Some cost relationships in lard processing

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SOME COST RELATIONSHIPS
IN LARD PROCESSING

by

Henry J. Hudek

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Agricultural Economics

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

Head of Major Department

Signature was redacted for privacy.

Dean of Graduate College

Iowa State College

1954
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INTRODUCTION

Scope

Framework

This study was conceived within the framework of a larger study. The larger study had wider goals of investigating the cost of producing lard by several processes, each process used by plants differing greatly in size. It was hoped to discover how cost varied with size of plant and by process used. It was also hoped that some information would be obtained as to how quality of product varied from process to process and whether cost was related to the quality of lard produced by either the same or different processes.

Limits

While the larger study was concerned, therefore, with comparisons between plants and processes this study hopes to throw more light on the nature of costs within a given plant using a specific process. The author

1Relations between lard quality, processing procedures and equipment cost and revenues. Project number 1209. Iowa State College, Ames, Iowa.
in this study is not concerned to the same extent, therefore, with the
type of process nor with the size of the plant. The author is concerned,
however, with isolating the costs for a specific department. This then
is a case study in one phase of the larger study. It was hoped that it
would serve as a pilot study for further investigation in the field.

This study started out as an investigation of a rendering department.
It was found that it was both desirable and necessary to do some inves­tigating of the costs in the power department. The cost of power and
the variability in that cost are major factors affecting the cost of pro­duction of lard. The influence of these two factors on the inputs of
power used by the rendering department were not available from the data
of the rendering department alone. Therefore, data of the power depart­ment as well as the rendering department were obtained and investigated.

Definition

The attempt was made to find the variations of costs to outputs in
physical terms. This may be considered, then, as a study of input-out­
put relationships. Where necessary because of the nature of the data and
for aggregation, standard prices were used to place values on the inputs
and outputs. The values of the units of goods and services used will be re­ferred to as costs. When the units themselves are given in physical terms
they will be referred to as inputs. The production of the department
Objectives

Primary objectives

The primary objectives of this study were:

1. To determine the variations in costs that take place with variations in production during a year in a department of a plant. The study is concerned, therefore, with the empirical problem of determining what the short run cost schedule is for a specific department.

2. To develop methods which could be used to discover input-output relationships within a specific department of a plant. It was hoped that the knowledge gained would be useful in carrying out additional studies in the same and related fields.

Secondary objectives

The study had these additional objectives:

1. To determine what proportion of total costs is fixed costs and what proportion is variable costs.

2. To make estimates of marginal cost over the range of output of the department.
Assumptions

The following assumptions were made:

1. The process remained technically unvaried throughout the period.

2. The machinery maintained the same technical efficiency.

3. The raw materials used were assumed to be of the same quality during the period though it was realized that certain of the materials, e.g. fat, had seasonal variations in quality while others may have had variations due to chance or other causes.

4. The composition of labor was assumed to be either homogeneous or at least the proportions of each of the various levels of labor proficiency remained constant throughout the period. It was not assumed, however, that the average productivity remained constant. In point of fact it was found that productivity changed seasonally. Other changes in productivity were not associated with the seasons.

5. Over-time labor had the same efficiency as straight-time labor.

It is felt that the assumptions are justified because they simplify the study and because they approximate the actual situation and hence do not affect the results significantly.
METHOD OF PROCEDURE

Nature of the Data

The first type of data made use of in this study was the accounting data available from the records of the company. These data, in dollars, comprised the company's estimates of the amounts chargeable to each department for the operation of the department. In some cases they comprised the amounts actually paid out for an item of expense incurred by a department. The labor or material used by a department are examples of this type of expense. In other cases the data were the company's estimates of the share for a particular item of expense that was considered chargeable to a given department. Generally items of fixed cost such as depreciation or plant administration come under this category. A fixed proportion of such fixed costs was allocated to each department each period by the administration. In the case of an item like power, however, the proportion allocated to each department varied with the relative volume of production of each department. The method of allocating this item of expense of the rendering department will be dealt with

1Coded values are used here and elsewhere so the identity of the plant and its confidential information are not revealed.
more fully later.

A second type of data made use of in this study was also obtained from the records of the company. These data were the quantities, in physical units, of most of the items of expense. These quantities are referred to as inputs in contradistinction to costs which are the inputs valued at standard prices. The input data for labor were available by weeks for the departments studied. The other input data such as for fuel oil and other materials were available by accounting periods.

The production of the rendering department was also available for each accounting period. The production data consisted of the pounds of lard and cracklings produced each period. Separate estimates of the quantities of each produced were available. For purposes of this study, however, the combined weights of the two products are used and are referred to as the outputs. Outputs are coded in units representing a given weight.

Treatment of Fixed and Variable Costs

Those items of cost which would have been incurred even if no production had taken place are considered as fixed costs. Those items of cost which are directly dependent upon production taking place are considered as variable costs. Fixed costs, therefore, would include such items of expense as rent, insurance, taxes, depreciation, interest,
company and/or plant administration and wages and salaries of clerical help, janitors and others. They are allocated equally to each of the twelve periods of the year. Variable costs would include such items of expense as labor, materials, power, maintenance and supplies. The allocation of these costs is discussed under the analysis of each department treated below.

It was not always possible to allocate the various items of cost to the appropriate category of either fixed or variable cost. The data of the variable cost items in some instances contained elements of both fixed and variable costs. There was insufficient information available both to separate the two elements and to allocate the variable part properly in proportion to the production of a department. On what basis, for example, should the item maintenance and repair be allocated? To enter for a particular month the amount of maintenance and repair that is done that month in a department neglects several things. Maintenance and repair is necessitated by such factors as rusting and weathering. Rusting and weathering, perhaps, are most properly considered as functions of time. As such they should be treated like and even included with depreciation as fixed costs. Much of the repair that is done in plants is repair for just such time factors but the repair charge is included in the cost item of maintenance and repair. Moreover, it is included in the month that the repair is made and hence its allocation
may bear no close similarity to its actual incurrence.

Maintenance and repair is also necessitated because of such factors as wear and tear and breakage. Costs incurred on account of these factors are more properly considered as some function of production. That part of maintenance and repair cost which is a function of production should be treated as a variable cost, therefore, and allocated to each period of production according to that function.

It was not possible in this study to determine what function represents the relationship between the cost item repair and maintenance and the output item, production. It was not possible to determine, therefore, what part, if any of the cost of maintenance and repair is fixed cost and what part is variable. Nor was it possible to determine how the variable portion of maintenance and repair cost varied with production.

To have allocated any part or all of maintenance and repair somehow in proportion to production would have given the data a regression. A regression between the cost item, maintenance and repair and the output of lard would have been "built-in". Such a relationship probably would not be the true relationship that does exist and hence is not of interest to this study. Moreover, in a regression then of total costs with output a more significant regression would have been obtained than the data justified. Such a regression could obscure, to some extent,
actual relationships that might be found for which we would want the appropriate regression coefficient.

For these reasons the cost item, maintenance and repair, was treated as a fixed cost and allocated equally to each of the twelve periods of the year. This was done for the rendering department as well as for the power department whose allocation of costs affects the costs of the rendering department.

Depreciation is a similar item of expense. Part of depreciation is also a function of production and hence a variable cost. Some machines and certain parts of buildings, for example floors, wear out and depreciate more rapidly with increasing use. As in the case of maintenance and repair it was not possible to determine what portion of depreciation was variable nor what function the variable portion was of production. The whole of depreciation was also treated as a fixed cost, therefore. There is the added justification for treating depreciation as a fixed cost in that the company's accounting practice was to treat it so. When costs which contain some elements of variable costs are treated as fixed costs it should be appreciated that the total cost curves which are obtained have lower regression coefficients by the amounts of the regression coefficients of such variable costs.

In several of the items considered as variable costs considerable elements of fixed costs were observed. Where possible the elements of
fixed costs were deducted from total costs for each item of expense where it occurred and treated as fixed costs. A fuller explanation of procedure is given under each item discussed separately below.

Adjustments and Methods of Analysis of the Data

General adjustments

Basis for the adjustments. It was necessary to adjust the data to take account of a number of exogenous factors. Some of the exogenous factors affected the data of all items. Such factors will be discussed in this section on general adjustments. Other factors affected the data only of particular items. These items required additional analysis and adjustment. They will be discussed in connection with the factors affecting them. The items of input and cost of the rendering department requiring additional analysis and adjustment will be discussed in the next section. The items of the power department will be discussed in the section following. Until adjustments appropriate for each factor were made, the exogenous factors caused variations and irregularities in the data sufficiently large to completely mask any relationship between inputs and outputs.

The output of most packing plants is subject to considerable seasonal variation but the seasonal variation in the plant studied was
quite small. The output of the plant each day was fairly constant and the plant operated about the same number of days each week. The output of the plant, therefore, was fairly constant throughout the year. Adjustments are particularly important in those plants that operate at a fairly uniform volume of output. Any discrepancies in the data from such plants become relatively more important since discrepancies that might constitute a small per cent of a large variation in production could become a large per cent of a small variation in production.

Accounting period adjustments. The accounting data for most items consisted of twelve observations during a year for each item of information. Each observation gave the information for a period comprising either four or five weeks. It was necessary to adjust the data for accounting period variation, therefore. It was found that adjusting the data to amounts representing an equal number of weeks was not an adequate adjustment for most of the data. The variation in production between periods that did occur came about to a considerable extent because of the variation in the number of days worked each week. This variation was seasonal to some extent and so accounts for much of the seasonal variation in output. It was also due to national holidays as well as to work stoppages on account of weather or other causes.

Fluctuations in the inputs (or costs) and outputs as given by the original data were largely a function of the number of days worked during
each of the twelve accounting periods. For that reason the accounting data of inputs (or costs) and outputs were adjusted to correspond to periods having equal numbers of work days. The plant worked 289 days during the year or the equivalent of twelve equal periods of 24.1 days. If then the plant operated 28 days in one accounting period the original figure for input or output was reduced to the equivalent of a 24.1 day period by multiplying the actual input or output for the accounting period by $\frac{24.1}{28}$. If in some other accounting period the plant operated 23 days the adjusting factor was $\frac{24.1}{23}$. Similar adjustments of each item of the original data were made for each of the remaining accounting periods.

Where adjustments of the original data were made on account of one or more exogenous variables the adjustments were made simultaneously by means of multiple regression with an independent variable representing each of the exogenous factors. In such a case, therefore, one of the independent variables e.g. $X_1$ would represent the number of days worked each month.

**Standardization of prices.** During the period studied the values of the factors varied from month to month because of the effect of prices. To some extent and in certain cases these variations were caused by a rate schedule in which prices decreased by stages. Any amount of the factor used, up to a specified amount, was paid for at one rate. A second and lower rate applied to a second block. Successive blocks of
the factor were paid for at successively lower rates. Hence, in the
case of such a factor the average price tended to be lower in a month
of large consumption of the factor then it was in a month when less of
the factor was used because proportionately more of the factor was
paid for at the last and lowest rate. Electricity is an example of such
a factor.

