1953

The theory of psychrometry

John R. May

Iowa State College

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UMI
THE THEORY OF PSYCHROMETRY

by

John R. May

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Chemical Engineering

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The analogy between fluid friction, heat transfer and mass transfer is used in this paper to describe the behavior of the wet- and dry-bulb psychrometer. Equations are derived to describe the velocity, temperature and concentration profiles for the flow of fluids through round pipes based on a new correlation of the Prandtl mixing length with pipe diameter for fully turbulent flow. These equations are extended to apply to the case of flow of a fluid at right angles to a single cylinder.

The resulting psychrometric equation is

\[
\frac{(P_w - P_a) \Delta b}{P(1 + h_f/h_c)(t_a - t_{wb})} = \frac{M_e}{M_a} \frac{0.72 \text{Re}^{2/3} - 1 + 0.38 \text{Re}^{0.15}}{(Pr^{2/3} - 1 + 0.38 \text{Re}^{0.15})}
\]

where

\( P_w = \text{vapor pressure of the vaporizing liquid at the wet-bulb temperature} \)
\( p_a = \) partial pressure of the active component actually present in the gas stream  
\( P = \) total pressure of the system  
\( L_b = \) latent heat of vaporization of the active component at the wet-bulb temperature plus the sensible heat required to heat the vapor from the wet-bulb temperature to the temperature at the boundary of the fusion layer  
\( t_a \) and \( t_{wb} = \) true gas temperature and wet-bulb temperature, respectively  
\( h_r' = \) modified radiation heat transfer coefficient:  
\[
\frac{t_w - t_{wb}}{h_r \frac{t_a - t_{wb}}}
\]  
\( h_r = \) radiation coefficient found from modified form of the Stefan-Boltzmann equation:  
\[
h_r = 0.00692 (\epsilon)^{\frac{1}{3}} \left[ 1 + 0.25 \left( \frac{\Delta T}{T_{av}} \right)^2 \right]
\]  
\( t_w = \) temperature of enclosure surrounding the thermometers  
\( T_{av} = \) arithmetic mean of \( t_w \) and \( t_{wb} \) expressed in °K  
\( \Delta T = \) temperature difference, \( (t_w - t_{wb}) \), in °K  
\( \epsilon_w = \) emissivity of the thermometer taken here as 0.9 for the wick
\( h_c = \text{convection heat transfer coefficient found by:} \)

\[
h_c = \frac{k}{D} \cdot 4.4 \cdot (10 \cdot \text{Re})^{0.235}[1 + 0.1595 \log (10 \cdot \text{Re})]
\]

\( k = \text{thermal conductivity of gas} \)
\( D = \text{diameter of thermometer} \)
\( \text{Re} = \text{Reynolds number} = \frac{D \cdot \rho \cdot u}{\nu} \)
\( u = \text{linear velocity of gas flow} \)
\( \rho = \text{density of gas} \)
\( \nu = \text{viscosity of gas} \)
\( M_m = \text{molecular weight of gas plus vapor it contains} \)
\( M_a = \text{molecular weight of active component (vapor)} \)
\( c = \text{heat capacity of gas} \)
\( S = \text{Schmidt number} = \frac{\nu}{\lambda D_v} \)
\( D_v = \text{diffusivity of vapor through gas} \)
\( \text{Pr} = \text{Prandtl number} = \frac{c \nu}{k} \).

To determine the true gas temperature, \( t_a \), where the dry-bulb thermometer is subject to radiation error, it is necessary to use a heat balance expressed by

\[
h_r(t_w - t_t) = h_c(t_t - t_a)
\]

where

\( t_t = \text{temperature indicated by thermometer and all other symbols are as defined above.} \)
The emissivity, $\epsilon$, to be used in the $h_T$ term in this case would be that of a plain mercury-glass thermometer which is shown to be 0.73.

Equations are also given here to predict friction factors for the flow of a gas at right angles to a single cylinder.

All of the equations given apply whether the system used is the common air-water vapor system or any other combination of gas and vapor, providing the physical properties are known such as $\epsilon_0$ to make up the Schmidt and Prandtl groups. Conversely, if the properties are not known, this provides one way of determining them experimentally.
I. INTRODUCTION

The wet-bulb hygrometer is a valuable engineering and meteorological tool, the use of which dates back as far as 1792 when it was used by Sutton. Although well over a hundred papers dealt with this instrument over the next one hundred fifty years, no correct theory of its mechanism was proposed until 1932.

Prior to 1932, two distinct theories of the psychrometer had been proposed: (a) the convection theory, usually associated with the name of August [4], and (b) the diffusion theory proposed by Maxwell [19]. Both theories predict the same (correct) form of the humidity-temperature relationship; however, because of their physical incompleteness, neither form gives a correct value of the proportionality constant.

Empirical correlations by numerous workers over many years have demonstrated that for the system water vapor and air the ratio

\[(\rho_w - \rho_a)/\rho(t_a - t_{wb}) = A\]  \hspace{1cm} (1)
is constant at any given air velocity. The convection theory as advanced by Gay-Lussac [11a], Ivory [14], August [4], and Apjohn [2] predicts that the quantity $A$ is equal to $\frac{h_b C}{h_a \lambda}$, whereas the diffusion theory of Maxwell led to the quantity

$$A = \frac{h_b C}{h_a \lambda} \left(1 + \frac{n}{h_a \lambda}\right).$$

Because the value of $A$ in equation (1) is not, in general, found experimentally to be exactly equal the value of $A$ as given by either the convection or diffusion theories, it has long been the practice to determine $A$ experimentally and employ it in the form of tables. The form of the function needed to correlate values of $A$ is easily determined from such data. Thus Grossmann [12] proposed an empirical equation in which the $A$ quantity was expressed as

$$A = \frac{M_b C}{M_a \lambda} \left(\frac{a + bu}{1 + bu}\right)$$

(2)

where $a$ and $b$ are empirically determined constants, and $u$ is the air velocity.

Weber [31] and Svensson [30b] independently proposed the form

$$A = \frac{M_b C}{M_a \lambda} \left(\frac{S}{Pr}\right)^n$$

(3)
where $S$ is the Schmidt number $\mu/\rho D_{v}$ and $Pr$ is the Prandtl number of air, $C\mu/k$.

It is clear that a correct psychrometric theory should predict a value of $A$ that will be valid, not only for air and water vapor, but for mixtures of any solute vapor and carrier gas, a requirement that the foregoing equations were unable to meet.

In 1902, J. H. Arnold [3] proposed a psychrometric equation in which the effects of both molecular and eddy diffusion were considered. His equation took the form

$$\frac{(p_{w} - p_{a})}{P(t_{a} - t_{wb})} = \left(1 + \frac{h_{r}}{h_{c}}\right) \frac{M_{b} C}{M_{a} \lambda} \left\{\frac{rS + (1 - r)}{rPr + (1 - r)}\right\}$$

where $r = u_{b}/U$, ratio of velocity at the boundary of a laminar film, to average velocity in the body of a gas stream.

Using water and three different organic fluids, he was able to show that this form of equation accounted fairly well for the variations in $A$ observed with changes in velocity and also with changes in the chemical nature of the solute vapor and carrier gas.

In this equation, $(1 - r)$ is a term proportional to the resistance to both heat and mass transfer in the turbulent body of a gas stream, while the terms $rS$ and $rPr$ are proportional to the resistances to mass transfer and heat transfer, respectively, in the laminar film considered to be univer-
sally present between a solid surface and a turbently flowing fluid.

If the sum of the diffusional resistances is expressed as

\[ \varphi_D = rS + (1 - r) \quad (5) \]

and the sum of the heat transfer resistances as

\[ \varphi_H = rPr + (1 - r) \quad (6) \]

and their ratio as

\[ \frac{rS + (1 - r)}{rPr + (1 - r)} = \frac{\varphi_D}{\varphi_H} = \psi \quad , \quad (7) \]

Arnold's equation may be rewritten as

\[ \frac{p_w - p_a}{P(t_a - t_{wb})(1 + h_f/h_c)} = \frac{M_b C}{M_a \lambda} \psi \quad . \quad (8) \]

It is apparent that Arnold's contribution to the earlier psychrometric theories lay in the introduction of the quantity \( \psi \), and the idea that conduction of heat and mass by both molecular and eddy diffusion must be considered simultaneously.

The theory which leads to the expression of \( \varphi_D \) and \( \varphi_H \) by the expressions of equations (5) and (6) is based upon an
analogy between fluid friction and heat transfer first suggested by Reynolds [26] in 1874, and later modified by Prandtl [23] in 1910 and Prandtl and Taylor [25] in 1916. It has, however, been still further modified and improved by numerous subsequent writers, although with incomplete success. Experimentally, it is found that $\varphi_0$ and $\varphi_h$ are proportional to the two-thirds power of $S$ and $Pr$, respectively, a fact that led Colburn [6] to suggest his well-known empirical correlations of heat and mass transfer in terms of $j$ factors. Applying this discovery to psychrometry, Colburn [7a] suggested shortly after Arnold's paper, that $\varphi$ be written simply as

$$\varphi = \frac{S^{2/3}}{Pr^{2/3}}.$$ (9)

This expression ignores, however, any possible effect of velocity, and fails to predict $A$ as closely as does the Arnold equation.

Dr. B. F. Ruth, professor of Chemical Engineering at Iowa State College, became interested in the theory of the psychrometer, when, in 1939, he and Mr. Marvin Rodekohr [27] undertook to explain the non-adiabatic interaction of air and water vapor in a wetted wall tower. In this work they desired to determine air humidity and temperature at two-foot intervals between the top and bottom of a column four inches in diameter and 16 feet in height, through which
passed air in contact with a descending film of water. The air velocity varied from two to 25 feet per second, and as it passed through the tower it was cooled or warmed through considerable temperature ranges. This led to rather large radiation errors on both the wet- and dry-bulb thermometers. To correct these temperature readings, the value of the emissivity of a mercury-glass thermometer, of 0.632, as determined by Dropkin [9], was used. Shortly after Rodekohr's work was completed, however, it was found that if a thermometer were wrapped with a thin sheet of aluminum foil it would actually read a lower temperature when surrounded by hot walls than the supposedly true air temperature calculated using the emissivity value of 0.632. This indicated that Dropkin's value was too low, and, incidentally, that a small square of aluminum foil made a far better radiation shield than the complicated one used by Dropkin. Because of the interest thus aroused in the true value to be ascribed to this emissivity, several different attempts to determine it have been made in the Chemical Engineering Laboratory at Iowa State College since that time. Various procedures and types of equipment were used in the period from 1941 to the time of the present work. Fox [10] in 1942 reported a value of 0.79, which, if an experimental error had been corrected, would have been 0.74. Wolsted [35] in 1948 found 0.88, and
Anderson [1] in 1949 obtained 0.78. Since precise temperature measurements were necessary in the proposed psychrometric investigation, it was deemed advisable to make a new determination of the emissivity of a mercury-glass thermometer in order to see whether or not the results of these previous investigations could be brought into better agreement.
II. THEORETICAL CONSIDERATIONS

A. Emissivity of a Mercury-Glass Thermometer

In the flow of gas past a thermometer there will always be convective heat exchange between the gas and the thermometer if they are not at the same temperature. Similarly, there will always be radiant heat transfer between the thermometer and any surrounding surfaces if they are also not at the same temperature. For the case of surroundings at a higher temperature than the gas stream, the thermometer will assume an equilibrium temperature some place between the surroundings and gas so as to satisfy the equation:

\[ \frac{Q}{A\theta} = h_c(t_t - t_a) = h_r(t_w - t_t) \]  \hspace{1cm} (10)

where

- \( \frac{Q}{A\theta} \) = heat transferred per unit area and unit time
- \( h_c \) = convection coefficient of heat transfer
- \( h_r \) = radiation coefficient of heat transfer
- \( t_t \) = temperature indicated by thermometer
- \( t_a \) = air temperature
- \( t_w \) = surrounding wall temperature.
In the case where the wall temperature is considerably above the air temperature, an unshielded thermometer may indicate a temperature several degrees above the true air temperature. In such a case the true air temperature may be found if the wall temperature is known, by employing a thermometer of known emissivity, such as may be obtained by covering it with a thin surface layer of silver or aluminum foil for which the emissivity is well known. With the true air temperature thus determined from a standard thermometer, the emissivity of any other thermometer can be found. This is illustrated in the following equations, where the subscripts s and p denote the covered or shiny thermometer and plain thermometer, respectively. A heat flow balance over each thermometer for the case in which the wall temperature is higher than the air temperature shows

\[ h_{cp}(t_{tp} - t_a) = h_{rp}(t_w - t_{tp}) \]  \hspace{1cm} (11a)  

and

\[ h_{cs}(t_{ts} - t_a) = h_{rs}(t_w - t_{ts}) \]  \hspace{1cm} (11b)  

If the wall temperature is below the air temperature, then the temperature differences on both sides of the equation would be negative in sense, so the equation would still hold. Since \( t_a \) is unknown, it may be eliminated by solving for it.
in equation (11b) and substituting the result into equation (11a). From equation (11b), \( t_a \) is

\[
t_a = t_{ts} - \frac{\frac{h_{rs}}{h_{cs}} (t_w - t_{ts})}{h_{cs}}.
\]

Substituted into equation (11a) this gives

\[
h_{cs} [t_{tp} - t_{ts} + \frac{h_{rs}}{h_{cs}} (t_w - t_{ts})] = h_{rp} (t_w - t_{tp}).
\] (13)

Since the physical dimensions of both thermometers are the same and since the thermometers are both exposed to the same air stream, \( h_{cs} = h_{cp} = h_c \). This permits equation (13) to be rewritten as

\[
h_{c}(t_{tp} - t_{ts}) + h_{rs}(t_w - t_{ts}) = h_{rp}(t_w - t_{tp}).
\] (14)

The coefficient \( h_c \) may be determined from an equation obtained by Dr. B. F. Ruth [29a] from an examination of the recommended line for convection coefficients at right angles to single cylinders as reported by McAdams [20]. This equation applies over the range of Reynolds numbers 0.1 up to 500,000.

\[
h_c = \frac{k}{D} (0.45) (10 \text{ Re})^{0.235}[1 + 0.1595 \log (10 \text{ Re})]
\] (15)
The radiation coefficient can be found by employing the Stefan-Boltzmann radiation equation:

\[ h_r = \frac{0.173 F_A r \varepsilon \left( \frac{T_w}{100} \right)^4 - \left( \frac{T_t}{100} \right)^4}{t_w - t_t}. \]  

(16)

In this case \( F_A \) is unity since the thermometer bulb is essentially completely surrounded by surfaces all at a single temperature, for which case \( F_A \) reduces to \( \varepsilon \), the emissivity of the thermometer. Hereafter, for simplicity, the difference of the fourth power of absolute temperatures will be referred to as \( \Delta \phi_{wp} \) or \( \Delta \phi_{ws} \) as the case may be.

Equation (16) reduces to

\[ h_{rp} = \frac{0.173 \varepsilon_p \Delta \phi_{wp}}{t_w - t_p} \]  

(17a)

and

\[ h_{rs} = \frac{0.173 \varepsilon_s \Delta \phi_{ws}}{t_w - t_s}. \]  

(17b)

If equations (17a) and (17b) are substituted into equation (14) and rearranged, the result is
This is the equation that permits calculation of the emissivity of a plain thermometer when another of known emissivity $\varepsilon_s$ is available. The solution is obviously a trial and error one since $\varepsilon_p$ also occurs on the right; however, the computations are easily made.

B. Psychrometric Equation

The following three equations are basic to the theoretical treatment of fluid flow, heat transfer and mass transfer:

$$\tau = \left(\mu + \rho \varepsilon_m\right) \frac{du}{dy} \quad (19)$$

$$q = (k + c \rho \varepsilon_H) \frac{dt}{dy} \quad (20)$$

$$N = (D_v + \varepsilon_D) \frac{dc}{dy} \quad (21)$$

where

- $\mu$ = molecular viscosity
- $k$ = thermal conductivity
- $D_v$ = diffusivity
γ = shearing stress
q = heat transferred
N = mass transferred
ρ = density of fluid
c = heat capacity of fluid

\( \xi_m, \xi_h, \text{ and } \xi_D \) = eddy diffusivity in momentum transfer (fluid flow), heat transfer and diffusion (mass transfer), respectively

du/dy = rate of velocity change with distance measured from the wall, y
dt/dy = rate of temperature change with distance from the wall
dc/dy = rate of concentration change with distance from the wall.

It can be shown that each of the eddy diffusivities are proportional to the product of \( v' \), the cross-current velocity fluctuations, and \( \ell \), the Prandtl mixing length. This is the average distance which individual eddies are assumed to travel before they lose their identity. As there is some question among present day authorities as to whether all three eddy diffusivities are equal, they are subscripted here to differentiate between them.
1. Fluid flow

Considering first the case of fluid flow, the substitution of \( v' \ell \) for \( \xi_m \) in equation (19) gives

\[
\gamma = (\mu + \rho v' \ell) \frac{du}{dy} . \tag{22}
\]

According to the Prandtl mixing length hypothesis, \( v' \) may be expressed as:

\[
v' = \ell \frac{du}{dy} \tag{23}
\]

giving

\[
\gamma = (\mu + \rho \ell^2 \frac{du}{dy}) \frac{du}{dy} \tag{24}
\]

where \( \rho \ell^2 \frac{du}{dy} \) is the turbulent or eddy viscosity \( \eta \) of a fluid.

At high Reynolds numbers the molecular viscosity will be negligibly small, so that rearrangement of equation (24) on the basis of this assumption gives

\[
\ell = \frac{\sqrt{\gamma/\rho}}{\frac{du}{dy}} \tag{25}
\]

where \( \gamma \) is the shear stress in the fluid. Since \( \gamma \) is linear with the distance from the centerline of a pipe,

\[
\frac{\gamma}{\rho} = \frac{\gamma_0 x}{\ell} = u_s^2 x \tag{26}
\]

where
\[ x = \text{fractional distance from centerline to wall,} \frac{r}{r_0} \]
\[ \gamma_o = \text{shear stress at the wall of a pipe} \]
\[ u_* = \text{friction velocity defined by} \sqrt{\frac{\gamma_o}{\rho}}. \]

Substituting equation (26) into equation (25) gives

\[ L = \frac{u_* \sqrt{x}}{du/dy} \]  \hspace{1cm} (27)

but since

\[ \frac{u}{u_*} = \sqrt{2/ f} \]  \hspace{1cm} (28)

then

\[ \frac{L}{r_o} = \frac{(u/r_o) \sqrt{xf/2}}{du/dy}. \]  \hspace{1cm} (29)

This permits the determination of the variation of \( L/r_o \) from a velocity traverse. The derivation of all of the above equations may be found in any standard text on fluid mechanics and so will not be repeated here.

Nikuradse [21] reported the results of a large number of pitot tube traverses at different Reynold's numbers. His work is probably, on the whole, the best available source of such data. From these data it is possible to derive a relationship between \( L/r_o \) and the fractional pipe radius.
Two notable attempts at such a correlation have received attention in the past; those of Prandtl and of von Kármán. Prandtl [24], by assuming $l$ to be proportional to $y$, i.e., $l = Ky$, derived the equation

$$\frac{du}{dy} = \frac{u_\infty \sqrt{1 - y/r_o}}{K y}$$

(30)

which when substituted into equation (37) gives

$$l/r_o = K (y/r_o) \sqrt{1 - y/r_o} .$$

(31)

von Kármán [15] used the relationship that

$$l = K \frac{du/dy}{d^2u/dy^2}$$

(32)

and obtained

$$l/r_o = 2K \sqrt{1 - y/r_o} \left(1 - \sqrt{1 - y/r_o}\right).$$

(33)

Both of these equations agree in giving a value of $l/r_o = 0$ at both the centerline of the pipe and at the wall, contrary to Nikuradse's finding for highly turbulent flow, e.g., for an $N_Re = 105,000$, that although $l/r_o = 0$ at the wall, it has a value of 0.14 at the centerline. Nikuradse reported the following empirical equation as fitting the data very well:
A third correlation of Prandtl mixing length with fractional pipe radius was made in 1944 by Dr. Ruth. In making an approach to the problem Dr. Ruth decided to search for an equation form that would be readily susceptible to integration in the hope that this might shed further light on the theory. The equation obtained, as yet unpublished, was extremely successful in correlating fluid friction with the processes of heat and mass transfer, and forms the basis of a series of some ninety lectures given in the graduate course Chem. E. 661-662-663. The following is a brief description of the method of correlation employed by Dr. Ruth.

