INTRODUCTION

Polar backscattering from a fiber-reinforced composite which contains regions of porosity, has been investigated in a number of papers [1]-[7]. Several scattering mechanisms appear to contribute to the backscattered signal. Roughly speaking the components of the backscattered signal can be attributed to effects of finite beam width, to the structuring of the material and to the existence of porosity.

It has been shown experimentally, [3]-[6], that polar backscatter from a multi-ply composite shows an amplitude dependence on the azimuthal angle which is characterized by a pattern of peaks and valleys. The peaks occur when the propagation vector of the incident pulse is normal to the direction of the fibers in one of the plies of the multi-ply layup. A theoretical model which accounts for this effect was proposed in Refs.[4] and [7]. Since the wavelengths at which the effect occurs is large as compared to the cross-sectional dimension of a fiber, the scattering action of single fibers was discounted in the discussion of Refs.[4] and [7]. It was, however, noted that a cross-sectional area of a fiber reinforced composite tends to show that the fibers are not uniformly spaced. There appears to be regions where the fibers are closely packed, while in other areas there is greater spacing. In Ref.[4] it was postulated that the deviations from relatively uniform fiber spacing act as scattering centers, which produce the steep variations in backscatter with azimuthal angle. Since it may be expected that the mechanical properties of the material in these irregular regions are close to those of the surrounding material, the Born approximation was used to investigate the polar backscattering from these inhomogeneities.

One effect, which has not yet been considered in the context of polar backscattering, concerns the influence of reflections from the bottom surface of a specimen. Fiber-reinforced composites display a great deal of attenuation of propagating disturbances due to the structuring of the material, due to porosity and due to the constitutive behavior of the fiber and matrix materials. Hence in some composites the ultrasonic wave motion may not have a significant amplitude when it reaches the bottom face of a composite specimen. The work reported in
Refs. [4] and [7], which ignores bottom-face reflections then applies directly. On the other hand, in other cases, particularly when the specimen is thin and/or the attenuation is not that large, the insonification of the bottom face will produce a reflection, which may contain useful information. Hence, in this paper the backscatter produced by the bottom surface of a specimen is discussed in some detail. Since, as follows from Refs. [4] and [7], the "peaks and valleys" effect due to inhomogeneities in the structuring is additive, it is not necessary to consider it in the present work. It can be added if this should be desirable. The emphasis in the present paper is, therefore, focussed on the backscatter from the upper and lower faces of the specimen, and on backscatter from porosity.

It is assumed in this study that the wavelength is sufficiently larger than the ply thickness. Hence, scattering by individual fibers and by the layered structure can be ignored. Hence, the composite is considered as a homogeneous but anisotropic layer.

The analysis presented in this paper requires the synthesis of several separate components. The composite has to be characterized as an anisotropic solid, the effective constants have to be computed, and the relations between phase velocities, propagation vectors, vectors defining the particle motion, as well as the propagation directions of the energy fluxes must be computed in terms of the effective elastic constants. Next the reflection and transmission coefficients at the water-solid interface must be obtained. Here the appearance of certain critical angles imposes severe limitations on the nature of the wave motion that can generated in the composite. The presence of both an upper and a lower solid-water interface generates multiple reflections, which must be considered. Finally the backscatter from the porosity must be analyzed. Here we first consider a single flat pore, which is characterized as a penny-shaped crack. Next the total effect of a distribution of pores is obtained by integration around the central ray of the incident beam.

The calculations associated with the analysis of the backscattered field, as described above, are lengthy. None of the details can be included in this brief presentation.

The geometry is shown in Fig. 1. A layer of the composite of thickness h is submerged in a water bath. The position of the transducer is defined by a polar angle $\theta_w$.

**Equivalent Formulas**

For a transducer system, Auld [8] has derived a steady-state reciprocal relation which can be applied to flaw detection and characterization. For backscattering (pulse-echo system) this relation gives the following expression for the change of received electrical signal due to the presence of an imperfection:

$$\delta \Gamma = \frac{i\omega}{4P} \int_S (\sigma_{kl}^{in} u_k^{in} - \sigma_{kl}^{in} u_k^{in}) n_k dA .$$

(1)

Here S is an arbitrary surface which surrounds the imperfection, $n_k$ is the normal of the surface S ($n$ is defined positive inward), and P defines the power of the transducer. Also

