The combined effect of cognitive monitoring, heuristic strategies, and computer-based learning on mathematical problem solving skills

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The combined effect of cognitive monitoring, heuristic strategies, and computer-based learning on mathematical problem solving skills

by

Dawn Marie Sleger Poole

A Thesis Submitted to the Graduate Faculty in Partial Fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE

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Signatures have been redacted for privacy

Iowa State University
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CHAPTER I. INTRODUCTION

This study examined the combined effect of heuristic teaching strategies, cognitive monitoring, and computer-based learning on mathematical problem solving skills. Chapter one has six sections: 1) A brief literature review to provide background information about the impact of cognition, metacognition, and heuristics on mathematical problem solving, 2) a statement of the study's research problem, 3) a statement of the purpose of the study, 4) the research questions, 5) definition of terms, and 6) a chapter summary.

For the last decade, the National Council of Teachers of Mathematics (NCTM) has stated that more emphasis needs to be given to mathematical problem solving in schools. In fact, the 1982 NCTM Agenda for Action advocated that the teaching of problem solving skills have the highest priority in school mathematics (Creswell, 1983). The new standards developed by the NCTM in 1991 advocated problem solving and mathematical reasoning as mathematical tasks that needed to be taught in K-12 schools. Problem solving was considered so important that Schoenfeld (1982, p. 2) stated,

...We believe the primary responsibility of mathematics faculty is to teach their students to think: to question and to probe, to get to the mathematical heart of the matter, to be able to employ ideas rather than simply regurgitate them.

Even before the new NCTM problem solving standards, mathematical problem solving was being investigated by several groups. These investigations indicated that students in the United States were not making significant gains in problem solving skills (Carpenter, Lindquist, Brown, Kouba, Silver, & Swafford, 1988). The National Center for Education Statistics (1991) reported data that showed only a four percent increase nation-wide in students' mathematics problem solving skills between 1978 and 1990. According to NCES, the ability to apply math operations in problem situations only increased by one percent during this same time period.
Girls, especially, seemed to lack the ability to solve mathematical problems (Backman, 1972; Becker & Forsyth, 1990; Benbow & Minor, 1986; Benbow & Stanley, 1983; Caporrimo, 1990).

The 1991 NCTM report, in addition to the new problem solving standards, listed new standards regarding the use of technology in the mathematics curriculum. The Mathematical Association of America Committee on the Mathematical Education of Teachers (1991) stressed the appropriate use of calculators and computers in the teaching and learning of mathematics. However, this committee stated that computer use for the sake of using technology alone was not enough. They stated that effective instruction that incorporated technology as a part of instruction needed to be addressed.

The increased interest in mathematical problem solving and in the development of problem solving ability have placed demands on mathematics educators to give greater attention to problem solving instruction. Teaching methods need to be devised to assist students in learning mathematical problem solving skills, rather than just modeling how to solve specific problems. When analyzing teaching methods for effectiveness, technology should also be considered as a resource for helping students learn mathematical problem solving skills (National Council of Teachers of Mathematics, 1991).

Background

Cognition and Problem Solving

Problem solving is a complex cognitive process. Since problem solving, especially mathematics problem solving, is an important outcome of K-12 education, according to the NCTM and other organizations, the role of cognition in acquiring these skills has been extensively examined. This section discusses the theory of cognition and its relevance to problem solving.

According to White and Collins (1983, p234), "Problem solving skills are not innate,
but rather are a systematic way of processing information." Thus, for students to solve problems they must process information effectively. Information processing is a cognitive strategy involving the use of schemata to enable proper encoding. Schemata are the knowledge structures in permanent memory that contain elements of related information and provide plans for gathering additional information. In order for problems to be solved, schemata need to be formed so that a student can access the correct strategies needed.

Research has shown that the major difference between effective problem solvers and poor problem solvers was amount of domain-specific knowledge (Perkins & Salomon, 1989). This indicates that before students can apply schemata, however, a large data base of context-specific information must be stored. Without a sufficient knowledge base, problem solving will be unsuccessful.

An individual's knowledge base necessary for problem solving includes more than merely factual knowledge in a subject area. Mayer (1982) defined four types of knowledge that are relevant in mathematical problem solving:

1. Linguistic knowledge concerning how to encode sentences effectively;
2. Schema knowledge concerning mental relationships between different categories of problem types such as "work" or "motion" problems;
3. Algorithmic knowledge concerning how to perform well-defined procedures such as addition; and
4. Strategic knowledge concerning how to approach problems most effectively.