Using values of \( \ell/r_0 \) averaged from Nikuradse's tests, values of \( \ell/r_0 \) were plotted against \( 1-x^{3/2} \) and the resulting faired curve was then differentiated. The resulting differentials were plotted against \( 1-x^{3/2} \) semi-logarithmically, i.e., \( \log \frac{d(\ell/r_0)}{d(1-x^{3/2})} \) vs. \( 1-x^{3/2} \) giving a straight line having the equation

\[
\frac{d(\ell/r_0)}{d\xi} = 0.27 \left( 10^{-0.652 \xi} \right).
\]  

(35)

where \( \xi \) represents \( 1-x^{3/2} \).
Conversion of this to natural logarithms yields

\[ d(\ell/r_o) = 0.27 \, e^{-1.5\gamma} \, d\gamma \quad (36) \]

which can readily be integrated to give

\[ (\ell/r_o) = -0.18^{-1.5\gamma} + c \quad (37) \]

The integration constant \( c \) may be evaluated by remembering that at \( x = 1, \ \gamma \) or \( 1-x^{3/2} = 0 \) and \( \ell/r_o = c \), so \( c = 0.18 \) or

\[ \ell/r_o = 0.18 \left( 1 - e^{-5/2\gamma} \right) \quad (38) \]

If this expression is substituted into equation (29) and rearranged, the result is

\[ r_0 du/u dy = \frac{\sqrt{x}}{0.18 \left( 1 - e^{-5/2\gamma} \right)} \quad (39) \]

Further rearrangement yields

\[ du/u_x = \frac{1}{(3/2)^2 \left( 0.18 \right)} \left[ \frac{(3/2)e^{3\gamma/2}}{e^{3\gamma/2} - 1} \right] d\gamma \quad (40) \]

which may be integrated to give
\[
\frac{u}{u_*} = \frac{1}{K} \ln (e^{3/2} - 1) + c. \tag{41}
\]

The constant of integration is readily evaluated by substituting values of \(u/u_*\) and \(y\) at the centerline where \(x = 0\). At this point \(y = 1 - x^{3/2}\) or 1, and \(u/u_* = u_m/u_*\); hence

\[
c = \frac{u_m}{u_*} - \frac{1}{K} \ln (e^{3/2} - 1) \tag{42}
\]

and equation (41) becomes

\[
\frac{u_m - u}{u_*} = \frac{1}{K} \left[ \frac{e^{3/2} - 1}{e^{3y/2} - 1} \right]. \tag{43}
\]

Converting to base 10 logarithms and substituting the numerical value of \(e^{3/2} - 1 = 4.48 - 1 = 3.48\), and of

\[K = (3/2)^2 / \pi^{3/2} = 0.4045,\]

this becomes

\[
\frac{u_m - u}{u_*} = 5.70 \log \frac{3.48}{e^{3y/2} - 1}. \tag{44}
\]

The above equation describes the velocity number deficiency \((u_m - u)/u_*\) of a fluid flowing in turbulent flow from the centerline of a smooth round pipe up to within a short distance from the wall. A more convenient form to use
in the region of the wall results, however, when the term \((e^{3y/2} - 1)\) is replaced by \((3/2)^2y/r_0\), to which it reduces as \(x\) approaches unity. The result of this approximation is

\[
\frac{u_m - u}{u_*} = 1.075 + 5.70 \log \frac{r_0}{y}
\]

(45)

a form useful for determining \((u_m - u)/u_*\) in the vicinity of the wall \((y/r_0 = 0.15\) to \(0.0\)), where the quantity \(e^{3y/2} - 1\) is difficult to evaluate with sufficient precision.

By integrating the velocity profile given by equations (44) and (45) it is possible to determine the average velocity deficiency number \((u_m - \bar{u})/u_*\), hereafter designated by the symbol \(D_{ef}\). The volume of fluid \(Q\) passing any given point in unit time would be given by

\[
Q = 2\pi \int_0^{r_0} u r \, dr.
\]

(46)

However, knowing only the velocity deficiency \(u_m - u\) we can only integrate for the volume of fluid lying outside of the surface of revolution between its vertex at \(u_m\) and any point \(u\) on its surface. Representing the volume outside of this surface of revolution by the symbol \(Q_D\),

\[
Q_D = 2\pi \int_0^{r_0} (u_m - u) \, r \, dr.
\]

(47)
The volume deficiency number \( D_{sf} \) then becomes

\[
D_{sf} = \frac{u_m - \bar{u}}{u_*} = \frac{\omega_d}{\pi r_0^2 u_*} .
\]  (48)

Since \( r/r_0 = x \), and \( dr = r_0 \, dx \), equation (48) becomes

\[
D_{sf} = \frac{2\pi r_0^2}{\pi r_0^2} \int_0^{r_0} \left( \frac{u_m - u}{u_*} \right) x \, dx .
\]  (49)

When \( (u_m - u)/u_* \) as given by equation (44) is substituted into the last equation, and the variable \( x \) is expressed in terms of \( \gamma \), it becomes

\[
D_{sf} = \frac{2\pi^{3/2}}{(3/2)^3} \int_0^1 (1 - \gamma)^{1/3} \ln \left[ \frac{e^{3/2} - 1}{e^{3\gamma/2} - 1} \right] d\gamma .
\]  (50)

Expressing \((1 - \gamma)^{3/2}, e^{3/2}, \) and \( e^{3\gamma/2} \) in the form of expansions, equation (50) can be integrated to yield a value of \( D_{sf} \) of

\[
D_{sf} = 4.20 .
\]  (51)

Inasmuch as a weighted average of the values of \((u_m - u)/u_*\) reported by Nikuradse yield \( D_{sf} = 4.19 \), the agreement between theory and experiment can so far be regarded as very good.
If the figure 4.20 for $D_{*1}$ is placed on the left hand side of equation (44), it becomes possible to solve for the value of $\xi$, and subsequently, for the values of $x$ and $y/r_o$ at the point where $u = \bar{u}$. The values obtained are

$$x = 0.765 \quad \text{or} \quad \frac{y}{r_o} = 0.235. \quad (52)$$

This means that a pitot tube situated at a distance $0.235r_o$ from the wall of a pipe and sufficiently far away from any obstruction ought to register a reading from which the average velocity can be computed directly.

Since $(u_m - \bar{u})/u_* = D_{*f}$, $\bar{u}/u_* = (u_m/u_*) - D_{*f}$, and equation (45) may be written

$$\frac{\bar{u}}{u_*} = 1.075 - 4.20 + 5.70 \log \frac{r_o}{y_a} \quad (53)$$

where $y_a/r_o$ is the apparent fractional distance from the wall at which experimental values of $u_m-u$ pass through zero. Since $r_o/y$ becomes infinite at $y/r_o = 0$, it is apparent that neither equation (44) or (45) can hold in the immediate vicinity of the wall.

Writing equation (53) as

$$\log \frac{r_o}{y_a} = \frac{(\bar{u}/u_* - 3.125)}{5.70} \quad (54)$$

it becomes possible to solve for the critical distance at which this appears to occur. Examination of Nikuradse'...
data, amounting to some 125 tests, reveals that the distance \( y_a \) forms a critical dimensionless modulus with pipe radius and the shear velocity, given by

\[
y_a \rho u_\tau / \nu = C.14C \tag{55}
\]

so that \( r_0 / y_a \) is given by

\[
r_0 / y_a = \frac{r_0 \rho u_\tau}{C.14 \nu} = \frac{Re_\tau}{C.14} . \tag{56}
\]

Substituting this expression for \( r_0 / y \) in equation (45), it becomes

\[
u_\tau / u_\tau = 1.075 + 5.70 \log \frac{7.14}{\nu} \frac{r_0 \rho u_\tau}{\nu} . \tag{57}
\]

Upon subtracting equation (45) from equation (57) and removing \( \log 7.14 \) from the logarithmic term, the result is

\[
u / u_\tau = 4.87 + 5.70 \log \frac{\nu}{r_0} \left( \frac{r_0 \rho u_\tau}{\nu} \right) . \tag{58}
\]

The quantity \( \frac{\nu}{r_0} \left( \frac{r_0 \rho u_\tau}{\nu} \right) \) is called the "friction distance parameter" in Fluid Mechanics, and is customarily represented by the symbol \( y^+ \), a convention which will be adhered to throughout the remainder of the present discussion.
Equation (58) describes the velocity distribution of fluids in turbulent flow through round pipes between a value of $y^+$ of about 25 up to values of $y/r_o$ of from 0.10 to 0.15; beyond this point the exact relation is given by subtracting equation (44) from equation (57) to obtain

$$u/u_* = 2.855 + 5.7C \log \left[ e^{3(1-x^2/2)} - 1 \right] Re_* . \quad (59)$$

Equation (59) is a general equation for velocity distribution in turbulent flow valid from the centerline of a pipe ($x = 0$) out to within a very short distance from the wall. At a value of $x = 0.85$ to 0.90 it becomes identical with equation (58), and either equation may be used.

By inserting the value $x = 0.765$, the distance from the center according to equation (52), at which $u/u_*$ has the value $\overline{u}/u_*$, one obtains

$$e^{3(2/2)} - 1 = 0.645$$

which permits one to write equation (59) as

$$\overline{u}/u_* = 1.77 + 5.70 \log Re_* . \quad (60)$$

If, however, one retains a value $y/r_o = 0.235$ within the logarithmic term to make it a $y^+$ quantity of value 0.235 $Re_*$,
the equation becomes

$$\frac{\bar{u}}{u_*} = 5.35 + 5.70 \log 0.235 \text{Re}_* . \quad (61)$$

Comparing equations (58) and (61) it is seen that they differ only by 0.48 units of $u/u_*$. This means that if velocity distribution data within the range $y/r_0 = \infty$ up to 0.15 are plotted semi-logarithmically, a straight line of slope 5.70 and intercept 4.87 at $y^+ = 1.0$ is obtained, whereas, if only values of $\bar{u}/u_*$ are plotted against $\log 0.238 \text{Re}_*$, a line of the same slope but with an intercept of 5.35 results.

The validity of the equations so far derived is well illustrated by converting equation (60) into an expression for friction factor. Equation (23) expressed the relation

$$\frac{\bar{u}}{u_*} = \sqrt{2/f}$$

where $f$ is the Fanning friction factor, defined as

$$f = \frac{\Delta P g d}{2 L u^2 c} . \quad (62)$$

The relation between the friction velocity number

$$\text{Re}_* = r_0 (\bar{u}_*/\nu)$$

and Reynolds number $\text{Re} = D u_c/\nu$ is given by

$$\text{Re}_* = (\text{Re}/2) \sqrt{f/2} . \quad (63)$$

Hence equation (60) may be rewritten as

$$\sqrt{2/f} = 1.77 + 5.70 \log (\text{Re}/2) \sqrt{f/2} . \quad (64)$$
Simplifying this, it becomes

\[
\frac{1}{\sqrt{f}} = 4.03 \log \text{Re} \sqrt{f} - 0.569 \tag{65}
\]

which differs to only a negligible extent from the equation

\[
\frac{1}{\sqrt{f}} = 4.01 \log \text{Re} \sqrt{f} - 0.392 \tag{66}
\]

found by Nikuradse in the examination of his data.

Although the foregoing equations from equation (45) on describe the overall behavior of fluid flow through smooth round pipes quite nicely, they fail to explain the absolute magnitude of the flow achieved. When it is recalled that equation (44) was derived ignoring the effect of molecular viscosity, it becomes apparent that molecular viscosity must have a nearly negligible effect throughout most of the flow region. Nevertheless, the fact that \((u_m - u)/u_*\) goes to infinity when \(x\) is taken as unity in equation (44) indicates that although the effect of molecular viscosity may be small, it nevertheless cannot be ignored. The reason that \((u_m - u)/u_*\) goes to infinity as \(x\) approaches unity is apparent from equation (38), where it is seen that \(L/r_o\) becomes zero when \(\gamma = 0\), i.e., when \(x\) in the expression \((1 - x^{3/2})\) becomes unity.

When molecular viscosity is included as called for by the basic equation (equation 19, rearranged),
\[ \frac{du}{dy} = \frac{\tau}{\mu + \rho \varepsilon_m} \]  

(67)

the resulting expression,

\[ \frac{du}{dy} = \frac{\gamma_0 x}{\mu + r \phi u_* \sqrt{x}(1 - e^{-3 \theta/2}) / \eta^{3/2}} \]  

(68)

can no longer be formally integrated. Its suitability can, however, be easily investigated by numerical integration for assumed values of Re*. When this is done, it is found that the values of \( u_m/u_* \) obtained are very much less than the experimentally observed values. Moreover, the shape of the resulting velocity profile departs considerably from the experimentally observed profiles. This result may be taken to indicate that either the effect of molecular viscosity is absent in the central region of flow, or that it may be partially included in the term for turbulent or eddy viscosity,

\[ \eta = \rho \varepsilon_m = r \phi u_* \sqrt{x} \eta / r_0 \]  

(69)

If it is assumed that the effect of molecular viscosity varies as \( \sqrt{x} \), equation (68) can be formally integrated. When such an integration is carried out, the value of \( (u_m - u)/u_* \) at the wall becomes finite and equal to
However, when values of $u_m/u_*$ as given by this equation are compared with Nikuradse's experimental values, it is found that the values given by equation (7C), although larger than when molecular viscosity is allowed to remain full valued as in equation (68), are still smaller than the experimentally observed values by some seven to eight units. Further investigation shows that the true fault actually lies in the term for turbulent viscosity; that as given by equation (68) it is too large in the region of the wall, no matter what modifier of $\nu$ is utilized.

The solution of this difficulty was arrived at by Dr. Ruth a number of years ago by discarding the idea that a completely laminar film exists in the region of a pipe wall. He postulated instead, that in this region there exists a "fusion layer" in which the effects of both molecular and eddy viscosity fall off in a complementary fashion given by

$$\nu \text{ effective } = \nu e^{-Z^2} \quad \text{(71)}$$

and

$$\eta \text{ effective } = \eta (1 - e^{-Z^2}) \quad \text{(72)}$$
where $Z$ is the ratio $y/y_b$ of the distance from the wall to a characteristic distance $y_Z/r_o$ through the fusion layer. It was Dr. Ruth's idea that the distance $y_Z/r_o$ might be determined by a critical value of the ratio of eddy to molecular viscosity, at least in isothermal flow. In mathematical form this hypothesis can be expressed as

$$\left(\frac{n}{\mu}\right)_c = \frac{r_o\alpha u_*}{\mu} \sqrt{1} \left(\frac{y}{y_b}\right) = c \quad (73)$$

Since in the region of the wall $\sqrt{1}$ is nearly unity, $y_Z/r_o$ would be given by $K_y Z/r_o$. Hence,

$$Re_\star K(y Z/r_o) = c \quad (74)$$

or

$$y Z/r_o = c/K Re_\star \quad (75)$$

and $Z^2$ is given by

$$Z^2 = \left(\frac{y}{r_o}\right)\left(\frac{y_Z}{r_o}\right)^2 = \left(\frac{y}{r_o}\frac{c}{K Re_\star}\right)^2 = \left(\frac{y}{r_o}\frac{Re_\star}{c/K}\right)^2 = \left(\frac{y}{c/K}\right)^2. \quad (76)$$
This hypothesis modifies the classical expression of equation (67) to

\[ \frac{du}{dy} = \frac{\gamma}{\mu e^{-Z^2} + \eta (1 - e^{-Z^2})}. \]  

(77)

At the wall of a pipe \( y \) is zero; hence \( e^{-Z^2} \) is unity and molecular viscosity is full valued. At the distance \( y_Z \) from the wall \( z = 1.0 \) and \( e^{-Z^2} \) has a value of 0.368 and \( (1 - e^{-Z^2}) \) a value of 0.632. At twice this distance from the wall \( Z = 2 \), and \( e^{-Z^2} \) and \( (1 - e^{-Z^2}) \) have values of 0.0183 and 0.9817, respectively. Hence at the distance \( 2y_Z/r_o \), the effect of molecular viscosity has almost disappeared, and the effect of eddy viscosity has become almost full valued. It is apparent, therefore, that equation (77) has the desired property of producing complementary changes in the values of molecular and turbulent viscosity throughout a short distance from the wall of the pipe. The reason for terming the region from the wall up to the point where \( Z = 2 \) a "fusion layer" is obvious. Instead of regarding the region next to the pipe wall as one of purely laminar flow, the "fusion layer" hypothesis regards it as a region in which, proceeding away from the wall, the effect of molecular viscosity quickly decreases to a negligible value, while that of eddy viscosity increases in a complementary fashion.
If in equation (77), $\eta(1 - e^{-z^2})$ is replaced by its actual value $\rho_0 \mu u_\ast \sqrt{x}(\ell/r_0)(1 - e^{-z^2})$, and $\mu$ is factored out, it becomes

$$\frac{du}{dy} = \frac{\tau/\mu}{e^{-z^2} + \frac{\rho_0 \mu u_\ast}{\nu} \sqrt{x}(\ell/r_0)(1 - e^{-z^2})} \cdot (78)$$

Multiplying $\tau/\mu$ by $C/\nu$ one obtains $\frac{\tau}{\nu} \frac{C}{\nu}$. Since $\tau = \tau_0 \chi$, and $\tau_0 / \nu = u_\ast^2$, the numerator may be written as $u_\ast^2 \rho / \nu$. The quantity $dy$ can be replaced by $r_0 d(y/r_0)$, and $\sqrt{x}$ by $\sqrt{1 - y/r_0}$. Making these substitutions in equation (78) and rearranging, it becomes

$$\frac{du}{u_\ast} = \frac{(\rho_0 \mu u_\ast / \nu)(1 - y/r_0)}{e^{-z^2} + \frac{\rho_0 \mu u_\ast}{\nu} \sqrt{1 - y/r_0} \ell/r_0(1 - e^{-z^2})} \cdot (79)$$

The integration of this equation employing a proper fusion layer thickness should yield values of $u/u_\ast$ in agreement with experimentally observed velocity profiles of fluids flowing in smooth round pipes providing the hypothesis is valid.

Unfortunately, formal integration appears to be impossible, so that numerical integration must be employed in testing the equation. For large Reynolds numbers the boundary of the fusion layer lies so close to the wall that it is
permissible to express $\ell / r_o$ as $\kappa y / r_o$; also, the terms $1 - y / r_o$ and $\sqrt{1 - y / r_o}$ are so nearly unity that their variation with $y / r_o$ can be ignored. Multiplying the numerator and denominator by the ratio $y Z / r_o$ then permits the following grouping of terms

\[
\frac{du}{u_s} = \frac{(y Z^2 / \mu) d(y / r_o) / (y Z / r_o)}{e^{-Z^2} + \frac{y Z^2}{\kappa} \frac{y}{y Z} (1 - e^{-Z^2})} . \quad (8C)
\]

Since $Z = (y / r_o) / (y Z / r_o)$

\[
dZ = d(y / r_o) / (y Z / r_o) . \quad (81)
\]

Hence equation (80) may be rewritten as

\[
\frac{du}{u_s} = \frac{y Z^2 dZ}{e^{-Z^2} + \kappa y Z^2 (1 - e^{-Z^2})} . \quad (82)
\]

Equation (82) may be numerically integrated as soon as a value of $y Z^2 = c / \kappa$ has been assumed. The work of integration is considerably decreased if a table of values of $e^{-Z^2}$ and $1 - e^{-Z^2}$ at uniform increments of $Z$ up to $Z = 2.0$ is prepared beforehand from available tables of $e^{-x}$.

The problem is to ascertain a value of $c / \kappa$ that when utilized in equation (82) will cause the resulting integral curve to join smoothly with the experimentally determined
plot of $u/u_*$ against $y^+$ as given by equation (58). Two trial integrations employing values of $c/\kappa$ not too far off from the correct value are sufficient to permit the necessary value of $c/\kappa$ to be found by either interpolation or extrapolation. In Figure 1, the result of integrating equation (82) with a value of $c/\kappa = 19.0$ has been illustrated in a semi-logarithmic plot of $u/u_*$ against $yu_*/\nu$. On the same plot the line representing equation (58) has been drawn in. It will be noted that the flexed curve resulting from the integration of equation (82) joins smoothly with the experimentally determined line of equation (58), indicating that a $c/\kappa$ value of 19.0 is correct.

In view of the fact that $e^{-Z^2}$ never falls quite to zero and $(1 - e^{-Z^2})$ never rises quite to unity, it is apparent that no definite boundary actually exists at which the effect of turbulence can be said to be completely developed. However, inspection of Figure 1 shows that for all practical purposes it may be considered to be fully developed at some convenient point at little beyond the point $Z = 1.0$. Because $Z = 2.0$ provides a convenient limit to which to carry the integration of equation (82), and one at which the values of $\nu$ and $\eta$ have decreased and increased respectively to 0.0183$\nu$ and 0.9817$\eta$, the effective boundary of the fusion layer will hereafter be considered as situated at the point
FIG. 1. VELOCITY DISTRIBUTION IN ROUND PIPES NEAR THE WALL AT HIGH REYNOLDS NUMBERS (PREDICTED).
Z = 2. The point \( \frac{yZ}{r_0 \left( \frac{\rho \partial u_z}{\mu} \right)} = y^+_Z = 19.0 \) will hereafter be referred to as the half-thickness of the fusion layer. When it is necessary to indicate the effective boundary of the fusion layer, the symbol \( y^+_b \), with a numerical value of 38, will be employed to represent the moduli \( 2y^+_Z = 2y^+_Z \frac{\rho \partial u_z}{\mu} = 2c/\kappa \). When it becomes necessary to refer to Prandtl and Schmidt number groups evaluated using viscosity or thermal conductivity at the point \( y^+_b \), these groups will be written as \( Pr_b \) and \( S_b \).

Nikuradse's velocity profiles at high Reynolds numbers were not determined sufficiently close to the wall to provide any experimental check upon the shape of the velocity profile as predicted in Figure 1. His low Reynolds number profiles, on the other hand, were subject to an experimental fault of some kind. Hence, although Dr. Ruth developed his theory of the "fusion layer" merger of molecular and eddy viscosity early in 1945, it has been only within the last several years that any definite confirmation of Figure 1 has become available. It is true that von Kármán, Rouse, and numerous other early workers published diagrams quite similar to Figure 1 with Nikuradse's low Reynolds number velocity profiles plotted in positions very nearly agreeing with the curve of Figure 1; however, this was due to a misinterpretation of a certain table of calculated values appearing
in Nikuradse' paper, coupled with a gross negligence upon the part of such writers in failing to compute for themselves the sought-for values directly from the data as reported. The error has been discussed at length by Benjamin Miller [20a].

