Small order effects in beta decay and some decay schemes

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SMALL ORDER EFFECTS IN ALPHA DECAY
AND SOME DECAY SCHEMES

by

Arthur Pohn

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
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Approved;

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I. INTRODUCTION

One might say scientists have two aims in their work. From a theoretical point of view, they must formulate mathematical descriptions of natural occurrences; from an experimental point of view they must conduct experiments to check the validity of existing theories and to discover new facts for developing better theories.

This work is of an experimental nature, and accordingly it will primarily be concerned with the aim of revealing new facts and checking existing theories. In particular it will deal with beta decay.

In 1914 Chadwick (1) first discovered the important features of beta decay, but it was only 20 years ago that a fairly detailed theory of beta decay was formulated. This theory of beta decay, which agreed in many instances with experimental results, was postulated by E. Fermi (2) in 1934. In formulating his theory, Fermi made use of the Pauli hypothesis (3) to account for the continuous distribution of energies in beta decay. He also used a form of the quantum mechanical expression for transition probabilities (4) and chose a matrix element form which was independent of the energy of the emitted beta particle (2). As a direct consequence of his particular choice, Fermi's theory predicted that the energy distribution of the emitted beta particles should be determined by the density of final states and coulomb effects (2).

The first order agreement of experimental facts with the Fermi theory indicates that the density of final states and the coulomb
effects are indeed the predominant factors in determining the energy distribution of the emitted beta particles.

In meeting the reasonable requirement of relativistic invariance, the matrix elements in the transition probability expression can take one of five mathematical forms formally called: scalar, pseudoscalar, vector, tensor, and axial vector, or take any linear combination of these forms (5). Matrix elements with certain forms, such as the one Fermi chose, are independent of energy, but others are not. Experimental data indicate that energy dependent terms do arise in the matrix elements in some cases; consequently the simple Fermi theory is not correct (6). By interpreting the energy dependence which occurs in these cases, it is feasible to select certain combinations of matrix element forms as being probably correct (7, 8). All combinations of the forms are theoretically possible, but existing data seem to indicate that the scalar, tensor, and pseudoscalar combination (STP), or possibly the vector, tensor, and pseudoscalar combination (VTP) may be the correct one (8).

By examining small order deviations of actual spectra from spectra predicted by the simple Fermi theory, it is possible to further specify the mathematical form of the matrix elements which occur in the formula describing beta decay. These effects usually arise from so called Fierz interference terms (9). It is partly the purpose of this work to study various allowed and forbidden beta spectra in order to investigate these small deviations which
may occur.

It is also the purpose of this work to examine the decay of some nuclei so as to provide information on their nuclear states. In a general way, it is hoped that this information will be of value in formulating more accurate descriptions of nuclear structure.
II. REVIEW OF LITERATURE

The problem of small order deviations was recently investigated from a theoretical point of view by E. J. Konopinski and H. M. Mahmoud (7). Using existing experimental data, they concluded that the correct law of beta decay must involve one of two forms of the beta interaction, namely the (STP) or (VT) combination with the (STP) combination being preferred. They showed that for high energy first forbidden transitions from low Z nuclei energy deviations from a linear Kurie plot should occur. They analyzed some existing data but found it inadequate to give positive deviations within statistical error.

J. P. Davidson and D. C. Peaslee (8) analyzed the data for a number of allowed spectra which had been obtained by other investigators. Their analysis indicated that within statistical error, no deviations of the 1/w type occur. Davidson and Peaslee (10) also investigated the possibility of examining a Cl$^{34}$ transition to determine the possibility of an (SV) or (TA) mixture. Peaslee (11) also considered the theoretical explanation of first forbidden transitions with allowed shapes.

M. E. Rose and R. K. Osborn (12, 13) have reformulated the correction factors describing small order deviations in a representation which simplifies calculations.

The activity of Y$^{90}$ examined in this study has been examined by a number of other investigators (14-22). The results of these
investigations indicate that the 62.5-hour activity of \( Y^{90} \) has one beta group with a maximum beta energy of about 2.25 Mev. This beta group has the unique shape of an \( n \) forbidden transition with \( n+1 \) units of spin change. No gamma rays were found.

The 14.3-day-\( P^{32} \) activity has been investigated by several workers (23-42). A summary of this work indicates that the decay of \( P^{32} \) has a single beta group with a maximum energy of about 1.71 Mev and no gamma rays. The single beta group has essentially an allowed shape.

The 12.4-hour-activity of \( K^{42} \) is reported to have two or possibly three beta groups and one or two gamma rays (43-51). There is general agreement on two beta groups with maximum energies of about 3.6 and 2.0 Mev and one gamma ray with an energy of 1.50 mev. N. H. Lazar and P. R. Bell (51) report an additional gamma ray with an energy of about 310 kev with an intensity of about 1.5 percent of the intensity of the 1.50 Mev gamma ray. This indicates the possible presence of a third beta group. Koerts, Schwarzschild, Gold, and Wu (50) indicated that the 2.0 Mev beta group is roughly linear to about 500 kev. They set experimental limits of 15 percent on the possible admixture arising from the tensor interaction.

The 26-hour-activity of \( As^{76} \) is reported as having four beta groups of about 3.1, 2.5, 1.3, and 0.4 Mev (52-61) and four gamma rays of 0.55, 1.22, 1.77, and 2.06 Mev. The highest energy beta
group is found to have the characteristic shape of an n forbidden
transition with n+1 units of spin change (52-54). The first
excited state is assigned as a 2+ state and accordingly the 2.5
Mev beta transition is assigned as a transition from a 2- to a 2+
state. P. Hubert (60) found an additional gamma ray of 0.648 Mev
although its high intensity which was not noticed by other investi­
gators makes it questionable.

The 5 to 15-year-activities of Eu\textsuperscript{152} and Eu\textsuperscript{154} have been studied
by a number of investigators (62-78). F. B. Shull (65) and J. M. Hill
(73) report Eu\textsuperscript{152} has beta groups of 1.58 and 0.75 Mev plus others.
These groups are assigned to Eu\textsuperscript{154} by J. A. Marinsky et al. (74).
They also assign an additional beta group of 0.3 Mev to Eu\textsuperscript{154}. Eu\textsuperscript{152}
is found to decay also by K capture (69). No K capture was detected
in the decay of Eu\textsuperscript{154}. H. B. Keller and J. M. Cork (62) assign gamma
rays with energies of 0.122, 0.123, 0.244, 0.344, 0.720, 0.964, and
1.086 Mev to the decay of Eu\textsuperscript{152}. The 0.123 and 0.344 Mev gamma rays
are assigned to the beta part of the decay and the others are assigned
to the K capture part of the decay. Keller and Cork also assign gamma
rays of 0.336, 0.778, and 1.116 to the decay of Eu\textsuperscript{154}. They also
detect weak additional gamma rays which could belong to the decay of
either Eu\textsuperscript{152} or Eu\textsuperscript{154}. 

III. BETA DECAY THEORY

The primary theoretical work on beta decay was first formulated by Fermi (2). In his work, Fermi used the "neutrino" hypothesis advanced by Pauli (3) and a general form of the quantum mechanical expression for transition probabilities (4). He also introduced a four particle interaction between the neutron, proton, electron, and neutrino.

The neutrino hypothesis was essential for two reasons. Chadwick (1) first showed that beta particles have a continuous distribution of energies up to some maximum energy in a given decay. To account for the distribution of energies and to preserve the conservation of energy, it is necessary to assume the existence of a neutrino which shares the available energy with the beta particle.

The neutrino hypothesis is also essential if spin conservation is to be preserved. Experimentally it is found that nuclear spins change by integral values of $\hbar$ during the decay process. If only a beta particle with an intrinsic spin of $\frac{1}{2} \hbar$ were emitted in beta decay, conservation of spin would demand that nuclear spins change by half integral values of $\hbar$. (The quantity $\hbar$ is Planck's constant divided by 2 pi.) Therefore, the neutrino hypothesis is necessary to preserve the important concepts of conservation of energy and spin.

Quantum theory gives in general the following expression for transition probabilities (4):
\[
P = g \frac{2\pi}{\hbar} |M|^2 \frac{dn}{dw}
\]

where: \(|M|^2 = \int_{\tau''} \psi_f' H' \psi_i' d\tau''\).

The quantity \(|M|^2\) represents the interaction matrix element; \(\psi_f\) represents the final wave function for the system, and \(\psi_i\) represents the initial wave function. The quantity \(H'\) is the perturbation to the Hamiltonian giving rise to the transition, \(P\) the transition probability, \(dn/dw\) the density of final states, and \(\int \frac{2\pi}{\hbar}\) is a constant.

For a given \(H'\) it is a straightforward matter to obtain the beta decay probability distribution. Using box normalization or other means to evaluate the density of final states (4), one finds the probability of emission of a beta particle per unit time with an energy between \(w\) and \(w'\) \(dw\) is given by (6):

\[
P \ dp = \left(\frac{g^2}{2\pi^3} \hbar^7 c^3\right) \left\langle |M|^2 \right\rangle_{\nu'} (W_0 - W)^2 p^2 dp
\]

where: \(\left\langle |M|^2 \right\rangle_{\nu'} = \int d\nu' \sum_{e m j} \left| \int d\nu' \sum_{k} V^* H' U \right|^2\).

The first integration \(\int d\alpha\) is over the direction of the neutrino momentum; the first sum is over the angular momentum states of the beta particle; the second integration \(d\nu'\) is over the internal space and spin coordinates of all the particles; and the sum \(\sum_{k}\) is over the individual nucleons which can contribute to the process. \(V\) and \(U\) are the normalized wave functions of the initial and final
nuclei properly symmetrized so as not to distinguish between identical protons and neutrons.

The electron and neutrino wave functions are usually assumed to vary little over the nucleus and can therefore be factored out. After extracting $F_0 (Z, P)$ from the matrix element which accounts for coulomb effect on the amplitude of the electron wave function at the nucleus, in allowed transitions the probability expression becomes:

$$p dp = (g^2 / 2\pi \hbar^2 c^3) \int M'^2 \, F_0 (Z, P) \, p^2 \,(W_0 - W)^2 \, dp$$

where $M^2$ is the new matrix element. In all following discussions this will be the matrix element which is referred to.

Fermi chose as his general perturbing energy term:

$$H' = g \left\{ (O^L \phi \quad \varphi^\dagger) \, O^H \, Q + (O^L \phi \quad \varphi^\dagger) \, O^H \, Q \right\}$$

where $Q$ is an operator which changes a neutron into a proton; $\phi$ and $\varphi$ are the neutrino and electron wave functions; and $O^L$ and $O^H$ are arbitrary operators (2). As the simplest assumption, Fermi chose $(O^L \phi \quad \varphi^\dagger)$ to consist merely of a bilinear combination of the Dirac components. These can be arranged in five combinations with the transformation properties of a scalar, vector, tensor, pseudoscalar, or axial vector. The perturbing energy must remain a scalar under a Lorentz transformation and this can be done by choosing $O^H$ in proper manner for each choice of $(O^L \phi \quad \varphi^\dagger)$. This gives rise to five individual matrix element forms commonly designated $S$, $V$, $T$, $A$, $P$ representing the scalar, vector, tensor, axial vector, and pseudo-scalar choices of $H'$. These are (6):
\[ S = (V^\sigma \gamma_k U) \cdot (\psi^\sigma \phi) \]
\[ V = (V^\beta \gamma_k U) \cdot (\psi^\beta \phi) - (V^\gamma \gamma_k U) \cdot (\psi^\gamma \phi) \]
\[ T = (V^\beta \sigma \gamma_k U) \cdot (\psi^\beta \sigma \phi) + (V^\gamma \sigma \gamma_k U) \cdot (\psi^\gamma \sigma \phi) \]
\[ A = (V^\gamma \gamma \gamma_k U) \cdot (\psi^\gamma \gamma \phi) - (V^\gamma \gamma \gamma_k U) \cdot (\psi^\gamma \gamma \phi) \]
\[ P = (V^\beta \gamma \gamma_k U) \cdot (\psi^\beta \gamma \phi) \]

The quantities \( \beta, \gamma, \sigma, \) and \( \gamma_\gamma \) are bilinear combinations of Dirac matrices and Pauli spin matrices. Any linear combination of the five forms is also possible.

Led by the analogy with the electromagnetic field, Fermi chose the single vector form. This yielded the simple Fermi theory with energy independent matrix elements in the allowed approximation, and also led to Fermi's selection rules.

In the matrix element expressions, \( \alpha \) and \( \gamma_\gamma \) are known to have eigenvalues of \( v/c \) where \( v \) is the velocity of the particle concerned. This is generally about 1/10 for nucleons. \( \beta \) and \( \sigma \) have eigenvalues of the order of unity. Accordingly, terms containing \( (V^\gamma \gamma \gamma_k U) \) or \( (V^\gamma \gamma_\gamma \gamma_k U) \) will be about 1/10 those with \( (V^\beta \gamma_k U), (V^\beta \sigma \gamma_k U) \) or \( (V^\gamma \gamma_k U) \). In the allowed approximation only these larger terms are used. Also the spatial part of the electron and neutrino wave functions are taken as plane waves in a zero charge approximation:

\[ \psi = A e^{-i p \cdot r} \quad \phi = B e^{-i q \cdot r} \]

Then \( (\psi^L \phi) \) becomes \( A^* B (1 - i (p+q) \cdot r - \frac{1}{2} (p+q)^2 r^2 \ldots \ldots ) \)

where \( p \) is the electron momentum and \( q \) is the neutrino momentum. For allowed spectra the first term in the expansion is used, but for
forbidden transitions, higher terms in the expansion are used. Evaluated at the nuclear radius \( R \), the quantity \((p - q)^2 R^2\) equals about \( 1/100 \) and accordingly causes the transition probability to be reduced to about this order for each succeeding degree of forbiddenness. For actual calculations the electron and neutrino wave functions are computed in a coulomb field using the Dirac relativistic equation. The correction to the amplitude of the electron wave function at the nucleus as compared to the simple plane wave is indicated by the "Fermi function" \( F_0 \) (2).