Each of these four types of knowledge is called upon in solving mathematical problems.

Once the four types of background knowledge are acquired, then domain-specific knowledge is stored in memory structures and is accessed when a problem situation occurs. In other words, before a problem can be successfully solved, students need to be able to
understand what a problem is asking them to find, decide on a successful approach to solving the problem, categorize the problem into a specific type, and correctly perform any algorithms the problem requires. For example, students that can not add will not be able to learn the division process. Similarly, if students can not successfully visualize a triangular prism, it will be difficult for them to find the surface area of that object. Teachers must be sure that students have the background knowledge necessary for problems presented to students so that they can form the schemata required to solve them.

Although a great deal of controversy exists, most current problem solving research indicates a lack of transfer across context domains without specific cueing and guiding (Perkins & Salomon, 1989, Schoenfeld, 1985). Because of the lack of agreement among cognitivists about their views on transfer, this study did not address transferring mathematical problem solving skills to other academic domains. This study specifically addressed the issue of whether problem solving skills could be acquired within a mathematical context and be applied within a strictly mathematical context.

**Metacognition**

Although domain-specific knowledge is related to the ability to solve problems, the metacognitive viewpoint also suggests that another factor influencing problem solving ability is the awareness of thinking processes, often called metacognition, reflective thinking, or cognitive monitoring. Metacognition refers to the idea of understanding one's own cognitive processes or anything related to these processes. It refers to, "...the active monitoring and consequent regulation and orchestration of those processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete [problem solving] goal or objective" (Flavell, 1976, p232). In a broad sense, metacognition emphasizes the ability to analyze thoughts and processes effectively in order to learn.
It has been observed that effective problem solvers test and reject ideas, are aware of their thinking processes, make tentative explorations, generate possible solution strategies, and try new strategies when warranted. Poor problem solvers lack the ability to perform these processes (Caporrimo, 1990). Schoenfeld (1985) suggested that performance on many tasks was positively correlated with the degree of one's metacognitive ability. When children were taught to monitor their cognitive thoughts as they attempted to solve new problems, they learned to understand the strategies employed.

These results suggest that teaching students metacognitive skills may help them to develop problem solving skills. Understanding strategies should be a major goal of problem solving instruction according to Simon (1980). Students need to become aware of how these strategies (heuristics) are organized in memory to provide not only a repertory of problem solving actions, but also conditions that serve to index these actions and evoke them when they need to be used. Metacognitive strategies, then, can be tools to assist students learning problem solving skills.

Heuristics and Mathematical Problem Solving

Despite the emphasis given to problem solving, students are often expected to be successful problem solvers without being taught any formal problem solving skills (Norman, 1980). However, according to Dewey (1933, p15), 'Thinking is not a case of spontaneous combustion; it does not occur just on 'general principles'. There is something that occasions and evokes it.' Heuristics may be this "something". Heuristics help students become aware of and move toward goals, make decisions and discoveries, and learn how to learn what to do (rather than just learn what to do). They require the teacher to assume a role of guide, co-learner, and resource instead of purely lecturer (Nicely, 1985).
Polya (1957) was the person to make a significant push toward the widespread use of heuristics. He believed that problem solving ability was a set of general principles which was systematically applied to a relevant database of knowledge. He felt that once general problem solving skills were acquired and the students' background knowledge was sufficient, problems could be solved successfully provided background knowledge was sufficient. Polya suggested four broad heuristic strategies to help learners acquire problem solving skills: 1) define the problem; 2) develop a plan; 3) carry out the plan; and 4) check the solution. Although Polya believed his heuristic strategies could be applied successfully in any context, he specifically applied them in the mathematical domain. He expected these general skills to transfer to other contexts.

Polya's ideas were initially supported by the results of research. Eventually, however, the literature began to show that problem solving skills were not the general skills that Polya believed automatically transferred to other context-domains (Perkins & Salomon, 1989). Schoenfeld (1985) asserted that the heuristic strategies suggested by Polya were in large part simply labels for categories of related strategies, and did not lead to specific procedures. He also claimed that only after considerable mathematical conceptual and procedural knowledge was acquired did mathematically-based heuristics appear to generate useful strategies for solving mathematical problems.