Once the value of \( c/\kappa \) has been determined by integration of equation (82) for the case of high Reynolds numbers, it becomes possible to predict the shape of the profile at low velocity numbers. To perform such integrations it is advisable to prepare a plot of both \( \varphi/r_0 \) and \( \sqrt{1 - y/r_0}(\varphi/r_0) \) against \( y/r_o \). Such a plot is shown in Figure 2. Since \( c/\kappa \) has a value of 19.3

\[
c = 19 \kappa = (19)(0.4045) = 7.68 \quad (83)
\]

and

\[
\text{Re}_* \sqrt{1 - y/r_0}(\varphi_z/r_0) = 7.68 \quad (84)
\]

becomes a condition that must be satisfied at the point \( Z = 1.0 \). For any given value of \( \text{Re}_* \)

\[
\sqrt{1 - y/r_0}(\varphi_z/r_0) = 7.68/\text{Re}_* \quad . \quad (85)
\]

At this value of \( \sqrt{1 - y/r_0}(\varphi_z/r_0) \), on the left hand ordinate axis of Figure 2 one can read on the abscissa the value of \( y_z/r_0 \) that satisfies equation (35). Returning up again
FIG 2. VARIATION OF $y/l_0$ AND $\sqrt{x}l_0$ WITH $y/l_0$. 
at this value of $y/r_o$ to the curve of $\ell/r_o$, one can read off the corresponding value of $\ell Z/r_o$. Once $y Z/r_o$ has been found for an assigned value of $Re_*$, values of $\ell/r_o$ can be read from Figure 2 at the values of $y/r_o$ called for by the relation $y/r_o = Z(y Z/r_o)$. Values of $(1 - y/r_o)$ and $\sqrt{1 - y/r_o}$ can be computed and listed. As soon as values of $e^{-Z^2}$ and $1 - e^{-Z^2}$ have been determined, the numerical integration of equation (79) can be carried out. It should be made clear that since no terms are neglected and no approximations are made in this procedure, the integral value of $u/u_*$ secured in the integration of equation (79) is exact from the wall ($y/r_o = 0.0$) to the centerline where $y/r_o = 1.00$. Beyond a value of $Z = 2.0$, the influence of the terms $e^{-Z^2}$ and $1 - e^{-Z^2}$ disappears, and the integration becomes in effect that of the following simplified equation:

$$\frac{du}{u_*} = \frac{(1 - y/r_o) \, d(y/r_o)}{\sqrt{1 - y/r_o} \, (\ell/r_o)}. \quad (36)$$

In Figure 3 the result of carrying out the integration of equation (79) with a value of $c = 7.68$ for a series of low Reynolds numbers has been illustrated.

It will be noted that the lowest Reynolds number shown is 3170, corresponding to an $Re_*$ value of 97.3. The reason for the existence of a limiting value becomes apparent when
FIG. 3. THEORETICAL VELOCITY DISTRIBUTION NEAR THE WALL AT LOW REYNOLDS NUMBERS.
we examine the plot of $\sqrt{1 - \frac{y}{r_0}}(\ell/r_0)$ in Figure 2. At a value of $y/r_0 = 0.50$ the value of $\sqrt{1 - \frac{y}{r_0}}(\ell/r_0)$ attains a maximum of 0.079, meaning that both the eddy diffusivity $\xi_m$ given by

$$\xi_m = r_0 u_\infty \sqrt{1 - \frac{y}{r_0}}(\ell/r_0)$$

(87)

and the eddy viscosity $\eta$ given by

$$\eta = \rho \xi_m = r_0 u_\infty \sqrt{1 - \frac{y}{r_0}}(\ell/r_0)$$

(88)

pass through maxima at $y/r_0 = 0.50$, falling back to zero again at $y/r_0 = 1.0$. If one solves for the value that $Re_*$ (and hence, $Re$) must have for assigned values of $y_Z/r_0$ from equation (84) rearranged to the form

$$Re_* = 7.68 / \sqrt{1 - \frac{y}{r_0}}(\ell/r_0)$$

(89)

one finds that $y_Z/r_0$ begins to increase rapidly as the critical value $Re_* = 7.68 / 0.079 = 97.3$ is approached. This is illustrated in Table 1.
Table 1
Determination of Critical Value of $\text{Re}_*$.  

<table>
<thead>
<tr>
<th>$y/r_0$</th>
<th>$\sqrt{x(l/r_0)}$</th>
<th>$\text{Re}_*$</th>
<th>$\text{Re}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.0183</td>
<td>420</td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>0.0335</td>
<td>229</td>
<td>7230</td>
</tr>
<tr>
<td>0.15</td>
<td>0.0458</td>
<td>167.7</td>
<td></td>
</tr>
<tr>
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<td>0.0559</td>
<td>137.5</td>
<td>4120</td>
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<tr>
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<td>0.0635</td>
<td>121</td>
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<td>0.30</td>
<td>0.0698</td>
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<td>3300</td>
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<tr>
<td>0.35</td>
<td>0.0738</td>
<td>104</td>
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<tr>
<td>0.40</td>
<td>0.0772</td>
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<tr>
<td>0.50</td>
<td>0.079</td>
<td>97.3</td>
<td>3170</td>
</tr>
</tbody>
</table>

As the pressure difference producing flow of fluid through a pipe is diminished, the group

$$\sqrt{\frac{\Delta p g D}{4 l \rho}} = \sqrt{\frac{\tau_0 l}{\rho}} = u_*$$

diminishes, and therefore also the modulus $r_0 \rho u_*/\nu = \text{Re}_*$. 

Hence, it is apparent that as $\Delta p$ is progressively decreased, thereby decreasing $Re_*$, the point $y_Z/r_0$ begins to move towards the centerline of the pipe with increasing rapidity. As the value $Re_* = 97.3$ is approached, $y_Z/r_0$ begins to increase without bounds, making it appear that for a value of $Re_*$ infinitesimally less than 97.3 it would become infinity. This means that $Z = \frac{y/r_0}{y_Z/r_0}$ tends rapidly towards $Z = \frac{y/r_0}{y_Z/r_0} = 0$ at a value of $Re_*$ just below 97.3. Hence, below $Re_* = 97.3$ the terms $e^{-Z^2}$ and $e^{-Z^2} - 1$ approach unity and zero, respectively. As a result, equation (79) reduces to

$$\frac{du}{u_*} = \frac{r_0 u_*}{\nu} \left(1 - \frac{y}{r_0}\right) d\left(\frac{y}{r_0}\right)$$

which when integrated becomes

$$\frac{u}{u_*} = Re_* \left[\frac{y}{r_0} - 0.5 \left(\frac{y}{r_0}\right)^2\right] = \frac{Re_*}{2} \left(1 - x^2\right). \quad (91)$$

This will be recognized as the equation for velocity distribution in laminar flow. If $u_*$ is multiplied over to the right hand side, $\frac{Re_* u_*}{2}$ becomes $\frac{r_0 C u_*^2}{2\nu}$, which is $\frac{r_0 C \gamma_0}{2\nu}$. Replacing $\gamma_0$ by its explicit value of $\frac{\Delta p r_0}{2L}$, the equation becomes
which is the more usual form of the equation of the velocity profile of a fluid in laminar flow through a round tube.

a. Non-isothermal fluid flow. If a fluid is either heated or cooled while flowing through a smooth round pipe, its viscosity in the fusion layer will vary from \( \mu_w \) at the wall to \( \mu_b \) at the boundary of the fusion layer. Just how it varies with the distance \( y/r_o \) from the wall will depend upon the temperature distribution profile.

It will be observed that in deriving equation (73) molecular viscosity was factored out of the denominator in order to secure the dimensionless group \( Re_* \). If numerator and denominator had been simply divided by molecular viscosity at some suitable point, say by \( \mu_b \) corresponding to the fusion layer boundary at \( 2z \), the result would have been

\[
\frac{du}{u_*} = \frac{(r_o \rho u_* / \mu_b) x d(y/r_o)}{(\mu / \mu_b) e^{-2^2} + (r_o \rho u_* / \mu_b) \sqrt{x/l/r_o}(1 - e^{-2^2})} \tag{93}
\]

in which \( \mu / \mu_b \) is unity if flow is isothermal, and a variable approaching unity at the fusion layer boundary in non-isothermal flow. If, for example, heating were so intense as to produce a temperature difference between the wall and
the fusion layer boundary corresponding to a two-fold change in the viscosity, the value of \( \eta/\eta_b \) would increase from 0.5 to 1.0 over this distance. In a case of cooling with the temperatures \( t_w \) and \( t_b \) just reversed, it would decrease from 2.0 to 1.0 over the same distance. Since the value of \( u_Z/u_\epsilon \) achieved at the distance \( y_Z/r_0 \) from the wall (11.93 units in isothermal flow, as seen from Figure 1) is very sensitive to the term \( (\eta/\eta_b)e^{-Z^2} \), it is to be expected that plots of \( u/u_\epsilon \) against \( \eta u_\epsilon/\eta_b \) would lie above the plot of Figure 1 in heating, and below it in cooling. Moreover, it is quite possible that the criterion as to the position of the boundary of the fusion layer might change.

In psychrometric work the viscosity of air changes but little over the temperature ranges involved. Hence, it is satisfactory to employ equations based on isothermal flow. However, to complete the present discussion it should be pointed out that for non-isothermal flow the critical ratio of \( \eta \) to \( \nu \) is found to be a variable, so that in general equation (73) should be written

\[
\left( \frac{\eta}{\nu} \right)_C \left( \frac{u_{Z1}}{u_\epsilon} \right)/\left( \frac{u_{Z1}}{u_\epsilon} \right) = c
\]

(94)

where the subscripts 1 and nl indicate isothermal and non-isothermal conditions, respectively, and \( c \) is the same con-
stant, 7.68, as before. If the ratio of velocity number in isothermal flow to velocity number in non-isothermal flow is represented by the symbol $F$, i.e.,

$$\frac{u_{Zi}}{u_*} / \frac{u_{Zni}}{u_*} = F,$$  \hspace{1cm} (95)

equation (94) becomes

$$\left(\frac{\eta}{\nu}\right)_{Zni} = cF$$  \hspace{1cm} (96)

meaning that the critical ratio of eddy to molecular viscosity at the half-thickness of the fusion layer is only $F$ times as great in non-isothermal flow as in isothermal flow. Since $\sqrt{x} \ell / r_o$ can be approximated closely by $K(y/r_o)$ within a distance of $y_Z/r_o$ from the wall of 0.10 (corresponding to an $Re_*$ value of 229 and an $Re$ value of 7230 [see Table 1]), this means that

$$K \frac{y_{Zni}}{r_o} Re_* = K \frac{y_{Zi}}{r_o} Re_*$$

or simply that

$$y_{Zni} = Fy_{Zi}$$  \hspace{1cm} (97)
i.e., the half-thickness of the fusion layer in non-
isothermal flow is \( F \) times the half-thickness in isothermal
flow, becoming thinner in heating and thicker in cooling.

To prepare equation (93) for numerical integration the
numerator is multiplied by the ratio \( \frac{y_{Zn1}}{r_o} / \frac{y_{Zn1}}{r_o} \), after
which it is rearranged to the form

\[
\left( \frac{y_{Zn1}}{r_o} \right) \left( \frac{r_o \alpha u_s}{\mu_b} \right) x d \left( \frac{y/r_o}{y_{Zn1}/r_o} \right).
\]

Utilizing the relation given by equation (97), and the fact
that \( \alpha e_s = 19.9 \), it becomes simply \( 19FxdZ \).

Introducing the same ratio into the second term of the
denominator, this term rearranges to

\[
\left( \frac{y_{Zn1}}{r_o} \right) \left( \frac{r_o \alpha u_s}{\mu_b} \right) \mu \left( \frac{y/r_o}{y_{Zn1}/r_o} \right) \sqrt{x} \left( 1 - e^{-Z^2} \right),
\]

and upon further simplification, to \( 19\mu FZ \sqrt{x} (1 - e^{-Z^2}) \).

Equation (93) then becomes

\[
\frac{du_{n1}}{u_s} = \frac{19FxdZ}{(\mu/\mu_b)e^{-Z^2} + 19\mu FZ \sqrt{x} (1 - e^{-Z^2})}.
\]

Simplifying this still further by factoring out \( 19\mu F \) from
both numerator and denominator, one obtains
The manner in which \( \mu/\mu_b \) varies with \( y/r_c \) through the fusion layer must be known before equation (99) can be integrated. From heat transfer studies it is found to depend upon the Prandtl number level and the viscosity ratio \( \mu_w/\mu_b \) between the wall and the fusion layer boundary. When this information is available, it is possible to integrate using an assumed value of \( F \). It should be emphasized that \( F \) is not an independent quantity, but that it too depends upon Prandtl number and \( \mu_w/\mu_b \). The point of integrating equation (99) is to ascertain what value of \( F \) will result in a value of \( u_{Z_{ni}}/u_* \) satisfying equation (95). In other words, such a value of \( F \) must be employed for every assumed value of \( Pr \) and \( \mu_w/\mu_b \) as to give a value of \( u_{Z_{ni}}/u_* \) satisfying the condition

\[
F(u_{Z_{ni}}/u_*) = u_{Z_{ni}}/u_* = 11.93 . \tag{100}
\]

As a result, at least two trial and error integrations are needed to determine the value of \( F \) associated with each assumed value of \( \mu_w/\mu_b \) and \( Pr_b \). The work of ascertaining \( F \) for various values of \( Pr_b \) and \( \mu_w/\mu_b \) is very lengthy and
tedious, requiring upwards of several hundred numerical integrations in order to cover a sufficient range of values in both heating and cooling. The results when obtained can be plotted as a series of curves of $F$ against $\log \mu / \mu_b$, each curve being for a different Prandtl number (see Figure 4). With the aid of these curves the friction factor $f_{ni}$ in non-isothermal flow can be predicted, making possible in turn, the accurate prediction of pressure drop over a piping system in which a fluid is being heated or cooled.

2. Heat transfer

Without attempting to derive the basic relation cited earlier of

$$q = (k + a \phi \epsilon_h) \frac{dt}{dy} \quad (2C)$$

for heat transfer by both molecular and eddy conduction, it is apparent that if $\epsilon_h$ in heat transfer can be considered to be identical with $\epsilon_m$ in momentum transfer, one might expect to have considerable success in applying the same modifiers $e^{-2^2}$ and $1 - e^{-2^2}$ to molecular and eddy conductivity as were previously applied to equation (20). However, it will be well to first consider several points of difference.

There has been, and still is, considerable question among present day contributors to the literature of heat and mass transfer in turbulent flow as to whether or not $\epsilon_m$ and
FIG. 4. PLOT FOR THE DETERMINATION OF "F"
\( \epsilon_H \) are equal. Some writers are inclined to believe that the ratio \( \epsilon_H / \epsilon_m \) may be as high as 1.7, others are inclined to accept a value of 1.4 to 1.5, and still others a value of unity. From a theoretical viewpoint, it would be expected that \( \epsilon_m, \epsilon_H, \) and \( \epsilon_D \) would all be equal since they all represent the product \( v'\sigma \) of the average cross-current velocity fluctuations \( v' \), and the Prandtl mixing length.

Dr. Ruth is able to show [29b] from the unpublished work of Mather [18b] that the values of \( \epsilon_m, \epsilon_H, \) at least, are identical. For this reason he is inclined to believe that \( \epsilon_m, \epsilon_H, \) and \( \epsilon_D \) are all equal, and that the present uncertainty of other workers in the field is simply due to the lack of adequate experimental data.

Rearrangement of equation (20) to the form

\[
\frac{dt}{dy} = \frac{q}{k + \sigma \epsilon_H} \tag{101}
\]

illustrates the close analogy between the basic equations for heat and momentum transfer. It is to be seen that eddy conductivity is the product of eddy diffusivity, \( \epsilon_H \), specific heat \( c \), and fluid density \( \rho \), forming a quantity having the units of thermal conductivity. In equation (101) \( q \) is the rate of heat transfer through unit area of a cylindrical annulus of thickness \( dy \) lying any place between the axis and the wall of a round tube. By means of a lengthy theoretical
analysis which need not be given here, the relation between q and the net q₀ passing across unit area of pipe wall surface in unit time can be derived [29b]. This analysis establishes that through the fusion layer, q/q₀ first rises slightly and then falls again, so that its average value throughout this region may be taken as unity. Between the boundary of the fusion layer and the axis of the pipe it decreases almost linearly with increasing y/r₀, permitting the variation of q/q₀ with x to be expressed with fair precision as

\[ \frac{q}{q₀} = \frac{\text{Re}_*}{\text{Re}_* - 22.8} \cdot x = f(x) \]  \hspace{1cm} (102)

At high Reynolds numbers the fusion layer is so thin that it is permissible to write simply

\[ q = q₀x = q₀(1 - y/r₀) \] \hspace{1cm} (103)

Upon rearranging equation (101) with \( \xi \) expressed explicitly as \( r₀u* \sqrt{1 - y/r₀(\ell/r₀)} \), and writing \( r₀d(y/r₀) \) for dy, it becomes

\[ \frac{dt}{q₀} = \frac{r₀(q/q₀)d(y/r₀)}{k + qφr₀u* \sqrt{1 - y/r₀(\ell/r₀)}} \] \hspace{1cm} (104)
At this point one might expect to introduce the complementary modifiers $e^{-Z^2}$ and $1 - e^{-Z^2}$. However, a little consideration of the matter shows that whereas there was a good physical basis for introducing a factor to cause the disappearance of molecular viscosity at the outer boundary of the fusion layer, there is no similar basis for desiring the disappearance of molecular conductivity in heat transfer. In the case of momentum exchange (fluid flow), it is the mutual interference of small clumps or vortices of fluid in the core region that accounts for the high apparent "mechanical" or eddy viscosity. As one vortex passes another, it is clear that the molecular viscosity of the fluid at the boundaries of the vortices should contribute to the resisting force. However, considering that the fluid particles inside a given vortex are practically at rest relative to one another, and that most of the fluid is at any given time within the interior of such vortices, it seems reasonable to expect that the contribution of molecular viscosity would be of minor importance. In the case of heat transfer, however, even though the vortices are moving exactly as in isothermal flow, heat is transferred to the relatively quiescent interiors of them by molecular conduction, without regard to the intensity of the eddy conduction process. For this reason it seems reasonable to leave $k$ in equation (104) full valued
by omitting the modifier $e^{-Z^2}$ used in the case of molecular viscosity.

Because it is desirable to express all physical equations in terms of dimensionless moduli as far as possible, $k$ will be factored out of the first term of the denominator of equation (104). If the second term of the denominator is then multiplied by $\mu/\mu$, it can be rearranged so as to yield a product of two dimensionless groups, i.e.

$$\left(\frac{r_0\rho u_0}{k}\right)\left(\frac{c/\ell}{\ell}\right)\sqrt{1 - y/r_0\ell/r_0}$$

or

$$\operatorname{Re}_* \operatorname{Pr} \sqrt{1 - y/r_0} \ell/r_0 .$$

If both sides of the equation are next multiplied by $q\rho u_0$, the same groups can be obtained in the numerator on the right hand side, and the left hand side also becomes dimensionless. The final result then becomes

$$\frac{q\rho u_0}{q_0} \frac{dt}{q_0} = \frac{\operatorname{Re}_* \operatorname{Pr} (q/q_0) d(y/r_0)}{1 + \operatorname{Re}_* \operatorname{Pr} \sqrt{1 - y/r_0} \ell/r_0 (1 - e^{-Z^2})} . \quad (105)$$

If $q/q_0$ is replaced by $(1 - y/r_0)$ as indicated by equation (103), it will be seen that except for the presence of the Prandtl number group, and the absence of $e^{-Z^2}$ as a mul-
tiplier of molecular conductivity, the right hand side of this equation is the same as in equation (79) for fluid flow. It is convenient to represent the quantity \( q_0/\varphi \mu_u \) by the symbol \( \Delta \tau \), i.e.

\[
q_0/\varphi \mu_u = \Delta \tau
\]  

(106)

and to factor out the product \( \text{Re}_* \text{Pr} \). With these changes made, and the left hand side integrated for the case of flow of heat from the wall towards the centerline of the pipe \((t_w = \text{wall temperature}, t_c = \text{temperature at the axis of the pipe})\), equation (105) becomes

\[
\frac{t_w - t_c}{\Delta \tau} = \int_0^1 \frac{(q/q_0) \, d(y/r_o)}{\frac{1}{\text{Re}_* \text{Pr}} + \sqrt{1 - y/r_o(C/r_o)(1 - e^{-z^2})}}. \tag{107}
\]

The integration of the right hand side may be carried out numerically in the same manner as before, the actual mechanics of the integration varying somewhat with the conditions assumed. If heat transfer is assumed to be isothermal, i.e., the temperature difference \((t_w - t_c)\) promoting flow of heat is assumed so small that the fluid viscosities at the temperatures \(t_w\) and \(t_c\) are essentially identical, equation (98) may be integrated as it stands for cases of low \( \text{Re}_* \) or Reynolds numbers. When \( \text{Re}_* \) is large enough that the half-thickness of the fusion layer occurs within the distance \( 0.05 \)
\( \frac{y}{r_0} \) from the wall, the term \( \frac{q}{q_0} \) may be taken as unity, and \( \frac{l}{r_0} \) can be expressed as \( \kappa \frac{y}{r_0} \). This permits equation (98) to be integrated up to a distance of 2 or 3Z from the wall as

\[
\int_{t_w}^{t} \frac{dt}{\Delta t_*} = \frac{1}{\kappa} \left[ \frac{1}{7.68 \text{Pr}} + \frac{Z}{1 - e^{-Z^2}} \right] \quad (108)
\]

For non-isothermal (actual, or practical) heat transfer, the equations are similar, except that \( \text{Pr}_b \) is written where only \( \text{Pr} \) occurred before. The value of \( \text{Pr} \) to be employed in any given case is fixed by the temperatures \( t_w \) and \( t_b \) at the boundaries of the fusion layer. Its numerical value can be ascertained as a function of \( \frac{\mu_w}{\mu_b} \) and \( \text{Pr}_b \) from the semi-logarithmic plot (Figure 4) mentioned earlier as the result of making upwards of one hundred numerical integrations of equation (99) for non-isothermal fluid flow.

In the foregoing treatment it was assumed that the molecular thermal conductivity \( k \) was a constant, independent of temperature. This assumption was justifiable in view of the fact that although \( k \) actually does vary somewhat with temperature, its variation is negligible compared with that of viscosity, and hence can usually be ignored.

In the analysis of experimental heat transfer data it is customary to write
where $h$ is a film coefficient, and $t_{av}$ is the average temperature of the fluid as obtained by passing it through some sort of mixing device before observing its temperature. If the temperature $t_{c}$ at the centerline of the stream of flow were observed with a thermocouple used in the same manner as a pitot tube, one could write

$$q_{o} = h(t_{w} - t_{c})$$  \hspace{1cm} (11c)

where $h_{c}$ would have a somewhat smaller numerical value. As values of $t_{c}$ are obtained more easily and more accurately, it might appear that film coefficients based on the second definition would preferable. This would indeed be true if it were not for the fact that it is customary to measure the quantity of heat transferred in a given time by means of the relation

$$\frac{q}{\theta_{A}} = q_{o} = wc(t_{2} - t_{1})$$  \hspace{1cm} (11l)

where $t_{2}$ and $t_{1}$ are the average temperatures of fluid stream. Obviously, if $t_{2}$ and $t_{1}$ are already available from an experiment, equation (109) is naturally selected as the equation defining the film coefficient.
In the theoretical treatment of heat transfer the use of a coefficient \( h_\|^\) permits a more simple treatment than does the use of an average \( h \). Since in psychrometric work the coefficients obtained correspond to the \( h_\|^\) variety, the discussion to follow will be based on that type of coefficient only.

