On the application of operations research techniques in agricultural economics

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ON THE APPLICATION OF OPERATIONS RESEARCH TECHNIQUES IN AGRICULTURAL ECONOMICS

by

Heinrich Kopetz

A Thesis Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of MASTER OF SCIENCE

Major Subject: Economics

Signatures have been redacted for privacy

Iowa State University Of Science and Technology Ames, Iowa

1966
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INTRODUCTION

Over recent years mathematicians have been developing a number of new procedures which make it possible to solve problems of management and economic decision-making faster and more accurately than ever before. These procedures, customarily summarized under the heading operations research techniques, are being used in agricultural economics to an increasing extent and already are so numerous and diverse that the newcomer to the field is hopelessly confused as to the particular significance of any single method. Moreover, as the toolkit of the scientist continues to expand, the gap between the researcher and the supposed beneficiary, the farmer, has not yet been bridged, but rather seems to widen. It is the goal of this study to attempt an evaluation of these tools and a summary of their applications to agriculture, with the purpose of facilitating judgement as to which techniques are appropriate for a particular research endeavor.

Operations research can be described as the combined application of different branches of science to solve ad hoc problems, which are formulated in terms of mathematical models. Although it is often emphasized (60) that operations research is mainly concerned with the ad hoc formulation of mathematical models, there are a large number of already formulated models available. Only the applications of these latter models will be the subject of the discussion.
In the chapter on linear programming a historical sketch of
of the cautious acceptance of the technique by agricultural
economists in the five years after the first publication of the
method will be given, followed by a mathematical statement of
the problem. Applications of linear programming to various
areas of agricultural economics such as feed mixing problems,
farm management applications, market studies and studies of
interregional patterns of resource use and product speciali-
ization then will be presented. The section on nonlinear pro-
gramming will deal with a few of the more important modifica-
tions of the original model including an indication of their
significance for economic analysis and of the mathematical
difficulties involved. The ranking of dynamic programming in
this family of programming techniques and its significance for
economic decision making will be outlined in the following
chapter. Various other operations research tools, some of them
having been introduced into the economics of agriculture fairly
recently, will be presented in the subsequent parts. In the
concluding section, a comparison of the various techniques and
a discussion of their possible implications on teaching,
research, and extension in agriculture will complete the pres-
entation.
LINEAR PROGRAMMING

Before describing the mathematical structure of the linear programming problem, the acceptance of the method in agricultural economics during the first years after its inception will be outlined.

Historical Sketch

The history of linear programming in agricultural economics is almost as old as the method itself. In 1951, shortly after the development of the linear programming technique by Dantzig, Koopmans, and associates (127), Waugh (313) made use of it in determining the least expensive combination of feeds for dairy cows. Two years later, Fisher and Schrubens (75) extended his example to two or more products under alternative price structures, and presented the effects of price changes on the combination of the minimum cost ratio in simple graphs. In 1952 the first solution of a linear programming problem was obtained on a computer and Swanson (197), three years later was the first to prescribe the use of a computer in solving a minimum cost feed mixing problem. He gave information about the time required to run his problem on a computer, and thus the way to the commercial application of linear programming in the milling industry was nearly open.

Besides an article in Koopman's book (127) dealing with the choice of a crop rotation plan, the first discussion of
linear programming in relation to farm problems was given by King (123). At the outset of his article he compares linear programming or activity analysis, as it is often called, with older techniques that could be used to determine the optimum profit combination of two resources for producing one output. He then cites three examples of applications of activity analysis to farm management problems and tries, finally, to draw certain implications of this technique to agricultural economics.

The decisive impetus for the fast acceptance of linear programming in the economics of agriculture was due to two excellent presentations, in which the nature and scope of the method was communicated to a wide audience in an easily understandable manner. One of these publications is Ready's (97) article "Simplified Presentation and Logical Aspects of Linear Programming Technique" published in 1954, and the second, Boles' (19) contribution on "Linear Programming and Farm Management Analysis" published in 1955.

In 1955 a few articles appeared that pointed to new problem areas into which the technique was to intrude in subsequent years. Fox and Taeuber (77) contributed an application of linear programming to the study of spatial equilibrium problems, French (78) discussed the impact of activity analysis on agricultural marketing and Babbar et al. (7) examined possible variations in input coefficients and the resulting output estimates in order to cope with planning problems under technical
uncertainty. This last contribution led to stochastic pro-
gramming. Apart from these explorations into new problem
fields, in the year 1955 some further improvements of the tech-
nique in its usefulness for farm management problems were set
forward. Swanson (188) provided a valuable contribution in
showing how activity analysis could be used for planning farm
operations where the crop and livestock enterprises were inte-
grated. A preliminary evaluation of linear programming was
given at this time by McCorkle (154). He compared the tech-
nique with conventional marginal analysis and budgetary proce-
dures and made suggestions about possible applications and fur-
ther needed research. Thus, by the end of 1955 linear pro-
gramming had already come to the fore in the thoughts of many
agricultural economists.

Mathematical Statement of the Models

The maximization or minimization of a linear function,
called the objective function, subject to a set of linear con-
straints is referred to as linear programming. The general
linear programming problem is to find the vector \((x_1, x_2, \ldots, x_n)\) which maximizes the objective function

\[
\sum_{j=1}^{n} c_j x_j
\]  

subject to the linear constraints
\[ x_j \geq 0 \quad j = 1, 2, \ldots, n \]  

\[ \sum_{j=1}^{n} a_{ij} x_j \{ \leq, =, \geq \} b_i \quad i = 1, 2, \ldots, m \]  

where \( a_{ij}, b_i \) and \( c_j \) are given constants and \( m < n \) so that the systems of equations forming the constraints is underdetermined. Assuming that Equation 2 is multiplied by \(-1\), where necessary to make all \( b_i \geq 0 \) and that the slack variables are included, Equation 3 can be written as

\[ \sum_{j=1}^{n} a_{ij} x_j = b_i \quad i = 1, 2, \ldots, m \]  

In terms of an agricultural activity analysis problem, \( m \) is the number of different resources available, \( n \) is the number of real and disposal activities, \( a_{ij} \) is the input coefficient telling how many units of resource \( i \) are required to produce one unit of activity \( j \), \( b_i \) is the number of units of resource \( i \) available, \( c_j \) is the net price per unit of activity \( j \) and \( x_j \) is the level of activity \( j \).

In order to express the problem more compactly, let \( x = (x_1, x_2, \ldots, x_n) \) be the column vector of activity levels, \( b = (b_1, b_2, \ldots, b_m) \) be the column vector of resource restrictions, \( c' = (c_1, c_2, \ldots, c_n) \) be the row vector of net returns and \( A = (a_{ij}) \) be the \( m \times n \) matrix of input coefficients. The problem may then be expressed as
\[
\begin{align*}
\text{max} & \quad c'x \\
\text{subject to} & \quad Ax = b \quad \text{and} \quad x \geq 0
\end{align*}
\]

A slight modification of this general linear programming problem is the parametric linear programming problem, whose discussion will be included in this chapter. This model includes parametric variations in the coefficients of the objective function, and in its dual version, parametric variations in the coefficients of the resource vector; sometimes a variation of the elements in the \( A \)-matrix is considered. With a parametric objective function the problem can be expressed as follows:

\[
\begin{align*}
\text{max} & \quad c' (\Theta) x \\
\text{subject to} & \quad Ax = b (\Theta), \quad \text{and} \quad x \geq 0 \\
\text{where} & \quad c' (\Theta) = c' + \Theta d'
\end{align*}
\]

and \( d' \) is a \( n \)-tuple row vector and \( \Theta \) is a scalar.

With a parametric resource vector it can be written as

\[
\begin{align*}
\text{max} & \quad c' x \\
\text{subject to} & \quad Ax = b (\Theta) \quad \text{and} \quad x \geq 0 \\
\text{where} & \quad b (\Theta) = b + b_0
\end{align*}
\]

and \( b_0 \) is a \( n \)-tuple column vector and \( \Theta \) is a scalar. If \( \Theta = 0 \), the parametric problem is reduced to the general case.

Some of the applications to be discussed in this chapter make use of a third kind of linear model, the so-called transportation model. The transportation problem can be stated as follows (81, p. 193):
"A homogeneous product is to be shipped in the amounts $a_1, a_2, \ldots, a_m$, respectively, from each of $m$ shipping origins and received in amounts $b_1, b_2, \ldots, b_n$, respectively by each of $n$ shipping destinations. The cost of shipping a unit amount from the $i$th origin to the $j$th destination is $c_{ij}$ and is known for all combinations $(i, j)$. The problem is to determine the amounts $x_{ij}$ to be shipped over all routes $(i, j)$ so as to minimize the total cost of transportation."

The mathematical statement is as follows:

\[
\begin{align*}
\text{minimize} & \quad \sum_{j=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \\
\text{subject to} & \quad \sum_{j=1}^{n} x_{ij} = a_i \quad i = 1, 2, \ldots, m \\
& \quad \sum_{i=1}^{m} x_{ij} = b_j \quad j = 1, 2, \ldots, n
\end{align*}
\]

All programming problems which differ from the above presented models by additional restrictions, nonlinear relations or sequential formulations will be classified as nonlinear programming problems to be discussed in the following chapters.

In the subsequent paragraphs various applications of the discussed models will be presented.

Feed Mixing Problems

Up to the present time the majority of the agricultural applications of linear programming has been in the feed mill industry rather than at the farm management level. It is not surprising that linear programming is being used commercially
in this field to such a large extent. The numerous complex feed-mixes produced, the often changing prices of ingredients and the considerable size of the enterprises in the mill industry explain the more frequent employment of the technique in this branch of agriculture. Furthermore, no serious conceptual difficulties are involved in calculating the minimum cost of a feed after the technological requirements are formulated. In addition to the already mentioned publications, Katzmann (118), Reddy et al. (96), Futton and Allison (111) and Futton and McAlexander (112), (152) contributed to this field by examining various operational and nutritional aspects of the linear programming formulation. A good introduction to this topic for the feed mill operator not familiar with the technique was published by Seagraves (173). Van de Panne (205) used a stochastic programming model to derive the minimum cost feed in a situation where the nutritive content of the inputs varies considerably. By now the problem can be regarded as nearly satisfactorily solved and (e.g. in California) the calculations of leastcost feed ratios by computers is already offered as a service to farmers.

Farm Management Applications

In 1956 linear programming was firmly recognized as a valuable addition to the tools available to farm management workers. By many it was looked upon as a formalized extension of budgeting capable of solving much more complex problems than
the latter. Several problems encountered in the early attempts of applying activity analysis to farm management questions were overcome and the task of developing optimal farming plans under limited resources could be handled successfully. Two detailed presentations of the state of art of the method at this time with respect to farming are given by Heady and Gilson (102) and Swanson (187). The first article not only contains a comprehensive description of the various steps involved in applying the technique to a farm but also arrives at an important conclusion about the significance of optimal plans (102, p. 712):

"There is not an optimum set of livestock enterprises or management practices (i.e., level of grain feeding) for all farms, but that recommendations should differ between farms depending on their capital and labor situations, as well as on their ability to stand risks." Swanson (187) elaborates on the importance of determining the sensitivity of an optimum plan to price changes. Puterbaugh et al. (163) presents the first thorough analysis of a linear programming solution by explaining the post optimum calculations necessary to determine the stability and feasibility conditions of an optimum solution.

Later on, this problem was to receive much attention.

In subsequent years the literature dealing with linear programming in agriculture can be divided into those publications devoted to the applications of the original model to various other farm problems and into those modifying the original model in order to increase its information content.
A few studies belonging to the first category will be mentioned subsequently. Bishop (17) put forward an economic evaluation of farm-nonfarm resource allocation of a small part-time farm in North Carolina. The growing proportion of part-time farmers in regions with small farms underlines the actuality of this study. The position of beginning farmers is characterized by the limited amount of capital available. Particular problems arising from this situation were analysed by Heady et al. (104). As can be expected, several studies tried to evaluate the implications of the rapid structural and technological changes in agriculture on the decisions of farmers. Examples in point are a publication by Faris and Kadishay (73) on adjustment possibilities of cotton farms in California and a study on the economy of innovations in dairy farming by Barker and Heady (9). Pavellis and Timmons (161) applied the method to a problem of water resource administration and de Benedictis and Timmons (53) developed procedures to appraise inefficiencies in leasing arrangements. It can be seen from this small survey that linear programming proved to be much more versatile and powerful than the budgeting methods with which it competed during the first years of its use in agriculture. The technique enables researchers to analyze problems that could not be studied previously.

After this short digression on various applications of the standard model the discussion will be continued with the already mentioned article of Puterbaugh et al. (163). Their efforts to
gain more information from an optimum solution can be interpreted as an answer to early critics of linear programming which were directed against the assumption of single and fixed input-output coefficients and prices. Recalling the wide yield variabilities, the frequent price variations, the technological changes, and many other sources of uncertainty in data these criticisms deserve to be taken seriously. In addition, examination of different solutions showed that very often trifling changes in coefficients, prices, or resource levels can cause considerable alterations in the optimum solution. As a result, at an early stage of linear programming two approaches were forged to mitigate the lack of certainty in the data. The one is stochastic programming which allows the coefficients to be random variates of a particular frequency distribution. Although this technique was outlined as early as 1955 in the mentioned article by Babbar et al. (7) and found notable further development in contributions by Tintner (201), van Noesenke (206), and Sengupta (175) it is still rather seldom used in agriculture, partly as a consequence of its fairly complicated mathematical structure.

A more popular acceptance in the agricultural field was given to the second means designed to cope with data uncertainty; these techniques are known as sensitivity analysis and parametric programming. Sensitivity analysis concerns itself with changes in the optimum solution due to changes in the data. In the context of this thesis the term will be used for investiga-
tions as to how much the price of an activity, the level of one of the resources, or a particular input coefficient may vary without changing the optimum solution. When discussing the range of allowable variations of any of these coefficients, constancy of all other data is always assumed. A detailed description of the computations involved is given in the article by Puterbaugh et al. (163). The calculation of these valid ranges is also necessary for a meaningful interpretation of the marginal costs (shadow prices) of the limited resources. It might be added that the constancy of the marginal costs over a certain range is implied by the linear nature of the assumed production function as opposed to the differentiable and continuous production function in common marginal analysis. The relation between common marginal analysis and linear programming is pointed out by Dorfman, et al. (61, p. 133) in the following way: "It would be misleading to contrast linear-programming with marginal analysis in general. Linear programming is marginal analysis, appropriately tailored to the case of a finite number of activities. Traditional marginal analysis is tailored to the case of a differentiable production function."

A recently developed computer program by IBM, the so-called Rangex report, permits the performance of all post optimum calculations on the computer so that the calculations by hand, as outlined in (163), become redundant. The post optimum analysis reveals much of essential economic information and is currently added to nearly any linear programming study at the
farm level. It permits distinction between those activities in the optimum solution sensitive to slightest changes in data as opposed to other ones showing a wide stability range. This knowledge is a decisive asset in interpreting optimum solutions with regard to uncertainty. Among the numerous studies using sensitivity analysis, articles by Faris and Kadiashay (73), by Faris and McPherson (74), and by Petit and Dean (162) should be mentioned.

