to isolate a general trend line and not to explain every annual change, only simple linear regressions are investigated.

One might consider identifying a single trend to adjust all yield data for productivity. Alternatively, it could be argued that the trend is not the same throughout the nation and that it would be proper to investigate the specific trend in each state or in each county. The objective here is to bring historical yields up to a level which reflect the 1997 production possibility levels while maintaining the influence of good and poor growing conditions in the data. Using national data to identify the trend limits the influences of local weather conditions on the regression. If, for example, there were a few years of very poor conditions in Iowa, a regressed trend of Iowa data might produce detrend indexes which remove some of the effects of weather rather than removing the effects of technological growth. As the U.S. is internally a free-trade economy, technology available in one part of the nation is generally available in all other parts. Therefore, national yields are used to obtain a single trend line for the entire nation. (The effect of using county-specific trends is investigated in the appendix). The form used is
\[ \ln(y_i) = \beta_0 + \beta_1 t + \varepsilon, \]
where \( y \) represents U.S. yield, \( t \) is a time index representing the crop year, \( \varepsilon \) is a random disturbance, and the \( \beta \)'s are parameters to be estimated. Use of this form suggests that productivity increases at a constant proportion each year. The estimated parameters for the three crops is given in table 1.

Table 1. Yield versus time regression results.

<table>
<thead>
<tr>
<th>Crop</th>
<th>Intercept</th>
<th>Time parameter</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn:</td>
<td>4.5276</td>
<td>0.01803</td>
<td>0.4302</td>
</tr>
<tr>
<td></td>
<td>(169.1)</td>
<td>(5.051)</td>
<td></td>
</tr>
<tr>
<td>Soybeans:</td>
<td>3.422</td>
<td>0.01453</td>
<td>0.6143</td>
</tr>
<tr>
<td></td>
<td>(194.1)</td>
<td>(6.183)</td>
<td></td>
</tr>
<tr>
<td>Wheat:</td>
<td>3.4158</td>
<td>0.00875</td>
<td>0.4417</td>
</tr>
<tr>
<td></td>
<td>(223.1)</td>
<td>(4.448)</td>
<td></td>
</tr>
</tbody>
</table>

Using the estimated equations, a prediction is made for the 1997 crop year for each crop. This expected yield for 1997 is then used with the expected yields for all other years to compute an index used to detrend the county-level observations. That is, \( I_i = \hat{y}_{1997,i} / \hat{y}_{1997} \). The historical county yields are all multiplied by the corresponding index so as to obtain 26 1997-equivalent yield observations.

In light of this method of adjusting for changes in productivity, an additional comment might be made in regard to the model selected for detrending the yield data. With a natural log regression, the implication is that the trend increases by a constant rate. The objective is to identify the trend. It is not argued here that the logarithmic
regression is necessarily the best explanation of changes in yields. It must be kept in
mind, however, that the purpose of the regression in this case is to determine the
productivity adjustment factors to use with the county data. More complex models
which provide a better fit will generate productivity adjustment factors which reduce
the variance in the adjusted data. Since the ultimate objective is analysis of risk,
paths which may understate risk should be avoided. An alternative where a perfect fit
for the national yields is assumed is considered in the appendix.

The Functional Form of the Distribution

It was decided to simulate county yields by drawing from the beta family of
distributions. An advantage in the beta distribution is that the values of the
parameters can be set so that there will be a negative, positive, or no skew to the
distribution. Skewness in yields has been identified (Gallagher 1987, Ramirez 1997)
and is particularly important for this study in which insurance payments result from
lower-than-average yields. As a means of capturing potential skewness, the beta
distribution has been used to model the behavior of yields in various studies (Nelson
and Preckel 1989, Babcock and Hennessy 1996). In a recent article, Just and
Weninger (1999) raised concerns regarding the methods others have used to reject
normality in the distribution of yields. Because the beta distribution can take on
various forms of skewness and kurtosis, it is useful here because the shape can be
determined by the data and by the estimator rather than by assumption. The beta
distribution has the density function
\[ f(y) = \frac{\Gamma(p + q) \, (y - \text{min})^{p-1} \, (\text{Max} - y)^{q-1}}{\Gamma(p) \Gamma(q) \, (\text{Max} - \text{min})^{p+q-1}} \text{ where } \text{min} \leq y \leq \text{Max} \]

where \text{min} and \text{Max} represent the limits of the range in which the random variable \( y \) may fall. \( \Gamma() \) represents the gamma function and \( p \) and \( q \) are parameters which will influence the shape of the density function.

Estimating the Parameters of the Distribution Function

Following Babcock, Hayes, and Hart (1996), the parameters \( p \) and \( q \) are estimated for each county using the method of moments equations

\[ p_i = \left( \frac{\mu_i - \text{min}_i}{\text{Max}_i - \text{min}_i} \right)^2 \left( 1 - \frac{\mu_i - \text{min}_i}{\text{Max}_i - \text{min}_i} \right) \left[ \frac{\sigma_i^2}{(\text{Max}_i - \text{min}_i)^2} \right]^{1/2} - \frac{\mu_i - \text{min}_i}{\text{Max}_i - \text{min}_i} \]

\[ q_i = \left( \frac{\mu_i - \text{min}_i}{\text{Max}_i - \text{min}_i} \right) \left( 1 - \frac{\mu_i - \text{min}_i}{\text{Max}_i - \text{min}_i} \right) \left[ \frac{\sigma_i^2}{(\text{Max}_i - \text{min}_i)^2} \right] - 1 - p_i \]

found in Johnson and Kotz (1970). The \( i \) subscripts are added to represent counties 0, 1, 2, ... \( n \).

In order to compute the values for \( p \) and \( q \) for the beta distribution, it is necessary to determine the values of all variables in the two equations above.

Standard deviation and mean can be estimated from the productivity-adjusted data for each county. The minimum value for yields is taken to be zero. The reasoning is that extreme conditions do actually have the potential of eliminating all production within a certain area. It should be noted, however, that this does not necessarily imply that farm, county, or state yields will equal zero with any frequency. If the historical yield
data do not contain any instances of near-zero yields, then the parameters p and q
generated from this data will ensure that near-zero draws from this distribution will
be very rare. This assertion holds up in simulation. Initial work in simulating corn
yields in Iowa counties used the beta distribution with the minimum set at zero. In
this work there were no cases of near-zero yields although the possibility for such
yields, however slight, remains. In areas where much fewer acres are in production,
where history has demonstrated that such low yields are possible for entire counties,
setting the minimum above zero would downplay the variability in production.

Determining the upper limit for the distribution is handled in a different
manner (The following draws heavily from suggestions given by Sergio Lence). The
maximum value among a set of draws from a beta distribution is certain to be less
than the upper limit of the distribution. The difference between the highest possible
value and the maximum observed will depend on the parameters of the distribution
and the number of draws made. If measured as a percentage, however, simulations
indicate that the shortfall between observed and potential is not dependent on the
upper limit of the distribution when other parameters are fixed.

To examine what is the likely shortfall, a simulation is run in which sets of
twenty-six draws are made from beta distributions with one thousand iterations. The
number of draws corresponds to the number of years of county data available. The
distributions all have the same minimums and upper limits. They have different
values for p and q. The values chosen for p and q are all between two and fifteen
inclusive. This was the range observed in initial tests on the county data when these
parameters were estimated using the method of moments equations above and setting
the upper limit of the distribution at values between 100% and 150% of the maximum
observation in each county. In all cases the value for \( p \) is greater than the value for \( q \)
but the difference between \( p \) and \( q \) is never more than seven. The result, then, is one
thousand observed maximums for each of the fifty-six sets of \((p, q)\) parameter values.

The shortfall of each of these simulated maximums is computed as a
percentage of the observed maximum. That is, the value computed is the percent by
which the observed maximum among twenty-six observations would need to be
increased in order for it to equal the upper limit of the distribution from which the
draws were made. The mean shortfall for each set of parameters is then computed.

The next step is to use regression analysis. Higher values for \( p \) tend to move
the concentration of observations to the right—closer to the upper limit of the
range—for a given value of \( q \). Higher values for \( q \) have the opposite effect for a
given value of \( p \). Thus, the shortfall should vary inversely with \( p \) and should increase
with \( q \). The relation appears not to be linear, however. A bit of experimentation
finds a least squares regression of the log of the mean shortfall on the log of \( p \) and on
the log of \( q \) to be a good predictor given the specification of the parameters. (It could
be mentioned here that a simple linear regression of the shortfall on the values of \( p \)
and \( q \) has high explanatory power—a high R-squared—but the logarithmic regression
yields smaller prediction errors. See figures 1 and 2.) The estimated equation is

\[
\ln(M) = -2.5426 - 0.9442 \ln(p) + 1.7203 \ln(q) \\
(-56.61) (-36.49) (82.94)
\]
Figure 1. Average multiplier needed to bring maximum observation to the distribution's upper limit.
Figure 2. Average multiplier needed to bring maximum observation to the distribution's upper limit.
with t-statistics given below the parameter estimates. The R-squared value for the regression equation is equal to 0.9926.

So there is now an equation from which the upper limit of the beta distribution can be estimated given the values for \( p \), for \( q \), and given the historical data. The difficulty which remains, however, is that the values for \( p \) and \( q \) are determined by the method of moments equations in which the maximum is a parameter. The method of moments equations and the regression equation are all nonlinear ones sufficiently complex enough to make a solution to the system difficult to find. Convergence, in this case is found by an iterative process. For the great majority of cases, a convergence is found yielding values of \( p \) and \( q \) within the range mentioned above and a value for the maximum of the range usually between 5% and 20% above the observed maximum.

For a small number of cases, there is no convergence. For these instances, the difference in maximums is minimized. Beginning with the observed maximum, \( p \) and \( q \) are computed using the method of moments equations. These values are then used in the estimated regression to produce an estimated maximum. The estimated is compared to the observed maximum. If they are not equal, the observed maximum is increased and values computed again. So long as the difference is decreasing, the process continues. When the difference ceases to diminish, the increased value of the observed maximum is taken to be the upper limit of the distribution and the values for \( p \) and \( q \) which are derived from this upper limit are used as the parameters of the distribution.
The Insurance Programs

Four insurance programs are examined here. Actual Production History (APH) catastrophic coverage and buy-up coverage are similar in their structures. Each guarantees a specific level of production which is based on a percentage of an average of the individual’s previous production. Catastrophic coverage pays indemnities when yields fall below 50% of the average. In such a case, a payment is made to the producer in the amount of the yield shortfall multiplied by 60% of the FCIC expected price. Mathematically, the per acre indemnity is written

\[ \text{indem}_{\text{cat}} = \max(0, 0.6 \cdot P \cdot (0.5 \cdot Y - y)) \]

where \( P \) is the FCIC expected price, \( Y \) is the FCIC expected yield, and \( y \) is the producer’s yield realization. Note that under catastrophic coverage there is no indemnity paid unless the harvest yield is less than half the expected yield. Note also that the market price does not figure into the formula. It affects neither the amount of the indemnity nor the probability that an indemnity will be paid.

Computation of indemnities for APH buy-up coverage is similar except the fixed parameters are replaced with choice variables. The producer is permitted to choose the level of yield protection and the price election within certain ranges:

\[ \text{indem}_{\text{bup}} = \max(0, A \cdot P \cdot (B \cdot Y - y)) \]

where \( P \), \( Y \), and \( y \) are defined as before. The parameter \( A \) is the price election chosen by the producer and \( B \) is the yield coverage selected. The price election can be
chosen from a range of 0.6 to 1.0 in increments of 0.05. Coverage level can be 0.5, 0.65, or 0.75.