Because of the research of Schoenfeld and others interested in mathematical problem solving skills, Polya's heuristics were refined for use in purely mathematical contexts. Krulik and Rudnick (1980) developed their own more specific mathematical heuristics. Krulik and Rudnick's heuristics were used in this study to help students increase their mathematical problem solving skills. Five broad categories were identified in this set of heuristics. Each category was further divided into more specific strategies. The first heuristic was to read the problem. Strategies included noting the key words, getting to know the problem setting, and
finding what was being asked for. The second heuristic was to explore the problem. Strategies identified included drawing a diagram, making a chart, and looking for patterns. The third heuristic Krulik and Rudnick defined was to select a strategy. Strategy choices included experimentation, conjecturing or guessing, and working backwards. Next, students solved the problem by carrying through the selected strategy. Last, students reviewed and extended by verifying the answer and looking for variations of the problem.

Because of the previous research dealing with heuristics as strategies within a specific context, this study did not examine potential transfer of problem solving skills to other domains. Rather, it concentrated on the acquisition of skills that could be applied in mathematical situations. Further research would need to be conducted to determine whether the acquisition of these skills in a mathematical domain would transfer to other domains.

Statement of the Problem

George Bush, in his America 2000 plan, set the goal of having U.S. students rank first in the world in terms of mathematics and science achievement by the year 2000 (U.S. Department of Education, 1991). If this or any similar goal is to be fulfilled, computational skills can no longer be the only emphasis of math courses. Problem solving skills need to be effectively taught and learned. Mathematics educators should give greater attention to problem solving instruction, since it is unlikely that problem solving skills will be learned through modeling alone (Krulik & Rudnick, 1980).

Prior to formal arithmetic instruction, almost all children exhibit reasonably sophisticated and appropriate problem solving skills. As children get older, however, they often solve problems by choosing a single arithmetic operation based only on the surface details of the problem (Carpenter, 1985; National Assessment of Educational Progress, 1983). This suggests that something happens to teach children to abandon their initial correct skills in favor of an
algorithmic replacement that is often incorrect. Most likely there is a problem with the instruction of mathematical problem solving skills.

Educators should examine teaching strategies that enable students to learn the necessary skills in solving mathematical problems. Use of heuristic strategies to teach these skills is widely documented. Within a domain-specific context, heuristic strategies can be used effectively to teach mathematical problem solving (Krulik & Rudnick, 1980; Schoenfeld, 1985). Also well documented is the relationship of cognitive monitoring to problem solving skill acquisition.

The NCTM (1991) adopted a standard regarding the use of technology in the mathematics classroom, when appropriate. The NCTM pointed out that computer use does not guarantee learning. If computers are to become meaningful assets to education, teachers need to be trained in techniques of using technology most effectively (Branscum, 1992). A wide array of well-written mathematical problem solving software is available, but it can not be effective without appropriate problem solving instruction.

Heuristic strategies used in the acquisition of mathematical problem solving skills have been examined extensively during the last decade. However, little research has been reported that determines whether mathematical problem solving computer software, used in conjunction with the teaching of these strategies and cognitive monitoring activities, can be used to increase mathematical problem solving skills.

Purpose of the Study

The purpose of this study was to determine whether employing heuristic strategies, using commercially produced mathematical problem solving software, and exercising cognitive monitoring activities increased the mathematical problem solving skills of seventh grade students in a rural Iowa school.
Research Questions

1. Does the combination of heuristic strategies, cognitive monitoring activities, and computer-based learning increase mathematical problem solving ability more than general problem solving instruction combined with computer-based learning?

2. Does the teaching strategy effect problem solving skills equally for girls and boys?

3. How does attitude toward mathematics change with respect to treatment?

4. How does attitude toward mathematics change among genders during the treatment?

5. Do the students who learn heuristic strategies actually use those strategies when they are problem solving?

Definition of Terms

Problem

A problem is a situation when an individual wants to do something but does not know the course of action needed to get what he or she wants (Newell & Simon, 1972).

Problem Solving

Problem solving consists of the mental and behavioral activities that are involved as the solution to a problem is formulated.

Cognitive Monitoring

Cognitive monitoring is the act of recording the mental processes that are occurring as a problem is solved.
Heuristics

Heuristics are the specific strategies problem solvers employ when finding the solution to a given problem.

Summary

Changes in the emphasis of mathematics education in the last decade from computational skills to problem solving skills have imposed a need to evaluate how to most effectively teach mathematics. Considerable research has been reported that analyzes the teaching of heuristic strategies as a method for helping children learn problem solving skills. This research indicates that heuristic strategies are effective when taught within a specific context-domain.