The chief variations in the amounts paid for factors which can be
ascribed to prices were due, however, to changes in the market prices
of the factors themselves. To obviate both these price effects, the in­
puts of the various factors were obtained in physical units wherever
possible. Where the costs of the inputs were required, therefore, for
comparison between various inputs or for an aggregation of costs, the
inputs were priced at constant prices throughout the period of the study.
This kept out the effect of market price changes as well as the effect of
the changing rate schedule. It was felt that the elimination of the ef­
fact of changing market prices was justified in the first place to simplify
the study. The effect of changing prices could be studied later, if
desirable, by simply changing the prices of the factors. Removing the
effect of the changing rate schedule is justified because the plant usually
operated within the same block of the factor. Hence, effectively it was
operating at a constant marginal cost for the factor. Additional in­
crements of the factor were being obtained at a constant price within the
range of operations of the plant each month.

Analysis and adjustment of rendering department data

Supplies and sundries. In the previous section the adjustments, common to all data used in this study, were made and discussed. In this section those items of input and cost in the rendering department which require additional analysis and adjustment for their final allocation will be inspected and discussed. Supplies and sundries will be discussed in this first part of the section. The items of labor and power will be discussed in the following parts.

Supplies and sundries themselves small are aggregates of still smaller items of expense in the rendering department. Aggregates of input data would be meaningless. The data obtained are the values of the supplies and sundries that were obtained by the department each period rather than the actual amounts used. The following factors besides output would affect such data: (a) price changes during the year, (b) variations in the accounting periods, (c) changes in inventory of supplies and sundries within the department, (d) chance variations in the quantities used.

It was considered that the amounts of supplies and sundries used varied directly with production. The data were allocated to each of the twelve periods of the year, therefore, in proportion to output after output
had been adjusted for accounting period variation. Allocating the cost of supplies and sundries in this manner had the effect of obviating the effect of the above four factors. It also had the effect of giving the data a relationship directly proportional to output. This, however, was felt to be the proper relationship.

Labor. The data pertaining to labor included information on the following:

1. The physical inputs of labor.
2. The amounts paid to labor.

The general nature and method of treatment of labor data will be given first. The methods of analysis of each type of labor data will be discussed under their separate headings which follow.

The physical inputs of labor consisted of the hours of labor used each week by the two departments of rendering and refining combined. There was no record of the hours of labor used in either department alone. Most of the hours of labor consisted of the straight-time and over-time hours put in by the regular employees. This straight-time and over-time labor is referred to as regular labor. The remaining hours of labor were for student and miscellaneous help. The hours of regular labor as well as the hours of student and miscellaneous help were available separately by weeks. Data on the number of men employed each week were also obtained.
The data of the amounts paid to labor consisted of the wages paid each month to all employees in each department separately. Thus, the amounts paid to labor used in rendering were available for each month. Similar information was available for all departments dealt with in this study. No consistent relationship through time seemed to exist between the raw data and the output of product in either the rendering or the refining department. Adjustments as previously described were made on the physical input data and on the amounts paid to labor to account for accounting period variation. Wage rates in the plant were increased during the period covered by the study. Adjustments for wage increases were made or constant wage rates were used throughout the study where comparisons between cost data were made.

Another factor which affected the possible relationships was the inclusion of student and miscellaneous help. The employment of this help was sporadic and haphazard and was not determined by the immediate demands of production. Though the hours they worked were available separately their wages were included in the amounts paid to labor in the rendering department in the periods during which they were employed. The teaching of employees, however, is an overhead or fixed cost. An adjustment of the original labor data was made, therefore, by subtracting the hours worked by student and miscellaneous help from the physical inputs of labor data and by subtracting the wages they
were paid from the amounts paid to labor. The amount of the student and miscellaneous wages were then allocated equally to each month of the year under fixed costs.

The physical inputs of labor. The physical input data was then correlated with the output of lard. Since the hours of labor used by the rendering and the refining departments were not available separately, the total number of hours of labor, excluding student and miscellaneous labor, used in the two departments each month was correlated with the output of lard. It was considered that if any relationship existed between inputs of labor and outputs of product it would be shown since the same quantities of the product go continuously through the two related departments of rendering and refining. The correlation coefficient was found to be \( r = -0.317 \). Testing \( r \) by means of the t test we find that \( t = 1.057 \) with 10 degrees of freedom. Such a value of \( t \) is at about the 32 per cent point. Therefore, \( r = -0.317 \) is not significant and we cannot reject the hypothesis that \( r = 0 \). The relationship between the quantities of labor used in the two departments and the output of lard could have occurred by chance.

An attempt was then made to estimate the amounts of labor used each month in rendering alone and to relate these to the outputs of lard. Estimates of the inputs of labor used in rendering were made by dividing the hours of labor between the two departments each month in the same
proportion as their wage bills for the same months. The correlation coefficient of the hours of labor used in rendering so calculated and the output of lard was \( r = 0.469 \). Testing \( r \) by means of the t test we find that \( t = 1.679 \) with 10 degrees of freedom. The value of \( t \) is at about the 14 per cent point. An \( r \) value of 0.469 could have occurred by chance and is not significantly different from zero. This correlation coefficient like the preceding one does not indicate that a linear relationship between the inputs of labor and outputs of lard exists. A closer relationship is indicated when the labor inputs for the two departments combined are examined by quarters. The hours of labor varied with the volume of production but it is also apparent that the hours varied during the year independently of the volume of production. The relationships that existed between inputs of labor and quantity of lard produced for each of the twelve months of 1950-1951 are shown on Figure 1.

Each three month period shows an almost exact linear relationship existing between inputs of labor and outputs as represented by units of product. There is considerable change in that relationship from one three month period to the next. Each succeeding quarter required a larger labor input, \( Y \), for any given volume of production, \( X \), within the range of the observations. That is, regression lines fitted to the three observations of each quarter are appreciably higher for each succeeding quarter. Moreover, the slopes, as given by the regression
Fig. 1. Relationship between inputs of labor and outputs of lard by months and by quarters for 1950-1951.
coefficients, appear to get significantly greater.

The shift in the fourth quarter is particularly marked. Comparable data for the following year were obtained as a check. The same linear relationship is evident in each quarter and again the relationships change from quarter to quarter. The extra shift upward that occurred in the fourth quarter of the first year, however, was maintained in the year following. The shifting that occurred between quarters in the second year was carried out at that somewhat higher level. See Figure 2.

It seems evident that the productivity or the efficiency of labor is not only a function of output but is quite closely controlled by other factors. The planning of the business operations is done on a quarterly basis. Labor is budgeted by three month periods according to the production planned for each period. A change in labor productivity occurs each season of the year with changes in the numbers and types of hogs slaughtered. The changing pattern of hog slaughter and the planning of business activity by quarters account for the seasonal shift in the relation between inputs of labor and outputs of product.

Adjustments in the agreements with labor were made during the first yearly period. A lower productivity resulted. A definite break in the productivity of labor is evident at the beginning of the last quarter of the first of the two years. The decrease in productivity is sustained through the second year. See Figures 3 and 4, page 22.
Fig. 2. Relationship between inputs of labor and outputs of lard by months and by quarters for 1951-1952.
Fig. 3. Comparison of the inputs of labor with the outputs of lard by months for 1950-1951 and 1951-1952.

Fig. 4. Units of output per unit of labor by months for 1950-1951 and 1951-1952.
It seems evident then that labor input is a function of production but certain factors change the relationship each quarter. The method of covariance was used to analyse the data for the two years 1950-1951 and 1951-1952. The data for 1950-1951 are in Table 1A, page 24. The data for 1951-1952 are given in Table 1B. The input data is coded in units representing a given amount of time measured in hours, the output data in units representing a given number of pounds.

The object then is to learn what relationship exists between inputs of labor and outputs of lard and whether each quarter is associated with changing labor inputs.

The analysis of variance given in Table 2A indicates highly significant differences between quarters in the number of hours worked each month for the amounts of lard produced. $F = 16.14$ with three and eight degrees of freedom. In the following year, given in Table 2B, the differences between quarters in the number of hours worked each month for the amounts of lard produced are not significant. $F = 0.155$. The number of hours worked per month is substantially the same for each quarter. A decrease in the productivity of labor took place during 1950-1951. The lower productivity was maintained during the following year.

Table 3A gives the mean square from regression within quarters as 355.96. The mean square appropriate for testing the significance of the differences among the adjusted means of hours worked is 13,725.36.
Table 1A

Output X (lard) and input Y (labor) by months for four quarters of 1950-1951.

<table>
<thead>
<tr>
<th>Quarters</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Month</td>
<td>X</td>
<td>Y</td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>1</td>
<td>2027</td>
<td>1124</td>
<td>2283</td>
<td>1193</td>
</tr>
<tr>
<td>2</td>
<td>2260</td>
<td>1130</td>
<td>2128</td>
<td>1170</td>
</tr>
<tr>
<td>3</td>
<td>2439</td>
<td>1147</td>
<td>2432</td>
<td>1236</td>
</tr>
<tr>
<td>Sums</td>
<td>6726</td>
<td>3401</td>
<td>6843</td>
<td>3599</td>
</tr>
</tbody>
</table>

Table 1B

Output X (lard) and input Y (labor) by months for four quarters of 1951-1952.

<table>
<thead>
<tr>
<th>Quarters</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Month</td>
<td>X</td>
<td>Y</td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>1</td>
<td>2085</td>
<td>1284</td>
<td>2328</td>
<td>1373</td>
</tr>
<tr>
<td>2</td>
<td>2362</td>
<td>1342</td>
<td>2270</td>
<td>1326</td>
</tr>
<tr>
<td>3</td>
<td>2219</td>
<td>1307</td>
<td>2078</td>
<td>1239</td>
</tr>
<tr>
<td>Sums</td>
<td>6666</td>
<td>3933</td>
<td>6676</td>
<td>3938</td>
</tr>
</tbody>
</table>
Table 2A

Analysis of variance of hours worked for four quarters of three months each, 1950-1951.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Sums of squares</th>
<th>Mean square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Months within-quarters</td>
<td>8</td>
<td>6,882.67</td>
<td>860.33</td>
</tr>
<tr>
<td>Quarters</td>
<td>3</td>
<td>41,650.00</td>
<td>13,883.33***</td>
</tr>
</tbody>
</table>

\[
F = \frac{13,883.33}{860.33} = 16.14 \text{ with three and eight degrees of freedom.}
\]

Table 2B

Analysis of variance of hours worked for four quarters of three months each, 1951-1952.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Sums of squares</th>
<th>Mean square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Months within-quarters</td>
<td>8</td>
<td>28,729.34</td>
<td>3,591.17</td>
</tr>
<tr>
<td>Quarters</td>
<td>3</td>
<td>1,668.91</td>
<td>556.30</td>
</tr>
</tbody>
</table>

\[
F = \frac{556.30}{3,591.17} = 0.155 \text{ with three and eight degrees of freedom.}
\]

\(^{**}\) indicates significance at the one per cent level.
Table 3A

Analysis of covariance and test of significance of adjusted quarter means, 1950-1951.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>df</th>
<th>( Sx^2 )</th>
<th>( Sxy )</th>
<th>( Sy^2 )</th>
<th>( \text{Sum of squares} )</th>
<th>( \text{Mean square} )</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>11</td>
<td>415,658.00</td>
<td>-44,968.00</td>
<td>48,532.67</td>
<td>43,667.82</td>
<td>4,366.78</td>
<td>10</td>
</tr>
<tr>
<td>Quarters</td>
<td>3</td>
<td>232,560.66</td>
<td>-73,322.33</td>
<td>41,650.00</td>
<td>18,532.77</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Within-quarters</td>
<td>8</td>
<td>183,097.34</td>
<td>28,354.33</td>
<td>6,882.67</td>
<td>2,491.74</td>
<td>355.96</td>
<td>7</td>
</tr>
</tbody>
</table>