The numerical integration of equation (108) up to the point \( Z = 2.0 \) for various assumed values of \( \text{Pr}_{p_b} \) provides a theoretical prediction of the manner in which the dimensionless temperature number \( \frac{t_w - t}{\Delta T_s} \) varies with \( y^+ \) out to the point \( y_b^+ \). These data comprise a family of curves which can be brought into coincidence in the region where molecular conduction is predominate, by plotting the dimensionless moduli

\[
\left( \frac{t_w - t}{\Delta T_s} \right)^{2/3} \left( \frac{\text{Pr}_{p_b}}{Z} \right)^{1/3}
\]

as illustrated in Figure 5. It will be noted that the correlation is perfect up to a value of \( Z(\text{Pr}_{p_b})^{1/3} \) of 0.7, beyond which point separate curves for each different Prandtl number are required.

The procedure for using Figure 5 to predict the actual temperature existing at any point between the wall and the boundary of the fusion layer is quite simple. Having been assigned a particular fluid and the temperature limits \( t_w \)
FIG. 5. MASTER TEMPERATURE DISTRIBUTION PLOT.
and $t_b, \nu_w, \nu_b$ and $Pr_b$ are evaluated. If the ratio $\mu_w/\mu_b$ departs much from unity, the value of $F$ may be read off from Figure 4. The value of $F Pr_b$ then determines which of the curves shown in the plot of Figure 5 is to be employed.

Stations in $Z$ varying by suitable uniform increments from $Z = 0$ to $Z = 2.0$ are then multiplied by $(F Pr_b)^{1/3}$. At the stations of $Z(F Pr_b)^{1/3}$ thus listed, values of

$$\frac{(\Delta t)}{(\Delta T^*_b)}/(F Pr_b)^{2/3}$$

are read off from the appropriate curve and recorded. Upon multiplying these last values by the constant value of $(F Pr_b)^{2/3}$, values of $\Delta t/\Delta T^*_b$ result. Since the terminal (last) value of $\Delta t/\Delta T^*_b$ in the list is $(t_w - t_b)/\Delta T^*_b$, the ratio of $\Delta t/\Delta T^*_b$ at any point in $Z$ is also the ratio of $(t_w - t)$ to $(t_w - t_b)$. Hence the actual temperature $t$ at any point in $Z$ is given by

$$t = t_w - \left(\frac{\Delta t/\Delta T^*_b}{(\Delta t/\Delta T^*_b)_b}\right)(t_w - t_b) \tag{112}$$

where $(\Delta t/\Delta T^*_b)_b$ is the terminal or last value in the table, corresponding to the boundary of the fusion layer at $Z = 2.0$.

It should be pointed out that the circumstance of whether heat transfer is substantially isothermal or decidedly non-isothermal has no influence upon the correlation shown in Figure 5, since this is provided for by the factor.
F. For this reason Figure 5 may be termed a "Master Plot of Temperature Number Distribution". Once this plot has become available, the investigation of non-isothermal fluid flow can be made that results in the correlation of \( F \) as a function of \( \text{Pr}_b \) and \( \mu_w/\mu_b \) in Figure 4.

When the integration of equation (108) for any given Prandtl number is carried out beyond a value of \( Z = 2.6 \), it is found that the plot of \( (t_w - t)/\Delta T^* \) against log \( y^+ \) has exactly the same slope as found for the plot of \( u/u_* \) against \( y^+ \) in fluid flow. Comparison of equation (82) written in the following form

\[
\int \frac{du}{u_*} = \frac{1}{K} \int \frac{dZ}{\frac{e^{-Z^2}}{7.68} + Z(1 - e^{-Z^2})} \quad (82a)
\]

with equation (108a)

\[
\int \frac{dt}{\Delta T^*} = \frac{1}{K} \int \frac{dZ}{\frac{1}{7.68\text{Pr}_b} + Z(1 - e^{-Z^2})} \quad (108a)
\]

reveals the reason. At the wall \( e^{-Z^2}/7.68 \) has a value of 0.1302, and \( Z(1 - e^{-Z^2}) \) a value of zero, giving \( (du/u_*)/dZ \) in equation (82a) an initial value of \( 0.1302/K = 0.322 \). For any value of Prandtl number greater than unity the corresponding value of \( (dt/\Delta T^*)/dZ \) in equation (108a) is larger than
the corresponding value of \((du/u_*)/dZ\) in equation (32a) so that the curve of \((t_w - t)/\Delta T_\infty\) in heat transfer rises more rapidly with \(y^+\) than does the corresponding curve of \(u/u_*\) with \(y^+\). It is true that as \(y^+\) increases, the value of the term \(e^{-Z^2}/7.68\) rapidly approaches zero, whereas the term 
\((1/7.68Pr_b)\) remains finite. However, in the region of 
\(y^+ = 19.0\), the effect of the term \(Z(1 - e^{-Z^2})\) begins to predominate, and beyond the boundary of the fusion layer 
\((y^+ = 38)\) it exerts exclusive control on the rate of increase in the value of the integral.

In view of the fact that the rate of increase in the integral values of the equations for both temperature number and velocity number distribution is the same beyond the fusion layer boundary, it becomes of interest to ascertain whether or not the integral values up to the boundary of the fusion layer can be correlated in some fashion.

In Table 2 the integral values of \((t_w - t_0)/\Delta T_\infty\) from the wall up to the fusion layer boundary \((Z = 2.0)\) as computed from equation (108a) for a variety of Prandtl numbers have been listed. In a third column the value of 
\((t_w - t_0)/\Delta T_\infty\) as computed from the following empirical correlation
Table 2

Comparison of Values Calculated by Equations (103a) and (113)

<table>
<thead>
<tr>
<th>$F \hat{P}_{p}$ assumed</th>
<th>Integral value</th>
<th>Computed from eq. (113)</th>
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<tr>
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<td>85.35</td>
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<td>160.36</td>
<td>161.2</td>
</tr>
<tr>
<td>100.0</td>
<td>254.12</td>
<td>254.6</td>
</tr>
</tbody>
</table>
\[
\frac{(t_w - t_b)}{\Delta T} = \left[11.5C + 1.34/(FPr_b)^{1/3}\right] (FPr_b)^{2/3} \tag{113}
\]

are given. It will be noted that the average difference between the integral value and that given by equation (113) is only 0.2 to 0.3 percent.

It will be noted in equation (113) that the term \(1.34/(FPr_b)^{1/3}\) makes only a minor contribution to the value of the integral. Thus the coefficient of \((FPr_b)^{2/3}\) is 11.50 for an infinite Prandtl number, 12.28 for a Pr of 1.0, and 13.09 for a Pr of 0.60. We will hereafter express equation (113) in the form

\[
\frac{(t_w - t_b)}{\Delta T} = M_H (FPr_b)^{2/3} \tag{114}
\]

understanding that \(M_H\) is a quantity that actually varies slightly with Prandtl number, but considering it to be constant whenever it suits our purpose.

It will be recalled that equations derived in the section on fluid flow provided a means of computing the velocity deficiency number \((u_m - u)/u_w\) backwards from the centerline of a pipe to any point in \(x\). The exact relation was
\[ \frac{u_m - u}{u_*} = 5.7C \log \frac{3.48}{(e^{38/2} - 1)} \]  

but for points near the wall the equation

\[ \frac{u_m - u}{u_*} = 1.075 + 5.7C \log \frac{r_o}{y} \]  

was equally accurate. If, in this last equation, we substitute the value of \( y_b/r_o \) corresponding to the boundary of the fusion layer, i.e., \( y_b/r_o = 38/Re_* \), equation (45) becomes

\[ \frac{u_m - u_b}{u_*} = 1.075 + 5.7C \log \frac{Re_*}{38} \]  

Now the value of \( u_b/u_* \) is 13.88 in isothermal flow and 13.88/F in non-isothermal flow, i.e.,

\[ u_{bni} = 13.88/F \]  

Bearing in mind that the velocity deficiency number \( (u_m - u_b)/u_* \) given by equation (115) is the same in both isothermal and non-isothermal flow, it is evident that if one desires, the value of \( u_m/u_* \) for the general case can be expressed as the sum of equations (115) and (116), i.e.,

\[ u_m/u_* = 13.88/F + [1.075 + 5.7C \log (Re_*/38)] \]  

(117)
Since the bracketed portion of equation (117) always represents the velocity number deficiency difference 
\[(u_m - u_b/F)/u_*\] between the axis of the pipe and the boundary of the fusion layer, the shorter expression will be used hereafter. It will also be convenient to express the quantity \(13.88/F\) by means of the symbol \(M_F\), understanding that \(M_F\) is actually a variable which under extreme conditions of heating or cooling of a fluid of very large Prandtl number may vary two or three fold, but which for fluids of low Prandtl number (\(F\) of near unity) can be regarded as constant with a value close to 13.88. In terms of the symbols just defined, the maximum velocity number \(u_m/u_*\) may now be expressed as

\[
u_m/u_* = M_F + (u_m - u_b/F)/u_* . \quad (118)
\]

Returning to the discussion of a few pages back, where it was shown that the integral value of \(dt/\Delta T_*\) between the fusion layer boundary and the centerline of the pipe was the same for heat transfer as it is for fluid flow, it is apparent that a corresponding equation may be written for heat transfer, i.e.,

\[
(t_w - t_\ell)/\Delta T_* = M_H(FPr_b)^{2/3} + (u_m - u_b/F)/u_* . \quad (119)
\]
The outstanding point to observe in the comparison of equations (118) and (119) is that the term \( (u_m - u_b/F) / u_* \) is common to both.

The relation between the transfer of heat and that of momentum becomes even more apparent if the \( t_w - t_e \) and the \( \Delta T_e \) in the heat transfer equation are made explicit. Thus \( \Delta T_e \) was introduced as a symbol for \( q_o/c\rho u_* \) in equation (106) and equation (110) was

\[
q_o / (t_w - t_e) = h_4 . \tag{120}
\]

Hence

\[
(t_w - t_e) \Delta T_e = c\rho u_* / h_4 \tag{121}
\]

and equation (119) may be rewritten as

\[
c\rho u_* / h_4 = h_B (\tilde{F} Fr_b)^{2/3} + (u_m - u_b/F) / u_* . \tag{122}
\]

The group \( c\rho u_* / h_4 \) has no recognized name among workers in the theory of fluid flow and heat transfer. Since the quantities on the right hand side of equation (122) are dimensionless quantities proportional, respectively, to the resistance of the fusion layer and the turbulent core, it seems logical to call \( c\rho u_* / h_4 \) a temperature number, and to view it as representing the sum of a fusion layer tempera-
ture number, \( M_h (FPr_b)^{2/3} \), plus a turbulent core temperature number, \( (u_m - u_b/F)/u_\infty \). The relation between temperature number and \( (y/r_0)Re_\infty \) is shown graphically in Figure 6. Note that for any given value of \( u_\infty \), the ratio of the change in either velocity or temperature number from the wall out to \( y_b^+ = 38 \) to the total change from the wall out to \( y^+ = Re_\infty \) is simply the ratio of \( u_b/u_m \) or \( t_w - t_b / (t_w - t_{\infty}) \), i.e., ratios of the resistance of the fusion layer to the total resistance. In the same way, the ratio of the change in either number from the boundary of the fusion layer out to the centerline to the total change is not only the ratio of change in velocity or temperature to the total change, but the ratio of the resistance of the turbulent core to the total resistance for the respective processes as well. Thus, at a medium Reynolds number, upwards of half of the total resistance to momentum exchange may reside in the fusion layer. If the flow is non-isothermal (i.e., transfer of heat is also occurring), half of the resistance to heat transfer will also be present in the fusion layer if the fluid in question has a Prandtl number of unity. However, if the Prandtl number is large (corresponding to a large value of \( M_h (FPr_b)^{2/3} \)), the resistance of the fusion layer may be such a large fraction of the total that only a small portion of the temperature change \( t_w - t_{\infty} \) will occur in the turbulent core.
FIG. 6. COMPARISON OF PREDICTED VELOCITY AND TEMPERATURE PROFILES AT HIGH Re.
It should perhaps be pointed out that neither velocity, temperature, nor (as we will see later) concentration numbers constitute the actual resistance to transfer of momentum, heat, or mass. For example, the true resistance to heat transfer is 1/h, as seen from the relation

\[ q = \frac{\Delta t}{\mu} = h\Delta t = \frac{\Delta t}{1/h} . \]

Hence resistance to heat transfer is given by rearranging equation (122) to

\[ h = \frac{1}{h} = \frac{h_u(Fr_{rb})^{2/3} + (u_m - u_d/F)}{c\rho u_s} + \frac{(u_m - u_d/F)}{c\rho u_s^2} . \]  

The first term on the right is the resistance of the fusion layer; the second, that of the turbulent core. The quantity \( \rho u_s^2 \) is \( \rho \gamma /\rho = \frac{\Delta P_{en} D^2/4}{nDL} \), the frictional resistance exerted on unit area of the pipe wall. This factor, measurement of which is ordinarily omitted in the experimental determination of heat transfer coefficients, is seen to be of prime importance in fixing resistance to heat transfer.

**a. Role of friction factor in correlating heat transfer data.** The Fanning friction factor was defined in equation (62) as
\[ f = \frac{\Delta \rho g D}{2L \rho U^2} \quad (62) \]

A similar definition based on maximum centerline velocity could just as well be written

\[ f_m = \frac{\Delta \rho g D}{2L \rho u_m^2} \quad (123) \]

and would be just as useful if it were not that the measurement of fluid flow in terms of the average velocity \( \bar{U} \) is more common. Since the quantity \( u_* \) is \( \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{\Delta \rho g D}{4L \rho}} \), it is apparent that \( u_* \) may be replaced by either

\[ u_* = u_m \sqrt{\frac{f_m}{2}} \quad (124) \]

or

\[ u_* = \bar{U} \sqrt{\frac{f}{2}} \quad (125) \]

depending upon which substitution is desirable. Replacing \( u_* \) in equation (122) by equation (125) and inverting, it becomes

\[ \frac{h_\tau}{\rho \bar{U}} = \frac{\sqrt{f/2}}{m_H (FPr_b)^{2/3} + (u_m - u_b/F) / u_*} \quad (126) \]

At either low Reynolds numbers or large Prandtl numbers the term \( (u_m - u_b/F) / u_* \) is small compared with \( m_H (FPr_b)^{2/3} \), so that if it is ignored, \( (FPr_b)^{2/3} \) may be multiplied to the left hand side. The result
\[
\frac{n \Pr^{2/3}}{c \rho u} = \frac{f/2}{K_h} \quad (127)
\]

is of considerable interest because it reveals why the empirical "film temperature" correlation

\[
\frac{n \Pr^{2/3}}{c \rho u} = f/2 \quad (128)
\]

discovered twenty years ago by Colburn met with such success.

The relation between equation (122) and the ordinary correlations of heat transfer in terms of the Nusselt number \( h_\text{D} D/K \) can be arrived at by introducing appropriate dimensionless ratios into the resistance number \( c u_*/h \) as follows

\[
\frac{c u_*}{h_\text{D}} = \left( \frac{\nu_b}{k} \right) \left( \frac{k}{h_\text{D} D} \right) \left( \frac{D \bar{u} \rho}{\nu_b} \right) \frac{f}{2} \quad (123)
\]

With this expression substituted for the heat transfer number in equation (122), followed by rearrangement and inversion, there is obtained

\[
\text{Nu}_\text{D} = \frac{\sqrt{f/2} \text{Re}_b \text{Pr}_b}{K_h (F \text{Pr}_b)^{2/3} + (u_m - u_b/F)/u_*} \quad (130)
\]

If the contribution of the temperature number for the turbulent core is similarly ignored in this case, it rear-
ranges to

\[ \frac{\text{Nu} \epsilon / \text{Pr}^{1/3}}{\text{Re}^{1/3}} = \frac{V_1/2}{N_H(F)^{2/3}}. \quad (131) \]

The similarity between equation (131) and such standard empirical heat transfer correlations as the Dittus-Boelter equation (4b)

\[ \text{Nu} / \text{Pr}^{1/3} = C.C225 \text{ Re}^{0.8C} \quad (132) \]

or the Seider-Tate equation (3Ca)

\[ \text{Nu} / \text{Pr}^{1/3} = C.C27(\mu_b / \mu_w)^{0.14} \text{ Re}^{0.8C} \quad (133) \]

reveals at once both the reason for the success of these equations, and at the same time the reason for their failure to provide more satisfactory correlations.

3. Mass transfer

The corresponding equations for transfer of mass by the combined mechanisms of molecular and eddy diffusion was cited earlier as

\[ N = (D_v + \xi_D) \frac{dc}{dy} \quad (21) \]

the form in which the theoretically derived expression
\[ N = (D_v + v' \ell) \frac{dc}{dy} \]  

is ordinarily presented. As pointed out earlier, current workers in the fields of fluid friction and heat and mass transfer have encountered difficulty in trying to show that eddy viscosity, eddy conductivity, and eddy diffusivity are exactly equal to the theoretical expressions \( \rho v' \ell, \rho v' \ell, \) and \( v' \ell. \) Hence they have written them as \( \rho \varepsilon_m, \rho \varepsilon_H, \) and \( \varepsilon_D, \) believing that the ratios \( \varepsilon_m/\varepsilon_m \) and \( \varepsilon_D/\varepsilon_m \) might differ from unity by as much as 1.7 in some cases.

This has no doubt been due to three factors: (a) a lack of adequately precise data, (b) a lack of knowledge as to the mathematical form of the fundamental relation between Prandtl mixing length and fractional pipe radius, and (c) the lack of an adequate mechanism such as the "fusion layer" hypothesis for explaining the dampening of eddy diffusivity in the region of the wall.

Because the superlative heat transfer data of Mather [18b], ranging in Reynolds number from 3,000 to 450,000, have been correlated [29b] with the equations presented in this thesis with an average deviation of less than five per cent, and because Mather's simultaneous measurements of temperature and velocity distribution in non-isothermal flow have been shown [29b] to demonstrate conclusively that the ratio
\( \xi_N / \xi_m \) is unity, it seems safe to conclude that if adequate mass transfer data were available, and were treated in the same manner, it would be found that the ratio \( \xi_D / \xi_m \) is also unity.

On the basis of this belief, then, the integration of equation (134) will be carried out in a manner identical with that employed in heat transfer. Because molecular diffusion goes on within the interior of clumps of fluid in turbulent motion just as does molecular conduction of heat in heat transfer, the effect of molecular diffusivity will be allowed to remain full valued. However, the effect of eddy diffusivity will again be modified in the fusion layer by use of the dampening factor \( (1 - e^{-Z^2}) \).

In equations (21) and (134) the symbols have the following meaning, applicable in consistent units of either the metric or English system:

- \( \Delta c \) = concentration difference, mols per unit volume
- \( N \) = rate of diffusion of component \( a \) through unit area of an element of cylindrical surface of thickness \( dy \) situated at any point between the wall and axis of a round pipe, mols/(unit area)(unit time)
- \( N_0 \) = rate of diffusion of component \( a \) through unit area at the wall
- \( \xi_D \) = eddy diffusivity, \( L^2/\bar{e} \)
$D_v = \text{molecular diffusivity, } L^2/\theta$

$y = \text{distance normal to the axis of a round pipe, measured from the wall toward the axis}$

$x = r/r_0$, fractional radial distance measured from the axis toward the wall.

If according to our assumption, $\xi_D$ is equal to $v' \ell$, then the following relation is true

$$\xi = u_* \sqrt{x} \ell = r_0 u_* \sqrt{x} \ell/r_0.$$ (135)

Just as in the case of heat transfer, it can be shown, by a rather lengthy derivation in which the Poisson equation is applied to an annular section of differential length and thickness, that $N/N_0$ can be treated as substantially constant throughout the fusion layer, and as given by

$$N = N_0 x$$ (136)

through the turbulent core if the fusion layer is thin. Substituting the right hand sides of equations (135) and (136) for $\xi_D$ and $N$ into equation (134), and at the same time introducing the fusion layer damping factor $(1 - e^{-Z^2})$, the result upon rearranging becomes

$$dc = \frac{r_0 x d(y/r_0)}{D_v + r_0 u_* \sqrt{x} \ell/r_0 (1 - e^{-Z^2})}.$$ (137)
It should be pointed out that because numerical rather than formal integration has to be employed, it is immaterial whether we write $x$ or $1 - y/r_o$; the use of the former here is simply a matter of convenience.

If the molecular diffusivity is factored out of the first term of the denominator, and both numerator and denominator are multiplied by $(\mu/\nu)$ and $(\rho/\rho)$, it is possible to obtain a group of dimensionless moduli in the second term of the denominator. If both sides of the equation are multiplied in addition by $u_*$ to make the numerators on both sides dimensionless, the result is

$$u_* d_c = \frac{(r_o \rho u_*/\nu) (\mu/\rho D_v) x d(y/r_o)}{1 + \left(\frac{r_o \rho u_*}{\nu}\right) (\rho/\rho D_v) (l/r_o) \sqrt{x} (1 - e^{-2^2})} \quad . \quad (138)$$

The new dimensionless group $(\mu/\rho D_v)$ appearing here for the first time is known as the Schmidt number, and is commonly represented by the symbol $S$.