$\sigma_{kl}^{in}, u_k^{in}$ correspond to the incident field

$\sigma_{kl}^{in}, u_k^{in}$ correspond to the perturbed field
For backscattering by a crack, Eq. (1) reduces to

$$\delta \Gamma = \frac{i \omega}{4 \rho} \int_A \sigma_{k3} u_k^\infty \Delta u_k \, dA$$

(2)

where \( \Delta u_k \) is the crack-opening displacement, and the + sign defines the insonified crack face. From the top surface of the specimen we obtain

$$\delta \Gamma_t = \frac{i \omega}{4 \rho} \int_{\Omega_t} (\sigma_{k3} u_k^\infty - \sigma_{k3} u_k) \, d\Omega$$

(3)

where \( \Omega_t \) is the relevant area of interjection. Similarly for the bottom surface we have

$$\delta \Gamma_b = \frac{i \omega}{4 \rho} \int_{\Omega_b} (\sigma_{33} u_3 - \sigma_{33} u_3) \, d\Omega$$

(4)

Here \( \sigma_{k3}, u_k^\infty \) corresponds to field transmitted into solid.

WAVE MOTION IN THE EFFECTIVE TRANSVERSELY ISOTROPIC MATERIAL

We assume that the mechanical behavior of the fiber reinforced composite may be represented by a transversely isotropic solid, whose axis of symmetry is in the \( x_3 \)-direction. The origin of the coordinate system is located in the top (insonified) plane of the layer, as shown in Fig. 1. The material behaves in an isotropic manner in the \( x_1 x_2 \)-plane.

Let us consider a plane wave of the form

$$u = A \, d \exp (ikp \cdot x), \quad k = \sqrt{\frac{\omega}{c}}$$

(5a,b)
An interesting feature of wave propagation in an anisotropic solid is that the direction of the energy flux vector, which defines the direction of energy flow, generally does not coincide with the direction of the phase propagation. The energy flux vector is defined by

\[ \langle F_j \rangle = -\frac{1}{T} \int_{t_0}^{t_0+T} \text{Re}(\sigma_{ij})\text{Re}(v_i)\,dt, \quad (6) \]

where \( T = 2\pi/\omega \) is the period and \( v_i = -i\omega_\alpha \) is the particle velocity. By the use of Eq.(5), \( \langle F_j \rangle \) can be expressed as

\[ \langle F_j \rangle = -\frac{\Delta A^2}{2c} \omega^2 \text{Re}(\mathbf{C}_{\alpha\mu\nu}^{*} \mathbf{F}_\alpha \mathbf{F}_\nu), \quad (7) \]

where the asterisk indicates a complex conjugate, and \( \mathbf{C}_{\alpha\mu\nu} \) are the elastic constants of the anisotropic solid.

In the sequel we have sometimes used the following two-dimensional (plane strain) stress-strain relation

\[
\begin{pmatrix}
\sigma_{11} \\
\sigma_{33} \\
\sigma_{31}
\end{pmatrix} =
\begin{pmatrix}
C_{11} & C_{13} & 0 \\
C_{13} & C_{33} & 0 \\
0 & 0 & 2C_{55}
\end{pmatrix}
\begin{pmatrix}
\epsilon_{11} \\
\epsilon_{33} \\
\epsilon_{31}
\end{pmatrix}, \quad (8)
\]

TRANSMISSION ACROSS THE WATER-SOLID INTERFACE

The reflection and transmission of a plane time-harmonic wave at an interface defined by \( x_3 = 0 \) of water and a transversely isotropic solid has been considered in Ref.[4]. The waves transmitted into the solid may be expressed as

\[ u = T_{\alpha} \mathbf{d}_{\alpha} \exp(i\mathbf{p}_\alpha \cdot \mathbf{x}), \quad \alpha = L, TV, TH \quad (9) \]

Here \( T_{\alpha} \) are the transmission coefficients. The transmission of wave motion \( u \) into the solid has some interesting features at critical values of the angle of incidence \( \theta \) (see Fig. 1). As \( \theta \) increases beyond a first critical value, the propagation vector of the transmitted pseudo longitudinal wave becomes parallel to the water-solid interface, and the amplitude decays exponentially with depth into the solid. Similar effects happen at higher values of \( \theta \) for the transmitted pseudo transverse waves. Hence at higher values of \( \theta \) relatively little wave motion will penetrate to any depth into the composite.

