PROBABILITY OF TIGHT CRACK DETECTION VIA EDDY-CURRENT INSPECTION

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INTRODUCTION

Among current NDE investigations, one of the important topics is the development of so-called probability-of-detection (POD) models. This activity is important because, given a POD model, one can examine inspection systems quantitatively in terms of flaw detectability. In the area of the eddy-current (EC) testing, there are a few applicable POD models in the literature [1,2]. In this paper, we will report on a generalization of the model constructed by one of the present authors [2]. After the generalization is done, the model becomes applicable to a wider variety of flaws than before.

It is necessary to employ a probabilistic approach in discussing flaw detectabilities because any measured signals, such as probe impedance, should be subject to some degree of fluctuation, caused, e.g., by the presence of noise. Building POD models requires prior knowledge of both expected flaw signals and their variabilities arising from such origins. It is possible to obtain sufficient information through a purely empirical approach. Namely, both flaw signals and their fluctuations may be determined by calibration, using a large number of sample measurements over a variety of flaws. In Ref. [2], however, a different approach was employed. There, theoretical predictions replaced a significant part of determining expected flaw signals and their variabilities. (See Ref. [3] for the theory used there.) In addition, a certain amount of experimental data was used to adjust noise parameters and an overall calibration constant. It was demonstrated in [2] that such a combined theoretical and experimental approach, after being implemented into a computer simulation, can actually save us from a large number of calibration measurements. The combined approach works well as long as an adequate theory is available, and has a clear advantage over the totally empirical method in its cost-effectiveness.

When dealing with fatigue cracks, the advantage of the combined approach is particularly pronounced for the various reasons: First, a purely empirical method will cost significantly in this case because it is hard to obtain many tight-crack samples of known crack sizes. Second, a
theory being appropriate for tight cracks is available [4]. Third, noise measurements can be done relatively inexpensively with a small number of slot measurements because noise parameters are practically independent of flaw types. And, after all, fatigue cracks constitute an important class of flaws for the eddy current testing. It hence is highly desirable to have a computer POD model which simulates fatigue-crack detection. It should be pointed out that tight cracks were excluded in the earlier POD model because of a technical reason [2].

The objective of this work is therefore to generalize the POD model of Ref. [2] to the extent that it can predict probabilities of fatigue-crack detection, in addition to those of open-slot detection. For this purpose, a tight-crack theory was implemented into the POD model. The extent of the modification was kept minimal so that the fatigue-crack result can be compared directly with the open-slot result.

MODEL INSPECTION SYSTEM

Let us first recapitulate the basic configuration of the model inspection system [2]. The model system is basically a half-space problem. Namely, a metal specimen is occupying the half space below a flat surface, on which is a surface-breaking flaw.

As mentioned in Introduction, it is the type of flaw that distinguishes the present work from the earlier one. In Ref. [2], several rectangular slots with finite openings were studied. Four flaws were actually used, and their lengths (depths) were respectively 0.25 mm (0.12 mm), 0.5 mm (0.25 mm), 0.75 mm (0.37 mm), and 1.0 mm (0.5 mm). The slot widths were 10% of the slot lengths. In contrast, this work is devoted to tightly closed cracks, i.e., cracks with infinitesimally small widths. To make a one-to-one correspondence, these cracks were assumed to have the same rectangular shapes and sizes, except for the infinitesimal widths.

Other system components were chosen to be identical to those used in Ref. [2]. For instance, the sample surface was scanned by an EC probe via a parallel X-Y scan with a finite line spacing. The probe was assumed to be a cylindrical, air-core coil, placed parallel to the surface. Also, the sample metal was aluminum, the coil diameter was 1.8 mm, and the operation frequency was 1.7 MHz.

THEORY OF TIGHT CRACKS

For the finite-slot case, the boundary element method (BEM) was used to obtain electromagnetic fields on the specimen surface [2,4]. From these fields, the impedance signal $\Delta Z$ was evaluated via Auld's reciprocity formula [5]

$$\Delta Z = \frac{1}{i} \oint_S d\overline{S} \cdot (\overline{E} \times \overline{H}' - \overline{E}' \times \overline{H}).$$

where the integral is over a surface $S$ enclosing the slot, and where $\overline{E}$ and $\overline{H}$ (respectively $\overline{E}'$ and $\overline{H}'$) are electric and magnetic fields in the absence (presence) of a flaw.