The Group Risk Plan (GRP) is another alternative available to producers. GRP pays indemnities based on county yields. This may be an attractive option for producers who have had abnormally poor yields in their recent history which has driven down the guarantee. As the FCIC makes an assumption of rather low yields when a farmer lacks production records, GRP may also be an attractive option for a producer who has not kept records. In addition, since indemnity payments are based on the yields of a large area which includes many producers, much of the moral hazard problem of typical insurance contracts is eliminated—individual producers have little control over county yields. Also, verification costs of Multiple Peril Crop Insurance is eliminated for the insurance company as indemnities are based on county yields reported by NASS (Skees et al. 1997).

The effectiveness of GRP depends on a positive correlation between the yield of the individual producer and the county yield. For producers whose yields are not highly correlated with that of the county, GRP is ineffective as an insurance tool.

Indemnities for GRP are computed somewhat differently from indemnities for the APH policies. Using $X$ to represent the expected county yield and $x$ to represent the realized county yield, indemnities for GRP can be expressed as

$$\text{indem}_{\text{GRP}} = \max(0, A \cdot P \cdot \left(\frac{CX - x}{CX}\right) \cdot X).$$
Note here that the coverage level, \( C \), does not define the upper limit of indemnification as it does under APH buy-up. In the unlikely event of a county yield of \( x = 0 \), per acre indemnities for those insured with GRP would be equal for all producers who have chosen the same value for the scaling factor \( A \) regardless of their choice of coverage level \( C \). This is not true for realized county yields greater than zero. Under GRP the producer can select coverage levels between 0.7 and 0.9 in increments of 0.05. The scaling parameter \( A \) ("protection" level) must be between 0.9 and 1.5 in increments of 0.05.

A recent addition to the policies available is Crop Revenue Coverage (CRC). This is a revenue protection product. A revenue guarantee is made based on 95% of futures price levels prior to planting (the base price) multiplied by a fraction of the producer's expected yield. At harvest, futures prices are reexamined. If futures prices have fallen, the revenue guarantee remains unadjusted. If, on the other hand, prices have risen, the higher values are used to compute the revenue guarantee.

Indemnities for CRC can be stated as

\[
\text{indem} = \max_{\text{CRC}}(0, 0.95 \cdot \max(S, H) \cdot B \cdot Y - H \cdot y)
\]

where \( S \) and \( H \) are measures of the planting and harvest prices as defined by the CRC policy, \( Y \) and \( y \) are the expected and realized yields respectively and \( B \) is the coverage level selected by the purchaser. The measure of the planting price (or base price) is the average daily settlement price in the month prior to planting of the contract of the month immediately following harvest. The harvest price is the
average daily settlement price in the harvest month of the contract of the month immediately following harvest. Coverage elections range from 0.5 to 0.75 in increments of 0.05.

**Calibration**

**Calibrating Catastrophic Coverage**

The mechanism for generating county yields has been explained above. But, as seen in the above paragraphs, only one policy insures county yields and only a small portion of insurers' liability depends upon the Group Risk Plan. The vast majority of indemnities result from shortfalls in individual producers' yields and revenues. Generating yields for each participant in the crop insurance programs would be an extremely burdensome task. Instead, since the county yield is merely an average yield of all production within the county, farm yields are assumed to be distributed around this mean. Nonrandom samples are then selected from the distribution of farm yields. By drawing nonrandom samples it can be certain that certain points along the continuum of yields will be measured in each county and in each iteration. Since it is yields in the left hand tail which generate most indemnities it is important that these be represented consistently in each iteration.

Nonrandom draws are selected in the following manner. The participating farm acres are divided into deciles. The yield for each decile of farm acres is a percentage of the mean (county) yield. This percentage is assumed to be normally distributed and centered at one. The percentage for the first decile, \( x \), will be where
the value for x which satisfies the equation \( F(x) = \frac{1}{11} \) where \( F(.) \) is the density function for the normally distributed variable x. For the second decile, the value will be the x which satisfies \( F(x) = \frac{2}{11} \). This pattern continues up to the tenth decile where x satisfies \( F(x) = \frac{10}{11} \). With probability levels chosen in this way, the probability for each decile is equal distance from the next decile's probability level. Thus the probability of the first decile is 0.091 above zero and is 0.091 less than the probability for the second decile.

Since it has been assumed that the mean of x is one, what is needed to solve for x is the coefficient of variation of farm yields within each county. It is assumed that the FCIC has priced each policy such that the expected loss ratio (indemnities divided by premiums) equals one. Manipulation of the aforementioned coefficient of variation will alter the loss ratio for each yield drawn for the county. Coefficient of variation is therefore chosen for each county such that the expected loss ratio for the APH catastrophic loss policies in that county equals one (Credit must be given to Bruce Babcock and Dermot Hayes who suggested this method). The calibrated coefficient of variation determines each decile's yield in relation to the average for the county. If, for example, the coefficient of variation is determined to be 0.25, the lowest ten percent of the county acres will have a yield which is 66.6% of the county average. A coefficient of variation of 0.5 means that the yield for these same acres will be 33.2% of the mean. Upper and lower limits are set at twice the mean and zero respectively. Recall that there are no choice variables with catastrophic coverage so
that there is only one value for the coefficient of variation that will satisfy the equality.

In almost all cases, a figure for the coefficient of variation can be found to equate expected indemnities to premiums. For a small number of counties, no number can be found to satisfy the equality. When no coefficient of variation can be found to bring expected indemnities down to premiums, a figure of 0.1 is assigned. Reductions below this level have a negligible effect on loss ratios. For the cases of expected loss ratios below one for all coefficients of variation, a value of 8.75 is used. Increases beyond this figure result in almost no increase in expected loss ratio.

Investigating Variability in the Distribution of Yields within a County

Up to this point, the discussion has proceeded under the assumption that the coefficient of variation of farm yields within a county is a constant. While it is reasonable to assume that individual farm yields may be represented as being distributed about a county average, it is likely less plausible that this distribution remains the same year to year. Obviously, the mean of this distribution will change. The shape, it would seem, may also vary depending upon, among other factors, the effects of weather conditions. One can consider various weather patterns that would alter the deviation of individual yields from the average. Under drought conditions, for example, it may be the case that all producers are affected similarly and yields may have less deviation from the county average than is normally experienced. Alternatively, it may be the case that some production units experience very low yields while others are not severely affected. In this scenario the variance of yields
within the county will be high. During flood years, which can also drive average yields to low levels, some land may have extremely low yields while others, located on higher ground or further from overflowing banks, may not experience much of a decline in production. In such a case, the mean is low and the variance is high.

Under favorable growing conditions it would also seem that superior results may be nearly universal or rather unevenly distributed. However, if the idea of an upper limit on productivity of land is accepted, then this has an implication for the distribution of yield values within a county. If the county yield is near its production possibility limit, it must be the case that individual units are near their own upper limit with very few exceptions. Therefore, the coefficient of variation must be rather small. If it were large then fields with yields far below their maximum would need to be offset by fields with yields above their maximum. This is obviously not possible assuming the premise is accepted.

In order to investigate the hypothesis of a relation between average and CV, individual farm yield data is compared to county yields. The farm-level data comes from FCIC records of APH participants. The data for corn covers the crop years 1983-1994. Using counties where there is data for at least 100 producers, the CV of yields within the county is calculated for each year. This figure is then divided by the average CV for that county across the series. The result is an index for each county for each year. If a long series of data were available, the CV index for year t could be compared to the county yield for year t to provide evidence for or against the hypothesis. However, since the series is short, combining the time-series with the
cross-sectional data may better demonstrate the behavior of CV when county yields change.

Of course, each county has a different yield distribution. Therefore, rather than comparing a county's CV index for a particular year with the county yield for each year, an effort is made to place all county yields on a similar scale so that the low end of the scale is not dominated by counties with low expected yields and the high end is not exclusively populated by counties with high yield potential. To do this, each county's (productivity-adjusted) yield for the years 1983-1994 is used in the estimated beta function for that county. This will produce a number between 0 and 1 which is the value of the cumulative distribution function and can be interpreted as a probability value. Note that this is the reverse of the process by which yields are drawn in the model.

A scatter diagram of the CV ratio versus the beta value appears to support the suggestion that there is an increase in CV when yields are low and that the dispersion of yields within a county is reduced when the county yield is high (see figure 3). In addition, the scatter of data points is visibly attenuated as the beta value increases. A regression is run with the form

$$\ln(CVR_i) = \alpha_0 + \alpha_1 \cdot b + \epsilon_i$$

where CVR is the ratio of the county's observed CV to its average CV, b is the value of the county's beta function, the \( \alpha \)'s are estimated parameters and \( \epsilon \) is a random error term. The estimated equation is given by
Figure 3. CV index versus the value of the beta function: corn.
\[
\ln(CVR) = 0.3983 - 0.8280 \cdot b \\
(23.62) (-31.75)
\]

with t-statistics given below estimated parameter values. The R-squared statistic for the regression equation is 0.5101. Since there appears to be heteroskedasticity present, a regression is run with the squared errors as the dependent variable and the beta value as the only independent variable. This returns the regression equation

\[
e = 0.1087 - 0.0792 \cdot b \\
(12.86) (-6.06)
\]

with an R-squared statistic equal to 0.03657. Using the White test for heteroskedasticity, the R-squared statistic is multiplied by 971, the number of observations. If the errors are homoskedastic, this statistic has a chi-squared distribution with one degree of freedom. In this case the value of the statistic is 35.51 so homoskedasticity can be rejected.

The problem of heteroskedasticity is addressed by weighting the observations by the square root of the value of the second regression equation. Running the weighted least squares regression yields the estimated equation

\[
LCVR^* = 1.3907 - 3.3074 \cdot b \\
(22.48) (-34.58)
\]

where \(LCVR^*\) is equal to the natural log of the CV ratio divided by the square root of the predicted squared error. The R-squared statistic for this equation is 0.5527. When the dependent variable of this equation is multiplied by the weights used to
adjust the observations, the fitted equation is quite similar to the one obtained from
the ordinary least squares regression.

The static value for each county's CV of individual yields can now be
replaced with a value which is multiplied by a random variable. The mean of the
multiplier is determined by the weighted least squares equation for the CV ratio. The
standard deviation of the multiplier is given by the square root of the estimated
equation for the squared errors.

The scatter diagrams for soybeans and wheat appear similar to the one for
corn exhibiting downward trends and decreasing variance. Likewise the regression
equations for these two crops are similar to those derived from the corn data. The
ordinary least squares regression for the soybean data is estimated as

\[
\ln(CVR) = 0.3473 - 0.9938 \cdot b \\
(18.99) \quad (-27.41)
\]

with an R-squared statistic of 0.5079. The estimated equation for the wheat data is
given by

\[
\ln(CVR) = 0.3127 - 0.8279 \cdot b \\
(18.06) \quad (-26.02)
\]

with an R-squared value equal to 0.4101. Again, the squared residuals are regressed
on the beta values to test for heteroskedasticity. The intercept and slope parameters
for the soybean data are equal to 0.0954 and -0.0619 respectively with t-statistics
equal to 14.75 for the intercept and equal to -4.83 for the slope parameter. The chi-
squared statistic for the regression equation is equal to 22.66 permitting the null
hypothesis of homoskedastic residuals to be rejected. Adjusting the observations as

\[ LCVR^* = 1.2520 - 3.9242 \cdot b \]

\[ (18.96) (-29.98) \]

which has an R-squared statistic equal to 0.5526.

Using the same weighting process on the wheat data results first in the

estimated equation for the squared residuals with an intercept equal to 0.1048 and a

slope parameter of -0.0597. T-statistics are equal to 15.06 and -4.67 respectively.

The high value for the chi-squared statistic (21.37) again indicates the presence of

heteroskedasticity. The weighted least squares regression which follows results in the

estimated equation for the adjusted data given by

\[ LCVR^* = 1.0745 - 3.0749 \cdot b \]

\[ (17.70) (-27.57) \]

with an R-squared statistic equal to 0.4383.

