INTRODUCTION

Porosity assessment of flat smooth surface cast aluminum samples by ultrasonic attenuation spectroscopy were found to be in good agreement with other porosity assessment methods, e.g., weight-density, and optical techniques [1]. Ultrasonic results were also obtained for samples with their as-cast rough surface intact. The volume fraction of porosity in these samples were unacceptably underestimated [2]. Since it is not feasible to maintain a smooth surface, especially for large castings prior to ultrasonic assessment, a technique to compensate for surface roughness was considered.

Initial results were obtained using ultrasonic pulse-echo immersion technique at normal incidence. The samples were first evaluated with the as-cast rough surface intact and secondly, machined smooth. The total attenuation was in each case measured by comparing the front and back-wall echoes of the sample.

The total attenuation for the smooth surface sample, $L$, is a combination of many factors, and will be considered first:

$$ L = 20 \log \frac{A_F}{A_B} = L_{\text{IMP}} + L_{\text{DIFF}} + L_{\text{GRAIN}} + \ldots + L_p $$  \hspace{1cm} (1)

$A_F$ and $A_B$ are the front and back-wall amplitudes of the sample. $L_{\text{IMP}}$ and $L_{\text{DIFF}}$ are due to the impedance mismatch between solid-liquid interfaces and diffraction loss due to beam spread, respectively. Both factors can be calculated from known parameters or even completely eliminated by comparing the back-wall echo of the porous sample to that of a porosity free sample. The grain scattering induced attenuation, $L_{\text{GRAIN}}$, has no feasible correction and limits this technique to materials in which the grain scattering induced attenuation is negligible. The remaining term, $L_p$, is the porosity induced attenuation from which the pore size and volume fraction of porosity in the sample can be calculated.

The technique described above is valid for samples having a smooth surface. Thus, when calculating the total attenuation for a rough surface sample, the surface roughness induced attenuation, $L_{\text{SURF}}$, must be introduced into Eq. 1.

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\[ L = 20 \log \frac{A_F}{A_B} = L_{\text{IMP}} + L_{\text{DIFF}} + L_{\text{GRAIN}} + L_{\text{SURF}} + L_p \]  

From Eq. 2, it can be seen that effective evaluation of porosity in a rough surface sample requires differentiating between the attenuation components induced by the rough surface and porosity inside the sample.

A simple unified approach based on first order phase perturbation is introduced to calculate the surface roughness induced attenuation for both reflected and transmitted components at normal incidence on a liquid-solid interface. Results using this analysis are shown for a Gaussian distributed surface.

SURFACE ROUGHNESS INDUCED ATTENUATION

In the following, we shall introduce a very simple technique to account for the effect of slight surface roughness on both the reflection and transmission coefficients of an otherwise flat interface at normal incidence. Let us have a randomly rough liquid-solid interface \( h(x,y) \) positioned in the \( z = 0 \) plane of an \( x,y,z \) coordinate system, as it is shown in Fig. 1.

The rough interface under study is supposed to be geometrically flat, i.e.

\[ \iint_A h(x,y) \, dx \, dy = 0 \]  

and the surface quality is characterized by a single effective (rms) roughness parameter \( \epsilon \)

\[ \epsilon^2 = \frac{1}{A} \iint_A h^2(x,y) \, dx \, dy, \]  

where \( A \) is an infinitely large area of the \( z = 0 \) plane. The incident plane wave propagates along the \( z \) axis:

\[ E_i = E_1 e^{i z k_1}, \]  

where \( E_1 \) is the complex amplitude and \( k_1 \) denotes the wave number in the first medium. Without surface roughness, the reflected and transmitted fields

![Fig. 1. Coordinate System.](image_url)
would be simple plane waves propagating in opposite directions:

\[ E_0 = R_0 E_1 e^{-izk_1} \]  

and

\[ E_0 = T_0 E_1 e^{izk}, \]  

where \( R_0 \) and \( T_0 \) are the well-known reflection and transmission coefficients for normal incidence on a smooth plane interface, and \( k_2 \) denotes the wave number in the second medium. In the presence of surface roughness, both reflected and transmitted waves become complex scattered fields. We are interested in the modified plane wave reflection \( R \) and transmission \( T \) coefficients, so we shall completely disregard the so-called incoherent components of the scattered fields and base our calculations simply on the reduced strength of the coherently scattered "specular" components.