For test of significance for adjusted means of hours worked

\[ F = \frac{41,176.08}{355.96} = 38.56 \text{ with three and seven degrees of freedom.} \]

Table 3B

Analysis of covariance and test of significance of adjusted quarter means, 1951-1952.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>df</th>
<th>( Sx^2 )</th>
<th>( Sxy )</th>
<th>( Sy^2 )</th>
<th>( \text{Sum of squares} )</th>
<th>( \text{Mean square} )</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>11</td>
<td>232,422.00</td>
<td>52,140.00</td>
<td>30,398.25</td>
<td>18,701.51</td>
<td>1,870.15</td>
<td>10</td>
</tr>
<tr>
<td>Quarters</td>
<td>3</td>
<td>114,502.66</td>
<td>2,374.34</td>
<td>1,668.91</td>
<td>1,619.68</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Within-quarters</td>
<td>8</td>
<td>117,919.34</td>
<td>49,765.66</td>
<td>28,729.34</td>
<td>7,726.67</td>
<td>1,103.81</td>
<td>7</td>
</tr>
</tbody>
</table>

For test of significance for adjusted means of hours worked

\[ F = \frac{10,974.84}{1,103.81} = 3.31 \text{ with three and seven degrees of freedom.} \]
F, therefore, of 38.56 with three and seven degrees of freedom is highly significant and the quantity of lard produced does not explain the differences between the hours worked. The hours worked per month still differ significantly between quarters even after they are adjusted to a common output of lard. Evidently the productivity of labor changes between quarters. The mean square from regression within quarters in 1951-1952 given in Table 3B is 1103.81. F, therefore, is 3.31 with three and seven degrees of freedom. With these degrees of freedom F at the five per cent point is 4.35. F, therefore, is not significant at the five per cent point but with a value of 3.31 there is some indication that the hours worked may differ after they are adjusted to a common output of lard, but that they differ is not as probable as it was the previous year.

Table 5A shows that the mean square within-quarters is reduced from 860.33 to 355.96 by regression. F = 12.34 with one and seven degrees of freedom. The reduction in the sum of squares due to regression is highly significant. Within-quarters, therefore, there is a highly significant regression of hours worked each month with quantity of lard produced. In the analysis of the error variance in the rendering data for 1951-1952 the mean square within-quarters is reduced from 3591.17 to 1103.81 by regression (Table 5B). F = 19.03 with one and seven degrees of freedom is highly significant. As in the previous year, there is a highly significant regression of hours worked each month with the
Table 4A

Calculation of adjusted mean hours of labor worked, 1950-1951.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Mean output of fat</th>
<th>Deviation from experiment mean</th>
<th>Product bx</th>
<th>Mean hours worked</th>
<th>Adjusted mean hours worked</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2242</td>
<td>73</td>
<td>11.30</td>
<td>1134</td>
<td>1123</td>
</tr>
<tr>
<td>2</td>
<td>2281</td>
<td>112</td>
<td>17.34</td>
<td>1200</td>
<td>1183</td>
</tr>
<tr>
<td>3</td>
<td>2222</td>
<td>53</td>
<td>8.21</td>
<td>1247</td>
<td>1239</td>
</tr>
<tr>
<td>4</td>
<td>1931</td>
<td>-238</td>
<td>-36.86</td>
<td>1293</td>
<td>1330</td>
</tr>
<tr>
<td>Mean</td>
<td>2169</td>
<td>0</td>
<td>0</td>
<td>1218</td>
<td>1218.33</td>
</tr>
</tbody>
</table>

Table 4B

Calculation of adjusted mean hours of labor worked, 1951-1952.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Mean output of fat</th>
<th>Deviation from experiment mean</th>
<th>Product bx</th>
<th>Mean hours worked</th>
<th>Adjusted mean hours worked</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2222</td>
<td>91</td>
<td>38.41</td>
<td>1311</td>
<td>1273</td>
</tr>
<tr>
<td>2</td>
<td>2225</td>
<td>94</td>
<td>39.68</td>
<td>1313</td>
<td>1273</td>
</tr>
<tr>
<td>3</td>
<td>2082</td>
<td>-49</td>
<td>-20.68</td>
<td>1286</td>
<td>1307</td>
</tr>
<tr>
<td>4</td>
<td>1995</td>
<td>-136</td>
<td>-57.41</td>
<td>1315</td>
<td>1372</td>
</tr>
<tr>
<td>Mean</td>
<td>2131</td>
<td>0</td>
<td>0</td>
<td>1306</td>
<td>1306.25</td>
</tr>
</tbody>
</table>
Table 5A


<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Sums of squares</th>
<th>Mean square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Within-quarters, unadjusted hours</td>
<td>8</td>
<td>6,882.67</td>
<td>860.33</td>
</tr>
<tr>
<td>Reduction due to regression</td>
<td>1</td>
<td>4,390.93</td>
<td>4,390.93</td>
</tr>
<tr>
<td>Error for adjusted hours</td>
<td>7</td>
<td>2,491.74</td>
<td>355.96</td>
</tr>
</tbody>
</table>

\[ F = \frac{4,390.93}{355.96} = 12.34 \text{ with one and seven degrees of freedom.} \]

Table 5B

Analysis of error variance in rendering data, 1951-1952.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Sums of squares</th>
<th>Mean square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Within-quarters, unadjusted hours</td>
<td>8</td>
<td>28,729.34</td>
<td>3,591.17</td>
</tr>
<tr>
<td>Reduction due to regression</td>
<td>1</td>
<td>21,002.67</td>
<td>21,002.67</td>
</tr>
<tr>
<td>Error for adjusted hours</td>
<td>7</td>
<td>7,726.67</td>
<td>1,103.81</td>
</tr>
</tbody>
</table>

\[ F = \frac{21,002.67}{1,103.81} = 19.03 \text{ with one and seven degrees of freedom.} \]
quantity of lard produced. It is accepted, therefore, that a definite relationship between inputs of labor and outputs of lard does exist for the two departments combined and that this relationship is linear.

Table 6A gives the sums of squares of errors of estimate from each of the regression equations for quarters. The total of these yield a mean square of 121.42 for 1950-1951 in Table 7A. The mean square of differences among-quarter regressions is 668.68. The value of F with three and four degrees of freedom is 5.51. The value of F at the five per cent point is 6.59. Such a value for F would indicate that there is a significant difference between the regressions of the four quarters. The F value of 5.51 therefore indicates, but with a lower probability, that there may be a difference between the regressions of the four quarters.

The same test performed on the data of 1951-1952 in Table 7B gives an F of 1.05. The differences within the individual regressions are of about the same magnitude as the differences among the quarter regressions. The regressions do not differ significantly and probably are from the same population.

Table 6A also gives the regression coefficient for each of the four quarters of 1950-1951. Since there may be a significant difference between them, the regression equation for each quarter of 1950-1951 was calculated and is given on page 93. The table also gives a weighted
### Table 6A

Regression and correlation data in four quarters of labor data, 1950-1951.

<table>
<thead>
<tr>
<th>Q.</th>
<th>df</th>
<th>$Sx^2$</th>
<th>$Sxy$</th>
<th>$Sy^2$</th>
<th>Correlation coefficient</th>
<th>Regression coefficient</th>
<th>Sum of squares</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>85,358.00</td>
<td>4,639.00</td>
<td>284.67</td>
<td>0.9411</td>
<td>0.05435</td>
<td>32.55</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>46,214.00</td>
<td>10,012.00</td>
<td>2,244.67</td>
<td>0.9830</td>
<td>0.21664</td>
<td>75.63</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>31,620.67</td>
<td>6,397.67</td>
<td>1,628.67</td>
<td>0.8915</td>
<td>0.20233</td>
<td>334.26</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>19,904.67</td>
<td>7,305.67</td>
<td>2,724.67</td>
<td>0.9920</td>
<td>0.36704</td>
<td>43.25</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.7987</td>
<td>0.1549</td>
<td>485.69</td>
<td>4</td>
</tr>
</tbody>
</table>

**Sums** 8 183,097.33 28,354.33 6,882.67

### Table 6B

Regression and correlation data in four quarters of labor data, 1951-1952.

<table>
<thead>
<tr>
<th>Q.</th>
<th>df</th>
<th>$Sx^2$</th>
<th>$Sxy$</th>
<th>$Sy^2$</th>
<th>Correlation coefficient</th>
<th>Regression coefficient</th>
<th>Sum of squares</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>38,378.00</td>
<td>8,051.00</td>
<td>1,706.00</td>
<td>0.9950</td>
<td>0.2098</td>
<td>17.0513</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>34,242.67</td>
<td>17,643.33</td>
<td>9,244.67</td>
<td>0.9916</td>
<td>0.5152</td>
<td>154.0531</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2,954.00</td>
<td>3,093.00</td>
<td>3,858.00</td>
<td>0.9162</td>
<td>1.0471</td>
<td>619.4517</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>42,344.67</td>
<td>20,978.33</td>
<td>13,920.67</td>
<td>0.8641</td>
<td>0.4954</td>
<td>3,527.6225</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.8550</td>
<td>0.4221</td>
<td>4,318.1786</td>
<td>4</td>
</tr>
</tbody>
</table>

**Sums** 8 117,919.34 49,765.66 28,729.34
### Table 7A

Analysis of errors of estimate from average regression within-quarters, 1950-1951.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Sum of squares</th>
<th>Mean square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deviations from average regression within-quarters</td>
<td>7</td>
<td>2,491.74</td>
<td></td>
</tr>
<tr>
<td>Deviations from individual quarter regressions</td>
<td>4</td>
<td>485.69</td>
<td>121.42</td>
</tr>
<tr>
<td>Differences among quarter regressions</td>
<td>3</td>
<td>2,006.05</td>
<td>668.68</td>
</tr>
</tbody>
</table>

\[
F = \frac{668.68}{121.42} = 5.51 \text{ with three and four degrees of freedom.}
\]

### Table 7B

Analysis of errors of estimate from average regression within-quarters, 1951-1952.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Sum of squares</th>
<th>Mean square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deviations from average regression within-quarters</td>
<td>7</td>
<td>7,726.67</td>
<td></td>
</tr>
<tr>
<td>Deviations from individual quarter regressions</td>
<td>4</td>
<td>4,318.18</td>
<td>1,079.54</td>
</tr>
<tr>
<td>Differences among quarter regressions</td>
<td>3</td>
<td>3,408.49</td>
<td>1,136.16</td>
</tr>
</tbody>
</table>

\[
F = \frac{1,136.16}{1,079.16} = 1.05 \text{ with three and four degrees of freedom.}
\]
average regression coefficient for the four quarters of 0.1549. From this the regression equation:

\[
\hat{Y}_1 = 882.44 + 0.1549 X
\]

was obtained. It represents the average within-quarter regression of hours worked on output of lard for 1950-1951. It is plotted on Figure 1, page 19, along with the regression equation of each of the four quarters for that year.