In order to further refine the analysis of optimum solutions, continuous changes in prices or resource levels were examined in their influence on optimum solutions. This procedure is termed parametric linear programming and can be defined as "the study of the behavior of solutions when the coefficients are allowed to vary" (81, p. 123). Most of these investigations deal with variations in elements of the b-vector (resources), of the c-vector (prices), and only a few examples with variations of elements in the A-matrix (input coefficients) are being discussed. A good explanation of these procedures is given in Heady and Caniler (99, Chap. 7, 8, 16).

In the first presentation of this method in 1956, Caniler (25) studied the variations of optimum plans under continuous varying capital restrictions and presented the results in graphical form. On the basis of this contribution, extension personal can use the optimum program derived for a typical farm of a given production region to impart individual recommendations to farmers in accordance with their capital situation.
This technique, often referred to as variable resource programming, has proved to be useful for studies of adjustment possibilities of farms, as was illustrated, in contributions by Love and Ready (138) and by Bolton (20).

Similar to the variable resource programming is the variable price programming, first presented by Candler (26). A good example of an application of this technique is given in a study by McPherson and Faris (158) in which the effect of different wage-price ratios on the optimum organization of dairy farms in North Carolina was examined. The results are presented in price maps. In the following years many other publications appeared using this method for a wide range of problems. Krenz et al. (129) computed profit maximizing combinations of enterprises for dairy farms and then, derived milk supply functions for these farms. Weeks (214) calculated the best composition of dairy feed ratios for farms in the Pacific Northwest.

A few studies dealing with particular facets of farm management problems remain to be mentioned. In many linear programming studies the outcome is dominated by the level of fixed resources. Edwards (65) criticized this overwhelming influence of assumed fixed resources and defined an asset as fixed if its return per unit, (its marginal value product), is below its acquisition price but above its salvage price. A detailed discussion of this point is also contained in a book by Weinschenck (216). In an interesting article by Miller and Nauheim (146) the relationship between profit maximizing and cost minimizing
models of a Kansas wheat farm is studied and the predictive value of these models is compared.

Most of the publications cited have only research value, dealing in many cases with hypothetical farms and do not present the application of linear programming to solve management problems of actual existing farms. The extent to which the techniques are actually used by farmers, either via commercial management agencies or extension services of universities, is outlined in a study by Swanson (191).

Marketing Studies

Most of the linear programming applications pertinent to marketing problems will be discussed in the next sections on interregional competition. In this paragraph only three studies, dealing with management and marketing problems of processing plants, shall be specified. Snyder and French (182) made use of the method in evaluating the adjustment problems of a fluid milk plant. Since this plant faced downward sloping demand curves, profit maximization required market research as well as an intensive study of the possible activities and restrictions within the firm. In a similar study, Sorenson (184) examined the plant operations in relation to market conditions of a cherry processing plant in Michigan. Using the transportation model, King and Logan (184) studied the best size and location of packing plants in California by considering simul-
taneously the cost of shipping raw materials, and processing and shipping the final products.

Spatial Research

The last few years witnessed an explosion in studies of spatial equilibrium analysis and of interregional production and consumption patterns. This is not too surprising, as a rigorous analysis of these problems was not feasible before the development of the modern analytical tools. The basic content and goal of spatial research is to find an answer to the question as to how an economy should be organized spatially, in order to maximize the returns on given human and natural resources, considering demand restrictions, transportation cost and various regional factors. In short, a spatial model may be broadly defined as "any theoretical construct having space as one component" (174, p. 1372).

Spatial equilibrium analysis is largely based on two well established economic theories. Trade theory, as early developed by the classical writers Ricardo and Mill in their discussions about the comparative advantage of different production regions, provides one origin. Despite some modern contributions by Meaberler and others, trade theory neglects almost completely, the space aspects of economic activity. Logically it requires to be supported by location theory, which emphasizes the role of transportation costs in their influence on production and consumption patterns. Von Thuenen was the first to develop a
formal mathematical model of this type.

The results of almost all spatial research studies reveal that redistribution and reallocation of economic activity would be desirable in order to increase the performance of an economy. In this, most of spatial research has severe policy implications, and thus, as a rule, these studies are closely related to problems in welfare economics.

In more detail, spatial research deals with such questions as the regional location and level of production, the regional consumption of final goods, and the relative and absolute price level between regions. The many different models used in these studies, such as interregional competition models, transportation models, spatial equilibrium models, and plant location models can be broadly classified into two groups, the standard equilibrium models (SE) and the activity analysis models (AA). The equilibrium models use defined demand and supply functions while the activity analysis models generate their own functions. Yet, these two constructs are not mutually exclusive and the standard transportation model can be classified as either one. Typical examples for the first type model are given by Emke (70) Samuelson (166) and Takayama and Judge (191), contributions exhibiting the use of activity analysis models are presented by Schrader and King (170) and Takayama and Judge (193). A general discussion of spatial economics in agriculture has recently been presented by Seaver (173) and Bawden (11) and Fassler (94). The
first thorough presentation of the transportation model for agricultural economist with regard to the computations involved and its significance for spatial studies was prescribed by Snodgrass and French (181). This article illustrates the transportation model in a clear manner similar to the presentation of the general linear programming procedures in the articles by Heady (95) and Boles (19). It shows the derivation of lowest transportation costs when satisfying fixed consumption needs of a given region from fixed production levels within the same region. In a similar study Stamberger (185) applied the transportation model to determine the best market for North Carolina eggs and the locational advantages of North Carolina eggs marketing agencies compared to competitors in other areas.

A basic study about regional adjustments in the grain production if the US was undertaken by Heady and Fghbert (100). They derived the grain production patterns in the US corresponding to an economic equilibrium, that would prevail unless government programs had interfered into the resource allocation over the last 30 years. How should the regional production be organized to be optimal at a national level? this is the main question to be answered. In this analysis the US is broken down into $10^4$ production regions, each having 3 activities, which finally yields a general linear programming model with a matrix size of $106 \times 310$. The relaxation of some simplifying assumptions in this study is reported in a later publication by the same authors (99). The effect of different soil types was
incorporated into a regional study as described by Whittlesey and Heady (64, 13, 218). Randhawa and Heady (164) illustrated the application of a linear programming model for interregional agricultural planning in India. Whittlesey and Skold (219) devoted a paper to illustrate the interpretation of the dual solution of the above characterizes models, which permits the derivation of equilibrium product prices and imputed values to limiting resources. The optimum allocation of resources for a whole country was analyzed by Blyth and Crothall (18) for the case of New Zealand.

Among the many studies deriving optimum plant locations, plant sizes and shipping routs of various products by using the transportation model the study by King and Logan is mentioned (126).

A new approach to spatial analysis taking time explicitly into account was proposed by Day (50).
NONLINEAR PROGRAMMING

At the beginning of the last chapter the linear programming problems were defined, and all other programming models were simply called nonlinear. It is now time to introduce a clearer distinction. In its original meaning, the term "nonlinear" applies to programming problems in which the objective function or the constraints, or both, contain nonlinear algebraic expressions. Among these strictly nonlinear problems, the class with linear constraints and a nonlinear objective function has been studied most widely. A general problem of this kind can be written as follows:

max or min \[ z = f(x_1, x_2, \ldots, x_n) \]

subject to \[ \sum_{j=1}^{n} a_{ij}x_j = b_i \quad i = 1, 2, \ldots, m \]
\[ x_j \geq 0 \quad j = 1, 2, \ldots, n \]

Even in this simplified case computational procedures are only available for problems where the objective function displays specific properties. Three such examples will be mentioned here. Firstly, consider an objective function which is the sum of \( n \) functions, each of which is a function of only a single variable. It can be written as

\[ z = f(x_1, \ldots, x_n) = f_1(x_1) + f_2(x_2) \ldots + f_n(x_n) \]

Such a function is called separable, in the sense that each of its components depends only on one of the \( x_j \)'s. In this case a
computational solution is possible, and the procedure could be called an approximate method involving separable objective functions. For further detail refer to Hadley (88, Chap. 4).

A second computationally feasible case occurs when the objective function consists of a linear and a quadratic form, so that the problem can be expressed as below:

$$\max \text{ or } \min \quad z = f(x_1, \ldots, x_n) = c' + x' Q x$$

subject to \( Ax = b, \quad \text{and } x \geq 0 \)

where \( Q \) is a \( n \times n \) matrix. This problem is called a quadratic programming problem and will be discussed in more detail.

A third approach, permitting the analysis of a much broader class of problems with nonlinear objective functions and nonlinear constraints, is known as convex programming. It allows the solution of a minimization problem with a convex objective function and a convex feasible region which can be formed by \( m \) convex functions. Mathematically, the problem can be stated as follows:

$$\min \quad z = f(x)$$

subject to \( g_i(x) = 0 \quad i = 1, \ldots, m \)

where \( g_i(x) \), \( g_2(x) \), \ldots, \( g_m(x) \) and \( f(x) \) are differentiable convex functions defined for all \( x \). Since this approach will not be discussed in this chapter, further reference is given to the original contributions by Hartley (91) and Hartley and Rocking (92).

Aside from these strictly nonlinear programming problems, a
class of models will be included where the additional restriction imposed on a general linear programming problem, is that some or all of the variables must take on integer values. It can be written as seen below:

\[
\begin{align*}
\text{min} & \quad z = c^T x \\
\text{subject to} & \quad Ax = b, \quad \text{and} \quad x \geq 0
\end{align*}
\]

some or all elements of \( x \) are integers.

As the fifth and last model in this chapter, a presentation of dynamic linear programming will be given.

General Discussion

The preoccupation with nonlinear programming is nearly as old as the study of its linear counterpart. However, with the advance of more powerful computational techniques, interest in this area has recently begun to grow rapidly, especially outside the field of pure mathematics. This is not too surprising, as it was recognized early that the rigid assumptions of linear programming, in some cases considerably deteriorate the meaningfulness of its results. In applying a linear programming model, the existence of a linear homogenous production function, not allowing for economies or diseconomies of scale, and perfect competition in the factor and product market are always assumed. Experience in agriculture often contradicts these assumptions. It can be seen for instance, that a double increase in inputs frequently does not double the output, that farms face sloping demand curves and that some farms take advantage of price reduc-
tions by buying bulk inputs. All these cases can be classified as problems where either increasing or decreasing returns to scale or outlays are present. Nonlinear programming provides a framework for an exact treatment of these problems, as opposed to the rough approximations necessary to force these relations into a linear model.

The only reason why these techniques are not yet in wider use are the conceptual and computational difficulties they involve. In order to portray these difficulties in an understandable manner, the nonlinear case will be confronted with the two earlier existing optimization techniques: the classical optimization technique based on calculus and the linear programming approach.

The classical optimization techniques, using calculus and Lagrange multipliers, are only amenable to problems where there are either no constraints or only equality constraints, where the variables are continuous functions possessing partial derivatives at least through the second order, and where the feasible region is convex. Due to these assumptions, applications of the technique, even to simple numerical problems, encounter such great difficulties, that these methods can be only classified as of theoretical importance without operational significance. Their computational infeasibility is one of the reasons for the fast acceptance of linear programming.

Linear programming can be described as having the following properties:
(1) The feasible region which is bounded by the linear constraints is a convex set i.e. a line connecting any two points of the feasible set lies entirely in the set. The boundary of this set, formed by the linear constraints, is referred to as the convex hull. The convex region has a finite number of corners, normally called the extreme points or vertices.

(2) The objective function is a straight line in the two-dimensional case, a plane in the three-dimensional case and a hyperplane in the n-dimensional case.

(3) From (1) and (2) it follows that a local optimum will always be a global optimum; and, this optimum must always be found at an extreme point of the convex hull. Hence, the search for an optimum can be restricted to a testing of cornerpoints, while the boundaries between the points and the region inside the hull, can be neglected. (For a proof of the subject matter see e.g. Gass, Chapter 3 and 4 (81).

These properties are the reason for the easy and straight forward way of finding an optimum solution in linear programming. It is only necessary to start at one extreme point and to walk from vertex to vertex, until each adjacent extreme point yields less revenue than the one under consideration.

In the nonlinear programming case, some or all of these properties might be violated, and a simplex-like "walking from corner to corner" might be completely unapplicable. Consider
for instance, the case of a linear objective function with non-linear constraints. Visualizing a two-dimensional problem, the feasible region will consist partly of curved lines instead of straight lines. This can result in a nonconvex feasible region which might be made up of several disconnected parts. This non-convex feasible region might lead to the existence of a local optimum (suboptimum), being different from the global optimum (total optimum). However, any procedure whose test for optimality only consists in checking whether a few steps in any direction will reduce profits, cannot distinguish between a local and a global optimum. Thus, differential calculus, whose second derivative test only concerns a very narrow part of the function, and the simplex method in linear programming, which involves only a check of adjacent vertices, do not necessarily provide the attainment of a global optimum. This is the reason why Baumol (10, p. 137) calls these techniques nearsighted optimization procedures; sometimes they are also termed myopic procedures. In fact, when applied to nonconvex feasible regions, most of the computational techniques, with the exception of dynamic programming, will only yield local optima and thus fail to solve these problems.

A closely related and more widely discussed case is the already mentioned problem with linear constraints and a nonlinear objective function. Rather than being a straight line, plane, or hyperplane, the objective function might be like a hill, a
valley, or any other irregular shape. The shape of the function is of importance insofar as a distinction between convex and concave functions has to be made. A concave function from below is defined as a curve whose linear approximations always under-estimates the real function. The convex case is the opposite. In order to avoid the problem encountered in the previous example with the nonlinear constraints, the objective function must have the right shape, or, as it is often formulated, must fulfill the concavity conditions. These conditions state that the objective function must be concave in a maximization problem, and, vice versa, convex in a minimization problem. Only if these conditions hold, and if the feasible region is convex, can the nearsighted maximization techniques be used to obtain a global optimum.

The economic interpretation of the concavity conditions says that these techniques are only amenable for problems involving diminishing returns, e.g. for situations where the cost per additional unit increases or the productivity per additional unit decreases.

Even when these conditions are fulfilled, these problems still lack the straightforwardness of linear programming as can be seen in a simple example (61, p. 187): Given a firm with linear constraints and a downward sloping demand curve, it can be shown graphically that the objective function is now a curve and the optimum can occur at any point of the boundary region of the convex set. Actually, the optimum might even lie inside the set
if there is more than one variable with a nonlinear objective function.

By now the main difficulties of nonlinear programming can be grasped, and it is seen that the two existing optimization techniques fail to provide solutions to nonlinear programming problems because of one or several reasons:

(a) The La Grange constraint maximization technique fails because the limiting constraints are usually not known in advance, and/or because the continuous function condition might be violated.

(b) The simplex-like procedure of linear programming fails because the optimum of nonlinear problems does not necessarily occur at one of the finite number of vertices, and/or the existence of nonconvex feasible regions gives rise to local optima.

Kuhn and Tucker (130) provided the way out of this dilemma. In their famous paper dealing with necessary and sufficient conditions for optimum solutions to programming problems, they have proved, among other things, that it is possible to form a La Grange expression for a large number of programming cases with the same properties as in the classical case. This generalization of the classical optimization technique forms the basis for further progress in cases in which the constraints may require inequalities, and the variables have to be nonnegative.

