ISOLATING FRACTURE-INDUCED ANISOTROPY
FROM BACKGROUND ANISOTROPY

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INTRODUCTION

A material may appear homogeneous and anisotropic when the scale of its fabric is smaller than the wavelengths that measure it. These structures can result from a variety of causes such as the thin layering in composites or stress resultant oriented microcracking. In the case of a stressed composite, the resultant anisotropy is a complex combination of the two component anisotropies. Hood and Schoenberg [1] showed theoretically that the effects of vertical fractures can be separated from a background that is already anisotropic. We propose an ultrasonic experiment designed to verify their theories. The vertical fracturing is scaled and simulated by the plate method of Hsu and Schoenberg [2] and superposed in a transversely isotropic (TI) material. The resultant material is presumed to have orthorhombic symmetry. Contact measurements are made to determine the elastic moduli. The results give the additional compliance that the fractures add to the system as well as the elastic properties of the original background material as if it had no vertical fractures.

BACKGROUND

Thin layering alone in composites often generates TI symmetry. The elastic properties of a TI medium do not vary with azimuth. However, azimuthal variations in composites have been widely detected. A common cause for the azimuthal variations in material properties is from stress-induced microcracking. Realistic material characterization models must include all the significant constituent anisotropies of the fracture systems and the backgrounds in which these systems are embedded.
Hsu and Schoenberg [2] showed that a system of stacked, bonded plates is equivalent to a homogeneous TI system in the long wavelength limit. Vertical fractures can be generated in a similar fashion. A stack of bonded isotropic plates is cut into thin vertical wafers and bonded back together to simulate a vertically fractured, transversely isotropic (VFTI) medium. In the long wavelength limit (approximately ten fractures per wavelength) a VFTI material appears homogeneous and anisotropic with orthorhombic symmetry. The goal of this paper is to experimentally verify the method presented by Hood and Schoenberg [1] that estimates the amount of compliance that fracturing adds to a system that is already anisotropic.

EXPERIMENTAL DESIGN

The forward problem is to create a VFTI material. This is accomplished in two stages. The first stage is to build the TI background. For long wavelengths,\( \lambda \geq 10h = h_g + h_e \) (\( h_g \) is the thickness of the glass plates and \( h_e \) is the thickness of the epoxy bonds), this inhomogeneous conglomerate of isotropic components will appear homogeneous and anisotropic with TI symmetry (see for example [4]). The TI material is produced by stacking and adhering thin glass plates with a UV cured epoxy bonding agent (see Fig. 1). The epoxy between each plate adds additional compliance to the system and acts as a set of horizontal planar fractures. Schoenberg modeled fractures as linear slip interfaces [3]. Because the epoxy bond is much more compliant and thinner than the glass plates, the extra tangential and normal compliance that these “horizontal fractures” add to the layered system (the epoxy) can be modeled with the parameters

\[
E_{Tsf} = \frac{h_e \mu_g}{\mu_e} \quad \text{and} \quad E_{Nsf} = h_e \frac{\mu_g}{\lambda_e + 2\mu_e},
\]

where \( E_{Tsf} \) and \( E_{Nsf} \) are the dimensionless compliances tangential and normal to the horizontal fracture planes, respectively. There is an ambiguity in determining the elastic moduli (\( \mu_e \) and \( \lambda_e \)) and the thickness (\( h_e \)) of the epoxy bonds: both can not be uniquely determined. Using this fracture model, only the excess compliance that the epoxy adds to the system can be determined from measured moduli. In order to estimate the thickness of the epoxy bonds, values for the epoxy’s elastic properties must be assumed. Similarly, if \( \mu_e \) and \( \lambda_e \) are desired, one must estimate the epoxy thickness \( h_e \) in the TI system.

![Fig. 1. TI: thinly layered glass-epoxy composite.](image-url)
Stage two is the addition of vertical fractures to the TI material (see Fig. 2). The vertical fractures are simulated by cutting the TI stack of bonded plates into thin wafers \(h_w\) is the wafer thickness, such that \(h_w < \lambda\) to satisfy the long wavelength criteria) and then rebonding the wafers using the same epoxy bonding agent \(h_{ef}\) is the epoxy thickness between wafers). The extra compliance that the vertical fractures add is modeled by parameters \(E_{Tef}\) and \(E_{Nef}\) where

\[
E_{Tef} = h_{ef} \frac{\mu_g}{\mu_e} \quad \text{and} \quad E_{Nef} = h_{ef} \frac{\mu_g}{\lambda_e + 2\mu_e}.
\]

The system of cut, stacked, bonded plates is a VFTI material and appears homogeneous and anisotropic with orthorhombic symmetry when measured at long wavelengths. The anisotropic moduli of this orthorhombic homogeneous VFTI material are complicated functions of all the constituent moduli: the \(\lambda_g, \mu_g\) of the isotropic plates and the additional compliances, \(E_{Tbf}, E_{Nhf}, E_{Tef}, E_{Nef}\), added by the various fracture sets. Therefore the fracture properties and the background elastic moduli can be obtained as functions of measured homogeneous orthorhombic moduli of the VFTI material.