Our approach is based on the angular frequency representation of the scattered fields. We shall presume that the incident energy is divided into reflected and transmitted parts in the same way as in the case of a smooth plane surface, but these components are perturbed by \( \phi_r(x,y) \) and \( \phi_t(x,y) \) random phase modulations, respectively. The resulting plane wave attenuation can be derived directly from the angular frequency representation of the perturbed specular field. The angular frequency distribution of the (reflected or transmitted) field in the \( z = 0 \) plane can be written as

\[ F(k_x,k_y) = \frac{E_0}{A} \iint_A e^{i\phi(x,y)} e^{-i(xk_x + yk_y)} \, dx \, dy, \]  

where \( k_x \) and \( k_y \) are the spatial frequency components in the \( x \) and \( y \) directions, \( E_0 \) is the complex amplitude of the undisturbed field without surface roughness, and \( \phi(x,y) \) is the random phase modulation due to the surface roughness. The coherent specular wave

\[ F(0,0) = \frac{E_0}{A} \iint_A e^{i\phi(x,y)} \, dx \, dy \]  

can be further approximated for weak phase modulations by using the first three terms only from the Taylor series of the integrand

\[ F(0,0) = E_0 \left[ 1 - \frac{1}{2A} \iint_A \phi^2(x,y) \, dx \, dy + \frac{i}{A} \iint_A \phi(x,y) \, dx \, dy \right]. \]  

The third term on the right side of Eq. 10 is zero because of the boundary's being flat (see Eq. 3), and the second one is simply \( \frac{i}{2} \phi_e \phi^2 \) where \( \phi_e \) is the rms value of the rough surface induced phase modulation. The reflection and transmission coefficients can be written by Eq. 10 as follows:

\[ R = R_0 (1 - \frac{1}{2} \phi_e \phi^2) \]  

and

\[ T = T_0 (1 - \frac{1}{2} \phi_e \phi^2). \]  

The surface roughness induced phase modulations can be easily derived from \( h(x,y) \), \( k_1 \) and \( k_2 \):

\[ \phi_r(x,y) = -2h(x,y)k_1 \]
and
\[
\phi_t(x,y) = -h(x,y)(k_1 - k_2). \quad (14)
\]

Of course, a similar relationship exists between the corresponding effective values too, therefore from Eqs. 11 and 12,
\[
R = R_0(1 - 2h^2k_1^2), \quad (15)
\]

and
\[
T = T_0(1 - \frac{3}{2}h^2(k_1 - k_2)^2). \quad (16)
\]

The feasibility of these modified reflection and transmission coefficients is badly limited by the strength of the random phase modulation. The simple three term approximation of Eq. 10 is insufficient for phases above one radian, therefore higher order terms connected to the skewness, kurtosis, etc. of the actual roughness distribution must be taken into account, also.

The area integral of Eq. 9 can be expressed by the probability density distribution \( p(\phi) \) of the random phase modulation:
\[
F(0,0) = E_0 \int_{-\infty}^{\infty} e^{i\phi p(\phi)} d\phi \quad (17)
\]

Eq. 17 is well known \([3,4]\) for determining the coherent specular part of the reflected field, but, to the knowledge of the authors, it has never been applied to the transmission problem.

The following experimental results were obtained from sandblasted aluminum surfaces. We checked the surface by a mechanical profilometer and found its distribution to be close to Gaussian. Therefore, we evaluated Eq. 17 for this distribution, by using the following probability density function:
\[
p(\phi) = \frac{1}{\sqrt{2\pi\phi_e}} e^{-\phi^2/2\phi_e^2}, \quad (18)
\]

where \( \phi_e \) is the rms value, or spread of the distribution. Substituting Eq. 18 into Eq. 17 gives
\[
F(0,0) = E_0 \frac{1}{\sqrt{2\pi\phi_e}} \int_{-\infty}^{\infty} e^{i\phi e^{-\phi^2/2\phi_e^2}} d\phi. \quad (19)
\]