After this discussion, it should not be surprising to find
a host of different methods for particular nonlinear programming problems. Most of these deal with the case of a nonlinear objective function and linear constraints. These methods can be roughly grouped into those based on linear approximations, into the gradient methods, and the so called hopping methods (10, p. 142). It is characteristic for nonlinear programming, that no one single general valid method exists that would be comparable to the simplex method in linear programming.

Applications

The interest of agricultural economists in nonlinear programming stems from problems encountered in practical work, Gleave and Seagraves, (82) being among the first to tackle these problems, were concerned that linear programming solutions frequently include more enterprises than would seem reasonable, and theorized the reason for this to be the complete neglect of economies of size in linear programming. Their observation is certainly true, and can be extended to a more general statement.

According to the mathematical structure of the linear programming model, each solution, including the optimum solution, lies at a vertex of the feasible region, thus forming a basis in the n-dimensional vector space. Hence, it must have as many positive variables, i.e. real or slack activities, as there are constraints. Using the chosen notation, this means that there will always be m positive variables. Baumol (10, p. 138) calls
this property of linear programming solutions the basic theorem of linear programming.

As was explained above, in nonlinear programming the optimum solution does not necessarily occur at a corner point, but can be situated at the boundary of the feasible region or in its interior, permitting real activities and slack activities to be positive. Therefore the basic theorem does not hold. However, even in nonlinear programming, an important relationship exists between the number of constraints and the number of nonzero variables in optimum solutions. In nonlinear problems with diminishing returns (or increasing returns) the number of possible variables will tend to be greater (smaller) than the number of constraints. This relationship has important consequences for cases where we approximate nonlinearities by linear programming techniques. It implies that the use of linear programming for situations of increasing returns (decreasing returns), yields too many (not enough) activities in the optimum solution. This is exactly what Ciaever and Seagreaves observed. They concerned themselves mainly with maximization problems under increasing returns, or, as they call them, under economies of size. As examples for economies of size in agriculture, they cite economies due to specialization of labor and management, market economies, and cases where fixed charges or set-up times occur. As was explained in the previous section, these problems involve many local optima, causing nearsighted computational techniques to
fail. The authors present four methods for handling nonlinearities, the first being designed to cope with problems under decreasing returns is a modified simplex procedure, the other three dealing with economies of size cases are based on post-optimal adjustments.

Candler and Manning (30) illustrated the solution for a problem where one or two input coefficients exhibit decreasing average cost (economies of size case). In their numerical example they discuss a case where average labor requirements for dairying is a function of output level, and where machine services can be obtained from 3 different sources, all showing continuous nonlinear cost functions. The solution procedure is based on parametric linear programming with variable input coefficients. The computations are rather involved but permit the attainment of a total optimum. The case of a firm experiencing diminishing returns due to negatively sloped demand curves, is explicitly illustrated in a paper by Yaron and Ready (222). The demand function is assumed to be separable and hence linear approximation can be applied. After having attained the approximative solution, and thus the information about the effective constraints, they reformulated the problem in terms of the classical optimization technique using a La Grange expression, and arrived at an exact solution.

Some further contributions to the problem with linear constraints and a nonlinear objective function, are given by Candler and Musgrave (31) and Edwards (64).
Quadratic Programming

Quadratic programming is a subset of the previously discussed nonlinear programming problems, where the constraints are linear and the objective function consists of linear and quadratic elements. The problem can be stated as below:

\[
\begin{align*}
\text{max or min} & \quad z = f(x_1, x_2, \ldots, x_n) = \sum_{j=1}^{n} c_j x_j \\
& \quad + \sum_{j=1}^{n} \sum_{l=1}^{m} d_{lj} x_l x_j \\
\text{subject to} & \quad \sum_{j=1}^{n} a_{ij} = b_i \quad i = 1, 2, \ldots, m \\
& \quad x_j \geq 0 \quad j = 1, 2, \ldots, n
\end{align*}
\]

or, using vector notation:

\[
\begin{align*}
\text{max or min} & \quad z = c^T x + x^T \mathbf{d} x \\
\text{subject to} & \quad A x = b \\
& \quad x \geq 0
\end{align*}
\]

As in most nonlinear problems, the developed numerical techniques provide an optimum solution, if, and only if, any local optimum is also a global optimum; i.e., if the concavity conditions are fulfilled. Having linear constraints the feasible set is convex and thus fulfills these conditions. The objective function consists of a linear part \(c^T x\) and a quadratic part \(x^T \mathbf{d} x\). The linear part \(c^T x\) can be interpreted as concave or convex and fulfills always the concavity conditions. The quadratic
form expressed by the symmetric n x n matrix P, as x^T P x, will be
concave only if the matrix P is of negative semidefinite or nega-
tive definite form. (A definition is given e.g. in 88, p. 41).

Given an objective function as specified above, the Kuhn-
Tucker constraint qualification is satisfied, and the results of
the Kuhn-Tucker theorem can be applied to define necessary and
sufficient conditions for an optimum (88, Chap. 8).

Based on this reasoning, several algorithms for solving
quadratic programming problems were developed. The methods of
Wolfe (231) and Beale (12) use a slightly modified simplex pro-
cedure, while an algorithm by Theil and van de Panne (198) is
directly derived from the Kuhn-Tucker conditions. Finally
Lemke's method (134) resembles, in some respects, the dual meth-
od of linear programming. A concise but thorough evaluation of
these algorithms is given by Borr (62).

The economic interpretation of solutions in quadratic pro-
gramming is the same as in the linear case. Yet, the economic
use of this technique has proved to be rather limited. Concept-
tually, it provides a framework for considerations of the more
realistic concepts of diminishing marginal rate of substitution
between factors and of diminishing utility, providing that the
data can be fitted to a quadratic function. The use of quad-
tratic programming for investment decisions, where a maximum
utility or a minimum expected loss is desired, was illustrated
by Markowitz (142) in his portfolio selection problem.

In the agriculture literature, several applications of the
technique are reported, dealing both with aggregated problems as well as with microunits. Louwes et al. (137) studied the problem of surplus milk in the Netherlands by formulating a quadratic objective function, \( p_1x_1 \), in which the quantity \( x_1 \) was expressed as a function of the price \( p_1 \). After finding effective constraints, they imputed, by use of La Grange formulation, the cost of certain social considerations which dominate the milk market policy in Holland. Takayama and Judge (195) indicated how spatial equilibrium models of the Yake-Samuelson type can be converted to a quadratic programming framework. The cost minimization of a bean processing plant was explored by Reed and Boles (165). McFarquhar (156) reports the use of a Markowitz portfolio formulation to a farm problem in England.

Integer Programming

It is fairly obvious that in many practical problems, linear programming solutions do not make much sense unless the variables are integers. In general, when the number of units per activity is large (e.g. 553.6 pigs to be included in an optimum solution), the problem can be easily overcome by rounding the value to the nearest integer (in this example 554). Yet, this procedure fails completely if the decision variable is supposed to assume the value 0 or 1, e.g. if it symbolizes the decision whether a grain harvester shall be bought or not. For situations of this kind involving indivisibilities, integer programming, or discrete programming as it is sometimes called,
provides a reasonable solution. In addition, integer programming exhibits great potential for solving nonlinear pro-
gramming problems. In fact, the main reason for the large in-
terest in integer programming in the last few years, is attribu-
table to the possible solution of nonlinear problems by this method. Dantzig (19) summarized the significance of integer
programming for a variety of nonlinear cases. Before mentioning
a few of these formulations, pertinent to agriculture, the
structure of the integer problem shall be presented. Integer
programming can be characterized as a linear programming problem
with the additional constraint that some or all of the variables
must be integers. The problem can be written as below:

\[ \text{max or min } z = c^T x \]

subject to \[ Ax = b \]

\[ x \geq 0 \]

\[ x_j \text{ an integer, } j \in J_4 \]

If \( J_4 \) contains all \( j \) it is an all integer problem, if \( J_4 \) con-
tains some \( j \) it is a mixed integer problem, and if \( J_4 \) is empty,
it is a linear programming problem.

The development of the first algorithm for solving all-in-
teger problems by Comory (85) in 1958, can be regarded as some-
what of a breakthrough. Two years later, he put forward a modi-
fication of the original procedure, making solutions of mixed in-
teger problems possible (84). Intuitively described, Comory's
method eliminates parts of the feasible region which do not con-
tain any points with integer coordinates by the addition of new efficient constraints, so called cutting planes, which are being applied to the original linear programming problem. These points, with integer coordinates, are termed integer lattice points. By adding new constraints, the boundary of the feasible region is finally being reduced to the convex hull of feasible lattice points, and hence the optimum solution of this redefined programming problem provides necessarily an integer solution.

Unfortunately, the addition of new constraints generally results in tedious and cumbersome computations, making large sized problems sometimes infeasible. Musgrave (150) for example, reports good results in a problem where the restrictions were less than \( \frac{1}{2} \) (m - \( \frac{1}{2} \)). A good intuitive presentation to the Comory technique is given by Baumol (10, Chap. 8), and a mathematical treatment of the technique by Hadley (88, Chap. 8) and Vajda (20, Chap. 11). In addition to the technique by Comory, Land and Doig (131) and more recently Maruyama and Fuller (143), presented alternative ways to obtain solutions of integer programming problems. Due to the addition of new constraints, the interpretation of the dual prices (shadow prices) in integer programming solutions is more difficult than in the general case. Their interpretation is discussed in contributions by Comory and Baumol (86) and by Alkaly and Klevorick (3).

Following in part, a presentation by Edwards (67), the economic significance of integer programming for a number of non-
linear problems will be illustrated. Integer programming can be
c conducive in handling the following problems:

(1) In situations where the variables with fractional val-
 ues are meaningless this technique can be used to provide opti-
mum solutions with integer variables. As an example a study by
Musgrave (150) might be mentioned in which he examined the opti-
mum number of sows to be farrowed per month.

(2) It can permit the incorporation of all or nothing-at-
all restrictions, such as when considering the purchase of a
combine, there is the problem of fixed cost involved in owning a
combine. Generally, the problems of fixed cost or fixed charge,
and of setup labor time, fall into this category. These formul-
ations require integer variables of the size 1 or 0. As an ex-
ample let $c_j$ be the net revenue per unit corn, $x_j$ the level of
corn and $A_j$ the annual fixed cost due to the maintenance of
equipment for corn growing. The problem can be expressed as

$$\max \quad z = f(x_j) = \sum_{j=1}^{n} c_j x_j + A_j y$$

where

$$y = \begin{cases} 
0 & \text{if } x_j = 0 \\
1 & \text{if } x_j > 0 
\end{cases}$$

Now, let $d$ be a large finite number indicating the upper bound
for $x_j$. As a result, in addition to the usual linear pro-
gramming constraint $(Ax = b)$ the restriction

$$x_j \leq dy$$

which can be written as
\[ x_j - dy \geq 0 \]

must be included into the program, where \( y \) is a dummy variable that must assume the values 1 or 0.

(3) It allows the treatment of economies of size problems, by including enterprises of various discrete sizes in a program, e.g., dairy enterprises with herds of 0, 10, 30, or 80 cows without considering any intermediate activity levels. A restriction of this type can be written as

\[ x_j = 10y_1 + 30y_2 + 80y_3 \]

where \( y_1, y_2, \) and \( y_3 \) are integers. If only one of these dairy activities shall be included in the optimum plan, the additional constraint

\[ 1 \geq y_1 + y_2 + y_3 \]

could be added, although this is often not necessary.

(4) It makes it possible to include either-or choices into a program, as when one would either keep dairy cows or hogs, but not both. This restriction can be expressed in the form

\[ x_1x_j = 0 \]

and can be imposed on a program by the two linear inequalities

\[ x_1 \leq d_iy \]
\[ x_j \leq d_j(1-y) \]

where \( y \) is an integer confined to the values 1 or 0. These few examples give an impression of the versatility of integer programming.

A discussion of the significance of integer programming for
various problems in agriculture, was first given by Chou and Feady (33). Despite the convincing advantages of the method, applications in agriculture are rather seldom. This is partially due to the lack of easily available computer programs, but an improvement of this situation can be expected in the future when the demand for these algorithms rises.

In a survey note on Comory's work, Musgrave (150) reports on the determination of an optimum farrowing schedule by use of integer programming. Koehne (127) developed a convex programming approach based on linear approximations that allows for nonlinear relationships to be included in the objective function and in the constraints. The method rests upon the use of dummy activities subordinated to integer restrictions. Seagraves (172) explains a method which does not require specific computer algorithms, and which uses regular linear programming to arrive at partial integer solutions. This approach is based on the contribution by Land and Doig (131).

As exemplified in the previous paragraphs, integer programming provides answers to many management problems that cannot be cast into a linear model, and it is likely that it will be as generally used in farm management in the future, as the general linear programming model is being used now.
Dynamic Linear Programming

The expression "dynamic programming" is used in the literature for two different procedures. Loftsgard and Heady (136) introduced this term when presenting a problem in which they took time explicitly into consideration. This method is based on a linear model, and is referred to as dynamic linear programming.

Bellman used the term "dynamic programming" for a computational technique which they developed by studying various types of sequential decision problems. This technique will be the subject matter of the next chapter. In this section the concern will be with dynamic linear programming.

Dynamic linear programming can be classified as an extension of the general linear programming model. It is dynamic in the Hicksian sense i.e. the input and output coefficients are dated so that the identification of any coefficient in the matrix refers to row, column, and year. The problem can be written as

$$\max \quad z = \sum_{j=1}^{n} \sum_{k=1}^{t} c_{jk} x_{jk}$$

subject to

$$\sum_{j=1}^{n} \sum_{k=1}^{t} a_{ijk} x_{jk} = b_{ik} \quad i = 1, 2, \ldots, m$$

$$x_{jk} \geq 0 \quad j = 1, \ldots, n$$

$$k = 1, \ldots, t$$

Problems of this kind are characterized by a block diagonal in-
put-output matrix. The input coefficients in each year are usually the same, unless changes in technology are to be included in the program. The blocks correspond to the single years and are connected with each other by intertemporal capital and, maybe, labor vectors and rows.

This type of model permits a more realistic formulation of farm problems, since they allow the incorporation of the time element in the model in an explicit way. Using the standard linear programming model, the optimum is always computed for one discrete point in time, being at present or in the future, dependent upon the choice of the coefficients. In any case it is always a static analysis. Dynamic linear programming, however, permits the calculation of optimum plans for a sequence of years, and these optimum plans warrant the maximization of the objective function for the whole period considered, and not only for one year. The income of one year is the input of capital in the following year, and thus, the crucial role of working capital for a farm is clearly expressed.

Among the applications, the article by Loftsgard and Ready (136) is mentioned first, since it introduced the technique to the public. In this paper the authors studied the close relationship between the consumption pattern of the farm family, and the further development of the farm over a period of several years. The same concept was used by Bean and de Benedictis (51) in an investigation into development plans for small peasant farms in Southern Italy. Candler (29), in a note on the
Loftsgard and Heady article, tried to relate the concept of dynamic linear programming to programming with variable capital restrictions.
DYNAMIC PROGRAMMING

From the foregoing discussion it will be recalled that optimization techniques can be classified into three major stages, the calculus approach, the linear programming approach, and the nonlinear programming approach. Dynamic programming has to be regarded as a fourth approach to optimization. It may be described as the "application of functional equations and the Bellman principle of optimality to sequential decision problems" (115, p. 53b). These three elements, functional equation, principle of optimality, and sequential decision, are characteristic for each dynamic programming problem. The sequential decision might occur in time, in space, or generally in any n-dimensional setting. The technique can also be applied to a static allocation problem expressed in a sequential framework.

