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Analysis of steady state voltage stability in large scale power systems

Colin David Christy

Iowa State University

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Analysis of steady state voltage stability
in large scale power systems

by

Colin David Christy

A Thesis Submitted to the
Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
Department:  Electrical Engineering and Computer Engineering
Major:  Electrical Engineering

Signatures have been redacted for privacy

Iowa State University
Ames, Iowa
1990
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CHAPTER 1. INTRODUCTION

The Voltage Stability Problem

In the operation of an interconnected electric power system, certain conditions may be encountered wherein the bus voltage magnitudes in a part of the system become uncontrollable or even collapse. When this set of conditions occurs, this resultant voltage instability or voltage collapse can have a drastic impact on both the reliability and economy of the system. Past incidents of voltage collapse have caused millions of dollars of equipment damage and have produced service interruptions to thousands of customers at a time [1]. Even though these grave consequences have motivated an increased level of precaution on the part of system operators, the occurrence of voltage collapse has increased in recent years. This increase, to a large extent, is because the increased stress that is being placed on interconnected systems. In particular, most utilities are serving higher and higher peak loads, but their capacity is expanding at a rate slower than their increase in peak load. In addition, more and more of the existing transmission line capacity is being used to transfer large quantities of power from one utility to another [2]. Each of these trends results in an interconnected system that is operating closer to its voltage stability limits.

While the trends that endanger the voltage stability of interconnected power
systems are reversible, their reversal would come at an enormous economic cost. Each new generating facility and major transmission line can cost millions of dollars. Also, the curtailment of sending inexpensive power from one utility to another would mean halting an activity that is profitable to both the buying utility and the selling utility. Given this trade-off between economy and reliability, there is a very real need to use the existing system capacity to the fullest, without sacrificing reliability or paying the price of a catastrophe brought on by a loss of stability. To do this, operators and planners must fully understand the voltage collapse phenomenon and prevent it from occurring.

Analysis Problems

One problem that has arisen in the analysis of voltage stability is that the analytical tools that have been used in the past to plan and operate the system are not sufficient for investigating voltage stability. A case in point is the Newton-Raphson power flow, which becomes numerically unstable in the region of the power flow solution trajectory that is of the greatest interest in voltage stability analysis. Since the power flow equations are a key part of the steady state model of a power system, an alternative to the Newton-Raphson method is needed to fully investigate steady state voltage stability.

Scope and Objective

In this thesis, a viable alternative to the Newton-Raphson power flow is presented and applied to the investigation of steady state voltage stability. This method, called the "continuation power flow" is more numerically robust than existing methods.
It also produces sensitivity information that can be readily used to calculate voltage stability indexes and to identify the areas of the system most prone to voltage collapse. Once the continuation power flow is used to solve the power flow equations in the region of interest, system stability theory can be applied to investigate the stability of the solutions in various regions of operation.

At this point, it should be pointed out that an analysis of the power flow equations reveals part, but not all of the picture of steady state voltage stability. This point is substantiated by looking at an accepted definition of the steady state stability of a power system [3].

A power system is steady state stable for a particular steady state operating condition if, following any small disturbance, it reaches a steady state operating condition which is identical or close to the pre-disturbance operating condition. This is also known as small disturbance stability of a power system.

Here a small disturbance is defined as

a disturbance for which the equations that describe the dynamics of the power system may be linearized for the sake of analysis.

Since the power flow equations describe only a portion of the dynamics of the power system, the machine, control system, and any other relevant dynamics have to be included to make final conclusions about the ability of the composite system to withstand small disturbances. However, an analysis of the power flow equations is still quite relevant since an inability in this portion of the system to withstand small perturbations necessarily implies steady state instability of the power system.
Given this narrowed scope, the objectives of this research can now be concisely defined. These objectives can be summarized by the following three goals:

1. Develope a numerically well-conditioned method of calculating power flow solutions in the operating region where the power system is approaching its steady state voltage stability limits.

2. Develope an index that relates how far a given operating point is from the voltage stability limits of power flow of the system.

3. Identify those buses or areas in the system that are most prone to experience voltage collapse.

**Plan of Development**

Chapters 2 and 3 of this thesis further develope the problem at hand by describing some general principles of voltage stability and presenting the limitations of the techniques presently used in steady state voltage stability analysis. This lays the foundation for Chapter 4, which contains a mathematical development of the proposed “continuation power flow”. In Chapters 5 and 6, this method is expanded upon by demonstrating its use on a 30 bus test system and then using sensitivity analysis to obtain a voltage stability index and an indicator of the buses most prone to voltage collapse. Finally, Chapter 7 presents the application of the continuation power flow to a 2353 bus power system (a task that requires the application of sparse programming techniques) and Chapter 8 contains conclusions drawn from this research and possible avenues for future work.
CHAPTER 2. SOME GENERAL PRINCIPLES OF VOLTAGE STABILITY

Introduction

In this chapter, some general principles of voltage stability are presented by using a very simple two-bus system. While the analysis of such a system is far simpler than that of an actual power system, the principles revealed are helpful in understanding a system of any size. Using the two-bus system, one can:

1. demonstrate voltage instability

2. point out differences and similarities between voltage instability and angle instability

3. view the general affect of reactive power on voltage stability limits

The Two-bus Power System

A simple two-bus power system can be constructed using a source, a load, and a lossless transmission line as shown in Figure 2.1. Two classic configurations can be formed from this basic system by making certain assumptions. The first configuration, (Figure 2.2) which assumes a constant source voltage magnitude and angle (or an infinite source), is used to demonstrate "pure" voltage instability. On the
other hand, the second configuration assumes a limited source serving an infinite bus load and is used to demonstrate "pure" angle instability (Figure 2.3). The adjective "pure" is used because each of these configurations represent extremes that would not appear in an actual system because there are no actual "infinite" sources.

![Figure 2.1: Basic two-bus power system](image)

![Figure 2.2: Single load served by an infinite bus](image)
Voltage Instability

When an infinite source is used in the 2-bus system, the voltage at the load bus is dependent upon the size and power factor of the load. If a constant power load model is used, the voltage at the load bus is governed by the rather complicated equation (derived in Appendix A)

\[ V' = \sqrt{\frac{(E^2 - 2Q_L X) - \sqrt{(2Q_L X - E^2)^2 - 4X^2(Q_L^2 - P_L^2)}}{2}} \]  

(2.1)

When this equation is evaluated for an increasing load of a given power factor, the basic phenomenon of voltage instability can be demonstrated. This phenomenon is shown by the so-called “P-V curve”, shown in Figure 2.4 where the load bus voltage is plotted against the load active power as load is increased at unity power factor.

As one would expect, increasing the load causes a gradual drop in the load voltage. However, there comes a point in the trajectory of the P-V curve where the voltage drops vertically as the load active power reaches a maximum. This point in the curve, called the critical point, represents the point where voltage stability may be lost. To understand this condition, consider what would happen if one were to try
Figure 2.4: P-V curve for the infinite source serving a single load with unity power factor
to increase the voltage at the load bus by adjusting the load. Above the critical point, backing off on the load increases the voltage, but when the critical point is reached, reducing the load could send the voltage on the decreasing path toward zero. Thus, the critical point is where the system reaches a maximum tolerable voltage difference between the load and source.

**Angle Instability**

Now that the phenomenon of voltage instability has been introduced, it is compared to the type of instability that can occur on the second system configuration. When the voltage magnitude of the source and load are held constant, and the load voltage angle is fixed at $0^\circ$, the power delivered to the load is governed by the following equation [5]

$$P_L = \frac{EV}{X} \sin \delta$$

(2.2)

As Figure 2.5 shows, the source angle $\delta$ increases as the active power transferred to the load increases, but the power transfer reaches a maximum when the source angle is $90^\circ$. The concept of angle instability can be seen by understanding what happens if the source is a synchronous machine and $\delta$ represents the rotor angle displacement of the machine. If the load increases above the maximum shown on the curve, the rotor angle would advance as the machine attempts to serve the additional load. However, any angle displacement above $90^\circ$ causes a decrease in the machine output instead of an increase, and the angle advance would be a fruitless effort. Thus, angle instability is similar to voltage instability in that it imposes a limit on the power that can be transmitted to the load, but the constraint is angle-oriented instead of voltage-oriented.
Figure 2.5: Power versus source angle for a single source serving an infinite bus load

- $E = 1.0$
- $V = 1.0$
- $X = 0.10$
Reactive Power and Voltage Stability

Returning to the earlier configuration of Figure 2.2, a set of P-V curves that correspond to different load power factors can be used to establish some general relationships between reactive power and voltage instability. First, by looking at such a set of curves (Figure 2.6) one can see that the critical point moves to higher and higher voltage levels as the power factor of the load becomes more leading. Thus, when less reactive power is consumed by the load (or more reactive power is generated by the load), the critical voltage increases. This general principle is quite important since it shows how overcompensating the load bus with excessive reactive power could cause the critical point to increase into the range of acceptable operating voltages. If this happened, a system would appear quite secure in terms of voltage magnitude even though the system was about to reach its voltage stability limit.

A second relationship, which can be observed from the set of P-V curves, is that the load power at which the critical point occurs increases as the load power factor becomes more leading. Thus, supplying more reactive compensation to the load increases the transfer limits of the system. When this relationship is coupled with the previous one, it appears that it would be good to compensate the load to increase the transfer limits of the system, but not so much as to move the critical point up into the acceptable range of operating voltages.
Figure 2.6: P-V curves for loads with various power factors
Now that the critical point has been shown to change position with changes in the load reactive power, the question arises as to how to determine how far the system is from collapse. The load voltage magnitude itself is by no means a reliable indicator since voltage collapse could occur at acceptable voltage levels, depending upon the level of reactive power compensation. This question will be addressed in Chapter 6 of this thesis when a voltage stability index is developed.

**Additional Remarks**

In the preceding discussion, a two-bus system in two extreme configurations was used to describe some basics of voltage and angle stability. The difference between angle instability and voltage instability was defined and the importance of reactive power to the subject of voltage stability was introduced. However, it should also be mentioned that the problem is not nearly as simple in a real power system with hundreds of buses. In modeling an actual power system, the minimum and maximum reactive power output limits have to be included and other devices, such as voltage regulating transformers, will have to be modelled. These extra considerations are addressed in later chapters.
CHAPTER 3. PROBLEMS WITH EXISTING METHODS OF ANALYSIS

Introduction

In the previous chapter, a power versus voltage curve was used to analyze the voltage stability limitations of power flow in a two-bus system. In this chapter, the problems of doing the same analysis for a multi-bus system are be analyzed. This discussion will be carried out by first presenting the equations that describe the power flow in a multi-bus power system, and then showing the numerical limitations of the presently used solution methods. This provides a solid background for the presentation of a proposed method called the "continuation power flow" which is the subject of the next chapter.

The Power Flow Problem

In the case of a two-bus power system, a closed form solution for the voltage magnitude at the load bus is possible if the active and reactive power of the load are given. In contrast, a multi-bus power system becomes mathematically too complicated to solve in such a manner. For this reason, the set of nonlinear algebraic equations that describe the power flow of the system must be solved using an iterative technique. The two most common methods of solving the problem are the
Newton-Raphson (N-R)\textsuperscript{1} method and the Gauss-Seidel method. However, the N-R method is preferred for large systems because it offers substantial benefits in terms of convergence, speed, and memory requirements. Given this fact, and the fact that the N-R method is very amenable for use in the continuation power flow, only the N-R method will be described. Descriptions of the other methods can be found in [6] and [7].