Specifying the Relation between each Decile and the County Average

Incorporating all these results and summarizing the process of determining

yields within a county, the farm acres each county has with catastrophic coverage is

divided into ten equal parts. The ten divisions or deciles are ranked by their ex post

yield realizations from worst to best. The yield for each decile is calculated as a

percent of the county yield:

\[ y_i = Y \cdot r_i \]
where \( i = 1, 2, \ldots, 10 \) and \( Y \) is the realized county yield. Farm yields are distributed normally about the county yield. The percentage that is multiplied by the county yield to obtain the decile's yield is given by

\[
r_i = \Phi^{-1}\left(\frac{i}{11}, 1, s\right)
\]

where \( \Phi^{-1} \) is the inverse of the normal probability distribution function, \( \frac{i}{11} \) is the probability, and 1 is the mean. The variable \( s \) is the standard deviation in this equation and the coefficient of variation of farm yields within the county. As discussed above, it appears that this number tends to have a lower mean and variance when growing conditions are favorable and a higher mean and variance under adverse growing conditions. These effects are incorporated by giving \( s \) the form

\[
s = \bar{s} \cdot \text{Exp}(Q)
\]

where \( \bar{s} \) is a fixed value calibrated such that the county's expected loss ratio for APH catastrophic coverage is equal to one. \( Q \) is a normally distributed random variable with mean \( \alpha_0 + \alpha_1 \cdot \beta \) and variance \( \gamma_0 + \gamma_1 \cdot \beta \). The \( \alpha \)'s here are the WLS estimated parameters for the CV. The \( \gamma \)'s are the estimated parameters for the squared errors. The variable \( \beta \) in both equations is the correlated probability draw from the uniform distribution used in the county beta function to obtain the county's yield. Making substitutions, the yield for each decile is given by

\[
y_i = Y \cdot \Phi^{-1}\left(\frac{i}{11}, 1, \bar{s} \cdot \text{Exp}(\Phi^{-1}(p, \alpha_0 + \alpha_1 \cdot \beta, (\gamma_0 + \gamma_1 \cdot \beta)^{0.5}))\right)
\]

where \( p \) is a probability draw from a uniform distribution with bounds zero and one and all other variables are as defined above.
Calibrating Buy-Up Coverage

Having determined the manner for calibrating catastrophic coverage, buy-up coverage is considered next. APH buy-up is calibrated by adjusting price elections ("A" in the above equation describing APH buy-up indemnities) and yield coverage levels (B). An attempt is first made to equate expected losses to premiums by adjusting the price election and assuming a coverage level of 0.65. If there is no price election within the permitted range (0.6 to 1.0) which will accomplish this, coverage level is changed to either the lower (0.5) or higher (0.75) level as necessary with price election then being adjusted to equate losses to premiums. In some instances, there are buy-up policies but no catastrophic coverage purchased within the county so that standard deviation has not been determined. When this is the case, a coverage level of 0.65 and a price election of 0.75 is chosen and standard deviation is selected so that expected buy-up loss ratio is one under these conditions. Sometimes, because of limits on the choice variables, the coverage levels needed to equate losses to premiums is in between those levels available (0.5, 0.65, 0.75). When this occurs, price elections are set at 1 and total indemnities are given by a linear combination of indemnities at the lower level and of indemnities at the higher level. I.e.

\[ \text{indem}_{\text{BUP}} = x \cdot \text{indem}(1, B_i) + (1 - x) \cdot \text{indem}(1, B_j). \]

where \( x \) is a number between zero and one and the B's are each one of the three available coverage levels. The implicit assumption in such a case is that a certain
portion, \( x \), of the acres in the county are insured at the first coverage level and the remaining acres are insured at the other coverage level.

Calibrating Group Risk Plan

GRP is handled in a similar manner. As with APH buy-up, there are two choice variables for GRP policies, the coverage variable and a scaling variable. Calibration begins by finding any value for the scaling factor within permitted bounds (0.9 to 1.5) which equates the expected loss ratio to one when coverage is at the highest level permitted, 0.9. If this cannot be accomplished, it is attempted at the next coverage level, 0.85. This process is repeated until the premiums and losses are equated. As with APH buy-up, it is sometimes necessary to use a linear combination of indemnities at two different levels of coverage.

The three preceding policy calibrations are handled by calculating yields at probability levels 0.001, 0.002, ..., 0.999. Indemnities are then figured for each probability level. Expected indemnity is assumed to be the average of these. The parameters are adjusted as described above until the mean of indemnities is equal to premiums.

Calibrating Crop Revenue Coverage

Calibration of CRC policies is handled in a different manner. The purchaser is permitted to select a coverage level between 0.5 and 0.75 in increments of 0.05. A simulation is run and indemnities calculated for each county at each level of coverage. Higher coverage levels result in higher indemnities when indemnities are paid. The mean of indemnities for each coverage level is computed and compared to
premiums. The coverage levels which result in mean indemnities immediately above and immediately below premiums are used to represent the county's CRC policies. The linear combination of the indemnities from these two coverage levels which equates them to premiums is found. Again, the logic is that x percent of acres are covered at the first coverage level and the remaining (100-x) percent are at the other level.

Systemic Risk

The Effect of Systemic Risk on Indemnities

One of the more important challenges in making draws for county yields lies in imposing reasonable correlation levels among the draws. The level of correlation among county yields will have dramatic effects on the shape of the indemnity distribution. Preliminary tests supported this as discussed below. Dramatic changes in indemnity statistics result when low levels of correlation are imposed compared to outcomes when high levels of correlation are imposed.

Yields being independent would imply that high or low yields observed in one county would provide no information about yields in another county. Likewise, if it were the case that indemnities are paid in one county this would reveal nothing about the likelihood that indemnities will be paid in the neighboring county. If, however, yields are positively correlated, as one would expect, this should increase the variance of total yield-based indemnities. The intuition is that below-average yields will tend to occur among counties in the same years. The result will be increased instances of
years when total indemnities are quite low and increased instances of years in which indemnities are quite high. This should serve to fatten the tails of the distribution.

To examine the effect of correlation, indemnity levels for GRP are estimated for Iowa corn using a spreadsheet simulation. Again, the beta distribution is used to model county yields with parameter estimates obtained as described above. As the interest here is an examination of correlation levels' effect on variability of returns, policy choice variables (coverage level and scale) were not calibrated but given constant values. A coverage of 0.9 and a protection level of 1.4 is used for all policy indemnities. Thus the mean value for indemnities is not expected to equal premiums at this point but changes in the standard deviation and in the VAR as imposed correlation levels are altered will demonstrate the importance of imposing correlation on the simulations.

The means by which correlation is imposed will be discussed in greater detail below. In short, the counties are separated into divisions and subdivisions based on geographic location within the state. Random draws are made each iteration to represent state, divisional, subdivisional, and county yields. Correlation is imposed between each sequential level of draws. That is, a certain correlation, $r$, is imposed between the state draw and the draw for each division. The value of $r$ need not be the same for each division. Likewise, the draw for each subdivision is also assigned a correlation value, $s$, between the parent division and the subdivision. The same is done for each subdivision and the counties within that subdivision. For purposes of examining the asset correlation-indemnity variance relationship, three separate
correlations are tested. Three simulations are run. Each time, the selected correlation level is imposed at all geographic levels.

Each simulation uses 5,000 iterations. The first level imposed is $\rho = 0.1$ where $\rho$ represents Spearman's rank correlation coefficient. At this level, the mean level of indemnities is $2.4$ million. Indemnities range from a low of $0.7$ million to $5.0$ million. The standard deviation of indemnities is equal to $0.58$ million. VAR at the 5% level is $3.5$ million.

The second test uses a correlation of 0.5 at all levels. Under these conditions, the mean is unchanged but the range of results increases. The lowest result is $0.2$ million. The highest is $6.0$ million. Standard deviation increases to $0.99$ million. VAR ($0.05$) is now $4.2$ million.

When a correlation of $\rho = 0.9$ is used, the measures again indicate increased risk while the mean remains at $2.4$ million. Indemnities now range from zero to $11.9$ million. The standard deviation jumps to $2.56$ million. VAR at the 5% level also increases substantially--to $7.6$ million at this correlation level.

From these results it can be seen that large changes in the level of correlation imposed can result in sizable differences in risk. Since the FCIC's total obligations increase at increasing rates as indemnities rise, measures of the FCIC's risk may be even more sensitive to the correlation level.

Positive correlation among risks reduces diversification potential. In the crop insurance market, correlation among indemnities is a result of correlation among
yields. Therefore, it is important for the simulation to maintain an appropriate relationship among yield draws.

**Imposing Positive Correlation on Yield Draws**

Johnson and Tenenbein (1991) demonstrate a method of imposing correlation on draws from two marginal distribution functions using linear combinations of draws from the marginals. If it is desired that draws be made from two correlated distributions for random variables A and B, draws are first made from these two distributions for preliminary variables a and b. The variable A is then merely assigned a value equal to a. The variable B is computed as a function of a linear combination of a and b. $B = h(c \cdot a + (1 - c) \cdot b)$. Johnson and Tenenbein give the specification for $h(.)$ depending on the distributions from which A and B are drawn and give the values of c for the level of correlation between variables A and B. They do not discuss extensions beyond the bivariate case. Their method is used here in the following manner.

A single draw is made from a uniform distribution with a range of zero to one which can be viewed as a probability level for the nation's corn yields for a single year. This value is not directly transformed into national yield. Its purpose is to permit the imposition of correlation on yield draws at the county level. Probability statistics are drawn for each state included in the simulation. The linear combination method is used to impose correlation between the national and each state's draw. The state is divided into a small number of divisions. A probability statistic is drawn for each of these divisions using the method of Johnson and Tenenbein to impose a level
of correlation between every division's draw and the draw for the state. Each division is divided into three to five subdivisions which also receive draws that are correlated with their parent division. Each subdivision contains a number of counties. Draws are made for the counties to represent $F(y)$ as described above but each of these draws are correlated with the draws for the parent subdivision. The effect is that all county yields are correlated with all other county yields but counties share the highest levels of correlation with counties in the same subdivision. Lower levels exist between counties in the same division but different subdivisions and still lower levels between counties in different divisions.

The historical correlation levels between national, state, divisional, subdivisional, and county yields is examined using the productivity-adjusted yields, (since there is a general upward trend in yields, using the unadjusted figures would overstate correlation levels). In general, correlation levels are quite high in high-production areas and vary widely in other areas. In a small number of instances where there are fewer years of data, a negative correlation level is computed. These cases are all in low-production counties where there are only a small numbers of acres going in and out of production over the years. Since it does not seem reasonable that negative correlation would actually exist, these are assigned a correlation level of zero in the simulation.

A summary of the process by which county yields are obtained is now in order. Each iteration begins with a draw for a variable, $U$, from a uniform distribution with limits of zero and one. This draw does not, by itself, determine the
national yield. Similar draws are made for each state. The draw for each state has correlation with the variable U imposed on it. Draws from uniform distributions are made for divisions within each state. Each of these is correlated with the draw for the state variable. Within each division, draws correlated with the divisional variables are made for subdivisions. Draws are again made from uniform distributions for each county. These have correlation imposed on each with the draws for the subdivision.

The correlated county values from the uniform distributions, \( h_c \), are used as probability values to obtain county yields from the estimated beta distributions specific to each county. When the value of a random variable is desired and the probability is known, the inverse of the variable's distribution function is required. In this case, the Excel BETAINV function is used.