Experimental measurements of the transmission of wave motion into unidirectionally reinforced composites of the kind considered here are often for the transmission of energy flux through the thickness of a plate. Corresponding expressions for coefficients of energy transmission can be derived by the use of Eqs.(7)-(9). We define

\[ T_{\alpha}^E = \frac{\text{transmitted energy flux for } \alpha \text{ wave}}{\text{incident energy flux}}, \quad \alpha = L, TV, TH \quad (10) \]

The direction of energy flux in the solid is defined by the unit vector

\[ \mathbf{\alpha}_\alpha = \frac{\langle F_\alpha \rangle}{||F_\alpha||}, \quad \alpha = L, TV, TH \quad (11) \]

Let us consider an area \( \Delta A \) on the interface. The corresponding normal cross-sectional area of the incident wave (propagation vector \( \mathbf{p}_\alpha^{\text{in}} \)) in the water is \( \Delta A_{\alpha}^{\text{in}} \), while the corresponding normal cross-sectional area in the solid is \( \Delta A_{\alpha} = \Delta A s_3^{\alpha} \). We may then write Eq.(10) as

\[ T_{\alpha}^E = \frac{\Delta A}{\Delta A_{\alpha}} \mathbf{c}_{\alpha}^T \mathbf{s}_3^{\alpha} \exp \left( \frac{\mathbf{C}_{\alpha\mu\nu}}{\lambda} \mathbf{d}_{\alpha} \mathbf{p}_\mu^{\text{in}} \right) \Re \left( \mathbf{C}_{\alpha\mu\nu} \mathbf{d}_{\alpha}^{\text{in}} \mathbf{p}_\mu^{\text{in}} \mathbf{s}_j^{\alpha} \right) \quad (12) \]
where $T$ are the regular transmission coefficients defined in Eq.(9) and computed in Ref.[4].

**MULTIPLE REFLECTIONS IN THE LAYER**

Since the angle of incidence is taken to be beyond the critical angle, only pseudo-transverse waves, with propagation vector $\mathbf{p}^T$ and displacement vector $\mathbf{d}^T$, will penetrate to any depth in the layer. This simplifies the analysis of multiple reflections considerably, since only one kind of wave motion needs to be considered, and mode conversion can be neglected. It can then be shown that the relevant stress components in the solid can be expressed as

$$\sigma_{13} = AC_{55} \frac{ik}{\Gamma} T_{13} \mathbf{E}_R \mathbf{E} \mathbf{x} \mathbf{Z}_{-}$$

$$\sigma_{33} = AC_{55} \frac{ik}{\Gamma} T_{33} \mathbf{E}_R \mathbf{E} \mathbf{x} \mathbf{Z}_{+}$$

where $A$ is an amplitude function which can include the distribution across a possible beam width. The remainder of Eqs.(13)-(14) represent the plane wave part of the displacement. The constant $C_{55}$ is a material constant, while

$$Z_{\pm} = \frac{1 \pm \frac{T}{R} H \mathbf{x}^{-1}}{1 - (\frac{T}{R})^2 H},$$

where

$$H = \exp(2ik_{p} \frac{T}{T} h) \exp[-2(\frac{1}{2} N \sigma + \gamma)h/p_{3}]$$

$$X = \exp(2ik_{p} \frac{T}{T} x_{3}) \exp[-2(\frac{1}{2} N \sigma + \gamma)x_{3}/p_{3}]$$

Also

$$\Gamma_{13} = (p_{1}d_{3}^T + p_{3}d_{1}^T) \frac{k_{T}}{k_{w}} + i(d_{1}^T/p_{3}) \frac{1}{2} N \sigma + \gamma) / k_{w}$$

$$\Gamma_{33} = (C_{13} p_{T}^T d_{1}^T + C_{33} d_{3}^T) (k_{T}/k_{w}) / C_{55}$$

$$+ i(d_{3}^T/p_{3}) \frac{1}{2} N \sigma + \gamma)(C_{33}/C_{55}) / k_{w}$$

$$E_{R} = \exp(ik_{w} P_{in} \cdot \mathbf{R})$$

$$E_{x} = \exp(ik_{p} P_{x} \cdot \mathbf{x})$$

$$\varepsilon_{x} = \exp[-(\frac{1}{2} N \sigma + \gamma)x_{3}/p_{3}]$$

The terms in Eqs.(16)-(17) include damping due both to scattering and material behavior. Generally there would be quantities $\sigma \pm \sigma$, which would be the scattering cross sections for forward scattering and backscatter from porosity. Here they are taken equal to $\sigma$; also, $N$ is the number of pores per unit volume, and $\gamma$ is the attenuation coefficient of the bulk composite.