Each sequential decision process so formulated, can be subdivided into single stages. At a particular state, a process can be characterized by the so called state variables. As the system changes the state variables also alter their values. These state variables are determined within the system as opposed to the decisions variables which are subject to direct control by the decision maker. The feasible sequence of decisions that leads to the maximization of the objective function is called optimal policy. The decision at each state has to follow the principle of optimality: "An optimal policy has the property that, whatever the initial state and the initial deci-
sion, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision." (204, p. 245). By applying this rule, a multidimensional problem is reduced to a single dimensional problem. The core of each problem is the formulation of the recurrence relationship. It implies that the optimum decision to be made at a particular stage of the process depends solely on the state of the process at that stage.

If the new state is unique, once the previous state and the decision are known, the problem is referred to as a deterministic case. Yet, the new state might be influenced not only by the previous state and the decision, but also by a random variable. If the probability distribution of this random variable is known, the problem is called a stochastic case. A higher degree of complexity occurs when the estimate of a random variable is reassessed as additional information about it accumulates in the course of computation. This is the adaptive case.

Dynamic programming possesses several considerable advantages in comparison to other methods. First of all, the mathematical structure is so conceived as to deal with maximization over a time horizon, and it permits the inclusion of stochastic elements, or adaptive processes, into a multistage computation. Such problems cannot be attacked by the more traditional programming techniques. With regard to the nature of the functions used to specify the problem, no special restrictions have to be satisfied. The return function, for instance, can display non-
linearities, discontinuities, or kinks and does not need to be differentiable. When compared to the set of assumptions to be fulfilled in the linear and nonlinear programming case, this property of dynamic programming exemplifies above all, the versatility of the method. This flexibility is due to the computer orientation of the technique. The application of the method depends completely on the availability of computers, and the writing of the computer program for a particular problem is as a rule, rather straight forward. Lastly, the nature of dynamic programming seems to correspond to a large extent with the manner in which real decisions are made.

One of the major disadvantages of the method is the computational difficulties that occur as soon as the number of decision variables exceeds one or two. This problem is often called the multidimensionality problem. The data requirements present another obstacle. While in linear programming only a single coefficient of a production function has to be specified, dynamic programming requires the definition of the whole function in numerical terms. Furthermore, dynamic programming is a very specific approach that must be tailored and qualified for each individual problem, providing numerical answers but so far no general formulations have been given. Thus, the technique does not satisfy the predilection of many researchers for general rules, but forces him to become deeply involved in statistical, mathematical, computational and real-world aspects of his problem.
The literature on dynamic programming is dominated by the many contributions of Bellman and his associates who developed the technique in the fifties. Their work is published in two books, (13), (14). Howard related their work to the concept of Markov chains (108). In the agricultural literature, Johnston (115) presented a well written introduction to the subject by discussing dynamic programming in comparison with other optimization techniques, in their relevance to the theory of the firm. Similar introductory studies were put forward by Throsby (199), (200). Burt and Allison (23) used a dynamic programming model of the Howard type for a study of rotation problems in Kansas. The same kind of model was used by Burt (22) to study involuntary replacement due to chance elements. A study of the optimal policy in selling butter from New Zealand to the United Kingdom was undertaken by Townsley (203). Dynamic programming is a new and yet unexplored research tool in agricultural economics. A good deal of thorough research will be necessary in the years to come, before a sound evaluation of the technique can be attempted. Indeed, the great potentials of the technique in its bearing on various inadequately solved problems in the economics of agriculture, let such an inquiry appear exceedingly desirable.

Suggested areas of applications are problems where the time element and data uncertainty play a key role. As the consideration of time in our models frequently implies data uncertainty, the possibility to combine both aspects of decision making into
one model, will make the studies much more realistic. Inventory and replacement studies and also allocation problems can be formulated in this framework.

Yet, in the immediate years ahead, it might be expected that the technique will be used only as a research tool without practical significance. Further investigations will probably be restricted to countries and research stations which can afford to allocate large funds and highly trained scientists into basic research, rather than trying to bring the other developed tools to practical use. After this developmental stage the situation might change. The characteristics and future prospects of dynamic programming are very similar to those of simulation which will be discussed in the next chapter. Both tools are rather complex but versatile, and appear to be in the early stages of development.
"Simulation is a general approach to the study and use of models" (159, p. 893). Conceptually, then, this is not a particularly new approach. Ever since economists tried to describe and analyze the functioning of economic systems, they constructed models to facilitate these studies. Be they highly intricate or very simple, these models are designed to simulate the real economic world or parts of it. What is changing is the language which is used to express these models. In the early beginning of economic analysis, ordinary prose was the sole language which was used. It was supplemented by pictorial geometry and later, by formal mathematical methods, both of which proved to be powerful tools in sharpening the economic investigations. Simulation carries this development one step further by exploiting fully the extraordinary capacities of high speed computers. Prose, geometrical presentation and formal mathematical methods are now being supplemented by computer languages and flow diagrams.

The new technique differs in many ways from the conventional methods. All formal mathematical models provide the possibility for a deductive derivation of a general solution. However, this generality is frequently attained at the expense of an accurate depiction of reality and attempts to present reality more closely are doomed to failure because the problems become mathematically intractable. Simulation techniques lack this
generality. Each simulation model, also termed simulator, is constructed to fit the particular economic system under investigation. The role of a particular simulation model for economic analysis may be compared with the significance of an experimental set-up in the natural sciences. Each computer run may be considered as an experiment performed with the model. In this sense, simulation provides the economist with a method of running laboratory experiments on economic systems.

Advantages

The direct communication with the computer supplies the simulation technique with several advantages over other more conventional methods. The models can deal with more complex and realistic situations and in better accordance with the particular problem under study. All kind of non-linearities, time-delays and discontinuities can be included. This fact imparts to simulation, unique properties for studies considering uncertainties and dynamic relationships. Orcutt (159, p. 905) asserts that "simulation is the only known approach to the satisfactory study and use of any of the existing dynamic models of economic systems for which any pretense of realism can be claimed."

Weinschenck uses simulation in combination with programming procedures for farm planning under uncertainty, and considers the simulation models "as the most important bridge between practical extension work and microeconomic theory" (215, p. 50).
Futton further stresses simulations importance (110, p. 1426): "In some extremely complex problems, particularly those of stochastic nature, simulation ... is about the only resource that we have for quantitative analysis".

Another advantage of simulation stems from the fact that qualitative aspects of human decision making can be included in computer programs. Several computer programs have been written that are able to handle non-numerical symbols, and others can modify their own program in an adaptive way and account for experience and heuristic processes. These aspects endow simulation with a much broader scope for economic analysis than mathematical models can provide.

A few other advantages of this technique are often pointed out in the literature including the approach it provides in tackling the aggregation problem. Another benefit of simulation seems to be that it can be more easily used than conventional methods by those lacking a higher degree of mathematical training. Yet, this point is questionable.

Disadvantages

Simulation techniques also have a few disadvantages. As was already pointed out, each simulation run provides the answer to a specific numerical problem, but fails to provide an insight into how the answer would be influenced by a change in one of the parameters. However, the main interest of the researcher
frequently centers around the question as to how an economic system responds to changes in certain conditions. A large number of computer runs has to be performed to procure this information.

It has been mentioned that simulation procedures are apt to cope with very complex problems. This also means that the abilities required from a researcher must be very high. The adequacy of the model depends largely on his competence and skill to select the decisive variables and to recognize the crucial relationships between the different variables. This is the most difficult task in building a simulator. The studies necessary to select the right variables and to establish the proper relationships require a sizable amount of research. During this preparatory analysis, the researcher has to gain an understanding of the operation of the system, to specify the relevant relationships and to gather the data for their quantification. Only when this part of the simulation study which represents a major portion of the project, has been carried out thoroughly, will the results of the model prove to be valuable. This considerable amount of time to be spent on supporting research is perhaps one of the most undesirable characteristics of simulation. In addition, the researcher must not only become thoroughly entrenched with minute details, but he also must have command over the technical knowledge involved in working with flow diagrams and writing a computer language. This high requirement might be responsible for the limited applications in
agriculture.

In applying programming procedures, the solution usually provides a local or global optimum, whereas a single run on a simulator yields only a point on the solution surface. In order to obtain an idea about the overall shape of the solution surface it is necessary to perform several runs on a computer each time changing the values of variables under control.

Simulation has been described as a technique permitting management to experiment and analyze the consequences of pursuing different policies. This analysis can be performed without putting the policies into effect. In predetermined the consequences of various actions, the avoidance of expensive mistakes becomes possible. This represents one of the main benefits of simulation.

Two slightly modified variants of simulation, Monte Carlo methods and Operational Gaming shall be mentioned briefly.

Monte Carlo Methods

Monte Carlo Methods are special simulation techniques which are used to study probability models. The computer is used to generate a stream of random data which are fed into the model. In this way a simulation of stochastic processes is possible.

Operational Gaming

Operational Gaming, Business Games, or Briefly Gaming, can
be described as simulation with human intervention. Using a normal simulation procedure, the effect of the initial set of data is traced to the final stage of the system. A new decision about changes in certain variables requires a new computer run. In operational gaming however, the simulation process is periodically interrupted, and human decision making interferes with the system. This interference of the human decision making element distinguishes between simulation and operational gaming. Operational Gaming serves mainly as a device for training of executives and for teaching management principles in colleges and professional workshops. As a research tool, it has not been significant. It should be emphasized that it is not an optimization procedure. Rather, many games might be necessary before distinction between good and bad decision is possible.

Applications

Among the models and techniques considered in this survey, simulation is by far the most versatile approach. To convey an impression of the scope of simulation, it should be mentioned that all problems dealt with in this thesis could be expressed by simulation techniques. Simulation applications have been applied to replacement, queueing and inventory problems, to various managerial areas and to interindustry studies. A fast growing interest in this technique can be observed, not only in economics but also in such disciplines as psychology, sociology and geography (93). A few publications will be cited below.
Firstly, some articles containing a general introduction to the topic, covering so called computer philosophy will be mentioned. Articles dealing with the methodology per se, and describing technical details of formulating a simulator will be found in the second part. Lastly, those articles reporting on applications in agricultural economics will complete the survey.

Basic ideas and concepts which offer a good insight into the technique of simulation were brought to the attention of economists in a series of three articles in the American Economic Review (159), (177), (38). In an earlier publication Dorfman (60) outlines the relation of simulation to other operation research techniques. An attempt to define simulation in a formal way was undertaken by Churchman (34). He relates the method to philosophical considerations. A good introduction into model building with regard to microeconomic problems is given by Cohen and Cyert (39). The Journal of Farm Economics published several general discussions about the significance of pure simulation and man-machine games for the Agricultural Economist (186), (5), (79), (6).

Turning to the second part of this section, we mention first the excellent paper by Fisgruber (68) which describes the procedure for carrying out a specific simulation of a farm. It contains flow diagrams and the corresponding computer statements. The developed program is remarkable because of its flexibility which allows it to be used as a simulator or as a management game. When using it as a management game, the com-
plex structure of the model permits the pursuit of a wide variety of teaching objectives. Fisgruber lists five possible teaching objectives, such as the effect of limited capital on farm planning, uncertainty in agriculture, price cycles and farm income, specialization versus diversification and finally, problems of introducing new technology to a farm. He emphasizes that any use of the model must be preceded by a detailed briefing session with the participants. The computations, performed by the model, may be grouped into nine categories, some of them being mentioned for illustrative purposes. They are a) cropping operations, b) livestock operations, c) availability of labor, d) machinery and building requirements and e) generation of stochastic variations. A less technical introduction to this simulator is given by Fisgruber (69) in another article where he mainly discusses its possible application in farm management teaching. The Carnegie Tech Management Game is explained in an article by Cohen et al. (41). It is a man-machine game in which teams of players compete against other teams. As the decisions of each team are fed to the computer during the process of the game, the relative profit positions of the teams are changing. Cohn et al. (40) also developed a comprehensive model of price and output determination under oligopoly.

For further information on general and technical aspects of simulation the reader is referred to various books on this subject such as for instance "The Behavioral Theory of the Firm" by Cyert and March (48) or "System Analysis: a computer approach to
decision models" written by McMillan and Gonzales (157).

Agricultural economists have been studying simulation techniques for several years but the first applications of these techniques to agricultural problems were published only one and a half year ago. As is conveyed by these few studies the main interest in using simulation procedures is presently directed towards the study of large agricultural markets and the developing of management games for farms. Crom and Maki (147) developed a model simulating the pork and beef sector of the economy. Their model yield a very good fit between the predicted and reported sizes of such variables as prices, consumption, slaughter and livestock inventories, for the last nine years. In another publication they (146) they point out how this recursive model can be modified to improve its predictive ability. A more detailed analysis of this problem area is given by Crom (145). The only published management game is the already mentioned simulator by Eigruber (68). Even though no other management games have been yet published as of the present time, at various universities such games are either being developed or already tentatively in use. Working at Purdue, Babb (5) reports the use of two business games, one for fluid milk processors and one for farm supply businesses. Both games are primarily marketing games involving decisions about pricing of goods, advertising, investments, and loans. Futton (110) in his address at the annual meeting of the AFFA, August 1965, announced that a model
which is designed to be used as a simulator and as a farm management game is presently in developmental stages in Pennsylvania. Another recently developed computer game by Wildermuth and Faris (220) is being used for teaching purposes in California. Weinschenck (215) reports on investigations into the usefulness of man-machine games for agricultural extension work in Germany.

Simulation techniques are frequently visualized as being of positive nature, exclusively, and of not securing the attainment of an optimum solution. However, this rather narrow view of simulation was disproved by the remarkable articles in which simulation techniques were combined with optimizing procedures to solve farm management problems. Zusman and Armiad (223) studied the optimal organization of a Kibbutz located in an arid region in Israel with low and unstable rainfalls by simulating the decision process. After having determined the main features of the response surface, they used the steepest ascent procedure to approach an optimum solution. The second article, which used simulation techniques with optimization to cope with price and weather uncertainties is prescribed by Halter and Dean (89). This study deals with a large California feedlot operation.

When suggesting various areas where simulation techniques would be useful in agriculture, it is tempting to propose nearly any size problem because of the vast potentials of the method. Unfortunately however, the method is extremely time consuming, tedious and requires highly trained staff. Consequently, simulation of larger economic systems, mainly at the macro level de-
serve first attention. The studies of the livestock meat economy can be regarded as a first step into this direction. Many other useful applications seem possible. Proposed changes of agricultural policy could be simulated in order to study their impacts on the whole socio-economic environment. Simulators of this kind would permit the evaluation of highly consequential decisions before they are made. Possible changes in taxes, agricultural prices, subsidy systems, market orders, trade policies are examples for questions which could be examined. If it is necessary to assess the impact of large projects on the economy of a region, it is quite feasible that developing an expansive simulator would be economically justifiable, too. Examples in this area are the planning of a big irrigation project or the construction of completely new transportation facilities.