In addition to changes in emphasis, mathematics education has also undergone a shift toward the integration technology into the curriculum. Researchers are currently studying how technology use in mathematics education will help students in their acquisition of problem solving skills. Computers have become quite common in schools, (Becker, 1991) and have the potential to become effective teaching tools (Kulik, Bangert, & Williams, 1983; Kurshan & Williams, 1985).

The focus of this study was to examine whether students could learn mathematical problem solving skills when computer-based learning was used as a part of instruction. It also examined whether teaching heuristic strategies and cognitive monitoring skills, followed by computer-based instruction, was an effective teaching method.
CHAPTER II. LITERATURE REVIEW

The purpose of this study was to determine the combined effect of using heuristic strategies, cognitive monitoring, and computer-based learning on mathematical problem solving skills. This chapter addresses the key areas of concern in this study and investigates prior research about each topic. This literature review is organized into the following five sections: 1) mathematical problem solving, 2) metacognition in problem solving, 3) heuristics in mathematical problem solving, 4) mathematics anxiety, and 5) computer-based instruction for mathematical problem solving.

Mathematical Problem Solving

Mathematical problem solving was the skill investigated in this study. This ability, or the lack of it, has recently received a great deal of attention from national organizations, parent groups, and even the President of the United States. Since mathematical problem solving is so broad and poorly defined, researchers have made slow progress in developing useful information about the acquisition of this skill.

It is apparent, however, that U.S. students have not developed problem solving skills to the extent expected. Performance in mathematical problem solving was low in 1988, as based on a National Assessment of Educational Progress instrument (Carpenter, Lindquist, Brown, Kouba, & Silver, 1988). Although the overall proficiency in math increased slightly from 1978 to 1990, problem solving ability among 13- and 17-year olds remained virtually the same (National Center for Education Statistics, 1991). The Mathematical Sciences Education Board National Research Council (1991, p3) suggested:

Other industrialized countries awakened 20 years ago to the significance of mathematics and science proficiency for their national well-being and began efforts to strengthen these components of education. We in the United States have been too slow to respond. As a result, we face these harsh realities today:
- Public attitudes encourage low expectations in math and science. Poor performance is socially acceptable.
- Curricula and instruction are years behind the times; they do not reflect the increased demand for higher order thinking skills or the best ways for students to learn math and science.
- Calculators and computers have had very little impact on math and science instruction, in spite of their potential to enrich, enlighten, and expand learning.

Although overall performance among U.S. students is low, an even greater discrepancy exists between genders. It is generally agreed that the Scholastic Aptitude Test (SAT) mathematics section evaluates mathematical problem solving skills (Messnick & Jungeblut, 1981). Boys on the average outperform girls by fifty points on the mathematics portion of this test (Navarro, 1989). For gifted seventh graders, the results are even more startling. In their Study of Mathematically Precocious Youth (SMPY), Stanley and Benbow (1983) reported that the number of seventh grade boys who scored 500 or above on the SAT's compared to the number of girls who scored 500 was a two to one ratio. The ratio of boys to girls scoring 600 or above was 4 to 1. Boys outnumbered girls at the 700 or above level by a ratio of 15 to 1. The gender difference found on the SAT-mathematics section in grade seven persisted into the high school level. From grade seven and grade twelve, boys increased on the average of 155 points, while girls increased only 145 points (Benbow & Stanley, 1983). Benbow and Stanley also found that despite the discrepancies in SAT scores, females received better overall math grades than males. So although females were able to do well in math classes, they lacked problem solving skills.

Becker (1990) tested over 3,000 Iowa students in a longitudinal study. He found that for the mathematics problem solving sections of the Iowa Test of Basic Skills and the Iowa Test of Educational Development, there was a small and fairly constant male advantage in grades 3 through 8, but there was a substantial male advantage that emerged in grade 9. Although the exact point where the large male advantage becomes significant is disputable, it is evident that
gender differences do exist in mathematical problem solving ability beginning in the middle school years and increasing with age (Backman, 1972; Benbow & Minor, 1986).

Several researchers have examined this gender difference and have tried to suggest reasons for the differences in ability levels. One possible explanation is that girls have been reported to lack spatial visualization skills and therefore have difficulty in problem solving (Gray, 1991). Other research supports physiological differences in the brain (Battista, 1990; De Lacoste-Utamsing & Holloway, 1982). Much of the literature suggests social reasons for this phenomena (Creswell, 1983; Johnson, 1984; Meece, Parsons, Kaczala, Goff, & Futterman, 1982; Rosser, 1989). According to feminist beliefs, the lack of females in mathematics careers can not be explained by an innate lack of ability, but a reaction to the prescriptions and proscriptions of society (Caporrimo, 1983). The social reasons for gender differences in mathematical problem solving ability may not be as apparent as some of the other reasons for these differences, but they are possibly more pervasive.