Table 6B gives the regression coefficients for the four quarters of 1951-1952. For comparison with the previous year the regression equations for the four quarters were calculated. They are given on page 93 and are plotted on Figure 2, page 21. The weighted average regression coefficient of 0.4220 is also given in Table 6B. This gives the within-quarter regression of hours worked on output of lard for 1951-1952,

\[
\hat{Y}_2 = 406.90 + 0.4220 X
\]

also plotted on Figure 2. Since the quarter regressions do not differ significantly, the within-quarter regression is considered as the average or best relationship between hours worked and output of product.

The correlation coefficients for each quarter of 1950-1951 are also given in Table 6A. A high correlation of inputs of labor to outputs of fat is indicated for each quarter. Because of the difference between the regressions of each quarter, however, the within-quarters correlation coefficient of 0.80 is somewhat less but still substantial. When these
correlation coefficients are compared with the total correlation coefficient of -0.317 the result of eliminating the effect of the variation due to quarters is seen.

Similar correlations for the four quarters of the following year are given in Table 6B, also. The average within-quarter correlation is 0.855. This compares with the total correlation of 0.620 for the twelve months of the year. The "improvement" though substantial is not as marked as for the previous year.

The correlation of the quarter means in 1950-1951 is $r_{1q} = -0.7450$. The regression $b_{1q} = -0.31528$. The regression equation for quarter means is:

$$q_{1q} = 1902.17 - 0.3153 X.$$  

This equation is plotted on Figure 5A along with the quarter means of hours worked.

The regression for quarter means decreases the sum of squares by 23,117.23. The large reduction in sum of squares from 41,650 to 18,532.77 shows that there is a pronounced trend. By quarters, mean hours of labor input per month increased over time even with a decreasing output of product. This effect is brought about mainly by the effect of the fourth quarter but it is apparent that changing agreements with labor brought about, during the year, increases, by quarters, in the hours of labor worked per month. This is verified in Table 8A, page 37,
Fig. 5. Within-quarter regression, regression of quarter means and quarter means of inputs of labor charted against outputs for 1950-1951 and 1951-1952.
where the mean square for quarter means is shown to be highly significant when compared with the mean square within-quarters. $F = 26.0$ with two and seven degrees of freedom.

The correlation of quarter means for 1951-1952 is $r_{sq} = 0.1718$. The regression $b_{sq} = 0.0207$. The regression equation for quarter means therefore is:

$$\hat{Y}_{sq} = 1262.14 + 0.0207X.$$ 

This equation is plotted on Figure 5B together with the quarter means of hours worked. In this case the sum of squares is reduced by regression by only 49.23. There was no trend during the year. By quarters mean hours of labor input per month remained almost constant even though the output of product decreased during the last half of the year. The value of $F = 0.73$ indicates that the variation of the quarter means was less than the variation within-quarters regression. The indication is that the increased hours of pay gained by labor the previous year are maintained in 1951-1952.

The mean square for remainder in Table 8A also indicates that for 1950-1951 the average hours worked per month by quarters has a different trend than the hours worked per month within the quarters. $F = 63.6$ with one and seven degrees of freedom is highly significant. The same is true the following year where $F = 8.48$ is significant (Table 8B).

The analysis of covariance demonstrates that there is a linear
Table 8A

Analysis of errors of estimate from three regressions, 1950-1951.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Sum of squares</th>
<th>Mean square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total from table 3A</td>
<td>10</td>
<td>43,667.82</td>
<td></td>
</tr>
<tr>
<td>Quarter means</td>
<td>2</td>
<td>18,532.77</td>
<td>9,266.38**</td>
</tr>
<tr>
<td>Average within-quarters</td>
<td>7</td>
<td>2,491.74</td>
<td>355.96</td>
</tr>
<tr>
<td>Remainder</td>
<td>1</td>
<td>22,643.31</td>
<td>22,643.31**</td>
</tr>
</tbody>
</table>

\[
F = \frac{9,266.38}{355.96} = 26.0 \text{ with two and seven degrees of freedom.}
\]

\[
F = \frac{22,643.31}{355.96} = 63.6 \text{ with one and seven degrees of freedom.}
\]

Table 8B

Analysis of errors of estimate from three regressions, 1951-1952.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Sum of squares</th>
<th>Mean square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total from table 3B</td>
<td>10</td>
<td>18,701.51</td>
<td></td>
</tr>
<tr>
<td>Quarter means</td>
<td>2</td>
<td>1,619.68</td>
<td>809.84</td>
</tr>
<tr>
<td>Average within-quarters</td>
<td>7</td>
<td>7,726.67</td>
<td>1,103.81</td>
</tr>
<tr>
<td>Remainder</td>
<td>1</td>
<td>9,355.16</td>
<td>9,355.16*1</td>
</tr>
</tbody>
</table>

\[
F = \frac{809.84}{1,103.81} = 0.73 \text{ with two and seven degrees of freedom.}
\]

\[
F = \frac{9,355.16}{1,103.81} = 8.48 \text{ with one and seven degrees of freedom.}
\]

*1** indicates significance at the five per cent level.
relationship within-quarters between the hours of labor worked in the lard processing departments of rendering and refining and the output of the product, lard. The inputs of labor required for given quantities of lard changes from quarter to quarter. The quarterly regression equations give the relationships that exist.

The inputs of labor were available only for the two departments combined. The analysis does not establish, therefore, the relationship that exists for the rendering department alone. The company did have figures, however, of the amounts they charged to the rendering department for labor each month. In the next section these data are examined in an effort to discover if any relationship exists between the company's figures of the amounts paid to labor and the output of product.

The amounts paid to labor. The data on the amounts paid to labor were adjusted for accounting period variation and for changing factor prices. The wages paid for student and miscellaneous help were also deducted. The amounts paid to labor in the rendering department so adjusted were then correlated with the output of lard. The correlation coefficient was found to be \( r = 0.516 \). The value of \( t = 1.904 \) is significant at about the nine per cent point with ten degrees of freedom. Again \( r \) is non-significant.

An examination of the adjusted cost figures for rendering did not reveal the linear relationship of labor cost on output existing in each
quarter that existed for labor inputs on outputs for the two departments combined. The shift in cost from quarter to quarter also is not apparent.

The company's figures of the amounts paid to labor in the refining department were adjusted in a similar manner. A marked shift is shown particularly after the first quarter of the year. The adjusted costs for the two departments combined were then obtained. A marked shift is again apparent similar to the shift that is evident on Figure 1 where the inputs of labor for the two departments are combined. There is considerably more irregularity in the wages figures than there is in the input figures of Figure 1, however. A number of factors whose effects may not be measured directly or accurately probably influenced the relationships. One such factor may be a labor market less than perfectly elastic. Additional suitable men cannot always be obtained readily when needed. Over-time labor is then used but this is available only at a fifty percent increase in cost per hour. When additional inputs of labor are obtained through over-time labor rather than by the hiring of additional straight-time labor, the increase in wages is proportionately greater than the increase in the inputs of labor.

Considerable amounts of over-time labor were used in the two fat processing departments of rendering and refining. Neither the hours of the over-time nor the amounts paid for over-time labor were available
separately. They were included with straight-time hours of labor and wages. The number of hours of regular labor and the number of men employed were available for each week from the company's records, however. From this information, estimates of the number of hours of straight-time and over-time labor were made. The proportion of each type used each month varied considerably.

Though no way was discovered to allocate the hours of either straight-time or over-time labor accurately to each department, the estimates do indicate that some of the irregularity or lack of relationship in the labor costs can be laid to the variation in the percentage that over-time labor is of the total labor used each month and to the fact that such over-time labor is paid for at one and a half times the rate for straight-time labor. That is, any deviation from linearity occasioned by a change in the labor required would be accentuated in the cost figures by the fact that this labor is paid for at a premium of fifty percent.

It should also be noted that a change in the number of men in either department, for any reason other than a change in production, would affect the hours of over-time of the men who are working. So, though the actual hours worked may not be noticeably different for a given volume of production, the labor cost could change substantially. Such a change in labor costs in a plant where the volume of production
is fairly constant throughout the year could produce a small correl-
ation coefficient of labor cost on output when there is actually a high
correlation between inputs of labor and outputs of product.

There was considerable shifting between quarters in the amounts
paid to labor in the refining department and little if any shifting in the
rendering department. It could be that increasing amounts of labor
were used in the refining department alone as the year progressed.
From the information available and from observation of the plant it
is felt, however, that the company's method of allocating the labor
costs between the two departments account for the increasing amounts
paid to labor in the refining department only. The assumption was
made and acted upon that the increasing quantities of labor used during
the year were used in both of these closely related departments.

On the basis of the assumption and because of the lack of regular-
ity in the labor cost data for the two departments taken separately, the
average within-quarter relationship between inputs of labor and outputs
of product for the two departments combined, that is $b_{10} = 0.1549$, was
used as the slope in determining the regression equation of inputs of
labor on outputs in the rendering department. The regression equation
$\hat{Y}_1 = 882.44 + 0.1549X$ represents the average within-quarter regression
of inputs of labor on outputs of lard for 1950-1951. From it were obtained
the $\hat{Y}$ values of the inputs of labor. These $\hat{Y}$ values are accepted as the
best estimates of the quantities of labor that were required to produce the quantities of lard manufactured each month. They represent the inputs of labor used in rendering and refining each month after allowance has been made for the changes taking place in productivity between quarters. They include a productivity which is the average of the among-quarters productivities. The $\hat{Y}$ values times a standard wage rate give the labor cost figures used in this study and entered on page 95.

Power. The rendering department used an item of expense referred to as power. Power is supplied to the rendering department by and through the power department. Power is used in a number of forms such as steam, electricity, water and others which we will refer to as components of power.

The inputs of the various power components used by the rendering department could not be measured directly. Facilities were not available to measure the quantities of steam or electricity, for example, that were used in rendering. The company kept account of the cost of operating the power department each month and then allocated the cost to the various departments using power. The company's method of allocating the power cost is based on their experience and knowledge of the amounts of power required by the various processes and departments.

The data obtained on power from the rendering department records comprised data of the estimated cost of the power used in rendering.
during each accounting period. These data indicated that the cost of power used in rendering is more than half the cost of lard production. The estimates of the cost of power used in rendering are obtained by a method of allocation used by the company.

The method of allocation was as follows. The volume of production of each department was weighted according to some factor previously determined by the company. The cost of operating the power department was then allocated to each of the other departments in the proportions that the weighted production of each was of the aggregate of the weighted productions of all the departments. On this basis from about 14 per cent to about 18 per cent of the cost of operating the power department was assigned to the rendering department each month.

The cost of power for rendering so calculated bears a high linear relationship with the output of lard ($r = 0.798$). This does not mean, however, that the actual amount of power used by the rendering department each month is approximated nor that the true relationship is linear. The linear relationship evident in the data could have been established by the method of calculation. Thus, if lard manufacture is a larger (or a smaller) proportion of plant activity in one month it assumes that larger (or smaller) proportion of power cost by the method of allocation. Though it is logical that power costs for rendering should vary with production there is no proof that its share changes in the
proportions determined by the method used.