**Imposing Negative Correlation Between Yields and Prices**

Recall that CRC guarantees revenues rather than production. The peculiar mechanism by which the price component of the revenue guarantee is determined requires two price levels for each iteration. The first represents an average of futures prices early in the crop year. This price level is known by the insurer and by the producer before entering into the insurance agreement. It is therefore not represented by a random variable but by a constant. The second is an average of the futures prices in the harvest month. The relevant data was assembled from the FCIC Managers Bulletins and calculated by Chad Hart of the Center for Agricultural and Rural Development at Iowa State University who made the figures available.
It is assumed that the natural log of prices is normally distributed. This is an assumption commonly made for commodity prices. The assumption of lognormality means that the distribution can be completely described by mean and standard deviation alone. These statistics were computed for the 1989-1998 data series and used as parameters for the price distributions.

An important factor to maintain is the negative relationship between prices and quantity. Correlation coefficients are figured for productivity-adjusted yields and futures prices using the following formula:

\[ \rho = \frac{\frac{1}{n} \sum (p_h - p_f)(y - y_t)}{\left( \frac{1}{n} \sum (p_h - p_f)^2 \right)^{0.5} \left( \frac{1}{n} \sum (y - y_t)^2 \right)^{0.5}}. \]

In this formula, \( p_f \) represents futures prices at planting. Futures prices at harvest are represented by \( p_h \). The regression equation for national yields is used to calculate expected yield, \( y_e \), and actual yield, \( y \), is the productivity-adjusted historical yield. As predicted by economic theory, for each of the three crops the correlation between yields and harvest prices is negative. The values are -0.788 for corn, -0.499 for soybeans, and -0.649 for wheat.

These negative correlation levels are imposed on price draws and the national yield draws for the corresponding crops using the method of Johnson and Tenenbein as described above. While harvest price is obtained by a draw from a lognormal distribution, there is no single draw which represents national yield. National yield is
figured by summing total production and dividing by total acres after county yields have been calculated. Johnson and Tenenbein provide the formulas necessary for imposing correlation between two marginal distributions. While their method can still be used here, some experimentation is necessary to find the value of \( c \) which generates the desired level of correlation between price and national yield. Different levels for \( c \) are used in multiple simulations until the output price and yield data consistently exhibit the correct level of negative correlation. The value for \( c \) that achieves this price-yield relationship is then used in the simulation model.

**Standard Reinsurance Agreement**

The obligations of the FCIC are determined from indemnity levels as described in the Standard Reinsurance Agreement. Insurance companies are permitted to allocate each policy they write to one of three different funds which determine the level of risk ceded to the FCIC and the level maintained by the insurer. In order to motivate the method by which the allocation is made for simulation purposes, a description of the SRA is given here.

**Attributes of the Three Reinsurance Funds**

As mentioned above, loss ratios are calibrated to one in the simulation. This is equivalent to setting expected profit to zero. Rates are set so that producers face policies that are actuarially fair. This leaves little incentive for the insurer to sell policies. The Standard Reinsurance Agreement creates the incentive. The agreement
is set up such that the greater portion of losses are borne by the FCIC. Under the SRA expected profits to the primary insurer are positive.

Under the SRA, there is also opportunity for the insurer to rid itself of its most undesirable policies and to attenuate the variance in the returns of the policies it keeps. Responsibility for undesirable policies can be ceded to the FCIC outright. Policies kept by the firm are designated to one of three reinsurance funds.

The highest-risk policies which the insurance company elects to keep will be placed in the Assigned Risk Fund (ARF). A limit for each state is placed upon the total value of policies which can be placed in the ARF. Policies allocated here have the greater portion of losses covered by the FCIC. For loss ratios between 1.0 and 1.6, the FCIC takes on 95% of the losses of all policies designated to the ARF. At higher loss ratios, the FCIC takes responsibility for an increasing portion of the indemnities. Likewise, when policies designated ARF return a profit, the lion’s share is transferred to the FCIC. For lower levels of profits, where the loss ratio for ARF policies lies between 0.65 and 1.0, the firm retains 15% of the difference between premiums and indemnities. At the most extreme, should there be no indemnities, only 7.6% of the gain is retained by the insurer.

Policies which have lower levels of risk may be allocated to the Development Fund (DF) for the state. Unlike the ARF, policies designated DF will be segregated by type into one of three categories: Fund C for APH catastrophic coverage, Fund R for policies like CRC which guarantee revenues rather than production level, and Fund B for all other policies such as APH buy-up coverage and GRP. The insurer
will be responsible for a greater portion of any losses among the policies designated DF. Again, examining loss ratios between 1.0 and 1.6, the FCIC is responsible for 75% of the losses for the B and C funds and for 70% of the losses in the R fund. As with ARF, the government assumes responsibility for an increasing share of losses at higher loss ratios. In exchange for taking on a higher level of responsibilities for losses, the primary insurer is entitled to greater portions of any profits that accrue. When indemnities range from 65% up to 100% of premiums, the firm keeps 60% of profits for DF policies in B fund or in R fund and keeps 45% of the profits from the catastrophic (C) coverage policies. Should a situation where there were no indemnities ever arise, the B and R funds would yield 31.5 cents to the insurer for every dollar of premium revenue. Policies in the C fund leave the insurer with 22.25% of the profit if there are no indemnities paid.

The third designation is the Commercial Fund (CF). The insurer will designate policies CF which the firm views as posing the least amount risk to the firm and/or stand to yield the greatest profits. Like policies designated DF, those in the Commercial Fund are divided into catastrophic, revenue, and other policy groups. Looking at losses which reach up to 60% of premiums within the group (i.e. loss ratios up to 1.6), the FCIC is responsible for half the difference in indemnities and premiums for policies in the B and C funds and for 43% of the losses in the R fund. As with the ARF and DF, FCIC responsibilities increase as losses move beyond this range. In the event of profits, CF policies in the B and R funds yield 94% of the gain to the primary insurer when the loss ratio is between 0.65 and 1.0 while those in the C
fund yield 75% of their profit to the firm. The maximum potential for CF policies is 48.9% of premiums for B and R funds and 37.75% of the premium in C fund.

Allocation of Policies to Reinsurance Funds

For the simulation, policies are allocated to funds according to levels of risk. 1997 data on total premium allocation among the reinsurance funds was obtained from the RMA. This data presents the allocation in premium dollars to each of the CF, DF, and ARF for every state. In general, states which account for very high levels of crop production have a large portion of policies designated CF by insurers. Other states have varying allocations to the three funds. The allocations seen in the data are mimicked by state in the simulation. This is accomplished by ranking the policies in each state by the standard deviation of the loss ratio. (This is obtained for each policy during calibration.) Beginning with the policies with the highest standard deviation, policies are designated ARF until the proportion of the sum of premiums so designated to the sum of all premiums in the state reaches the percentage of premiums designated ARF in the RMA data. This is then repeated for CF policies except that the process begins with the policies having the lowest standard deviation of loss ratios. All policies yet to be designated are then assigned to DF. There are a small number of policies which can not be calibrated such that the expected loss ratio is one within the choice ranges defined within the policies. Any policies which have an expected loss ratio above one, the insurer cedes to the FCIC.
CHAPTER 4
SIMULATION RESULTS

The value of the results depend upon the validity of the simulation model. If the model is incapable of replicating real world yield-indemnity combinations, there is no reason to trust the simulated reinsurance costs. The simulated indemnity payments can be compared to the yield-indemnity combinations that have occurred in the past. CRC, GRP, and Catastrophic coverage unfortunately have very short histories with only four years of results. APH Buy-up coverage has a longer history and comparisons are therefore made for indemnities under this program.

There are, however, problems with using APH Buy-up for comparison as well. The program has changed over the years in premium structure and in participation levels. Subsidies and other incentives have not remained fixed for farmers. It appears that actuarial soundness has in general improved over time. Thus, it must be remembered when comparing per acre indemnities time series data that different acres are involved in the experiment each year. As discussed in the review of the crop insurance literature, if there were more extensive adverse selection problems in the earlier years of the data series, this will make the likelihood of high average indemnities more predominant in the series. An indemnity level seen in one of the earlier years may not be realistic in a simulation which replicates the current program.
At least two adjustments can be made to make comparisons of historical and simulated data better. The first is to use per acre indemnity comparisons rather than total indemnity comparisons. The main reason for this is because of large differences in participating acres in various years. The second is to account for the changes that are made in the FCIC expected price. For each year, 1986-1998, the historical indemnity is divided by that year's expected price and then multiplied by the 1997 expected price. The adjusted per acre indemnities for each crop are then plotted with the simulated indemnities against yields. In all cases the historical points appear within the bounds of the simulated data (see figures 4-6).

An additional point is worth mentioning prior to examination of simulation results. Subsidies account for a significant portion of the government's costs in the crop insurance program. In all discussions up to this point, analysis has proceeded without regard for sources of premium revenue. When premiums are greater than indemnities, part to the insurance company's gain is transferred to the FCIC as outlined by the SRA. This is not, however, a profit to the FCIC. A large portion of the premiums paid to the insurers are actually paid by the FCIC. In all simulation results there are no cases where a positive return from the reinsurance business conducted by the FCIC comes up to the value of the premium subsidy paid by the FCIC.

The FCIC also pays an administration and operations expense (A&O) subsidy to the insurer for policies sold. For APH buy-up policies, the insurance company
Figure 4. APH Buy-up simulated and historical indemnities: corn.
Figure 5. APH Buy-up simulated and historical indemnities: soybeans.
Figure 6. APH Buy-up simulated and historical indemnities: wheat.
receives a payment equal to 27% of the premiums. For GRP the subsidy is 25%. For CRC it is 23.25%. No A&O subsidy is paid for APH catastrophic coverage.

As these two subsidies depend on demand by producers and are not affected by yields or harvest prices, they are not considered in the simulations. They can, however, be calculated and added to simulated results of the reinsurance business to obtain a measure of total cost to the government (less operating costs). Premium subsidies are included in the FCIC summary of business data. A&O subsidies are calculated using the subsidy rates mentioned in the previous paragraph and the premium amounts for each policy given in the summary of business data. The subsidy total is given with the results for each crop. Since these subsidies are unaffected by yields and harvest prices, their inclusion merely shifts the distributions horizontally. Thus, the mean, minimum, maximum, and VAR statistics would all be reduced by the same value. Standard deviation of returns is unaffected.

In a simulation of 2,500 iterations, the expected value for the FCIC is a net outflow of $36.3 million from reinsuring the corn crop. A net outflow occurs 40.5% of the time. The minimum and maximum values among the results are -$1,383.8 million and +$277.7 million. Standard deviation of returns is equal to $295.5 million. The value at risk (VAR) at the 5% level is -$633.8 million. The VAR at the 10% level is -$487.2 million. The total premium subsidy for the corn crop for the included insurance programs is $202.3 million. The A&O subsidy is calculated to be $98.3 million. Taking these into account, the minimum, mean, and maximum return values become -$1,684.4 million, -$336.9 million, and -$22.8 million respectively. The
VAR figures at the 5% and 10% levels fall to -$934.4 million and -$787.8 million respectively.

The results for reinsuring soybeans are similar although on a somewhat smaller scale. The expected value for this crop is a net outflow of $24.6 million for the FCIC in its capacity as reinsurer. Outflows are greater than inflows in 42.2% of all iterations. Results range from a minimum of -$1,078.8 million to a maximum gain of $221.1 million. The standard deviation of returns is equal to $227.7 million. The VAR at the 5% level is -$498.6 million. The figure for the 10% level is -$344.3 million. The premium subsidy for the soybean crop is $141.5 million. The A&O subsidy is $56.6 million. Inclusion of these bring the minimum, mean, and maximum net returns to -$1,276.9 million, -$222.7 million, and $23.0 million respectively. VAR (.05) falls to -$696.7 million. VAR (.10) falls to -$542.4 million.