In simulating the changing structure of agriculture, the economist might join the rural sociologist to draw conclusions about future conditions in farming with regard to both the economical and sociological aspects of this transition.

At the microlevel the further development of management games seems very promising mainly as a tool to improve the teaching of management principles. Regarding applications at the farm level the combination of simulation techniques with various other operations research tools show great potentials. This will enable the researcher to cope with various problems of uncertainty and dynamic relations, problems that cannot be
solved by other methods alone. Research in this direction is already on the way at several universities.
Like many other decision makers, farmers usually face an uncertain future. This uncertainty presents one of the main obstacles to efficient decision making. It is introduced into the decision making environment of the farmer by various forces such as technical and technological change, price variation, and unpredictable human actions. The unpredictability of yields, weather, and other natural phenomena is the reason for technical uncertainty, whereas the rapid change in production techniques gives birth to the various technological dubieties. Price variations due to the interaction of several uncontrollable economic, political, and natural phenomena provide a further source of ignorance. Changes in the goals and relations of individuals involved in farming add further to the aggravating aspect of uncertainty.

The fact is that farmers must make decisions in their uncertain environment. Hence, the assumption of full information, underlying many methods of analysis, is often a strong oversimplification. Recognition of these problems is not new. Due to the lack of theories leading to a normative approach to these problems, early research work was directed towards the attaining of an understanding of decisions made under uncertain conditions. These investigations supplied highly valuable insight into the decision making procedure of farmers and revealed a few valid arguments as to the basis of farmer's decisions. (103), (107),
(106). Yet, this purely descriptive approach, despite some further contributions by Shackle (176) and Simon (179), could not attain a lasting importance in Agricultural Economics.

Another early attempt to tackle these problems was the elaboration of numerous rules of thumbs, generally based on the projection of some weighted averages of past results (96, Chap. 16).

All these early approaches were subsumed under the more general theories of decision making as developed in Von Neumann and Morgenstern’s Theory of Games (207), Wald's Theory of Statistical Decision Functions (209), and Savage’s Subjective Probability Approach (168). These contributions intend to maximize the utility of a rational behaving decision maker. They opened a new area in Statistical Theory, that provides a normative approach to the problems under uncertainty. In recent years statistics has even been called the science of decision making under uncertainty.

When classifying the approaches of modern decision theory to problems under uncertainty, distinction can be made between the game theoretic approach, which is strictly based on game theory as developed by Von Neumann and Morgenstern, and the probabilistic approach based on the contributions of Wald and Savage. The game theoretic models, commonly referred to as games against nature, assume complete uncertainty about the decisions of the other player, while the probabilistic approach emphasizes the decision maker’s expectations and the use of
probabilistic measures by attaching weights to the possible states of nature.

After a short introduction to the two-person zero-sum games, the game against nature approach and the probabilistic approach will be presented briefly.

Two-Person Zero-Sum Games

Generally, game theory deals with situations characterized by independent players whose interests conflicts and in which no one player is in the position to exert full control over the situation. Among the various types of games, the two-person zero-sum games received most attention in the literature since these are the only games to which a complete solution in the sense of maximizing the assumed utility function of the two players is attainable. They are also best in elucidating the connections between game theory and decision theory. (Luce and Raiffa (140) and Neady and Candler (99) provide a complete treatment of game theory in this framework).

In this type of game two players oppose each other. Each of them has a finite number of alternative courses of action, the so called strategy set, available to him. Corresponding to each pair of strategies $a_i$ and $b_j$, a payoff $o_{ij}$ exists. All possible pairs of strategies form a matrix of outcomes, $(o_{ij})$, which is referred to as a payoff matrix. Under a set of rules of the game and an array of assumptions about the players, game
theory makes use of the minimax principle to point out which strategy a player has to choose to obtain the highest sure return or the lowest sure loss. The underlying assumptions about the players are:

(a) that they behave as rational and intelligent beings
(b) that each tries to maximize his expected utility and thus, behaves in a malicious way towards his opponent and
(c) that each knows the strategies available to him and to the other player and the corresponding payoffs.

The use of the minimax criterion seems reasonable in this model because of these specific assumptions. On the other hand, these rigid assumptions prevented the model from receiving wider attention in the agricultural field.

Two structural features of this game will be present in each decision theoretic model. These are first, the payoff matrix presenting strategies available to the decision maker and his opponent and second, the utility function of the decision maker.

Games against Nature

Games against nature are often classified as belonging either to decision theory or game theory. The basic distinction between games against nature and two-person zero-sum games is that, in the game against nature case the set of assumptions,
cited previously, holds only for one player. The other player
is a non-conscious, indifferent adversary choosing passively in
a completely unpredictable manner among his strategies. In
terms of decision theory, the rational decision maker faces na-
ture, where nature may reflect a wide range of phenomena from
actual states of nature such as various weather situations to
mere ignorance. Since the decision maker only knows the set of
strategies open to nature but is completely ignorant as to which
state of nature will occur, these problems are also referred to
as the true decision problems under uncertainty (RPUU).

Since nature does not plot and think against its opponent
the assumption that the worst possible course of action on part
of the adversary must be expected, does not hold and thus, the
use of the minimax criteria is not necessarily reasonable any
more. Rather, under conditions of uncertainty no optimal solu-
tion and no best criteria exist, which are generally accepted.
The pecuniary situation of a decision maker and his individual
attitude towards risk will determine, to a large extent, the
strategy he will choose. According to the various positions of
individuals towards uncertainty several theories of decision
making have been suggested (14,7), four of which will be pres-
ented below.

The Wald or maximin criterion treats the RPUU strictly as
a two-person zero-sum game calling for the selection of a plan
that allows the greatest minimum return. As it insinuates that
nature will do its worst, it applies to farmers being concerned with positive short-run outcomes because of financial commitments. It is sometimes termed the pessimistic approach.

The Hurwicz criterion chooses the plan with the highest pessimist-optimist index (59) which makes this criterion particular adequate for farmers who want and can afford to gamble.

The Savage regret criterion provides a solution which minimizes the regret that might be felt, after having attained export cognizance as to the true state of nature. This regret might occur if the outcome shows that another choice had brought a higher payoff.

The Laplace principle of insufficient reason is the oldest of all decision theories. Due to the lack of information the decision maker assumes each state of nature to be equally likely and selects the strategy that yields the highest expected utility, i.e. \( \max_{j=1}^{m} \sum_{i=1}^{m} o_{ij} / m \) where \( m \) is the number of possible states of nature. It is pertinent to farmers who are financially strong enough to strive for highest long-run profits.

A more detailed discussion of these criteria is presented by Heady et al. (71), (59).

Subjective Probability Approach

In recent years the game against nature approach was criticized because it assumes less information than is actually available, thus, making the decision process more difficult than
necessary. In many situations the decision maker has some knowledge about the likelihood of the occurrence of the various states of nature but this information is neglected when using the game-theoretic model. When dealing with weather uncertainty for instance, the meteorological forecasts and the records of past years enable the farmer to attach at least subjective probabilities to the possible states of weather in the immediate future. The same holds if the opponent is represented by policymakers. Some of their possible decisions might appear more likely to the farmer than others and this fact places him again in the position of assigning subjective probabilities to the possible moves of his policy opponent.

The Bayes criterion permits the incorporation of this information into the model by assigning subjective probabilities $P_j$ to each state of nature so that

$$\sum_{j=1}^{m} P_j = 1 \quad \text{for all} \quad i = 1 \ldots n.$$  

Then, the decision maker chooses this strategy with the largest expected value

$$\sum_{j=1}^{m} c_{ij} P_j \quad \text{for all} \quad i = 1 \ldots n.$$  

The expected value of each strategy is computed by multiplying the payoff of this strategy by the probability of each state of nature and summing up the products of this strategy. If the decision maker has not enough knowledge to assign any subjective
probabilities to each state of nature, he will assume the occurring
ence of each state of nature to be equally likely. In this
case the Bayes criterion becomes the same as the Laplace cri-
tion. Therefore the Laplace criterion is sometimes classified
as a special case of the Bayes rule.

In many situations the Bayes decision rule seems to be the
most preferable one. The increasing number of applications of
this criteria over the most recent years supports this asser-
tion.

Before proceeding to consider specific applications of sta-
tistical decision theory to agricultural problems, one point re-
mains to be noted. Although these theories are designed to cope
with conditions under uncertainty, a considerable amount of
knowledge is assumed to be available to the decision maker in
all cases discussed. First of all, he must be able to describe
his choice situation in terms of a payoff matrix knowing his own
and his opponents set of strategies and the expected outcome for
each pair of strategies. Also, he is assumed to know his own
utility function. Good record keeping systems aimed at provid-
ing these particular data might reduce the rigidity of these as-
sumptions.

Applications

The majority of the farm management applications have been
in the vein of games against nature. Ready and Dillon (59) as-

sumed such a model in a study aimed at assessing its descriptive and normative usefulness in terms of the Wald, Savage, Hurwicz, Simon and Shackle criteria. The study is based on a population of 77 farmers in Marshall County, Iowa and revealed that the farmers could have increased their expected profits by at least 21% if they had followed a normative approach. Further discussion is given in (56), (61, p. 158). The most successful applications of the game against nature formulation are the studies dealing with production decisions under weather uncertainty. In an extensive study of this type Heady and Walker (211) demonstrated how the various decision criteria could be applied to obtain alternative recommendations suited to a wide range of farmer's goals and attitudes. A less comprehensive study of the same kind has been carried out by Swanson (190). The model has been used to study several other decision situations of farmers. Dillon (56) suggested to consider the farmer as a free competitor who selects his alternatives relative to an aggregate opponent made up of other members of his industry and including such sources of variation as the climate. Dillon also proposed (55) to analyze vertical integration and the growth of cooperatives in a gametheoretic framework. Other applications used this model to tackle such problems as the adoption of innovations by farmers (57) and bilateral bargaining (120). An interesting field open to gametheoretic research are questions of policy. Suggestions in this direction are made by Luce and Raiffa (140),
p. 97) and Hady (95, p. 364-374).

In the last few years a slight decline of interest in the game against nature approach could be observed, accompanied by a rising curiosity in the subjective probability concept. As a starting point of this development an article by Dillon (54), containing an excellent survey of applications in agriculture, could be mentioned. In this article Dillon, a long time user of game theory in agricultural economics, arrives at the conclusion that "... little success has rewarded these efforts. Only the game against nature approach to climatic uncertainty appears to have any immediate practical value" (54, p. 31). Without contesting the validity of this judgement, a definite merit of the application of game theory to agricultural problems is certainly the better understanding of the decision processes under uncertainty. Hence, these studies can be regarded as prerequisites for the use of other concepts of statistical decision theory.

A few applications of the subjective probability approach have been reported in the agricultural literature, recently. McConnen (153) illustrates briefly the use of a modified Bayes criterion in determining "best" stocking rates and selection of the "best" maturity class of corn for farm situation in the great plains involving climatic variability. In a study by Langham (132) the question as to whether rice farmers in Florida should request an acreage allotment increase is expressed in a decision theoretic model in which various government policies
present the states of nature. Since these rice farmers have
certain estimates about the likely government policy the author
uses a probability matrix based on a modified Bayes solution.
After applying the four previously discussed decision criteria
to the complete ignorance case and the modified matrix, the va-
rious outcomes are compared. Tedford (196) reports the use of
decision theoretic concepts in analyzing a replacement model.
An application of Bayesian statistics in combination with linear
programming procedures is shown in a recent study by Dean, C. W.
et al. (52). They analyze the optimum stocking rates for feed-
lots in California under weather uncertainty.

These few applications certainly do not exhaust the poten-
tials of statistical decision theory in agricultural economics.
Bearing in mind that the basic objective of statistical decision
theory is to cope with problems under uncertainty some addi-
tional areas of possible applications in agriculture can be sug-
gested. First of all these concepts might be useful as a re-
search tool in helping to define decision rules under uncertain-
ty and in classifying various farm systems in their sensitivity
to uncertainty. They might be of help to pinpoint limitations
in solutions arrived at by other operational techniques which
assume full information and to modify these solutions. The con-
ceptual framework offered by this theory should form the basis
when other computational techniques such as simulation are used
to deal with uncertainty. They also afford better insight into
the data requirements that are needed for a sound evaluation of
choice situations under uncertainty. In this sense this theory lies at the core of the various attempts to improve our decision making under uncertain conditions and should help to choose the right computational approach for a particular problem.
Economic situations and institutions as well as the structure of economic aggregates are continually changing. It is often of great interest to analyze changes in the past in order to draw conclusions about the possible future structure of an economic system. Markov chains provide a technique for analyzing and describing problems of this kind.

In order to develop the concept of Markov chains three basic terms will first be explained. These are state, transition and equilibrium. In applying Markov chains it is assumed that any population of firms or other economic entities can be classified into distinct categories or states. In a specific period of time, an economic entity can occupy only one particular state. During the course of several time periods the economic unit may move through various states. It is essential that the term state is defined unambiguously and that each economic entity is assigned to one of the different states for each time period. In a practical example, a farm could be defined as an economic entity and two size categories could be the two different states.

As time passes, the farm changes from one state to another. It is this procedure of moving from state to state which is referred to as transition. Transition occurs between time periods of equal length. In order to exemplify a transition probability we turn our attention to a population of farmers being exposed
to the same choice possibilities. The recording of their size changes over several time periods provides a basis for the derivation of the probabilities for a single farm of staying in one state or moving to another state. By computing these transition probabilities the movement of a farm from one state to another is regarded as a stochastic process.\(^*\)

Suppose that the underlying tendencies that led to the transition probabilities will prevail in the future. In this case it can be observed that the fraction of farms belonging to state I and, concomitantly, the fraction of farms belonging to state II are approaching a stable proportion. This situation is called the equilibrium of a Markov chain. In this equilibrium situation the system is stable in terms of the states within the whole population but it is dynamic for the individual entity.

Properties of Markov Chains

After this short introduction it seems desirable to present the properties of Markov chains in a more rigorous manner. Kemeny et al. define them in the following way (121, p. 148):

A Markov chain process is determined by specifying

the following information: There is given a set of states \(s_1, s_2, \ldots, s_p\). The process can be in one and only one of these states at a given time and it moves successively from one state to another. Each

\(^*\)A stochastic process can be defined as a sequence of events whose outcome depends upon some chance element.
move is called a step. The probability that the process moves from \( s_i \) to \( s_j \) depends only on the state \( s_i \) that it occupied before the step. The transition probability \( p_{ij} \), which gives the probability that the process will move from \( s_i \) to \( s_j \), is given for every ordered pair of states. Also an initial starting state is specified at which the process is assumed to begin.