The large value of \( r \) does not measure the reliability of the data as estimates of the true relationship that exists between the inputs of power and the outputs of lard. It simply means that the method establishes an almost exact linear relationship between power inputs and lard outputs. If there is, in fact, a close linear relationship between inputs of power and outputs of lard the method could be a suitable method of allocating power cost provided a number of other conditions are approximated. The original weighting of the production of each department must have been correct. The relative prices of labor and materials used and the relative quantities of each type of input would have to remain constant. Thus, no great changes in techniques or in the efficiency in the use of the factors of production should have occurred over the period since the method was established.

If the actual relationship between inputs of power and outputs of lard is non-linear, the method of allocation could none-the-less yield estimates that are linear. Moreover, the estimates would show a high degree of correlation though the actual inputs could have little correlation with outputs. The same situation would hold if conditions would have changed over the period. It must be concluded, therefore, that the reliability of the estimates obtained by the method cannot be established or tested from the data available.
A check on the reliability of the power cost data was obtained for the rendering department from another source, however. The plant engineer made estimates of the cost to the rendering department of each of the components of power supplied by the power department during an average day's run. His estimate so obtained differed by less than five percent from an average day's cost as obtained by the company's method of allocation.

It is assumed that both estimates are sufficiently reliable to give estimates of average cost. To give estimates of cost at various levels of output both methods may be inadequate. The engineer's estimate, being an estimate for an average day, could not serve as a check on the variation in cost that occurs with a variation in production. The company's method of allocating power costs is subject to the limitations described above. It is also limited by such factors as changing input prices, over-time pay and the other factors discussed earlier for which adjustments should be made. In order to check into such factors and discover others, if present, the individual items of cost in the power department should be examined.

It was pointed out that it was not possible to obtain the actual quantities of the various components of power used by the rendering department. Since power constitutes such a considerable portion of the cost of rendering lard, the lack of information on the actual amounts
used is a major limitation of the study. It should be pointed out, however, that it does not suffice to measure the inputs of the various power components in the rendering department. It is necessary to know the variation in the costs of these components throughout their range of use by the whole plant. Obtaining the physical inputs of the power components and valuing those inputs at a standard price is not an adequate measure of the cost nor of the variation in cost that occurs with changes in the output of lard.

Even if the relationships between the inputs of each of the components of power and the outputs of lard were found, it would not be possible to obtain the true relationships between costs and outputs for those components. The reason for this is that it is not proper to apply standard or average prices to each component at all its levels of use. The costs of producing the components may and probably do vary with variations in the quantities produced. Moreover, the quantities produced must vary not only as a result of variations in the quantities used by the rendering department but also as a result of variations in the quantities used by all the departments throughout the plant. In reality the cost of producing each component at all its levels of use should be determined. The rendering department should then be charged for the quantities it uses, that price which is the cost of the component for the level of production at which the plant is operating. The costs of all the components so obtained
would then be aggregated into a total cost for power. Only then would a complete picture be obtained of the relationship between cost and output of lard.

It may be concluded that in order to get a true picture of the cost of operation of any one department and the variations in cost that occur with variations in outputs of that department estimates of the costs of the various inputs obtained from the departments supplying them would have to be obtained. The costs for each input should be the costs at the levels or over the range of output of the supplying department. Though such costs could be average costs, more interesting for policy considerations of management would be the marginal costs of each item of input. Marginal costs could be obtained from the same data.

It was felt that a greater knowledge of the cost of power than the company estimates provided was desirable. In order to adjust for factors already discovered and discussed above and to discover and adjust for others, if present, the accounting data for the power department were examined. The manner in which this was done and the specific adjustments that were made are described under the heading of each item of expense in the section which follows. It was hoped thereby that more reliable estimates could be obtained each month and that a better idea of the variation in cost which occurs with variation in output could be arrived at.
Analysis and adjustment of power department data

Labor. The information on labor in the power department comprised data on the number of men employed by weeks, the number of hours worked, also by weeks and the wage bill which was available for the twelve accounting periods of the year. The labor in the power department did not seem to be affected solely by the same factors that obscured the relationship between inputs and outputs in the rendering department. No clear relationship seemed to exist between the inputs of labor in the power department and the measures of the outputs of power that were available. This may be due either to discrepancies arising in the input data or to discrepancies in the output data.

Data pertaining to the output of power will be examined first for discrepancies. The actual output of power each month was not available. Nor were there adequate estimates of the individual components of power output available. For example, there was no record of the amount of steam produced each month. To serve as a measure of power output a number of other gauges were examined instead.

The total of the dressed weight of beef and hogs slaughtered by the plant each month was used as a gauge of the relative amounts of power required. This gauge does not take into account the variation in power requirements occasioned by the variation in processing that occurs at
different seasons of the year. The discrepancies that occur on this account should not be too great, however. A relationship approaching one to one should exist between pounds of slaughter and power used. Pounds of slaughter should be an adequate gauge, therefore, of power output. A clear relationship between labor cost and dressed weight of slaughter is not evident, however.

Another gauge of power output was obtained by using the value of the materials used in the power department. The total of the quantities of fuel oil, electricity, water, gas and ammonia used each month valued at average rates was used as a measure of power output. No clear-cut relationship seemed to exist between the total cost of materials used and the inputs of labor.

A third gauge of power output is the quantity of fuel oil used each month. Whereas electricity, water and other materials obtained outside the plant are used by the various departments with little or no additional processing, fuel oil is obtained and used in the power department to produce steam. Fuel oil, then, should represent quite accurately the activity of the power plant. Even here, however, there does not seem to be a close relationship of labor with fuel oil used.

It would seem that there should be a relationship also between the amounts of materials used in the power department and the weights of the animals slaughtered. It appears again that no close relationship
exists. This would indicate that the lack of relationship between inputs of labor and outputs of power may not be caused solely by discrepancies in the labor figures. Since a close relationship between materials used and pounds slaughtered does not appear to exist, there probably is discrepancy in one or other or both of these gauges. Materials used will be examined first.

An examination of each of the types of materials used in the power department does seem to reveal logical reasons why the total cost of materials used each month does not bear a clear linear relationship to the weights of animals slaughtered nor to the inputs of labor. In the case of items like water and electricity that are metered, the date of reading the meter is important. The meter periods do not coincide exactly with the company's accounting periods, nor are the periods necessarily of the same length. These two factors alone could completely mask any relationship that might exist and to a large extent probably do account for much of the lack of smoothness in the series of the materials used each month for the two years 1949-1950 and 1950-1951.

Another factor also makes its effect felt. No seasonal pattern is apparent in the total of all materials used but when each material is examined separately a distinct but differing seasonality is evident for each of them. In the two years for which the information is available a large amount of fuel oil is used in January and February while the
smallest amount is used in August. This pattern of use does not seem to be the result of the pattern of slaughter alone. The units of physical material used per 1000 pounds live weight of hog kill were calculated. A high use of fuel oil in the winter months with a low in the summer months was evident and is consistent with our knowledge of the climate.

The data indicate that larger amounts of electricity are used in the summer months than at any other seasons. The same high use, on a per unit basis, from May to October is also indicated. In addition to its other uses the year around, electricity is used for cooling and ventilation as well as for refrigeration. These uses are particularly heavy in the summer months. Water has two high periods of use in this plant. One occurs in December, the other in June.

It is considered logical to assume that the seasonal pattern evident in the use of fuel oil, electricity and water is in large part due to climatic conditions. The seasonal effect of climate would have to be removed or controlled if the effects of other factors are to be measured. This is attempted later in the study under the heading of each material or input.

Other causes which are seasonal could account for some of the seasonal pattern. During certain periods of the year special processes come into operation or some departments assume greater or lesser importance. Luncheon meats, for example, are made just prior to the
summer season. These special processes, being seasonal, could induce a similar seasonal use of the materials they require. Their influence would further affect and confound the seasonal variation in the data. It should be noted at this point, however, that the volume of production in each department is taken into account in the allocation of power cost by the company. If the production of each department is properly weighted, the method of allocating the power cost to each department should largely compensate for the seasonal effect of these processes but not necessarily for the effect of seasonal factors such as ventilation, heating and refrigeration.

Because of the seasonal effect on all the materials used and the fact that this seasonal effect is felt at different seasons of the year on each material there seems to be no reason why we should expect the quantity of all materials used to show a close relationship to volume of slaughter. Nor does it seem that quantity of materials used would be an adequate gauge of power required unless the seasonal effect due to temperature can be removed from each of the materials.

The number of pounds slaughtered also may be an inadequate gauge of the power requirements for a number of reasons. In the first place the type of animal slaughtered is not constant throughout the year. During certain seasons more of particular types of animals are slaughtered and the proportions of all types changes to some extent throughout
the year. Such variations may require varying power requirements which would not be reflected by the number of pounds slaughtered. A greater degree of processing also takes place at certain times of the year with the production of specialty meats. This also results in changing power requirements. The changing importance of different processes partly as a result of changes in the proportions of each type of animal and partly as a result of the seasonal production of certain kinds of meats could make pounds slaughtered an inadequate gauge of the power produced. It would appear therefore that a lack of close relationship between pounds slaughtered and materials used in the power department is probably due to variations in each which are not caused by variations in production. Both would seem to be inadequate gauges of power used or required.

The inadequacy of pounds slaughtered or materials used as a measure of power output may account for the lack of relationship between the inputs of labor and the outputs of power. The discrepancies that appear in the output of power data do not seem to account completely for the lack of relationship, however. When the inputs of labor are examined and the same factors that caused discrepancies in the labor input data in the rendering department are taken into account a logical relationship still does not seem to exist. No clear relationship between inputs of labor and outputs of power is indicated. Discrepancies in the labor input
data are suggested.

For the above examination of the data the inputs of labor in the power department were adjusted to take account of the fact that the original data were given in four week and five week periods. The data were adjusted, as explained under general adjustments, so that they represented periods of equal numbers of work days. The labor in the power department does not seem to fluctuate directly in proportion to the number of days of operation of the plant each month, however. Labor in this department unlike the labor in the rendering department tends to be a more fixed factor. For that reason the original monthly data were then adjusted so that they represented the 52 weeks of the year in twelve equal length periods of \( \frac{52}{12} \) weeks.

The labor data, so adjusted, adopted a much smoother pattern. The fluctuations are not violent and erratic as they were under the adjustment of an equal number of working days per period. What is more significant, however, is that the labor inputs follow the pattern of use of fuel oil much more closely. It seems logical to expect that the amounts of labor required would be related to the quantity of fuel oil used and that both would be related to the output of power. Fuel oil in turn is a function not only of production but also of heating. It is suggested, therefore, that the labor in the power department is also a function of climate.
It would appear, therefore, that the labor in the power department may be a function of at least the following factors:

1. Temperature. When more power in the form of heating is required, more labor is necessary.

2. The number of days the plant is operating each month. That is, during a five week period there would normally be more hours operated and hence more hours of time put in by the labor force.

3. The number of days the plant is idle each period. Some labor is required in the power department even when the plant is not operating such as on week-ends and holidays and during shut-downs.

4. Level of production of the plant. When more (less) power is required for an increase (decrease) in production of the plant more (less) labor is required.