The magnitudes of the transitions depend on how much the perturbing or coupling operator can make the initial and final wave functions overlap. Because angular momentum and parity are always conserved (6), selection rules occur with respect to these quantities. The eigenfunctions \( U \) and \( V \) are orthogonal unless \( I_1 = I_f \) and the parities of the two functions are the same. If the states differ in these respects, then the matrix elements vanish unless the coupling operator alters the symmetry of the \( U \) to one corresponding to \( V \) and with the same parity. This operation corresponds to the light particles carrying off the difference in the spin and parity.

For allowed transitions, if only a single form, that is just one of the \( S, T, V, A, \) or \( P \) forms is used in determining the transition probability, the matrix elements are independent of energy. For first or higher forbidden transitions, single matrix elements can give rise
to energy dependent matrix elements; the most prominent examples of this are the energy variations of the matrix elements characterizing n forbidden transitions with n/2 units of spin change. These energy variations have been tabulated as coefficients which multiply the normal probability expression (79) (80).

For allowed transitions a linear combination of forms $S, V, T, P,$ and $A$ can give rise to energy dependent matrix elements. In particular, if both the $S$ and $V$ forms ($SV$) or the $T$ and $A$ ($TA$) forms are present in the allowed transition then the simple Fermi theory distribution must be corrected by terms proportional to $1/w$ which are commonly called Fierz interference terms (9).

The fact that no deviations of this kind have been found has possibly indicated that not both the $T$ and $A$ forms ($TA$) nor both the $S$ and $V$ forms ($SV$) can be part of the beta coupling.

In first forbidden transitions the $(SA), (VT),$ and $(AP)$ linear combinations also produce Fierz interference effects. The $(STP)$ combination can also produce an energy deviation of the type of $p^2/w$ but this effect is masked by energy independent terms with the coefficient $(\kappa Z/2R)^2$ where $R$ is the nuclear radius, $Z$ is the nuclear charge, and $\kappa$ is a constant. Therefore the effect should be most prominent in those first forbidden transitions with low $Z$ and high energy.

From experimental data now available certain tentative deductions can be made about which forms of the interaction are essential (7). First, the $T$ or $A$ form must be part of the correct law of beta interaction to account for the unique spectrum shapes found in $n$
times forbidden transitions with $n/2$ units of spin change. All other forms give zero contributions for these transitions. The $T$ or $A$ form is also essential to give Gamow-Teller selection rules which are needed to understand the short life of many unit spin change transitions.

The $S$ or $V$ form must be included to explain the short life of zero spin change transitions in $C^{10}$ and $O^{14}$.

The $P$ form is needed to explain the spectrum of RaE, or actually in combination with the $T$ form.

As previously indicated, it is also indicated that not both the $T$ and $A$ form or both the $S$ and $V$ forms can be part of the beta coupling because of the apparent absence of Fierz interference in allowed transitions.

Taking all these results into consideration, one finds two alternative combinations which are possible, (VTP) and (STP). Presently, the (STP) combination of forms is favored because it would make the energy dependence of the matrix elements much smaller in first forbidden transitions.

Peaslee (8), assuming an (STP) combination, investigated first forbidden transitions from $2^-$ to $2^+$ states. He indicated that one might expect a small amount of the energy dependence normally characteristic of $n$ forbidden transitions with spin changes of $n/2$ units.

The correlation between nuclear shell structure and beta decay characteristics was noted with the development of the shell model (81), and it has become quite customary to refer to the shell model in
considering decay schemes. The shell model helps to assign spins and parities which characterize a given decay.

A number of experimental facts support the idea of nuclear shell structure. For example, the magnetic moments of odd A isotopes fall into two groups which roughly result from the parallel and anti-parallel coupling of the spin and angular momentum. (Commonly called Schmidt groups) (81). (The quantity A is the atomic number.) Also it is known that marked regularities are associated with the even A shell numbers 8, 20, 28, 50, 82, and 126, and for "even-even" nuclei, it is found that the spin is zero in every known case. The expression "even-even" referring of course to nuclei having an even number of both neutrons and protons.

A number of methods have been proposed to obtain nuclear shell structure. Most of the methods involve a single particle model, and employ two basic assumptions (82). The first assumption is that the ground state of the nucleus has the same parity as the last odd nucleon. Secondly it is assumed that the spin of the nucleus is the same as that of the last odd nucleon. For odd A nuclei, experimental results are in good agreement with the predictions of the shell model (83).

M. G. Mayer (82), and Haxel, Suess, and Jensen (84) independently proposed strong spin-orbit coupling to obtain the "magic numbers" and the proper level arrangements in the single particle model. Mayer selected a potential somewhere between a harmonic oscillator potential and a square well, although the model is not critical to
well shape. The strong spin orbit coupling was chosen to give the
gunic numbers at the proper points in the periodic table, and this
of course also indicates the order in which the levels supposedly
fill at other points. The order in which the levels fill as
predicted by the shell model has been conveniently tabulated (82).

One can, with exceptions, use the shell model to determine the
character of a given beta decay by noting the spins and parities of
the initial and final nuclei, and from this determine spin and
parity changes.

Nordheim (81) extended the shell model to odd-odd nuclei by
assuming the neutron and proton levels fill independently. He
proposed this rule for determining the resultant spin. If the
spins of the odd particle groups are $J_1 = l_1 \pm \frac{1}{2}$, and $J_2 = l_2 \mp \frac{1}{2}$,
then the resultant spin is $I = |J_1 - J_2|$. If $J_1 = l_1 \pm \frac{1}{2}$ and
$J_2 = l_2 \pm \frac{1}{2}$, the $I > |J_1 - J_2|$. 
IV. APPARATUS AND PROCEDURE

In obtaining data, three instruments were used primarily. To make beta-gamma coincidence measurements and to obtain simple beta spectra an intermediate-image spectrometer was used. This instrument is basically of a design developed by H. Slatis and K. Siegbahn (85). The details of its construction and performance have been described in the literature (86).

The instrument has a transmission of 10 percent with a resolution of about 6 percent. A greater resolution can be obtained by sacrificing transmission. Scintillation crystals in conjunction with 6292 photomultiplier tubes are used as the detectors. An anthracene crystal is used in the beta detector and a NaI (Tl) crystal is used in the gamma detector.

The gamma detector is capable of detecting up to 15 percent of the total number of gamma rays emitted and this is sufficient to give fairly good coincidence rates.

The circuitry associated with the instrument is illustrated in Figure 1. As indicated in the block diagram, the gamma pulses are amplified and passed into a pulse height analyzer. The pulse height analyzer gives an output pulse only when the input pulses have a maximum voltage amplitude within a certain range. The range is selected to correspond to pulses from the desired gamma ray. These pulses are then sent to a coincidence circuit and to a pulse stretcher that operates the scaler recording the number of pulses.
FIG. 1 BLOCK DIAGRAM OF THE INTERMEDIATE IMAGE SPECTROMETER CIRCUITRY
Pulses from the beta detector photomultiplier tube are amplified and sent to the coincidence circuit and to a pulse stretcher which is used to eliminate after pulses. The output of the pulse stretcher is used to operate the beta scaler and a gating circuit.

If pulses from both the beta amplifier and pulse height analyzer enter the coincidence circuit within about 0.5 micro-seconds of each other, an output pulse is obtained. The output pulse is gated by the pulse from the beta pulse stretcher to limit the coincidence pulses to at most one for each beta pulse. It is also gated by the gamma pulse for this reason. The coincidence circuit is normally set to be sensitive to pulses smaller than those which will trigger the beta or gamma pulse stretchers. The gating circuits eliminate any error that might be introduced by this difference in sensitivity.

In analyzing complex spectra with the intermediate-image spectrometer, one usually starts by either selecting the highest energy beta particles and scanning with the gamma pulse height analyzer or by setting the gamma pulse height analyzer on the highest energy gamma ray and scanning with the beta spectrometer.

Because of the Compton distributions, it is possible to get only the highest energy gamma ray by itself. Similarly it is possible to get only beta particles from the highest energy beta group by themselves.

By selecting beta particles from only the highest energy beta group, one can determine the gamma rays that are in coincidence with these beta particles by scanning with the gamma pulse height analyzer. Next, by selecting beta particles from the two highest beta groups,
one can observe what additional gamma rays appear in coincidence. Extending this procedure, one can determine decay schemes.

The K to L shell conversion electron ratios of the various transitions are examined whenever possible to determine the multipole order and the half-lives of the excited states of the transitions. If the half-lives of the excited states are too long, coincidences may not be detectable.

For examining gamma-gamma coincidences a gamma ray spectrometer was employed. The block diagram of the instrument is illustrated in Figure 2. As detectors, NaI (Tl) crystals mounted on 6292 photomultiplier tubes are used. Within narrow limits the voltage amplitudes of the photo tube pulses are proportional to the gamma ray energies. Pulses from the two detecting tubes are amplified and shaped. Pulses in one channel are analyzed (87) and passed to a coincidence circuit. All pulses in the other channel go to the coincidence circuit.

The coincidence pulses are used in a synching circuit to trigger the sweep of an oscilloscope. The pulses from the channel not analyzed are sent to the vertical deflection plates of the oscilloscope. Thus, the pulses arising from gamma rays in coincidence with those selected by the analyzer in the other channel are displayed. A photographic record is made of the pulse height distributions.

By switching the synching circuit, the total pulse height distribution in each channel and the part of the distribution selected out in the analyzed channel can be displayed. Photographic records are also made of these distributions. The photographic negatives
Fig. 2 Block Diagram of $\gamma$-Ray Coincidence Spectrometer
are examined with a photo-densitometer which uses a Brown recorder to graphically indicate the results.

A thin lens spectrometer was also used to examine total beta spectra (88, 89). The instrument is equipped with an automatic current control and an automatic readout device. This made it possible to examine isotopes continuously for long periods of time without interruption and without the use of an operator.

A scattering problem was encountered in the thin lens spectrometer. The holder which contained the counter window was of such a size and shape that the beta particles could scatter into the counter window. At lower energies where large angle scattering is much more probable, a detectable number of beta particles were scattered into the counter. This caused a distortion of the low energy end of the beta spectra. By shaping the holder properly and positioning the window it was possible to eliminate this difficulty.

In making up the radioactive sources which were used, care was taken to minimize the source thickness and the thickness of the source backings. Back-scattering from the backing and energy loss within the source material itself may seriously distort the spectra (90).

Whenever possible the sources were made by evaporating the source materials onto the mounting films at elevated temperatures in a vacuum. This was accomplished by placing the source materials into small tantalum boats. The films on which the sources were to be deposited were placed above the boat on a metal plate which had a hole of the right diameter to give the proper source size. After
the bell jar surrounding the film and boat was evacuated, the
tantalum boat was heated by an electrical current to the boiling
point or sublimation point of the source material. The evaporated
material, in this way, deposited uniformly on the films in areas
defined by the hole in the supporting plate.

With this procedure, most sources used in the intermediate
image spectrometer had surface densities under twenty micrograms
per square centimeter. In some instances, in particular with
sources for the thin lens spectrometer which were necessarily
of a much greater intensity, sources were made by putting the
source material into solution. A few drops of the solution were
placed on a film and the water was then evaporated off. These
sources generally were not of uniform density, for the source
materials tended to crystalize out in clumps.

The source materials in general had a high specific activity,
so the sources prepared by evaporating off the water still had
reasonably low average surface densities.

As a criterion for determining when source effects would become
appreciable, the formula given by D. R. Hamilton and L. Gross (90)
was used. This gives an energy as a function of source thickness
and material below which source effects become appreciable.
Specifically the formula is:

\[ E = 1700 \left( \frac{Z^2 \ D}{A} \right)^{1/2} \]

Where \( E \) is the energy in kev, \( Z \) is the average nuclear charge, \( A \) is
the average atomic weight, and $D$ is the surface density in grams per square centimeter. The formula was obtained empirically. As a safety factor only measurements at energies well above this "scattering" energy were used.

In mounting the various beta sources, films with surface densities of about 30 micrograms per square centimeter were employed. These films had a thin coating of aluminum evaporated onto one of their sides to prevent the gradual accumulation of positive charge.

In processing the data a number of corrections was applied. First, because the counters used have a certain dead time, a correction was applied to the measured count. The probability of counting a particle per second, neglecting decay, is constant. However, when a count does occur, the counter is unable to detect another count until a certain period or dead time has elapsed.

Because the probability per unit time of a count occurring is uniform, just as many counts are lost in the period immediately after a count as would be lost if the counter were dead for the same period at any randomly selected time. Thus the actual number of counts is diminished by the percentage of time that the counter is dead. Let $N$ be the measured number of counts per second, $N_t$ the true number of counts per second, and $b$ the dead time of the counter per count. The true and measured number of counts are then related by:

$$N = (1 - N_b) N_t$$

$$N_t = \frac{N}{(1 - N_b)}.$$
Normally Nb is much less than one and therefore:

\[
\frac{1}{1 - Nb} = 1 + Nb
\]

\[N_t = (1 + Nb)N.\]

This last formula was used to correct for the counter dead time.

The data were also corrected as usual for background and decay. At the upper end of the spectrum a small background arose from scattering. Because this background was small and occurred only at the upper end of the spectrum an approximation was used to correct for it. This background was assumed constant for the upper half of the spectrum and then assumed to decrease linearly to zero at the lower end of the spectrum. Experimental evidence indicates that this is a satisfactory first approximation.