The information about the set of states \( s_1, s_2, \ldots, s_n \) and the transition probabilities \( p_{ij} \) can be exhibited in two different forms. The first way is to construct a probability tree with attached branch weights describing the Markov chain process as it moves through a finite number of steps. However, more commonly the second form is used. The transition probabilities are arranged into a matrix. This transition matrix, \( p \), can be written as:

\[
\begin{bmatrix}
    s_1 & s_2 & \cdots & s_m \\
    s_1 & p_{11} & p_{12} & \cdots & p_{1m} \\
p = s_2 & p_{21} & p_{22} & \cdots & p_{2m} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    s_m & p_{m1} & p_{m2} & \cdots & p_{mm}
\end{bmatrix}
\]

The elements of \( P \) denote the probability that a process being in state \( s_i \) will enter the state \( s_j \) in the next period. The transition matrix lies at the core of each Markov chain process and hence deserves a more detailed discussion. Each element of the matrix is nonnegative and since it expresses a relative distribution, the condition
\[ \sum_{j=1}^{m} p_{ij} = 1 \]

must hold for each \( i \). Since a vector having nonnegative entries and adding up to 1 is defined as a probability vector each row of \( P \) represents a probability vector. The entire matrix is then called a probability matrix or a stochastic matrix. A stochastic matrix of this kind and an initial starting state, i.e. a row vector, must be given in order to define a Markov chain process completely and to find its outcome after the \( n \)th period.

According to the shape of the stochastic matrix \( P \) different types of Markov chains can be distinguished. For most economic analysis, it will be required that all states are accessible, i.e. a process starting in any state \( i \) must have a non zero probability of moving from \( i \) into any other state \( j \) in a finite number of steps. A Markov chain satisfying this condition is called a regular Markov chain. Or, as Kemeny et al. define it (121, p. 391): "A Markov chain is called a regular chain if some power of the transition matrix has only positive elements."

In relation to these regular chains Kemeny et al. discuss two important theorems (121, pp. 392, 393):

**Theorem 1.** If \( P \) is a transition matrix for a regular chain, then

1. The powers \( P^n \) approach a matrix \( T \) (that is, each entry of \( P^n \) approaches the corresponding entry of \( T \))
2. Each row of \( T \) is the same probability vector \( W \).
3. The components of \( W \) are all positive.
Theorem 2. If $P$ is a transition matrix for a regular chain, and $T$ and $W$ are as Theorem 1, then the vector $W$ is the unique probability vector such that $WP = W$.

These theorems state that there exists an equilibrium solution to this Markov process and that, this equilibrium is unique and independent of the initial configuration. Thus, the starting state is without influence on the equilibrium distribution at the time $n$. Furthermore, these theorems permit the derivation of two procedures for obtaining the equilibrium solution.

One method is the repeated application of $P$ to the initial vector or starting state. Denoting the initial vector as $w_j^0$ and the equilibrium vector as $w$ we have

$$w_j^0 \cdot P = w_j^1$$
$$w_j^0 \cdot P^2 = w_j^2$$
$$\ldots$$
$$w_j^0 \cdot P^n = w_j^n = w$$

A less tedious way for deriving the equilibrium situation follows from Theorem II. Theorem II states that in equilibrium the distribution of firms among states must be invariant. The equilibrium situation for the equilibrium vector $w$ might be rewritten as:

$$WP = W$$

or:

$$w(P - I) = 0$$
This is a system of \( n-1 \) linearly independent equations in \( n \) unknowns. Since \( w \) represents a relative distribution it is known that

\[
\sum_{j=1}^{n} w_j = 1
\]

This information is sufficient to solve our system of \( n \) linear equations for \( w \). Hence, the equilibrium structure of the Markov process is found. A numerical example of these computations can be found in an article by Padberg (160).

Although the so far discussed regular Markov chains are one of the more important Markov processes they only form a subset of the ergodic chains. "A Markov chain is called an \textbf{ergodic Markov chain} if it is possible to go from every state to every other state" (121, p. 394). This definition says that all powers of the transition matrix may have some zero elements.

Quite different from the ergodic Markov chains are the \textbf{Absorbing Markov chains}. They are defined as follows (121, p. 404):

A state in a Markov chain is an absorbing state if it is impossible to leave it. A Markov chain is absorbing if (1) it has at least one absorbing state, and (2) from every state it is possible to go to an absorbing state (not necessarily in one step). In an absorbing state Markov chain the probability that the process will be absorbed is 1. The outcome of such an absorbing chain after \( n \) steps will depend not only on the number of absorbing and nonabsorbing states but also the states at which the process starts.
Economic Significance of Markov Chains

The forces determining economic processes are often so complex that the economist is faced with choosing between economic models which are either highly complicated or drastically simplified. The Markov technique belongs to the latter category and involves a few fundamental assumptions.

The first assumption is that the evolution of an economic entity through several states can be regarded as a stochastic process. When discussing the changing structure of an industry this means that the emergence of a new firm to another state is represented by a probability distribution, rather than by a complicated system which incorporates such variables as resource endowment, ambition and energy of the management, and various others. The growth of a firm is assumed to be statistical in nature and depends only upon its absolute size at the beginning.

Secondly it is assumed that this probability for the economic units moving from one state to another remains constant over time. In other words, the forces operating during the sample period are supposed to be invariant until the equilibrium is reached. Thirdly, accepting the two previous assumptions it is further taken for granted that the sample period considered provides a satisfactory estimation of the true transition probabilities. It is assumed that data from the sample period can be assimilated for each firm and for each time interval.

To which extent these assumptions restrict the practical
application of Markov chain models depends largely upon the problem under investigation. In industries where the growth rate depends largely on the personality of the entrepreneur this model is not very applicable. In situations however, where the structural development within an industry is largely dominated by general forces or where technical innovation plays a key role, such a stochastic model might very well be appropriate.

In very complex situations the Markov chain model with its simplifying assumptions might provide the only possibility for a quantitative analysis at all.

In this economic context the equilibrium situation needs some further clarification. In equilibrium the number of firms in each state is stable. It can be defined as that distribution where the number of firms entering any single state equals the number of firms leaving it. Hence the equilibrium situation is dynamic for the individual firm but static for the industry as a whole. It should be emphasized that the stochastic conception of equilibrium requires that firms move in and out. The equilibrium generated by the model is not a real forecast. Rather, it describes the structure of an industry which would result if the observed historical tendencies were to continue.

As a changeover to the last part of this discussion it should be noted that the main difficulty encountered by research workers applying this method is the search for detailed data in order to get good estimates for the transition probabilities.
The Applications of Markov processes are mainly geared to the analysis of problems where dynamic considerations play a key role. This being the case in almost all branches of agriculture, the technique deserves certainly some attention by Agricultural Economists. Given that the current trends continue, the Markov chain analysis can provide answers to such questions as what will be the size distribution of farms of various types, of agricultural processing plants, within a region, a country and so on. A short review of some of the existing literature should be helpful in an assessment of the method.

The concept was developed as early as 1907. For many years it was primarily used in Physics and Chemistry. In 1951 Solow applied this probabilistic approach first to the analysis of income and wage distributions (183). Bart and Prais employed the same technique in an investigation of business concentration (90). This study was extended by an article of Adelman in 1958 in which she successfully handled the phenomena of exit and entry of firms into the industry (1). At this time several publications appeared discussing the Markov chain concept from the mathematical point of view (121), (122). The proofs for the cited theorems are given in (121). A significant extension of the Markov chain concept was developed by Howard (108). By supposing that each process moving from state i to state j earns a reward $r_{ij}$ and by presenting these rewards in a reward matrix $R$, he relates the Markov chain concept to the principles of dynamic
programming. As such he demonstrated how it can be used for sequential decision processes of stochastic nature. In particular he analyzed the optimal replacement period for used automobiles.

In the various agricultural sources, a considerable number of articles using this method of analysis are presented. The first publications which were concerned with introducing the method and suggesting its potentials with regard to agriculture appeared in 1962. An article by Don Postwick (71) as well as one by Judge and Swanson (117) offer a good explanation of the concept. In the latter one (117) the method is applied to an analysis of adjustments of the size of hog enterprises in Illinois. The use of regular and absorbing Markov chains is demonstrated. A similar study is reported in (116). Several research projects have been devoted to the analysis and projections of farm number and farm sizes (189), (128) as well as to the study of the development of firms processing agricultural products. Fisberg (160) reports an investigation directed towards the fluid milk industry in California and Ternoy (197) made use of this technique in projecting the number of dairies in a region in France.

A weakness of the technique frequently mentioned in publications is the lack of a useful statistical method for evaluating the importance of errors in the estimation of the transition probabilities. Lee et al. (133) discussed such estimation procedures in their paper. Finally, an application of the opti-
mizing technique as developed by Howard is given by Burt and Allison in a paper dealing with rotation problems in dry land wheat farming (23).

From this review it follows that the Markov chain concept may prove to be useful in two areas of agricultural economics. Firstly it has an immediate bearing on the study of dynamic changes in the structure of the agricultural industry and its various sectors. These applications can be referred to as studies at the microeconomic level. Suggested problems to be examined might be the study of changes in farm size within a country, or within different production regions of a country, in order to draw conclusions about the expected future structure or about the efficiency of governmental policies. Investigations of this kind might be extended to such areas as the study of structural changes in the dairy industry, the hog industry and various other groups. Even the proportion of agricultural population within a country in the future can be predicted or the probable income distribution between farm and nonfarm sectors of an economy.

A second area of application lies at the microeconomic level. The explicit consideration of the probability concept provides a framework for studies under uncertainty. The combination of the Markov chain concept with dynamic programming permits operational formulations for these problems.
NETWORK ANALYSIS

The continuously declining labor force in the agricultural industry raises serious problems for the individual farmer. The most efficient use of the existing labor force and the careful organization of interfarm cooperation seems more necessary than ever. Recently developed techniques promise to be helpful tools in tackling those problems. The last years have witnessed a fast growing interest in these techniques commonly known under various names such as CPM (Critical Path Method), PERT (Program Evaluation and Review Technique), or more generally, Network Analysis or Integrated Project Management. These methods are being adopted by a fast growing number of various industries and government agencies.

Succinctly, the new methods can be described as presenting the many activities of a project in a network form revealing in a clear-cut manner their interrelationships and sequences. After assigning duration times to each activity an arithmetical procedure is used to identify those activities critical against the completion of the job in the shortest time. More advanced modifications of the method permit the calculation of the least cost schedules.

Historical Background

Originally these techniques were developed to assist in the planning of large scale one-time projects. When it developed
its Polaris Missile System, the Navy stipulated that this weapon system had to be constructed in a minimum period of time. A special operations research team formed to investigate existing scheduling techniques rejected the traditional methods and in one month devised a completely new approach. The results of this work became known as PERT. Reportedly, as first benefit of PERT the Navy succeeded in making the Polaris operational two years ahead of schedule.

At about the same time a team of mathematicians and engineers of the Du Pont Corporation devised a similar scheduling technique, that became known under the name CPM. It was aimed at reducing the costs of plant overhauls and of scheduling construction work. The methods are based on the same principles: the use of a network (arrow diagrams) to portray the activities (jobs or tasks) and the use of certain computational procedures to identify the critical path. Differences, apart from terminology, concern some computational details and extensions. Whereas the Navy team used three time estimates to take care of uncertainties in estimating the duration time of the activities, in the CPM approach only one time estimate is required. Yet, already at an early stage of development cost considerations were introduced in the CPM approach and in this case the estimation of a second duration time, the so called crash time, becomes necessary. Over the last few years both methods have become increasingly similar and thus, no further distinction will
be made between them.

To give some idea about the underlying theoretical work a few important contributions shall be mentioned. Many aspects of the new techniques are based on graph theory. The concepts of graph theory have evolved over many years, major contributions being publications by Koenig (126), Berge (15) and Ford and Fulkerson (76). Despite this early work, it was not until 1958 that Project Networking found a formal definition by the work of the two research teams. The original work of the Navy team is published in an article in The Journal of the Operations Research Society of America (151). Theoretical background and proof of the subject-matter have been illustrated by Clark (37) and Kelley (119). Fulkerson (80) and more recently Berman (16) pioneered various cost aspects of the technique while several publications deal with statistical aspects (180), (155).

Theoretical Concepts*

The following discussion presents a few terms of graph theory, that are underlying all more advanced treatment of networking.

Assume, that are points in a plane, called nodes and lines, called arcs (also edges or arrows) connecting certain pairs of

*This chapter is largely based on the following references (204 Chap. 4), (119) and (148 Chap. 6).
nodes. Such a collection of nodes and arcs is referred to as a graph.

A path connecting nodes $i$ and $j$ is an ordered set of arcs such that each node is the endpoint for two arcs in the set, with the possible exception of the last and first node.

A loop is defined as a path where $i = j$.

Introducing a sense of direction, the origin of an arc is defined as a source and the end as the sink. Such an arc is said to be orientated.

Now, consider a graph and assign numbers to each arc which may be interpreted as flow capacities. Such a graph is called a network. Thus, an orientated network is a graph with attached flow capacities and a sense of direction.

A project network is defined as "an oriented network that has an oriented path from a single source node to every other node in the network, and from every node in the network to a single sink node, but contains no oriented loops (148, p. 148)".

Assume further, that each arc has a capacity restriction denoted by $c_{ij} \geq 0$. The flow labeled as $y_{ij}$ from node $i$ to $j$ must satisfy the constraint $0 \leq y_{ij} \leq c_{ij}$.

Given this information let it be required to compute the maximum possible flow from source to sink. This task can be formulated as a linear programming problem.

Before continuing the discussion from this point, a few terms common to scheduling techniques shall be introduced, in
order to show the significance of graph theory for the networking problem. The new scheduling techniques require the project to be described completely by two terms, activity and event.

An activity (also job, task) is any well defined, distinguishable portion of a project. It has definable beginnings and endings and cannot start until certain other activities are completed.

An event is the beginning or endpoint of one or several activities. It has not been achieved until all activities leading to that event are completed. Theoretically an event is an instantaneous point in time. If it represents the joint completion of more than one activity it is referred to as a "merge" event, while the joint initiation of more than one activity from an event signifies a "burst" event.

A network serves to present the sequential order of activities and events in a schematic way. It is fundamental to distinguish between two different ways a network can be used to portray activities and events.

In the activity-on-arrow-system, as the term implies, the arrows represent the activities and correspondingly the nodes the events. A time estimate is attached to each activity. Arrows representing merely a dependency of one activity upon another are called dummy activities and their attached time estimates are zero. Each arrow lies between two events, the tail of the arrow indicating the beginning and the head signifying the
completion of that activity. Hence, the arrow implies logical precedence only. Every activity preceding a particular event must be completed before any activity emanating from that event can begin.

In the activity-on-node-system the activities are graphically portrayed by nodes and the arrows serve only to express the dependency relationships among the nodes. By using this system the idea of events as such becomes superfluous. Dummy activities also will not occur, unless all arrows are considered as dummies since they represent only precedence relationships. The main advantage of this system lies in its simplicity and although it is not yet in wide spread use it seems possible that it will displace the activity-on-arrow representation in the future.

After this short introduction to a few definitions of graph theory and to some terms used in scheduling we can put together both concepts and continue the previous discussion. (The following formulations pertain to the activity-on-arrow system).

Given a project network having a source labeled 0 and a sink labeled n there are \( n + 1 \) nodes which are representing events. If event \( i \) precedes event \( j \) it can be written as \( i < j \). Attached to each event \( i \) is its occurrence time \( t_i \). Thus, if \( i \) precedes \( j \), \( t_i < t_j \).