The relationship of labor, the dependent variable to each of the above four factors or independent variables, is what it is desired to discover. The method of multiple regression could be used if adequate data for each of the variables were available. Data on the dependent variable, the physical inputs of labor, were available from the company data. Data on the first factor, temperature, were derived from meteorological records of the temperature in the area. A unit of measure called a degree day\(^1\) measures the variable, temperature. The number of

\(^1\)A degree day represents a daily temperature one degree below a
degree days during each of the twelve periods of the year were computed.

Data on the second factor, the number of days the plant operated and the third factor, the number of days idle, were also available from the company data. As was discussed earlier, however, a suitable measure of the level of production of the plant during each period was not available. Data, representative of the fourth factor, therefore, were not obtainable. A possible measure, volume of slaughter, does not take into account variability in the amount of processing. As well, it is highly correlated with $X_3$, the number of days operated. A second measure, volume of fuel oil used, is affected by heating requirements. Both these measures are probably affected by several other lesser factors to an unknown extent. For example, they are probably correlated with each other and with temperature since slaughterings are somewhat higher in the winter months.

It was not possible, therefore, to determine directly how labor in the power department is affected by the changing volume of production of the plant. It was felt, however, that something could be gained by determining the effect of the first three factors. The regression equation:

$$Y_L = a + b_1 X_1 + b_2 X_2 + b_3 X_3$$

was used.

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specified temperature of 65° F. and hence is a measure of the heating requirement. Thus, an average temperature of 52° F. one day would yield 13 degree days for that day.
The variables represent the following for each of the twelve periods in 1950-1951:

- $X_L$ - units of labor used (units are in time),
- $X_1$ - number of degree days,
- $X_2$ - number of days the plant operated,
- $X_3$ - number of days the plant was idle.

The estimating equation was found to be:

$$\hat{Y}_L = -45.52 + 0.1012 X_1 + 51.0134 X_2 + 46.2893 X_3.$$ 

The partial regression coefficients were tested for significance.

- For $b_1$ the value of $t = 3.38$.
- For $b_2$ the value of $t = 7.16$.
- For $b_3$ the value of $t = 4.81$.

With eight degrees of freedom the partial regression coefficients are all highly significant. The value of $R$ is 0.969 and with eight degrees of freedom is highly significant. Nearly 94 per cent of the amount of labor used is "explained" by the three independent variables.

The regression equation ascertains how much of the variation in labor inputs is due to the three independent variables, temperature, days operated and days idle. The remaining variation would be due to changes in production as well as other factors and error. The remaining variation will be regarded as being due mainly to changes in production.

Estimates of the variations dependent mainly upon production can be
obtained by removing the effect of the first three independent variables.

The physical inputs of labor, $Y_L$, are adjusted successively for each of these three variables. The adjustment on account of $X_1$, temperature, is made by subtracting the number of units of labor required each month because of temperature, that is $b_1X_1$, and adding in the average number of units of labor required each month on account of temperature, that is $b_1\bar{X}_1$. This adjustment is justified because the greater number of degree days in the winter months requires additional inputs of labor independent of the volume of production. Since the climate for the year in the area of the plant is a fixed item, the costs of operation which are caused by that climate should be made fixed costs. Thus the cost in any month should not be the actual cost incurred during that month because of climate but rather one-twelfth the total cost due to climate for the whole year.

$Y_L$ is also adjusted each month in a similar manner for days operated, $X_2$, and days idle, $X_3$, by subtracting $b_2X_2$ and $b_3X_3$ and adding $b_2\bar{X}_2$ and $b_3\bar{X}_3$. This adjustment accounts for the accounting period variation. When the three above adjustments are made values for $Y_L$ which should reflect only residual variations have been obtained. The residual variations would include a component for error and a component for each of the other factors that affect the quantity of labor used. To
the extent that the volume of production of the plant affects the quantity of labor in the power department it would be included in the residual variation.

An attempt was made to divide the labor into that which varies with production and that which would be required even if no production took place. That is, the adjusted values of $Y_L$ (which are referred to as $Y_L^{123}$) are divided into two parts, variable labor and fixed labor. The amount of labor dependent upon $X_1$, temperature, is a fixed cost i.e. $b_1\bar{X}_1$ per month. The regression shows also that labor is necessary in the power plant even when the plant is idle such as on weekends and holidays and shut-downs. The amount of labor required for such days, that is $b_3\bar{X}_3$, is also a fixed cost. Some of the labor used each day the plant is operating must also be considered a fixed cost. It would still require as much labor for this day if the plant were idle as would be required for a day when the plant actually is idle. Only the additional labor above the amount required on an idle day, can be considered as variable labor. The total labor per day dependent upon $X_2$ is $b_2$ or 51.013. The amount of labor per day which is fixed is $b_3$ or 46.289. Therefore, the amount of fixed labor during the days the plant is operating would be 46.289 $\bar{X}_2$ per month. Total fixed labor per month would, therefore, be $b_1\bar{X}_1 + b_2\bar{X}_2 + b_3\bar{X}_3$ or $(0.101248)(625) + (46.289, 270)(24.125) + (46.289, 270)(6.2083) = 1467.4$ units of fixed labor input per
month. The variable labor per month would be the difference between 1467.4 and the adjusted values of total labor, $Y_{l,123}$. The fixed labor inputs and variable labor inputs so derived for each month and multiplied by a standard wage rate yield the fixed labor costs and the variable labor costs in the power department entered on page 94.

Fuel oil. The data on fuel oil that were available for this study are the gallons of fuel oil that were used each month. The data were coded in units of a certain number of gallons. The data are subject to the limitation of errors of measurement mentioned earlier in that the quantities given may not be precisely for the identical periods that were used for other data. This may be a limitation of the data for all items to a greater or lesser extent. It was considered that the amount of fuel oil used was a function of at least the following factors:

1. The power requirements related to the level of production. No measure for this factor is available.

2. Heating requirements. This factor is measured in terms of degree days. See footnote page 55.

3. Number of days operated by the plant each month.

4. Number of days the plant is idle.

Factors three and four take into account the effect of accounting period variation but four is also included because some fuel oil is used at all times to keep up pressure. The fuel oil required for this purpose
would be the amounts used for all the days of each month calculated at
the rate that the fuel oil is being used when the plant is idle. The in-
clusion of the number of days the plant is idle as a factor should give us
a coefficient for that factor which would be the amount of fuel oil requir-
ed for each day the plant is idle. This again should be considered as a
fixed cost for each day of the month.

Since no measure of the first factor, level of production, was avail-
able it had to be left out as in the case of labor above. Again it was felt
that the residual effect would be left when the quantity of fuel oil was
adjusted for the remaining factors and that it would appear in the total
costs. It was considered that the company's method of allocation would
then allocate the cost in the appropriate amounts to the respective depart-
ments. The regression equation \( Y_F = a + b_1 X_1 + b_2 X_2 + b_3 X_3 \) was again
used. The variables represent the following for each of the twelve per-
iods in 1950-1951.

\[
Y_F - \text{units of fuel oil used (each unit coded in gallons)}.
\]

\[
X_1 - \text{number of degree days}.
\]

\[
X_2 - \text{number of days the plant operated}.
\]

\[
X_3 - \text{number of days the plant was idle}.
\]

The estimating equation was found to be:

\[
\hat{Y}_F = 33.21 + 0.1052 X_1 + 26.3221 X_2 + 2.3849 X_3.
\]

The partial regression coefficients were tested for significance.
For $b_1$ the value of $t = 5.43$

For $b_2$ the value of $t = 5.72$

For $b_3$ the value of $t = 0.38$

With eight degrees of freedom $b_1$ and $b_2$ are highly significant. $b_3$, however, is non-significant. The value of $R$ is .951 and with eight degrees of freedom is highly significant. With no means of checking the reliability of the data and with no alternate source of information on the quantities of fuel oil that are used on days when the plant is idle the variable $X_3$ was dropped from the equation and the regression was recalculated. The estimating equation was now found to be:

$$\hat{Y}_F = 35.41 + 0.1044 X_1 + 26.87 X_2.$$  

The values of $t$ for $b_1$ and $b_2$ were calculated and now found to be 5.70 and 6.45 respectively. The value of $R$ is still .951 and with nine degrees of freedom is also highly significant.

The values of $Y_F$ were now adjusted to represent conditions of an average number of degree days and an average number of working days in the same manner described under the section on labor. Thus, each value of $Y_F$ was adjusted to an average number of degree days by subtracting $b_1X_1$ (where $X_1$ is the number of degree days for the particular month) and adding $b_1\bar{X}_1$. The adjustment for $X_2$ was made in the same manner by subtracting $b_2X_2$ for each month and adding $b_2\bar{X}_2$. The values of $Y_F$ adjusted for these two factors are referred to as $Y_{F,12}$. 
As in the case of labor the values of $Y_{F_{12}}$ were divided into fixed inputs and variable inputs. The amount of fuel oil used on account of $X_1$, degree days, is considered as fixed inputs. The average of this amount for one month, that is $b_1 \bar{X}_1$, is the appropriate fixed input for each month. $b_1 \bar{X}_1$ was then subtracted from $Y_{F_{12}}$ for each month to yield the variable inputs. The values of fixed and variable cost were calculated by multiplying the fixed and variable inputs by the price of the input, fuel oil. The resulting values were then entered under fixed costs and variable costs on page 94.

Some fuel oil is used on a day when the plant is idle to keep up pressure. This fuel oil is a fixed input for each day since it would be required whether or not production took place. The total of the fixed input of fuel oil for one month, therefore, would be the rate of use of fuel oil on this account per day times the average number of days per month. In dividing the adjusted values of fuel oil between fixed costs and variable costs above some amount should have been included for this factor under fixed costs. This was not done. The regression coefficient for the factor is the rate of use. In the first equation above the coefficient for the factor was found to be non-significant. When the calculations were made, it was decided not to use the first equation and the non-significant coefficient. The second equation which does not include the factor, days idle, was then used. It is considered that
discrepancies in the original data probably account for the non-significance of the coefficient, however, rather than that the plant does not have a fixed input for fuel oil due to days idle.

Electricity. The data on electricity comprised the number of kilowatt hours of electricity used by the plant each month and the amounts paid for this electricity. In the mathematical analysis the coded physical units were used. As in the previous sections multiple regression was used as the method of analysis. The regression equation is:

\[ Y_E = a + b_1X_1 + b_2X_2. \]

The variables represent the following factors for 1950-1951:

- \( Y_E \) - units of electricity used, where each unit represents a specified number of kilowatt hours,
- \( X_1 \) - number of refrigeration days\(^1\),
- \( X_2 \) - number of days the plant operated.

The estimating equation was found to be:

\[ Y_E = -95.37 + 0.7647X_1 + 142.0980X_2. \]

The partial regression coefficients were tested for significance.

For \( b_1 \) the value of \( t = 10.37 \).

\(^1\)A refrigeration day represents a daily temperature one degree above a specified temperature of 30° F. and hence is a measure of cooling requirement. Thus an average temperature of 38° F. one day would yield eight refrigeration days for that day. The number of refrigeration days per period were calculated from the meteorological data for the area.
For $b_2$ the value of $t = 9.18$

With nine degrees of freedom $b_1$ and $b_2$ are both highly significant. The value of $R = 0.980$ and with nine degrees of freedom is highly significant.