The mean in the case of wheat is an expected loss to the FCIC of $17.6 million. In a majority of cases (50.6%), the expenses to the FCIC are greater than revenues from reinsurance activities. The results have a range with the limits smaller than those of the other two crops (-$524.9 million, $190.8 million). The standard deviation of returns is equal to $111.8 million. VAR for wheat reinsurance at the 5% level is -$226.2 million. At the 10% level the VAR is -$171.5 million. Inclusion of the premium subsidy ($151.2 million) and of the A&O subsidy ($71.2 million) bring the minimum, mean, and maximum to -$747.3 million, -$240.0 million, and -$31.6 million respectively. VAR (.05) becomes -$448.6 million. VAR at the 10% level falls to -$393.9 million.
The second portion of this study is to investigate the hedging potential of area yield and commodity price contracts for the crop insurance reinsurer. In order to do this, an addition is made to the simulation making national yield an output of each iteration. This figure is merely the sum of the production for all counties divided by all acres in production. Note that these must also include acres not insured.

**Hedging Corn Reinsurance Risk**

With this additional yield figure as an output, the relationship between yields and reinsurance cost per acre can be examined. Analysis will proceed on a scale of dollars per acre. A scatter diagram of the FCIC cost for corn graphed against the U.S. corn yield shows a trend starting with yield near 75 bushels per acre and net cost near $30 per acre (see figure 7). The worst-case point has a yield of 86.7 bushels/acre and a cost of $28.62 per acre. From here, the trend appears to decline at a decreasing rate until net reinsurance costs become negative, with positive returns above $5 and yields reaching almost 150 bushels per acre. While the trend is obvious even to the naked eye, there is also obvious variation in returns at every yield. Instances of positive net costs can be noted at all yield levels. There are cases in which the FCIC reaps a positive return even when yields fall as low as 94 bushels/acre.

The effect of hedging with yield futures contracts is first investigated. These contracts do currently exist for corn and are traded on the Chicago Board of Trade with a slightly different measure of yield than used here. The existing contracts are based on NASS yield estimates of harvested acres. The analysis used here assumes
Figure 7. Reinsurance costs: corn.
that contracts are available based on planted acres. It is assumed that contracts are purchased far enough in advance so that they are priced at their unconditional expected value. The mean value of corn yields is 117.6 bushels per acre. It will be assumed that it is at this value that the reinsurer takes its position in the futures market.

As noted above, a diagram of reinsurance costs versus national yields reveals a downward trend as should be expected. Therefore an offsetting position would be one in which declining yields return increasing financial gains. This is the case for a short position in corn yield futures. What remains is to determine the size of the short position. A trendline can be constructed by regressing costs on yields. As the naked SRA position of the reinsurer has a slope which appears to decrease at a decreasing rate, a linear trendline is not the best summary of the data. As a contract in the futures market has a linear payoff, however, the linear regression gives a good indication of the position which can be offset by taking a position in yield futures. The slope of the regression line is about -0.26. This means that over the range of the sample per acre cost will fall by about $0.26 for every one bushel per acre decrease in corn yields. The offsetting position would there for be one which returns $0.26 for every unit decrease in corn yields.

Assuming that the short position is entered at the unrestricted expected value of 117.6 bushels per acre, the expected return of the contracts will be zero. Over the range of simulation yields, however, the return ranges from a loss of $7.89 at the highest yield to a gain of $10.70 at the lowest yield. The standard deviation of returns
is $5.12. Adding the return from shorting corn yield futures to the return from reinsuring corn policies gives the net position.

To the extent that the short reduces deviations from the mean of costs, the futures position is an effective hedge. The expected net cost is unchanged from the unhedged position of the SRA ($0.75). Compared to the original, standard deviation of costs is reduced considerably ($6.11 vs. $3.34). The maximum is reduced in magnitude from $28.62 to $20.64—still significant but improved from the naked position. The minimum net cost is actually increased by the short position rising from a net gain of $5.74 to $8.14. VAR at the 5% level is reduced in absolute terms from $13.11 to $6.58. The figure for the 10% level falls from $10.08 to $4.44.

A scatter diagram of the net costs reveals a trend which appears to be parabolic and convex to the x-axis (see figure 8). The trough of the pattern appears to be between yields of 95 and 105 bushels per acre. As with the unhedged costs, the dispersion of costs appears to decrease as yields increase. When compared to the diagram of the original costs, the benefits of the hedge is apparent (see also figure 9). A great many instances of cost above the maximum and below the minimum of the net position can be found on the diagram representing reinsurance cost without hedging instruments. The tradeoff of hedging can also be seen in the diagram. The linear return of the short futures position, while reducing the level of losses for the lowest yields, also causes a large number of positive returns from the SRA to become net losses resulting from high yields.
Figure 8. Corn reinsurance net costs with yield futures hedge.
Figure 9. Value at risk: corn reinsurance costs.
It may be the case that the reinsurer is only concerned with downside risk. If there is no great aversion for variance of costs so long as there are no traumatically high losses, it is possible that this may be achieved by holding put options on corn yields. The put option will begin to return to the holder when yields fall below the strike value. The return is linear over the range of yields below the strike so that each one bushel drop in per acre yields increases the payment to the holder of the option by a constant amount. Should the final yield be greater than the strike at settlement, the option expires with no value. The loss in this case is limited to the initial payment for the purchase of the option.

A strike value of 120 bushels per acre is selected as this is near the mean value of yields. Holding put options at this strike value will only attenuate losses which occur when yields fall below 120 bushels per acre. Again, to determine the investment necessary to offset the losses, a linear summary of the slope of the costs from the SRA must be figured. This time, however, only the slope of the trend below 120 on the x-axis is estimated since the payoff of the option will not affect results above this figure.

The slope of the regression is -$0.38. Thus, on average, for every one bushel drop in per acre yields below 120, costs of the reinsurer are increased by about $0.38 per acre. The offsetting put position will therefore return a like amount for a similar fall in yields. This means that the slope of the put position must be -$0.38 over this range. Above this range the slope of the position will be zero.
An essential part of determining what the net position will be is knowing what
the cost of the hedge will be in this case. It is assumed here that the market for these
contracts is efficient and the cost of the options is equal to the expected return of
holding the options. To do this the return for each of the 2,500 simulated yields and
the mean of these returns is computed. The formula for the settlement payment is
\[ \text{max}(0, 120-y) \times 0.38 \] where \( y \) is the realized national corn yield. The mean of these
gross payoffs is $3.75 which is taken to be the cost of the contracts in an efficient
market. The return of the options will not be positive until this cost is covered. This
happens at a yield of about 110.25 bushels per acre.

Again, the mean for the net is unchanged from the unhedged mean cost
($0.75). The standard deviation is reduced from $6.11 to $3.17. Note that this
reduction is only slightly greater than that seen with the short position in yield
futures. This is somewhat unexpected. Standard deviation here is a measure of
dispersion about the mean. While the futures position is used as a tool for reducing
this dispersion, the put option is used only to reduce the downside risk. Also
perplexing are the comparisons of the downside risk reduction by the two
instruments. While the position in yield puts does a better job of reducing standard
deviation of costs, it appears that the futures position is better at suppressing
downside risk. \( \text{VAR}(0.05) \) goes from $13.11 to $6.58 with the short position in the
futures market and to $7.20 with the put contracts. The potential culprit here may be
the high costs incurred in purchasing the yield puts ($3.75). One way of reducing this
cost without increasing the downside risk at low yields is for the reinsurer to
simultaneously sell call options. With this strategy, up-front total expenditures on the hedge portfolio is reduced. This comes at the cost of potential losses if yields are high. The use of such a put-call combination is examined below.

A scatter diagram of the net position compared to the unhedged shows points to the right of the strike value lowered vertically by the cost of the option (see figure 10). Points to the left of the strike are moved vertically by an amount equal to the payoff of the options less the fixed cost of the options. Note that in some cases (yield > 110.25) the cost is greater than the payoff so that these points are lowered. As with the case with the futures hedge, the risk-reducing benefits are visible.

While a great many positions could be examined depending on objectives and motivations, a single addition will be explored for this crop before adding commodity price contracts to the analysis. It was noted earlier that the short position in yield futures does a fair job of reducing the measures of risk used here. It was further noted that part of the limitation is the fact that the return on the futures position is linear with respect to changes in yield whereas the position that needs to be hedged appears to be nonlinear with respect to yields. To some extent nonlinear hedge positions can be constructed using options on futures. Adding a second contract to the portfolio may have the benefit of reducing total variation if it targets the upper portion of the range of yields.

The relevant contract for targeting costs where yields are above the strike is a call option. This is because of the fact that returns for the call option change only when the strike is exceeded. The fact that costs are decreasing as yields increase
Figure 10. Corn reinsurance net cost with yield puts.
means that the offsetting position will be the sale of call options. This means that the reinsurer will owe the purchaser of the call options when yields rise above the strike value. What remains is to determine the amount of the sale. A regression of the points with yields above 120 bushels is run. The summary slope is -$0.13 per bushel. The sale of contracts which reduces costs by this amount generates $0.93 in revenues. Therefore the net cost of the portfolio of options is $2.82. The addition of the second contract reduces the standard deviation of costs to $3.12. VAR is likewise reduced to $6.63 at the 5% level and to $4.13 at the 10% level. A scatter diagram shows what appears to be a roughly horizontal trend—a significant change from the original position determined by the SRA (see figures 11 and 12).

It could be hypothesized that traditional derivative contracts on commodity prices have potential risk management uses for the reinsurer. Price figures directly into the indemnity computation of one of the insurance products available to producers (CRC). Hedging only against low yields therefore leaves the reinsurer exposed to potential losses due to price changes. In addition, there is a high negative correlation between price and yield. If one is primarily concerned about yield risk, one might consider the use of price contracts as a substitute if yield contracts are not available.

Assuming that only contracts on corn prices are used to hedge reinsurer risk, separate regressions are run on outcomes with harvest prices above and below $2.75 per bushel. This strike price is selected because it is near the expected price of $2.73. Points on the low end of the price range have an estimated slope of $9.62. For those
Figure 11. Corn reinsurance net cost with yield puts and calls.
Figure 12. Value at risk: corn reinsurance net costs.
above the strike price, there is an estimated increase in cost of $12.03 for every dollar increase in the harvest price. Purchasing the put contracts and writing the call options which offset these positions yields a net gain to the reinsurer of $2.25 (the put options cost less than the gain from selling the call options). Not surprisingly, the position in the commodity price derivatives is not as effective at reducing risk as is the position in the yield options (see figures 13 and 14). With the price options, standard deviation is $4.74 compared to $6.11 with the naked position and $3.12 with the position in yield contracts. VAR(.05) is equal to $10.07, falling in between the unhedged and yield-hedged figures of $13.11 and $6.63. Likewise, the figure for the 10% level which is $7.14 is better than the result with no hedge ($10.08) but not as good as the result using yield contracts ($4.13).

An initial glance at a scatter diagram of unhedged costs to the reinsurer plotted against the harvest price gives one the impression that costs to the reinsurer tend to increase as the harvest price increases. Closer inspection, however, reveals what may be a dramatic change in the slope. While the costs to the reinsurer appear to generally increase as price increases, the increase becomes more pronounced as price rises above $2.75. If the effects of yield can be removed, it may even be the case that the slope is negative for prices below $2.75. This pattern makes sense under certain conditions. Examination of the indemnity function for CRC illustrates the relation between indemnities and harvest price. The indemnity function can be written
Figure 13. Corn reinsurance net cost versus harvest price.
Figure 14. Corn reinsurance net cost with price options.
\[ I = \max (0, I') \]

where \( I \) is the per-acre indemnity level and \( I' \) is the difference in the guaranteed level of revenue per acre and the realized revenue per acre:

\[ I' = \overline{R} - R = cPy' - P^*y. \]

The guaranteed level of revenue is represented here by \( \overline{R} \) while the realized revenue is given by \( R \). The guarantee is the product of the selected coverage level, \( c \), the price component, \( P \), and the expected yield, \( y' \). Realized revenue is calculated as the product of the commodity price at harvest, \( P^* \), and the realized yield, \( y \). The price component of the guarantee, \( P \), is the larger of the expected price and the harvest price multiplied by 0.95:

\[ P = 0.95 \max (P^*, P^*). \]

The effect of changes in the harvest price can now be investigated. The condition for an indemnity to be paid while the harvest price is greater than or equal to the expected price can be given by

\[ I = I' = 0.95cP^*y' - P^*y > 0 \quad (4.1) \]

and therefore,

\[ 0.95cy' - y > 0 \quad (4.2). \]

Differentiating equation 4.1 with respect to harvest price gives

\[ \frac{\partial I}{\partial P^*} = \frac{\partial I'}{\partial P^*} = 0.95cy' - y. \]
Note that this can be signed positive since it is identical to condition 4.2. Therefore increases in harvest prices will increase indemnities when an indemnity is being paid and harvest price is above the expected price. It follows that an increase in the harvest price under these conditions will increase the costs of the insurer and therefore will also increase the costs of the reinsurer. This somewhat counterintuitive outcome is due to the unique feature of the CRC contract in which the revenue guarantee is increased if harvest prices rise above the expected prices. Under the conditions above, increasing the harvest price raises the guaranteed revenue at a rate greater than the rise in the realized revenue.