Metacognition in Problem Solving

Because of the widely recognized lack of problem solving skills, especially among girls, it is important to examine ways of teaching mathematical problem solving effectively to both genders. Metacognition, as explained in chapter one, is a method of examining how one thinks. Metacognition instruction typically involves the recording, either written or verbal, of the thoughts involved as a student solves a problem. Generally, research indicates that knowledge and awareness of one's own thinking strategies develop with age and that problem solving ability is positively correlated with metacognitive skills (Schoenfeld, 1981 & 1985). Lester (1983) suggested having students discuss and think about the processes they incorporated in an effort to make them aware of the complex processes they used, and to make them more aware that many problems allowed multiple solutions.
If problem solving skills are to be learned, then skills in thinking about the processes involved in solving problems also need to be acquired (Caporrimo, 1990). According to Silver (1982), problem solving analyses have concentrated too much on cognitive actions and not enough on the behavior relevant to strategy selection, cognitive monitoring, and the evaluation of cognitive processes. Silver believed that purely cognitive explanations of complex problem solving behavior were insufficient and that many of the "driving forces" that determined success or failure were, in fact, metacognitive in nature.

White and Collins (1983) reported a study where students were first asked to describe a process in detail (such as making a peanut butter and jelly sandwich), and then work on a computer programming activity. The researchers speculated that problem solving skills were not innate, but were rather a systematic way of processing information, and their experiment supported this idea. As students described a process, they had to carefully reflect on the pieces of the process, even though the processes were often routine tasks. This attention to detail was useful to the students as they learned programming skills. The cognitive monitoring activity was found to help them increase their reasoning skills, based on their ability to write a successful computer program.

Another study that examined the difference between expert mathematicians and inexperienced problem solvers found that cognitive monitoring and self-regulating behaviors were characteristics of the experts as they solved mathematical problems (Schoenfeld, 1987). Inexperienced problem solvers did not employ metacognitive behavior, and were limited in their problem solving endeavors. Slife (1985) found similar differences between regular students and learning disabled students. The learning disabled subjects were less skilled in knowing about the cognitive processes employed in solving mathematical problems.

Over 670 secondary students in Singapore participated in another study that addressed the metacognitive behaviors of learners (Wong, 1989). The subjects from three courses of study
(general, science, and the arts) were given a 20-item questionnaire to determine their metacognitive beliefs and strategies during problem solving. Each of the courses of study was divided into a special track, a normal track, and an express track. No significant differences were found among students from different courses of study, yet within each course of study, students in the normal track exhibited less frequent usage of metacognitive strategies. This was consistent with other research that suggested successful problem solvers employed metacognitive strategies.

Lester, Garofalo, and Schoenfeld are just three of many researchers interested specifically in the development of mathematical problem solving skills who have begun to advocate metacognitive strategies in mathematics instruction. Other research has shown that successful problem solvers engage in metacognitive activities as they work. Because of the interest among mathematicians and other researchers, metacognitive strategies were employed as a teaching method in this study.

Heuristics in Mathematical Problem Solving

Heuristics are broad strategies, independent of any topic or subject matter, that help problem solvers approach and understand their resources in solving problems. Polya's heuristics began the study of a large number of strategies designed to help students learn to be successful problem solvers. When Polya developed his strategies, he felt that problem solving was a skill in itself. He believed that once problem solving was learned, it could take place in many domains (Polya, 1957). Current research, however, has shown that transfer across domains is not likely to occur without specific prompting in other contexts (Perkins & Salomon, 1989).

Schoenfeld (1985) asserted that heuristics such as those Polya suggested were in large part simply labels for categories of related strategies and did not lead to specific procedures. He
also claimed that many mathematical algorithms were so complex and consisted of so many phases that a general strategy was unlikely to be effective. Polya's strategies have become more refined throughout the years, and are now a series of more specific ideas that can be used to solve problems.

Many researchers have studied the value of teaching heuristic strategies to improve problem solving skills in one domain, such as mathematical problem solving. Schoenfeld (1982) studied students in a month-long elective college course on solving non-routine mathematical problems. The results indicated that students could learn to use some general problem solving heuristics to solve different types of unrelated mathematical problems, but that general transfer to other contexts was not observed. He also discovered that the students' assessments of their problem solving performance became more favorable when the heuristic strategies were employed.