The N-R Method

The N-R method is an iterative technique for solving a set of nonlinear algebraic equations of the form

$$F(X) = 0$$

(3.1)

where $F$ is used to represent the set of equations and $X$ is the vector of unknowns and has the same dimension as $F$.

This method for finding $X$ is derived by performing a Taylor series expansion on $F$ about an initial estimate, $X^{(0)}$, of the solution.

$$F(X) = F(X^{(0)}) + F_X(X^{(0)})(X - X^{(0)}) + \text{higher order terms} = 0$$

(3.2)

where $F_X(X^{(0)})$ is the Jacobian of $F(X^{(0)})$. Neglecting the higher order terms and solving for $X$ results in

$$X = X^{(0)} - F_X^{-1}(X^{(0)})F(X^{(0)})$$

(3.3)

Since only the first term of the Taylor series expansion was used, this is a better approximation of the solution of equation (3.1). To reach an acceptable solution,\textsuperscript{1}

\textsuperscript{1}Hereafter, Newton-Raphson will be abbreviated N-R.
multiple iterations must be performed. Generalizing equation (3.3) for the $k^{th}$ iteration gives the following:

\[
X^{(k+1)} = X^{(k)} - [F^{-1}(X^{(k)})]F(X^{(k)}) \\
= X^{(k)} + \Delta X^{(k)}
\]

Formulating The Power Flow Problem with the N-R Method

In order to apply the N-R method to the power flow problem, the equations must be expressed in the form of equation (3.1) where $X$ represents the set of unknown voltage magnitudes and angles. The general approach is to write equations stipulating that the load, generation, and injection at each bus must add to zero for both the active and reactive components of power. For each bus $i$ of an $n$ bus system, these can be written as

\[
\Delta P_i = P_{G_i} - P_{L_i} - P_{T_i} = 0, \quad P_{T_i} = \sum_{j=1}^{n} V_i V_j y_{ij} \cos(\delta_i - \delta_j - \nu_{ij})
\]

\[
\Delta Q_i = Q_{G_i} - Q_{L_i} - Q_{T_i} = 0, \quad Q_{T_i} = \sum_{j=1}^{n} V_i V_j y_{ij} \sin(\delta_i - \delta_j - \nu_{ij})
\]

where the subscripts L, G, and T denote bus load, generation, and injection respectively. The voltage at bus $i$ is $V_i/\delta_i$ and $y_{ij}/\nu_{ij}$ is the $(i, j)^{th}$ element of the system admittance matrix ($Y_{BUS}$).

Before solving these equations using the N-R method, it should be realized that not every bus needs two equations. There are four basic types of buses, each of which has different requirements. Theses types are summarized in Table 3.1.
Table 3.1: Basic bus types used in the N-R power flow

<table>
<thead>
<tr>
<th>BUS TYPE</th>
<th>KNOWNS</th>
<th>UNKNOWNS</th>
<th>EQUATIONS USED</th>
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<tr>
<td>SWING</td>
<td>$V_i, \delta_i$</td>
<td>NONE$^a$</td>
<td>NONE</td>
</tr>
<tr>
<td>GENERATOR (pv)</td>
<td>$V_i, P_{Gi}$</td>
<td>$\delta_i$</td>
<td>$\Delta P_i$</td>
</tr>
<tr>
<td>GENERATOR (pq)</td>
<td>$Q_{Gi}, P_{Gi}$</td>
<td>$V_i, \delta_i$</td>
<td>$\Delta P_i, \Delta Q_i$</td>
</tr>
<tr>
<td>LOAD (pq)</td>
<td>$Q_{Li}, P_{Li}$</td>
<td>$V_i, \delta_i$</td>
<td>$\Delta P_i, \Delta Q_i$</td>
</tr>
</tbody>
</table>

$^a$The power consumed or supplied by the swing bus is not initially known. However, this (and the power flow in each branch) is solved for after finding a solution for the voltage magnitude and angle at each bus.

As the table shows, each pq bus requires two equations and each pv bus requires just one. Thus, if $n_{pq}$ is the number of pq buses in the system and $n_{pv}$ is the number of pv buses, the total number of buses is

$$n = n_{pq} + n_{pv} + 1$$

(3.7)

where the one corresponds to the swing bus. Then the system of equations to be solved is of dimension

$$m = 2n_{pq} + n_{pv}$$

(3.8)

When the power flow iterations are actually performed, $\Delta X$ from equation (3.4) is usually determined using the equation in the standard $Ax = b$ form.

$$- F_X(X^{(k)}) \Delta X^{(k)} = F(X^{(k)})$$

(3.9)

This is a set of linear algebraic equations that can be solved using any acceptable method such as Gaussian-elimination. When the power flow equations just described are placed in this form, the result is

$$\begin{bmatrix} H & N' \\ J & L' \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

(3.10)
where $\Delta P$ and $\Delta Q$ are the mismatch vectors whose elements are calculated from (3.5) and (3.6), $\Delta \delta$ and $\Delta V$ are sub-vectors that together comprise the solution update that corresponds to $\Delta X^{(k)}$ in (3.4). $H$, $N$, $J^0$, and $L'$ are submatrices of the Jacobian. The elements of these submatrices are evaluated as follows:

$$H_{ij} = -\frac{\partial \Delta P_i}{\partial \delta_j}$$  \hspace{1cm} (3.11)

$$N'_{ij} = -\frac{\partial \Delta P_i}{\partial V_j}$$  \hspace{1cm} (3.12)

$$J_{ij} = -\frac{\partial \Delta Q_i}{\partial \delta_j}$$  \hspace{1cm} (3.13)

$$L'_{ij} = -\frac{\partial \Delta Q_i}{\partial V_j}$$  \hspace{1cm} (3.14)

In order to make the calculation of the Jacobian more amenable for computer solution, equation (3.10) is often written in the alternate form

$$\begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$  \hspace{1cm} (3.15)

where

$$N_{ij} = -V_j \frac{\partial \Delta P_i}{\partial V_j}$$  \hspace{1cm} (3.16)

$$L_{ij} = -V_j \frac{\partial \Delta Q_i}{\partial V_j}$$  \hspace{1cm} (3.17)

The details of calculating the Jacobian elements are given in Appendix B.
Using the N-R Power Flow in Voltage Stability Studies

Although the N-R power flow works well for calculating power flow solutions under normal loading conditions, it is numerically unstable when used to calculate power flow solutions under certain heavy load conditions. This can be a problem when performing voltage stability analysis. To further explain this point, it is useful to describe how such analysis might be carried out.

The voltage stability analysis of a multi-bus power system can be performed using the same method used for the two-bus system in the previous chapter. In the multi-bus case, a P-V curve could be plotted for each bus in the system. If a conventional N-R power flow is being used to complete this task, the following steps are performed [8].

1. Find a power flow solution for a given nominal system load. Each unknown voltage and angle are usually set for a flat start (1.0/0°) before the solution process begins.

2. Increment the system load by some amount.

3. Find a power flow solution at the new load. Before the solution process begins, initialize the unknowns to their respective values found for the previous load.

4. Repeat steps 2 and 3.

Although these steps are simple, and have been programmed, the process has several inherent problems. One problem is that a maximum in the system load occurred just as it did for the two-bus example. Thus, the system load cannot be
increased indefinitely. The second problem is that, assuming a standard constant-power load model, the Jacobian of the power flow becomes singular at the point where system load reaches a maximum. When this occurs, a unique solution for $\Delta X$ cannot be found and the method fails. In fact, as the system load approaches the peak, the method becomes more and more ill-conditioned as the Jacobian approaches singularity. This may result in longer solution times or a divergent solution before the Jacobian actually becomes singular.

**Existing Alternatives**

With the onset of voltage stability problems in power systems, several researchers have proposed ways of overcoming the numerical problems associated with the N-R power flow. Among them, the following three methods show promise for use in investigating the voltage stability of large scale power systems.

1. The method of multiple power flow solutions [9]

2. Use of an optimal multiplier with each N-R correction [10]

3. The direct method [11], [12]

The first method can be explained by referring back to the P-V curve for the two bus system. In such a curve, there is a high and low voltage solution for any given load level. In the method of multiple power flow solutions, the high voltage solution and the low voltage solution are found for each power level as the load power is increased. By the time the load reaches the point where the power flow is becoming unstable, a good approximation has been obtained for the location of the solution at peak power.
The second method is very similar to the damped Newton's method [13] except that the amount of damping used at each step is optimized for the best possible correction. Thus, many solutions that exist but can't be found with the conventional N-R can be found with this method.

The object of the third method is to use a single calculation to find the solution corresponding to peak power. To do this, the original power flow equations are augmented by a set of equations that specify singularity of the power flow Jacobian. Then, if a solution near the critical point is known, it is used as the initial guess for solving the set of equations using Newton's method.

Improvements Needed

While each of the alternatives just described offer improvements over the conventional N-R power flow, each has room for improvement. In the first two methods, although solutions can be found in the area of interest, the formulation remains ill-conditioned in this area. Thus, the methods are prone to error and numerical instability. On the other hand, the third method is a well-conditioned, but the size of the problem is effectively doubled. Also, a good estimate of the solution is needed for convergence. Thus, even with these methods, there remains a need for a well-conditioned and robust method of calculating power flow solutions in the region where the system is approaching its steady state stability limits.
CHAPTER 4. THE CONTINUATION POWER FLOW

Introduction

As discussed in previous chapters, the physical constraints of power flow impose a constraint on the maximum load that a power system can serve. Near this maximum load, the analysis of steady-state voltage stability is hampered by the inadequacy of conventional power flow methods. For this reason, the development of a well-conditioned and robust method is in order.

In this chapter, a new method of calculating power flow solutions is described. It is based on a locally parameterized continuation algorithm and has thus been named "the continuation power flow". This method is well-suited for use in voltage stability analysis because it remains well-conditioned over the entire solution path. The following paragraphs explain the principles on which this method is based and show the details of its development.

Locally Parameterized Continuation

Consider what would happen if the set of nonlinear algebraic equations that describe the power flow of a system were reformulated to include a load parameter. If it was desired to find the solution of the set of equations over a range of the load parameter, one might use a step-by-step process whereby the load parameter is
increased by small amounts and the solution of the equation along a given solution path is found for each value of the load parameter. A problem arises with this method because a solution doesn’t exist above some maximum possible value of the load parameter. (This can be visualized on a P-V curve.) At this point, an alternate scheme could be used where one of the state variables is effectively used as the parameter to be varied. For instance, by looking at a P-V curve, one sees that voltage is continually decreasing as the load nears a maximum. Thus, the voltage magnitude at some particular bus could be changed by small amounts and the solution found for each given value of the voltage. Here, the load parameter would be free to take on any value it needed to satisfy the equations. The principle just described is that which is used in locally parameterized continuation. Local parameterization allows not only the added parameter but also the state variables to be used as the so-called continuation parameter. By doing this, the system of equations can be augmented by one equation. If the continuation parameter is chosen correctly, the Jacobian of this augmented set of equations will remain nonsingular and the solution process will be well-conditioned. As shown in Figure 4.1, the method couples this principle with a predictor-corrector scheme so that the solution path can be found efficiently.