If indemnity payments are made when harvest prices are below expected prices, equation 4.1 is changed to

\[ l = l' = 0.95cP' y' - P^* y > 0 \quad (4.3) \]

where the expected price has now been substituted for the harvest price in the revenue guarantee portion of the equation. Differentiating equation 4.3 with respect to harvest price now yields

\[ \frac{\partial l}{\partial P^*} = \frac{\partial l'}{\partial P^*} = -y \]

which is obviously negative assuming a nonzero yield. Therefore, increases in harvest prices will decrease indemnity payments as long as the harvest price does not move above the expected price. Again, it follows that returns to the insurer and to the reinsurer will increase as harvest price increases over this range.
Putting these two results together indicates that returns to the reinsurer should tend to decline as harvest price moves away from the expected price. With this analysis, one would expect that the appropriate hedge combination would be a portfolio consisting of call purchases and put purchases with strike prices near the expected commodity price (a long straddle). The net effect of this hedge will be to reduce net returns when the harvest price is near the expected price and to increase returns when the harvest price moves away from the expectation.

To examine the additional benefit of including commodity price contracts, the net return on the yield-hedged portfolio is used. A regression equation is first estimated for price-return combinations in which price is below $2.75. (The expected price for the 1997 corn CRC program is $2.73.) The slope of this line is -$2.14. Therefore, the offsetting position will decrease returns by -$2.14 per acre for every $1 increase in the harvest price of corn. The regression equation for results with price above $2.75 per bushel has a slope of $2.41. The offsetting position increases returns by this amount for each dollar increase in the harvest price.

It appears that the incorporation of price contracts into the hedge portfolio has very little effect on the distribution of results. The cost of the put-call combination with these slopes is $0.78. With their inclusion, the standard deviation is reduced only slightly to $3.07 compared to $3.12 when using yield options only. Value at risk falls from $6.63 to $6.58 at the 5% level but moves from $4.13 to $4.17 at the 10% level (see figure 15). Looking at the range of outcomes, the minimum moves from a net gain of $8.96 to $8.65 and the maximum cost level falls from $18.61 to $18.08.
Figure 15. Value at risk: corn reinsurance cost.
Histograms of net returns with and without hedging instruments in figures 16 and 17 demonstrate two points. The first is that there is a wide variation in potential returns with small probabilities of very high costs. This shape of the density function is altered considerably when price and yield contracts are included in the portfolio (see figures 18 and 19). Where there was previously a long negative tail with a gradual increase and an abrupt end near the peak, there is with the hedge a density with a more compact look to it and rapidly decreasing positive and negative tails. The second point, however, is that there remains a substantial amount of variation in the returns. While the likelihood of the worst results is much reduced or even eliminated, the range appears no smaller that it was originally (in fact, it is slightly greater).

Hedging Soybean Reinsurance Risk

A scatter diagram of the returns from the reinsurance activities with the soybean reveals a pattern similar to the one seen in the corn crop (see figure 20). There is, of course, a difference in the ranges over the yield figures. The curvature of the trend also appears somewhat less pronounced. As with the corn output, it appears that the heteroskedasticity in this series takes the form of decreasing variance as yields increase. The low end of the pattern begins with national yields just below 30 bushels per acre and costs to the reinsurer as high as $25 per acre. From there the
Figure 16. Distribution of unhedged corn reinsurance costs.

Figure 17. Distribution of corn reinsurance costs with price and yield options.
Figure 18. Cumulative distribution of unhedged corn reinsurance costs.

Figure 19. Cumulative distribution of corn reinsurance costs with price and yield options.
Figure 20. Reinsurance costs: soybeans.
trend appears to decline at a decreasing rate to yields just under 48 bushels per acre and positive net returns above $5 per acre.

A short position in yield futures is again used to construct a hedge to the original position. As done for corn, a simple linear regression is performed on the simulation results. The slope of the regression line is -$0.88. A hedge position is constructed with a short futures position having a slope equal to this regression and crossing the x-axis at the unrestricted expected yield of 39.7 bushels per acre.

Plotting the net position reveals a change similar to the one for the first hedge construction for corn (see figure 21). The trend of the net position appears slightly parabolic with most cost results between -$5 and +$10 although a small number of points fall outside of this range. Analysis of summary statistics shows an unchanged mean cost of $0.58 per acre. Standard deviation falls from $5.39 to $3.37. The VAR statistics decrease from $11.81 and $8.16 to $6.95 and $4.15 for the 5% and 10% levels respectively (see figure 22).

Replacing the short futures position with a put purchase and a call sale results in some slight changes in these statistics. Selecting the strike value at 40 bushels per acre for both contracts, two separate regressions are run; one for all instances with yields below the strike and another for all with yields above the strike. The slope for the lower end is -$1.16. The slope for the upper portion is -$0.51. The net cost of the contracts to eliminate these slopes is $1.60. The position diagram of the option portfolio looks like the short position in the futures market except that there is a kink at the strike value of 40.
Figure 21. Soybean net reinsurance cost with yield futures.
Figure 22. Value at risk: soybean reinsurance costs.
The scatter diagram of the net position also has changes from the diagram of the net position with the futures short. The points representing the net returns look to have a generally horizontal trend with variations tending to remain in the range -$5 to $10 with a small number of deviations outside of these bounds. The summary statistics are somewhat improved. Compared to the futures hedge, standard deviation is slightly lower at $3.29. VAR at the 5% level is somewhat decreased ($6.60) while it is also a bit better at the 10% level ($4.33).

Usage of the more familiar commodity price options is also investigated here as it is with the position in the corn market. The strike price for the option contracts is at $6.95 per bushel—near the expected price of $6.97. Regression analysis on the unhedged returns yields a slope of $2.69 per dollar change in harvest prices below the strike. The trend for the yield-hedged results with harvest prices above $6.95 is 6.22. The initial net gain from taking the offsetting position is $2.41. The net effect is a moderate, though still visible, attenuation of risk. Standard deviation is reduced from $5.39 without hedging to $5.01 when the price contracts are included. The value at risk figures move from $11.81 and $8.16 to $10.90 and $7.52. The extreme points also move with the maximum cost figure falling from $25.56 to $23.06 and the minimum decreasing from -$5.24 to -$6.68 (see figure 23).

When the use of price options in addition to the position in yield options is investigated, the sign of the slope of points below $6.95 changes to negative as is expected. The estimated slope of this portion of the diagram is -$0.34. For points
Figure 23. Soybean reinsurance net cost with price options.
above $6.95 the least squares regression estimates a slope of $2.91. The cost of the straddle is $0.39 per acre. The results of purchasing the price puts and calls are ambiguous (see figure 24). There is a negligible reduction in standard deviation vis-à-vis the position with yield contracts alone ($3.28 per acre versus $3.29 per acre) but the VAR figures are slightly worse ($6.67 versus $6.60) at the 5% level and unchanged at the 10% level ($4.33).

Inspection of the change in the VAR graphs demonstrates the effectiveness of the various hedge instrument combinations (see figure 25). Usage of price options alone makes a small improvement in VAR. With the availability of yield options, which, target the source of risk more directly, risk is reduced more significantly. Adding price options to the yield hedge has an imperceptible effect on the VAR. Examination of the histograms and distribution schedules of costs reinforce the results of the VAR graph (see figures 26-29). Holding price and yield options transfers the frequency of extreme results toward the mean.

**Hedging Wheat Reinsurance Risk**

A plot of the results for the wheat simulation has some similarities to the plot of returns for the other two crops. The scatter conforms to the other patterns in that the dispersion seems wide at the lower yield levels and attenuates as yields rise (see figure 30). One difference that is visible, however, is that the pattern of the wheat data shows considerably less curvature than is visible in the case for the plots for the other two crops. In addition, the range of cost possibilities appears to be much
Figure 24. Soybean reinsurance net cost with price and yield options.
Figure 25. Value at risk: soybean reinsurance cost.
Figure 26. Distribution of unhedged soybean reinsurance costs.

Figure 27. Distribution of soybean reinsurance costs with price and yield options.
Figure 28. Cumulative distribution of unhedged soybean reinsurance costs.

Figure 29. Cumulative distribution of soybean reinsurance costs with price and yield options.
Figure 30. Wheat reinsurance cost.
smaller for wheat. The pattern begins with a small number of simulated observations
having yields between 25 and 28 bushels per acre and net costs to the reinsurer from
zero to $10. The trend then falls at what appears to be a nearly linear rate to yields
just under 42 bushels per acre and negative costs (positive returns) nearing $4 per
acre.

Constructing a hedge with a short position in the yield futures market is
handled here in the same manner as previously. The summary slope of the naked
position is found to be -$0.46 per acre. The short position which counterbalances this
nets a reduction in standard deviation ($1.61 versus $2.21) and reduced VAR figures
(from $4.47 to $3.44 at the 5% level; from $3.39 to $2.47 at the 10% level). As with
the other crops, a visual inspection of the net position appears to have a curve convex
to the x-axis though it is a very subtle one for the wheat data. The bulk of
observations fall into the range of -$4 per acre to +$4 per acre (with some exceptions
above this range; see figure 31).

As done previously with soybeans, the benefits of permitting a kink in the line
which describes the hedge position is investigated here. The change in slopes is
achieved by replacing the futures contracts with a long put position and a short call
position. The strike value chosen is at 34 bushels per acre which is near the mean
yield. The regression slope for points to the left of this value is -$0.56. The slope for
the upper portion is -$0.41. The subtlety of the difference in these slopes compared
to the one for the futures hedge suggests that the benefits of allowing the change in
slope may not be large. Indeed, the measures of risk used here are not greatly
Figure 31. Wheat reinsurance net cost with yield futures.
affected. Standard deviation falls from $1.61 to $1.60. VAR(.05) realizes a slight improvement falling from $3.44 to $3.40. VAR(.10) declines from a value of $2.47 to $2.44 (see figure 32).

The potential benefit of price contracts is also examined with the wheat crop. The strike price for these contracts is selected at $4.00 which is close to the expected price of $3.99 per bushel. Regression on the lower-price results indicates a summary line with slope equal to $1.51. Analysis on the second portion of the data points yields a regression equation with slope equal to $0.84. The position in the options market which has slopes equal to these yields an up-front net gain of $0.61 per acre.

The net effect of including the price contracts is inferior in terms of risk-reduction to the wheat portfolio which contains yield options as hedging instruments (see figure 33). Standard deviation of returns to the reinsurer is reduced somewhat from $2.21 to $1.96. The VAR figure for the 5% level is a bit improved ($4.07 versus $4.47) while the figure for the 10% level is also improved ($3.39 versus $3.00). The maximum cost has gone from $10.37 to $8.97. The minimum cost to the reinsurer also falls slightly moving from -$3.77 to -$3.93.