All practical spectrometers have a finite resolution and accordingly introduce some error in the measurements of the beta distributions. The spectrometers used have transmission functions which have a constant percentage half-width with regard to the average momentum of the focused beta particles; therefore the transmission functions can be written as:

\[f\left(\frac{I - I_0}{I_0}\right)\]

where \(I_0\) is the instrument magnet current and \(I\) is the variable of integration which represents the range of current (momentum) through which particles are detected for a given current setting.
If \( N(I) \) is the true beta distribution, then the measured number of counts per second \( N_m(I_0) \) at a particular current setting is given by:

\[
N_m(I_0) = A \int_{I_0 - E I_o}^{I_0 + E I_o} f(I - I_0 / I_0) N(I) \, dI
\]

where \( A \) is a constant including the collecting geometry of the instrument. For convenience it is taken as 1. If \( f(I - I_0 / I_0) \) were a delta function, then the measured distribution would correspond exactly to the correct one. However, the actual transmission functions have half-widths of three to six percent. \( N(I) \) is a well behaved function and can be expanded about \( I_0 \) in a Taylor series:

\[
N(I) = N(I_0) + N'(I_0)(I - I_0) + N''(I_0)/2 (I - I_0)^2 - -
\]

Substituting into the expression for \( N_m(I_0) \):

\[
N_m(I_0) = \int_{I_0 - E I_o}^{I_0 + E I_o} f(I - I_0 / I_0) \left[ N(I_0) + N'(I_0)(I - I_0) + \frac{N''(I_0)}{2} (I - I_0)^2 - - \right] \, dI.
\]

Let one consider the new variable \( y \):

\[
y = \frac{I - I_0}{I_0}
\]

\[
dy = \frac{dI}{I_0}
\]

and for the limits of integration a to b:
\[ y_b = \frac{I_0 + EI_0 - I_0}{I_0} = E \]
\[ y_b = \frac{I_0 - EI_0 - I_0}{I_0} = -E \]

Rewriting the expression in terms of the new variable of integration \( y \) and solving for \( \frac{N_m(I_0)}{I_0} \):

\[ \frac{N_m(I_0)}{I_0} = \int_{-E}^{E} f(y) \left[ N(I_0) + I_0 N'(I_0) y + \frac{I_0^2 N''(I_0)}{2} (I_0)y^2 \right] \, dy. \]

Normalizing the transmission function:

\[ \int_{-E}^{E} f(y) \, dy = 1 \]

and neglecting terms above the second order:

\[ \frac{N_m(I_0)}{I_0} = N(I_0) \left[ 1 + I_0 \frac{N'(I_0)}{N(I_0)} \int_{-E}^{E} yf(y) \, dy + \frac{I_0^2 N''(I_0)}{2 N(I_0)} \int_{-E}^{E} y^2 f(y) \, dy \right]. \]

The transmission function \( f(y) \) was determined by examining a conversion line which is essentially a delta function. If one lets this delta function be \( K \int (I - I') \) then the measured count can be expressed as:
\[ N_m(I_0) = K \int_{-E}^{E} f \left( I - I_0 / I_0 \right) \delta (I - I') \, dI \]

\[ N_{m1}(I_0) = K \frac{f(I' - I_{oi})}{I_{oi}} = K f(y_1). \]

The quantity \( I_0 \) was varied through a range of values so as to give the values \( f(y_1) \) for a sufficient number of points to accurately describe the transmission function. The function \( f(y) \) goes to zero beyond plus or minus \( E \). The quantities:

\[ \int_{-E}^{E} f(y) \, dy = c \]

\[ \int_{-E}^{E} f(y) \, y^2 \, dy = d \]

were evaluated numerically. For most instrument settings the quantity "c" was nearly zero because \( f(y) \) is an almost symmetric function and "y" is an anti-symmetric function. To second order terms the measured counting rate divided by the current setting may then be written as:

\[ \frac{N_m(I_0)}{I_0} = N(I_0) \left[ 1 + \frac{I_0 N'(I_0)}{N(I_0)} c + \frac{I_0^2 N''(I_0)}{2 N(I_0)} d \right]. \]
and:

\[ N(I_o) = \frac{N_m(I_o)}{I_o \left[ 1 + \frac{I_o N''(I_o)}{C(I_o)^2} c + \frac{I_o^2 N''''(I_o)}{2 C(I_o)} d \right]} \]

Corrections arising from the terms in the denominator of the above expression were applied only as long as the corrections did not exceed about five percent. Points with greater error than this were discarded. Therefore the following approximate relations exist:

\[ N(I_o) = \frac{N_m(I_o)}{I_o} = C(I_o) \]

\[ N'(I_o) = \frac{d (N_m(I_o)) / I_o}{d I_o} = C'(I_o) \]

\[ N''(I_o) = \frac{d^2 (N_m(I_o)) / I_o}{d I_o^2} = C''(I_o) \]

Then the expression for \( N(I_o) \) becomes:

\[ N(I_o) = \frac{N_m(I_o)}{I_o \left[ 1 + \frac{I_o C'(I_o) c}{C(I_o)} + \frac{I_o^2 C''(I_o)}{2 C(I_o)} d \right]} \]

The derivatives \( C'(I_o) \) and \( C''(I_o) \) were graphically determined from the measured spectra and the final formula given above was used to determine the corrections.

By discarding points which needed more than a five percent correction one was reasonably sure that the points were accurate to
one percent. The correction factors converge quickly since the terms:

\[
E \int_{-E}^{E} f(y) y^n \, dy
\]

diminish very rapidly. The quantity \( E \) is the order of 0.05 and therefore:

\[
E \int_{-E}^{E} f(y) y^n \, dy \leq (0.05)^n .
\]

Because \( f(y) \) is almost symmetric the odd terms disappear leaving the second derivative correction as the only major one as long as the corrections are small.

Fermi functions are normally computed by assuming that the electron is in a purely coulomb field. This of course is not strictly true since the orbital electrons modify the potential. In accurate calculations, it is necessary to correct for this screening effect, particularly at high \( Z \) where \( Z \) is the nuclear charge. When necessary, data obtained in this work were corrected by interpolating in the table of J. R. Reitz (91). Reitz used the Thomas-Fermi model to calculate the potential distribution and then used this to accurately evaluate the Fermi functions. He then tabulated the correction factors for converting Fermi functions calculated in a coulomb field to the more accurate ones obtained by considering screening.
In first and higher forbidden transitions it is necessary to multiply the simple distribution formula obtained by Fermi by a correction factor. The formulas for these have been worked out exactly by Grueling (79) for the various cases. Rose, Perry, and Dismuke (80) using a relativistic wave equation have numerically computed the values of the corrections arising in the different cases and have published them in a tabulated form. Their calculations were based on an old value of $r_o$ where the nuclear radius is given by:

$$r = r_o A^{1/3}.$$  

In this work the numerical results of Rose, Perry, and Dismuke were corrected to a new and more accurate value of $r_o$ ($r_o = 1.15 \times 10^{-13}$ cm) which was obtained by Pidd (92) and others in recent studies on high energy scattering of electrons. The corrected values of the corrections of Rose, Perry, and Dismuke were then applied to the data of this work.

In accurately analyzing spectra of high Z nuclei it is also necessary to consider the effect of the cutoff of the coulomb potential at the nuclear radius. M. E. Rose and D. K. Holmes (93) have computed these corrections. For Z below sixty they are negligible; they were neglected in this work because all of the isotopes accurately analyzed had values of Z equal to or below forty.
V. RESULTS ON SMALL ORDER EFFECTS

In examining spectra for small order deviations it is essential that the measuring instruments do not add any significant distortions to the spectra. To check the two beta spectrometers which were employed in this study, the beta spectrum of $^{90}$Y was examined. One spectrometer was iron free while the other was not. The use of two spectrometers with different types of focusing also helped to check possible instrumental errors.

The 62.5-hour activity of $^{90}$Y consists of a single beta group with an endpoint energy of about 2.27 Mev. It has the unique shape of a first forbidden transition with a spin change of two (14-22). Because the matrix element involved in this transition is uniquely known, it is possible to determine the exact correction necessary to the simple Fermi formula. Thus the theory is capable of completely describing the distribution of beta particles. If a measured spectra deviates from the one predicted by the theory, one can be reasonably sure that it arises from an instrumental error.

The sample examined in the intermediate image spectrometer was evaporated in a vacuum on a collodion film with a surface density of 30 micrograms per square centimeter including a thin coating of aluminum evaporated on the rear of the film to keep it from collecting a charge. From the known solid angle and resolution of the spectrometer and the specific activity of the sample, it was possible to determine the surface density of the source material. The source density was found to be less than twenty micrograms per square centimeter. Using the
empirical relation of Hamilton and Gross (90), one finds that scattering should not seriously effect the beta distribution above 40 kev. As a safety factor, no measurements were used for energies below 100 kev.

The source material was made by bombarding spectrographically pure \( \text{Y}_2\text{O}_3 \), obtained from F. H. Spedding of the Ames Laboratory, in the high flux density pile of the Argonne National Laboratory. The average bombarding flux was about \( 1.5 \times 10^{13} \) neutrons per second per square centimeter. The oxide was converted into a chloride with HCl. This chemical form was used in the actual sources examined.

The beta spectrum obtained with the intermediate image spectrometer is shown in Figure 3. The measured spectrum was corrected for counter dead time, decay, counter background, and the finite resolution of the instrument. Measurements requiring more than a five percent finite resolution correction were discarded.

The Kurie plot obtained from this spectrum is illustrated in Figure 4. The "a" correction factor in \( \left( \frac{N}{I_0 F} \right)^2 \) was obtained from the accurate tabulated data of Rose, Perry and Dismuke (80). Their values were corrected to the new and more accurate value of \( r_0 \) obtained by Pidd. Also a slight correction was applied to the lower end of the spectrum to account for screening. The screening correction was obtained from the calculations of J. R. Reitz (91).

One observes that the Kurie plot is straight within very small limits. To facilitate the detection of any possible slight deviations, a deviation analysis plot was made. Normally on a Kurie plot the
FIG. 3 $\beta$ SPECTRUM OF $\gamma^{90}$ OBTAINED WITH THE INTERMEDIATE IMAGE SPECTROMETER
\[ a = L_0 (W_0 - W)^2 + 9 L_1 \]

2.275 \pm 0.010 MEV.

FIG. 4 KURIE PLOT OF Y^{90} OBTAINED WITH INTERMEDIATE IMAGE SPECTROMETER
experimental data are manipulated to conform with a linear equation:

\[
\left( \frac{N_i}{I_i F_1} \right)^{\frac{1}{2}} = A (W_0 - W_i)
\]

where \( W \) is the energy in \( mc^2 \) units at which the \( i \)th measurement is performed and \( A \) is a constant describing the slope of the line. Dividing both sides by \( (W_0 - W_i) \) and squaring one obtains the expression for a deviation plot:

\[
\frac{N_i}{I_i F_1(W_0 - W_i)^2} = A^2
\]

which is in the form of a constant plotted against energy. Because \( A^2 \) is determined from the experimental points it will have roughly a mean value of the various \( \frac{N_i}{I_i F_1(W_0 - W_i)^2} \). If there is variation, of course the plot will not appear as a constant. Because \( W_0 \) is determined from the least squares line through the experimental points, it would tend to make any curve describing a consistent variation of the \( \frac{N_i}{I_i F_1(W_0 - W_i)^2} \) lie partly below and partly above the value of \( A^2 \), but it would not destroy the variation.

A deviation plot of the \( ^{90}Y \) spectrum obtained with the intermediate image spectrometer is indicated in Figure 5. The probable error of the various points was assigned by considering the probable error in the measured number of counts which is equal to 0.675 \( N/\sqrt{t} \) and the probable error due to an uncertainty of the background. The quantity \( t \) is the counting time on a point. The background error is
FIG. 5 DEVIATION ANALYSIS PLOT OF $Y^{90}$ OBTAINED WITH INTERMEDIATE IMAGE SPECTROMETER
one which can add a consistent error; that is it may influence a series of experimental points all in one direction. Therefore, although one finds that in Figure 5 a few more points lie within the probable error than out, it does not necessarily mean that an excessively large probable error has been assigned.

From Figure 5 it is seen that the intermediate image spectrometer does not introduce any deviations of the spectrum within experimental error.

A $^{90}\text{Y}$ source was also examined with a thin lens spectrometer. This source was made by placing a drop of the source material on a thin film and evaporating off the water. Although the average source thickness was estimated to be less than 1.0 milligrams per square centimeter, irregularities in the source probably made the surface density higher in some parts and lower in other parts. The supporting film had a surface density of about 30 micrograms per square centimeter which was negligible in comparison with source thickness. The formula of Hamilton and Gross indicates that scattering would become noticeable for energies below 300 kev. As a safety factor points were discarded for energies below 350 kev.

The data were corrected for background, decay, screening, instrument resolution, and counter dead time. Also the revised values of the Rose, Perry, and Dismuke corrections were employed. The Kurie plot of the spectrum obtained with the thin lens spectrometer is illustrated in Figure 6. The endpoint energy is measured to be
FIG. 6 KURIE PLOT OF $\gamma^{90}$ SPECTRUM OBTAINED WITH A THIN LENS SPECTROMETER

$2.27 \pm 0.01$ MEV
2.27 0.010 Mev and within experimental error the plot is straight. A deviation plot of the spectrum is illustrated in Figure 7. Within experimental error the instrument does not introduce any error in the shape of the spectrum. The probable error was assigned, as in the case of the spectrum obtained with the intermediate image spectrometer, by considering the probable error in the measured count and the probable error in the background count.

The beta spectrum of $^{32}$P was examined for the possible presence of a Fierz type of deviation with both a thin lens and an intermediate image spectrometer. Sources were prepared by evaporating the source material on thin films in a vacuum. The films had a total surface density of about 30 micrograms per square centimeter and were aluminized to prevent source charging. Both sources had a uniform surface density which was less than 30 micrograms per square centimeter. The source material was carrier free phosphorous obtained from the Oak Ridge Isotope Division. The source material was made by a neutron-proton reaction on a sulphur sample. As a result, a small amount of the activity of $^{33}$S was present in the sample. The beta spectrum of $^{33}$S has an endpoint energy of about 249 kev and accordingly only points taken at energies above about 300 kev were used in the analysis of the $^{32}$P spectrum. The experimental data were corrected for counter dead time, decay, counter background, and for the finite resolution of the instruments. Points at the upper end of the spectrum needing more than a 5 percent resolution correction were discarded. It was not necessary to correct for screening or for the finite nuclear
Fig. 7 Deviation plot of $\gamma^0$ obtained with a thin lens spectrometer.

$N \overline{\sigma F (W_0 - W)^2}$

Expanding scale

Probable error

W energy in MeV units
radius effects. The spectrum obtained with the intermediate image spectrometer is illustrated in Figure 8 and the Kurie plot of the spectrum is illustrated in Figure 9. The maximum beta energy was found to be $1.712 \pm 0.005$ Mev, and within experimental error the Kurie plot was straight. A deviation plot was also made and is illustrated in Figure 10.

A Kurie plot of the $p^{32}$ beta spectrum obtained in the thin lens spectrometer is illustrated in Figure 11. Within experimental error the points lie on a straight line. A deviation analysis plot of the spectrum is illustrated in Figure 12.

