INTRODUCTION

In this paper, the dispersion of surface acoustic waves (SAW) in case carborized steel samples was calculated. The calculations were based on the Thomsen-Haskill matrix formalism. This method has the advantage that it treats a given profile as a stack of several layers, each with arbitrary thickness, density and elastic moduli. This treatment allows for a realistic representation of real material effects by allowing, for example, fluctuations in these parameters with depth, either from material variability or measurement error. In order to demonstrate the capability of the formalism we measured hardness, density and bulk velocity in homogeneously carbonized specimens. We then used these data to synthesize several different hardness profiles, such as might be encountered in actual industrial applications. For these profiles the SAW dispersion curves were then calculated. The results provided valuable insight into such questions as the range of SAW velocity extremal values and the frequency bandwidth over which the velocity changes are maximum. Thus the Thomsen-Haskill matrix method is shown to provide a useful tool for simulating the effect of hardness on SAW velocity with potential applications for a wide variety of physical property gradients.

BACKGROUND

When surface waves travel over planar homogeneous lossless isotropic samples they do not disperse; i.e., their phase velocity is constant with frequency. However, if the material properties of the sample change with depth, the acoustic parameters are often affected causing the SAWs to disperse in a manner dependent on the material property gradient. This arises because the elastic waves penetrate the solid to depths proportional to their wavelength. Low frequency waves reach deeper into the solid summing up the contributions of the upper layers under some weighting criteria defined by the nature of the wave itself. Mathematically this dispersion can be expressed as an integral equation. Thus a dispersion curve can be predicted if the variation in elastic values is known.

Many researchers have reported relationships between ultrasonic measurements and hardness. In 1952, Roderick and Truell [1] found that ultrasonic attenuation was quite sensitive to differences in 4150 steels prepared under various heat treatments, and that the attenuation could be used to differentiate between states of hardness. Ultrasonic velocity
measurements have been suggested as a method for sorting malleable cast iron based on a hardness correlation [2]. Borzorg-Grayeli [3] made longitudinal velocity measurements on a variety of carbon and alloy steels. Rosen [4] found a quadratic correlation between changes in hardness and ultrasonic velocity, whereas he found a linear correlation between hardness and ultrasonic attenuation in age hardened 160 Al-Cu alloy. Krautkramer and Krautkramer [5] cite a number of German articles to support the claim that hardness can be related to acoustic velocity or attenuation and empirically determined from them when all other variables are kept constant. They also assert that treatments of steel, such as quenching and tempering have a greater effect on acoustic velocity than do variations in composition of alloying elements. Recently Singh et al.[6] have reported on the effects of carbon and microstructure on ultrasonic velocities in 81XX steels.

Considerable effort has been spent on the Inverse Problem of determining hardness gradients from surface acoustic wave dispersion.

Using the integral equation derived by Auld [7], Tittmann and Thompson [8] predicted the dispersion of Rayleigh waves in a case-hardened steel. A solution was obtained by perturbing an energy functional description of the elastic wave through changes in a single stiffness constant, c44. Subsequent work focused on predicting property gradients from measured dispersion data; the inverse problem. The studies [8-12] assumed that the perturbed elastic and density values were mutually proportional. These methods have been reviewed and reconsidered by Tittmann et al. [13] in light of hardware advances: broadband ultrasonic transducers and digitizers which facilitate more convenient dispersion data collection.

APPROACH

In this work we use an approach which was not constrained to variations in a single material parameter or to an assumed material mutual proportionality. The mathematical treatment for prediction and inversion of hardness gradients in 81XX case carburized steel is based on a layered model. A special case of the propagator matrix method due to Thomson [14] and Haskell [15] is used to determine the surface wave dispersion of isotropic layers stacked on a half space. These methods are well developed and have found extensive use in the seismological community. The features of our approach can be summarized as follows: i) a solution to the forward problem can be attempted for any vertical spatial variations among the total set of acoustic parameters used to characterize uniform isotropic material under the assumptions of linear elastic theory. ii) the procedure is efficient because the propagator matrix method does not require matrices larger than 4 x 4 regardless of the number of layers. iii) a priori information such as a delta V/V versus hardness correlation [3], layer acoustic signatures and model smoothness assumptions can be implemented as specific cases dictate. iv) the procedure is easily modified to account for geometric dispersion produced by cylindrical or spherical shapes, increasing its usefulness to industry.

THEORETICAL SIMULATION

A simulation was performed to check practical aspects and the sensitivity of the layered model approach. Since the extent of martensite development and therefore hardness is correlated to the sample’s carbon content [16], five normalized, hardened and tempered 81XX steel samples, with levels of 0.25, 0.43, 0.64, 0.84, and 1.03 weight percent carbon, [6] were prepared and characterized. These measurement values were assigned to layers in six model profiles in order to approximate continuous carbon gradients.