The values of $Y_{E_1}$ for each month were adjusted to represent conditions of an average number of refrigeration days and an average number of days operated by the plant in the same manner as the adjustments for the input, labor. Thus $Y_{E_1}$ was adjusted each month by subtracting $b_1X_1$ and $b_2X_2$ and adding $b_1\bar{X}_1$ and $b_2\bar{X}_2$. The values of $Y_{E_1}$, the amounts of electricity used, adjusted for these two factors are referred to as $Y_{E_1}$. The values of $Y_{E_1}$ were divided into fixed inputs and variable inputs. The amounts of electricity used on account of $X_1$, refrigeration days, are considered as fixed inputs. The monthly average of these amounts, that is $b_1\bar{X}_1$, is the appropriate fixed input for each month. The fixed input $b_1\bar{X}_1$ was subtracted from $Y_{E_1}$ for each month to yield the variable inputs. The values of fixed and variable cost are obtained by multiplying the fixed and variable inputs by a standard price for electricity. The resulting values are entered under fixed costs and variable costs on page 94.

As in the case of fuel oil some electricity is used when the plant is idle. This is also a fixed cost and a coefficient for its rate of use should have been obtained. A reliable estimate of its amount was not
obtained, however. The estimates of variable cost for electricity would be too high by the amount of the electricity used on this account.

**Water.** The data on water comprised the number of cubic feet of water used by the plant each month and the amounts paid for this water. The figures were checked against separate estimates of the cost of water used, made by the plant engineer. The relationships between these estimates of water used and a number of factors were examined by means of simple, multiple and graphic curvilinear regression. One factor, pounds of slaughter, gave a value for simple correlation of \( r = 0.636 \) with quantity of water used which is significant at about the three per cent point. No other factor was found which yielded significant correlation coefficients with the quantity of water used. The factors tried and their simple correlation coefficients are as follows:

- \( X_1 \) - pounds of slaughter, \( r = 0.636 \)
- \( X_2 \) - degree days, \( r = 0.045 \)
- \( X_3 \) - refrigeration days, \( r = 0.193 \)
- \( X_4 \) - days operated, \( r = 0.460 \)

The physical inputs of water, \( Y_w \), correlated with \( X_2 \), \( X_3 \) and \( X_4 \) gave the following regression coefficients, their corresponding \( t \) values and points of significance.

- \( b_3 = 1.23 \) with \( t = 1.88 \) at about the 10\% point.
- \( b_3 = 1.38 \) with \( t = 1.95 \) at about the 9\% point.
b₄ = -1.82 with t = 0.04 at less than an expected value.

When X₃ was dropped from the regression equation the following values were obtained.

b₃ = 0.10 with t = 0.48 at less than an expected value.

b₄ = 64.34 with t = 1.50 at about the 15% point.

Yₜ was then correlated with X₁, X₂ and X₃. The regression coefficients, their corresponding t values and points of significance are as follows:

b₁ = 1.37 with t = 1.11 at about the 30% point.

b₂ = 0.67 with t = 1.01 at about the 35% point.

b₃ = 0.85 with t = 1.26 at about the 25% point.

When X₃ was again dropped from the regression equation the following values were obtained:

b₁ = 0.23 with t = 2.76 at about the 3% point.

b₂ = 0.19 with t = 1.13 at about the 29% point.

Yₜ was then correlated with X₁, X₂ and X₄. The regression coefficients, their corresponding t values and points of significance are as follows:

b₁ = 0.45 with t = 2.61 at about the 3% point.

b₂ = 0.30 with t = 1.71 at about the 12% point.

b₄ = -104.10 with t = 1.43 at about the 19% point.

It was felt that the relationship between the amounts of water used
and one or more independent variables may have been curvilinear rather than linear. The method of graphic curvilinear analysis was used to examine the data to see if such a relationship existed on account of variables $X_2$ and $X_3$, degree days and refrigeration days. From the analysis no curvilinear relationship for either of these factors appeared to exist.

As a result of the above examination of the data it was decided to adjust the data only for accounting period variation. Accordingly the input data were adjusted in the manner described on page 12 under accounting period adjustments. The inputs of water adjusted to correspond to equal numbers of work days were then valued at a standard price for water. The values obtained are entered under variable costs on page 94.

It is felt that some part of the inputs of water is a function of factors other than the volume of production. The analysis that was carried out does not supply substantiation for this point of view. However, errors in the observations or the fact that the proper factors which affect the used of water in packing plants were not selected may be the reasons why significant correlations were not obtained.

Miscellaneous. Several other items of expense charged to the power department are grouped under miscellaneous. Included are gas, ammonia, receiving, supplies and sundries.
The reliability with which they can be adjusted to take account of exogenous factors affecting their use may be quite low. Since they are of considerably less importance than the ones already examined, however, it is presumed that their influence will not affect the results unduly.

The data on gas and ammonia comprised the quantities of these commodities that were used by the plant each month together with the amounts paid for them. The data did not reveal relationships with any factors that might have affected their use. It was felt that this may have been due to inaccuracies in the data. The actual amounts used each month may not have been accurately given in the data of these two relatively unimportant items. The data on the amounts of each that were used were adjusted for accounting period variation. The cost of each was obtained by multiplying the adjusted quantities by standard prices.

No satisfactory way to treat the two items of expense, supplies and sundries, was found. More information on the composition and use of these two items of expense should have been obtained as a guide in their analysis. The data available included information only on the amounts paid each month for these two items. The variation in the amounts paid from month to month was large and seemed to bear no relationship to the length of each period nor to the output of power as measured by the volume of slaughter or the amount of fuel oil used. As in the case of supplies and sundries in the rendering department, it was
decided that supplies and sundries in the power department were items of expense that would vary closely in proportion to the output of the department. The best measure of that output appears to be fuel oil adjusted for accounting period variation and heating requirements. The quantities of fuel oil so adjusted were used, therefore, and the cost of supplies and sundries was allocated to each month in the same proportions that the adjusted quantities of fuel oil for each month were of their aggregate for the year.

The data on the expense item, receiving, were the amounts paid out each month. The data were adjusted for accounting period variation. The cost of each of the items discussed in this section and calculated in the manner described were then aggregated. The resulting values are entered under variable costs on page 94.
RESULTS

This study attempted to compute the costs of operation of the power department to obtain the variations in those costs which are dependent only upon changes in the output of the department and are independent of the effects of exogenous factors. The costs which are adjusted to take account of the exogenous factors are referred to as adjusted costs. The adjusted costs of each item of expense are aggregated each month to yield the total adjusted costs of operating the power department.

The total adjusted costs of operating the power department each month for 1950-1951 are charted on the Y axis of Figure 6. The dressed weights of slaughter are charted on the X axis. A curved regression line is hand-fitted through the points. For comparison the original data of power costs, adjusted only for accounting period variation, and charted against dressed weights of slaughter on Figure 7. The variability on Figure 7 appears to be considerably more than in the case of the adjusted data of Figure 6. Figure 6 indicated that total costs of power increase at a decreasing rate with increases in output and, therefore, that marginal costs, as measured by the slope of the total cost line, are less at larger volumes of slaughter and, therefore, at larger volumes of output of power. From the slope of the hand-fitted curve at 99 units of slaughter it appears, in fact, that the marginal cost is only about one-fifth the amount it is at 82 units of slaughter. Marginal
Fig. 6. Relationship of adjusted total cost of operating the power department to total dressed weight of slaughter, 1950-1951.
Fig. 7. Relationship of original total cost of operating the power department to total dressed weight of slaughter, 1950-1951.
costs of power to the rendering department operating at a given level of production are substantially less, therefore, when the plant is operating at the larger volumes of production than when the plant is operating at smaller volumes of production. The same may be said of the total cost per unit and the variable cost per unit. No further investigation of the costs in the power department was undertaken.

Estimates of the shares of the power costs which are chargeable to the rendering department must now be obtained. Additional research would be necessary in order to make independent estimates. Further investigations would have to be carried out to obtain information on the variations in costs that arise from variations in the amounts of each component of power put out by the power department. More would also have to be done to obtain data on the actual amounts of each component used by the rendering department at various levels of output. It was not possible to do either of these two things in this study. In the absence of information on the actual amounts of each component used by the rendering department at its various levels of production the company's method of allocating a portion of the total power cost each month to the rendering department was used.

The company used the following method to make the allocation. Every month the output of each department was weighted by a coefficient fixed for each department. The products of outputs times coefficients of
all the departments are aggregated. The proportion that the product for each department is of the aggregate each month is the proportion of total power cost for that month which is allocated to each department by the company. The proportions so obtained for the rendering department each month were used to calculate the rendering department's share of the total adjusted costs of power. The shares obtained in this manner from our data are given in Table 11, page 95. They are plotted against the outputs of the rendering department in Figure 8. Except for the fifth month which appears to be unusually low the cost of power for the rendering department appears to be largely dependent upon the output of product. As explained under the section on power commencing page 42 this would tend to obtain because of the method of allocation. The method of allocation appears reasonable but the reliability of the estimates would depend mainly upon the reliability of the weighting coefficients used and the other factors discussed previously.

Table 11 gives a summary of the costs of the rendering department for each of the twelve periods of the year. The variable costs include the costs of power, labor, supplies and sundries. The regression of variable costs on outputs gave the estimating equation

\[ Y = 3249.1 + 1.1202X^1. \]

\[ ^1 \text{In the case of the fifth month the proportion was low for no apparent reason. It is believed that an error may have been made in calculating the original proportion. No check was possible so it was omitted from the data when regressions were run.} \]
Fig. 8. Relationship between estimates of adjusted costs of power used in rendering and outputs of lard by months, 1950-1951.
The value of $r = 0.903$ with nine degrees of freedom is highly significant. The marginal cost of producing lard, therefore, is 1.12 units of cost per unit of output.

The regression of power cost on output is

$$\hat{Y} = 1929.067 + 0.8537X.$$  

The regression of labor cost on output is

$$\hat{Y} = 1323.4 + 0.2324X.$$  

The regression of supplies and sundries on output is

$$\hat{Y} = 0.0326X.$$  

Fixed costs are equal to 1628. The total costs each month from Table 11 are charted against the outputs of the rendering department in Figure 9. The regression of total costs of producing lard on output is given by the equation

$$\hat{Y} = 4877.1 + 1.1202X$$  

also charted on Figure 9.

The total cost of operating the rendering department for the year was 87,102 units of cost. The output of the department was about 26,028 units of lard during the year. The average cost of lard rendering in the plant therefore, was about 3.35 units of cost per unit of output.
Fig. 9. Relationship between estimates of adjusted total costs of producing lard and outputs by months, 1950-1951.
The marginal cost of lard production will bear additional examination. The value of 1.12 units of cost per unit of additional output could be useful in arriving at decisions. However, a number of considerations impinge upon the situation. In making the estimates of cost there was no satisfactory way of taking into account the changes in upkeep (due to maintenance and repair) and in depreciation of the plant that take place with changes in the volume of production. Maintenance and repair and depreciation, though affected by the wear and tear of production, were treated as fixed costs instead of as partially fixed and partially variable. In consequence the slopes of the total cost curves in the power department and the rendering department and the regression coefficients of the equations they represent are less than they would otherwise be. The marginal cost of 1.12, therefore, is less than it should be by whatever the regression coefficients of maintenance and repair and depreciation on output in both departments would amount to.