Another study involved students in grades five to seven, and their ability to perform non-routine problems dealing with number theory (Schultz, 1984). The study took a close look at the role of heuristics, concrete models, and the relationships between these variables. Students were given lessons about number theory, heuristic strategies, concrete models, and microcomputers. The computer presented the problem, provided hints from a menu, and recorded students' work. Results showed that the problem solving ability of students increased as a result of the treatment.

Using heuristics in a variety of learning situations was examined by Leighton (1989). Seventh grade students were separated into three treatment groups: working individually, working in groups, and working as cooperative teams. Each of the treatment groups received instruction on heuristic strategies, and then took a problem solving posttest. Results of the tests at the end of the five-week experimental period indicated no difference between the three
treatment groups, but students in all three conditions significantly outperformed a control group which did not receive heuristic instruction.

Despite the positive results reported in literature, simply providing heuristic strategies alone does not insure success in problem solving. Students must be able not only to understand the heuristics, but also to apply them. Instructors need to spend time showing students how and when to use the heuristics (Krulik & Rudnick, 1980). Students must become aware of how heuristics are organized in memory, so they may be used when necessary (Simon, 1980). And although research has been supportive, more attention needs to be given to problem solving to determine how heuristics and experimental strategies fit within current teaching practices (Weigand, 1991).

Mathematics Anxiety

Regardless of the method used to teach mathematical problem solving, another factor may effect instruction even before teaching begins. Math anxiety is a reality for many people. Aspects of mathematics such as precision, logic, and an emphasis on problem solving make it particularly anxiety provoking for some individuals (Richardson & Suinn, 1972).

The results of a longitudinal study that dealt with spatial visualization, confidence, and mathematics achievement indicated that girls lowered their expectation for success in math during grades 6-8, while boys' expectations increased during this same period (Fennema, 1983). Results of Caporrino's study (1990) were similar. She found that eighth grade girls lacked math confidence although they had the ability to do as well as boys. Regardless of the reason, girls do not expect to do as well as boys on problem solving activities once they get to the middle school level.

Another study examined 564 children in grades 5 - 12 (Wigfield & Meece, 1988). Students were given an attitude questionnaire and a math anxiety questionnaire. Worry,
which was defined as self-deprecatory thought about one's own performance, was highest in grade 9, intermediate in grades 7, 8, 10, 11, 12, and lowest in grade 6. However, the averages were rather high in all groups on the 7 point scale used. Overall, girls reported experiencing significantly more negative attitudes about mathematics than did boys. This affect was observed at each grade level as well.

Although it is often not considered, math anxiety may be a very important factor in a student's ability to solve mathematical problems. When anxiety is present, a student may give up before even attempting to solve a problem or learn from instruction. Thus, it is necessary to use instructional techniques that limit mathematics anxiety.

Computer-Based Instruction for Problem Solving

Computer-Based Learning

Because of the availability of computers and computer software, this study examined the effectiveness of using computer-based instruction that proclaimed to teach mathematical problem solving skills. Software is now available that can alter the way mathematics is taught and learned. When used properly, computers can make the learning of mathematics the investigative journey it was meant to be, make mathematics more relevant to learners, provide new approaches for teaching problem solving skills, and free the learner from time-consuming computational skills to provide more time for learning higher-order thinking skills (Bazak & Bazak, 1987/88).

The meta-analysis reported by Kulik, Bangert, and Williams (1983) showed that computer-based teaching had a moderate effect on the attitudes of students toward instruction and toward the subjects they were studying. In light of the research on mathematics anxiety, it would seem beneficial to affect attitudes in a positive way using computer-based instruction. It is possible that computers affect attitudes toward mathematics because students become so
involved in what they are doing on the computer, that they "forget" it is mathematics (McLeod, 1986). Although Clark (1983) claimed that instructional media use was no more effective than more traditional modes of instruction, it has been reported that computer use can at least enhance traditional teaching (Ferrell, 1986; Kulik, Kulik & Bangert-Drowns, 1985; Kurshan & Williams, 1985; Viteli, 1989).

The effects of "problem solving computer software" on problem solving ability and attitudes toward mathematics were studied using secondary students as subjects (Funkhouser, 1990). The experimental group received computer-augmented math instruction while the control group learned from more traditional instruction. The experimental group demonstrated significantly better performance on tests of mathematical content. They also had positive gains in attitudes towards mathematics.