**BACKSCATTER FROM A CRACK**

The signal strength of the backscattered field follows from Eq.(2). When the wavelength of the incident wave is much longer than the crack radius, the crack opening displacement for a crack located in a plane normal to the $x_{3}$-axis may be written as
Fig. 2 Ray geometries for transmission across the water-solid interface.

\[ \Delta u_j = \sigma_{j3} F_j(x_1, x_2; \omega), \quad (23) \]

where \( \sigma_{j3} \) is the (uniform) effective stress acting on the crack faces.

Equation (2) then simplifies to

\[ i\omega \delta \Gamma = - \frac{i \omega}{4P} (\sigma^{\hat{\Delta}}_{13} \sigma^{\hat{\Delta}}_{13} V_1 + \sigma^{\hat{\Delta}}_{33} \sigma^{\hat{\Delta}}_{33} V_3) \]

\[ (24) \]

where \( V_j \) is the crack-opening volume per unit applied stress \( \sigma^{\hat{\Delta}}_{3j} \). For low frequency (long wavelength), the crack opening volume can be estimated.

The contributions from a number of cracks can be added to obtain the total signal, and this summation can subsequently be replaced by an integration, as discussed in Refs. [4] and [7]. These integrations are carried out in the immediate vicinity of the central ray of the propagating beam. It is, therefore, necessary to calculate ray lengths near the central ray of the beam.

Figure 2 shows the geometry in a Cartesian coordinate system. Ray POQ is the central ray of energy propagation. The lengths \( R \) and \( r \) are the ray lengths in the water and in the solid. It is necessary to express the length \( PO_2 Q_2 \), that is \( R + r \), when the point \( Q_2 \) is slightly displaced from the point \( Q \) by a length \( (x_1^2 + x_2^2)^{\frac{h}{2}} \). The analysis is divided into two steps. The first step is to express the length \( PO_1(-R) \) and \( Q_1 O_1(-r_1) \) in terms of \( R \) and \( r \), using the geometry shown in Fig. 2b. Next \( PO_2(-R) \) and \( Q_2 O_2(-r) \) are expressed as functions of \( R_1 \) and \( r_1 \), using the geometry of Fig. 2c.

The total backscattered signal consists of three components: backscatter from the porosity. The result is

\[ \delta \Gamma_t = - \frac{i \omega}{4P} \int \left[ T_{13}^2 I(x_3) \epsilon_2^2 Z_{-1}^2 V_1 + T_{33}^2 I(x_3) \epsilon_2^2 Z_{+3}^2 V_3 \right] dx_3 \]
where all quantities have been defined earlier, except for $I(x_3)$ which is defined as

$$I(x_3) = \int E^2 F^2 dx_1 dx_2$$

This integral can be further simplified by using the Fresnel integral.

**BOTTOM-SURFACE EFFECT**

The results have been normalized by the backscatter from the top surface, which may be calculated as

$$|\delta \Gamma_o| = \frac{\bar{R}\pi}{k_w}$$

Numerical calculations were carried out for

$$C_{11} = 99.45 \text{ GPa}, \quad C_{13} = 3.39 \text{ GPa}, \quad C_{33} = 10.79 \text{ GPa}$$

$$C_{55} = 6.16 \text{ GPa}, \quad \rho_s = 1.57 \text{ g/cm}^3$$

The term $Na/2$ was approximated as

$$Na/2 = (Na^2)(k_w^2)\frac{a^4}{4},$$

while $\gamma$ was taken as

$$\gamma = 5 \times 10^{-4} \text{ mm}^{-1}$$

The geometrical parameters are

- Slab thickness $h = 0$ to 5.0 (mm),
- Sensor distance $R = 10$ (mm),
- Flaw radius $a = 0.07$ (mm),
- Incident angle $\theta_w = 20$ (deg.),
- Frequency $f = 2$ (MHz)

Results for $|\delta \Gamma/\delta \Gamma_o|$ are shown in Figs. 3a,b and for low $Na^2$, the results show strong oscillations when $|\delta \Gamma/\delta \Gamma_o|$ is plotted versus $h$. As the volume density of the porosity increases, i.e., as $Na^2$ becomes larger, the oscillatory behavior disappears, because the reflection from the back surface disappears. Larger values of $Na^2$ correspond to more damping, and a signal is attenuated when it has traversed the distance $h$ twice.

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Backscatter signal versus plate thickness and pore density at a frequency of 2 MHz.

REFERENCES


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