Writing equation (138) in the same form as employed for equation (107) it becomes

$$u_* d_c = \frac{(N/N_o) d(y/r_o)}{(1/Re_* S) + \sqrt{1 - y/r_o} (l/r_o) (1 - e^{-2^2})} \quad . \quad (139)$$
From the similarity of equations (139) and (1C7) it is obvious that the result of integration should be the same as that for the heat transfer equations. If diffusion is isothermal, i.e., diffusion takes place in the absence of heat transfer, then \( \rho \) is constant throughout the fusion layer and \( \mathcal{F} = 1 \). If Re* is large enough to move the mid-point of the fusion layer to within \( y/r_0 = 0.05 \) of the wall, \( \mathcal{K}/r_0 \) can be expressed as \( \mathcal{K}(y/r_0) \), permitting the equation to be written as

\[
\frac{u_*}{N_0} \int_{c_w}^{c_b} dc = \frac{1}{\mathcal{K}} \int_{0}^{Z=2.0} \frac{dZ}{(1/7.83) + Z(1-e^{-Z^2})}
\]  

(140)

in which form it is similar to equation (1C8) in heat transfer. Now the Schmidt number in gaseous diffusion is limited to a relatively narrow range, e.g., from around 0.50 to perhaps 2.5. However, if it were to vary as widely as Prandtl number does in heat transfer, and if the variation of diffusivity with temperature could be ignored as was that of molecular conductivity with temperature in the case of heat transfer, then the same correlation that was illustrated in Figure 5 as a "master temperature plot" would apply to diffusion if the coordinates were interpreted as

\[
\left( \frac{c_w - c}{N_0} \right) \left/ \left( \frac{\mathcal{F}S_b}{\mathcal{W}} \right)^{2/3} \right. \text{ instead of } \left( \frac{t_w - t}{\Delta T_*} \right) \left/ \left( \frac{\mathcal{F}Pr_b}{\mathcal{W}} \right)^{2/3} \right.
\]
For the same reason the terminal points on the master temperature plot, which corresponded to the value of the integral of equation (108) out to the point \( Z = Z_c \), also represent the value of the integral of equation (140) out to \( Z = Z_c \) for various values of \( \text{Pr}_D \), and hence are correlated by an equation entirely analogous to equation (113), i.e.,

\[
\frac{(c_w - c_D) u^*}{N_0} = \left[ 11.56 + 1.34/\left( \text{Fr}_D \right)^{1/3} \right] \left( \text{Pr}_D \right)^{2/3} \quad (141)
\]

In this connection it might be well to point out that Lin and his co-workers [17b] demonstrated that the electrolytic migration of ions at constant temperature in aqueous electrolytes in forced convection through pipes at Reynolds numbers varying from 300 to 50,000 gave every indication of obeying the same laws of diffusion as apply in the diffusion of gases. Since the Schmidt numbers of ions diffusing through liquids is of the order of 300 to 3,000, it would appear that experiments in which electrolysis was carried out non-isothermally, i.e., simultaneously with the transfer of heat, might provide an excellent means of checking equation (141) and the theory on which it is based.

It is apparent from the analogy between heat transfer and mass transfer that the shape of the plot of the diffusion
number \( \frac{u_* (c_w - c)}{N_o} \) against \( y^+ \) has the same slope beyond the point \( y^+_b = 38 \) as do the plots of velocity number and temperature number, so that the integral of equation (139) from the wall to the axis of a pipe can be expressed in a similar manner by writing

\[
\frac{u_* (c_w - c)}{N_o} = \frac{h_S (FS_b)^{2/3} + (u_m - u_B/F)}{u_*}.
\]

The left hand side of equation (142) may be expressed in terms of a film coefficient in the same way as was done in heat transfer. Writing

\[
N_o = \frac{D_V P}{B_{g_b} P_{b_m}} (c_w - c_\xi) = k_{gm_\xi} (c_w - c_\xi)
\]

defines a film coefficient \( k_{gm_\xi} \) having the units of mols/(unit area)(unit time)(unit concentration difference). Replacing the ratio \( (c_w - c_\xi)/N_o \) in equation (142) by \( k_{gm_\xi} \), it becomes

\[
u_{u_\xi} = \frac{h_S (FS_b)^{2/3} + (u_m - u_B/F)}{u_*}.
\]

It will be observed that the group \( u_{\xi}/k_{gm_\xi} \) in mass transfer is analogous to the group \( \phi u_{\xi}/h_k \) in heat transfer. The group analogous to the Nusselt number \( h_k D/k \) in heat trans-
ier would in mass transfer be \( \frac{k_{\text{ext}} D}{D_v} \). This group can be obtained by multiplying \( u_* / \xi_{\text{ext}} \) by the four dimensionless ratios \( D/D, D_v/D, \rho/\rho' \) and \( \mu/\nu \), to obtain

\[
u_* / \xi_{\text{ext}} = \left( \frac{D_v}{D} \right) \left( \frac{\rho}{\rho'} \right) \left( \frac{\mu}{\nu} \right) \left( \frac{D_v}{\xi_{\text{ext}} D} \right) .
\] (145)

Upon substituting \( \bar{u} \sqrt{f/2} \) for \( u_* \), inverting and rearranging, equation (144) becomes

\[rac{k_{\text{ext}} D}{D_v} = \frac{\sqrt{f/2} \text{Re}_b S_b}{k_S (FS_b)^{2/3} + (u_m - u_p/F) / u_*}
\] (146)

a form analogous to equation (130) for heat transfer. If the term \( (u_m - u_p/F) / u_* \) in the denominator is considered negligible compared with the partial diffusion number for the fusion layer, equation (146) simplifies to

\[rac{k_{\text{ext}} D}{D_v} \frac{1/3}{S_b} = \frac{\sqrt{f/2} \text{Re}_b}{M_S(k)^{2/3}}
\] (147)

a form analogous to equation (131) for heat transfer, and sufficiently like that of the equation

\[rac{k D}{D_v} \left( \frac{1}{S_{0.44}} \right) = 0.023 \text{Re}^{0.83}
\] (148)

found empirically by Sherwood and Gilliland [11b] from their
examination of a large amount of mass transfer data, to explain why their correlation worked as well as it did.

If in equation (144) \( u_* \) on the left is replaced by \( \bar{u} \sqrt{f/2} \), and the equation inverted, it becomes

\[
\frac{k_{\text{gm}}}{\bar{u}} = \frac{\sqrt{f/2}}{n_S (FS_p)^{2/3} + (u_m - u_0/F) / u_*}.
\]  \hspace{1cm} (145)

If the term representing resistance in the turbulent core is again neglected, \((S_p)^{2/3}\) may be multiplied over to the left hand side to obtain

\[
\frac{k_{\text{gm}} (S_p)^{2/3}}{\bar{u}} = \frac{\sqrt{f/2}}{n_S (F)^{2/3}}.
\]  \hspace{1cm} (150)

If this is rewritten as

\[
\frac{k_{\text{gm}} (S_p)^{2/3}}{\bar{u}} = \frac{f/2}{n_S (F)^{2/3} \sqrt{f/2}}
\]  \hspace{1cm} (151)

and the denominator is evaluated for some specific case, e.g. using the Prandtl number of air \((0.72)\) at an average value of \( f = 0.0118 \) in isothermal flow (corresponding to an \( \text{Re} \) of about \( 5,000 \)), the denominator becomes unity as seen in the following computation

\[
(11.50 + 1.54 / \sqrt{0.72}) \sqrt{0.0118/2} = 1.00
\]
and equation (151) becomes simply

$$k_g \frac{S_D^{2/3}}{\bar{u}} = f/2 \quad (152)$$

This explains why the empirical correlation suggested by Colburn [7b] some twenty years ago, i.e.,

$$k_g \frac{S^{2/3}}{\bar{u}} = \frac{j_D}{2} \quad (153)$$

where \(j_D/2\) is approximately equal to \(f/2\), works so well in practice, and why the use of Colburn's \(j_h\) and \(j_D\) factors has become so universally accepted.

4. Equilibrium during the simultaneous exchange of heat and mass between the wall of a cylinder and a fluid in turbulent flow

To illustrate the application of the foregoing equations to the equilibrium attained in the evaporation of a fluid from a surface in contact with a current of gas, we may consider the following hypothetical experiment. Let us imagine the interior surface of the lower portion of a long vertical round pipe to be covered smoothly with a porous wicking in such a fashion that liquid at any given temperature can be fed to the top of the wicking. By adjusting the feed rate of the liquid it will be possible to compensate for loss due to evaporation and still keep the lower rim of the wicking
sufficiently wet that an occasional drop of liquid will drip off. Let the entire column be so well insulated with insulation of zero heat capacity that the column will operate adiabatically, i.e., the heat transferred from the gas stream to the wall will be used solely to supply all of the heat needed for vaporization of the liquid.

Let us further imagine a resistance thermometer consisting of a few turns of platinum wire to be installed behind the wicking at the lower end of the tube, thereby permitting the temperature of the wicking at the exit to be measured with any desired degree of precision. Let a second resistance thermometer consisting of platinum wire wrapped around the outside of a narrow glass tube be available to measure the temperature and velocity of the gas stream at the axis of the pipe at the exit, and at the same time permit drawing off gas samples for determination of the moisture content by gravimetric means.

Through the apparatus thus envisioned, let us imagine a stream of heated gas to be passed downwards, emerging from the wick section at the point where the measuring equipment has been installed. If the wick section has a length of at least some fifty pipe diameters or more, the profiles of velocity, temperature, and concentration distribution that will have been flat at the entrance to the pipe will have become rounded out to the equilibrium shape called for by
the equation
\[ \frac{u_m - u}{u_*} = \frac{i}{N} \ln \frac{e^{3/2}}{e^{3\bar{e}/2} - 1} \] (43)

by the time the stream reaches the exit. In this way, all three of the equations

\[ \frac{u_d}{u_*} = \frac{n_F}{F} + \left( \frac{u_m - u_b}{F} \right) / u_* = \varphi_D \] (118a)

\[ \varphi_F = \frac{n_F}{F} \left( \frac{F_P}{b_b} \right)^{2/3} + \left( \frac{u_m - u_b}{F} \right) / u_* = \varphi_D \] (122a)

\[ \frac{u_*}{k_S} = \frac{M_S (F_S b)}{2/3} + \left( \frac{u_m - u_b}{F} \right) / u_* = \varphi_D \] (146a)

will apply with the assurance that the profile of temperature and concentration distribution through the turbulent core has become fully developed.

Let us suppose the resistance thermometer situated behind the wicking at the exit of the pipe to be connected to a combined indicating and servo-mechanism control system capable of quickly refrigerating the water fed to the wick to exactly the temperature indicated by the resistance thermometer. We may consider that before any water is fed to the wick the reservoir temperature will have been brought up to that of the uncooled air stream. Obviously, hot water fed to the wick will tend to vaporize very rapidly. Since complete adiabaticity of the column has been postulated,
the latent heat necessary for vaporization must at first come entirely from the sensible heat content of the water itself, thereby causing a rapid evaporative cooling. As soon as it attains a temperature sensibly below the air temperature, the transfer of heat at an increasing rate from the air to the wet wicking will begin. When the wick has cooled to such a temperature that rate of transfer of heat from the air to the wick is just sufficient to meet the demand for heat imposed by the rate of vaporization, there will be no further cooling of the wick; i.e., it will have attained an equilibrium temperature, $t_{\text{eq}}$.

Let us consider in detail the mechanism whereby equilibrium in the short section of wick at the bottom of the tube is attained. The temperature of the water in the wick at this point is $t_{\text{eq}}$, and because of the automatic action of the servo-operated refrigerating mechanism postulated in a previous paragraph, this temperature is also that of the water fed to the top of the wick. The vapor pressure of water at the temperature $t_{\text{eq}}$ is $p_{\text{eq}}$, and the vapor pressure of water in the air in the exit stream as measured gravimetrically is $p_a$. The rate of diffusion from the wick into the air stream in weight units is given by
where \( k_e \) is a convenient and commonly used film coefficient having the units weight/unit area \( \times \) unit time.

Because the evaporated moisture is swept out of the exit of the tube before it has warmed up beyond the temperature \( t_b \) corresponding to the boundary of the fusion layer, the heat that must be supplied per unit weight of fluid evaporated is \( \lambda + 0.48(t_b - t) \), a quantity which we shall define as

\[
\lambda_b = \lambda_{we} + c(t_b - t_{we}).
\]  

The rate of escape of heat from the wet wick is, therefore,

\[
\frac{Q_e}{A\delta} = \frac{\lambda_b w}{A\delta} = \lambda_b k_e \frac{(P_{we} - P_a)}{P_{bm}}.
\]  

At equilibrium, heat will be supplied from the surroundings at exactly the same rate. Since it has been assumed that the water coming to the wick was introduced at the temperature \( t_{we} \), and that no heat can be exchanged with the tube...
wall, the mechanisms by which heat can be supplied to the wick are limited to convection from the air stream and radiation from the surroundings. Indicating by the symbol \( \frac{Q_1}{A \theta} \) the rate of incoming heat supply, this will be given by

\[
\frac{Q_1}{A \theta} = h_c(t_a - t_{we}) + h_R(t_s - t_{we})
\]

where \( h_c \) is the film coefficient for forced convection, and \( h_R \) a radiation coefficient. By factoring out the temperature difference \( (t_a - t_{we}) \), this becomes

\[
\frac{Q_1}{A \theta} = (h_c + h'_R)(t_a - t_{we})
\]

where \( h'_R \) is a modified radiation coefficient defined by the expression

\[
h'_R = h_R \frac{t_s - t_{we}}{t_a - t_{we}}.
\]

In the case being considered the wick is able to "see" surfaces at the temperature \( t_s \) throughout half its sphere of vision. If it were situated farther upstream in the pipe, it would be able to see only surface at the temperature \( t_{we} \), so that although \( h_R \) would still be a finite quantity determined by the temperature of the wick, there would be no net difference in the radiant heat exchange. In such a
and the coefficient \( h'_r \) becomes zero.

Setting the heat rate \( \dot{Q}_1/\Delta \theta \) equal to the rate \( \dot{Q}_o/\Delta \theta \) given by equation (157)

\[
\dot{Q}_o = \left( h_c + h'_r \right) (t_a - t_{we}) . \tag{162}
\]

Upon rearrangement this becomes

\[
\frac{(p_{we} - p_a) \lambda_b}{P_{bm} (1 + h'_r/h_c) (t_a - t_{we})} = h_c/k_g . \tag{163}
\]

Now the quantity \( h_c \) occurring in this expression is the same film coefficient obtained in equation (122a)

\[
\psi_{qu} = h_c/k_g . \tag{122a}
\]

The \( k_g \), however, possesses units somewhat different from those of the coefficient \( k_{gm} \) occurring in the corresponding equation (145a)

\[
\psi_{u} = k_{gm} . \tag{145a}
\]
The necessary relation between $k_e$ and $k_{gm\ell}$ can be deduced as follows.

If equation (165) is divided on both sides by $k_a$, molecular weight of the active, or diffusing component, and if the partial pressures $p_{we}$ and $p_{at}$ are replaced by concentrations employing the relation that

$$c_{we} - c_\ell = \frac{p_{we} - p_{at}}{nT},$$

(164)

it may then be written as

$$n_0 = \frac{k_g RT (c_{we} - c_\ell)}{p_{bm} k_a}.$$  

(165)

Comparing this with the right hand member of equation (145) in which $k_{gm\ell}$ was defined simply as

$$N_0 = k_{gm\ell} (c_\ell - c_\ell)$$

(166)

it is apparent that

$$k_{gm\ell} = \frac{k_g RT}{N_0 k_a p_{bm}}.$$  

(167)

From the gas law $p_v = nRT$ written for unit volume of gas it is evident that the density of a gas can be expressed as
\[ \rho = \frac{nM}{V} = \frac{\rho_m}{RT}. \] (168)

Hence, by introducing the ratio \( \frac{\rho_m}{RT} \) into equation (167) we can eliminate the term RT in favor of a density.

In equation (122a) for heat transfer the symbol \( \rho \) represented the actual density of the medium exchanging heat with the walls. Since in the case of our hypothetical experiment the gas passing the short test section at the end of the pipe contains a very considerable amount of water vapor evaporated from the calming section of wick ahead of the test section, the density of the main stream of fluid at this point will be given by

\[ \rho_m = \frac{M_m}{RT} \] (169)

where \( M_m \) is the mean molecular weight as determined by the relative amounts of air and water vapor at this point. Because it will prove to be desirable for the following equation to contain this particular density, \( M_m \) is used in the transformation of equation (167) to obtain
\[ K_{\text{umf}} = \frac{K_{\text{c}}}{M_a P_{\text{bu}} P_{\text{um}}} \]

\[ = \frac{K_{\text{c}} P_{\text{um}}}{M_a P_{\text{bu}} P_{\text{um}}} \cdot (17c) \]

Having ascertained the relation between \( k_{\text{gm}} \) and \( K_{\text{c}} \), equation (14ia) may now be written

\[ \frac{u_s M_a P_{\text{bu}} \rho}{K_{\text{c}} P_{\text{um}}} = \phi_D \cdot (171) \]

Dividing this by equation (122a), i.e.

\[ \frac{c P_s u_s}{h_\ell} = \phi_r \cdot (122a) \]

we obtain

\[ \frac{h_\ell}{c P_s u_s} \frac{u_s M_a P_{\text{bu}} \rho}{K_{\text{c}} P_{\text{um}}} = \phi_D \cdot \phi_r \cdot (172) \]

Cancelling \( \rho \) and \( u_s \) from numerator and denominator, and rearranging to yield \( h_\ell/K_{\text{c}} \), there results

\[ \frac{h_\ell}{K_{\text{c}}} = \left( \frac{c M_a}{h_\ell} \right) \left( \frac{P}{P_{\text{bu}}} \right) \left( \frac{\phi_D}{\phi_r} \right) \cdot (173) \]
Equation (173) is the theoretically predicted value of the right hand side of equation (163) for the equilibrium attained in the simultaneous exchange of heat and mass between a stream of gas and a liquid wetting the walls of a smooth round pipe.

Replacing the right hand side of equation (163) by equation (173) there is obtained

\[
\frac{(P_{we} - P_a) \lambda_b}{r(1 + \frac{h_a}{h_c})(t_a - t_{we})} = \left( \frac{c_{m_a}}{h_a} \varphi_D \right) \frac{Q}{\theta_H}. \tag{174}
\]

Since \( P_{we}/P \) and \( P_a/P \) constitute mol fractions expressible by the symbols \( y_{we} \) and \( y_a \), equation (174) may also be written

\[
\frac{(y_{we} - y_a) \lambda_b}{(1 + \frac{h_a}{h_c})(t_a - t_{we})} = \left( \frac{c_{m_a}}{h_a} \varphi_D \right) \frac{Q}{\theta_H}. \tag{175}
\]

In the absence of any net exchange of heat by radiation (true of the interior of a pipe away from the exit) it would simplify to

\[
\frac{y_{we} - y_a}{t_a - t_{we}} = \frac{c_{m_a} \varphi_D}{\lambda_b \sigma_H} \tag{176}
\]

from which it is seen that the equilibrium temperature attained some distance inside the pipe would be somewhat lower than that attained at the exit.
5. Application to the wet-dry-bulb psychrometer

A wet-bulb thermometer consists of a cylinder (the thermometer bulb and stem) covered with a layer of two or three thicknesses of cloth. When this is dipped in a liquid and exposed to a gas stream flowing past it at right angles, it rapidly cools to an equilibrium temperature that we will designate by the symbol $t_{wb}$ (temperature, wet-bulb) to distinguish it from the equilibrium temperature $t_\text{we}$ attained in flow through a pipe. If the initial temperature of the liquid in which it is dipped is considerably above or below the temperature $t_{wb}$, the approach to equilibrium may require such a relatively long period of time $t$ that a true equilibrium is not actually achieved. This is because the limited amount of fluid stored in the porous membrane of the wick may not be enough to supply the evaporative loss over such a period of time and still remain sufficiently wet as equilibrium is neared. To overcome this difficulty, one needs only to pre-heat or cool the wetting fluid to such a temperature that when the freshly wetted wick is exposed to the gas stream, it is already within a degree or so of the equilibrium temperature. This permits equilibrium to be attained so quickly that the wick is still saturated with liquid. In this event, a long period of completely adiabatic exchange of heat and mass occurs, during which the minimum constant temperature
can be read with precision, and with the assurance that it is the true equilibrium temperature.

During the approach to equilibrium, the rate of heat loss by evaporative cooling, as well as the rate of heat gain by forced convection and radiation, are given by the same equations as were written for the case of the interior of a cylinder. It should be noted, however, that in applying them to flow at right angles to a cylinder, the film coefficients of heat transfer and diffusion are likely to be different and hence should be distinguished from those for flow through cylinders. Thus, in applying the heat balance of equation (163) to flow at right angles to a cylinder we should write

\[
\frac{(P_{wb} - P_a) \lambda_b}{P_{dm}(1 + h^i_{pc}/h_c)(t_a - t_{wb})} = h_{cc}/k_{gc}
\]  

(163a)

where the additional subscript c on the ratio \( h_c/k_g \) is intended to indicate flow around a cylinder.

The mechanism of fluid flow at right angles to a cylinder is so complex that a mathematical treatment of it similar to that presented for flow through a pipe appears almost hopeless, if not impossible. However, whatever the true form of the expressions analogous to equations (118a), (122a), and (125a) for flow at right angles to a cylinder
may be, one can always write \( \phi_{FC}, \phi_{HC}, \) and \( \phi_{DC} \) as symbols for the right hand members of the respective, analogous equations.

Since the ratio \( h_{CC}/h_{G} \) will still be given by equation (173) written with \( \phi_{DC} \) and \( \psi_{HC} \) instead of \( \phi_{D} \) and \( \phi_{H} \), the equations for a wet-bulb thermometer will, except for this difference, be the same as before. Thus for the wet-bulb thermometer we may write

\[
\frac{(y_{wb} - y_{a}) \lambda_{b}}{(1 + h_{c}'/h_{cc})(t_{a} - t_{wb})} = \left( \frac{c_{H}}{M_{a}} \right) \left( \frac{\phi_{DC}}{\phi_{HC}} \right) = \left( \frac{c_{H}}{M_{a}} \right) \psi_{C}.
\]

(175a)

The fact that the ratio \( (\phi_{DC}/\phi_{HC}) \) cannot be much different than the ratio \( (\phi_{D}/\phi_{H}) \) is shown by the experiment in which a wet-bulb thermometer is placed at the axis of a small pipe through which air is drawn. Since the air flows parallel to the surface of the wick in such a case, it comes close to being the same condition of flow for which equation (175) was derived. After correcting for the difference in radiation exchange, the difficulty of detecting a consistent difference between the value of \( \psi \) obtained in this manner and \( \psi_{C} \) for flow at right angles to the same bulb would argue that, although the numerical values of \( \phi_{HC} \) and \( \phi_{DC} \) may be markedly different from those of \( \phi_{H} \) and \( \phi_{D} \), their ratio is, neverthe-
less, very nearly the same.