The formulation thus developed for finite slots [4] is not directly applicable to tight-crack problems. It was in fact observed that the numerical code became unstable when the width of a slot was chosen too small. Below, we will explain the origin of this instability first, and then present a remedy for the problem later.
To study the source of the numerical instability, let us consider a cross section of a crack with a finite opening (Fig. 1). We will show that a certain component of the electric field $\mathbf{E}$ inside the crack volume will diverge in the tight-crack limit: This statement can be verified by integrating $\mathbf{E}$ along the contour $(C_1+C_2+C_3)$ illustrated in Fig. 1, which encloses a crack cross section. Using Maxwell's equation and Stokes' formula, we see that

$$\int_{C_1} d\mathbf{x} \cdot \mathbf{E} + \int_{C_3} d\mathbf{x} \cdot \mathbf{E} = i\omega \mu_0 \int_{S_0} d\mathbf{S} \cdot \mathbf{H}. \quad (2)$$

Fig. 1. Illustrations of a flaw and of the integration contours and surfaces appearing in Eqs. (1) and (2): (A) is a cross sectional view of the flaw, and (B) illustrates the surfaces enclosing the flaw.

Here, $S_0$ is the cross sectional area of the crack, and $C_3$ is the part of the closed contour, bridging over the mouth of the crack. In the tight-crack limit, the r.h.s. of Eq. (2) will vanish because $S_0$ becomes infinitesimally small, and because, intuitively, the magnetic field $\mathbf{H}$ will not have a $\delta$-function-like singularity. Since the first term of the l.h.s. of Eq. (2) does not vanish in the limit, one must conclude that the second integral in the l.h.s. should remain finite in the limit. In order that the integral over the infinitesimal path $C_3$ may remain finite, the integrand, i.e. the component of $\mathbf{E}$ perpendicular to the crack face, must diverge along the path $C_3$. This singularity may cause a problem because it appears in Eq. (1) explicitly. To see this clearly, let us separate the closed surface $S$ into $S_1+S_2$, where $S_1$ is the part of the surface covering the mouth of the crack. Then, it is clear that the aforesaid component of $\mathbf{E}$, which becomes singular in the tight-crack limit, is contained by the $S_1$ integral. Notice that the formula (1) itself is still well-defined because the singularity is integrable. The problem, however, is that explicit use of the singular $\mathbf{E}$ component in the numerical procedure may cause instability.
We will show next that the instability problem can be avoided by eliminating the singular integrand from Eq. (1). This can be accomplished with the help of Bowler's potential method [3,6]: Consider the integral over $S_2$ introduced above. The integrand contains field variables on the crack faces. When the crack is closed tightly, the two sides of the crack approach each other to form a single surface. Let $S_c$ denote the limit surface of the two sides. Then, in the limit, Eq. (1) reads

$$
\Delta Z = \frac{1}{f^2} \left[ \int_{S_c} d\overline{S} \cdot (\overline{E}' \times \overline{H}) + \int_{S_c} d\overline{S} \cdot (\text{disc} \overline{E}' \times \overline{H}) \right].
$$

(3)

where \(\text{disc} \overline{E}'\) denotes the discontinuity of $\overline{E}'$ across $S_c$. In Eq. (3), only the divergent integrand was retained in the infinitesimal $S_1$ integral. Similarly, in the $S_1$ integral, we kept only $\text{disc} \overline{E}'$ because all the other discontinuities should vanish. Furthermore, it was pointed out [3,6] that the tangential components of $\text{disc} \overline{E}'$ can be written as tangential derivatives of a scalar potential $\phi$,

$$
\text{disc} \overline{E}_t = -\overline{\nabla}_t \phi.
$$

(4)