Application to the Power Flow Problem

In order to apply locally parameterized continuation to the power flow problem, the power flow equations must be reformulated to include a load parameter. This can be done by expressing the load and generation at a bus as a function of the load parameter. The details of two re-formulations are discussed in subsequent chapters, but it is sufficient for now to say that the general form of the new equations for each
Figure 4.1: An illustration of the predictor-corrector scheme used in the continuation power flow
bus i is

\[
\Delta P_i = P_{G_i}(\lambda) - P_{L_i}(\lambda) - P_{T_i} = 0 \tag{4.1}
\]

\[
\Delta Q_i = Q_{G_i}(\lambda) - Q_{L_i}(\lambda) - Q_{P_i} = 0 \tag{4.2}
\]

where \( \lambda \) is the previously mentioned load parameter and the subscripts used have the same meaning as in the previous chapter. When the equations for the whole power system are considered collectively, the addition of a load parameter results in

\[
F(\delta, V, \lambda) = 0 \tag{4.3}
\]

In the continuation power flow, continuation starts at \( \lambda = 0 \), which corresponds to some base case, and proceeds until the solution path passes through the region of interest. This is illustrated in Figure 4.1. For each solution found along the path both a predictor step and a corrector step are required. Each of these is described separately.

**Predicting the Next Solution**

Once a base solution has been found (\( \lambda = 0 \)), a prediction of the next step can be made by taking an appropriately sized step in a direction tangent to the solution path. Thus, the first task in the predictor process is to calculate the tangent vector. This tangent calculation is derived by first taking the derivative of both sides of the power flow equations.

\[
d[F(\delta, V, \lambda)] = F_\delta d\delta + F_V dV + F_\lambda d\lambda = 0 \tag{4.4}
\]
Factoring;

\[
\begin{bmatrix}
F_{\delta} & F_V & F_\lambda
\end{bmatrix}
\begin{bmatrix}
d\delta \\
dV \\
d\lambda
\end{bmatrix}
= 0
\]

(4.5)

On the left side of this equation is a matrix of partial derivatives multiplied by a vector of differentials. The former is the conventional power flow Jacobian augmented by one column \((F_\lambda)\), while the latter is the tangent vector being sought. There is, however, an important barrier to overcome before a unique solution can be found for the tangent vector. The problem arises from the fact that one additional unknown was added when \(\lambda\) was inserted into the power flow equations, but the number of equations remained unchanged. Thus one more piece of information is needed. This problem can be solved by using additional information from the problem being solved to assign a nonzero magnitude (say 1.0) to an appropriate tangent vector component. In other words, if \(t\) is used to denote the tangent vector and \(k\) is the subscript of the appropriate tangent vector component;

\[
t = \begin{bmatrix}
d\delta \\
dV \\
d\lambda
\end{bmatrix}
\]

\(t_k = \pm 1\)

(4.6)

This results in the augmented system of equations;

\[
\begin{bmatrix}
F_{\delta} & F_V & F_\lambda \\
e_k
\end{bmatrix}
\begin{bmatrix}
t \\
\pm 1
\end{bmatrix}
= \begin{bmatrix}
0 \\
\pm 1
\end{bmatrix}
\]

(4.7)
where $e_k$ is an appropriately dimensioned row vector with all elements equal to zero except the $k^{th}$, which equals one. If the index $k$ is chosen correctly, letting $t_k = \pm 1.0$ imposes a nonzero norm on the tangent vector and guarantees that the augmented Jacobian will be nonsingular at the point of maximum possible system load [14]. Whether $+1$ or $-1$ is used depends on how the $k^{th}$ state variable (or $\lambda$ if $k$ corresponds to $\lambda$) is changing as the solution path is being traced. If it is increasing a $+1$ should be used and if it is decreasing a $-1$ should be used. A method for choosing which tangent element should be assigned the value of $\pm 1.0$ will be presented later in this chapter.

Once the tangent vector has been found by solving equation (4.7), the prediction can be made as follows:

$$
\begin{bmatrix}
\delta^* \\
V^* \\
\lambda^*
\end{bmatrix} = 
\begin{bmatrix}
\delta \\
V \\
\lambda
\end{bmatrix} + \sigma 
\begin{bmatrix}
d\delta \\
dV \\
d\lambda
\end{bmatrix}
$$

(4.8)

where "**" denotes the predicted solution and $\sigma$ is a scalar that designates the step size. The step size should be chosen so that the predicted solution is within the radius of convergence of the corrector. While a constant magnitude of $\sigma$ has been used in the research done for this thesis, more elaborate methods of choosing the step size are described in [14] and [15].

**Parameterization and the Corrector**

Now that a prediction has been made, a method of correcting the approximate solution is needed. Actually, the best way to present this corrector is to expand on
parameterization, which is vital to the process.

Every continuation technique has a particular method of identifying each solution along the path being traced. The scheme used in this work is referred to as local parameterization.

In local parameterization the original set of equations is augmented by one equation that specifies the value of the parameter or the value of one of the state variables. In the case of the reformulated power flow equations, this means specifying either a bus voltage magnitude, a bus voltage angle, or the load parameter $\lambda$. In equation form this can be expressed as follows: Let

$$X = \begin{bmatrix} \delta \\ V \\ \lambda \end{bmatrix} \in \mathbb{R}^{2n_{pq} + n_{pv} + 1}$$ \hfill (4.9)

and let

$$x_k = \eta$$ \hfill (4.10)

where $\eta$ is an appropriate value for the $k^{th}$ element of $X$. Then the new set of equations would be

$$\begin{bmatrix} F(X) \\ x_k - \eta \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$ \hfill (4.11)

Now once a suitable index $k$ and value of $\eta$ are chosen, a slightly modified N-R power flow method (altered only in that one additional equation and one additional state
variable are involved) can be used to solve the set of equations. This provides the corrector needed to modify the predicted solution found in the previous section.

By inserting the augmented system of equation (4.11) into the general Newton form of equation (3.9), the general form of this iterative corrector process is

\[
-F_X(X^{(k)}) \Delta X^{(k)} = F(X^{(k)}) - 0
\]

where \((k)\) refers to the iteration number and should not be confused with the subscript \(k\) used in the augmenting equation. When this Jacobian is expanded by using the definition of \(X\) given in equation (4.9), the corrector Jacobian can be seen to have the same form as the predictor Jacobian.

\[
\begin{bmatrix}
F_X \\
e_k
\end{bmatrix} = \begin{bmatrix}
F_\delta & F_V & F_\lambda \\
e_k & e_k
\end{bmatrix}
\]

Actually, the index \(k\) used in the corrector is the same as that used in the predictor and \(\eta\) will be equal to \(x_k^*\), the predicted value of \(x_k\). Thus, the state variable \(x_k\) is called the continuation parameter. In the predictor it is made to have a nonzero differential change \((dx_k = t_k = \pm 1)\) and in the corrector its value is specified so that the values of other state variables can be found. How then does one know which state variable should be used as the continuation parameter?

Choosing the Continuation Parameter

There are several ways of explaining the proper choice of continuation parameter. Mathematically, it corresponds to the state variable that has the largest tangent
vector component. More simply put, this corresponds to the state variable that has the greatest rate of change near the given solution. In the case of a power system, the load parameter $\lambda$ is probably the best choice when starting from the base solution. This is especially true if the base case is characterized by normal or light loading. Under such conditions, the voltage magnitudes and angles remain fairly constant under load change. On the other hand, once the load has been increased by a number of continuation steps and the solution path approaches the maximum possible load, voltage magnitudes and angles will likely experience significant change. At this point $\lambda$ is a poor choice of continuation parameter since it may change only a small amount in comparison to the other state variables. For this reason, the choice of continuation parameter should be re-evaluated at each step. Once the choice has been made for the first step, a good way to handle successive steps is to use

$$x_k : |t_k| = \max \{|t_1|, |t_2|, \ldots |t_m|\}$$  \hspace{1cm} (4.14)

where $t$ is the tangent vector with a corresponding dimension $2n_{pq} + n_{pv} + 1$ and the index $k$ corresponds to the component of the tangent vector that is maximal [16]. When the continuation parameter is chosen, the sign of its corresponding tangent component should be noted so that the proper value of $+1$ or $-1$ can be assigned to $t_k$ in the subsequent tangent vector calculation.

Actually, this method of choosing the continuation parameter is most effective when the state variables and load parameter are evenly scaled. Since voltage is used in p.u. and angle is used in radians, the state variables of the power flow problem are not evenly scaled. In fact, voltage may typically change by 0.10 to 0.35 p.u. between the base case and the critical point while the change in angle might be as
high as 1.0 radian (approximately 60°). To compensate for this scaling problem, the tangent components corresponding to voltage magnitudes used in equation (4.14) are divided by their per unit magnitude. Since the voltage magnitudes of the buses with the largest tangent components typically drop below 1.0 as the critical point is approached, this slightly favors the voltage components and compensates for the discrepancy in scaling. This modification of the method suggested in equation (4.14) would not be effective if the voltage magnitudes of these buses didn’t drop below 1.0 per unit. An alternate method of handling the problem might be to express the angles in per unit on an appropriate base, but this did not prove necessary.

Knowing When to Stop

The only thing left to do amid the predictor-corrector process is to decide when to stop the process. This depends, to a large extent, on the region of the solution path that is of interest. In this research, it is desired to find the portion of the solution path that corresponds to steady state stability of power flow in the system. Thus, when the stability limit has been passed, the continuation should be halted. The criteria for this halt is developed in Chapter 6 where the stability of some solution paths are analyzed.

Summary of the Process

Now that the continuation power flow has been described in some detail, a meaningful summary of the process can be made. Figure 4.2 provides such a summary in the form of a flow chart. In addition, Appendix C contains a more detailed flow chart patterned after the actual FORTRAN programs used in this research. These
two flow charts describe the general solution procedure used with both the constant power and nonlinear load models. The details of these load models are presented in the following chapter.
Figure 4.2: A flow chart of the continuation power flow
CHAPTER 5. THE CONSTANT POWER AND NONLINEAR LOAD MODELS

In the development of the continuation power flow, the power flow equations were reformulated into the form

\[ \mathbf{F}(\delta, \mathbf{V}, \lambda) = 0 \]  \hspace{1cm} (5.1)

and the general form for each bus \( i \) was

\[ \Delta P_i = P_{Gi}(\lambda) - P_{Li}(\lambda) - P_{T_i} = 0 \]  \hspace{1cm} (5.2)

\[ \Delta Q_i = Q_{Gi} - Q_{Li}(\lambda) - Q_{T_i} = 0 \]  \hspace{1cm} (5.3)

In this chapter, the details of two methods of reformulating the power flow equations are given. The first method is based upon a constant power load model and the second method uses a nonlinear load model where the dependence of the load on voltage is accounted for. In both cases, the results of the method are analyzed using a similar test case.
The Constant Power Load Model

In the constant power load model, the load and generation at each bus are independent of voltage. Thus, both are made to vary in direct proportion to any change in $\lambda$, with $\lambda = 0$ corresponding to the base load. This is written as

\[ P_{Li} = P_{Lio} + \lambda(k_{Li}S_{\Delta BASE} \cos \psi_i) \]  
(5.4)

\[ Q_{Li} = Q_{Lio} + \lambda(k_{Li}S_{\Delta BASE} \sin \psi_i) \]  
(5.5)

where the following definitions are made:

- $P_{Lio}, Q_{Lio}$ - original load at bus $i$, active and reactive respectively
- $k_{Li}$ - multiplier to designate the rate of load change at bus $i$
- $\psi_i$ - power factor angle of load change at bus $i$
- $S_{\Delta BASE}$ - a given quantity of apparent power which is chosen to provide appropriate scaling of $\lambda$

With this load model and any given value of $\lambda$, the change in active power load from the base load can be calculated and the load change distributed among the generators.

\[ P_{TOTAL} = \sum_{i=1}^{n} (P_{Lio} + \lambda k_{Li}S_{\Delta BASE} \cos \psi_i) \]  
(5.6)

\[ = \sum_{i=1}^{n} P_{Lio} + \lambda S_{\Delta BASE} \sum_{i=1}^{n} k_{Li} \cos \psi_i \]

\[ = \sum_{i=1}^{n} P_{Lio} + \lambda PMULT \]
where, for a given set of $k_L$,’s and $\psi$,’s, $PMULT$ is a constant defined as:

$$PMULT = S_{\Delta BASE} \sum_{i=1}^{n} k_L \cos \psi_i$$  \hspace{1cm} (5.7)$$

Then, if $GFRAC_i$ is the fraction of the load change to be served by the $i^{th}$ generator, the general expression for the generation at bus $i$ becomes

$$P_{G_i} = P_{Gio} + \lambda(GFRAC_i)(PMULT)$$  \hspace{1cm} (5.8)$$

where $P_{Gio}$ is the generation at bus $i$ in the base case.