The difficulty in predicting the absolute value of $\dot{q}_{\infty}$ or $\dot{q}_{Dc}$ for the wet-bulb thermometer arises from the fact that the frictional shear at the surface of a cylinder past which fluid is flowing at right angles varies widely with the position of any given element of surface with respect to the direction of flow. Since the square root of the ratio of shear to density is proportional to $u_*$, it is apparent that the modulus $\sqrt{r_{\infty} u_* / \nu}$ varies from small to large values as a function of the relative position of a given element of area.

An excellent demonstration of the fact that this must be the case is available in the work of Winding and Cheney [32], and of Drew and Ryan [8a]. Winding and Cheney passed air at right angles to cylinders of naphthalene made by casting it in molds. By measuring the local decrease in thickness around the cylinder they were able to determine the quantity of naphthalene sublimed, and from this, the local value of the heat transfer coefficient. Drew and Ryan divided the interior of a cylinder into 18 parallel compartments, from which condensed steam used in heating the cylinder while exposed to an air stream could be collected individually from each compartment. In this way they were able to establish the local value of the heat transfer coefficient in relation to the direction of fluid flow past it. Both groups of ex-
experimenters found substantially the same results.

To assist in showing how these results apply to the present problem, let us imagine the cross section of a cylinder to be represented by a circle on which relative position is indicated by marking in degrees from 0° to 360°. Let the diameter extending from 0° to 180° be considered as parallel to the direction of flow of an air stream, with the air impinging on the 0° element, and with flow parallel to the surface elements at 90° and 270°.

In relation to such a diagram, it is found that the maximum rate of heat transfer occurs at the 0° and 180° positions. Between these points it falls to a minimum at about 90° and 270°. Winding and Cheney found the ratio of the local value of the heat transfer coefficient at the 90-270° positions to that at the 0° position to vary from 23 per cent at a Reynolds number of 3,800 up to only 29 per cent at a Reynolds number of 12,000. Similar results were obtained by Drew and Ryan.

Perhaps the most remarkable feature of the findings of both teams of researchers was the fact that at the 180° position, where only the eddies circulating in the backwash behind the cylinder can be acting to produce surface shear, the local value of the coefficient above a Reynolds number of 3,800 actually exceeded that observed at the 0° or impact position. Since the film coefficient of heat transfer is
Influenced so greatly by the intensity of fluid shear, this can only mean that the value of $u_\tau$, and hence that part of the drag responsible for surface shear, is no greater at the front of the cylinder than it is at the rear.

The minimum value of heat transfer observed at the 90° and 270° positions can be explained as due to the presence of a stagnation zone. As is well known, the inertia of an air stream contacting the front of a cylinder causes the stream to diverge, thus creating a vacuum around the rear half of the cylinder. Air from the eddies behind the cylinder must then flow forward from the 180° position to the 90° and 270° positions in order to fill the void, from which it eventually flows outward to merge with the inner surfaces of the outer diverging flow streams. Hence, at the 90° and 270° positions there is no net flow relative to the surface of the cylinder and $u_\tau$ probably falls to practically zero.

If one can assume that flow of air from the 0° and 180° positions toward the 90° and 270° positions is parallel to the surface of the cylinder throughout the fusion layer, the differential expression found for Prandtl mixing length, equation (36), should still apply. Integrating this across the film, and following a procedure similar to that employed for the derivation of heat and mass transfer equations, expressions similar to equations (118a), (122a), (125a), and those which led up to them should result. If we can assume
that the integral curves representing \( u/u^*, c_f u^*/h \) and \( u^*/\varepsilon_m \) have the same general shape as before, i.e., a rapid rise proportional to \( y^+ \) in the first half of the fusion layer, bending over to become nearly flat beyond the outer boundary of the fusion layer, it becomes possible to give at least a qualitative explanation of the manner in which the overall or average coefficients for heat and mass transfer around a cylinder might be expected to vary with Reynolds number.

That there must be a considerable difference between the mechanics of flow around a cylinder as compared with flow through a pipe is obvious simply from a comparison of the manner in which the Nusselt number \( \frac{h_d}{k} \) varies with Reynolds number for the two types of flow. Thus for air flowing through a round pipe, \( \frac{h_d}{k} \) increases from a value of about 13 at an \( \text{Re} \) of 3,000 up to a value of 200 at \( \text{Re} = 100,000 \), a rate of increase proportional to the eight-tenths power of Reynolds number over the entire range. Below an \( \text{Re} \) of about 1,500 \( \frac{h_d}{k} \) becomes constant with a theoretically predicted value [29a] of 4.365 in both heating and cooling. The outstanding feature of this behavior is that the abrupt transition from turbulent to laminar flow at a Reynolds number of 1,500 to 3,000 so well known to occur in the flow of fluids through pipes, is in the case of heat transfer fully reflected by the abrupt transition of
$\frac{hd}{k}$ from a quantity proportional to $Re^{0.8}$ above $Re = 3,000$ to a value independent of $Re$ below $Re = 3,000$.

When we examine the variation of $\frac{hd}{k}$ with $Re$ in the case of heat transfer at right angles to cylinders, we find an entirely different state of affairs. The correlation developed by Dr. Ruth [29a], presented earlier as equation (15), was

$$\frac{hd}{k} = 0.45(10Re_f)^{0.235}[1 + 0.1595 \log (10Re_f)] \quad (15)$$

When $\frac{hd}{k}$ as computed from this equation is plotted logarithmically against $Re$, it yields a curve which has a slope of 0.235 at $Re = 0.10$, and which increases as a function of $Re$ up to a value of only 0.46 at an $Re$ of 100,000. At an $Re$ of 3,000 the $\frac{hd}{k}$ for flow around cylinders is 28; at 100,000 it is 260.

Comparing the values of $\frac{hd}{k}$ for flow around cylinders with those through pipes, two characteristics are seen to be outstanding: (a) there is no abrupt change in the variation of $\frac{hd}{k}$ with $Re$ in the case of flow outside of cylinders to indicate the existence of critical region in which flow might change from the laminar to the turbulent regions, (b) the values of $\frac{hd}{k}$ for flow outside of cylinders vary with Reynolds number to a much smaller extent than they do for flow through pipes.
Reynolds number affords no basis for comparing the absolute magnitude of \( \frac{h_d}{k} \) for the two cases. Such a comparison requires the expression of \( \frac{h_d}{k} \) in terms of \( \text{Re}_* \) or \( u_* \). Since the frictional drag \( \Delta P_f \) for flow around cylinders is completely masked by the impact drag \( \Delta P_i \), no direct comparison is possible, and we can only infer what the average frictional drag must be from an examination of heat and mass transfer, rather than friction drag data.

Since \( u_* \) varies from a maximum value at the 0° and 180° positions to near zero at the 90° and 270° positions, it is apparent that the average values of \( \frac{c_p u_*}{h} \) and \( u_* / k_{gm} \) for flow around a cylinder can be obtained only by summing up the individual values of these quantities around the periphery of the cylinder. Because in such an integration we must always pass through the regions at 90° and 270°, where both eddy conduction and diffusivity have fallen to practically zero, the integral values of \( \frac{c_p u_*}{h} \) and \( u_* / k_{gm} \) will not exhibit the abrupt change in their rate of increase with \( y^+ \) that was so characteristic of these quantities in the case of flow through pipes. Instead, the semi-logarithmic plots of \( \frac{u_m}{u_*} \), \( \frac{c_p u_*}{h} \), and \( u_* / k_{gm} \) against log \( \text{Re}_* \) can very well be imagined to take the form of a set of three quite similar upward swinging curves, the slope of which approaches a value of 5.7 only at very large values of
Due to the fact that they represent averages which are taken over a region which always extends down to small values of $y^+$ no matter how large the value of $Re_*$ may be, they may not be quite congruent; i.e., if they were to be plotted on separate sheets of graph paper, the vertical shifting of any two plots would not quite bring them into coincidence over their entire length. However, we would expect that their relative positions on the arithmetic axis would be fairly proportional to the quantities $Pr^{2/3}$ and $S^{2/3}$ since these groups may reasonably be expected to still characterize the relative magnitude of the quantities $c/u_*/h$ and $u_*/k_{gm}$ for the curves describing the behavior of individual elements of cylinder area. Hence, although the fusion boundary limit of $y^+ = 38$ for flow through pipes may not be expected to have any significance in the plots of average velocity, temperature, and diffusion numbers for flow around cylinders, it should still be possible to find some unique point $Re_{*c}$ on the $Re_*$ axis of $(c/u_*/h)_{av}$ and $(u_*/k_{gm})_{av}$, which when divided by the corresponding value of $(u_0/u_*)_{av}$ read from the velocity number plot at $Re_{*c}$, would come close to satisfying the conditions

$$\frac{(c/u_*/h)_{av}}{u_0/u_*} = Pr^{2/3} \quad (176)$$
and

\[
\left( \frac{u_*/k_{r\text{m}}}{u_c/u_*} \right)_{av} = \varepsilon^{2/3} .
\]  

(177)

Since all three curves are supposed to be fairly congruent, they should all exhibit very nearly the same numerical difference in \((c_\rho u_*/h)_{av}\) and \((u_*/k_{r\text{m}})_{av}\) between any other value of \(Re_*\) and the critical value \(Re_{*c}\). Hence, one should be able to express the total average temperature and diffusion numbers at a given value of \(Re_*\) by means of the relations

\[
(c_\rho u_*/h)_{av} = \frac{u_c}{u_*} \varepsilon^{2/3} + \frac{u_m - u_c}{u_*} = \phi_{Hc} \quad (178)
\]

and

\[
(u_*/k_{r\text{m}})_{av} = \frac{u_c}{u_*} \varepsilon^{2/3} + \frac{u_m - u_c}{u_*} = \phi_{Dc} . \quad (179)
\]

Indicating the particular value of \(u/u_*\) that satisfies equation (176) and (177) by the symbol \(M_c\), and factoring it out of equations (178) and (179), they become

\[
(c_\rho u_*/h)_{av} = M_c(\varepsilon^{2/3} - 1) + \frac{u_m}{u_*} = \phi_{Hc} \quad (180)
\]
and

\[
\left(\frac{u_s}{k_{gm}}\right)_{av} = \\
\text{Re}_c (S^{2/3} - 1) + u_m/u_s = \phi_{Dc}. \quad (181)
\]

Now the ratio \( u_m/u_s \) should be related to the frictional drag or surface shear around a cylinder in exactly the same way as in flow through pipes, i.e.

\[
u_m/u_s = \frac{u_m}{\sqrt{f/\rho}} \quad (182)
\]

\[
= \sqrt{f/2} \quad (182a)
\]

\[
= \sqrt{D_f S/A} \quad (182b)
\]

where \( D_f \) is the frictional drag alone.

Hence, if one chooses, equations (178) and (179) may be rewritten in still another form, i.e.

\[
(c\ell u_s/h)_{av} = \\
\text{Re}_c (Pr^{2/3} - 1) + \sqrt{2/f} = \phi_{He} \quad (183)
\]

and

\[
\left(\frac{u_s}{k_{gm}}\right)_{av} = \\
k_c (S^{2/3} - 1) + \sqrt{2/f} = \phi_{Dc}. \quad (184)
\]
It now becomes apparent that the quantity \( \psi_c \) in equation (176a) may be written

\[
\psi_c = \frac{\phi_{dc}}{\phi_{hc}} = \frac{(s^{2/3} - 1) + (\sqrt{2/f})/\text{Mc}}{(\text{Pr}^{2/3} - 1) + (\sqrt{2/f})/\text{Mc}}
\]  

(186)

where the term \( \sqrt{2/f} / \text{Mc} \) is a variable which, although it comprises a ratio of two unknowns, neither of which is susceptible of direct measurement by itself, we may nevertheless expect to be able to correlate in terms of Reynolds number.

In view of the fact that in the case of flow through pipes the quantities \( \text{Mc} \) and \( \text{M}_D \) were found to depend somewhat upon the magnitude of the Prandtl or Schmidt numbers involved, giving values of \( \text{Mc} \) and \( \text{M}_D \) at the boundary of the fusion layer correlated by the expression

\[
\text{M}_H = (11.50 + 1.34 / \sqrt{\text{Pr}})(\text{Pr}_{b})^{2/3}
\]

(113)

in the case of Prandtl number, and by a similar expression containing \( S \) in the case of Schmidt number, the possibility should be borne in mind that the curves of \( (c\phi u_\ast / h)_\text{av} \) and \( (u_\ast / \tilde{e}_m)_\text{av} \) as functions of \( \text{Re}_\ast \) may be spaced in such a way that no single value of \( u_\ast / u_\ast \) can be found that will satisfy both equations (176) and (177) simultaneously for all values of Prandtl and Schmidt number. However, two separate points
in $Re_*$ can always be found that will satisfy equations (176) and (177) independently. Let us assume that at two such points in $Re_*$ the value of $u_c/u_*$ that satisfies equation (177) for diffusion is $b$ times larger than the value of $u_c/u_*$ that satisfies equation (176) for heat transfer. In this event equation (177) may be written as

$$\frac{(u_*/k_{gm})_{av}}{u_{CH}/u_*} = bS^{2/3} \quad (177a)$$

and equation (179) as

$$\frac{(u_*/k_{gm})'_{av}}{u_*} = \frac{u_{CH}}{u_*} bS^{2/3} + \frac{u_m - u_{CH}}{u_*} \quad (179a)$$

A possibly confusing point here is the fact that the increment $\Delta (c'_{u_*/h})_{av}$ between $Re_{* CH}$ and any other value of $Re_*$ is the same as the corresponding increment $\Delta (u/u_*)$ on the temperature number curve between the same values of $Re_*$, due to the fact that the curves of velocity, temperature, and diffusion number are all assumed to be fairly congruent. Hence, the second term of equation (179a) remains as $(u_m - u_{CH})/u_*$ instead of $[u_m/u_* - b(u_{CH}/u_*)]$ as might at first glance be expected.

Factoring out the quantity $u_{CH}/u_*$ from equation (179a) it becomes
\[(u_*/k_{\text{av}}) = (u_{c*}/u_*)[bS^{2/3} - 1] + u_*/u_* \quad \text{(181a)}\]

Since \(u_{c*}/u_*\) is the same \(M\) quantity as obtained in the corresponding equation for heat transfer, the ratio of (181a) to (180) yields

\[
\psi_c = \frac{\phi(Re)}{\phi_{\text{HC}}} = \frac{(bS^{2/3} - 1) + \sqrt{2f/M_{\text{CH}}}}{(Pr^{2/3} - 1) + \sqrt{2f/M_{\text{CH}}}} \quad \text{(185a)}
\]

or, upon expressing the quantity \(\sqrt{2f/M_{\text{CH}}}\) as

\[
\sqrt{2f/M_{\text{CH}}} = \phi(Re) \quad \text{(186)}
\]

where \(\phi(Re)\) means simply "function of Reynolds number", as finally,

\[
\psi_c = \frac{(bS^{2/3} - 1) + \phi(Re)}{(Pr^{2/3} - 1) + \phi(Re)} \quad \text{(187)}
\]

It is obvious from equations (185) and (187) that if a solute vapor and carrier gas should have Schmidt and Prandtl numbers that are approximately equal, the quantity \(\psi_c\) will either be constant or vary so little with Reynolds number as to escape detection. Water vapor diffusing in air, for example, has a Schmidt number of about 0.60. Since the Prandtl
number of air is about 0.717, this gives a ratio of $S^{2/3}/Pr^{2/3}$ of $0.712/0.80 = 0.89$ when $\phi(Re)$ is assumed to be zero, increasing to only 0.905 when $\phi(Re)$ is assumed to be unity. On the other hand, if the coefficient $b$ of $S^{2/3}$ is not unity, the value of $\psi_c$ experimentally observed will depart widely from the simple ratio of $S^{2/3}/Pr^{2/3}$.

Hence, a solute vapor-carrier gas combination such as water vapor in air should be useful in determining whether or not the value of $b$ is unity, but should yield little information as to either the magnitude of the $\phi(Re)$ quantity, or as to how it varies with $Re$. To ascertain this variation requires experimental data on solute vapor-carrier gas combinations for which the difference in Schmidt and Prandtl numbers is as large as possible.

Given values of $\psi_c$ at the widest extremes of Schmidt number possible, the value of $b$ can be ascertained between any two tests on solute vapors of different Schmidt numbers determined at the same Reynolds number in the same carrier gas.

Writing

$$\psi_1 = \frac{(bs_{i}^{2/3} - 1) + \phi(Re)}{(Pr_{i}^{2/3} - 1) + \phi(Re)} \quad (188)$$

for solute 1, and
for solute 2, each equation is solved for $\phi(Re)$. By setting them equal, the unknown quantity $\phi(Re)$ can be eliminated to give

$$\frac{(bS_2^{2/3} - 1) - \psi_1(Pr^{2/3} - 1)}{\psi_1 - 1} =$$

$$\frac{(bS_1^{2/3} - 1) - \psi_2(Pr^{2/3} - 1)}{\psi_2 - 1} .$$

Upon reducing to a common denominator and solving for $b$ the following results:

$$b = \frac{(\psi_2 - \psi_1)Pr^{2/3}}{S_1^{2/3}(\psi_2 - 1) - S_2^{2/3}(\psi_1 - 1)} .$$

By comparing different solute vapors at a series of equal Reynolds numbers, an average value of $b$ can be obtained. Having found the best value of $b$, $\phi(Re)$ can be determined for each individual test by means of the equation previously ob-
tained when solving (188) or (189) for $\varphi(Re)$, i.e.,

$$
\varphi(Re) = \frac{b(S^{2/3} - 1) - (Pr^{2/3} - 1)}{\psi - 1}.
$$

(192)

It is of interest to note that if the function $\varphi(Re)$ deduced in this manner from psychrometric observations has any real physical significance, instead of being simply an empirical relation that happens to correlate psychrometric data, it should give values of $\sqrt{2/f}/M$ in agreement with heat transfer and mass transfer experiments as carried out separately.

Since the equation

$$
c_{f}u_{*p}/h_{p} = M_{b}(Pr^{2/3} - 1) + \sqrt{2/f_{p}} ,
$$

(183a)

in which the subscript $p$ stands for a differential element of surface area at a point, must be true for any given point on the periphery of a cylinder, the expression

$$
(c_{f}u_{*}/h)_{av} = k_{CH}(Pr^{2/3} - 1) + \sqrt{2/f}
$$

(183)

must be true for the cylinder as a whole.

Replacing the group $(c_{f}u_{*}/h)_{av}$ by its equivalent $Re \sqrt{f/2Pr}/Nu$, and dividing through by $M_{CH}^{2}$ and $\sqrt{f/2}$, this
becomes, upon rearrangement

\[
\left( \frac{\sqrt{2}/f}{h_{\text{CH}}} \right)^2 + (Pr^{2/3} - 1) \left( \frac{\sqrt{2}/f}{M_{\text{C}}^2} \right)^2 - \frac{RePr}{M_{\text{C}}^2 Nu} = 0 .
\] (193)

Solving this quadratic for \( \sqrt{2}/f / h_{\text{CH}} \) we obtain

\[
\frac{\sqrt{2}/f}{h_{\text{CH}}} = \frac{-(Pr^{2/3} - 1) \pm \sqrt{(Pr^{2/3} - 1)^2 + 4RePr/M_{\text{C}}^2Nu}}{2} .
\] (194)

It is true that one of the unknowns, \( h_{\text{CH}} \), still remains on the right hand side of the equation. However, by assuming values of \( h_{\text{CH}} \) arbitrarily, the known values of \( Nu \) as given by equation (15) can be employed to obtain a family of curves when plotted as \( \sqrt{2}/f / h \) against \( \text{Re} \).

When \( \Phi(\text{Re}) \) as obtained for various values of \( \text{Re} \) by means of psychrometric observations is plotted on the same diagram, it is seen that that line computed by means of equation (194) which most nearly coincides with the plot of \( \Phi(\text{Re}) \), must have been based upon a proper value of \( h_{\text{CH}} \). With the proper value of \( M_{\text{C}}^2 \) thus determined, it should be possible to solve for \( \sqrt{2}/f \), \( f \), and \( u_* = u \sqrt{f/2} \) individually.

The term \( f \) found in this fashion should represent the friction drag only, a quantity of much more theoretical significance than the total drag due to both friction and \( u_* \).
pact. The ability to distinguish between friction drag and total drag should permit us to compute the true surface shear, $\tau_0$, and thus afford theorists in fluid mechanics a means of investigating the exchange of momentum, heat, and mass between fluids and solid surfaces other than the interior of a simple cylinder.
III. EXPERIMENTAL

Since there were actually two phases to the experimental work, the emissivity determination and the psychrometric studies, the equipment (see Figures 7 and 8) was designed to serve both purposes. Many modifications and additions were made to the equipment as preliminary investigations revealed troubles that had not been anticipated in the original design. The two major parts of the equipment were the blower and the jacketed test section. The blower, B, was a squirrel cage type, manufactured by the North American Co., providing air velocities up to about 25 ft./sec. past the test thermometers. Control of air velocity was provided by a damper, D, on the inlet side of the fan.

The test section, T, consisted of two similar halves, each made of 14-gage mild steel, nine inches inside diameter and three feet long. The two parts were joined by flanges making the total vertical height six feet. Each part was jacketed, leaving a 3/4 inch annulus around the 9-inch pipe, through which water or steam could be circulated to provide controlled wall temperatures. Baffles were installed in the
FIG. 7. SCHEMATIC DIAGRAM OF MAIN APPARATUS.
FIG. 8. SCHEMATIC DIAGRAM OF TEST SECTION OF MAIN APPARATUS.
annulus to insure more complete circulation of the heating fluid. The section was mounted vertically to prevent convection currents from disturbing the velocity profile. The airflow was upward through the section into a plenum chamber, C, which helped eliminate possible unevenness in the flow due to an abrupt change of direction from the top of the test section to the blower. A galvanized iron conduit, in which the damper was located, led the air from the plenum chamber to the fan.