Physically, $\phi$ is the potential gap across $S_c$, and thus equal to the first term in the l.h.s. of Eq. (2). In addition, the boundary conditions imposed on $\phi$ (for the case of a surface-breaking tight crack) was also formulated [4]: It should vanish (i.e. $\phi = 0$) on the edge of the crack, while its normal derivative should vanish (i.e. $\delta_n \phi = 0$) at the mouth of the crack. We now use Eq. (4) in Eq. (3), and perform an integration by parts in the $S_1$ integral, retaining the surface term correctly as the boundary conditions of $\phi$ dictate. One then finds immediately that, thanks to Eq. (2) and its relationship with the potential $\phi$, the dangerous $S_1$ integral in (3) is cancelled exactly by the very surface term which just emerged after the integration by parts. The impedance formula (1) therefore reads [4]

$$
\Delta Z = \frac{1}{f^2} \int_{S_c} d\overline{S} \cdot \overline{E}' \phi = \frac{\sigma}{f^2} \int_{S_c} d\overline{S} \cdot \overline{E} \phi.
$$

(5)

where the last equality comes from Maxwell's equation. Evidently, the expression (5) has no instability problem.

The potential $\phi$ was obtained by using the formulas developed by one of the authors for the uniform-field problem [4]. The only change necessary here is that $\overline{E}$ and $\overline{H}$ should be replaced by the Dodd-Deeds solution [7]. This procedure is valid within the so-called "Born" approximation, where the magnetic field on the sample surface is assumed unaltered by the existence of a tight crack.

**PROBABILITY OF DETECTION**

To evaluate POD, one of the authors developed a software package [2]. Here, the earlier POD package was adapted for calculating the probability of tight-crack detection. We will describe the necessary modification: Let us consider the probe impedance $Z$, in the presence of a flaw. By definition, $Z$ is the sum of the background impedance $Z_b$ and the flaw signal $Z_f$. In Ref. [2], a parallel X-Y scan was considered, and the scan process was simulated on a computer to estimate the variability of
measured $Z_1$ values. There, the quantity $Z_1$ becomes a stochastic variable because there is a finite chance of missing the flaw during the scan, where individual scan lines are separated from each other by a finite spacing. The code developed for the simulation yields a probability distribution function (PDF) $f_0$, which quantifies the variability of $Z_1$. In addition, one should also consider the fluctuation of $Z_0$ caused by the presence of noise. Let another PDF ($f_n$) describe its variation. In [2], $f_n$ was actually determined experimentally. Then, the PDF for $Z_n$, to be denoted by $f_n$, is given by the convolution of the two PDF's, $f_0$ and $f_n$. Notice that, in Ref. [2], $f_n$ was equated approximately to $f_0$. This approximation was adequate there because the flaw signals were larger than the noise itself. If, however, flaw signals are much smaller than the noise, $f_n$ will be dominated by $f_n$ instead. This happened to be the case, in the present work, when the smallest and the second smallest cracks were considered. Several examples of the PDF's thus obtained are illustrated in Fig. 2.

The above function $f$, was implemented into the existing code, which then evaluates PODs and probabilities of false alarm (PFA) as functions of threshold values. The results are summarized in the form of relative operating characteristic (ROC) curves, and illustrated in Fig. 3. The figure should be compared with Fig. 6 of Ref. [2]. Qualitatively, the comparison confirms the natural expectation that the tight-crack detection is more difficult than that of open slots. Actually, the derived ROC curves not only provide the qualitative confirmation, but also quantitatively determine, in terms of POD, to what degree the fatigue-crack detectability is worse than the open-crack detection.

![Fig. 2. Probability density functions: (A) is for the 1 [mm] crack, and (B) for the 0.5 [mm] crack. The distribution (1) is the noise distribution in the absence of a flaw, and (2) is the signal distribution obtained from the simulated scan. The curve (3) is the convolution of (1) and (2).](image)
Fig. 3. The predicted relative operating characteristics: The curves (1), (2), (3), and (4) correspond, respectively, to 1 [mm], 0.75 [mm], 0.5 [mm], and 0.25 [mm] cracks.

CONCLUSION

The work reported here shows that the demonstrated ability [2] of replacing a large number of eddy current measurements by a computer simulation in assessing flaw detectability can be successfully extended to fatigue-crack detections. The generalized POD model was developed by implementing an adequate tight-crack theory [4] into the existing POD codes. The model predicts the operating characteristics given in Fig. 3. The result gives a quantitative assessment, in terms of POD, about the degree of difficulty the fatigue-crack detection imposes, compared with the volumetric-flaw detection.

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