An activity is defined by two events, \((i, j)\) such that \( i < j \) and each activity is characterized by its duration time \( y_{ij} \).
For \( y_{ij} \) the following condition must hold:
\[
y_{ij} \leq t_j - t_i
\]
or:
\[
y_{ij} - t_j + t_i \leq 0
\]

The duration time of an activity can be subjected to certain restrictions the simplest one being the following:
\[
d_{ij} \geq 0
\]

Thus, the duration time \( y_{ij} \) of the activity \((i, j)\) must satisfy the following constraint
\[
0 \leq y_{ij} \leq d_{ij}
\]  
(1)

The similarity of this problem to the previously discussed maximum flow problem can be seen. Here, it is required to find the longest path from source to sink. A linear programming formulation again is possible.

In the last few years this problem underwent divers extensions and modifications and various algorithms are reported in the literature. In addition to the already cited publications the reader, interested in more technical details, is referred to a few more articles listed in the bibliography (4), (105).

Practical Procedures

The actual work of applying a network technique to a particular problem consists of the three steps planning, time estimation and scheduling. A cost analysis might be added.
Planning

Planning can be characterized as defining activities, exploring their order relationships and presenting this information in an arrow diagram. Any project can be divided into several activities having the property that certain activities must be completed before other ones can be started. For instance, fertilizer cannot be spread before it is bought. In an initial examination of the project these activities must be defined and the precedence relationships among them recognized. After doing this, the researcher can move on to the actual planning by drawing the network that reveals the order relationship between all activities.

Time estimation

For each activity included in the network a duration time has to be estimated. The procedure to follow depends largely on the type of project under scrutiny and in any case experience will be a major factor in obtaining good estimates. Usually the breakdown of a complex project into many activities facilitates these estimation procedures.

The main requirement for a successful handling of these two steps, planning and estimating is a very close knowledge of the project on the part of the person executing these steps. The few networking rules necessary for the graphical presentation can be learned in a short time.

Scheduling

The third and major phase of the network analysis is refer-
red to as scheduling. It comprises several computations which yield the earliest and latest start and completion time for each activity, the slack time for the activities not on the critical path and finally, the critical path itself. The scheduling computations consist of a forward and backward pass through the network.

Having performed phase one and two, planning and estimation, the question arises as to which is the earliest possible completion time (EPC) for any activity. The forward pass provides an answer to this question for each single activity by adding together the time estimates of the activities along all possible paths leading to it. From this analysis it can be concluded that the earliest possible completion time for the whole project is the longest of all computed EPCs. Thus, the forward pass reveals the sequence of those activities that form the longest path through the network. By subtracting the duration time of each activity from its EPC the earliest possible starting time can be obtained.

Similarly, computing the backward pass yields the latest allowable start and completion times for each activity.

The difference between these two computations gives the total slack for each activity. Total slack is defined as the amount of time that a single activity can be postponed without affecting the completion of the total project. As opposed to the total slack, the free slack is that amount of time which an activity can be delayed without affecting any other activity.
The critical path is defined as the sequence of those activities whose slack is zero. It may also be defined, following Jellinger (114, p. 7) as "that sequence of operations in which the delay of any single activity will delay the final completion time. Conversely, the speeding up or earlier completion of any single activity in the critical path sequence will advance the final completion date of the total project." Sometimes, several critical paths might occur within a network, especially if similar operations are going on at the same time.

**Time-cost trade-offs**

The many attempts to include cost considerations into network analysis methods have resulted in one generally used computerized procedure. The underlying assumption is that by decreasing the duration time \( y_{ij} \) of an activity from the normal job time \( y_{ij} \) to the shortest possible time \( r_{ij} \) (crash time) the costs attached to that activity are increasing linearly. This condition imposes an additional restriction on the duration time and inequality (1) on page 76 might be rewritten as

\[
0 \leq r_{ij} \leq y_{ij} \leq d_{ij} \leq \infty
\]  

The purpose of this formulation is to reduce the project duration time with a minimum increase in costs by buying time along the critical path where it can be obtained at least cost.
An Example

To illustrate the discussed procedures a simplified example will be presented. A farmer decides to plant 10 acres barley. This will involve the five steps of ordering fertilizer, ploughing, harrowing, fertilizing, and drilling. These activities must follow in a certain sequence. Fertilizing cannot start before the fertilizer is procured and the ploughing is completed. The ploughing must also precede the harrowing and the drilling depends on all other four operations. The five activities, their duration time in days and the results of the scheduling computations are given in Table 1. In Figure 1 and in Figure 2 the two systems of representation are used to portray the project graphically.

In Figure 1 the squares represent the activities. The numbers in the left and right upper corner show the earliest possible start and finishing time, respectively. The numbers in the lower corners the latest start and finishing times. The duration time is denoted in the center of the square and the free slack below each square. The arrows express order relationships and the first square is a dummy activity to denote the start.

In Figure 2 the arrows show the activities, the nodes the events, while the slashed arrow is a dummy activity. This representation is added to allow a comparison of the two methods.

The results show that the speeding up of the ploughing or harrowing would permit an earlier planting time. The fertilizer
Figure 1. Activity - on-node representation

Figure 2. Activity - on-arrow representation
delivery could take two days longer without postponing any other activity or three days longer without delaying the planting time.

Table 1. Results of scheduling computations

<table>
<thead>
<tr>
<th>Activity</th>
<th>Symbol</th>
<th>Duration time</th>
<th>Earliest Start</th>
<th>Earliest Finish</th>
<th>Latest Start</th>
<th>Latest Finish</th>
<th>Slack Total</th>
<th>Free</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delivery of Fertilizer</td>
<td>A</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Ploughing</td>
<td>B</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Harrowing</td>
<td>C</td>
<td>2</td>
<td>6</td>
<td>8</td>
<td>6</td>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Fertilizing</td>
<td>D</td>
<td>1</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>8</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Drilling</td>
<td>F</td>
<td>1.5</td>
<td>8</td>
<td>9.5</td>
<td>8</td>
<td>9.5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Given these duration time it is clear that ploughing and harrowing are the two activities on the critical path.

Applications and Evaluations

During the past few years the use of critical path methods has grown very rapidly. Applications have been reported from various industries, military services and government agencies. In many of these applications projects with several thousands of activities were analyzed and considerable gains were realized due to the method. This experience has shown that the main benefit of the method is in encouraging long range and detailed planning of projects and the possible identification of the most
critical elements in a plan. This identification may allow a reallocation of resources from activities not on the critical path to activities on the path or in case of no time pressure the diminishing of costs by releasing resources from the project.

The few publications available concerning agriculture offer an introduction to the main ideas of the method and some applications of a primarily exploratory nature. In addition to a comprehensive explanation of the technique Cooke-Yarborough (42) described an example of scheduling the planting of wheat in Australia. Schroeder (171) applied the method to a rural community development program in Canada. Based on a variant of the PERT technique Morris and Nygaard (149) studied, in a complex system, the least cost way of hog manure handling. One of the most comprehensive studies is reported by Heiland et al. (105) from Germany. Two examples of the labor economy are used to illustrate the technique, the more complex problem being solved by use of linear programming.

It seems, that the potential benefits to be realized from these techniques within the agricultural field have barely been touched and a far range of potential uses can be suggested. Certainly, these methods will prove to be of help to agricultural administrations on both state and national level, for instance in launching educational projects, farm programs or budgeting. Similar gains might be realized by large firms proc-
essing and marketing agricultural products. One of the first applications of CPM was the scheduling of an advertising program to launch a new product.

When discussing the usefulness of these techniques on the farm level another characteristic of the critical path methods should be borne in mind in that the three basic steps can be equally successful when applied to small scale projects as many examples in industry have shown. Projects in the range from 10 up to 100 activities can be diagrammed and scheduled in a few hours by hand and only a short instruction period is required to obtain a working knowledge for problems of this size. Thus, it seems possible to use these techniques on the farm level as a research tool as well as a practical management help.

As a research tool it might be of value for the analysis of the various labor activities on a farm. The organization of activities constituting a farm operation in the most time saving sequence might often lead to diminishing labor costs. This problem is hardly recognizable on a highly specialized farm where the sequence of operations is often almost fixed due to biological reasons. However, it can be of great importance on diversified farms where frequently several different operations are required to be performed at the same time and the best sequence of these operations is not obvious. As an example we might think of a farmer who has a time period of six weeks in September and October, in which he has to fill his silos, to harvest his sugar beets and to plant wheat. The sequence of
these operations, which consist of many single activities, is often not at all evident and the use of networking techniques might possibly reveal considerable gains in working time.

In regions with small farms the effort of cooperation among several farmers in an attempt to reduce their machinery costs creates many organizational problems. Under these circumstances the use of networking might be suggested in order to investigate into the various forms of organizing this cooperation.

Exact investigations of rather short but daily occurring labor sequences such as the feeding of animals can also be undertaken by the use of this technique (105). The old-type farm time and motion studies or work simplification studies are to be mentioned in this context. A further field for the application of network methods might be the study and design of development plans for farms in regions of rapid structural change. The explicit consideration of time elements might offset certain other disadvantages. After defining a long run objective for a farm the various steps necessary to attain this goal can be presented synoptically in the network.

Since this graphic portrayal provides the farmer with a logical and clear framework along which he can organize his thoughts in order to save labor and since the basic procedures can be learned fast, the use of networking by interested farmers for solving practical management questions seems very possible. The farmer's close knowledge of the practical problems will be a valuable asset in setting up networks for farms. In addition to
the previously cited examples farmers could use this technique effectively to monitor the many facets of construction projects that occur on every farm in the course of a few years.

In view of all these potential benefits it seems that much greater use of the method would be very profitable. This will have to be combined with the design of better data collection methods, as the lack of appropriate labor data frequently imposes rigid limitations on these studies.
In this section models will be discussed that could be considered as classics in Operations Research. Inventory problems, replacement problems and to a lesser degree queueing problems have been of main concern to researchers as soon as the first mathematical based studies of business operations appeared. A series of standard publications such as the books by Churchman et al. (36), Sasieni et al. (167), or Fabrycky and Torgersen (71) give a comprehensive discussion of these models and the reader interested in more details is referred to these publications.

Replacement Models

Replacement theory is concerned with cases in which the adequacy of an asset tends to worsen with time. In order to obtain the initial level of adequacy the asset under scrutiny requires replacement. Assuming that the purpose of each replacement decision is to secure an economic advantage, replacement theory tries to define at which point in time replacement has to take place.

The declining adequacy of an asset might be caused by various reasons such as changing demand requirements, new technological development, diminishing productivity, or increasing maintenance expenses.

All replacement models whether taken from industrial or biological problem areas have the following structure. Two assets
require evaluation: The present capital good and its potential replacement. The evaluation of both items has to be based upon their expected future performance. The expected costs and outputs have to be discounted to the same point in time and the comparison of these estimates will reveal which course of action is the most profitable.

The reasoning can be illustrated with the following simple example. Consider a farmer who must decide when a tractor should be replaced. Assuming that the new tractor is of the same kind as the old one and that the farmer tries to minimize his average cost per tractor per year he must consider the following calculations. The capital cost per year due to the purchase of the old tractor will decrease the longer it is used. Its running cost will increase the longer the replacement decision is procrastinated. At that point in time where the increasing running cost more than offsets the saving in capital cost the replacement should take place. A solution to this problem can be obtained in the following way:

Let \( r \) be the year of replacement, \( C \) the purchase price of the tractor (Capital cost), \( i \) the interest rate, \( v \) the discount rate and \( R_n \) the running cost in year \( n \), which are increasing from year to year. \( P_r \) is the present value of expenditures for running costs after \( r \) years and \( TC \) are the total cost per year.

\[
v = \frac{1}{(1+i)}
\]
\[ P_r = R_1 v R_2 v^2 R_3 \ldots v^{r-1} R_r \]

\[ TC = \frac{C}{r} \frac{v^{r-1} R_r}{r} = \frac{C}{r} P_r \]

In the period where TC is a minimum the replacement should take place.

Unfortunately, most replacement models are of a much more complex nature involving changed capacity of the new item, inflation, and other variables.

Depending upon the life pattern of the asset under study a distinction can be made between two main categories of replacement models. Those where the equipment deteriorates with time as in the case of the tractor example and those where the assets either work of fall completely e.g. electric bulbs. To analyze the latter type of models the use of probabilistic concepts is required. The principle of dynamic programming provides a general model for most of the second type replacement problems.

**Applications**

Replacement models have an increasing significance for agriculture. As the process of substituting capital for other inputs in agriculture continues, it will become of increasing importance to formulate replacement policies for the various capital items on a sound economic basis. A brief discussion of a few applications in agriculture follows in order to illustrate how large a range of decision problems in agriculture can be formulated in terms of a replacement model.
Several studies were concerned with the problem of optimum annual rate of flock replacement in poultry production. Back and Becker (8) derived the optimal replacement pattern by analyzing various cost and return functions from several experiments. White (217) approached the problem by formulating an equation that represents the recurrent relationship in a continually operating egg production enterprise. In trying to find the optimum replacement policy he used the concept of dynamic programming as developed by Bellman. In all these studies the flock performance is assumed to be known with certainty.

Tedford (196) carries the analysis of the flock replacement problem one step further by considering imperfect knowledge of the expected output performance of a flock of birds. He makes use of an extended Bayes model.

Faris (72) presented some discounting and compounding procedures to determine a replacement criterion for longlived assets such as timber. The validity of his replacement criterion has been questioned (37). An example of tractor replacement similar to the one discussed previously was given by Dunford and Richard (63). Caniler studied (28) the influence of interest rate on farm machinery replacing.

Considering the second type of replacement models it can be argued that many items in agriculture are subject to chance destruction that calls for immediate and complete replacement. Burt (22) cited as examples dairy cows or livestock breeding herds where random failure due to disease, death, physical in-
jury or fire requires replacement. To cope with these questions or involuntary replacement due to random events a modified stochastic model, originally developed by Howard, was used.

As these applications show, the use of replacement models in agriculture can be suggested for many situations where investment decisions are to be made. Replacement theory and investment theory are closely related. Decisions about purchase of machinery, tractors, buildings, about replacing animals that either give a rise to a daily income such as hens or milkcows or to a single lumpsum income such as beef cattle of hogs are examples in point.

Inventory Models

During the last thirty years an impressing number of studies has been devoted to inventory problems. The primary objective of these studies was to define the most profitable inventory policy. The large interest of business analysis and economists in this problem is understandable for in almost all business and industrial operations a stock of certain goods has to be maintained to warrant a smooth running of the enterprise. This stock might be held in the form of raw material, as in-process inventory, or as finished goods inventory. In any case, keeping of such an inventory stock gives rise to expenses and the optimal inventory policy tries to define that level of inventory which should be carried in order to minimize the costs
involved in these operations.

In making decisions regarding inventory levels the advantages and disadvantages of a large inventory stock must be pondered. Possible advantages of a large inventory level are the economies of production due to large run sizes, the avoidance of shortage costs that have to be paid as penalty for not meeting a demand, and potential profits from speculation in a market of rising prices. On the other hand, the main disadvantage of a large inventory is the carrying costs that occur. These costs include insurance, taxes, deterioration and obsolescence, warehouse rent, various operating expenses, and the interest on the invested capital.

In trying to derive the best inventory policy, the supply of new items to the inventory stock and the demand pattern for these items have to be examined. The demand for the commodity in stock can either be known with certainty (deterministic) or may be subject to a probability distribution (probabilistic). Many agricultural products being supplied at a given level each year are facing a demand that is deterministic and infinitely elastic. In this case the inventory planner can easily control the level of inventory by either selling or storing his products and the policy to follow will depend upon his price expectations. On the other hand, when the demand for a commodity is beyond the influence of the operator, he might exert control over his stock level by changing the number of production runs and the size of each run. A simplified example of this type is
presented below.