To determine the wall temperature, a number of copper-constantan thermocouples were installed on the wall surface by cutting a groove in the metal wall and soldering the couple in the groove in such a manner as to leave a smooth surface. This arrangement was found to be unsatisfactory since with steam in the jacket at essentially atmospheric pressure, an inside wall temperature of about 99-100°C. would be expected from the calculated temperature drop through the steel wall and steam film. Actually, the thermocouples indicated temperatures ranging from about 90 to 96°C. with changes in air velocity of from 25 to 5 ft./sec. This indicated that even though intentionally sunk in the wall, the thermocouples were actually reading a temperature somewhere between the true wall temperature and the temperature of the air stream.
In all later work, therefore, the wall temperature was calculated using the steam temperature found from its absolute pressure and the calculated temperature drop through the wall and steam film. An assumed steam film coefficient of 1,200 BTU/hr. ft.² °F. was used. This is probably on the conservative side, but since the computed temperature drop through the film was only about 0.1 to 0.3°C. an error of 100 per cent in the assumed value of the steam film coefficient would not make an appreciable change in the calculated wall temperature.

Several methods of providing a uniform velocity and temperature profile were tried. This effort was hampered somewhat by the location of the equipment near one of the main outside doors of the building. The rather considerable traffic at practically all hours of the day created large fluctuations in ambient conditions, particularly during the winter months.

The first method tried was the installation of an orifice at the bottom or inlet end of the test section. This did not change the profile in the desired way so the method was discarded. Next, one or more layers of cloth were installed at the upper and/or the lower end of the section. The cloth at the upper end gave satisfactory results for the profile but decreased the attainable velocity to about 15 ft./sec. so this idea had to be abandoned.
The next method was the use of a conical entrance section mounted below the test section. This also had no appreciable effect on the profile. The last and most successful method was the installation of two unit-fin-tube heaters, \( H \), mounted with the fins of one at right angles to the fins of the other at the entrance to the test section. This served to smooth out the flow as desired, and, in addition, by circulating water at room temperature through the fin-tubes, fluctuations in the inlet air temperature were almost entirely eliminated. The water used for this purpose was pumped through the heaters from a 55-gallon drum, and then returned to the drum.

Another drum was provided to hold water to circulate through the jacket, \( J \). A steam coil was placed in the drum so that any reasonable water temperature could be maintained. However, when it was found that the thermocouples did not operate as desired, this system was abandoned, due to the fact that since the temperature drop through a water film is generally much greater than through a steam film, a more accurate value of the heat transfer film coefficient would have been needed. Hence, steam was the heating fluid used in all emissivity determinations.

Ports \( P_1 \) and \( P_2 \) were provided in the lower half of the test section to permit the installation of thermometers and
a pitot tube. One of the ports, $P_1$, located two inches below the flange, held the pitot tube in a track so that it could be pulled back against the wall when not in use. The tip of the pitot tube was on the same level as the thermometers so that a point velocity could be determined exactly where the thermometer bulb was to be situated. The pitot tube was made of two concentric copper tubes. The inside tube, $\frac{1}{8}$ inch outside diameter, formed the tip, and the outside tube, $\frac{1}{4}$ inch outside diameter, extended to within $\frac{3}{4}$ inch of the tip. The remaining space was filled with solder and ground smooth to give a taper to the end of the pitot tube. Static pressure holes were drilled through the outer tube three inches from the tip. The static pressure was transmitted through the annulus to a side take-off tube which was connected to one leg of a manometer. The impact pressure was transmitted through the central tube to the other leg of the manometer.

Two different manometers were used, one for high velocities and one for low. They were both of the null point type and both used methanol as the manometer fluid. The manometer for the high velocities had a large chamber filled with liquid which was connected by rubber tubing to an inclined glass tube held by a fixture which was mounted on a vertical machine screw having ten threads to the inch. Since there was a calibrated dial on the hand wheel which
turned the screw, readings could be made to the nearest 0.001 inch. An etched mark on the inclined glass tube provided a reference point to which the meniscus was returned for an impact pressure reading as well as a zero reading with both legs of the manometer open to the atmosphere.

The other manometer was an inclined tube device as indicated in Figure 9. The manometer was supported on three points, one fixed (B) and two movable (A and C). Both A and C had graduated dials and pointers to determine the number of revolutions of the dial and screw. In use, the manometer was elevated to any convenient small angle above horizontal by adjusting pivot C. With the manometer open to the atmosphere, pivot A was turned until the menisci were in coincidence and the reading of dial A was noted. After applying the differential pressure to the manometer legs, pivot A was again turned until the menisci were again brought into coincidence and the reading noted. The actual head of fluid was then determined from the number of net turns of pivot A and the physical measurements of the device. From the simple geometry involved it can be seen that the ratio of the height of manometer fluid (r) to the height change (H) of pivot A is the same as the ratio of the distance between the centerlines of the manometer tubes to the distance between pivots A and B. This is because the angles are so small that the
FIG. 9. MICRO-MANOMETER.
cosines of them are essentially unity. Thus

\[ \frac{r}{\bar{h}} = \frac{D}{\bar{E}} \] (195)

Since the physical dimensions were readily obtained, the height of fluid could be found quite accurately to 0.001 inch.

The other ports, \( P_2 \), (Figure 7) for the thermometers, were diametrically opposed and six inches below port \( P_1 \). Holders were made of glass tubing for the thermometers in order to eliminate the necessity for making stem corrections for the portions of the thermometers that extended through the steam jacket. The inside diameter of the glass tube was slightly larger than the thermometer, thus providing an annular space through which air could be drawn at a high velocity by means of a vacuum pump. The vacuum pump was connected to a side outlet of the glass tube. The outside end of the tube through which the thermometer extended was sealed by means of a short piece of rubber tubing of a diameter small enough to contact the thermometer stem after it had been slipped on to the glass tubing. Small nodules of glass were fused to the inside end of the glass tube to keep the thermometer centered. The tube extended out to about 1 1/2 inches beyond the hot wall inside of the test section so that the air drawn into the tube was at essentially the same temperature as that passing the bulb of the thermometer.
Side outlets to the atmosphere on the vacuum line permitted varying the air velocity past the thermometer bulb from zero to a high velocity.

The usual test procedure was to obtain a pitot tube reading at the centerline of the test section, and then, with the pitot tube pulled back out of the way, one thermometer would be inserted with the bulb on the centerline. After the equilibrium temperature had been ascertained, this thermometer would be pulled back and the one from the other side would be inserted in a similar manner. The procedure was repeated often enough until consistent readings were obtained. If the flow of air past a thermometer bulb while it was withdrawn into its tube was maintained constant, the indicated temperature might change several degrees from the temperature it had indicated while in the test point location. Upon returning it to the test position, several minutes might be required for it to return to an equilibrium reading. By adjusting the suction on the tube while a thermometer was waiting its turn, this temperature change could be prevented so that when it was reinserted to the test section, it was already within 0.1 or 0.2 degrees of the expected reading.

The thermometers were 0 to 100°C. range standard boiling point thermometers with 0.1°C. graduations. They were
calibrated in a constant temperature bath against a similar thermometer which had previously been calibrated by the U.S. Bureau of Standards. The maximum error over the range employed was about 0.05°C.

During emissivity tests, one thermometer would be wrapped with two or three layers of aluminum foil, while the other would be used plain but carefully wiped free of foreign films of moisture, dust, or grease before insertion.

The same plain thermometer was also used as the dry-bulb thermometer in the psychrometric tests. Calibrated 0.1°C. thermometers of 0 to 50°C. range were used as wet-bulb thermometers in order to secure a sufficient length of mercury thread to permit reading a wet-bulb temperature when the bulb of the thermometer was situated at the centerline of the duct. Wet-bulb wicks were made of clean cotton cloth. Tests made with two different weaves of cloth and with one, two, or three layers of cloth showed no apparent difference when the wet-bulb thermometer was introduced already cooled to the wet-bulb temperature.

As stated previously, steam was used to heat the walls. It came from the laboratory low pressure main at about 40 psig and was throttled at the main to slightly above atmospheric pressure before being led to the jacket. At first, parallel flow through the various parts of the jacket was used. The jacket on each half of the test section was di-
vided into two parts by partitions extending from top to bottom. Additional baffles also were installed to distribute the flow more thoroughly. Thus the steam originally went through all four parts in parallel, but when inconsistent results were obtained, the piping was changed to make the steam flow in series, first through the bottom halves, then through the top and to the drain.

The results obtained in this manner were better but still varied from day to day until it was noted that since the steam outlets were located 1 1/2 inch above the flange, there was room for condensate to collect, in only a narrow band it is true, but in the most critical part of the apparatus directly opposite the thermometers. Even though the water in this layer would have been at the same temperature as the steam, the lower film coefficient for the water would have lowered the wall temperature adjacent to this band by several degrees. The situation was corrected by inserting a small copper tube through a tee in the steam line discharge port of each section. This tube was bent so that its open end extended into the steam jacket to a point only a fraction of an inch above the bottom flange. Since the steam pressure was slightly above atmospheric pressure, this forced the condensate out through the tube to the drain before free passage of steam through the jacket could take place. The effectiveness of its action was observed by placing a length
of glass tube in the drain line. The rather high rate of condensate flow indicated that the modification had been really necessary.

To use the same equipment for psychrometric work it was thought desirable to make it into a closed system. Experience had shown that the humidity of the laboratory air varied with time in a random manner due to open floor drains, the use of heating steam, and the intermittent opening of the outside door. Consequently a large box, A, (Figure 7) was erected, 4 by 8 by 12 feet, made of gypsum wall board on a wooden framework. Suitable ducts connected this to the fan outlet and test section inlet. A large volume was needed so that the small amounts of water evaporated from the wet-bulb wick would not change the humidity appreciably over a test period. The duct leading to the test section was also made of wall board. Turning vanes, V, were installed where the direction of air flow changed abruptly from horizontal to vertical. These vanes were made of metal and curved so that the leading and trailing edges were at right angles to each other. They were placed one inch apart and fulfilled their purpose very well.

The portion of duct leading from the fan to the box was made of tempered Masonite and lined with asbestos paper since in it were placed five 600-watt cone electric heaters
to heat the air when desired. Provision was also made to
insert a bed of activated alumina, A, in the system at this
point when it became necessary to dry the air. All joints
were sealed with tape and any leaks which were found were
plugged with Permatex No. 2 gasket compound, which, since
it is non-hardening and impervious, proved ideal for the
purpose.

Glass-wool furnace-filters, F, which had previously
been installed at the inlet of the test section were now put
into the box in the shape of an inverted L to give maximum
flow area and minimum pressure drop. These had originally
been installed in the apparatus as used in emissivity deter­
minations after it was noted that a relatively few particles
of dust could change a thermometer reading considerably by
presumably changing the emissivity of its surface.

Upon testing the chamber, it was found that the humidity
varied quite inconsistently. If after determining the humid­
ity enough water vapor were added to have doubled the humid­
ity, it was found that the humidity at constant temperature
was practically unchanged after the addition. Humidity did
change, however, almost linearly with temperature, although
if the air were maintained at an increased temperature for
a period of time the humidity would decrease and reach a
constant value after about three hours. Hence, before making
tests it was necessary to allow the apparatus to run for
several hours to come to an equilibrium humidity. It was thought that the wall board contained a considerable amount of residual equilibrium moisture in the gypsum composing it, and that upon heating this was given off and upon cooling was readorsed. Acting on the basis of this hypothesis, all of the inside surfaces were coated with sealer and bakelite resin varnish. This reduced the moisture exchange but did not eliminate it, so that it was still necessary to operate for a considerable time before beginning an actual test run.

To enable the wet-bulb thermometer to be re-wetted and precooled to a desired temperature without removing it completely, a holder similar to the thermometer holders used in the emissivity tests was designed. The general shape is sketched below.
The diameter at A and F was large enough to allow the thermometer to be inserted easily and yet be sealed off at F with a rubber tube. Points B and E were connected to the vacuum pump by rubber tubing provided with pinch clamps so that the rate of air flow could be controlled. The opening D was connected to a source of distilled water, in this case a burette, so that the water could flow into tube C to provide a convenient reservoir in which to dip the wick. At A, small nodules of glass were welded on in three or four spots around the periphery to hold the thermometer centered in the tube and assure free passage of air around the entire stem.

The temperature of the water in C was maintained about one-half degree above the observed wet-bulb temperature so that when the thermometer was reinserted into the test section after having been dipped, there would be a definite observable fall in temperature, but a rapid attainment of the wet-bulb temperature. This made certain that the wick did not have an opportunity to dry out before reaching the wet-bulb temperature. The water for wetting the wick was either warmed or cooled, depending upon ambient conditions. Warming was accomplished by merely placing a small electric hot plate below C. Cooling was done by wrapping wet cotton cloth around C and blowing air from the laboratory compressed air line past it. The temperature could thus be maintained.
at any desired point so that the wet-bulb thermometer could be inserted back into the test section and come to an equilibrium wet-bulb temperature within a matter of 30 seconds or so.

It was considered advisable to check the results obtained upon the elaborate equipment just described with data as it is obtained under normal operation procedure. For this purpose a fan and duct situated in the unit operations laboratory were available. It provided a flow of room temperature air at velocities up to 40 ft./sec. in a nine-inch diameter pipe. The thermometers and pitot tube were situated at the outlet end of the duct and testing was done in much the same manner as described before. To circumvent the difficulty of changing humidity over a period of time, thermometer readings were recorded as a function of time and the humidity was determined at timed intervals during the testing period. A plot of humidity against time allowed the use of the proper humidity with each set of wet- and dry-bulb temperatures.

The method of determining humidity was to draw a sample of air through an absorption train consisting of two U-tubes, the first of which contained "Drierite" which is anhydrous calcium sulfate, and the second "Anhydrene", an anhydrous magnesium perchlorate. The dried air passed into a cali-
brated tank of about two cubic feet capacity, where it displaced water. The loss of water, along with the temperature and pressure in the tank, permitted the weight of air sample to be calculated. The weight gain of the absorption train, which was determined by an analytical balance, divided by the weight of air sample gave the humidity.
IV. Treatment of Data and Results

A. Emissivity of a Mercury-Glass Thermometer

For an emissivity test, the data taken were shiny and plain thermometer readings, pitot tube impact pressure, pressure of the steam in the jacket, barometric pressure, and temperature of the air into and out of the test section.

The best method of describing the method of calculation is probably to go through a sample of the steps involved. Taking the test of 11 November 1952 as an example, the data taken were:

Average plain thermometer temperature (corrected) = 38.05°C.
Average shiny thermometer temperature (corrected) = 32.53°C.
\[ \Delta t = 5.52°C. \]

Turns of manometer screw = 6.556
Manometer temperature = 23.8°C.
Steam pressure = 3.022 in. Hg
Barometer (corrected) = 28.986 in. Hg

Temperature of air in = 40.19°C.
Temperature of air out = 28.20°C.
\[ \Delta t_{\text{air}} = 11.99°C. \]
The calculations are as follows:

1. Determine manometer reading in feet of methanol using equation (195)

\[ r = \frac{H}{D/5} \]

where \( E = 6.65 \) in.
\( D = 0.816 \) in.
\( H = \text{(number of turns)} \frac{1}{28 \text{ threads per inch}} \)

or \( r = N \left( \frac{0.816}{6.65} \right) \left( \frac{1}{23 \times 12} \right) = 3.652 \times 10^{-4} \text{ N ft. of methanol.} \)

Since in this case \( N = 6.556 \) turns,
\( r = 2.094 \times 10^{-3} \text{ ft. methanol.} \)

2. Convert feet of methanol to feet of air:

At 23.8°C, the density of methanol is 49.22 lb./ft.³.

The density of air may be calculated as

\[ \rho_{\text{air}} = \frac{29}{359} \left( \frac{273.2}{T} \right) \frac{P}{29.92} = 0.7373 \frac{P}{T} \]

In this case \( P = 28.986 \) in. Hg, and
\( T = (32.53 + 273.2) = 305.7^\circ\text{R.} \)

Hence, \( \rho_{\text{air}} = 0.06994 \text{ lb./ft.}^3 \).

This gives

\[ \text{ft. air} = (2.394)(10^{-3}) \frac{49.22}{0.06994} = 1.6850 \]
3. Calculate air velocity:

For a pitot tube $u = \sqrt{2\frac{gH}{\eta}}$ or

$$u = \sqrt{(64.34)(1.6851)} = 10.41 \text{ ft./sec.}$$

4. Determine Reynolds number $\text{Re} = \frac{D u \rho}{\mu}$:

$$D = \frac{0.231}{12} \text{ ft.}$$

$$u = 10.41 \text{ ft./sec.}$$

$$\rho = 0.06994 \text{ lb./ft.}^3$$

$$\mu = (0.01878)(0.000672) \text{ lb./ft. sec.}$$

Therefore,

$$\text{Re} = 1118.1$$

5. Calculate Nusselt number $\text{Nu} = hD/k$ by equation (15):

$$\text{Nu} = 0.45(10 \text{ Re})^{0.235}[1 + 0.186 \log (10 \text{ Re})]$$

$$\log (10 \text{ Re}) = 4.04863$$

$$\text{Nu} = (0.45)(11.186)0.3868 = 16.94$$

6. Calculate $h_\text{c}$ by $h_\text{c} = \text{Nu}k/D$:

$$h_\text{c} = (16.94) \frac{12}{0.231} (0.01543) = 13.58 \text{ Btu/hr. ft.}^2 \ ^\circ\text{F.}$$

7. Determine steam temperature:

steam pressure = 28.986 + 3.022 = 32.008 in. Hg

from steam tables $t = 215.41^\circ\text{F.}$, or

$$t = 101.89^\circ\text{C.}$$
8. Calculate heat transferred to air:
\[ q = wc \Delta t \]
and
\[ w = ua \rho \]
so
\[ q = (10.41) \left( \frac{2}{12} \right)^2 \pi \left( \frac{1}{4 \times 144} \right) (0.06994)(0.242)(11.99)(1.8)(3600) \]
\[ q = \text{6624 Btu/hr.} \]

9. Calculate temperature drop through steam film and pipe wall:
   for steam film \[ q = h \Delta t \text{ or } \Delta t = \frac{q}{ha} \]
   \[ h = 1200 \text{ (assumed)} \]
   \[ a = \pi DL = \pi \left( \frac{9}{12} \right)(6) = 14.15 \text{ ft}^2 \]
   \[ \Delta t = \frac{6024}{(14.15)(1200)} = 0.35^\circ F. \]
   for pipe wall \[ \Delta t = \frac{q}{(k/x)a} \]
   \[ k = 26 \]
   \[ x = 0.0781/12 \text{ ft.} = 0.00651 \text{ ft.} \]
   \[ a = 14.15 \]
   \[ \Delta t = \frac{6024}{(26/0.00651)(14.15)} = 0.11^\circ F. \]
   total \[ \Delta t = 0.46^\circ F. = 0.26^\circ C. \]
   \[ t_{\text{wall}} = 101.89 - 0.26 = 101.63^\circ C. \]
10. Compute $\Delta \phi_{wp}$ and $\Delta \phi_{ws}$:

$$(T_p/100)^4 = 93.73$$

$$(T_s/100)^4 = 87.26$$

$$(t_w/100)^4 = 197.19$$

$$\Delta \phi_{wp} = 103.46 \quad \quad \Delta \phi_{ws} = 109.93$$

11. Compute $\xi_p$ by equation (18):

$$\xi_p = \frac{h_c \frac{\Delta t}{\Delta \phi_{wp}}}{1.0089 \left(1 - \frac{\xi_p \Delta \phi_{ws}}{\xi_p \Delta \phi_{wp}}\right)}$$

Assume $\xi_p = 0.73$

Take $\xi_s = 0.050$

$$\Delta t = 5.52^\circ C.$$  

$$\xi_p = \frac{(13.58) \frac{5.52}{103.46}}{1.0089 \left(1 - \frac{(0.05)(109.93)}{(0.73)(103.46)}\right)} = 0.774.$$

Since the results of the individual runs vary somewhat, a graphical method was employed to determine the average values. By inspection of equation (18) it is seen that since the term $1 - \frac{\xi_p \Delta \phi_{ws}}{\xi_p \Delta \phi_{wp}}$ is essentially constant, if $h_c$
is plotted against $\Delta \Phi_{w}/\Delta t$ a straight line should result going through the origin, with a slope of

$$(1.0089) \left[ 1 - \frac{\varepsilon_s \Delta \Phi_{ws}}{\varepsilon_p \Delta \Phi_{wp}} \right] (\varepsilon_p)$$

from which $\varepsilon_p$ can easily be calculated. The resulting plot is shown in Figure 10. The data from which the plot was made are given in Table 3.