The same strike price is used when considering the addition of price contracts to the hedge using yield options. Using the yield-hedged returns as the dependent variable, the slope of the lower portion of the scatter is estimated to be -$0.35. The estimate for the slope of the upper portion is equal to -$0.10. This slope has a negative sign while a positive one is expected. It should be noted that the t-statistic for this parameter estimate is not significant whereas all other slope estimates which
Figure 32. Wheat reinsurance net cost with yield options.
Figure 33. Wheat reinsurance net cost with price options.
have been used to construct hedge positions for corn, soybeans, and wheat have been significant without exception. This may be due, in part, to the fact that of the three crops, wheat has both the smallest allocation of CRC policies both in terms of premium dollars ($49.3 million versus $82.5 million for soybeans and $137.9 million for corn) and as a percentage of total crop premium dollars (15.7% versus 29.3% for soybeans and 30.3% for corn). The benefit of using the hedge constructed from these parameter estimates is small and ambiguous (see figure 33). Compared to the results from using the yield hedge alone, standard deviation is unchanged ($1.60). VAR(.05) is improved a small amount moving from $3.40 to $3.37 but the figure for the 10% level is slightly worse rising from $2.44 to $2.49 (see figure 34). Inspection of the VAR schedules and the distribution diagrams indicate that the hedging instruments are less effective for the wheat crop (see figures 35-39).

**Hedging Total Reinsurance Risk**

A final point well worth investigating is the usefulness of the hedge positions on the total portfolio of the reinsurer. In so doing, care must be taken to include the correlation across the different crops. As the level of correlation among areas of production greatly affects the ranges and frequency of returns, so does the level of correlation among the crops.

Recall that for each crop a series of (yield, return) points are generated. In this final simulation, each iteration draws one of these yield realizations all with equal likelihood and the corresponding return is noted. This time, however, the historical
Figure 34. Wheat reinsurance net cost with price and yield options.
Figure 35. Value at risk: wheat reinsurance net costs.
Figure 36. Distribution of unhedged wheat reinsurance costs.

Figure 37. Distribution of wheat reinsurance costs with price and yield options.
Figure 38. Cumulative distribution of unhedged wheat reinsurance costs.

Figure 39. Cumulative distribution of wheat reinsurance costs with price and yield options.
correlation levels among the three crops is imposed not with a variation on the method of Johnson and Tenenbein but with the correlation matrix which the @Risk software makes available to the user. For each iteration, then, correlated draws are made for national crop yields and, since the returns to the reinsurer are positively correlated with the yields, returns for each crop will also move together.

These correlated returns are summed for each iteration to give the total return to the portfolio. The result is an expected cost of $0.56 per acre and extreme values of 14.26 and -$4.55. The standard deviation is $3.36. Value at risk is computed for the 5% and 10% levels. The values for these two statistics are $7.13 and $5.52. For each sum, then, the hedge position for each crop is added. The net cost is then calculated with the hedge instruments included. The attenuation in variability is noteworthy. While the mean is unchanged, the standard deviation of returns is less than a half of what it was ($1.56). The VAR measures of risk are also reduced by large amounts (see figure 40). These are now $3.31 and $2.44 for the .05 and .10 levels respectively.

A visual inspection of the graphed results with and without hedging instruments gives the impression that the hedged distribution would be preferred by the risk-averse reinsurer (see figures 41-44). Comparison of summary statistics reinforces showing reduction in standard deviation and VAR statistics reinforces this. A more rigorous investigation, however, would include the testing for stochastic dominance.
Figure 40. Value at risk: total reinsurance net costs.
Figure 41. Distribution of unhedged total reinsurance costs.

Figure 42. Distribution of total reinsurance costs with price and yield options.
Figure 43. Distribution of unhedged total reinsurance costs.

Figure 44. Distribution of total reinsurance costs with price and yield options.
The concept of first-order stochastic dominance involves the comparison of
stochastic returns. One distribution has first-order stochastic dominance over another
if it yields "unambiguously higher returns." A given distribution, G(), is said to first-
order stochastically dominate distribution H() if for every nondecreasing indirect
utility function u(), it is the case that
\[ \int u(x) dG(x) \geq \int u(x) dH(x). \]
This is only the case when G(x) is greater than or equal to H(x) for all x (Mas-Colell
et al. 1995). Superimposing the cumulative distribution figures of the hedged and
unhedged returns reveals that the two functions cross. This coupled with the fact that
the two distributions have the same expected return indicates that neither distribution
has first-order stochastic dominance over the other.

It is possible for a distribution to have second-order stochastic dominance
over another distribution even though they may have equal expected values. Second-
order stochastic domination of distribution G() over distribution H() means that for
every indirect utility function, u(), that is nondecreasing and concave, it is the case
that
\[ \int u(x) dG(x) \geq \int u(x) dH(x). \]
In such a case, it can be said that distribution G() is a less risky alternative to H()
(Mas-Colell et al. 1995). Another look at the superimposed graphs suggests that the
hedged distribution may second-order stochastically dominate the unhedged
distribution. The graph of the unhedged distribution begins above the hedged and is
then overtaken. There is a higher probability of the low returns with the unhedged
distribution.

A formal test of stochastic dominance using sample data from different
distributions was developed by Gordon Anderson (1996). The output is first
partitioned. The partitioning is arbitrary except that at least five observations should
be in each partition. For these data, partitions are selected at returns of -$1 billion,
-$800 million, -$600 million, -$400 million, -$200 million, 0, $200 million, $400
million, and greater than $400 million. The proportion of the total number of
observations in each partition is taken to be an estimate of the probability of a draw
falling within that partition.

Following Anderson's procedures to approximate integrals, the following
matrices are defined:

\[
I_r = \begin{bmatrix}
1 & 0 & 0 & \ldots & 0 \\
1 & 1 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 1 & 1 & \ldots & 1
\end{bmatrix}
\]

\[
I_F = 0.5 \begin{bmatrix}
d_1 & 0 & 0 & \ldots & 0 \\
d_1+d_2 & d_2 & 0 & \ldots & 0 \\
d_1+d_2+d_3 & d_2+d_3 & d_3 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
d_1+d_2+d_3+d_4 & d_2+d_3+d_4 & d_3+d_4 & \ldots & d_k
\end{bmatrix}
\]

where the \(d_i\) figures are the interval lengths. Using these definitions, the existence of
first-order stochastic dominance implies that all elements in the vector \(I_f (p^H - p^U)\) is
less than or equal to zero where \( p^H \) and \( p^U \) are the vectors of estimated probabilities for the hedged and for the unhedged partitions of the respective distributions.

Second-order stochastic dominance implies that all elements will be less than or equal to zero for the vector determined by \( I_F I_F (p^H - p^U) \). The calculations for these two vectors are

<table>
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so first-order stochastic dominance is not indicated. Note that this is as expected since the mean returns from the two distributions are equal. Note further that second-order stochastic dominance is implied so that further evidence is given that the distribution of the hedged returns is less risky than that of the returns without any hedge.

The Standard Reinsurance agreement can be viewed as a derivative contract. Payments are made based on state loss ratios in each reinsurance fund which are in turn based on yields and futures prices. Option contracts are an exchange of a payment for a commitment. The writer of an option receives a payment and agrees to make a payment to the purchaser under a set of contingencies. In effect the purchaser
is paying the seller to accept the purchaser's risk. Under some circumstances the option contract will have a positive value at the delivery date. At worst, the contract will have a value of zero. Therefore, prior to the delivery date, the option will have some positive price.

The FCIC, however, does not receive an up-front payment from the firms that it reinsures. While the SRA does not have a payoff schedule exactly like a put contract since the reinsurer sometimes receives payments at the end of the crop year, it is found here that the expected return to the FCIC is negative. The bias against the reinsurer in the SRA can be demonstrated with a diagram of loss ratios and reinsurer costs. There is not an exact relationship between national loss ratios and the cost schedule. A scatter diagram of total loss ratio and reinsurer cost does demonstrate, however, that knowing the value of the loss ratio can give a very good indication of how much the payments will be and which party will be the net beneficiary. A linear trendline added to the diagram demonstrates that the relationship is not quite linear.

Government net payments increase at an increasing rate as the loss ratio rises (see figure 46).

The expected net payoff to the insurance companies can be interpreted as the fair market value of the contract. That is, this is an amount that the insurers would be willing to pay in order to obtain the reinsurance services of the SRA. This figure, as has been mentioned, is $36.3 million for the corn crop, $24.6 million for soybeans, and $17.6 million for wheat. For the three crops together the fair market value is estimated to be $78.7 million.
Figure 45. Returns versus loss ratio.
CHAPTER 5
CONCLUSION

While the topic of crop insurance has received a significant amount of attention from economists, there has been little attempt at quantifying the level of risk that has been accepted by the government in its role as reinsurer for this market. Research has gone instead into other worthwhile topics such as welfare analysis, adverse selection and moral hazard problems, and insurability. Academic inquiry has led to reform of policy structures and to the development of area-yield and revenue insurance products. Until the appearance of the 1997 article by Miranda and Glauber, however, there was little to be found on the subject of reinsuring crop insurance in the published literature.

The first objective of this work was to determine the level of risk which is accepted by the FCIC when reinsuring crop insurance for corn, wheat, and soybeans. This is accomplished by use of a Monte Carlo simulation in which correlated yields and prices are drawn and indemnities are then computed. Using the reinsurance obligations of the FCIC as described in the 1997 Standard Reinsurance Agreement, payments to and from the FCIC are calculated from indemnity levels. With this analysis, it is estimated that there is a five percent probability that at least $1 billion in reimbursements will need to be made to insurance companies based on results for these three crops alone. This is a number greater than the worst reinsurance year for the reinsurer, 1993, in which claims against the FCIC exceeded premiums for reinsurance by $822 million (GAO 1998).
Another important result is the derived value of the reinsurance services which are provided by the FCIC to the insurance companies. There is found to be a positive net transfer payment to the insurance industry due to the specifications of the SRA. In addition to the administrative and operations expense subsidy paid to insurance companies, there expected net transfer of $78.7 million. The actual transfer, of course, varies from year to year depending on indemnities and can sometimes be a net transfer from the industry to the government if indemnities are low enough. It should also be remembered that net transfers to firms in any year come in the form of reduced losses and not as increased profits. The figure of $78.7 million includes only the value of reinsurance services. It does not include other costs and benefits associated with the SRA. While this figure is small compared to the potential losses under the worst-case scenarios, it should be acknowledged that this is a real cost to the government and it is a real benefit to the insurers.

There are both systemic and nonsystemic components to crop insurance risk. As with other risky financial instruments, each portion of the insurer's portfolio is neither totally dependent nor totally independent of the other components. If there were no correlation, the insurer's risk could be managed by diversification alone. If the risk were completely systemic, derivatives markets could perfectly eliminate the uncertainty.

The reinsurer's risk is mostly systemic. Obligations or profits increase at increasing rates as loss ratios move away from one. In order for premiums and indemnities to deviate far from equality, indemnities must, for the most part, be
moving in the same direction. This is systemic risk. The net value of $78.7 million is more than just what the insurer's would be willing to pay for the reinsurance services of the SRA, it is what the competitive market would require to accept the insurers' systemic risk.

The question of risk reduction for the reinsurer was also investigated. Assuming the existence of national yield contracts, various positions in the market were examined for their potential to reduce the frequency and level of extremely negative results. While the level of risk reduction was found to be appreciable, the use of these contracts alone is clearly no panacea. While no assumptions have been made with regard to the risk attitudes of the FCIC, the movement of the VAR figure at the five percent level from -$1 billion to -$467.2 million would likely be seen by a private reinsurer as significantly beneficial.