Let \( R \) be the demand for a good per unit of time, and \( t \) the number of time units between two production runs. \( S \) is the set-up costs, occurring each time a production run starts and \( H \) the costs for holding one unit of inventory for one unit of time. It is assumed that the cost of holding inventory is proportional to the amount of inventory \( (I) \) and to the time \( (T) \) it is hold. In addition, assume that the production time (lead time) is negligible and that whenever inventory is at zero level a new production starts so that no delay in fulfilling the orders can occur, i.e. the demand is deterministic, the procurement lead time is deterministic and negligible and the penalty for shortages is infinitely high. It follows from these assumptions that at each run an amount of \( Rt \) must be produced to fulfill the demand in \( t \) time units.

\[
I = Rt \int_{0}^{t} dt = \int_{0}^{t}Rt dt = \frac{1}{2}Ht^2
\]

The holding costs of inventory per production are

\[
\frac{1}{2}Ht^2
\]

The average total costs per unit of time are

\[
C = \frac{1}{2}Ht^2 + \frac{S}{t}
\]

To minimize \( C \) we differentiate with respect to time and set equal to zero.
\[
\frac{dC}{dt} = \frac{1}{2}HRt \quad \frac{S}{t} = \frac{1}{4}ER - St^2 = 0
\]
The optimal timing for each production run is \( t = 2S/HR \) and the quantity to be produced at each run is \( q = Rt = 2RS/H \). These two expressions define the optimal inventory policy for this example.

As a rule real world problems are much more complicated. Such complications arise when the procurement lead time, defined as the time necessary to replenish an inventory stock, and the demand for the commodity are known only as probability distributions. Nonlinearities in the cost functions might add to the complexity of the problem. In many cases simulation techniques and Monte Carlo Methods are used to solve these complex problems.

**Applications**

Several articles in agricultural literature used inventory models for their analysis. Applications at the farm level have been reported so far only from Australia. Candler (24), (27) formulated the problem of optimum forage reserve in face of a possible draught as an inventory problem. In these studies the future demand is only known as a probability distribution. A similar study was put forward by Mauldon and Dillon (144). Tolley (202) described the use of inventory analysis in a meat packing plant, while Gislason (83) developed grain storage rules for public policy by using a similar model. A more recent investigation into the optimum feed reserves under
draught conditions assuming a probabilistic demand is presented by Afzal et al. (2).

In summary it can be said that whenever the storage of large inventory stocks occurs, the use of these types of models might be suggested to reduce the costs involved. In the agricultural field the storage of the various farm inputs and outputs is becoming more and more the concern of large supply firms, cooperatives, processing firms, and government agencies. These firms could use inventory models for determining the best inventory policy for such inputs as fertilizer, seeds, implements, and products, such as cheese, butter, meat, and grain.

As the few previously cited examples show, these models can also be of help in solving certain problems at the farm level. In addition to the determination of the optimal forrage stock in the face of weather uncertainty the amount of fertilizer or seed to be kept on the farm can be determined in this way if past experience proved that shortages of these inputs are likely to occur. A special case in point is the decision as to whether harvested grain should be sold immediately or be kept on the farm in the expectation that rising prices will more than offset the carrying costs.

Queueing Models

Queueing problems show a certain similarity with inventory problems. Both are concerned with the accumulation of items (or tasks) and with subtractions. In queueing problems the accre-
tion of items are usually random and the subtraction are under a certain control, whereas in inventory problems the accretions are partially controllable and the subtractions usually random. The study of both models shows certain similarities.

Waiting line or queueing theory models are widely used models in operations research. In everyday life waiting lines are encountered frequently. The customers in a supermarket waiting to pay their bill form a queue, as do the idle standing agricultural machines waiting for the repairman, or the planes awaiting permission to land at the crowded airport. Generally, any situation where arriving units have to wait for a desired service can be classified as a queueing problem.

Usually, costs are associated with waiting in a line. They are referred to as waiting costs and might be of quite different nature. The delay in repair of an agricultural machine might cause considerable loss to the farmer, the planes awaiting landing permission use fuel and time, the customer in the supermarket irritated by long waiting lines loose their good will towards the store and their switching to a competitor inflicts a loss to the proprietor. On the other hand, the provision of additional service facilities to reduce the waiting costs is expensive. These costs are referred to as service facility costs. They usually consists of the capital cost for a new service channel (e.g. costs of constructing a new landing strip) and the operating cost of this new facility.

Given such a problem the task of queueing theory is to min-
imize the sum of waiting costs and service facility costs. To attain this objective the capacity of the service facility and the characteristic of the arrival pattern must be analyzed.

The simplest problems in queueing theory are those where the amount of time necessary for service and the arrival pattern of the units, to be served, are known in advance, i.e. the future demand for service can be predicted with certainty. These models might be classified as deterministic while most real world problems are of a probabilistic nature. The service time is a variable taking on values of a specific probability distribution and the same holds for the items that require a service. Their arrival pattern depends upon the nature of the population that causes the demand for service. Further classification of queueing models is possible according to the existence of one or several service facilities, one or several queues and whether the arrivals come from a finite or infinite population.

A widely used queueing model will be characterized below by a few basic equations. Assume that there is one service facility and one queue. The arrivals emanate from an indefinite population. Both arrivals and services occur in accordance with a Poisson distribution.

Let \( w \) be the number of units in the queue, \( n \) the number of units in the system, made up from the items in the queue and the one being served, and \( t \) the time an arrival must wait for service. \( P_n \) is the probability of having \( n \) units in the system and \( \lambda \) is the mean arrival rate (the average number of customers
expected to arrive in one unit of time). \( \mu \) is the mean servicing rate (the average number of customers the service facility completes in one unit of time, assuming no shortage of customers). Given these definitions and assumptions the corresponding queueing model can be described by the following three equations:

\[
F(w) = \frac{\lambda^2}{\mu (\mu - \lambda)} \quad \text{average queue length}
\]

\[
F(n) = \frac{\lambda}{(\mu - \lambda)} \quad \text{average number of units in system}
\]

\[
F(t) = \frac{\lambda}{\mu (\mu - \lambda)} \quad \text{average waiting time of an arrival}
\]

The above equations apply only for \( \frac{\lambda}{\mu} \leq 1 \). The ratio \( \frac{\lambda}{\mu} \) is known as the utility factor. If it exceeds 1 the length of the queue will grow toward infinity. If the queue is to be kept short the utility factor must be substantially less than unity, i.e., the service facilities must be idle during a large proportion of time. Thus, idle capacity and prompt service go together.

**Applications**

A review of the literature reveals that the bulk of applications of queueing models was done in large industrial enterprises. The use of this model in agriculture is very limited. Cox et al. (144) studied the most efficient number of unloading facilities at a livestock market. Simmons (178) reported a sim-
ilar study of unloading facilities for fleet milk distribution. A demonstration of queueing theory is given in an article by Lu (139) in which he analyzed checkout facilities of a supermarket.

Applications of queueing theory to management questions of normal average-sized farms have not been reported in the literature. But it is conceivable that management problems of large scale farms could be formulated in this framework. The purchase of an additional machine might be looked upon as the installation of a new service facility and the completion of the harvest in a shorter time as the reduction of waiting costs.

The significance of the model might increase over the future as the "automation" of feed handling, and storage and livestock operations become more common. A development in this direction can be observed and the continuing trend to large farms might increase the importance of "automation." The optimum number of trucks for feed hauling, the best size and number of unloaders for silo filling operations, and the right capacity of conveyors for automatic cattle feeding might be formulated in this framework. The use of this model can further be suggested for cooperatives and other services providing firms in deciding such questions as the best number of repairmen in a tractor repair station or the optimum number of various machines required for custom operations.
CONCLUSIONS

The various mathematical tools presented in this study invaded agricultural economics in the last 15 years. The term "invasion" seems justified as the following summary will confirm.

Linear programming was introduced in agricultural economics in 1951 (213). The first application of game theory was reported in the literature in 1957 (190). In subsequent years the introduction of new techniques occurred at an increasing speed. The first study using nonlinear programming was described in 1960 (82), followed by several applications of Markov chains (117), (21), in 1962. One year later a dynamic programming problem was first described in the agricultural literature (23), and the year 1964 witnessed a comprehensive presentation of network analysis (39). In 1965, simulation techniques were initially applied to farm management problems (223), (89).

While researchers often praise the superiority of these new techniques over previously used methods and eagerly accept new and more sophisticated creations being supplied by mathematicians, the practical farmer who, in many cases, is supposed to be the final recipient of these tools hardly notices their existence. This situation has been criticized frequently (110), (113) and deserves a moment of consideration.

To do so, let us divide the agricultural field into three sections: research, teaching, and extension, where extension
shall represent the application of operations research techniques to management problems of individual farmers. The modern techniques have definitely conquered the research field and are revolutionizing the teaching of management. It is becoming evident that the education of the future agricultural economist has to be supplemented by training in research techniques, computer use, in mathematics and statistics (210). Until the time that people with this education are available in extension work a large communication of these methods to farmers cannot be expected. This might be partial explanation for the lagging applications of these tools.

Yet, the above situation can be interpreted in a different fashion. The fast growing family of decision tools in economics is nothing more than the manifestation of a large revolution taking place in many fields, the computer revolution. The use of complicated decision techniques for management problems is only one facet of the services computers render to agriculture. A second area is the field of data processing. In the immediate future, it is likely that the average farmer will gain more from this service than from the applications the various discussed decision tools, because there will be a shortage of qualified staff to apply these tools to the many farms. However, it would be a far-reaching omission on the part of agricultural researchers if they did not attempt to supervise the installation of these record keeping systems, because a design of data collecting systems without regard to the data requirements of the
available mathematical models is a waste of effort. Only a de-
tailed knowledge of the mathematical models can assure the de-
sign of proper data processing systems. This point is further
emphasized, since the successful application of modern decision
techniques to practical farm problems hinges on the careful re-
cording of specific data needed for these decision tools. These
data recording systems should not only fulfill such conventional
requirements as providing a basis for tax purposes, intrafarm
comparisons, and calculations of various indices but should also
satisfy the data requirements of specific decision models, such
as: input-output coefficients for linear programming, labor data
for networking, data to fit functions for dynamic programming
and detailed records about the transition of micronits in time
for Markov chains. Only researchers are in a position to war-
rent a smooth combination between the two services, data re-
cording and application of decision tools, and it is certainly a
challenging task that is waiting for them in the future.

Attention will now be given to a comparison of the relative
merits of the discussed research tools. This will be done in
two ways. First an attempt will be made to answer the question
as to which tools deserve the most attention in agricultural
teaching and extension when the general problem area is farm
management and available instruction time is limited. Secondly,
the models will be classified with regard to their appropriaten-
ness to specific research problems.

Turning to the first question, linear programming certainly
deserves to be mentioned at the beginning. The wide spread use of this model can be regarded as justified. Naturally, various criticisms have been raised against some of the underlying assumptions, e.g. Edwards (66), but its overall usefulness is not in doubt. Edwards concludes that the problems originating in mathematics, statistics, and economics of programming are surmountable and cites the deficiencies of programmers as a main bottleneck for successful applications. Musgrave (151) in a survey of linear programming applications in Australia, assessed its contribution as being very positive. The adequacy of the linear programming model has recently been emphasized in a study by Merrill (145) in which he reported that the solutions obtained from highly sophisticated planning models (a stochastic programming model and a linear team model were used) differ little from those obtained from a standard linear programming model. The significance of nonlinear programming at present lies primarily in providing a better understanding of the linear programming limitations. In many cases the information which could be obtained by one computer run with a nonlinear model can be gained in several runs using a linear model. Yet, as nonlinear computer programs become more easily available they will be used to encompass linear programming work. The significance of integer programming in particular, in combination with the linear model certainly deserves to be studied.

Another technique, worthy a closer examination with respect to farm management is network analysis. The advantages of this
method have been elaborated to sufficiency. A further ordering of the techniques depends on the particular situation. But, keeping the first question in mind, dynamic programming and simulation would certainly end the list, since the additional efforts for their application are not likely to be offset by the additional gains. On the other hand, if the objective was to investigate possible contributions of new techniques, dynamic programming and simulation would probably be the first methods to be considered.

The second question to be asked is which methods have the largest potential for particular problem areas, such as uncertainty and time and investment decisions. A mere enumeration shall suffice. The consideration of uncertainty is made possible by the following techniques: sensitivity analysis and parametric programming, decision theory, stochastic programming, dynamic programming, and simulation. The incorporation of the time element is possible in dynamic linear programming, dynamic programming, Markov chains, and simulation. In some manner all discussed techniques have a bearing on investment decisions. Dynamic programming, integer programming, and replacement models are of immediate significance to this problem. The intricate problems involved in investment decision have been approached from various standpoints, examples being contributions by Cotner (143), Lerner (135), and Wadsworth (208).

In discussing the adequacy of various models for problems
at the microlevel, the transportation model, the Markov chain concept, and simulation deserve mention.

As was pointed out, compared to the huge effort in the research area, the actual use of modern management tools on the farm level is still limited. A brief breakdown of these applications shall be given. There are four main types of agricultural problems to which computers and operations research techniques have been applied in the last years. Firstly, the calculation of least cost rations for beef and dairy cattle. This service is available (for instance) to farmers in California. In an experiment this service was made available to a group of farmers in Switzerland (87). Secondly, the calculation of optimum plans for individual farms by extension services or commercial consulting agencies (191). Thirdly, management games are used for teaching at several universities and tentatively at workshop with farmers. Lastly, the area of electronic data processing of farm records in which field work is being carried out by many extension services in the United States and in other countries (169).

In the years ahead the combined utilization of these techniques will certainly expand rapidly. A system of simulation and optimization techniques being set up on the computer, in which the farmer also maintains his accounts, might soon be available to solve readily management questions upon the request of the farmer. In a further step the computer might even be
used to monitor certain production processes.

However, it is likely that the proper use of the large amount of data soon available and of the information, gained from the utilization of operations research tools, will create new problems of interpretations. Maybe the concepts of information theory will have to be relied upon to define that amount of information that is adequate for the capacity of the information centers and channels that we encounter on farms.

This last prospect is typical for modern management research and the world of decision making (35). This area seems to be ever expanding in scope and complexity and, repeatedly it can be observed that the successful solution of one problem opens the door to a dozen new problems. The objective behind all these feverish efforts is to gain an understanding of our own decision making. This however, is still far beyond sight. Perhaps, it will forever remain as the attainment of the understanding of one's decision making implies the understanding of oneself. This fundamental problem of philosophy has reappeared under changing formulation in all ages of history.

The essence of this digression is that the potential of these tools should not be overestimated. They are mainly able to free one from routine decisions at lower levels of management. However, the conception of the problems, the recognition of those areas where the use of computers would reveal most potential gains, and the making of those decisions that have last-
ing impact of one's life remain as yet unmechanized. Or, as Waugh (212) put it: "A machine could figure today's least cost of dairy feed, and probably mix it for us. But I doubt if we should ever be satisfied to let electronic computers run our farm".
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