In another study, mathematics computer software was used with success along with cooperative learning in a non-traditional school for at-risk students (Brickle, 1990). Although the use of computer based instruction was only one aspect of this particular treatment, the combination of software use and cooperative learning strategies was very successful in increasing the problem solving skills of the students. The software was integrated into the classroom in a meaningful way to enhance learning.

Although they did not use a mathematics software application, Choi and Gennaro (1987) found some very interesting results in their study. Their research involved the comparison of computer simulations with hands-on labs for teaching the concept of volume displacement to junior high school students. They found that the computer-based experiences were as effective as hands-on lab experiences. No significant gender difference was found. By varying the mode of instruction, it was possible to eliminate the gender difference that existed when more traditional teaching methods were employed. It is possible that this effect may be true for mathematics as well.
Studies dealing with programming in Logo, BASIC, and Pascal have produced mixed results when programming was used to teach problem solving skills (Lynch, Fischer, & Green, 1989; McCoy & Orey, 1988; Reed, Palumbo, & Stolar, 1988; Swan & Black, 1988). Although many programming studies reported an increase in problem solving skills, the tests used in many cases were not ones that evaluated mathematical problem solving skills. More often, the instruments used tested only programming ability, an ability the researchers determined was an indication of problem solving skills. Even if programming was an accurate indication of problem solving skills, Perkins and Salomon (1988) claimed that transfer to other domains, such as mathematical problem solving, would be unlikely. For this reason, programming was not chosen as a vehicle to teach problem solving skills in this study.

Gender Differences and Computer-Based Learning

The literature suggested that males and females had equal interest in using computers as tools. In a study of 292 students from grades 1, 3, 5, 7, 9, and 12 where three different computer games were used as part of instruction, both genders were highly interested in all three of the computer applications that were presented to them (Johnson & Swoope, 1987). However, in the same study both sexes perceived boys' interest as significantly higher than girls' interest. Chen (1986) also found that males had more interest in computers and more confidence in their ability to program computers than females. Girls believed females were capable of doing as well as boys on programming activities, but individually did not believe they could do as well.

No gender differences were found with a computer task when eighth grade students that accessed an on-line video encyclopedia were instructed to write a science essay (Eastman & Krendl, 1984). Gender differences that were present on an attitude pretest about boys' and girls' ability to work with the computer were nonsignificant at the posttest. The authors
attributed this change to the positive effects of engagement with computers in reducing the sex stereotypes among male and female students.

Implications of Computer-Based Instruction on Problem Solving Skills

Since computer technology has become widespread, educators are faced with developing curricular materials that use the computer to its potential (Robinson, Moyer & Odell, 1984). "Teacher training on computers and technology alone is ineffective. Training needs to be based on strong, comprehensive research on the best techniques for using computers effectively," according to Branscum (1992). Software use by itself will not increase problem solving skills. If mathematical problem solving skills are to be learned, students need preparation and follow-up (Hansen & Zweng, 1984). This study addressed these issues.

Summary

Students' ability in mathematical problem solving has received an enormous amount of negative media attention in the last ten years. The perception is that teachers are not effectively teaching the skills needed by students, especially female students, so they can become successful problem solvers. Current research addresses this issue and offers suggestions for making mathematical problem solving instruction more meaningful.

Metacognition is one of the areas that recently has been associated with mathematical problem solving success. In problem solving, it has been said that possessing knowledge alone is insufficient; problem solvers need to exhibit metacognitive skills to be successful at problem solving (Gagne, 1985). Metacognitive skills enable students to understand how they think and to be better able to formulate solutions for problems. In fact, there is much mental activity underlying the application of algorithms and heuristics of
mathematical problem solving–metacognition may account for a significant amount of this activity (Garofalo & Lester, 1985).

Mathematical problem solving skills do not just "happen". Specific instruction must occur to teach students how to successfully solve problems. This instruction deals with heuristics, which are specific strategies in problem solving. Once students learn the strategies, they have some schema from which to base their problem solving endeavors. Although Polya's heuristic strategies were general skills thought to transfer to other contexts, this study did not address the issue of transfer. Heuristic strategies were taught strictly within a mathematical context with the intent of teaching students to be successful mathematical problem solvers.