When the newly formulated load and generation terms are inserted into the general form in equations (5.2) and (5.3), the result is

$$\Delta P_i = P_{Gio} + \lambda(GFRAC_i)(PMULT) - P_{Lio} - \lambda(k_L S_{\Delta BASE} \cos \psi_i) - P_T$$  \hspace{1cm} (5.9)$$

$$\Delta Q_i = Q_{Gio} - Q_{Lio} - \lambda(k_L S_{\Delta BASE} \sin \psi_i) - Q_T$$  \hspace{1cm} (5.10)$$

The set of equations that describe the entire system are made up of a combination of these two general equations by following the rules listed in Table 3.1. The details of calculating the augmented Jacobian, which has the form of equation (4.13), are given in Appendix D.

Using the formulation just described, the values of $k_L$, $\psi$, and $GFRAC_i$ can be uniquely specified for every bus in the system (with the exception of the swing bus, since no equations are written for it). As a result, almost any load change scenario can be simulated by the appropriate selection of these control parameters.

However, while choosing the $GFRAC_i$’s and $\psi$,’s is fairly straightforward, the way in which the $k_L$’s are chosen and scaled deserves further explanation.
In choosing $k_L$'s for a simulation of load increase, one can choose $k_L = 1$ to designate some standard load multiplier and then choose the others relative to this standard. For instance, if the load at bus 10 was to increase at twice the nominal rate then $k_{L10}$ should be set to 2.0. On the other hand if load on the same bus was to increase at half of the nominal rate, the $k_{L10}$ would be set to 0.5.

Once the load change multipliers are chosen to have the proper size with respect to one another, it may be necessary to scale them in order that the change in $\lambda$ during continuation has an acceptable order of magnitude. The general object here is to assure that the change in $\lambda$ has about the same order of magnitude as the change in the state variables. This keeps the tangent vector components on a comparable scale so that the correct continuation parameter can be chosen [14]. The scaling should be properly coupled with the choice of $S_{\Delta BASE}$. A method of scaling these multipliers is shown in Appendix E.

**Application to a Test Case**

In order to demonstrate the continuation power flow with a constant power load model, a scenario from the 30 bus New England test system was simulated. This system, shown in Figure 5.1, is commonly used in voltage stability research. A base case load that accompanies the system and the system data was taken from [17] and is shown in Appendix F.

The scenario to be simulated is a slightly modified version of one found in [18]. Here, the load increase is such that at any point in the continuation, the active and reactive power at any bus is a given multiple (say "l") times the active and reactive power that was specified in the base case. Thus, if the active power at a bus is double
Figure 5.1: The 30 bus New England system
its original value, the reactive power at the bus would be double its original value and the load would be doubled in the same manner at every bus in the system (where load existed). Similarly, the active power generation will be increased the same way by choosing \( GFRAC_i \) of each bus \( i \) to be equal to the fraction of the total generation that bus \( i \) provided in the base case. That is

\[
GFRAC_i = \frac{P_{Gio}}{\sum_{i=1}^{n} P_{Gio}} \tag{5.11}
\]

Reactive generation will vary within the specified limits so as to regulate the voltage at a given generator bus.

In terms of the control parameters used in the model, this load increase is set up by letting

\[
k_{Li} = \frac{S_{Lio}}{ADJ} \tag{5.12}
\]

where \( ADJ \) is some scaling factor common to each load multiplier and where \( S_{Lio} = \sqrt{P_{Lio}^2 + Q_{Lio}^2} \) is the apparent power at bus \( i \) in the base case. In addition, the power factor angle of the load change at each bus is chosen to be equal to the original power factor angle at the bus.

\[
\psi_i = \arccos \left( \frac{P_{Lio}}{S_{Lio}} \right) \tag{5.13}
\]

Then, from equation (5.4), the active load at a given bus will be

\[
P_{Li} = P_{Lio} + \lambda \left[ \frac{S_{Lio}}{ADJ} S_{\Delta BASE} \cos \left( \arccos \frac{P_{Lio}}{S_{Lio}} \right) \right] \tag{5.14}
\]

\[
= P_{Lio} + \lambda \left[ \frac{1}{ADJ} S_{Lio} S_{\Delta BASE} \frac{P_{Lio}}{S_{Lio}} \right] \n
= P_{Lio} + \lambda \left( \frac{1}{ADJ} \right) S_{\Delta BASE} P_{Lio} \n
= \left[ 1 + \lambda \left( \frac{1}{ADJ} \right) S_{\Delta BASE} \right] P_{Lio} = lP_{Lio}
\]
and from equation (5.5) the reactive load is

\[
Q_{Li} = Q_{Lio} + \lambda \left[ \frac{S_{Lio}}{ADJ} S_{\Delta BASE} \sin(\arccos \frac{P_{Lio}}{S_{Lio}}) \right]
\]

\[
= Q_{Lio} + \lambda \left[ \frac{1}{ADJ} S_{Lio} S_{\Delta BASE} \left( \frac{Q_{Lio}}{S_{Lio}} \right) \right]
\]

\[
= Q_{Lio} + \lambda \left( \frac{1}{1 - \frac{Q_{Lio}}{S_{Lio}}} S_{\Delta BASE} \right) Q_{Lio} = lQ_{Lio}
\]

Results

The test case just described was run using both a normal system configuration and a contingent configuration that involved the removal of the branch joining buses 9 and 30. The results are given in the form of P-V curves for two buses that lie in opposite corners of the system. The two P-V curves for bus 7 are shown in Figure 5.2 and the two for bus 24 are shown in Figure 5.3.

Since no step sizing algorithm was used, the step sizes had to be selected ahead of time and then adjusted as necessary after a solution attempt was made. For this example, in both the normal and contingent cases, the multipliers were set so that \(\lambda\) would be about 0.125 when the active power load doubled. Then, 0.05 was used for the step size when \(\lambda\) was the continuation parameter, and 0.02 when a state variable was the continuation parameter. This step size was modified to 0.001 for the 28th step of the contingent case when a convergence problem occurred. Once the cases were successfully run, no attempt was made to optimize the step size.

The most significant aspect of these results is the successful completion of the solution path around the peak loading condition. This is a feat that could not be completed with a standard Newton-Raphson power flow. In fact, during continuation,
the non-augmented Jacobian was verified to pass through a singular point at the peak power condition.

In addition to this basic result, the P-V curves for both buses show the additional power transfer limitation imposed on the system by the removal of a branch. This is of course an expected result.

It should also be noticed that the P-V curve for bus 24 tends to drop off rather sharply just before peak power transfer occurred. This sharp drop off actually occurs
Figure 5.3: P-V curves for bus 24 when a constant power load model is used after the generators in the area near bus 24 reach their maximum reactive power output limits. This is discussed in greater detail in Chapter 6 when the voltage stability index and indicator of weak buses are presented.

The Nonlinear Load Model

In order to obtain a more accurate model of the load in a power system, it is often desired to model the response of load to changes in voltage magnitudes.
However, developing such a model is far from trivial since the characteristics of a wide variety of motors, appliances, etc. must be considered and an estimate must be made of the composition of load at each bus. Fortunately, very sophisticated load modelling software, called LOADSYN, has been recently developed by the General Electric Company under contract with the Electric Power Research Institute [19]. This software makes it possible to obtain nonlinear load models by inputting the geographic area the load is in and the percent of each load class that makes up the load at each bus.

The LOADSYN program uses the user input information and a built in data base to calculate the coefficients for the steady-state load model. Although the full model includes terms that account for frequency deviation, only the voltage dependent terms are used here. This results in the load model

\[
P = P_o P_{a1} \left( \frac{V}{V_o} \right)^{KPV1} + P_o \left( 1 - P_{a1} \right) \left( \frac{V}{V_o} \right)^{KPV2}
\]

\[
Q = P_o Q_{a1} \left( \frac{V}{V_o} \right)^{KQV1} + (Q_o - P_o Q_{a1}) \left( \frac{V}{V_o} \right)^{KQV2}
\]

where the following definitions are made;

- \( P_o \) - initial active power consumed by the load (from base case)
- \( V_o \) - initial voltage at the bus (from base case)
- \( P_{a1} \) - frequency dependent fraction of active power load
- \( KPV1 \) - voltage exponent for frequency-dependent active power load
- \( KPV2 \) - voltage exponent for non-frequency-dependent active power load
- \( Q_o \) - initial reactive power consumed by the load (from base case)
- \( Q_{a1} \) - ratio of uncompensated reactive power load to active power load
44

\( KQV1 \) - voltage exponent for the uncompensated reactive power load

\( KQV2 \) - voltage exponent for the reactive power compensation

As these definitions show, the reactive portion of this model is broken into both load and compensation terms.

In order to use this load model in the continuation power flow, the load parameter and load increase multiplier must be somehow added so that various load changes can be simulated. For the active power portion, this is done as follows:

\[
P_{Li} = (1 + k_{Li}(\lambda)) P_{Lo} \left[ P_{al} \left( \frac{V_i}{V_{oi}} \right)^{KPV1} + (1 - P_{al}) \left( \frac{V_i}{V_{oi}} \right)^{KPV2} \right]
\]

(5.18)

where the \( Li \) subscript is used to denote load at bus \( i \). Now instead of \( \lambda \) corresponding directly to actual load in terms of MW as it was in the constant power formulation, \( \lambda \) in this formulation corresponds to the quantity of connected motors, appliances, etc., that has a given characteristic. As a consequence, in this model, it is probably more accurate to refer to \( \lambda \) as a connection parameter rather than a load parameter. The same holds for the reactive power portion of the model, where the resulting load model is

\[
Q_{Li} = (1 + k_{Li}(\lambda)) [P_{Lo} Q_{al} \left( \frac{V_i}{V_{oi}} \right)^{KQV1} + (Q_{Lo} - P_{Lo} Q_{al}) \left( \frac{V_i}{V_{oi}} \right)^{KQV2}]
\]

(5.19)

Now the problem of increasing generation must be dealt with. If the active power generation is to match the active power load, an expression for the total load change is needed. Once obtained, this expression can be used to divide generation increases among the generators available. The load change can be expressed as

\[
\Delta P_{TOTAL} = P_{TOTAL} - P_{TOTALo} = \sum_{i=1}^{n} P_{Li} - P_{TOTALo}
\]

(5.20)
where $P_{TOTAL}$ is the total active power load at any given instant and $P_{TOTALo}$ is the total active power load in the base case. If each generator is made to take up a fraction, $GFRAC_i$, of the load change, generation at a given bus $i$ is

$$P_{Gi} = P_{Gio} + (GFRAC_i) \frac{\Delta P_{TOTAL}}{\sum_{i=1}^{n} P_{Gio}}$$  \hfill (5.21)

Details of how the Jacobian is calculated for the nonlinear load model just described are given in Appendix D.