Since the elastic moduli of each constituent component of the VFTI system can be measured during stages of the sample preparation, this provide a means of determining how well our method of separating constituent anisotropies works. Prior to stacking and bonding, the elastic constants \(\mu_g, \lambda_g\) and \(\mu_e, \lambda_e\) of the isotropic glass plates and of the epoxy, respectively, were determined from \(P\)-wave and \(S\)-wave velocity measurements. We measured \(h_e\) and calculated the expected amount of \(E_{Tbf}\) and \(E_{Nhf}\) the epoxy provided the TI stack. The elastic moduli were determined from contact ultrasonic travel time measurements made coincident with the symmetry planes of the TI stack. From these moduli the \(\mu_g, \lambda_g\) for the glass and the excess compliance \(E_{Tbf}\) and \(E_{Nhf}\) that the horizontal fractures add to the system were inverted for using the “laws of mixing” [5]. For a stack of \(n\) plates with \(H\), the total thickness of the stacked bonded plates, \(n h_g + (n - 1) h_e = H\). We measured \(h_g\) and \(H\) and counted \(n\) and from these values \(h_g\) was obtained. The elastic properties \(\lambda_g, \mu_g, E_{Tbf}, E_{Nhf}\) were determined directly from the \(c_{ij}^{ef}\). For \(h_e = 8.38 \times 10^{-4}\) cm and \(h_g = 0.4\) cm, \(\lambda_e\) and \(\mu_e\) were obtained from \(E_{Tbf}\) and \(E_{Nhf}\) (see Table 2). The \(\mu_g\) was determined from \(c_{66}^{ef}\), and the dimensionless tangential compliance \(E_{Tbf}\) was.
Table 1. Bulk glass and epoxy measurements with densities $\rho_g = 2.51 \text{ gm cm}^{-3}$ and $\rho_e = 1.08 \text{ gm cm}^{-3}$.

<table>
<thead>
<tr>
<th>Velocity $\times 10^5 \text{ cm sec}^{-1}$</th>
<th>Elastic constants $\times 10^{11} \text{ dyn cm}^{-2}$</th>
<th>forward calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{S_g} = 3.37$</td>
<td>$\mu_g = 2.85$</td>
<td>$E_{T_h} = 0.0242$</td>
</tr>
<tr>
<td>$V_{P_g} = 5.37$</td>
<td>$\lambda_g = 1.54$</td>
<td>$E_{N_{h f}} = 0.00441$</td>
</tr>
<tr>
<td>$V_{S_e} = 0.956$</td>
<td>$\mu_e = 0.987$</td>
<td></td>
</tr>
<tr>
<td>$V_{P_e} = 2.24$</td>
<td>$\lambda_e = 0.345$</td>
<td></td>
</tr>
</tbody>
</table>

Determined from $c_{44_TI}$:

$$\mu_g = c_{66_TI}, \quad E_{T_h} = \frac{\mu_g}{c_{44_TI}} - 1 \rightarrow \mu_e = \frac{\mu_e\mu_g}{E_{N_{h f}}}.$$  \hspace{1cm} (3)

From $c_{11_TI}$ with $c_{33_TI}$ the normal compliance $E_{N_{h f}}$ and the parameter $\lambda_g$ were determined from

$$E_{N_{h f}} = \frac{\mu_g - c_{33_TI} + \sqrt{c_{11_TI}c_{33_TI} - 2c_{33_TI}\mu_g + \mu_g^2}}{2c_{33_TI}} \rightarrow \lambda_e = \frac{\mu_e\mu_g}{E_{N_{h f}}} - 2\mu_e, \quad \lambda_g = \frac{\mu_g \left(1 + 2E_{N_{h f}}\right) \left(c_{11_TI} - 2\mu_g\right)}{\mu_g + 4E_{N_{h f}}\mu_g - E_{N_{h f}}c_{11_TI}}.$$  \hspace{1cm} (4)

(5)

These intermediate calculations for the moduli of the constituents help us determine how advantageous it is to apply this method of separation to fractured composites.