To statistically analyze the data for Fierz deviations, the following least square development was used. If Fierz interference effects are present, the quantity $\left( \frac{N}{IF} \right)^{\frac{1}{3}}$ is described by the equation (7):

$$\left( \frac{N_i}{I_1F_1} \right)^{\frac{1}{3}} = K = A \left( W_0 - W_1 \right) \left( 1 + \frac{r}{W_1} \right)^{\frac{1}{3}}$$

where $A$ is a constant, $W_0$ is the endpoint energy, $r$ is the parameter determining the magnitude of the Fierz interference, $N_i/I_1$ is the measured count at the point "i" divided by the momentum times a constant, and $F_1$ is the value of the Fermi function for point "i". The most rigorous least squares procedure would be to minimize the sum of the squares of the deviations from the line with respect to the three parameters $A$, $W_0$ and $r$. However, this presents an imposing computational problem. The problem can be reduced to a one parameter
Fig. 8 - β spectrum of P^{32} obtained with intermediate image spectrometer.
FIG. 9 KURIE PLOT OF $p^{32}$ OBTAINED WITH INTERMEDIATE IMAGE SPECTROMETER

$1712 \pm 6$ KEV
FIG. 10 DEVIATION ANALYSIS PLOT OF P³² OBTAINED WITH INTERMEDIATE IMAGE SPECTROMETER
50
40
30
20
1.712 \pm 0.006 \text{ MEV}

W ENERGY IN MC^2 UNITS

FIG. II KURIE PLOT OF P^{32} OBTAINED WITH A THIN LENS SPECTROMETER
Fig. 12 Deviation plot of P³² spectrum obtained with a thin lens spectrometer.
problem to a good approximation.

Let one consider now an actual Kurie plot in which the least squares line is determined from the points on the upper end of the distribution between \( W_1 \) and \( W_2 \) where \( W_1 \) is some arbitrary point and \( W_2 \) is the last usable point as illustrated in Figure 13a. Let one now suppose that a Fierz type of effect is present; then the Kurie plot is curved as is indicated in the exaggerated example of Figure 13b.

At the upper end of the spectrum the points are fairly uniformly spaced. A least squares line passes through the average value of the points and is rotated so as to minimize the sum of the squares of the deviations. Thus the least squares line has to a good approximation the average slope between points \( W_1 \) and \( W_2 \), and is displaced slightly from the line connecting these two points as indicated in Figure 13b. This may be numerically verified as was done or may also be shown by considering two continuous functions describing the straight line and the Fierz type line. The square of the deviations of the Fierz line from the straight line may be minimized with respect to the two parameters of the straight line. The procedure is simplified because the summation may be replaced by an integration. The approximation to the finite number of points case is good because the number of points is fairly large and they are almost uniformly spaced.

The error in the endpoint value is almost negligible even if some Fierz type deviation is present. If Fierz deviation outside of statistical error were found, this value of \( r \) could be used to
FIG. 13 EFFECTS OF FIERZ DEVIATION IN LEAST SQUARES LINES
recompute the endpoint energy and this in turn could be used to recompute the value of \( r \). Because the endpoint value is only slightly affected by a variation of \( r \), one is reasonably sure that the process is rapidly convergent.

The small error in the value of the endpoint energy may be easily shown. In Figure 13b one notes that the endpoint energy will be approximately in error by the difference at \( \frac{W_1 + W_2}{2} \) between the Fierz line and the straight line connecting \( W_1 \) and \( W_2 \) divided by twice the slope. \( W_2 \) is near \( W_0 \), so for simplicity \( W_2 \) is taken as \( W_0 \) and \( \Delta W_0 \) becomes:

\[
\Delta W_0 = \frac{\lambda (W_0-W_1)(1+x/W_1)^{3/2} - \lambda (W_0-W_1/2)(1+x/W_0+W_1/2)^{3/2}}{2 \lambda}.
\]

If one expands the quantity \((1 + x/W)^{3/2}\) in powers of \( x/W \), which for all known cases has a value less than 0.1, one obtains to the second order:

\[(1 + x/W)^{3/2} = 1 + x/2W_1 - x^2/8W_1^2.\]

One can expect the series to converge rapidly because of the small value of \( x/W \). Using this expansion in the expression for \( \Delta W \) one gets to the second order:

\[
\Delta W = \frac{1}{2}(W_0-W_1)(x/2W_1 - x/(W_1 + W_0) - x^2/8W_1^2 + x^2/2(W_1 + W_2) - - -).
\]
If one takes now a typical case and considers only the first order terms, which are the largest, where:

\[ W_1 = 3, \quad W_0 = 4.5 \]

then one gets:

\[ \Delta W_0 = \frac{1}{8} r \left( \frac{1}{W_1} - \frac{1}{W_1 + W_0} \right) \]

\[ = \frac{1}{8} r \left( \frac{1}{W_1} - \frac{1}{W_1 (1 + \frac{1}{4})} \right) \]

\[ \approx \frac{r}{8} \left( \frac{1}{W_1} - \frac{1}{W_1} \right) \]

\[ \approx \frac{r}{32 W_1} \approx \frac{r}{100} \text{ for } W_1 = 3. \]

For the case in which \( r = 0.1 \), \( W \) equals about 0.001 which usually is less than the statistically error in the value of \( W_0 \).

Returning to the problem of analyzing the deviations, one can greatly reduce the computational work by assuming all weight factors are unity. In general measurements were made so that weight factors were the same within a factor of two. The unity weight factor approximation accordingly gave good but slightly conservative results.

If one lets \( S \) equal the value of the slope obtained from the least squares line through the upper points of the spectrum and making use of the approximation that the least square line has the average slope, one can write:
\[ S = \frac{A (W_0 - W_1) (1 + \frac{r}{W_1})^{\frac{1}{3}} - A (W_0 - W_2) (1 + \frac{r}{W_2})^{\frac{1}{3}}}{W_2 - W_1} \].

If one used the power expansion of \((1 + \frac{r}{W})^{\frac{1}{3}}\), \(S\) becomes:

\[ S = \frac{A \left[ (W_0 - W_1) \left(1 + \frac{r}{2W_1} - \frac{r^2}{3W_1^2} \right) - (W_0 - W_2) \left(1 + \frac{r}{2W_2} - \frac{r^2}{3W_2^2} \right) \right]}{W_2 - W_1} \]

and collecting terms in powers of \(r\), \(S\) becomes:

\[ S = A \left[ 1 + \frac{W_0 r}{2W_1 W_2} - \frac{r^2}{3W_1^2 W_2^2} (W_0 W_2 + W_0 W_1 - W_1 W_2) \right]. \]

Let one make the following simplification in notation:

\[ \alpha = \frac{W_0}{2W_1 W_2} \]

\[ \beta = \frac{(W_0 W_2 + W_0 W_1 - W_1 W_2)}{S W_1^2 W_2^2} \]

then \(S\) can be written as:

\[ S = A \left[ 1 + \alpha r - \beta r^2 \right] \]

and:

\[ A = \frac{S}{1 + \alpha r - \beta r^2}. \]
If one substitutes this value for \( A \) in the expression for \( K \), one obtains:

\[
\left( \frac{N_1}{I_1 F_1} \right)_{\frac{1}{2}} = K_1 = \frac{S(w_0 - w_1)}{(1 + \alpha r - \beta r^2)}^{\frac{1}{2}} .
\]

Squaring both sides and dividing both sides by \( (w_0 - w_1)^2 \), one obtains:

\[
\frac{N_1}{I_1 F_1 (w_0 - w_1)^2} = Y_1 = \frac{s^2(1 + \frac{r}{W})}{(1 + \alpha r - \beta r^2)^2} .
\]

If one expands \( \frac{1}{(1 + \alpha r - \beta r^2)^2} \) in a power series of \( r \) and substitutes in the expression for \( Y \) and collects terms to the second order of \( r \), one obtains:

\[
Y = s^2(1 + \frac{r}{W}) \left[ 1 - \alpha 2 r + r^2 (3 \alpha^2 + 2 \beta) \right] .
\]

Let:

\[
3 \alpha^2 + 2 \beta = \sigma^-
\]

and again collecting terms to \( r^2 \), one obtains:

\[
Y = s^2 \left[ 1 + r \left( \frac{1}{W} - 2 \alpha \right) + r^2 \left( \sigma - \frac{2 \alpha}{W} \right) \right] .
\]

Let \( Y_{1e} \) be the experimental value of \( Y \) at the point "1". Then the difference \( D_1 \) between the theoretical value \( Y_{1t} \) calculated from the line equation of energy \( W_1 \) and the experimental value \( Y_{1e} \) is:
\[ D_i = Y_{ie} - Y_{it} = Y_{ie} - S^2 \left( 1 + r \left( \frac{1}{W_i} - 2\alpha \right) + r^2\left( \frac{2\alpha}{W_i} \right) \right). \]

The sum of the differences squared is:

\[ \sum_i D_i^2 = \sum_i Y_{ie}^2 - 2Y_{ie}S^2 \left( 1 + r(1/W_i - 2\alpha) + r^2(\sigma - 2\alpha/W_i) \right) + S^4 \left[ 1 + r(1/W_i - 2\alpha) + r^2(\sigma - 2\alpha/W_i) \right]^2. \]

Differentiating with respect to \( r \) and setting the derivative equal to zero to minimize the sum of the squares of the deviations, and solving for \( r \), one obtains:

\[
r = \frac{\sum_i (Y_{ie} - S^2) \left( \frac{1}{W_i} - 2\alpha \right)}{\sum_i S^2 \left[ (1/W_i - 2\alpha)^2 + 2(\sigma - 2\alpha/W_i) \right] - 2Y_{ie}(\sigma - 2\alpha/W_i)}.
\]

Normally \( r \) is less than 0.1 and therefore:

\[ S^2 \approx Y_{ie}. \]

Using this approximation, the expression for \( r \) becomes:

\[
r = \frac{\sum_i (Y_{ie} - S^2) \left( \frac{1}{W_i} - 2\alpha \right)}{S^2 \sum_i (1/W_i - 2\alpha)^2}.
\]

or

\[
r = \frac{\sum_i \left( Y_{ie}/S^2 - 1 \right) \left( 1/W_i - 2\alpha \right)}{\sum_i (1/W_i - 2\alpha)^2}. \]
The above expression for evaluating \( r \) was employed in analyzing the spectra obtained with both the thin lens and intermediate image spectrometers.

The intermediate image spectrometer data gave a value of \( r \) of \( +0.030 \pm 0.040 \). The thin lens spectrometer data gave a value of \( -0.032 \pm 0.045 \). Averaging these two results, one obtains:

\[
 r = 0.00 \pm 0.03 
\]

The probable error was determined in the following manner.

In the expression for \( r \):

\[
 r = \frac{\sum_i (Y_{ie} / s^2 - 1) (1/w_i - 2\alpha)}{\sum_i (1/w_i - 2\alpha)^2}
\]

or substituting for \( Y_{ie} \):

\[
 r = \frac{\sum_i \left( N_i / I_i F_i (W_0 - W_i)^2 s^2 - 1 \right) (1/w_i - 2\alpha)}{\sum_i (1/w_i - 2\alpha)^2}
\]

only the term \( (Y_{ie} / s^2 - 1) \) is subject to large error because it represents the difference between two nearly equal quantities. The other terms are in error at most by one percent, whereas this term may have errors of the order of 100 percent. Therefore in determining the probable error in \( r \), it was only necessary to consider the variation in the \( (Y_{ie} / s^2 - 1) \) term.
The procedure used in evaluating \( r \) was to determine the value of the slope \( S \) of the Kurie plot from the points of the upper end of the spectrum. One can designate these points by the subscript "u". The value of "r" was then determined using the above formula and the points of the lower end of the spectrum. One can designate these lower points by the subscript "i". Thus \( r \) can be written as:

\[
r = f(Y_1, Y_u) = g \left( \frac{N_1}{I_1F_1} \frac{(W_0 - W_1)S^2}{S_m} \right).
\]

One can apply the statistical formula that if \( f = f(X, Y) \), then

\[
\sigma_f^2 = \left( \frac{\partial f}{\partial X} \right)^2 \sigma_X^2 + \left( \frac{\partial f}{\partial Y} \right)^2 \sigma_Y^2 - \text{---},
\]

where \( \sigma_f, \sigma_X \) are the probable errors in these quantities.

Applying this formula to the expression for \( r \), one obtains:

\[
\sigma_r = \sqrt{\sum \left( \frac{1}{W_i} - 2\alpha \right)^2 \left( \sigma_{N_1} \right)^2 \frac{I_1F_1}{I_1F_1^2} \frac{(W_0 - W_1)^2 S^2}{S_2^2} + \frac{4N_1}{I_1^2 F_1^2} \frac{(1/W_i - 2\alpha)^2 [\sigma(W_0 - W_1)]^2}{(W_0 - W_1)^6 S^6}}
\]

\[
\sigma_r = \sum \left( \frac{1}{W_i} - 2\alpha \right)^2
\]

where \( W_i, I_i, F_i \) are regarded as having no error.

Noting that:

\[
\frac{N_1}{I_1F_1(W_0 - W_1)^2 S^2} \approx 1
\]
one can reduce the expression for the probable error to:

\[
\sigma_r = \sum_i \left( \frac{1}{W_i} - 2\alpha \right)^2
\]

\[
(1/W_i - 2\alpha)^2 \left[ \left( \frac{\sigma N_i}{N_1} \right)^2 + 4 \left( \frac{\sigma [(W_o - W_i)S]}{(W_o - W_i)S} \right)^2 \right]
\]

Noting that \( \sigma N_1 \) equals \( 0.675 \sqrt{N_1/t} \), one obtains:

\[
r = \sum_i \left( \frac{1}{W_i} - 2\alpha \right)^2
\]

\[
\left[ \frac{0.675^2}{N_1} + 4 \left( \frac{\sigma [(W_o - W_i)S]}{(W_o - W_i)S} \right)^2 \right]
\]

The expression \((W_o - W_i)S\) is actually the least squares line equation of the Kurie plot that gives the value of \( K_1 \) at the energy \( W_i \). One can determine the probable error in the value of the \( K_1 \) by writing it as a function of the \( K_u \) used to determine the least squares line. Writing the least squares line as a function of the measurements \( K_u \), one obtains:

\[
K_1 = \frac{\sum_u (w_u - \bar{w}) K_u (w_i - \bar{w})}{\sum_u (w_u - \bar{w}) w_u} + \bar{K}
\]
where: \[ \overline{W} = \frac{\sum u W_u}{\sum u} = \frac{\sum u W_u}{n} \] and \[ R = \frac{\sum u K_u}{\sum u} = \frac{\sum u K_u}{n} \].