**Application to a Test Case**

For the purpose of comparison, the performance of the nonlinear load model will be demonstrated on a case that is basically identical to that used for the constant power load model. That is, the amount of load connected at each bus will be increased at the same rate. If it is doubled at one bus, it will be doubled at all the buses. The prime difference between this demonstration and that for the constant power load model is that doubling the amount of load connected will not necessarily double the load in terms of MWs and MVARs. This is of course due to the fact that voltage is now allowed to impact the load. As in the previous demonstration, the generator multipliers used for this case are

$$GFRAC_i = \frac{P_{Gio}}{\sum_{i=1}^{n} P_{Gio}}$$  \hfill (5.22)

This increases generation at a given bus in proportion to its generation in the base case. On the other hand, the load increase multipliers are chosen differently than for the constant power load model. These parameters are set by ignoring the voltage...
dependence of the load as shown below. Let \( \frac{V_i}{V_{oi}} = 1 \), then

\[
\begin{align*}
P_{Li} & = (1 + k_i \lambda)P_{Lio}P_{a1} + (1 - P_{a1}) \\
& = (1 + k_i \lambda)P_{Lio} \\
& = lP_{Lio} 
\end{align*}
\]

Similarly for the reactive power component:

\[
\begin{align*}
Q_{Li} & = (1 + k_i \lambda)\left[P_{Lio}Q_{a1} + (Q_{Lio} - P_{Lio}Q_{a1})\right] \\
& = (1 + k_i \lambda)Q_{Lio} \\
& = lQ_{Lio}
\end{align*}
\]

Thus, by setting the \( k_i \) multiplier of each bus equal, the active and reactive load connected increase at the same rate. The method used to scale the multipliers so that \( \lambda \) will be of an acceptable magnitude is shown in Appendix E.

Results

The nonlinear load model just described was used on the same two system configurations used for the constant power model. The constants for the load model were obtained from the LOADSYN program. Each bus was assigned constants corresponding to load in the Northwest United States with a composition of 35% heavy industrial, 35% commercial, and 30% residential load. The resulting set of constants are shown in Table 5.1. In addition, the values used to scale the load multipliers and the step sizes used are shown in Table 5.2.
Table 5.1: Constants used in nonlinear load model

<table>
<thead>
<tr>
<th>CONSTANT</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{a1}$</td>
<td>0.70</td>
</tr>
<tr>
<td>$KPV1$</td>
<td>0.38</td>
</tr>
<tr>
<td>$KPV2$</td>
<td>1.78</td>
</tr>
<tr>
<td>$Q_{a1}$</td>
<td>0.44</td>
</tr>
<tr>
<td>$KQV1$</td>
<td>1.64</td>
</tr>
<tr>
<td>$KQV2$</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table 5.2: Step sizes and scaling constants used with the nonlinear load model

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>NORMAL</th>
<th>CONTINGENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>step size for $\lambda$</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>step size for $V$</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>step size for $\delta$</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>EST</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$\lambda_{DES}$</td>
<td>0.50</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Again, P-V curves were plotted for buses 7 and 24 and the same basic results were obtained (Figure 5.4 and Figure 5.5). However, two distinct results were noticed. First, for both system configurations the maximum power transfer increased by about 3.0 per unit over that obtained with the constant power load. Thus, the constant power load model is shown to be more conservative. The second distinct result is that the non-augmented Jacobian, which is simply the power flow Jacobian, did not become singular until after the peak power transfer had been encountered. For the normal configuration the active load peaked at about 110.6 per unit, but the non-augmented Jacobian does not pass through a singular point until the load had decreased to between about 96.0 to 98.0 per unit. Similarly, for the contingent
configuration, the active power load peaked at about 99.4 per unit, but the singular point does not occur until the load has decreased to between 79.5 to 81.7 per unit.

![Graph showing P-V curves for bus 7](image)

**Figure 5.4:** P-V curves for bus 7 when the nonlinear load model is used

The fact that the non-augmented Jacobian passes through a singularity after peak power transfer is significant in that the singular point has been shown to divide the stable and unstable parts of the solution path when a constant power load model is assumed. It is shown in the next chapter that this is also true for the nonlinear
Figure 5.5: P-V curves for bus 24 when the nonlinear load model is used

load model and in fact steady state stability is not necessarily lost at the point of peak power transfer.

Further Comparison Between Load Models

In addition to the difference in results discussed in the previous section, it is interesting to compare the load variation with respect to $\lambda$ for the two different load models. Figures 5.6 and 5.7 show this variation for the constant power load model
under normal and contingent conditions respectively and Figures 5.8 and 5.9 show the same thing for the nonlinear load model.

Figure 5.6: Lambda versus active and reactive load for the constant power load model under a normal system configuration

As expected, load changes linearly with a change in \( \lambda \) for the constant power load model. But hysteresis is apparent in the load change for the nonlinear model. Namely, for this particular set of load model parameters, power is shown to peak before a peak in the load connection parameter \( \lambda \) occurs. Physically, this means
that after the peak power transfer is encountered, more load is being connected but power consumption actually decreases due to the dominating effect of the voltage dependence of the load. The significance of the maximum in λ is explored when the stability of the solution path is discussed in the next chapter.
Figure 5.7: Lambda versus active and reactive load for the constant power load model under a contingent system configuration
Figure 5.8: Lambda versus active and reactive load for the nonlinear load model under a normal system configuration
Figure 5.9: Lambda versus active and reactive load for the nonlinear load model under a contingent system configuration
CHAPTER 6. STABILITY OF THE SOLUTION PATH

In the previous chapters, the continuation power flow was developed and demonstrated using both a constant power and a voltage dependent load model. In this chapter, the stability of the solution paths found in Chapter 5 are analyzed. The principles discovered in this analysis are used to develop both a voltage stability index and a method of identifying the buses most prone to voltage collapse.

Eigenvalue Analysis of the Power Flow Equations

Given a system of the form

\[ \dot{x} = Ax \]  

(6.1)

local stability of the system can be analyzed by observing the eigenvalues of \( A \). The eigenvalues of \( A \) are the roots of the characteristic polynomial of the system. The system is considered stable if the real parts of all the eigenvalues are negative. If the real part of any one of the eigenvalues is positive, the system is unstable and if the real part of one of the eigenvalues is zero, the system is critically stable.

Eigenvalue analysis is suitable for checking the steady state stability of a system because it tells whether or not the system can withstand small perturbations about some quiescent point. Along this line, when the power flow equations are linearized about a given quiescent point, the power flow Jacobian is the \( A \) matrix of the system.
This can be seen by expressing the power flow equations as the equilibrium solution of a set of dynamic equations

\[ F(X) = \dot{X} = 0 \quad (6.2) \]

Then, if \( X^o \) is assumed to be a solution of equation (6.2), expanding the power flow equations about \( X^o \) results in

\[ F(X) = F(X^o) + F_X(X^o)(X - X^o) \quad (6.3) \]

then if \( \dot{X} = F(X) \) and \( \dot{X}^o = F(X^o) \), this expression can be manipulated into

\[ (\dot{X} - \dot{X}^o) = F_X(X^o)(X - X^o) \quad (6.4) \]

which is in the general \( \dot{x} = Ax \) form with the Jacobian \( F_X \) corresponding to the \( A \) matrix. This is simply a linearization of the algebraic expression in equation (6.2). Since the Jacobian describes only the power flow portion of the system, the stability analysis done here is simplified in that it does not consider machine dynamics. This is in keeping with the narrowed scope of this research as described in Chapter 1.

Results

During continuation, an augmented Jacobian is calculated for each predictor step using a corrected power flow solution. The core of this augmented Jacobian is simply the Jacobian of the power flow equations. Thus, the power flow Jacobian is readily available for eigenvalue analysis.

Such an eigenvalue analysis was performed for the four solution paths found in the previous chapter using the EVCRG subroutine of the IMSL math library. This routine is based upon the EISPACK routines BALANC, ORTHES, ORTAN, and
HQR2. It first transforms the matrix into a real upper Hessenberg form and then uses the QR method to find the eigenvalues.

The results of this eigenvalue analysis are shown in Figures 6.1 through 6.4 where the smallest real part of the set of eigenvalues is plotted against $\lambda$. Figures 6.1 and 6.2 show the results for the constant power load model under normal and contingent conditions respectively and Figures 6.3 and 6.4 show the same for the nonlinear load model.

The most striking aspect of these results is that in all cases the stable and unstable regions are separated by a maximum in $\lambda$. In addition, it was noted that the singularity of the power flow Jacobian (mentioned in Chapter 5) also occurred at the maximum in $\lambda$. Of course, this was a known result for the constant power load model because the peak in $\lambda$ corresponds to the peak in power and a loss of stability was already known to occur just after the peak in power. However, for the nonlinear load model, the significance of the result comes from the fact that the maximum in both active and reactive power transfer came before the maximum in $\lambda$. Physically, stability in the region between the peak in power and the peak in $\lambda$ is reasonable because disconnecting load (decreasing $\lambda$) pushes the system in the stable direction. These results can be summarized by saying that the maximum in connectivity ($\lambda$) is what separates the stable and unstable operating regions.

Now that the most important result of the eigenvalue analysis has been discussed, it should be pointed out that a slight jump in the minimum real part of the eigenvalues occurred in both the cases that used the nonlinear load model. The exact reason for this jump is not known although both occurrences correspond to a place where at least one generator has reached its maximum reactive power output limit. When this
Figure 6.1: Eigenvalue analysis using the constant power load model and a normal system configuration
Figure 6.2: Eigenvalue analysis using the constant power load model and a contingent system configuration
Figure 6.3: Eigenvalue analysis using the nonlinear load model and a normal system configuration
Figure 6.4: Eigenvalue analysis using the nonlinear load model and a contingent system configuration
occurs the dimension of the Jacobian is increased by one row and column. While this may affect the eigenvalue calculation, the same change in dimension does not seem to impact the eigenvalue calculations for the constant power load model.

In addition, it is also noted that the eigenvalues changed rather dramatically near the critical point in the cases that use a constant power load model. For both system configurations the minimum real part drops gradually at first and drops suddenly near the critical point when the reactive power output limits of one or more generators are met. This is in contrast to the cases that used the nonlinear load model. For them, the trajectory is more smooth and rounded near the critical point. This may be because the somewhat self-correcting nature of the nonlinear model. When reactive power is constrained, a voltage drop occurs. This voltage drop in turn reduces the load and mitigates the impact of the reactive power constraint.

An Index and Indicator

One of the goals of this research was to develop both a voltage stability index and a method of identifying the buses most prone to voltage collapse (for brevity these will from here on be referred to as “weak” buses). It has already been pointed out that observing bus voltage magnitudes is not a sufficient indicator of the proximity to voltage collapse. In light of this fact, several voltage stability indexes have been proposed. In fact, in [20] it is proposed that the minimum real part of the set of eigenvalues be used. However, as the previous results showed, this value drops off sharply at the critical point instead of changing gradually. A gradual change is more desirable since it would more accurately relate the closer and closer proximity to collapse as the load increased. In addition, calculating the eigenvalues is a rela-
tively expensive process. Another proposed index, the minimum singular value of the Jacobian, was proposed in [21]. It changes more gradually but is an even more expensive alternative than using eigenvalue information. On the other hand, in this research it has been discovered that a very inexpensive and effective index can be obtained by using the sensitivity information available from the tangent vector of the continuation predictor. The index is actually derived by first using the same sensitivity information to identify the weak buses of the system.