Changes in densities between the samples were measured to ±0.01% by Archimedes principle. Hardness values were assessed with a Rockwell hardness machine using a diamond cone indenter. After a series of measurements an average value and error, ±1 Rc, was established for each sample. As an example, we present in Fig. 1 hardness values for each of the samples as a function of their carbon content. Shear and longitudinal velocities were measured by a pulse echo overlap method using a digital oscilloscope. The samples were
ground flat and parallel to an angular deviation of less than 0.16 degrees, which according to the criteria given by [1] will ensure that the returning signal will not be affected by wedging up to a frequency of 20 MHz and ten echoes. No corrections were taken for diffraction effects or transducer bond phase errors.

At the outset certain assumptions were made about the layered model:

i) no attenuation. Attenuation can produce dispersion, but the resulting amount of dispersion was considered to be insignificant in the present situation. This was justified by using a worst case value for steel attenuation from the literature [3] and determining the change in velocity from the relationship given by O’Donnell et al. [17]. Dispersion was found to be 5E-4 percent over a frequency range of 1 - 25MHz.

ii) homogeneous layers. Multiphase materials such as steel are not strictly homogeneous. However, since velocity is not affected by grain size [18], the variations in elastic properties from the various phases were assumed to sum into an effective stiffness value for each sample.

iii) isotropic layers. Anisotropy, due to sample texture or residual stresses, was ignored. In general, texture is absent in case carburized specimens and the contributions of residual stress to changes in velocity is relatively small.

iv) single hardening mechanism. We assumed that the changes in hardness were entirely due to the phase transformation of austenite (FCC) to martensite (BCT). The martensite phase transformation leads to changes in the internal strain energy of the material thereby modifying the stress-strain relationship. In addition, the density of the material is affected and these variations lead to changes in velocity.

v) no other compositional variations. Changes in composition other than carbon content were assumed to have negligible consequence on the gradient model.

A set of six models was formed from the information gained from the homogeneous samples. This was accomplished by using Fick’s second law of diffusion [16] to predict carbon concentration as a function of depth. Fig. 2 presents an example for the case of a layer model for a mean carbon concentration at a depth of 0.25 mm from the surface.

The Thomsen-Haskell method was then used to calculate the dispersion curves for each of the layer models. Fig. 3 is a summary of all the calculations. It shows the frequency dependent SAW phase velocity for six different profiles. The profiles are distinguished by the depth of their mean carbon value.

Apparent in the curves is the fact that at low frequencies the SAW velocity is high and then diminishes rapidly as the frequency is increased. This is consistent with the notion that at low frequencies the waves penetrate deeply beneath the surface and thus sample the region with low hardness or high bulk velocities and density. At the high frequencies the

![Figure 1](image-url)

Figure 1. Measured hardness as a function of carbon content in each of five homogeneously carbonized samples.
Figure 2. Example of hardness profile based on an assumed mean carbon level at 0.25 mm below the surface.

Figure 3. SAW Dispersion Curves for Layered Models

penetration is very shallow and only the high hardness region with low bulk velocities and low density is sampled by the wave. The small increase in phase velocity at high frequencies is anomalous and arose because of a fluctuation in the bulk velocities measured in the sample with highest carbon content, either because of materials variability or measurement error. Another noticeable feature is the shift of the curves towards high frequencies as the mean carbon level shifts towards more shallow depths. Thus all the features in the calculated dispersion curves are consistent with intuition and valuable in providing guidelines on how to optimize measurements of the dispersion curves on actual hardness profiles.
DISCUSSIONS AND CONCLUSIONS

This paper has presented results for the solution of the Forward Problem, i.e. the calculation of dispersion curves from a layer model of an arbitrary property gradient. The method employed was the Thomsen-Haskill matrix formalism. To demonstrate the method we have used data obtained from a set of differently carbonized samples to simulate a variety of possible hardness profiles, each with a different mean carbon depth level. The homogeneous samples exhibited a variation in longitudinal and shear wave velocity of 0.7% and 1.5% respectively and a density variation of 0.5%. These values lead to a surface wave dispersion of 1.2% over a frequency range of more than a decade for each simulated profile. An experimental measurement of the dispersion curve would require keeping the relative error below a level of about ±0.1%. Current state-of-the-art measurements [19] have shown that under well-controlled conditions a SAW velocity measurement precision of ±0.005% is now routine.

This work sets the stage for the ultimate goal of solving the Inverse Problem, in which data from an experimentally determined SAW dispersion curve are used to estimate the corresponding hardness profile.

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REFERENCES