The solution of Eq. 19 is well-known \([5]\)
\[
F(0,0) = E_0 e^{-\phi_e^2/2}. \quad (20)
\]

The modified reflection and transmission coefficients for a Gaussian distribution can be written from Eq. 20 as:
\[
R = R_0 e^{-2h^2k_1^2} \quad (21)
\]

and
\[
T = T_0 e^{-\frac{3}{2}h^2(k_1 - k_2)^2}. \quad (22)
\]
Eq. 21 is the well-known formula [3] often used in ultrasonic surface roughness studies, but Eq. 22 gives us the sought tool to handle transmission as well. According to this very simple analysis, the surface roughness attenuates the reflected and transmitted components in a similar way, and their attenuation ratio is independent of frequency, rms roughness, or even the surface profile. Fig. 2 shows the calculated reflection and transmission attenuation coefficients for a water-aluminum interface and two other characteristics of practical importance: the attenuation coefficients of the through-transmitted and back-wall signal for a sample with rough front and back surfaces of 14 μm rms.

It is very important to recognize that due to the substantial velocity difference between water and aluminum, the transmitted wave is much less affected by the surface roughness than the front-wall reflection. Furthermore, the through-transmitted and back-wall reflected signals are less attenuated than the front-wall reflection as well in spite of their multiple interaction with the attenuating rough surface.

![Diagram](image)

**Fig. 2.** Surface roughness induced attenuation versus frequency for an aluminum sample of 14μm rms roughness immersed in water: (1) front wall reflection, (2) transmission, (3) through-transmission, and (4) back-wall reflection.

**EXPERIMENTAL RESULTS**

This somewhat astonishing phenomenon is shown in Fig. 3 where we compared theoretical and experimental results for an aluminum sample of 18μm rms roughness.

The much stronger surface roughness induced attenuation of the front signal over that of the back-wall echo results in a disturbing effect: the surface roughness induced attenuation of the sample seems to be negative when the rough surface front-wall echo is used as a basis for comparison, as it is shown in Fig. 4.

In this way, we would substantially underestimate the attenuation of the ultrasonic wave interrogating the inside of the sample. According to Fig. 4, it is somewhat better to disregard the actual front signal and compare the back-wall signal simply to the smooth front-wall signal. In this way, the surface roughness induced attenuation error will be about three times lower and opposite in sign, which is often more acceptable.
We have found that more accurate results were obtained by using a smooth front reference signal as a basis of comparison for porosity assessment of rough surface samples. To illustrate this point, aluminum samples having a surface roughness of 20 μm rms were ultrasonically measured and compared with a smooth surface sample. The received front- and back-wall amplitudes are shown in Fig. 5, respectively.

The attenuation coefficient was then calculated and plotted as a function of frequency for the following combinations: (1) smooth front-/smooth back-wall, (2a) rough front-/rough back-wall, and (2b) smooth front-/rough back-wall, as shown in Fig. 6. For each combination, the volume fraction of porosity was calculated and is listed in Table 1. By disregarding the rough front surface and comparing the smooth surface reference to the rough back-wall, the measured volume fraction of porosity for the rough surface sample was properly estimated.
CONCLUSION

It was found for samples having as-cast rough surfaces that the back-wall echo was much less attenuated than the front-wall echo. By ignoring the surface roughness induced attenuation and using the rough surface front-wall echo as a basis for comparison, the volume fraction of porosity was
substantially underestimated. The porosity assessment results were improved by using a smooth front-wall reference echo; however, a slight overestimation of the volume fraction of porosity occurs using this technique. It is our opinion that some overestimation of porosity is more favorable than underestimation and for ease of analysis the latter method should be used. To avoid either overestimation or underestimation of the volume fraction of porosity, a more accurate correction for surface roughness was considered. In order to do so, we can take advantage of the fact that the surface roughness induced attenuation of the front-wall echo is three times higher than that of the back-wall echo (this is a material constant independent of frequency, surface profile or rms roughness). As opposed to the back-wall echo, the surface roughness induced attenuation of the front-wall echo can be measured easily, and the sought correction, L_{SURF}, will be minus 2/3 of the measured attenuation.

REFERENCES


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