In the continuation process, the tangent vector proves useful because it describes the direction of the solution path at a corrected solution point. A step in the tangent direction is used to estimate the next solution. However, if one looks at the elements of the tangent vector as differential changes in the state variables \( dV_i \) or \( d\delta_i \) in response to a differential change in load connectivity \( d\lambda \), the potential for a meaningful sensitivity analysis becomes apparent.

When it comes to spotting the weakest bus, it is helpful to think in terms of a P-V curve, or considering the more general situation, a \( \lambda-V \) curve. Heuristically speaking, the weakest bus is the one that is closest to the turning point or "knee" of the curve since that is where stability is lost. Equivalently, a weak bus is one that has a large ratio of differential change in voltage to differential change in connected load. The weakest bus is then the one with the largest \( \frac{dV}{d\lambda} \) ratio. Or, if \( j \) is the subscript used to denote the weakest bus, it is

\[
\text{bus}_j : \left| \frac{dV_j}{d\lambda} \right| = \max \left\{ \left| \frac{dV_1}{d\lambda} \right|, \left| \frac{dV_2}{d\lambda} \right|, \ldots, \left| \frac{dV_n}{d\lambda} \right| \right\}
\]  

(6.5)

Since \( d\lambda \) is common to each term, the above expression is equivalent to saying that the weakest bus is the one with the largest \( dV \) component. The weak areas can therefore be identified by merely finding out which buses have the largest corresponding \( dV \).
components. Thus, division by $d \lambda$ is not necessary. Of course, since the actual load change brought on by a given $d \lambda$ depends upon the set of $k_{Li}$'s and $GFRAC_i$'s used, the location of the weak buses depends upon the particular scenario being simulated.

When the weakest bus, $j$, reaches its steady state voltage stability limit, the ratio of $-\frac{dV_j}{d\lambda}$ becomes infinite, or equivalently, the ratio of $-\frac{d\lambda}{dV_j}$ is zero. It is this latter ratio that makes a good voltage stability index for the system. The index will be high when the weakest bus is far from instability, but will be zero when the weakest bus experiences voltage collapse. (The negative sign is used so that the index will be positive before the critical point is encountered and negative afterwards.)

When it comes to providing a meaningful scale for the index, relating the change in $\lambda$ to a change in active power can be used. For the constant power load model this makes the index $-\frac{PMULT d\lambda}{dV_j}$, but for the nonlinear load model there is no such constant as $PMULT$ because the power change is dependent upon voltage terms. Thus, for the nonlinear load model scaling can be accomplished by using the change in power that would occur if voltage did not decrease (see equation (5.22)). This would mean using $-\frac{\sum_{i=1}^{n} k_{Li} P_{Li,0} d\lambda}{dV_j}$. The results of these indexes and weak bus indicator are demonstrated in the following section.

**Results**

The indexes just described were calculated for the four cases from Chapter 5 and are shown in Figures 6.5 and 6.6. Figure 6.5 shows the trajectory of the indexes for the constant power load model and Figure 6.6 shows the indexes when the nonlinear load model was used. To demonstrate the effect of the reactive power output limits of generators, the number of the generator is placed on the graph at the point where
it reaches its limits.

In each of the four cases, the index decreases gradually until the reactive power supply is further limited by generator limits. Often, when one or more units reach their reactive output limit, the index drops sharply. This is desirable in that the index is effectively relating the impact of key changes in the system. It does, however, also display the limitation of the presently used method of modelling reactive power output limits. Although the common practice (as used here) is to model the limit as a single number, the actual limit is due to thermal limitations and should decrease gradually as the output of the unit is increased. The use of a more realistic model would probably produce a smoother trajectory of the index as the system approaches instability. To further demonstrate the effect of this type of system change, the reactive power output limits were removed from all the units and the simulation repeated using the constant power load model and a normal system configuration. The result, shown in Figure 6.7, is smooth in contrast to those shown previously.

The effect of limits on reactive compensation can also be seen in the movement of the weakest buses found via sensitivity. This is illustrated in Figures 6.8 through 6.11 where the five weakest buses found are marked for several different load levels. In each figure the weakest buses are enclosed in a diamond shape and the circle representing a generator is filled in when the generator reaches its reactive power output limits. It should be noted that the buses marked are the five weakest relative to the rest of the system. In each figure, the last frame is the one closest to the critical point.

In each of the cases illustrated the weakest buses can be seen to shift location with the onset of the limits on various generators. In particular, the results are very
Figure 6.5: Voltage stability index trajectories for the constant power load model
Figure 6.6: Voltage stability index trajectories for the nonlinear load model
Figure 6.7: Index trajectory for the constant power load model and normal system configuration when reactive power output limits on generators are removed.
Figure 6.8: Weak buses for the constant power load model and normal system configuration.
Figure 6.9: Weak buses for the constant power load model and contingent system configuration
a) 1.68 times original load.  
b) 1.96 times original load.  
c) 2.19 times original load.  
d) 2.17 times original load.  
e) 1.94 times original load.

Key

- Generator
- Generator at Maximum
- Reactive Power Output
- Weak Bus

Figure 6.10: Weak buses for the nonlinear load model and normal system configuration
a) 1.46 times original load.  
b) 1.78 times original load.  
c) 1.97 times original load.  
d) 1.91 times original load.  
e) 1.62 times original load.

Figure 6.11: Weak buses for the nonlinear load model and contingent system configuration

<table>
<thead>
<tr>
<th>Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>○ = Generator</td>
</tr>
<tr>
<td>• = Generator at Maximum Reactive Power Output</td>
</tr>
<tr>
<td>◇ = Weak Bus</td>
</tr>
</tbody>
</table>
reasonable in that the weakest buses tend to move toward the generators that have reached their reactive power output limits. These would naturally be the areas of the system where reactive compensation is most limited.

As Chapter 7 demonstrates, this same index and method of identifying weak buses is also effective for large power systems.
CHAPTER 7. APPLICATION TO A LARGE SYSTEM

No matter how well a stability analysis method works on small systems, in order for it to be truly practical it must also work effectively on the large system models that are typically used for today’s interconnected power systems. With this in mind, sparsity programming techniques were used with the continuation power flow so that it could be applied to a 2353 bus system model supplied by Iowa Power. This model was formed by adding a detailed representation of the Iowa Power system to the basic Mid-Continent Area Power Pool model. The load data used are based upon an estimate of the 1994 summer peak.

The Use of Sparsity Techniques

As with the conventional N-R power flow, the matrices used in the continuation power flow are highly sparse. Thus, in order to facilitate power flow solutions of large power systems, only the nonzero elements of these large matrices are stored (see for instance [6]). In the programming done for this research, this involved sparse storage of the system admittance matrix, two power injection matrices (\(c_{ij}\) and \(d_{ij}\) from Appendix B), and the Jacobian matrix.

Since only the nonzero elements of the Jacobian were stored, a suitable linear algebraic system solver was also required. Whereas the LINPACK solver used in
the earlier computations manipulated both zero and nonzero elements in the solution process, sparse storage requires a solver that is programmed to manipulate only the nonzero elements. The solver used for this purpose was MA28 from the Harwell math subroutine library [22]. In the Harwell package, two different means of decomposing the Jacobian are available. One method is to perform the decomposition without using prior pivoting information and the second method is to use pivoting information from a matrix having the same sparsity pattern that has already been decomposed. Thus, whenever a matrix was to be decomposed, this latter method was used whenever possible because it is four to seven times faster than performing the decomposition with no previous pivoting information.

Because of the large amount of time required to decompose a large sparse matrix, it is desirable to minimize the number of decompositions. One means of doing this is by using what [13] refers to as the “modified Newton method”. Here, the same Jacobian is used for each set of corrector iterations. When this method is used, decomposition of the Jacobian is only required when the dimension of the system changes (such as occurs when a generator reaches its reactive power output limits and the generator bus must be changed from pv status to pq status).

The implementation of these time-saving techniques requires a slightly different program structure than was used previously. In addition, the existence of tap changing transformers in the Iowa Power model required some logic that was not needed for the 30 bus test system. These differences are illustrated in a flow chart of the resulting program shown in Appendix G.
Test Case

In order to demonstrate the effectiveness of the continuation power flow on a large-scale power systems, four test cases from the Iowa Power system are presented. Each of these cases uses a constant power load model and involves increasing load on 38 buses in the Des Moines area, which is somewhat remote from base load generation (see Figure 7.1). As in the 30 bus example, the apparent power at each of these buses is used for its corresponding unscaled load increase multiplier. The corresponding generation increase was set up to come from units in Nebraska and the Dakotas. Load and generation increases were set to occur at the same location in each case but the system conditions were changed each time. First, the load increase was performed under the normal or base case system configuration. Second, the increase was repeated after removing the 345 KV line joining the Lehigh and Sycamore buses. Thirdly, with the same line outage, the six generators in Des Moines were removed from service. Finally, with the previous outages still in effect, 500 MW of load was added to the eastern side of the state and the generation at Council Bluffs was increased by 500 MW. This last modification was done so that the effect of the additional “through-flow” from west to east across the system could be observed. In particular, the load was added to the Davenport area so that the through-flow would specifically affect the same east-west lines that transmit power from the west to the Des Moines area.

In each of the cases described above, the power factor of the load added to each bus was set equal to the load power factor in the base case, or, if the original power factor was unity, it was set to 0.98 lagging. This was done to make the scenario more realistic since a load increase at unity power factor is not likely.
**Figure 7.1:** The Iowa portion of the MAPP system

**Results**

The four cases were simulated using $\lambda_{DES} = 0.25$ and $EST = 2.0$. The step size was 0.05 if either $\lambda$ or an angle were the continuation parameter and 0.03 if a bus voltage was a continuation parameter. The resulting index trajectories are shown in Figure 7.2.

In the first two cases, the large dip in the index occurs because two generating units in Des Moines reach their reactive power output limits at the second solution point. Such a dip does not occur in the third and fourth cases because the units in Des Moines are removed from service.
Figure 7.2: Index trajectories for the Iowa Power cases

In each of the second through fourth cases, the trajectory of the index is progressively lower. This effectively indicates the closer proximity to voltage collapse as the system conditions become more harsh. In the fourth case, the addition of the through flow across the system reduced the critical loading by 40 MW. This demonstrates the extra stress that is placed on the system when power is wheeled.

Since a detailed presentation of the movement of the weakest buses would be a sizable task, it will simply be pointed out that the five weakest buses found at the
critical point were the same for each case. All five were 69.0 KV load buses located on the west side of Des Moines. This is, of course, a reasonable result since the load increase took place in Des Moines.
CHAPTER 8. CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

Conclusions

The examples included in this work have demonstrated the ability of the continuation power flow to find solutions in the stable operating region of the system and near the critical point. In fact, while using this new method, it was discovered that the maximum in load “connectivity” separated the stable and unstable regions of the solution path. When this new-found principle was coupled with the sensitivity information available from the tangent predictor, a voltage stability proximity index and indicator of weakest buses were readily calculated. The index effectively indicated both proximity to collapse and changes in the reactive power supply such as those that occurred when a generator reached its reactive power output limit. In addition, the weakest buses found via sensitivity were reasonable in that they tended to congregate in the area of the system where the reactive supply was most limited by the output limits on generators.