The slope of the line is 0.6821. The average value of

$$\left[ 1 - \frac{\varepsilon_s \Delta \Phi_{ws}}{\varepsilon_p \Delta \Phi_{wp}} \right]$$

is 0.9276. Therefore, the emissivity is determined as

$$\varepsilon_p = \frac{0.6821}{(0.175)(1.8)^3(0.9276)}$$

$$= \frac{0.6821}{0.9359}$$

$$= 0.730.$$

This involved, of course, assuming a value of $\varepsilon_p$, but if the assumption was anywhere close to the right value a second trial was sufficient to fix the proper value.
FIG. 10. PLOT FOR EMISSIVITY DETERMINATION.
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<th>Date</th>
<th>Bar.</th>
<th>In. g</th>
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<th>t2 °C</th>
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of the Emissivity of a Mercury-Glass Thermometer

| Re  | \( \eta_C \) | \( \Delta \phi_{wp} \) | \( \Delta \phi_{ws} \) | \( \frac{\varepsilon_s \Delta \phi_{ws}}{\varepsilon_p \Delta \phi_{wp}} \) | \( \frac{\Delta \phi}{\varepsilon} \) | A |  \\
|-----|-------------|----------------|----------------|---------------------|----------------|---|
| 2036 | 18.43       | 101.47         | 106.16         | 0.9286              | 25              |  \\
| 1119 | 13.58       | 103.46         | 109.93         | 0.9275              | 18              |  \\
| 853.9 | 11.52       | 97.83          | 104.37         | 0.9269              | 16              |  \\
| 1313 | 14.66       | 96.36          | 101.88         | 0.9279              | 21              |  \\
| 1375 | 17.72       | 94.30          | 98.86          | 0.9285              | 20              |  \\
| 616.1 | 9.78        | 95.42          | 103.65         | 0.9259              | 14              |  \\
| 2938 | 22.35       | 93.11          | 101.56         | 0.9294              | 33              |  \\
| 2764 | 22.49       | 91.45          | 94.88          | 0.9292              | 33              |  \\
| 2673 | 21.93       | 54.00          | 57.48          | 0.9293              | 33              |  \\
| 2245 | 19.86       | 94.13          | 97.97          | 0.929C              | 30              |  \\
| 2453 | 20.55       | 106.43         | 109.96         | 0.9295              | 33              |  \\
| 1686 | 16.68       | 106.26         | 110.75         | 0.9289              | 27              |  \\
| 1624 | 16.60       | 102.99         | 107.58         | 0.9285              | 24              |  \\
| 1032 | 12.60       | 102.83         | 109.25         | 0.9275              | 16              |  \\
| 1322 | 14.59       | 100.14         | 106.59         | 0.9281              | 21              |  \\
| 725.5 | 10.53       | 100.81         | 108.49         | 0.9266              | 15              |  \\
| 1947 | 13.21       | 97.50          | 101.99         | 0.9237              | 26              |  \\
| 1302 | 14.55       | 97.27          | 102.80         | 0.9279              | 21              |  \\
| 656.2 | 10.05       | 99.61          | 107.56         | 0.9264              | 14              |  \\
| 533.3 | 3.89        | 102.96         | 112.12         | 0.9257              | 12              |  \\
| 2896 | 32.98       | 92.63          | 96.19          | 0.9211              | 32              |  \\

\[ \text{AV} \quad 0.9276 \]

*Omitted from average.
Activity of a Mercury-Glass Thermometer

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* $\Delta t_{ps}$

Av 0.3276

Av 0.728

and from average.
B. Psychrometric Tests

Since the values of ψ for the water vapor-air system calculated from data taken from the literature [3, 8b, 9, 18a, 27] did not agree too closely, it was thought advisable to concentrate the experimental work on this system.

The data taken in a psychrometric test were wet- and dry-bulb temperatures, pitot tube impact pressure, barometric pressure and humidity. The thermometer readings were corrected for calibration error and the barometer reading was corrected for temperature, giving the atmospheric pressure as inches of mercury at 32°F. In these tests, no radiation correction was necessary for the dry-bulb thermometer because the walls were at the dry-bulb temperature. However, if a correction had been necessary, it could have been readily determined using equation (10) if the wall temperature were known, and the emissivity of C,73 determined in the previous section. In case the wall temperature were not known, employing a second thermometer, shielded as was done in the emissivity tests, would permit the true air temperature to be determined. This would be a trial and error procedure since, although the wall temperature, \( t_w \), could be eliminated between equations (11a) and (11b) giving
\[ h_c(t_c - t_a) = h_{rp}[(t_{tc} - t_{tw}) + \frac{c_w}{\alpha_{rs}}(t_{ts} - t_a)] \quad (196) \]

the \( h_{rp} \) and \( h_{rs} \) are functions of the wall temperature so that equation (196) is not independent of the wall temperature as it might appear. When the proper value of \( t_w \) is assumed, then equation (14) will be satisfied and \( t_a \) can then be easily determined.

To determine the \( \psi \) values, employing equation (175a), it was necessary to find \( y_a \) and \( y_w \). This was done by taking the partial pressure, \( p_w \), from a handbook table of vapor pressures [23] at the wet-bulb temperature. The partial pressure of the water vapor actually present was calculated from the humidity of the air by

\[ \frac{P_a}{P - P_a} = \frac{29}{18} \alpha \quad (197) \]

The value of \( y_w - y_a \) was then given by \( (p_w - p_a)/P \).

The convection coefficient, \( h_c \), was determined by equation (15). The modified radiation coefficient, \( h_r' \), was calculated by equation (160) after having found \( h_r \) by means of the following simplified form of the Stefon-Boltzmann equation

\[ h_r = 0.00692 \cdot \frac{\varepsilon_w \tau_av}{100} \cdot \left[1 + 0.25 \left(\frac{\Delta t}{Tav}\right)^2\right] \quad (198) \]
where $T_{av}$ is the arithmetic mean of the Rankine temperatures of the two surfaces, $\Delta t$ is their difference in temperature, and $\epsilon_w$ is the emissivity of the wet-bulb wick, which was taken as 0.90.

The quantity $\lambda_b$ was taken simply as $\lambda$ at the wet-bulb temperature as a first approximation. This value was taken from the steam tables [16]. Later, in accord with the definition of $\lambda_b$ as given by equation (156), it was increased by the sensible heat term. Before this could be done, however, it was necessary to know the fraction of the total wet-bulb depression to be ascribed to the fusion layer. This will be discussed further, later on.

The value of $K_a$ was taken as 18. Although the presence of water vapor in the main air stream reduced the value of $K_m$ from 29, the variation was small enough to be neglected. The heat capacity, $c$, of the air was considered constant at 0.242 B.U/^\circ F.

From these data a provisional value of $\psi$ was calculated. Fortunately for actual use, but not for experimental evaluation of the equation, the value of $\psi$ for water is very nearly unity due to the fact mentioned earlier, that the Schmidt and Prandtl numbers for the air-water vapor system are not greatly different, being 0.602 and 0.717 respectively. Hence for a proper testing of the psychrometric equa-
tion it was necessary to resort to some data from the literature on organic vapors.

The work of Arnold previously cited \([3]\) was taken as the most consistent and reliable. He used three substances for the major portion of his work: toluene, chlorobenzene and \(m\)-xylene. He operated over a range of velocities from 500 to 2,000 ft./min., corresponding to reynolds numbers of about 1,200 to 7,700. His data showed a definite velocity effect on \(\psi\) as would be expected from the theory \(\text{[equation (187)]}\). The form of this equation would indicate that as the velocity increased, the value of \(\psi\) for the organic vapors would decrease and approach unity as the velocity approached infinity. This is exactly the trend indicated by Arnold's data. Similarly for water, the \(\psi\) value should increase toward unity with increasing air velocity.

As was described earlier, the use of two different solute vapors of widely differing Schmidt numbers enables the value of \(b\) in equation \(\text{(137)}\) to be found. The value thus determined was about 1.08. Using this value the \(\phi(\text{ae})\) could be calculated by

\[
\phi(\text{ae}) = \frac{(bS^{2/3} - 1) - \psi(Pr^{2/3} - 1)}{\psi - 1}.
\] (199)

This was done for the three organic vapors and the \(\phi(\text{ae})\) values thus determined were found to correlate nicely with
Reynolds number when plotted logarithmically, i.e. \( \log \phi(Re) \) against \( \log Re \). The magnitude of the \( \phi(Re) \) was from about 1.0 to 1.5 in the range of Reynolds numbers used. This now permitted a better value of \( \lambda_b \) to be found. Since the total resistance to heat transfer from the air stream to the evaporating liquid on the wick is proportional to \( \left[ \Pr^{2/3} - 1 + \phi(Re) \right] \), and the resistance of the fusion zone is proportional to \( \Pr^{2/3} \), then the fraction of the total wet-bulb depression which occurs in the fusion zone is found by the ratio \( \frac{\Pr^{2/3}}{\left[ \Pr^{2/3} - 1 + \phi(Re) \right]} \). It can be seen that since the value of \( \phi(Re) \) is just slightly greater than unity, most of the temperature drop occurs within the fusion layer.

Using the fraction of the total temperature drop thus found, \( \lambda_b \) was redetermined. Using this, the values of \( \psi \), \( b \), and \( \phi(Re) \) were recomputed. These were not sufficiently different from the original values to warrant a third calculation.

The final results obtained for \( b \) and \( \phi(Re) \) were

\[
b = 1.072 \quad \text{(200)}
\]

\[
\phi(Re) = 0.38 \, Re^{0.15} \quad \text{(201)}
\]
This permits writing equation (137) as

$$\varphi = \frac{1.072 \frac{S^2}{3} - 1 + 0.38 \frac{He}{Re^{0.15}}}{Pr^{2/3} - 1 + 0.36 \frac{Re}{C.15}}.$$  \hfill (202)

Figures 11 and 12 are a demonstration of the validity of the correlation. Equation (137) may be rearranged to

$$bS^{2/3}/\varphi = (Pr^{2/3} - 1) + [(1 - \frac{1}{C})(\varphi(He) + \frac{1}{C})].$$  \hfill (203)

It may be seen that if $bS^{2/3}/\varphi$ is plotted against the term in brackets, $[(1 - \frac{1}{C})(\varphi(He) + \frac{1}{C})]$, a straight line having a slope of unity and an intercept of $(Pr^{2/3} - 1)$ or $-C.199$ should result. This is the form of Figure 11. To overcome the objection of plotting the same variable, $\varphi$, on both axes which sometimes leads to erroneous conclusions, equation (137) may be further rearranged to give

$$bS^{2/3} - 1 = \varphi(Pr^{2/3} - 1) + \varphi(1 - \frac{1}{C})(\varphi(He)) .$$  \hfill (204)

This form of the equation indicates that a single point for each substance ought to result from plotting $(bS^{2/3} - 1)$ against the entire right hand side of equation (204). These points should fall on a line through the origin with a slope of unity. This is shown in Figure 12. The length of the horizontal line for each substance shows the maximum deviation which occurred in the data and the symbol near the
Fig. II. Correlation of Psychrometric Data.

DATA OF THIS AUTHOR
- WATER-AIR
- TOLUENE-AIR
- CHLOROBENZENE-AIR
- M-XYLENE-AIR

DATA OF ARNOLD (3)
- WATER-AIR
- TOLUENE-AIR
- CHLOROBENZENE-AIR
- M-XYLENE-AIR

$\frac{bS^{2/3}}{\psi}$ vs $\phi(R\psi)(1-1/\psi) + 1/\psi$
FIG. 12. CORRELATION OF PSYCHROMETRIC DATA.
center of the horizontal line is located at the average value of the abcissa quantity. The data for water vapor, which represents the results of 32 runs at velocities which ranged from 5.5 to 36.3 ft./sec. or a Reynolds number range of about 700 to 5,100, is quite good. The data and calculated values are found in Table 4. For each of the three organic vapors seven runs were used. The agreement here, too, is good.

It will be noted in Figure 11 that even though there was a fairly wide range of velocities used for water vapor, all of the points fail practically on top of each other. This is due to the fact mentioned before, that $\psi$ for the water-air system is practically constant, since the ratio $\frac{bs^{2/3}}{Pr^{2/3}}$ is nearly unity itself. This explains why the various theories which have been proposed usually work so well with water vapor and air, but fail to work for other combinations of gases and vapors.

Other data from the literature were also investigated. One of the first reported researches using organic vapors was that of Mark [18a]. His data for toluene exhibit an exactly similar trend with velocity as do the data of Arnold. Unfortunately, the data are not complete enough to treat with the derived equation. Mark's data on other organic vapors did not cover a sufficiently wide range of velocities to detect a velocity effect.
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<td>0.906</td>
<td>1.074</td>
<td>0.968</td>
<td>1.105</td>
<td>-0.228</td>
<td>0.261</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.906</td>
<td>1.075</td>
<td>0.960</td>
<td>1.11</td>
<td>-0.235</td>
<td>0.266</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.906</td>
<td>1.078</td>
<td>0.964</td>
<td>1.13</td>
<td>-0.242</td>
<td>0.292</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Bedingfield and Drew [5] have recently reported values of "wet-bulb" readings using such subliming substances as naphthalene, camphor, p-dichlorobenzene and p-dibromobenzene. They could detect no velocity effect with their data. Upon investigation it is found that their data are so scattered as to be useless for the present purpose. It is possible that their method of determining the "wet-bulb" temperature is faulty in that a single thermocouple was imbedded in the center of a comparatively large cast cylinder of the test substance. Even though the surface of their cylinders might have been at the true wet-bulb temperature (lower than the ambient air temperature), there could have been sufficient conduction of heat along the thermocouple leads from the point where they emerged from the cylinder back to the junction, to have given erroneous temperature readings. Since the materials tested by them had low thermal conductivities, the thermocouple junction would have indicated a somewhat higher temperature than the true "wet-bulb" temperature. This in turn would have made their computed $\Phi$ values too high, as is found to be the case. The use of these substances does, however, afford a new approach to the investigation of the psychrometer in that the Schmidt numbers obtainable are higher.
The physical data used in the various calculations came from several sources. It was mentioned previously that the vapor pressure data and latent heat values of water were taken from Perry [22] and Keenan and Keyes [16], respectively. The viscosity and thermal conductivity of air were calculated from equations given by Keyes [17a]. These values at 20°C, 0.01710 centipoises and 0.0096 BTU/hr. ft. °F., respectively, when combined in the proper units with the heat capacity of air, 0.242 BTU/°F.°F., give a Prandtl number of 0.717 or \( Pr^{2/3} = 0.801 \).

The Schmidt numbers were the same as used by Arnold [3] and are tabulated in Table 5. These, of course, are determined for mixtures containing primarily air.

### Table 5

<table>
<thead>
<tr>
<th>Substance</th>
<th>( S )</th>
<th>( s^{2/3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chlorobenzene</td>
<td>1.983</td>
<td>1.578</td>
</tr>
<tr>
<td>Toluene</td>
<td>1.917</td>
<td>1.543</td>
</tr>
<tr>
<td>m-Xylene</td>
<td>2.087</td>
<td>1.653</td>
</tr>
<tr>
<td>Water</td>
<td>0.601</td>
<td>0.712</td>
</tr>
</tbody>
</table>
Both Schmidt and Prandtl numbers are practically invariant with moderate changes in temperature and have been taken as constants throughout this work.
V. DISCUSSION

The preceding treatment, while it adequately describes the behavior of the psychrometer over the Reynolds number range of 500 to 8,000 for systems of various vapors and air, should also be tested for other conditions.

One recommendation would be to take psychrometric data at extremely high velocities such as could be obtained in a wind tunnel. This would enable a future worker to ascertain whether or not equations (200) and (201) would apply over the full range of attainable conditions. It might be possible that the $\bar{\phi}(\tilde{\Phi})$ is actually a gentle curve which can be represented adequately by a straight power function over a relatively narrow range of conditions but which might not apply if the velocity were increased to perhaps 100 ft./sec.

Studies should also be made using other gases in place of air. Helium would be a good choice if equipment to hold it could be easily built since it would give widely different Schmidt numbers. The Schmidt numbers, incidentally, offer one source of error for the treatment. There is fair-
ly wide divergence among the values reported in the literature. The Prandtl number is not subject to as much error, although there is some difference of values for this reported also.

The work of Bedingfield and Drew [5], mentioned previously, offers additional substances for investigation. It might be a better technique to use a resistance thermometer imbedded barely under the surface of the subliming solid rather than a thermocouple in the center as they used.

It was pointed out in the theory section that if the expression of $\psi$ by the relation

$$\psi = \frac{bs^2/3 + \phi(\text{Re})}{pr^2/3 + \phi(\text{Re})}$$

had real physical significance, it should be possible to identify $\phi(\text{Re})$ with the quantity $\sqrt{2/3}/K_0$ obtainable from the examination of data for heat or mass transfer alone.

In Table 6 values of $hd/k$ as given by equation (15) have been listed for a series of Reynolds numbers, and from these, by means of equation (194), values of $\sqrt{2/3}/K_0$ for assumed values of $M = 7$ and $M = 8$ have been calculated. In Figure 13 these two lines have been plotted, together with the line for $\phi(\text{Re})$ as determined from the analysis of the psychrometric data. It will be observed that
Table 6

Comparison of $\sqrt{\frac{2}{f}}/M_c$ Values as Given by Equation (194) and $\varphi(Re)$

<table>
<thead>
<tr>
<th>Re</th>
<th>$\frac{hD}{\kappa}$ (Eq. 15)</th>
<th>$\sqrt{\frac{2}{f}}/M_c$</th>
<th>$\varphi(Re)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>15.57</td>
<td>1.08 0.96</td>
<td>1.07</td>
</tr>
<tr>
<td>2,000</td>
<td>22.7</td>
<td>1.24 1.10</td>
<td>1.19</td>
</tr>
<tr>
<td>4,000</td>
<td>34.0</td>
<td>1.42 1.26</td>
<td>1.32</td>
</tr>
<tr>
<td>6,000</td>
<td>42.5</td>
<td>1.54 1.36</td>
<td>1.40</td>
</tr>
<tr>
<td>8,000</td>
<td>51.1</td>
<td>1.62 1.43</td>
<td>1.46</td>
</tr>
<tr>
<td>10,000</td>
<td>57.6</td>
<td>1.69 1.50</td>
<td>1.47</td>
</tr>
</tbody>
</table>
FIG. 13. COMPARISON OF \( \phi(Re) \) AND \( \sqrt{\frac{\xi}{2}} \frac{M_C}{M} \) OR \( \phi(Re) \) WITH \( M = 7 \) AND \( M = 8 \).
for a $M_c$ of about 7.5 the line of $\phi(Re)$ is nearly coincident with the plots of $\sqrt{2f/\mu_c}$ obtained for neat transfer only.

This would seem to indicate that the correlation presented is essentially correct.
VI. CONCLUSIONS

1. The emissivity of a mercury-glass thermometer in the temperature range 36-46° C is 0.730.

2. The behavior of the wet-bulb psychrometer is described by the equations

\[
\frac{(P_w - P_a) \lambda_b}{\rho(1 + h_r'/h_c)(t_e - t_{wb})} = \frac{\dot{m}_e}{h_a} \psi
\]

where

\[
\psi = \frac{(bs^{2/3} - 1) + \phi(\bar{\text{Re}})}{(Pr^{2/3} - 1) + \phi(\text{Re})}
\]

and

\[
b = 1.072
\]
\[
\phi(\bar{\text{Re}}) = 3.38 \bar{\text{Re}}^{0.15}.
\]

3. The variation of the Nusselt number with Reynolds number for the flow of gas past a single cylinder can be approximated by the method outlined in the discussion.
4. The form of the variation of the friction factor excluding the impact drag can be determined for the flow past a single cylinder.
VII. LITERATURE CITED


X. Acknowledgments

The writer wishes to express appreciation to Dr. E. F. Huth, whose treatment of the mechanism of mass and heat transfer in turbulent flow as presented in Chem. E. 661-2-3, has been freely abstracted in establishing the theoretical background necessary to an understanding of the theory of the psychrometer as presented in this thesis.

Appreciation is also expressed to H. H. Kuehl and J. B. Dahms for assistance in constructing equipment and taking data.
IX. Appendix

Nomenclature

\( A = \) psychrometric constant defined by eq. (1)

\( a = \) area

\( b = \) ratio of \( \frac{K_{CH}}{K_{CD}} \)

\( C = \) concentration

\( c = \) heat capacity

\( D = \) diameter

\( D_v = \) diffusivity

\( e = \) base of natural logarithms

\( F = \) ratio of \( \frac{u_{Z1}}{u_{Zni}} \)

\( F_a = \) angle factor

\( F = \) emissivity factor

\( f = \) Fanning friction factor = \( \frac{\Delta p D}{2 \rho u^2} \)

\( G = \) mass velocity = \( u \)

\( g = \) gravitational constant

\( H = \) humidity \( \frac{\text{vapor}}{\text{vapor-free gas}} \)

\( h = \) heat transfer coefficient

\( h_c = \) for convection

\( h_r = \) for radiation
\( h' = \frac{h}{r} \frac{t_s - t_t}{t_a - t_t} \)

\( k = \) thermal conductivity

\( k_s = \) mass transfer coefficient

\( L = \) length

\( = \) Prandtl mixing length

\( M = \) molecular wt.

\( M_a = \) diffusing component

\( N_m = \) mean value

\( N = \) mols of mass transferred

\( P = \) pressure

\( p = \) partial pressure

\( P_w = \) \( p \) at wet-bulb temperature

\( P_a = \) actual \( p \)

\( Q = \) heat transferred Btu

\( q = \) heat transferred Btu/hr. ft.\(^2\)

\( R = \) gas constant, or resistance

\( r = \) partial radius

\( r_o = \) radius

\( T = \) absolute temperature

\( t = \) temperature °C or °F

\( t_t = \) observed temperature of thermometer

\( t_a = \) air temperature = dry-bulb temperature

\( t_{wb} = \) wet-bulb temperature

\( t_s = \) temperature of shiny thermometer
\( t_w \) = wall temperature
\( t_{we} \) = equilibrium wick temperature
\( u \) = velocity
\( u_* \) = friction velocity = \( \frac{c}{\nu} \), \( u/u_* = \frac{2}{f} \)
\( v' \) = cross-current velocity of eddies
\( w \) = wt. of material
\( x \) = \( \frac{r}{r_0} \)
\( y \) = \( r_0 - r \)
\( Z = \frac{(y u/)}{(y u/)_b} \)
\( z = \frac{(y u/)}{(y u/)_b} \)

**Greek letters**

\( = 1 - x^{3/2} \)
\( \Delta \) = difference
\( = \) emissivity
\( = \) eddy diffusivity
\( \varepsilon \) = time
\( = \) latent heat of vaporization or sublimation
\( b = + c(t-t_b) \)
\( = \) molecular viscosity
\( = \) eddy viscosity
\( = \) density
\( = \) shearing stress
\( \phi \) = function
$\phi_D = \text{diffusion function}$

$\phi_H = \text{heat transfer function}$

$= \phi_D / \phi_H$

**Subscripts**

$a = \text{value at average conditions}$

$b = \text{value at boundary of fusion layer}$

$c = \text{convection or cylinder}$

$i = \text{isothermal}$

$ni = \text{non-isothermal}$

$o = \text{value at wall of pipe}$