The various results of this study should be of interest for budgeting. In order to budget for a stochastic cost, it is necessary to know about the distribution of the cost variable. The output data of these simulations can be examined to determine the maximum amount that will be needed for reinsurance payments for any given level of confidence. Agricultural policymakers can also make use of these results. If policymakers determine that, for example, too much of the risk burden is borne by either the FCIC or too much by the insurers, it is relatively easy to examine alternative structures for the SRA by altering the appropriate parameters and re-simulating. It should be kept in mind, however, what the simulation model does not do. If for example one wishes to examine the effects of changes in the structure of
one or more of the insurance programs, in addition to re-calibrating one must also
determine in advance what changes this will make in premium revenues and number
of acres participating. This is necessary because the simulation model does not make
any predictions with regard to demand for insurance. Participation is fixed at the
1997 level.

One of the more important assumptions that is made in the process of
constructing the simulation model is that policies are actuarially fair. This means not
just that aggregate premiums are equal to aggregate expected indemnities but also that
there is equality between premiums and expected indemnities at the level of the
individual policy. This is almost certainly not the case for the actual crop insurance
program as the current goal for the expected aggregate loss ratio is 1.075 (GAO
1998). As noted earlier, a significant share of the crop insurance literature has been
devoted to adverse selection and moral hazard problems. An investigation into the
effects of alternative assumptions regarding the actuarial properties of the policies
requires an assumption for each individual policy and re-calibrating to reflect the
assumptions.

These are costs associated with analyzing portfolio risk analysis by
simulation. For this particular problem, however, the labor-intensive nature of these
costs compare favorably with the costs of using typical portfolio risk analysis. The
general estimation of variance for the portfolio in matrix notation is given by $w'\Sigma w$
where $w$ is the vector of portfolio weights and $\Sigma$ is the variance-covariance matrix of
the assets in the portfolio (Jorion 1997). An attempt to use this formula on the
reinsurer's portfolio is met with significant difficulty. The components of the
variance-covariance matrix are generally estimated using historical data. While there
is some historical data on returns from the reinsurance activities of the FCIC, it
cannot be said that this is equivalent to having data on, say, stock prices. While a
share of stock remains an equal share of a firm over time (assuming no splits), the
reinsurance of policies in a particular state in one year is a different liability than the
reinsurance for the same state in any other year. Not only will premiums be different
for the state differ over time, but allocations to reinsurance funds, structure of
insurance programs, and the SRA itself may change. A calculation of variance and
covariance based on these data would be highly suspect. In addition, the above
portfolio variance estimator assumes a normal distribution of returns. Simulation
results here do not appear to support this assumption for the current structure of the
SRA. The development of means to overcome these problems would be of high
value if the benefits of the simulation method and the comparable ease and
convenience of the single equation method could be found in one approach.
APPENDIX
SENSITIVITY ANALYSIS

In adjusting for changes in productivity regressions based on national yield data was used to detrend the data for all county yields. As with all choices, there are both costs and benefits to selecting this method. Previously mentioned arguments included the assumption that new innovations and new technologies can be adapted quickly throughout a free-trade economy. Furthermore, local disruptions will affect regressions on county data more severely causing greater disturbances in the indexes used to detrend the data. This is especially true for counties which have less data available.

Some of the reasons in favor of using county-specific regressions to adjust the data should also be acknowledged. The most important of these is that counties probably in fact do have differing yield trends. This is not, however, likely due to differing rates of technology advances. It is more probable that some areas have marginal land falling out of production as opportunity costs rise thus raising the average yield for the county at a rate higher than the national trend. It is also possible that the more-productive land is being removed from crop production if, for example, the better land happens to be closer to an expanding urban area. The effect, in this case, would be to reduce the average yield growth for the area.

Whether or not the selection of a national trend versus county-specific trends makes a difference in the results of this study is a topic worthy of investigation. In order to do this, data for 2,638 corn-producing counties was used. The natural log of
the county yield is regressed on a simple time-trend variable as is done with the national yield in the main portion of this study. The explanatory power of these regression estimations is wide-ranging with some effectively explaining all variation in the county's yield (R-squared > 0.999) and others having virtually no explanatory power (R-squared < 0.001). The simple average of the R-squared statistics is equal to 0.276. Some of the regressions fit the data well only because there are very few data points. Despite this, eliminating all counties with fewer than ten years of data does not seem to alter the summary much. The elimination of these 187 counties leaves a range of R-squared values from 0.000 to 0.942 which have an average of 0.263.

Among the regression equations, 295 (11.2%) have a negative slope. The average of the time parameters is 0.024. The values fall in a range of -0.741 to 0.542 but extreme values tend to be associated with counties with little data. Restricting attention to instances where at least 10 years of data are available makes little change in the average slope but brings the extremes to -0.112 and 0.232.

The regression estimates are used to construct index numbers which are used to detrend the county data as was done with the national index numbers in the methodology section. In some cases the index numbers are clearly absurd. One county, for example, has an index number for 1972 of 759,336. Many other have index numbers which are near zero. All such cases appear to be a result of a lack of data. In general, when there is a suspiciously large or small index number for a particular year for a county, there is rarely any data present for that year which will be affected by the index number which has been generated. There is, for example, no
1972 yield value to multiply by 759,336 in the county which produced this index number. The existence of counties with numbers like these causes the average to be much higher than seems reasonable, however. The mean value for all 1972 index numbers is 300.3 (eliminating this one outlier brings the average down to a somewhat more reasonable 12.6). In general counties which have higher levels of production and, therefore, more data, the index numbers produced are more reasonable. Restricting attention to counties in Illinois, Indiana, and Iowa, for example, gives an average 1972 index number of 1.40 with a range of 1.04 to 2.02.

Parameters for the beta distributions which govern county yields are determined in the same manner as previously explained. Calibration is likewise handled using the methods in the original work. Correlation levels must be recalculated for yields. Simulation proceeds as before with 2500 iterations. A comparison of the results for this simulation with those of the original simulation is worth examining.

It is not unexpected that the summary statistics for the price variable is little changed. The same distribution is used in both simulations so there is no reason to expect that the results would be different. The simulated yield figures do appear to differ with the mean yield for the second simulation being 1.63 bushels per acre greater than the mean yield for the first simulation. While standard deviation of yields is not too large, greater difference exists at the low end of the distribution of yields. With the single-regression simulation, five percent of all yields are below
84.1 bushels per acre. Under the county-specific regression, the five percent level is at 88.2 bushels per acre.

As loss ratios are calibrated to equal one, the similarity in the means of the indemnity figures is logical. It is worth noting that the measures of dispersion are also not too dissimilar despite the fact that no direct effort is made to arrive at this outcome. Estimated standard deviation of APH buy-up indemnities, for example, is increased by 2.6% when the county-specific regression is used. The same figure for catastrophic coverage is reduced by 1.2%. The decrease is 6.9% for the small GRP program. The estimate for CRC increases by 1.2%.

Of greater interest is the change in reinsurance costs. Using national yields to adjust for productivity changes in the simulation yields an expected loss of $36.3 million to the reinsurer under the SRA. The estimation of the expected net transfer to the insurers is $34.8 million under the alternative specification. There is also a small change in the estimate of the standard deviation of returns to the reinsurer ($295.5 million versus $311.2 million). The VAR figures have also been altered by the use of county-specific regressions. At the 5% level VAR moves from -$633.8 million down to -$661.7 million. The change in the 10% level is a decline from -$487.2 million to a figure of -$496.3 million.

The regression model used in the study contained only a single independent variable, a time trend, to describe the trend of yields. This resulted in R-squared statistics between 0.43 and 0.61. As stated in the text, the objective of this regression was not to identify the reason for variation in yields, but solely to identify the trend
over time. Use of a more complex model which would obtain a better fit of the data, would make changes in the output of the simulation. It was hypothesized that overfitting of the data would result in an underestimation of risk. To investigate this, behavior of the output was analyzed assuming that a regression model was used which fits the data perfectly. Since there are 26 data points, this could be accomplished with a polynomial of order 25. While it would be difficult to justify this model theoretically, this can be viewed as an extreme assumption which may help to identify the robustness of the results with respect to choice of the regression's functional form.

Rather than using expected yield for 1997 divided by expected yield for year t, a ratio of actual yields is used to create an index for each crop year. This index is then used in the same manner as described in chapter 3 to raise state, divisional, subdivisional, and county yields from previous years to 1997 levels. The remainder of the analysis proceeds exactly as previously.

As with the county-specific regressions, statistics for price are very similar to the original figures. The variability of national yields is less by 17% than that which appeared in the original simulation as expected. Standard deviation, however, is slightly greater for indemnities for each of the four different insurance programs. Paradoxically, the standard deviation for reinsurance returns a bit (5%) smaller although the range of outcomes is greater. With the exception of the worst-case results, the similarity of the statistics for the returns to the reinsurer is notable. In summary, it can be said that while estimates are not completely immune to
specification, general conclusions hold up fairly well under the three alternatives examined here.

**Comparison of Simulation Results Using a Single Regression Equation, a Perfectly Fitted Regression, and County-Specific Regressions to Adjust Historical Yields for Changes in Productivity**

<table>
<thead>
<tr>
<th>Summary statistics using a single regression</th>
<th>price ($/bushel)</th>
<th>yield (bu/acre)</th>
<th>APH</th>
<th>CAT</th>
<th>GRP</th>
<th>CRC</th>
<th>SRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>2.50</td>
<td>117.6</td>
<td>2.484</td>
<td>0.672</td>
<td>0.066</td>
<td>1.357</td>
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</tr>
<tr>
<td>st. dev.</td>
<td>0.36</td>
<td>19.8</td>
<td>2.306</td>
<td>0.678</td>
<td>0.107</td>
<td>1.295</td>
<td>2.955</td>
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<td>min</td>
<td>1.42</td>
<td>76.1</td>
<td>0.016</td>
<td>0.005</td>
<td>0</td>
<td>0.002</td>
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<tr>
<td>max</td>
<td>4.18</td>
<td>148.2</td>
<td>11.02</td>
<td>3.62</td>
<td>0.511</td>
<td>7.54</td>
<td>2.777</td>
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<tr>
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<td>84.1</td>
<td>7.017</td>
<td>2.052</td>
<td>0.312</td>
<td>3.944</td>
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<tr>
<td>VAR(.10)</td>
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<td>88.0</td>
<td>6.039</td>
<td>1.675</td>
<td>0.247</td>
<td>3.31</td>
<td>-4.872</td>
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<table>
<thead>
<tr>
<th>Summary statistics using a perfectly fitted regression</th>
<th>price ($/bushel)</th>
<th>yield (bu/acre)</th>
<th>APH</th>
<th>CAT</th>
<th>GRP</th>
<th>CRC</th>
<th>SRA</th>
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<tbody>
<tr>
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<td>0.075</td>
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<tr>
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<td>0.123</td>
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<td>2.807</td>
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<tr>
<td>min</td>
<td>1.59</td>
<td>87.6</td>
<td>0.003</td>
<td>0.001</td>
<td>0.000</td>
<td>0.001</td>
<td>-17.084</td>
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<tr>
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<td>148.5</td>
<td>13.425</td>
<td>4.598</td>
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<td>8.684</td>
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<tr>
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<td>1.708</td>
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<table>
<thead>
<tr>
<th>Summary statistics using county-specific regressions</th>
<th>price ($/bushel)</th>
<th>yield (bu/acre)</th>
<th>APH</th>
<th>CAT</th>
<th>GRP</th>
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<td>0.065</td>
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<td>0</td>
<td>0.019</td>
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<td>0.478</td>
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<tr>
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<td>2.033</td>
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<td>6.108</td>
<td>1.638</td>
<td>0.237</td>
<td>3.319</td>
<td>-4.963</td>
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REFERENCES


Mahul, Oliver. "Optimum Area Yield Crop Insurance." American Journal of Agricultural Economics. 81 (February 1999): 75-82.