Finally, the continuation power flow was shown to be effective on a large power system. The index indicated the closer proximity to voltage collapse that was brought on by the progressively harsher system conditions. This capability to handle large systems is paramount to the practicality of any proposed power flow method.
Suggestions for Future Work

Although the continuation power flow has proven practical for the steady-state analysis of voltage stability, several things could be done to enhance and verify this work. Perhaps the most needed enhancement is a method of automating the step-size selection. Although it was not difficult to choose a workable step size, some trial and error was required before the appropriate step size was found. For example, in Chapter 5, the step size for the 28th step of the contingent case using a constant power load model had to be reduced for successful completion of the calculation. If a means of automatically selecting the step-size was available, such a special step-size selection would not be required of the operator. In addition, automating the step size selection might allow the solution path to be found using fewer steps. This would help to optimize the efficiency of the continuation process.

Another enhancement that would be beneficial is an efficient means of setting the load change and generation change constants. This is largely a programming problem, but will be helpful if a scenario involves changes on thousands of buses at a time.

Additional work could be done to verify the indication of weak buses that is produced via sensitivity. (One deterministic means would be to calculate participation factors for the eigenvalues (see [23]).) These factors identify the state variables that are most significant to a particular eigenvalue. The voltage variables of the weakest buses found via sensitivity should be among the state variables most significant to the zero eigenvalue at the critical point.

Further development could include interfacing the techniques used in this research with optimization techniques to determine the best way to dispatch reactive
power. Knowing the location of the weak buses would divulge the best place to locate reactive power compensation. The objective would be to maximize the power transfer of the system without allowing the critical point to move up into the range of acceptable operating voltages. Since this type of optimization would imply operating the system closer to its physical limits, additional stability analysis would be needed to verify this process. The machine dynamics that were not considered in this research would require analysis and the constraints of transient stability would require monitoring.

Lastly, it will be pointed out that the type voltage stability analysis completed in this work requires a realistic load model. While the EPRI load model used is claimed to be accurate down to ten percent below nominal voltage, bus voltages at the critical point are typically well below this minimum value for uncompensated systems. Thus a load model that is valid over a broader range of voltage is needed for a more accurate analysis. The load-model problem is, however, inherent to the problem at large and is not peculiar to the solution algorithm used in the analysis.
BIBLIOGRAPHY


APPENDIX A. EXPRESSION FOR VOLTAGE IN A TWO-BUS POWER SYSTEM

The following is a derivation of the expression for the load bus voltage in Figure A.1.

\[
\begin{align*}
E \angle 0 & \quad jX \\
& \quad P_L + jQ_L \\
& \quad V \angle \theta
\end{align*}
\]

Figure A.1: Single load served by an infinite bus

Active power at load bus:

\[
0 = P_{GENERATION} - P_{LOAD} - P_{INJECTION} \tag{A.1}
\]

\[
= -P_L - V E \left( \frac{1}{X} \right) \cos(-\theta - 90)
\]

Reactive power at load bus

\[
0 = Q_{GENERATION} - Q_{LOAD} - Q_{INJECTION} \tag{A.2}
\]

\[
= -Q_L - V E \left( \frac{1}{X} \right) \sin(-\theta - 90) - V^2 \left( \frac{1}{X} \right) = 0
\]
Solve for $\cos(-\theta - 90)$ and $\sin(-\theta - 90)$:

$$\cos(-\theta - 90) = \frac{P_L X}{V E} \quad \text{(A.3)}$$

$$\sin(-\theta - 90) = \frac{-(Q_L X + V^2)}{V E} \quad \text{(A.4)}$$

Use $\cos^2 x + \sin^2 x = 1$ identity:

$$1 = \left(\frac{Q_L X + V^2}{V E}\right)^2 + \left(\frac{P_L X}{V E}\right)^2 \quad \text{(A.5)}$$

Solve for $V^2$:

$$V^4 + V^2(2Q_L X - E^2) + Q_L^2 X^2 + P_L^2 X^2 = 0 \quad \text{(A.6)}$$

$$V^2 = \frac{(E^2 - 2Q_L X) \pm \sqrt{(2Q_L X - E^2)^2 - 4X^2(Q_L^2 + P_L^2)}}{2} \quad \text{(A.7)}$$

$V$ is the positive square root of $V^2$ (since voltage should be positive);

$$V = \sqrt{\frac{(E^2 - 2Q_L X) \pm \sqrt{(2Q_L X - E^2)^2 - 4X^2(Q_L^2 + P_L^2)}}{2}} \quad \text{(A.8)}$$
APPENDIX B. POWER FLOW INJECTIONS AND JACOBIAN ELEMENTS

From equations (3.5) and (3.6), the general form of the power flow equations is

$$\Delta P_i = P_{G_i} - P_{L_i} - P_{T_i} = 0, \quad P_{T_i} = \sum_{j=1}^{n} V_i V_j y_{ij} \cos(\delta_i - \delta_j - \nu_{ij})$$  \hspace{1cm} (B.1)

$$\Delta Q_i = Q_{G_i} - Q_{L_i} - Q_{T_i} = 0, \quad Q_{T_i} = \sum_{j=1}^{n} V_i V_j y_{ij} \sin(\delta_i - \delta_j - \nu_{ij})$$  \hspace{1cm} (B.2)

From Chapter 3, the elements of the Jacobian submatrices are given by

$$H_{ij} = \frac{-\partial \Delta P_i}{\partial \delta_j}$$  \hspace{1cm} (B.3)

$$N_{ij} = -V_j \frac{\partial \Delta P_i}{\partial V_j}$$  \hspace{1cm} (B.4)

$$J_{ij} = \frac{-\partial \Delta Q_i}{\partial \delta_j}$$  \hspace{1cm} (B.5)

$$L_{ij} = -V_j \frac{\partial \Delta Q_i}{\partial V_j}$$  \hspace{1cm} (B.6)

The injection terms and Jacobian elements can all be calculated at the same time by using the following terms

$$\tilde{y}_{ij} = G_{ij} + jB_{ij} = y_{ij} / \nu_{ij}$$  \hspace{1cm} (B.7)
\[ \tilde{V}_i = e_i + j f_i = V_i / \delta_i \] (B.8)

\[ \tilde{V}_j = e_j + j f_j = V_j / \delta_j \] (B.9)

\[ a_{ij} = G_{ij} e_j - B_{ij} f_j \] (B.10)

\[ b_{ij} = B_{ij} e_j + G_{ij} f_j \] (B.11)

\[ c_{ij} = e_i a_{ij} + f_i b_{ij} \] (B.12)

\[ d_{ij} = f_i a_{ij} - e_i b_{ij} \] (B.13)

Then

\[ P_{T_i} = \sum_{j=1}^{n} V_i V_j y_{ij} \cos(\delta_i - \delta_j - \nu_{ij}) = \sum_{j=1}^{n} c_{ij} \] (B.14)

\[ Q_{T_i} = \sum_{j=1}^{n} V_i V_j y_{ij} \sin(\delta_i - \delta_j - \nu_{ij}) = \sum_{j=1}^{n} d_{ij} \] (B.15)

For the \( H \) submatrix:

\[ H_{ij(i \neq j)} = -\frac{\partial \Delta P_i}{\partial \delta_j} = V_i y_{ij} V_j \sin(\delta_i - \delta_j - \nu_{ij}) \] (B.16)

\[ = d_{ij} \]

\[ H_{ii} = -\frac{\partial \Delta P_i}{\partial \delta_i} \] (B.17)
For the $N$ submatrix:

$$N_{ij(i\neq j)} = -V_i \frac{\partial \Delta P_i}{\partial V_j} = V_i y_{ij} V_j \cos(\delta_i - \delta_j - \nu_{ij})$$

$$= c_{ij} \quad \text{(B.18)}$$

$$N_{ii} = -V_i \frac{\partial \Delta P_i}{\partial V_i} = \sum_{j=1}^{n} V_i y_{ij} V_j \cos(\delta_i - \delta_j - \nu_{ij})$$

$$+ V_i y_{ii} V_i \cos(\delta_i - \delta_j - \nu_{ii})$$

$$= c_{ii} + P_{Ti} \quad \text{(B.19)}$$

For the $J$ submatrix:

$$J_{ij(i\neq j)} = -\frac{\partial \Delta Q_i}{\partial \delta_j} = -V_i y_{ij} V_j \cos(\delta_i - \delta_j - \nu_{ij})$$

$$= -c_{ij} \quad \text{(B.20)}$$

$$J_{ii} = -\frac{\partial \Delta Q_i}{\partial \delta_i} = \sum_{j=1, j\neq i}^{n} V_i y_{ij} V_j \cos(\delta_i - \delta_j - \nu_{ij})$$

$$= -c_{ii} + P_{Ti} \quad \text{(B.21)}$$

For the $L$ submatrix:

$$L_{ij(i\neq j)} = -V_j \frac{\partial \Delta Q_i}{\partial V_j} = V_i y_{ij} V_j \sin(\delta_i - \delta_j - \nu_{ij})$$

$$= d_{ij} \quad \text{(B.22)}$$
\[ L_{mn} = -V_t \frac{\partial \Delta Q_i}{\partial V_i} \]  
\( = \sum_{j=1}^{n} V_i y_{il} V_j \sin(\delta_i - \delta_j - \nu_{ij}) + V_i y_{il} V_i \sin(\delta_i - \delta_j - \nu_{ii}) \)  
\( = d_{ii} + Q_{T_i} \)
APPENDIX C. DETAILED FLOW CHART OF THE CONTINUATION POWER FLOW
Figure C.1: Data input
Figure C.2: Base case power flow
Figure C.3: Predictor
Figure C.4: Corrector
APPENDIX D. CONTINUATION POWER FLOW JACOBIANS

Predictor Jacobian versus Corrector Jacobian

As shown in Chapter 4, the Jacobian used in the predictor has the same form as the one used in the corrector. However, it is most convenient in the corrector iterations to absorb the negative sign of equation (3.9) into the Jacobian. Then, to maintain the similarity between the predictor Jacobian and the corrector Jacobian, it would be useful to negate terms in the predictor Jacobian. As the following derivation shows, such negation does not affect the tangent calculation.

Negating equation (4.4) results in

\[-F_\delta d\delta - F_V dV - F_\lambda d\lambda = 0\]  \hspace{1cm} (D.1)

Factoring;

\[
\begin{bmatrix}
-F_\delta & -F_V & -F_\lambda \\
\end{bmatrix}
\begin{bmatrix}
d\delta \\
dV \\
d\lambda \\
\end{bmatrix} = \begin{bmatrix}
0 \\
\end{bmatrix}
\]  \hspace{1cm} (D.2)

Then, when the definition for $t$ of equation (4.6) is used, and the augmenting equation $t_k = \pm 1.0$ is used, the system becomes

\[
\begin{bmatrix}
-F_\delta & -F_V & -F_\lambda \\
e_k \\
\end{bmatrix}
\begin{bmatrix}
t \\
\end{bmatrix} = \begin{bmatrix}
0 \\
\pm 1 \\
\end{bmatrix}
\]  \hspace{1cm} (D.3)
In this equation, all but the $e_k$ portion of the Jacobian has been negated without affecting the predictor calculation. However, since the mismatch of the augmenting equation, $x_k - \eta = 0$, will always be zero, the last row in the corrector equation (4.12) will simply stipulate that $\Delta x_k^{(k)} = 0$. Thus, leaving $e_k$ positive in the corrector Jacobian does not affect the outcome of the iterative process.