THE TI BACKGROUND

From the constitutive relation, the $6 \times 6$ matrix of elastic moduli for the background isotropic material is $\mathbf{C}_b$ where

$$c_{11_b} = c_{22_b} = c_{33_b} = \lambda_g + 2\mu_g, \quad c_{44_b} = c_{55_b} = c_{66_b} = \mu_g,$$  \hspace{1cm} (6)

and $c_{12_b} = c_{13_b} = c_{23_b} = c_{11_b} - 2c_{44_b}$ and all other $c_{ij_b} = 0$. The inverse of the stiffness matrix is the $6 \times 6$ compliance matrix $\mathbf{S}_b$ which has entries

$$s_{11_b} = s_{22_b} = s_{33_b} = \frac{\lambda_g + \mu_g}{D}, \quad s_{12_b} = s_{13_b} = s_{23_b} = \frac{-\lambda_g}{2D}, \quad s_{44_b} = s_{55_b} = s_{66_b} = \frac{1}{\mu_g}.$$  \hspace{1cm} (7)

Where $D = \mu_g \left(3\lambda_g + 2\mu_g\right)$, $s_{12_b} = s_{13_b} = s_{23_b} = s_{11_b} - 1/2s_{44_b}$, and all other $s_{ij_b} = 0$.

Horizontal fracturing is incorporated into the isotropic background using the method of Nichols et al. [6]

$$\mathbf{S}_{T I} = \mathbf{S}_b + \mathbf{F}_{h f}$$  \hspace{1cm} (8)

where a set of axisymmetric horizontal fractures is represented by the $6 \times 6$ matrix $\mathbf{F}_{h f}$ where

$$F_{33_{h f}} = \frac{E_{N_{h f}}}{\mu_g}, \quad F_{44_{h f}} = F_{55_{h f}} = \frac{E_{T_h}}{\mu_g}.$$  \hspace{1cm} (9)
Table 2. TI data with density $\rho_{TI} = 2.48 \text{ g/cm}^3$.

<table>
<thead>
<tr>
<th>measured velocities $\times 10^5 \text{ cm/ sec}$</th>
<th>associated moduli $\times 10^{11} \text{ dyn cm}^2$</th>
<th>from TI stack data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{11, TI} = 5.64$</td>
<td>$c_{11, TI} = 7.89$</td>
<td>$\mu_e = 2.83 \times 10^{11} \text{ dyn cm}^2$</td>
</tr>
<tr>
<td>$V_{33, TI} = 5.28$</td>
<td>$c_{33, TI} = 6.92$</td>
<td>$\lambda_e = 2.91 \times 10^{11} \text{ dyn cm}^2$</td>
</tr>
<tr>
<td>$V_{44, TI} = 2.93$</td>
<td>$c_{44, TI} = 2.13$</td>
<td>$E_{T_{x_f}} = 0.331$</td>
</tr>
<tr>
<td>$V_{66, TI} = 3.38$</td>
<td>$c_{66, TI} = 2.83$</td>
<td>$E_{N_{y_f}} = 0.055$</td>
</tr>
</tbody>
</table>

and all other $F_{ij,k} = 0$. The $E_{N_{y_f}}$ and $E_{T_{x_f}}$ are the dimensionless compliances normal and tangential to the horizontal fracture planes, respectively. Therefore the $6 \times 6$ compliance matrix $C_{TI}$ representing an isotropic material with horizontal fracturing has terms

\[
s_{11, TI} = s_{22, TI} = s_{11}, \quad s_{33, TI} = s_{33} + F_{33, f}, \quad s_{44, TI} = s_{55, TI} = s_{44} + F_{44, f}, \quad s_{66, TI} = s_{44},
\]

(10)

where $s_{12, TI} = s_{11, TI} - \frac{1}{2}s_{66, TI}$ and all other $s_{ij, TI} = 0$. The inverse of $C_{TI}$ gives the $6 \times 6$ stiffness matrix $\mathcal{S}_{TI}$ which is term by term identical to the stiffness matrix presented by Schoenberg [3] for a horizontally fractured material.

The form of Eq. (10) shows that the resulting system has TI symmetry with $x_3$ as the axis of symmetry. There are only four independent parameters in this TI system: $\gamma$ and $\mu_g$ of the background and the additional compliance that the bonding agent adds to the system in the form of $E_{N_{y_f}}$ and $E_{T_{x_f}}$. The $E_{T_{x_f}}$ and $E_{N_{y_f}}$ calculated in Table 1 differ from those inverted from the TI data in Table 2. The results show that the excess compliance which the epoxy is expected to add to the system (Table 1) is less than that obtained from the TI stack (Table 2).