Designating \[ \sum u (W_u - \overline{W}) W_u \] as \( \alpha \) one obtains the following expression after applying the formula for statistical error:

\[
\sigma_{K_1} = \sqrt{\sum u \left[ \frac{(W_1 - \overline{W})^2 (W_u - \overline{W})^2}{\alpha^2} + \frac{(W_1 - \overline{W}) (W_u - \overline{W})}{2 \alpha} \right] \frac{1}{n^2}} \sigma_{K_u}^2
\]

which may be rearranged in the computational form:

\[
\sigma_{K_1} = \sqrt{\sum u \left[ \frac{(W_1 - \overline{W})^2 (W_u - \overline{W})^2}{\alpha^2} (\sigma_{K_u})^2 + \frac{(W_1 - \overline{W}) (W_u - \overline{W})}{2 \alpha} (\sigma_{K_u})^2 + \frac{1}{n^2} \right]} \sigma_{K_u}^2
\]

The above formula was used to determine the value of:

\[
\frac{(\sigma[ (W_0 - W_1) S ])^2}{[(W_0 - W_1) S]^2} = \frac{(\sigma_{K_1})^2}{K_1^2}
\]

in the expression for the probable error of \( r \). With the use of this and the probable error in the \( N_1 \) the errors in "r" were determined. They have been listed previously.

The 12.4-hour activity of \( K^{42} \) was examined with both a thin lens and an intermediate image spectrometer. The source for the intermediate image spectrometer was prepared by evaporating the source material onto an aluminized 30 microgram per square centimeter collodion film in a vacuum. This gave the source a fairly uniform surface density.
From the known specific activity of the source, the surface density was calculated to be about 20 micrograms per square centimeter. The formula of Hamilton and Gross indicates that scattering should be negligible for energies above 35 kev. As a safety factor, measurements performed at energies below 50 kev were discarded.

The source for the thin lens spectrometer was prepared by evaporating off the water from a drop of source solution placed on an aluminized 30 microgram per square centimeter collodion film. The source accordingly did not have a uniform surface density. The average surface density was about 0.5 milligram per square centimeter. The formula of Hamilton and Gross indicates that scattering effects should be negligible for energies greater than 120 kev. As a safety factor, and to account for non-uniformity of the source, measurements made at energies below 200 kev were discarded.

The data were corrected for decay, counter dead time, background, and the finite resolution of the instruments. The screening correction was negligible because of the low value of Z. The correction factors of Rose, Perry, and Dismuke were adjusted to the new value of the nuclear radius and were applied to the analysis of the highest energy beta group. The correction due to the cut off of the coulomb potential at the nuclear radius was found to be negligible.

The total beta spectrum of K\(^{42}\) obtained with the intermediate image spectrometer is illustrated in Figure 14. The spectrum of photoelectrons from a 17.2 milligram per square centimeter Thorium
FIG. 14 $\beta$ SPECTRUM OF $K^{42}$ OBTAINED WITH INTERMEDIATE IMAGE SPECTROMETER

- $\beta_1 3.535 \pm 0.015$ MEV, 82%
- $\beta_2 1.98 \pm 0.02$ MEV, 18%
- $\gamma_1 1.53 \pm 0.01$ MEV, 0+
- $\gamma_2 320 \pm 5$ KEV, 2+
- $\delta 0.4-0.5$ MEV, ~15%
radiator was obtained also with the intermediate image spectrometer. A strong gamma ray with an energy of 1.53 $0.01 \text{ Mev}$ and a weak gamma ray with an energy of 320 $0.5 \text{ kev}$ were detected. The two photoelectron lines are illustrated in Figure 15. The weak gamma ray has been also observed by Holazar and Bell (51).

Gamma ray energies were determined by adding the K binding energy of Thorium and the energy loss of the photoelectrons in passing through the radiator to the most probable energy of the photoelectrons. The energy loss was obtained from the data of E. N. Jensen (94).

A 0.063 inch copper cap was placed over the source material and the Thorium radiator was fastened on top of this. The data of Davisson and Evans (95) were used to calculate the attenuation of the gamma rays in passing through the copper cap. The photoelectric cross section for the two gamma ray energies were also determined from the data of Davisson and Evans. Applying a correction for the difference in the photoelectric cross sections and correcting for the absorption in the copper cap, one finds that the 320 kev gamma ray has an intensity of $0.8 \pm 0.4$ percent of that of the 1.53 Mev gamma ray. The internal conversion coefficients were neglected in this calculation because they are small in comparison to the error assignments.

Two beta groups with the possibility of one more very weak beta group were found in the decay of K$^{42}$. The data from the intermediate image spectrometer gave energies of $3.54 \pm 0.01 \text{ Mev}$ and $1.98 \pm 0.015 \text{ Mev}$. 

GAMMA RAY ENERGY
320 ± 5 KEV
INTENSITY 0.8 ± 0.4 % OF
1.53 MEV GAMMA RAY
$H_p = 1650$ GAUSS-CM

$H_p = 6178$ GAUSS-CM
GAMMA RAY ENERGY 1.53 ± 0.01 MEV

FIG. 15 GAMMA RAYS OF $^4K$ WITH A 17.2 mg/cm$^2$ THORIUM RADIATOR
Mev to the two prominent beta groups. The Kurie plot of the total
spectrum obtained with the intermediate image spectrometer is
illustrated in Figure 16. To obtain a straight Kurie plot for the
3.54 Mev beta group, it was necessary to apply the "a" correction
factor. This identified it as an "n" forbidden transition with
n+1 units of spin change. The \( \log (W_0^2 - 1) \) ft value of about
9.75 identifies it as a first forbidden transition, which is in
agreement with previous investigations (43-51).

The 1.98 Mev beta group was obtained by subtracting the higher
energy beta group from the total beta spectrum. The Kurie plot of
the second beta group is illustrated in Figure 17. These data were
obtained from measurements with the intermediate image spectrometer.
The Kurie plot appears to be straight to about 0.4 or 0.5 Mev and
then deviates upward. The \( \log ft \) value obtained for this spectrum
is about 7.5 which is in good agreement with the average \( \log ft \) value
for a transition from a 2\(^-\) to a 2\(^+\) state (96). The upward
development of the Kurie plot at 0.5 Mev may indicate the existence
of a weak third beta group. Assuming that it represents a weak
third beta group, one finds that it has roughly an intensity of one
percent of the 1.98 Mev beta group. The relative intensities of the
3.54 Mev and 1.98 Mev beta groups were found to be 82 and 18 percent
respectively.

A deviation plot of the 1.98 Mev spectrum obtained with the
intermediate image spectrometer is illustrated in Figure 18. The
value of \( W_0 \) used in calculating the points of the deviation plot
\[ a = L_0 (W_0 - W)^2 + 9 \text{ Li} \]

FIG. 16 KURIE PLOT OF TOTAL $\beta$ SPECTRUM OF $^{42}$K OBTAINED WITH INTERMEDIATE IMAGE SPECTROMETER
FIG. 17 KURIE PLOT OF $^{42}K$ LOWER $\beta$ GROUP OBTAINED WITH AN INTERMEDIATE IMAGE SPECTROMETER
FIG. 18 Deviation plot of $K^{42}$ activity obtained with intermediate image spectrometer.
the determination plot is straight within experimental error and lower
of the thin lens spectrometer is indicated in Figure 21. The upper end
a determination plot of the 1.99 Mev beta group obtained with
corresponding error. As indicated the new axis from a week third beta
the Kube plot is nearly straight back to 0.5 Mev and then
the spectrometer. The Kube plot of the spectrum is indicated in Figure
agreement with the value obtained with the intermediate image
subtraction is found to have an energy of I .99 ± 0.015 Mev in
the lower energy beta group which was obtained by performing a
mediate image spectrometer
which is in good agreement with the value obtained with the
with high energy beta group is found to have an energy of 3.5 ± 0.01 Mev
the spectrometer is indicated in Figure 19. From this spectrum the
the Kube plot of the total beta spectrum obtained with the thin

* New gamma ray

indicated that the 1.99 Mev beta was in countenance with the 1.53
insufficient to give an accurate spectrum shape, although it clearly
and the I.99 Mev beta group. The countenance counting rate was
countenance counts were detected between the I.53 Mev gamma ray
possibility of a weak beta group

* The rapid rise at the low end may be due to the previously mentioned
upward at the low end may possibly the absorption of the upper end.
determination plot indicates that the correction factor decreases markedly
was determined from about the upper fourth of the spectrum. The
FIG. 19 KURIE PLOT OF THE TOTAL $\beta$ SPECTRUM OF $K^{42}$ OBTAINED WITH A THIN LENS SPECTROMETER.
FIG. 20 KURIE PLOT OF 1.9 MEV $\beta$ GROUP OF $K^{42}$
OBTAINED WITH A THIN LENS SPECTROMETER

$\left( \frac{N}{IF} \right)^{1/2}$

W ENERGY IN MC$^2$ UNITS

1.99 ± 0.015 MEV
FIG. 21 DEVIATION PLOT OF 1.99 MEV $\beta$ GROUP OF $^{42}$K
OBTAINED WITH A THIN LENS SPECTROMETER
end deviates upward as in the case of the data obtained from the intermediate image spectrometer.

The points of the 1.98 Mev beta group were determined by performing a subtraction, and therefore it was necessary to consider the errors arising in the total number of counts and the errors in the values determined from the extrapolated line. The measurements used to determine the extrapolated line of the 3.54 Mev beta group are independent of those measuring the total spectrum up to 2 Mev. Thus the two errors which contribute to the total error of the points of the 1.98 Mev spectrum are independent of each other. The error in the values determined from the extrapolated line were computed from the previously derived formula:

\[
\sigma_{K_8} = \sqrt{\frac{(W_s-W)^2 \sum_u (W_u-W)^2 (\sigma_{K_u})^2 + \sum_u (W_s-W)(W_u-W) (\sigma_{K_u})^2} {\alpha^2} + \frac{\sum_u (W_s-W)(W_u-W) (\sigma_{K_u})^2} {2 \alpha} + \frac{\sum_u (\sigma_{K_u})^2} {n^2}}
\]

where \( \alpha = \sum_u (W_u-W) W_u \). The notation is the same as that previously used. The error in the number of counts per minute \( N_1 \) was determined by the formula:

\[
\sigma N_1 = 0.675 \sqrt{\frac{N_1}{t}}
\]

where \( t \) is the counting time at a given momentum setting of the spectrometer.
Part of the beta spectrum associated with the 26.5 hour activity of $\text{As}^{76}$ was examined with an intermediate image spectrometer. Just the upper end of the total spectrum was examined because the lower end was obscured by the superposition of the various beta groups.

The source was prepared by evaporating the water from a drop of the source solution placed on an aluminized 30 microgram per square centimeter collodion film. The average surface density was about 0.5 milligram per square centimeter. The formula of Hamilton and Gross indicates that scattering would become significant for energies below 150 kev. However, since no measurements were made for energies below 1 Mev, source thickness effects were negligible.

Data were corrected for decay, counter dead time, background, and for the finite resolution of the instrument. For the highest energy beta group, the adjusted values of the corrections of Rose, Perry, and Dismuke were used. The corrections for screening and the cutoff of the coulomb potential at the nuclear radius were negligible.

The upper end of the total beta spectrum is shown in Figure 22. A coincidence spectrum obtained from coincidence measurements between gamma rays above 100 kev and the beta particles also is indicated in Figure 22. The coincidence data were obtained with the intermediate image spectrometer.

A Kurie plot of the upper end of the total spectrum of $\text{As}^{76}$ is indicated in Figure 23 along with the Kurie plots of the lower energy beta groups obtained by performing a subtraction. Three beta groups
FIG. 22 TOTAL AND COINCIDENCE SPECTRUM OF As$^{76}$ OBTAINED WITH AN INTERMEDIATE IMAGE SPECTROMETER.
FIG. 23 KURIE PLOT OF THE TOTAL $\beta$ SPECTRUM OF $^{76}$As
were obtained with energies above 1.5 Mev. The energies are 2.97 ± 0.01, 2.43 ± 0.02, and 1.79 ± 0.05 Mev. The "a" correction factor was applied to the points of the highest energy beta group in order to obtain a straight Kurie plot. This is in agreement with the assignment of a first forbidden transition with a spin change of two which is generally given in the literature (52-61). The energies obtained are in very good agreement with the energies cited in the very recent paper of P. Hubert (96).

The Kurie plots of the 1.75 and 2.44 Mev beta groups obtained by coincidence measurements are shown in Figure 24. The endpoint energies of these two beta groups determined from these measurements are 2.41 ± 0.015 and 1.74 ± 0.02 Mev. These values are in fairly good agreement with the values obtained by performing subtractions on the total spectrum.

Deviation plots of the 2.4 Mev beta group obtained from both the subtraction and coincidence measurements are indicated in Figure 25. Only a limited range of points could be used because of the presence of the 1.75 Mev beta group. Probable errors were assigned to the points of the deviation plot by considering the error in the number of counts and also the error in the values determined from the extrapolated line when the spectrum was obtained by performing a subtraction.
FIG. 24 KURIE PLOTS OF THE COINCIDENCE SPECTRUM OF $^{76}$As OBTAINED WITH AN INTERMEDIATE IMAGE SPECTROMETER.
FIG. 25 DEVIATION PLOTS OF 2.41 MEV $\beta$ SPECTRUM OF As$^{76}$
The deviation plots indicate that over the range which was examined, the 2.41 Mev beta group had an allowed shape. It would indicate that the contribution from the pure tensor terms is less than 15 percent.
VI. DISCUSSION ON SMALL ORDER EFFECTS

In the case of allowed transitions one can write the following general shape factor $C_o$ which corrects the simple Fermi distribution (7):

$$C_o = \left[ G_s^2 + k^2 G_v^2 \pm \frac{2K}{W} G_s G_v \right] |\int \beta| ^2$$
$$+ \left[ G_t^2 + \lambda G_a^2 \pm \frac{2\lambda}{W} G_t G_a \right] |\int \beta \sigma| ^2.$$

Where $\lambda$ and $K$ are real parameters defined by the following equation:

$$\int 1 = -K \int \beta$$

and

$$\int \sigma = -\lambda \int \beta \sigma.$$

The integrals represent the nuclear matrix elements which are defined by Konopinski (6). The G's represent the coupling constants for the various forms of interaction and $W$ is the energy of the beta particle in $mc^2$ units.