Now that negation of most of the Jacobian has been performed without consequence to the predictor calculation, the same Jacobian can be used for both the predictor and corrector. The rest of this appendix deals with the details of calculating this Jacobian for the two load models used in this thesis.

**The Constant Power Load Model Jacobian**

The only term that contains voltage or angle in the constant power load model is $P_{Ti}$. Thus, the $-F_{\delta}$ and $-F_V$ terms are the same as those calculated in Appendix B for the conventional Newton-Raphson power flow (assuming the $\Delta V_i$ terms are changed to $\frac{\Delta V_i}{V_i}$ as in Appendix B). The Jacobian then takes on the form

\[ \begin{bmatrix} H & N & -\Delta P_{\lambda} \\ J & L & -\Delta Q_{\lambda} \\ e_k \end{bmatrix} \]  

where

\[ -\Delta P_{\lambda_i} = -\frac{\partial \Delta P_i}{\partial \lambda} = -(GFRAC_i)(PMULT) + k_{L_i}S_{BASE}\cos \psi_i \]  

\[ -\Delta Q_{\lambda_i} = -\frac{\partial \Delta Q_i}{\partial \lambda} = k_{L_i}S_{BASE}\sin \psi_i \]
The Nonlinear Load Model Jacobian

The Jacobian for the nonlinear load model takes on the form

\[
\begin{bmatrix}
H & N^* & -\Delta P_\lambda \\
J & L^* & -\Delta Q_\lambda \\
e_k
\end{bmatrix}
\]  \hspace{1cm} (D.7)

where \( H \) and \( J \) are the same submatrices derived in Appendix 2. The \( N^* \) and \( L^* \) terms differ from the \( N \) and \( L \) terms in that they also contain partial derivatives of the nonlinear load model. These terms and the \(-\Delta P_\lambda \) and \(-\Delta Q_\lambda \) terms are derived as follows.

\[
\frac{\Delta P_i}{P_{G_0} - P_{L_i} - P_{T_i}} \hspace{1cm} (D.8)
\]

where

\[
P_{G_i} = P_{G_{0i}} + GFRAC_i \left( \sum_{k=1}^{n} P_{Lk} - \sum_{k=1}^{n} P_{L_{ko}} \right) \hspace{1cm} (D.9)
\]

\[
P_{L_i} = (1 + k_{Li})P_{aoi}P_{ai} \frac{\left( \frac{V_i}{V_{ao}} \right)^{KPV_1}}{KPV_2} + (1 - P_{aoi})(\frac{V_i}{V_{ao}})^{KPV_2} \hspace{1cm} (D.10)
\]

so

\[
N_{ii} = -V_i \frac{\partial \Delta P_i}{\partial V_i} \hspace{1cm} (D.11)
\]

\[
= -V_i \frac{\partial P_{G_i}}{\partial V_i} + V_i \frac{\partial P_{L_i}}{\partial V_i} + V_i \frac{\partial P_{T_i}}{\partial V_i}
\]

\[
= -V_i GFRAC_i \sum_{k=1}^{n} \frac{\partial P_{Lk}}{\partial V_i} + V_i \frac{\partial P_{L_i}}{\partial V_i} + c_{ii} + P_{Ti}
\]

\[
= (1 - GFRAC_i)V_i \frac{\partial P_{L_i}}{\partial V_i} + c_{ii} + P_{Ti}
\]

\[
\mathcal{N}_{ij, i \neq j} = -V_j \frac{\partial \Delta P_i}{\partial V_j} \hspace{1cm} (D.12)
\]
\[-\Delta P_{\lambda i} = -\frac{\partial \Delta P_i}{\partial \lambda} \]

\[-\frac{\partial P_{Gi}}{\partial \lambda} + \frac{\partial P_{Li}}{\partial \lambda} + \frac{\partial P_{Ti}}{\partial \lambda} \]

\[-GFRAC_i V_j \frac{\partial P_{Li}}{\partial V_j} + c_{ij} \]

where

\[V_i \frac{\partial P_{Li}}{\partial V_i} = (1 + k_{Li})P_o [P_a (KPV1)(\frac{V_i}{V_{oi}})^{KPV1} + KPV2 (1 - P_{a1})(\frac{V_i}{V_{oi}})^{KPV2}] \]  \hspace{1cm} (D.14)

\[\frac{\partial P_{Li}}{\partial \lambda} = k_{Li} P_o [P_a (\frac{V_i}{V_{oi}})^{KPV1} + (1 - P_{a1})(\frac{V_i}{V_{oi}})^{KPV2}] \]  \hspace{1cm} (D.15)

and \(c_{ii}, c_{ij}, \) and \(P_T\) are defined in Appendix B. For the reactive power portion

\[\Delta Q_i = Q_{Gi} - Q_{Li} - Q_{Ti} \]  \hspace{1cm} (D.16)

where

\[Q_{Li} = (1 + k_{Li})[P_o Q_{a1} (\frac{V_i}{V_{oi}})^{KQV1} + (Q_o - P_o Q_{a1})(\frac{V_i}{V_{oi}})^{KQV2}] \]  \hspace{1cm} (D.17)

so

\[L_u = -V_i \frac{\partial \Delta Q_i}{\partial V_i} \]  \hspace{1cm} (D.18)

\[\frac{\partial Q_{Gi}}{\partial V_i} + V_i \frac{\partial Q_{Li}}{\partial V_i} + V_i \frac{\partial Q_{Ti}}{\partial V_i} \]

\[= V_i \frac{\partial Q_{Li}}{\partial V_i} + d_u + Q_{Ti} \]  \hspace{1cm} (D.19)
\[ L_{ij,i\neq j} = -V_j \frac{\partial \Delta Q_i}{\partial V_j} \]  
\[ = -V_j \frac{\partial Q_{gi}}{\partial V_j} + V_j \frac{\partial Q_{Li}}{\partial V_j} + V_j \frac{\partial Q_{Ti}}{\partial V_j} \]  
\[ = d_{ij} \]  

\[ -\Delta Q_{\lambda_i} = -\frac{\partial \Delta Q_i}{\partial \lambda} \]  
\[ = -\frac{\partial Q_{gi}}{\partial \lambda} + \frac{\partial Q_{Li}}{\partial \lambda} + \frac{\partial Q_{Ti}}{\partial \lambda} \]  
\[ = \frac{\partial Q_{Li}}{\partial \lambda} \]  

where

\[ V_i \frac{\partial Q_{Li}}{\partial V_L} = (1 + k_{Li}\lambda)[P_o Q_{a1}(KQV1)\left(\frac{V_i}{V_{oi}}\right)^{KQV1} + (Q_o - P_o Q_{a1})(KQV2)\left(\frac{V_i}{V_{oi}}\right)^{KQV2}] \]  

\[ \frac{\partial Q_{Li}}{\partial \lambda} = k_{Li}[P_o Q_{a1}\left(\frac{V_i}{V_{oi}}\right)^{KQV1} + (Q_o - P_o Q_{a1})\left(\frac{V_i}{V_{oi}}\right)^{KQV2}] \]  

and \( d_{ii}, d_{ij}, \) and \( Q_{Ti} \) are defined in Appendix B.
APPENDIX E. SCALING LOAD CHANGE MULTIPLIERS

Constant Power Load Model

The method used to scale the load change multipliers for the constant power load model is a three step process.

1. Estimate the multiple of original active power load for collapse. Call this $EST$.

2. Normalize the $k_L$’s so that $\sum_{i=1}^{n} k_L \cos \psi_i = 1$. The following derivation shows how this can be accomplished:

Let $k_{Li}'$ be the original, unscaled value of the load change multiplier for bus $i$. Then let

$$ADJ = \sum_{i=1}^{n} k_{Li}' \cos \psi_i$$

(E.1)

Now

$$\sum_{i=1}^{n} \left(\frac{k_{Li}'}{ADJ}\right) \cos \psi_i = 1$$

(E.2)

so if $k_{Li} = \frac{k_{Li}'}{ADJ}$ then $\sum_{i=1}^{n} k_L \cos \psi_i = 1$. With this scaling, $PMULT$ (defined in Chapter 5) is equal to $S_{BASE}$.

$$PMULT = S_{BASE} \sum_{i=1}^{n} k_L \cos \psi_i = S_{BASE}$$

(E.3)
3. Choose $S_{\Delta BASE}$ such that $\lambda$ has the desired magnitude of change at the estimated point of collapse. Let $\lambda_{DES}$ be the desired value of $\lambda$ at collapse.

$$\Delta P_{TOTAL} = (EST - 1)(\sum_{i=1}^{n} P_{Lio}) = \lambda_{DES}PMULT = \lambda_{DES}S_{\Delta BASE} \quad (E.4)$$

so

$$S_{\Delta BASE} = \frac{(EST - 1)(\sum_{i=1}^{n} P_{Lio})}{\lambda_{DES}} \quad (E.5)$$

Generally, voltage may change by 0.10 to 0.50 per unit before collapse and the change in angles would not be expected to exceed $\frac{\pi}{2}$ (radians). Thus $\lambda_{DES}$ should be set between 0.10 and $\frac{\pi}{2}$.

**Nonlinear Load Model**

Let $EST$ be the estimate of the multiple of original active power load for collapse. Then, if $\lambda_{DES}$ is the desired value of $\lambda$ at collapse, scaling can be accomplished by ignoring voltage dependence as follows:

$$\left(EST - 1\right) \sum_{i=1}^{n} P_{Lio} = \lambda_{DES} \sum_{i=1}^{n} \frac{k'_{Li}}{ADJ} P_{Lio} = \frac{1}{ADJ} \lambda_{DES} \sum_{i=1}^{n} k'_{Li} P_{Lio} \quad (E.6)$$

so

$$ADJ = \frac{\lambda_{DES} \sum_{i=1}^{n} k'_{Li} P_{Lio}}{(EST - 1) \sum_{i=1}^{n} P_{Lio}} \quad (E.7)$$

The scaled multiplier is then $k_{Li} = \frac{k'_{Li}}{ADJ}$. 
APPENDIX F.  NEW ENGLAND SYSTEM DATA
<table>
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<th>BUS NO</th>
<th>NAME</th>
<th>Angle</th>
<th>Real Power</th>
<th>Reactive Power</th>
</tr>
</thead>
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<td>1</td>
<td>Gamma</td>
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<td>0</td>
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</tr>
<tr>
<td>2</td>
<td>Delta</td>
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<td>500.00</td>
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<tr>
<td>3</td>
<td>Epsilon</td>
<td>3</td>
<td>0</td>
<td>750.00</td>
</tr>
<tr>
<td>4</td>
<td>Theta</td>
<td>4</td>
<td>0</td>
<td>1000.00</td>
</tr>
<tr>
<td>5</td>
<td>Iota</td>
<td>5</td>
<td>0</td>
<td>1250.00</td>
</tr>
</tbody>
</table>

Figure F.1: Bus data in IEEE common data format [17]
APPENDIX G. FLOW CHART FOR THE SPARSE VERSION OF THE CONTINUATION POWER FLOW
Figure G.1: Data input
Figure G.2: Base case power flow and corrector
Figure G.3: Decomposition for base case power flow
Figure G.4: Decomposition for corrector
Figure G.5: Predictor