The effect of the higher wage that must be paid to over-time labor would have to be taken into account. If, in order to expand production, additional regular labor could not be obtained on a straight-time basis the additional labor obtained by means of over-time would mean a marginal cost of labor of not 0.2324 but 0.349. This would increase the total
marginal cost of lard by a like amount at larger volumes of output.

Marginal costs, average variable costs and average total costs may be lower at larger outputs to a considerable extent because labor is satisfied to work harder at times of large production only because they make up for it by working less hard at times of lower production. If production was maintained at high levels consistently, labor would ask for higher pay or labor could only be obtained if higher wages were paid for the heavier work load. Therefore, marginal costs may be higher than the figure of 1.12 indicates at the higher levels of production. Since the cost of power includes labor, it also and for the same reasons, should have a higher marginal cost than the one given.

The results obtained of the adjusted costs of power used in rendering impinge in other ways upon the conclusions that may be drawn. It cannot be established positively that the method of allocation gives to rendering its proper share of power cost. It may be that the proportions should be either less or more. This would affect the total and average costs of lard production but not its marginal costs. At the same time it may be that the proportions should be larger at large outputs and smaller at small outputs in which case the marginal costs would be more. If the proportions are the other way around the marginal costs would be less.

It may be also that the adjusted total costs of operating the power department obtained each month are inaccurate. They could be inaccurate
because of errors in the original data of any of the components or because faulty adjustments were made or because all the proper adjustments were not made in the original data. It must be recalled, for example, that no satisfactory adjustments for exogenous factors could be made in the case of water. Yet the data on water consumption besides being subject to error probably are a function of several factors besides production. The adjusted data on fuel oil and electricity and perhaps other components probably still contain substantial elements of variability due to exogenous factors not discovered. The amounts charged to rendering would be inaccurate as well, therefore, and the marginal cost of power might thereby be affected.

The above considerations at least would have to be taken into account in basing any decisions on the estimate of marginal cost. The estimate indicates, however, that the plant should attempt to keep its output of lard at least as high as the maximum of the period if the price of the lard is greater by 1.12 units of cost than the price that could be obtained for its raw material, fat. In the case of this plant the price was substantially greater.
RECOMMENDATIONS AND SUGGESTIONS FOR FURTHER STUDY

One of the main points of interest observed in the plant examined in this study and obtained from a knowledge of the accounting methods in other plants visited is that at best management in most plants knew only the general level of the cost of producing lard. In some plants they did not know even that. It is felt that management knew very little about the variation in costs that occurred with variations in output even in plants having an accounting system.

It serves no purpose to have an expensive accounting department calculating the costs of operation each month if no more than an approximation of average costs is arrived at. Such an approximation is of no vital use for making short run decisions. Marginal costs and changes in marginal costs are the most important cost concepts for a plant operator to keep in mind for a plant that is in production. Cost accounting methods in the plant studied did not give this information to the operator. If cost accounting data are not further analyzed and investigated so that marginal costs are arrived at, little value is obtained from cost accounting data particularly for the short run. For long run decisions it is not necessary that average cost be calculated each month. Cost accounting records kept on a monthly basis for this purpose are an expensive luxury. Much of the cost of accounting could be eliminated if
the calculation of total and average unit costs were limited to a yearly undertaking. Such costs would be adequate for long term decisions and would be nearly as useful as present data for short run decisions.

It is interesting to note that in several plants studied the engineer on the job was consulted most frequently pertaining to any particular cost item. In the plant covered by this study the engineer's estimate of cost of operation for an average day made on the basis of a separate estimate for each item of expense varied by less than five per cent from the average cost as secured from the accounting records. For short run decisions his estimates made for a particular situation would probably be more useful than the accounting data. Moreover, he would also be in a position to give an estimate of the variation that would take place in cost as a result of a variation in production.

If with the accounting data no effort is made, therefore, to allocate the costs to the various factors on whose account they arise it would be money saved if the accounting procedure were simplified and the accounting staff reduced. As substantiation for this point of view it should be pointed out that one plant visited actually did just this while the study was in progress.

A simplified or less intensive accounting system in the packing industry is further justified by the fact that the processes and methods involved have been established and evaluated for a long period of time.
No substantial modifications of the main processes in use have been undertaken since their establishment. Under such circumstances very little is to be gained by an accounting system set up to yield an estimate of cost each month. What should be more important in the field of costs is a knowledge of the differences between the costs of operation of processes already established and the costs of operation of the new processes which are being tried in the packing industry. Two things are important here particularly in regard to the new processes: (a) the variations in those costs occasioned by variations in production, (b) the differences in the levels of costs between the new and the old processes. Until these things are discovered and the processes established or discarded even more detailed accounting data of the new processes and better analysed data of both the old and new processes would be in order.

The information from such investigations along with other considerations of the quality of the product, relative speed, simplicity and reliability of operation of the new processes in comparison with the old would be useful in promoting the acceptance or rejection of the new processes. A comparison between all processes would hasten into greater use and prominence the more efficient ones.

It is also recommended that the data should be collected for a period following the initiation of the study rather than for a past period. It is felt that much more reliable data could be obtained and more sup-
plementary information of an explanatory nature could be counted upon than is possible from data of a past period. The operation of the plant can be observed and the methods of accumulation of the data can be checked. A knowledge of the way the plant operated which led to particular data for each month and the methods of treatment of the data accumulated are important for a better interpretation and evaluation of the data. A more accurate and comprehensive analysis of the data could be made and more meaningful conclusions drawn. The analysis would be simpler. Much time could have been saved on calculations and in puzzling over reasons for apparent discrepancies. Many of the discrepancies could have been averted or their causes ascertained. Results would have been obtained that were not obtained and results that were obtained would have been more reliable. It is believed that the study could be completed at about the same point of time. As a result it would be for a more recent period and hence should be of considerably greater value.

A case in point pertains to the data on the quantities of materials used. If the data are collected for a current period a check on the lengths of the metering periods could be made and the assistance of the cooperating plant would enable the obtaining of data which corresponds more accurately to the quantities actually used during the accounting periods. In some cases it might be necessary and the cooperation of the
company could no doubt be obtained if the study were planned sufficiently ahead of time, to see that the time periods for all categories of data coincided precisely. If this were not feasible ancillary information would facilitate the compensating adjustment of the data. In other cases items of extraordinary costs were grouped with regular costs in certain months. A separate record of these costs could have been kept by the company if their complicating effect on the analysis had been known. It would be possible to obtain separate records if the current period of operation were studied.
SUMMARY

The study was designed primarily to discover the short-run input-output relationships in the rendering department of a packing plant employing a specific process. Input factors were valued at constant relative prices for purposes of aggregation as well as for comparisons between costs. It was hoped to discover the variations in costs that are dependent upon changes in production.

Emphasis was placed on methodology. This was required because of the nature of the data and because the study was to serve as a guide to similar investigation of other plants and as a basis for further investigation in similar or related fields. The methodology is discussed under the heading of Method of Procedure above. Separate sections are devoted to the treatment of the two departments of rendering and power and the individual items of cost that occur in each department. In the rendering department the method of covariance was used to analyse the labor data. The within-quarter regression equation was obtained as the best estimate of the relationship between the inputs of labor and the outputs of product. In the power department the various components of power were examined by means of multiple regression to determine their relationship to exogenous factors affecting the quantity of each used.
It was found that a large portion of the costs of operating a rendering department are fixed costs. Some costs such as rent and taxes and others are wholly fixed costs. Other costs are entirely variable. The labor employed for operating the department, for example, may all be treated as variable. Some labor, such as that of janitors and night watchmen, is to a large extent fixed, but accounting procedure was such that the cost of the operating labor was kept separate and so could be treated as a variable cost in its entirety.

Still other costs are heterogeneous in that they contain elements of both fixed and variable costs. In the case of two such heterogeneous costs it was not possible to divide them into their fixed and variable elements, nor was it possible to discover the relationship of the variable element to output. These two costs were depreciation and maintenance and repair. It was decided to treat them as fixed costs. In the case of other heterogeneous costs it was possible to obtain estimates of at least some of the elements which should be considered as fixed. Many such elements were functions of factors other than production. The variations in costs due to such exogenous factors were removed from the cost components and treated as fixed costs. One item of cost to the rendering department which contained many such components having both fixed and variable elements of cost was power. The power department was studied, therefore, with the purpose in view of removing the
effect of exogenous factors and determining the relationship of input of power (which is the output of the power department) to the output of the plant.

The summary of the power costs are given in Table 10 page 94. The total cost of power each month is charted on Figure 6 page 72 against the output of the plant gauged by the volume of the dressed weight of slaughter. The proportion of the cost of power used by the rendering department is then obtained for each month. These monthly cost data are charted on Figure 8 page 75b against the outputs of product for each month. The relationship is given by the equation

\[ Y = 1929.067 + 0.8537X \]

also charted on Figure 8.

The costs of operating the rendering department each month are summarized in Table 11 page 95. The total costs of rendering are charted on Figure 9 page 77 against the outputs of the department for each month of 1950-1951. The relationship is given by the equation

\[ Y = 4877.1 + 1.1202X \]

also charted on Figure 9.


Dean, J. P. and James, R. W. The long-run behavior of costs in a chain of shoe stores - a statistical analysis. School of Business, Univ. of Chicago, Univ. of Chicago Press. May 1942.


Greer, Howard C. and Smith, Dudley. Accounting for a meat packing business. Chicago, Ill., Institute of meat packing, the University of Chicago. 1943.


APPENDIX
### Table 9A

Regression equations for the four quarters of 1950-1951.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>First quarter</td>
<td>$Y_{11} = 1012.15 + 0.05435X$</td>
</tr>
<tr>
<td>Second quarter</td>
<td>$Y_{12} = 705.84 + 0.21664X$</td>
</tr>
<tr>
<td>Third quarter</td>
<td>$Y_{13} = 797.42 + 0.20233X$</td>
</tr>
<tr>
<td>Fourth quarter</td>
<td>$Y_{14} = 584.25 + 0.36704X$</td>
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</table>

### Table 9B

Regression equations for the four quarters of 1951-1952.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>First quarter</td>
<td>$Y_{21} = 844.82 + 0.2098X$</td>
</tr>
<tr>
<td>Second quarter</td>
<td>$Y_{22} = 166.68 + 0.5152X$</td>
</tr>
<tr>
<td>Third quarter</td>
<td>$Y_{23} = 894.06 + 1.0471X$</td>
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<tr>
<td>Fourth quarter</td>
<td>$Y_{24} = 326.68 + 0.4954X$</td>
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Table 10

Summary of the adjusted costs of the power department by months for 1950-1951.

<table>
<thead>
<tr>
<th></th>
<th>Variable costs</th>
<th>Fixed costs</th>
<th>Other</th>
<th>Total costs</th>
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</thead>
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<td>Water</td>
<td>Miscellaneous</td>
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<td>5004</td>
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<td>4154</td>
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<td>1812</td>
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Table 11

Summary of the adjusted costs of the rendering department by months for 1950-1951.

<table>
<thead>
<tr>
<th>Month</th>
<th>Power</th>
<th>Labor</th>
<th>Supplies and sundries</th>
<th>Fixed</th>
<th>Total</th>
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<td>7667</td>
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<td>1628</td>
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<td>6885</td>
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Total 44,788 21,930 848 19,536 87,102