It has generally been regarded that not both the $S$ and $V$ or $T$ and $A$ forms could be present in the correct form of the beta interaction because allowed distributions do not appear to have $1/W$ deviations. However, it has been only very recently that critical attempts have been made to set limits on the possible magnitudes of the $1/W$ terms.
For the case of a $\Delta I = 1$, "No" transition, the matrix elements $\int_1$ and $\int_3$ vanish and only the $T$ or $A$ terms (Gamow-Teller terms) contribute to the allowed correction or shape factor.

At the present time, there are probably two isotopes which are experimentally well suited for studying possible Fierz deviations in $\Delta I = 1$, "No" transitions. They are $^{32}\text{P}$ and $^{64}\text{Cu}$. The $^{64}\text{Cu}$ activity has the disadvantage of having both a positive and negative beta particle spectrum.

Konopinski and Mahmoud analyzed a $^{64}\text{Cu}$ spectrum and set the limits of $-0.2$ and $+0.06$ on the value of $r$. Davidson and Peaslee also analyzed some data of Langer and Price (36) on the beta spectrum of $^{32}\text{P}$ and evaluated $r$ as being $0.00 \pm 0.07$. They also examined other isotopes, but the $^{32}\text{P}$ activity set what they regarded as the lower limit on the value of $r$.

In this work, by careful experimental procedure, an attempt was made to lower the probable error in the value of $r$, or possibly establish a positive deviation by examining the beta spectrum of $^{32}\text{P}$. As stated in the section on results, this work indicates that $r$ has a value of $0.00 \pm 0.03$. Thus one is able to reduce by over a factor of two the probable magnitude of $r$ in allowed transitions with Gamow-Teller selection rules.

The results should be reliable, for precautions were taken to have uniform thin sources and backings and two separate instruments were used to obtain the values of $r$. This would indicate that the
\* \* is the fine structure constant and Z is the nuclear charge.

where

\[ Z \approx 1 \] for the shape factor in the approximation that [text cut off]

For convenience, so many and Kondo-such (7) are given for allowed transitions would be too long an expression for the shape factor for first forbidden transitions.

It necessary to introduce the form factors to evaluate the

uncertainty in light of new information. King and Pease have done

this because the value of certain parameters which they need now seen

exceeded the detection as arising from the P form may be incorrect.

while that the analysis of Kondo and Pease (56) which

singular spectrum of Kondo. King and Pease (57) have indicated

the probability the correct one. The P form was included to explain the

combination of specific 0 to 0 transitions. They concluded that the (g(4)p)

5 so a form must be included to account for the short half lives of

forbidden transitions with n + 1 units of spin change and that the

included for Cameron-Teller selection rules and the shape of n

combinations are incorrect. Nothing that the P a form must be

observed, Kondo-such and Kondo-such (7) have concluded that these

(8) of (7) would occur. Because they have not been experimentally

ox of (7) and (W/T) should occur. Large effects of the order

In the case of first forbidden transitions, if the (g(4)p)

\[ \| = 0 \text{ transitions} \]

\[ 1 \times 10^4 \]
They separated the shape factor \( C_1 \) into three parts:

\[
C_1 = C_1^{(0)} + C_1^{(1)} + C_1^{(2)}
\]

where \( C_1^{(0)} \) consists of the only terms which contribute to \( 0 \rightarrow 0 \) transitions and contribute to \( \Delta I = 0 \) transitions, \( C_1^{(1)} \) consists of terms which contribute to \( \Delta I = \pm 1 \) transitions but not to \( 0 \rightarrow 0 \) or \( \Delta I = \pm 2 \) transitions, and \( C_1^{(2)} \) which is the only term which contributes to \( \Delta I = \pm 2 \) transitions, and also contributes to \( \Delta I = 0, \pm 1 \) transitions.

For the \((S,T,P)\) combination, they obtained the following explicit expressions for the first forbidden correction terms:

\[
C_1^{(0)}(TP) = G_T^2 \int \beta \sigma \cdot r \left\{ \frac{\alpha z}{2 R(1 - Z_3)} + \frac{1}{3} \left( \frac{P^2}{3W} \right) + \left( \frac{P}{3W} \right)^2 \right\}
\]

\[
C_1^{(1)} = G_T^2 \int \beta \sigma \times r \left\{ \frac{\alpha z}{2 R(1 + X_1 Y_1)} + \frac{(1 + X_1) P^2}{3W} + \left( \frac{1 - X_1}{3} \right) \frac{P}{W} \right\}^2
+ \left( 1 + \frac{X_1}{3} \right) \frac{P}{W} + \left( 1 - 2X_1 \right)^2 \frac{P^2}{18} + \left( 1 + 2X_1 \right)^2 \frac{P^2}{18} \frac{C^2}{2}
\]

\[
C_1^{(2)} = G_T^2 \sum |B_{1j}|^2 \left( \frac{P^2 + C^2}{12} \right).
\]

The quantities:

\[
\int \beta \sigma \cdot r
\]

\[
\int \beta \sigma \times r
\]

and

\[
B_{i,j}
\]

are nuclear matrix elements defined by Konopinski (6). The quantities \( X_1, Y_1 \) and \( Z_3 \) are real parameters defined by Mahmoud.
and Konopinski (7), and \( p \), \( q \), and \( w \) are respectively the beta particle momentum, the neutrino momentum, and the beta particle total energy.

The study of \( ft \) values by King and Peaslee indicates that \( X_1 \) and \( Y_1 \) have a value of about unity and \( Z_3 \) has a value of about 0. Ahrens and Feenberg (99) have evaluated these parameters using a simplified nuclear model and reached the same conclusion. In these evaluation it is assumed \( G_w/G_x^1 = 1 \). One notes that for high \( Z \) nuclei and relatively low energy transitions the constant in the shape factor terms proportional to \( \alpha Z/2R \) dominates, and accordingly allowed spectral shapes are anticipated.

In the case of low \( Z \) high energy transitions though, this may not be true, and energy deviations may be observable.

With this idea in mind, the beta activities of \( K^{42} \) and \( As^{76} \) were examined.

The 3.54 Mev beta transition of \( K^{42} \) goes to the ground state of \( Ca^{42} \) and its log \( ft \) value and the shape of its Kurie plot identify it as a first forbidden transition with a spin change of two. \( Ca^{42} \) is an even-even nucleus with zero spin and plus parity. The ground state of \( K^{42} \) is then a 2- state. Even-even nuclei, in almost every case have \( 2^+ \) first excited states (100). One might reasonably assume then that the 1.53 Mev gamma ray of \( Ca^{42} \) to the ground state is an E2 or electric quadrupole transition. Angular correlation work, particularly that of Hamilton, Lemonich, and Pipkin (43) also
indicates that the 1.53 Mev gamma ray is an $E_2$ transition. The evidence is not conclusive, but it is strong.

The 1.98 Mev beta transition from the ground state of $K^{42}$ to the first excited state of $Ca^{42}$ would then correspond to a transition from a 2 to a 2$^+$ state, or a first forbidden transition with a zero spin change. The log ft value of 7.5 of this transition agrees very well with the average of such transitions as evaluated by King and Peaslee (97). The measured sum of the gamma ray energy and the 1.98 Mev beta group energy agrees within statistical error with the endpoint energy of the highest energy beta group. Coincidence measurements also show that the 1.98 Mev beta group is in coincidence with the 1.53 Mev gamma ray.

Let one consider what magnitude of deviation one might expect in the correction factor for the 1.98 Mev transition of $K^{42}$, accepting the strong but not conclusive evidence that it is a transition from a 2$^-$ to a 2$^+$ state.

The two major shape factor terms in the transition are $C_1^{(0)}$ and $C_1^{(1)}$. The ft values for transitions in which only the $C_1^{(2)}$ term is non zero are at least an order of magnitude larger than for those in which the $C_1^{(0)}$ and $C_1^{(1)}$ terms are present. This indicates that the $C_1^{(2)}$ term is about an order of magnitude smaller.

For $K^{42}$, $\alpha Z/2R$ equals about 5.1 using Pidd's value of $r_0$. $R$ is measured in \(\text{fm}/mc\) units. Let one take the best estimates of $X_1$, $Y_1$ and $Z_3$ in the approximation that $Gx/Gx^1$ equals one:
Using these values, the expressions for $C_1^{(0)}$ and $C_1^{(1)}$ become:

$$C_1^{(0)} = A \left\{ \left[ 5.1 + 1/3 \left( p^2/W - \varepsilon \right) \right]^2 + \left( p/3W \right)^2 \right\}$$

and

$$C_1^{(1)} = B \left\{ \left[ 5.1 + 2p^2/3W \right]^2 + 4p^2/9W^2 + p^2/18 + \varepsilon 2/2 \right\}$$

where $A$ and $B$ are constants including the matrix elements and coupling constants.

Noting that $p^2 = W^2 - 1$, and that $q = W_0 - W$, and that $W_0 = 4.9\text{mc}^2$ for this transition one can write the expressions as the following simple functions of $W$:

$$C_1^{(0)} \approx A(12 + 4.6W - 2.3/W + 4/9 W^2)$$

$$C_1^{(1)} \approx B(37.5 + W^2 + 1.9W - 6.8/W)$$

Let one consider how these functions vary as $W$ is varied from 1 to $W_0$ where $W_0 \approx 4.9\text{mc}^2$:

for $W = 1$

$$C_1^{(0)} \approx 15A, \quad C_1^{(1)} \approx 34B$$

for $W = 3$

$$C_1^{(0)} \approx 29A, \quad C_1^{(1)} \approx 50B$$
for \( W = 4.9 \)

\[
C_1^{(0)} = 45A, \quad C_1^{(1)} = 69B.
\]

Thus for the values of the parameter chosen, \( C_1^{(0)} \) and \( C_1^{(1)} \) are roughly linearly increasing functions which double or triple in value over the range of the variation of \( W \).

The experimentally determined deviation plots of the 1.98 Mev beta group of \( K^{42} \) shown in Figures 18 and 21 are precisely a plot of the shape factor for this beta group. One immediately notes that above 0.5 Mev the shape factor is almost constant within experimental error. The deviation below 0.5 Mev occurs very abruptly, and one might reasonably expect it to arise from a very weak third beta group.

If one examines the parameters \( X_1, Y_1, \) and \( Z_3 \) involved in the expression for the shape factors \( C_1^{(0)} \) and \( C_1^{(1)} \), one finds that a constant shape factor can only be achieved by assigning a large value to \( Y_1 \) and \( Z_3 \). The work of King and Peaslee and others indicates \( Z_3 \) is probably much less than one. It is still possible to obtain an essentially constant shape factor by assuming that the contribution from the \( C_1^{(0)} \) term is small and the value of \( Y_1 \) in the \( C_1^{(1)} \) term is large. On the other hand, if one assumes that the contribution from the \( C_1^{(1)} \) term is small \( Z_3 \) must have a large value to make the \( C_1^{(0)} \) term almost constant.
In both cases, assuming $x_1$ to have a value of about one, one must assign a value of 10 or greater to the magnitudes of $z_3$ and $y_1$ to account for an essentially constant shape factor in the case of the $K^{42}$ activity.

The $C^{(2)}_1$ term can possibly account for 10 percent of the transitions. This is the correction factor which characterizes forbidden transitions with $n + 1$ units of spin change. It has roughly a parabolic shape with a minimum at about $W_0/2$ which is approximately $\frac{1}{2}$ the maximum value which occurs at $W_0$. Figures 18 and 21 show that the contribution from the $C_1$ term was not experimentally detectable in the 1.98 Mev $K^{42}$ beta transition. A conservative estimate would assign the contribution from the $C_1$ term as less than 10 percent.

If one accepts the not entirely conclusive evidence that the 1.98 Mev beta transition is one with a zero spin change with a change in parity, one find that the estimates which give $x_1 = y_2 = \lambda$ and $z_3 = 0$ are inadequate to account for the essentially allowed shape of the spectrum. The signs of the parameters may be regarded as unknown in some cases but if $x_1$ and $y_1$ are assumed to have a magnitude of unity, any choice of sign will lead to a detectable variation in the calculated shape factor.

The theory is worked out in the approximation that $(\alpha \bar{z})^2 \ll 1$. For $K^{42}$ $(\alpha \bar{z})^2$ equals about 0.02, and therefore the approximation should be reasonably good.
Because the 321 kev gamma ray and what is assumed as a third beta group have roughly the same intensity, one might conclude that the two form a cascade as indicated in Figure 16. This would also imply the existence of an additional gamma ray with an energy of about 1.2 Mev and with an intensity about equal to that of the 321 kev gamma ray. With the methods employed, this was probably beyond detection. The 321 kev gamma ray might also indicate an additional beta group with an energy of 1.6 or 3.2 Mev. In either case, its very low intensity would not change the results obtained within experimental error.

The log ft value of the 2.42 Mev transition of $^{86}$As is a little higher than the average for a transition from a 2− to a 2+ state. The experimental evidence is fairly conclusive for this assignment (52-61). Because of the high ft value, one might expect the $C_1^{(2)}$ term to contribute more than in many other such transitions.

When this study was undertaken, an apparently incorrect decay scheme indicated that the next lower energy beta group had a maximum energy of about 1.27 Mev. This indicated that there might be a range from 1.27 to 2.42 Mev in which the shape of the 2.42 Mev beta group might be carefully measured by coincidence measurements. However, as indicated in the results, an additional beta group of about 1.75 Mev was detected and restricted the usable range. This is in agreement with the very recent work of Hubert (96).
Within this range, however, one finds that the measurements indicate that the possible addmixture from the $C^{(2)}_1$ term is conservatively less than 15 percent. The results illustrated in Figure 25 also indicate, as in the case of $K^{42}$, that the parameter $Y_1$ or $Z_3$ must have a magnitude considerably greater than unity depending on whether the $C^{(1)}_1$ term or $C^{(0)}_1$ term is dominant.
VII. RESULTS ON Eu\textsuperscript{152-154}

The 5 to 15-year activities of Eu\textsuperscript{152} and Eu\textsuperscript{154} were examined with an intermediate image and a scintillation spectrometer. The material for the source used in examining the beta distributions and conversion electron lines was obtained from the Oak Ridge Isotope Division and was prepared by bombarding Eu\textsubscript{2}O\textsubscript{3} with neutrons. Europium has two stable isotopes, Eu\textsuperscript{151} and Eu\textsuperscript{153}; the neutron gamma reaction consequently produced two activities, that of Eu\textsuperscript{152} and Eu\textsuperscript{154}.