ORTHORHOMBIC SYMMETRY BY ADDING VERTICAL FRACTURES

In the long wavelength limit, the cut stacked plates (sample two) will appear homogeneous and anisotropic with orthorhombic symmetry. The cut stacked plates can be represented by the $6 \times 6$ matrix

\[
\mathcal{S}_{VFI} = \mathcal{S}_{TI} + \mathcal{F}_{vnf},
\]

(11)

where $\mathcal{S}_{TI}$ is given in Eq. (10).

The vertical bonding thickness is not necessarily equal to the thickness of the horizontal bonding ($h_{v_f} \neq h_{e}$, see Fig. 2) so, generally, $E_{N_{y_f}} \neq E_{N_{y}}$ and $E_{T_{x_f}} \neq E_{T_{x}}$. A set of axisymmetric vertical fractures can be represented by $\mathcal{F}_{vnf}$ where

\[
F_{11, v_f} = \frac{E_{N_{y_f}}}{\mu_g}, \quad F_{55, v_f} = \frac{E_{T_{x_f}}}{\mu_g}, \quad F_{66, v_f} = \frac{E_{T_{x_f}}}{\mu_g},
\]

(12)

and all other $F_{ij, v_f} = 0$ and where $E_{N_{y_f}}$ and $E_{T_{x_f}}$ are the dimensionless compliances normal and tangential to the vertical fracture planes, respectively. Using Eq. (12) to represent the vertical fractures and Eq. (10) to represent the layered background, the VFTI elastic moduli for sample two are calculated using Eq. (11). The form of the
resulting elastic moduli indicates that the VFTI system has orthorhombic symmetry. General orthorhombic media have nine independent parameters. In $\mathbf{C}_{\text{VFTI}}$, however, $s_{12} = s_{13} = s_{23}$ and $s_{44} + s_{66} = 2(s_{22} - s_{12})$. These relationships reduce the number of independent parameters in this system to six. These relationships give us a means for determining how closely the subject medium conforms to the specified symmetries imposed by the fracture and background model.

SEPARATING THE VERTICAL FRACTURING FROM THE TI BACKGROUND

One must begin with moduli determined from the measured data in the form

$$\mathbf{C}_{\text{VFTI}} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}. \quad (13)$$

If the moduli indicate that the material is orthorhombic and a VFTI model is appropriate ($s_{13} = s_{23}$ [1]) then a method following procedures similar to Hood and Schoenberg [1] can be used to extract the effects of vertical fracturing from an anisotropic background. Shear and longitudinal contact velocity measurements made in the three symmetry planes ($x_1 - x_2$, $x_2 - x_3$, $x_1 - x_3$) are sufficient to provide all six diagonal elements of $\mathbf{C}_{\text{VFTI}}$. For this particular VFTI system, these six independent parameters are sufficient to characterize the material properties of all system components; all nine different elements of the orthorhombic $\mathbf{C}_{\text{VFTI}}$ are not required. Therefore, all six parameters, $\lambda_g, \mu_g, E_{N_{(f)}}$, $E_{T_{(f)}}$, $E_{N_{(f)}}$, $E_{T_{(f)}}$, can be unambiguously defined as functions of the experimentally determined elastic moduli.

From the measured shear velocities we obtained parameters $E_{T_{(f)}}$, $E_{T_{(f)}}$, and $\mu_g$:

$$\mu_g = \frac{c_{44}c_{55}c_{66}}{c_{55}c_{66} + c_{44}c_{55} - c_{44}c_{66}}, \quad E_{T_{(f)}} = \frac{\mu_g}{c_{66}} - 1; \quad E_{T_{(f)}} = \frac{\mu_g}{c_{44}} - 1. \quad (14)$$

From measured longitudinal velocities we determined $E_{N_{(f)}}$, $E_{N_{(f)}}$, and $\lambda_g$:

$$E_{N_{(f)}} = 2\mu_g (\lambda_g + \mu_g) - c_{11} (\lambda_g + 2\mu_g) + c_{11}c_{33}\lambda_g^2 + 4\mu_g^2 (\lambda_g + \mu_g)^2 \frac{4c_{11}}{c_{11}(\lambda_g + \mu_g)}; \quad (15)$$

$$E_{N_{(f)}} = 2\mu_g (\lambda_g + \mu_g) - c_{33} (\lambda_g + 2\mu_g) + c_{11}c_{33}\lambda_g^2 + 4\mu_g^2 (\lambda_g + \mu_g)^2 \frac{4c_{33}}{c_{33}(\lambda_g + \mu_g)}; \quad (16)$$