The beta source was mounted on a 30 micrograms per square centimeter aluminized collodion film. To examine the lower energy region of the spectrum, a source was prepared by evaporating the source material onto a thin film in a vacuum. For the higher energy region of the spectrum and for coincidence work a more intense source was prepared by evaporating off the water from a drop of source solution placed on a thin film. This source had an effective surface density of about one milligram per square centimeter. The source evaporated onto the thin film in a vacuum had a surface density of about 20 micrograms per square centimeter.

The data obtained were corrected for background and counter dead time. Because of the long half-life, it was not necessary to correct for decay.
An examination of the beta spectrum revealed a number of beta
groups and a number of conversion lines as indicated in Figure 26
and 27 for the Eu$^{152-154}$ activity. Conversion electrons were found
for the energies indicated in the following Table 1:

Table 1. Conversion lines of the Eu$^{152-154}$ sample

<table>
<thead>
<tr>
<th>Electron energy in kev</th>
<th>Gamma ray energy in kev</th>
<th>Shell and isotope</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>31.6 ± 1</td>
<td></td>
<td>Average of Sm and Gd</td>
<td>Auger K LL</td>
</tr>
<tr>
<td>37.5 ± 1</td>
<td></td>
<td>Average of Sm and Gd</td>
<td>Auger K LM</td>
</tr>
<tr>
<td>73.0 ± 0.5</td>
<td>122, 123 (64)</td>
<td>K, Sm; K, Gd</td>
<td>not resolved</td>
</tr>
<tr>
<td>114.2 ± 0.5</td>
<td>122, 123</td>
<td>L, Sm; L, Gd</td>
<td>not resolved</td>
</tr>
<tr>
<td>143 ± 5</td>
<td>193</td>
<td>K, Gd</td>
<td>very weak</td>
</tr>
<tr>
<td>196.7 ± 0.5</td>
<td>243.6</td>
<td>K, Sm</td>
<td></td>
</tr>
<tr>
<td>236.9 ± 0.6</td>
<td>244.1</td>
<td>L, Sm</td>
<td></td>
</tr>
<tr>
<td>293.5 ± 1</td>
<td>340, 343.3(64)</td>
<td>K, Gd; K, Gd</td>
<td>not resolved</td>
</tr>
<tr>
<td>336.5 ± 1</td>
<td>340, 344.2</td>
<td>K, Gd; K, Gd</td>
<td>not resolved</td>
</tr>
<tr>
<td>355 ± 4</td>
<td>405</td>
<td>K, Gd assumed</td>
<td></td>
</tr>
<tr>
<td>396 ± 4</td>
<td>446</td>
<td>K, Gd assumed</td>
<td></td>
</tr>
<tr>
<td>636 ± 6</td>
<td>686</td>
<td>K, Gd assumed</td>
<td></td>
</tr>
<tr>
<td>726 ± 4</td>
<td>776</td>
<td>K, Gd</td>
<td></td>
</tr>
<tr>
<td>834 ± 8</td>
<td>874</td>
<td>K, Gd assumed</td>
<td></td>
</tr>
<tr>
<td>1039 ± 5</td>
<td>1086</td>
<td>K, Sm</td>
<td></td>
</tr>
<tr>
<td>1065 ± 6</td>
<td>1115</td>
<td>K, Sm</td>
<td></td>
</tr>
<tr>
<td>1365 ± 10</td>
<td>1415</td>
<td>K, Gd</td>
<td></td>
</tr>
</tbody>
</table>
FIG. 26 \( \beta \) AND CONVERSION LINE SPECTRUM OF Eu\(^{152-154}\)
\[ \eta = 0.09775 \times 1 \]

**CONVERSION ELECTRONS 1365 KEV**

**FIG. 27** UPPER END OF Eu^{152-154} TOTAL \( \beta \) SPECTRUM
Three additional weak gamma rays were also detected. From coincidence measurements as indicated in Figure 38, a weak conversion line with an energy of about $530 \pm 10$ kev was detected. This indicated a gamma ray of $580 \pm 10$ kev assuming conversion in Gd. Weak gamma rays with energies of $520 \pm 20$ and $1250 \pm 40$ kev were also detected using a scintillation spectrometer. The scintillation spectrum also indicated the weak 190 kev line.

A sample of Eu$_2$O$_3$ obtained from Dr. F. H. Spedding of the Ames Laboratory was bombarded intermittently in the Iowa State College synchotron for a period of four months. This produced the activities of Eu$^{150}$ and Eu$^{152}$. Eu$^{150}$ has a half-life of 14 hours, but its daughter products have longer half-lives. Eu$^{152}$ has a 9 hour isomeric state which produces considerable activity immediately after bombardment.

After one of the intermittent bombardments of 10 hours the sample was examined with a scintillation spectrometer. The sample was also examined for a period of several weeks after the last bombardment. A sample of the photographed pulse height distributions which were obtained is shown in Figure 28.

The results of the investigation indicate that within experimental error, the 776 and 1115 kev gamma rays are not present in the synchotron produced activity. It was found that the 9 hour isomeric state of Eu$^{152}$ gives rise to intense gamma rays with energies of $960 \pm 20$ kev,
FIG. 28 PULSE HEIGHT DISTRIBUTIONS
A very intense gamma ray with an energy of 520 ± 10 keV was found which decayed with about a one or two hour half-life. The 1415 ± 20 keV and 405 ± 10 keV gamma rays were also detected in the 9 hour activity but with lower intensity.

After the short lived activity had decayed for a couple of weeks gamma rays with energies of 120 ± 5 keV, 190 ± 5 keV (unusually strong) 345 ± 5 keV, 405 ± 10 keV, 840 ± 20 keV, 960 ± 15 keV, 1415 ± 30 keV and possibly 1085 ± 20 keV were found and assigned to the decay of Eu$^{152}$.

Additional gamma rays with energies of 560 ± 20 keV, 610 ± 20 keV, 640 ± 10 keV, 740 ± 30 keV and 270 ± 20 keV were found. These probably represent the gamma rays present in the decay of Eu$^{150}$ and its daughter products.

Several beta groups appeared in the combined spectrum of Eu$^{152-154}$ as indicated in the total Kurie plot, Figure 29. The Kurie plot of the upper end of the total spectrum, Figure 30, indicates that the highest-energy beta group has an energy of 1860 ± 10 keV.

The second highest beta group was obtained by coincidence measurements. A NaI(Tl) crystal placed behind the source in the beta spectrometer detected the gamma rays. A pulse height analyzer was used to select out the desired gamma ray from the total distribution, Figure 31, and coincidences between these and the beta particles were measured. The pulse height analyzer was set to select out the
FIG. 29 KURIE PLOT OF TOTAL Eu$^{152-154}$ $\beta$ SPECTRUM
FIG. 30 Kurie Plot of the Upper End of Eu$^{152-154}$ Spectrum

$\left(\frac{N}{IF}\right)^{1/2}$ vs. Energy in $\text{MeV}$ units

$1860 \pm 10$ keV
Fig. 31 \( \gamma \) Pulse Height Distribution Obtained with Intermediate Image Pulse Height Analyzer on Eu\(^{152-154}\)
observed for several gamma-ray energies as indicated in Figure 27. Electrons from the 122 and 123 key gamma rays, co-existing with the beta spectrometer was set to detect only the L conversion electrons when the beta spectrometer was set to detect strongly coexisting beta particles as indicated in Figure 26. The 778 key gamma ray indicated that several higher-energy gamma rays were in coexistence with the two highest-energy beta groups. Again, the pulse height analyzer was then used to select the beta ray from the two highest-energy beta groups as indicated in Figure 24. The pulse height analyzer was then used to select the various gamma rays and co-existing beta particles were measured, yielding beta groups were detected by the pulse height analyser. The pulse height analyzer was then used to select the various gamma rays and co-existing beta particles were measured, yielding beta groups were detected by the pulse height analyzer. The pulse height analyzer was then used to select the various gamma rays and co-existing beta particles were measured, yielding beta groups were detected by the pulse height analyzer. The pulse height analyzer was then used to select the various gamma rays and co-existing beta particles were measured, yielding beta groups were detected by the pulse height analyzer.
FIG. 32 \( \beta \) SPECTRUM IN COINCIDENCE WITH 344 KEV \( \gamma \) OF Eu\(^{152-154}\)
$\left( \frac{N}{IF} \right)^{\frac{1}{2}}$

**Figure 33** Kurie Plot of Spectrum in Coincidence with 344 KeV $\gamma$ of Eu$^{152-154}$
FIG. 34 $\gamma$ RAY IN COINCIDENCE WITH HIGHEST ENERGY $\beta$ GROUP OF Eu$^{152-154}$
Figure 35: γ rays in coincidence with two highest β groups of Eu^{152-154}
FIG. 36 GAMMA RAYS IN COINCIDENCE WITH HIGHER GROUPS OF Eu\textsuperscript{152-154}
SPECTROMETER SET TO L CONVERSION LINES OF THE 122 AND 123 KEV GAMMA LINES

- COINCIDENCE POINTS WITH SPECTROMETER SET JUST ABOVE THE CONVERSION LINE COINCIDENCE SPECTRUM

FIG. 37 GAMMA IN COINCIDENCE WITH THE L CONVERSION ELECTRONS OF THE 122 AND 123 KEV GAMMA RAY
To determine what coincidences might arise from beta particles of that energy, coincidences were measured with the beta spectrometer selecting particles with energies just above that of the conversion lines. The coincidences are indicated in Figure 37.

After subtracting the coincidence counts arising from beta gamma coincidences, one finds that gamma rays with energies of approximately 170, 240 405 and 1415 kev are in coincidence with the conversion electrons of the 122 and 123 kev gamma rays. The 340-345 kev gamma ray probably is also in coincidence with these conversion electrons, although a good part of them arise from beta gamma coincidences. It also appears that other gamma rays are in coincidence with these conversion electrons, but there was not sufficient resolution to resolve them.

The gamma pulse height analyzer was set to accept all gamma rays with energies above 650 kev. Because of the 12 or 15 percent resolution, the separation was not sharp of course.

The coincidence distribution which was obtained is indicated in Figure 38. This indicates that the 340 kev, 244 kev, and probably the 450 kev and 580 kev gamma rays are in coincidence with gamma rays with energies above 650 kev. The Kurie plot of the beta distribution obtained, Figure 39, indicates a beta group of 838 ± 40 kev and probably two other groups with energies of about 630 ± 50 and 450 ± 40 kev.
COINCIDENCES WITH \( \gamma \) RAYS ABOVE 950 KEV (SMALL SCALE)

COINCIDENCES WITH THE ANALYZER SET ON THE 778 \( \gamma \) RAY (SMALL SCALE)

COINCIDENCES WITH \( \gamma \) RAYS ABOVE 650 KEV (LARGE SCALE)

FIG. 38 COINCIDENCE \( \beta \) GROUPS OF Eu\(^{152-154}\) WITH HIGHER ENERGY \( \gamma \) RAYS
FIG. 39 KURIE PLOT OF β GROUPS IN COINCIDENCE WITH GAMMA RAYS ABOVE 650 KEV

\( \frac{N}{I_F} \)

W ENERGY IN MC\(^2\) UNITS

630 \(\pm\) 50 KEV

838 \(\pm\) 40 KEV
The analyzer was set to select all gamma rays with energies above 960 kev and coincidences were measured as indicated in Figure 38. The analyzer was then set to window on the 776 kev gamma ray and the coincidence beta distribution was then again measured as indicated in Figure 38. The distributions obtained for the 960 kev and 776 kev window settings of the analyzer were normalized at the upper end of the spectrum. As indicated in Figure 38, the intensity of the 838 kev beta group was increased in relation to the lower energy group when the analyzer was to window on the 776 kev gamma ray.

The pulse height analyzer was set to select gamma rays above 1250 kev and the beta groups in coincidence were measured. These are indicated in Figure 40. The Kurie plot of this spectrum is indicated in Figure 41. The Kurie plot indicates two beta groups with maximum beta energies of $660 \pm 50$ kev and $440 \pm 40$ kev. Strong coincidences are also evident with the conversion electrons of the 123 kev gamma ray.

Due to the ten percent resolution of the scintillation detector, there is some overlapping of the gamma ray lines. To determine the effect of this, the analyzer was reset to select gamma rays with energies above 1350 kev. This indicated that the intensity of the 450 kev beta group was diminished percentage wise less than the decrease in the gamma count, whereas the 660 kev group was diminished more.
Fig. 40 β groups in coincidence with γ-rays above 1250 keV
FIG. 41 KURIE PLOT OF $\beta$ GROUP IN COINCIDENCE WITH $\gamma$ RAYS ABOVE 1250 KEV

$440 \pm 40$ KEV

$650 \pm 50$ KEV
The coincidences with the conversion electrons decreased in proportion to the decrease in the gamma count. Low counting rates made accurate measurements difficult.

A positron baffle was inserted to detect the possible presence of positrons. Within experimental error no positron group was found. This would indicate that no more than 0.1 percent of the transitions were by positron emission.

Gamma-gamma coincidences were measured with a scintillation spectrometer. A pulse height analyzer was used on the pulses from one detector to select out the ones corresponding to a given gamma ray. Whenever a coincidence occurred between the analyzed pulses and a pulse from the other detector, the sweep of an oscilloscope was triggered. The pulses from the second detector were connected to the Y deflection plates of the oscilloscope. The resultant coincidence pulse distributions which occurred were photographed. Samples of the total, windowed, and coincidence distributions are shown in Figure 23.

The results of the gamma-gamma coincidence measurements indicated that the 1415 kev gamma ray is in coincidence with the 122 or 123 kev gamma ray and that the 778 kev gamma ray is in coincidence with a gamma ray of about 340 kev. Also strong coincidences were found between the 122 or 123 kev gamma ray and the 244 kev and 960 kev gamma rays. Mild coincidences apparently occurred between the 122 or 123 kev gamma ray and the 344 kev gamma ray. Weak coincidences seemingly appeared between the 244 kev and 720 kev gamma ray.
Weak coincidences also apparently occur between the 405 kev gamma and other gamma rays. The 190 kev gamma and a 1100 kev gamma may also be in coincidence with other gamma rays.
VIII. DISCUSSION OF Eu$^{152-154}$ RESULTS

The Eu$^{152}$ activity formed by a synchotron bombardment produced no gamma rays of 776 and 1115 kev. Because these gamma rays are found in the combined Eu$^{152-154}$ sample but not in the Eu$^{152}$ sample, one can reasonably assign them to the decay of Eu$^{154}$.

The gamma-gamma coincidence measurements showed that the 776 kev gamma ray is in coincidence with a gamma ray with an energy of about 340 kev. The sum of the energies of the 776 and 340 kev gamma rays agrees well with the energy of the 1115 kev gamma ray.

Coincidence measurements showed that a $338 \pm 40$ kev beta group was in coincidence with gamma rays above 950 kev. When the analyzer was set to select predominantly the 776 kev gamma ray which increased the percentage of detected gammas coming from the decay of Eu$^{154}$, the intensity of this group increased in comparison to the intensity of the lower energy groups as indicated in Figure 38. Therefore one can assign this group to the decay of Eu$^{154}$.

In every measured case, even-even nuclei have a $0^+$ ground state (81). Also in almost every measured case the first excited state is a $2^+$ state. The second excited state is usually a $4^+$ state (100).

One might assign tentatively the first excited state of Gd$^{154}$ as a $2^+$ state and the second as a $4^+$ state. This assignment is consistent with the fact that coincidences were measured between the 340 and 776 kev gamma rays. The data of R. Montalbetti (101)
indicates the half-lives of the excited state would be long enough to destroy most of the coincidences if higher order transition than an E2 were involved.

An insufficient part of the spectrum of the 838 kev beta group was obtained by itself to determine its shape. However, its log ft value of about 9.7, assuming a 10 year half-life, places it between a first and second forbidden transition, the tentative decay scheme is shown in Figure 42.

One might attempt to include the 1507 kev beta group in coincidence with the 344 kev gamma ray as part of the decay of Eu\textsuperscript{154}. However, the energy agreement is very poor. Secondly, the synchotron Eu\textsuperscript{152} sample showed that there is a gamma ray of about 345 kev associated with its decay. Consequently a gamma ray of about 345 kev appears to be associated with both the decay of Eu\textsuperscript{152} and Eu\textsuperscript{154} and the single observed line of 344 kev must actually be a composite of two closely spaced lines. This agrees with the results of Cork et al. (64) who examined the conversion electron spectrum of Eu\textsuperscript{152-154} with a 180° spectrograph. The instrument had sufficient resolution to resolve the lines.

As indicated in Figure 42, the single beta transition of Eu\textsuperscript{154} very probably goes to the 1115 kev excited state of Gd\textsuperscript{154}, for it was found to be in coincidence with both the 1115 kev and 776 kev gamma rays. The 340 kev level was arbitrarily taken as the first excited state rather than the 776 kev because the first excited states of Sm\textsuperscript{152} and Gd\textsuperscript{152} are of considerably less energy than 776 kev.
FIG. 42 TENTATIVE DECAY SCHEMES OF Eu$^{152}$ AND Eu$^{154}$
If one applies Nordheim's systematics to Eu\textsuperscript{154}, the neutron group is either assigned as a f 7/2 or h 9/2 state and the proton group is assigned as a d 5/2 state. Using his composition rule, the state of Eu\textsuperscript{154} has an odd parity. If the neutron group is assumed as a f 7/2 state, Nordheim's systematics indicate that the spin should be greater than 1, and at most equal to 6. The h 9/2 choice would indicate a spin of 2. The f 7/2 choice seems more reasonable, for a spin of two with odd parity would indicate transitions should take place also to both the ground state and the first excited state. If this were the case, the log ft values should also be much lower. Nordheim indicated that the spin for the same Schmidt groups is usually near a maximum value. For the f 7/2 and d 5/2 choice this would indicate a spin near six.

If one assigns the ground state of Eu\textsuperscript{154} as a 6- state, then the beta transition to the 1115 kev excited state of Gd\textsuperscript{154} would correspond to a first forbidden transition with a spin change of two if the assignment of a spin of four and an even parity to the 1115 kev excited state is correct. The log ft value of 9.7 for the 838 kev beta group is high for this type of transition, but not unreasonably so. This would agree with the experimental result that no observable transitions were found to the ground state and first excited state. These transitions would involve two and three times the change in spin and consequently would be much less probable.
Because the same neutron and proton assignments also fit Eu$^{152}$, one by similar arguments would expect Eu$^{152}$ to have a negative parity and a large spin.

The mass spectrograph work of Hayden, Reynolds, and Inghram (69) shows that within experimental error, K captured is not present in the decay of Eu$^{154}$. Accepting this, one can assign the gamma rays associated with K capture to the decay of Eu$^{152}$.

The energy difference of 39.5 kev between the K shell and average L shell conversion electrons of the 244 kev gamma ray indicates that conversion takes place in Sm, and therefore this gamma ray results from K capture for no detectable positron emission was found. Its K to L conversion electron ratio identifies it as an E2 transition according to the empirical results of Goldhaber and Sunyar (102).

It is in coincidence with a gamma ray of 122 or 123 kev as coincidence results show and possibly with one of 720 kev.

The 1415 kev gamma ray transition was identified as occurring in the decay of Eu$^{152}$ and was found to be in coincidence with beta particles. It is also found in coincidence with a gamma of about 122 or 123 kev.

Therefore the single observed conversion line of 73 kev, which was obtained with a 3.5 percent resolution setting of the instrument, is actually a double line, one of which is associated with K capture and the other with beta decay. Cork et al. (64) using a 180° spectrograph had sufficient resolution to resolve these lines.
The sum of the 244 and weak 720 kev gamma ray energies agree well with the 964 kev gamma ray energy. Therefore the 964 kev transition is assigned to the K capture decay of Eu$^{152}$. The sum of the energies of the 964 kev gamma ray and the 122 kev gamma ray in coincidence with it agrees well with the energy of the 1085 kev gamma ray. Therefore the 1085 kev gamma ray is assigned to the K captured decay of Eu$^{152}$. This agrees with the assignment of Cork et al. who were able to determine the K and L conversion electron differences and assign these transitions to the K capture of Eu$^{152}$.

The high intensity of the 122 kev transition and the intensity of the 244 kev gamma ray, which is much greater than that of the very weak 720 kev transition, indicates that the first excited state of Sm$^{152}$ is 122 kev above the ground state. The second excited state would then be 366 kev above the ground state.

As indicated in Figure 34, the 1860 ± 10 kev beta group is in coincidence with the 123 kev gamma ray within experimental error. This gamma ray is in coincidence with the 1415 kev gamma ray which is known to result from the beta decay of Eu$^{152}$. Therefore the 1860 kev transition is associated with the beta decay of Eu$^{152}$.

As indicated in Figure 40 and 41 the 1415 kev gamma ray is in coincidence with a beta group of 440 ± 40 kev. The energy sum of 122 and 1415 kev gamma ray and the 440 kev beta group agrees within experimental error with the sum of the 1860 kev beta group and the
123 kev gamma ray. The weaker beta group of 660 ± 50 kev can be assigned as a transition to a lower excited state of Gd$^{152}$. This interpretation is substantiated by the fact that as the window of the analyzer was set up to increase the percentage of the total number of gamma ray pulses coming from the 1415 kev gamma ray, the intensity of the 660 kev beta group was diminished in comparison to the 440 kev beta group. Statistics, however, were too poor to reach absolute conclusions. The weak 190 kev and 1250 kev gamma rays fit in with this interpretation. Also the sum of the 876 and 344 kev gamma ray transitions fit in with this interpretation within experimental error.

The 1507 ± 10 kev beta group is found to be in coincidence with the 344 kev gamma ray and possibly in coincidence with the 123 kev gamma as indicated in Figure 35. Gamma-gamma coincidence measurements also indicate possible coincidences between the 123 and 344 kev gamma rays. This agrees with the coincidence results of Fowler and Shreffler (63). The energy sum of the 344 and 123 kev gamma rays and the 1507 kev beta group also agree within experimental error with the sum of the 123 kev gamma ray and the 1860 kev beta group.

The experimental information lead to the speculative decay scheme indicated in Figure 42.

Both the 122 kev and 123 kev gamma rays appear to represent transitions from the first excited states to the ground states of even-even nuclei. This would indicate that they both should correspond to E2 transitions. The combined K to L conversion electron ratio of
the two lines together gives a value which agrees very well with values of Goldhaber and Sunyar (102) for an E2 transition of this energy. This would indicate that either both are E2 transitions, that the predominant one is an E2 transition, or that the two gamma rays have just the right intensity and are of just the right order to give together an apparent E2 transition. The last possibility seems very unlikely.

The graphical data of R. Montalbetti (101) indicates that the half-life of anything higher than an E2 or M1 transition would destroy most of the coincidences. Therefore the 122 and 123 kev transitions must be M1, E1, or E2 transitions. This is in general true for the various lines found in coincidence. The highest energy lines could possibly be M2 or E3 transitions and still have a short enough half-life to give rise to coincidences.

For the combined line of the 340 and 344 kev gamma rays the average K to L ratio agrees very well with the values indicated by the Goldhaber and Sunyar data for an E2 transition. Therefore the predominant gamma ray is an E2 transition, or both gamma rays are E2 transitions, or the combination accidentally gives the K to L conversion electron of an E2 transition. The last possibility is probably very unlikely.

The combined K to L ratios are therefore not inconsistent with the assignment of 2+ and 4+ to the first and second excited states of Gd\(^{152}\).

The 9 hour isomeric activity of Eu\(^{152}\) gave rise to two strong gamma rays of 960 ± 20 and 840 ± 20 kev. One might be tempted to consider the 960 kev transition as being the same as the 964 kev
transition which was found in the 5 - 15-year activity.

However, the work of Hill and Shepherd (73) indicates that the 840 kev gamma ray is in coincidence with the 122 kev gamma ray and that the 960 kev transition goes to the ground state.

Some of the weaker gamma rays found have been left unassigned because experimental information was too meager to make an accurate assignment. It should be possible with instruments of high transmission and better resolution to accurately assign these transitions.

With present instruments it should be possible to make further assignments by using critical absorption techniques and measuring coincidences between gamma rays and X rays. Also one should be able to make coincidence measurements on the synchotron produced activities.

The beta decay results in this work on Eu$^{152}$ and Eu$^{154}$ differ considerably from previous results. Because of low intensities, previous investigators did not separate the two highest energy beta groups, but nearly obtain an average result. This yielded a single beta group of 1.58 Mev instead of two groups with energies of 1.507 and 1.860 Mev. Also the lower energy beta groups were not previously resolved.
IX. SUMMARY

The beta activities of $K^{42}$, $P^{32}$, and $As^{76}$ were examined for small order effects with a thin lens and an intermediate image spectrometer. The linearity of the spectrometers was checked by examining the unique beta spectrum of $Y^{90}$.

The 12.4-hour activity of $K^{42}$ was found to have 2 beta groups with energies of $1.985 \pm 0.015$ Mev and $3.545 \pm 0.010$ Mev. The 3.54 Mev beta group was found to be a first forbidden transition with a spin change of 2 from its shape and log ft ($W_0^{2-1}$) value. The 1.985 Mev transition was found to have essentially an allowed shape and apparently corresponds to a first forbidden transition with a zero spin change. Its allowed shape indicates that the nuclear parameters $Z_3$ or $Y_1$ (7) must have values greater than 10. The possible addmixture from the $C_1^{(2)}$ term (7) is less than 10 percent. What appeared to be a very weak third beta group of 0.4 or 0.5 Mev was also found.

Gamma rays with energies of $1.53 \pm 0.01$ Mev, $0.32 \pm 0.05$ Mev were found.

The upper end of the beta spectrum of $As^{76}$ indicates three beta groups with energies of $2.975 \pm 0.01$ Mev, $2.42 \pm 0.015$ Mev, and $1.75 \pm 0.02$ Mev. The 2.97 Mev transition was identified as a first forbidden transition with a spin change of two. From the known $2^+$ first excited state of $Se^{76}$ and the $2^-$ ground state of $As^{76}$, the $2.42$ Mev transition can be identified as a first forbidden transition
with a spin change of zero. It is found to have an allowed shape within experimental error. The possible contribution from the $C_1^{(2)}$ term (7) is less than 15 percent.

The beta spectrum of $F^{32}$ was examined for possible Fierz type deviations. The beta spectrum was found to have a maximum energy of $1.712 \pm 0.006$ Mev. The parameter $r$ describing the magnitude of the Fierz type deviation as defined by Davidson and Peaslee (8) was found to be equal to $0.00 \pm 0.03$.

The 5 - 15-year activities of Eu$^{152}$ and Eu$^{154}$ were examined with a scintillation and an intermediate image spectrometer. The decay of Eu$^{154}$ was assigned a single beta group of $838 \pm 40$ kev and three gamma rays with energies of $340 \pm 3$, $776 \pm 4$, and $1115 \pm 5$ kev as indicated in the decay scheme of Figure 42.

The decay of Eu$^{152}$ was assigned four beta groups with maximum beta energies of $1860 \pm 10$, $1507 \pm 10$, $660 \pm 50$, and $440 \pm 40$ kev. Gamma rays with energies of $123 \pm 0.5$, $344 \pm 0.6$, $1415 \pm 10$, $1250 \pm 40$, and $876 \pm 8$ are assigned as following the beta decay of Eu$^{152}$. Gamma rays with energies of $122 \pm 1$, $244 \pm 0.6$, $720 \pm 20$, and $1085 \pm 5$ kev are assigned to the K capture decay of Eu$^{152}$. The 244 kev gamma ray is found to be an E2 transition. The decay scheme is illustrated in Figure 42.

Additional weak gamma rays with energies of $405 \pm 4$, $520 \pm 20$, $580 \pm 10$, $686 \pm 6$, $840 \pm 20$ and $960 \pm 20$ kev probably occur in the decay of Eu$^{152}$. They were not fitted into the decay scheme because of the lack of experimental information.
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