$$\lambda_g = \frac{\sqrt{(A4/A1) + (\mu_g^2A3^2/A1^2)} - (\mu_1A3/A1)}{2} \quad (17)$$

where

$$A1 = (c_{11} - 4c_{22})^2 - 2c_{11}c_{33} - 8c_{22}c_{33} + c_{33}^2 + 16c_{11}\mu_1 - 64c_{22}\mu_1 + 16c_{33}\mu_1 + 48\mu_1^2,$$

$$A2 = \sqrt{c_{11}c_{22} - 2c_{11}\mu_1 + \mu_1^2},$$

$$A3 = 8(2\mu_1 - c_{22})(c_{11} + c_{33}) + 32c_{22}^2 - 128c_{22}\mu_1 + 112\mu_1^2 - 4A2(c_{11} + 4c_{22} + c_{33} - 8\mu_1),$$

$$A4 = -16\mu_1^2(c_{11}c_{22} + 4c_{22}^2 - c_{22}c_{33} - 2\mu_1 [c_{11} + 8c_{22} - c_{33} - 8\mu_1] + 4A2[c_{22} - 2\mu_1])$$.
The off-diagonal elements of $Q_{ij}$, $c_{12}$, $c_{13}$, and $c_{23}$, remain to be determined. Although the diagonal elements alone supply a complete description of this medium, the off-diagonal elements make available constraints on the complexity of the background symmetry and the properties of the vertical fractures. For example, the assumption that the material consists of a TI background with parallel vertical fractures can be tested by examining whether constraint $s_{13} = s_{23}$ holds; the TI background is the result of axisymmetric horizontal fractures in an isotropic matrix only if $s_{12} = s_{13}$ holds; the horizontal and vertical fractures have identical properties only if $s_{11} = s_{33}$. The relationships $2(s_{22} - s_{11}) = s_{44} + s_{66} - s_{55}$, $s_{12}$ or $s_{13} = s_{23}$, and $s_{44} = s_{66}$, will further constrain the complexity of the background and fracture properties.

Table 3. VFTI data with density $\rho_{\text{VFTI}} = 2.46$ $\text{g/cm}^3$.

<table>
<thead>
<tr>
<th>Velocity $V_{ij}$</th>
<th>Moduli $c_{ij}$</th>
<th>from VFTI data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{11}=5.05$</td>
<td>$c_{11}=6.26$</td>
<td>$\mu_g = 2.55 \times 10^{11} \text{dyne/cm}^2$</td>
</tr>
<tr>
<td>$V_{22}=5.56$</td>
<td>$c_{22}=7.59$</td>
<td>$\lambda_g = 2.92 \times 10^{11} \text{dyne/cm}^2$</td>
</tr>
<tr>
<td>$V_{33}=5.17$</td>
<td>$c_{33}=6.56$</td>
<td>$E_{Tk} = 0.214$</td>
</tr>
<tr>
<td>$V_{44}=2.91$</td>
<td>$c_{44}=2.08$</td>
<td>$E_{N_{K}} = 0.040$</td>
</tr>
<tr>
<td>$V_{55}=2.58$</td>
<td>$c_{55}=1.63$</td>
<td>$E_{T_{K}} = 0.330$</td>
</tr>
<tr>
<td>$V_{66}=2.78$</td>
<td>$c_{66}=1.90$</td>
<td>$E_{N_{K}} = 0.077$</td>
</tr>
</tbody>
</table>

DISCUSSION

The foregoing separation method derives explicit formulae for the fracture compliances and the background elastic moduli. The ability to separate constituent anisotropies will allow us to use more complex models to represent additional anisotropy generated by stress in homogeneous material and composites. Our preliminary results indicate that the material properties of the individual components obtained from the VFTI sample data are more similar to those determined from the TI stack data rather than agreeing with the properties of the individual bulk glass and epoxy. This suggests that this particular epoxy is more compliant while in a thin bond than in bulk form.

Determination of $c_{ij}$ where $i \neq j$ requires a measurement oblique to the coordinate planes. Mignogna [7] presented an experimental procedure designed to determine all the elastic constants for a homogeneous anisotropic medium by using oblique angle measurements and performed a physical model study [8] on a sample of cubic spinel which showed that the off-diagonal elastic moduli could be successfully determined using this method. Once the complete elastic stiffness matrix $C_{ij}$ is determined, the compliance matrix $S_{ij}$ can be obtained. Although the diagonal elements alone are sufficient in resolving all the unknowns, the additional off-diagonal elements offer constraining relationships for background and fracture properties.
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REFERENCES