One of the reasons for poor mathematical problem solving skills is mathematics anxiety. Research has shown that many students lack the confidence necessary to solve mathematical problems successfully. Students, especially girls, often give up before even trying to solve problems. Educators need to look at instructional treatments and settings that seem less threatening to students.

Computers may provide such an environment. Boys and girls are both interested in computers, and when used correctly, a computer can be a non-threatening instructional tool. A vast amount of computer software is currently available for teaching mathematical problem solving. Educators need to examine how to most effectively use this software in instruction.

This study examined how mathematical computer based lessons along with heuristic strategies and cognitive monitoring affected mathematical problem solving skills among seventh grade students. It also examined whether boys and girls responded equally to the treatment and whether attitude toward mathematics was affected by the treatment.
CHAPTER III. METHODOLOGY

The purpose of this chapter is to describe the methodology used to examine the study's research questions. This summary of the research methodology includes five sections: 1) subjects; 2) instruments; 3) research design; 4) research procedures; and 5) analysis of data.

Subjects

The subjects used in the study were seventh grade students from a small, rural Iowa middle school. The subjects were 40 students in two classes, 19 females and 21 males. Based on the 1991-92 Iowa Test of Basic Skills mathematics section, the group as a whole was of slightly above-average mathematics ability (The national percentile rank for the students was 64%), although there was some variation among the groups. These subjects had explored problem solving with their regular classroom teacher regularly prior to the study; however, they had not learned specific strategies for solving mathematical problems. The subjects used the computer lab on occasion for mathematics instruction, and half of the students had received keyboarding instruction in seventh grade prior to the study.

Instruments

Iowa Test of Basic Skills--Total Math

The Iowa Test of Basic Skills (ITBS) provided for the comprehensive measurement of growth in several fundamental skill areas, including mathematics. This test reported performance of the basic skills in objective terms that could be used to compare students, schools, and states. Test scores were reported as national percentile ranks, and were used to show that the students in the study were of near-average mathematical ability. The test scores were also used to show that the treatment groups within the study were of equal mathematical ability.
Mathematical Problem Solving

The Collis-Romberg Mathematical Problem Solving Battery

The Collis-Romberg Mathematical Problem Solving Battery, Senior Level (1992) was used as the primary measure of mathematical problem solving ability (Appendix A). The test consisted of five question stems, each with four sub-questions, for a total of twenty test problems. Each of the sub-questions evaluated students’ ability to work with a specific aspect of mathematical problem solving. The questions dealt with non-routine mathematical problems in algebra, chance and data, measurement, numbers, and space. The Collis-Romberg test was given three weeks before the study began and again three days after the treatment ended. (The test will be referred to as the C-R pretest or posttest for the remainder of the study.)

The measures obtained on the C-R test were interpreted using both the SOLO (Structure of Observed Learning Outcome) taxonomy (Biggs & Collis, 1982) and the number of correct responses out of the twenty problems. The SOLO levels are basic levels of performance based on the norms for different ages. The SOLO levels are the Unistructural level, the Multistructural level, the Relational level, and the Extended Abstract level. The Unistructural level is the use of one obvious piece of information from the problem stem. Generally, students perform at this level at age 9. It was level 1 in the study. Level 2, the Multistructural level, is the general level of children around age 13. Students at this level are able to use two or more separate pieces of information contained in the stem of the problem. At approximately age 17, many students are at level 3, the Relational stage. Use of an integrated understanding of two or more pieces of information contained in the problem stem identifies students at this level. Extended Abstract is the fourth and final level, occurring sometime after the age of 17. Students at this stage are able to use an abstract general principle or hypothesis which is derived or suggested by the information in the stem. Although these four levels are clearly
defined, children may fall between stages as they grow in mental ability. The ages given are merely guidelines; children develop problem solving abilities at different rates and may perform above or below their age level.

Homework Assignments

Homework was collected and graded daily during the study. The homework assignments consisted of three mathematical problems each day. Students were required to successfully solve two out of the three problems to obtain a score of five points, the value of all daily homework in the classes. Students had the opportunity to earn an additional extra credit point by correctly solving the third problem. All of the homework problems came from the Tops Problem Solving Decks, levels AA and BB and D (1980). The experimental group assignments contained reflective questions about heuristic strategies (Appendix C). The problems were the same for both the control group and the experimental group.

Quiz

A teacher-made quiz was used to measure mathematical problem solving ability. The quiz contained 19 homework problems. To earn a perfect score of 50 points, students were required to answer any ten of the problems correctly. An additional point was earned for each problem over ten that was correct.

Mathematics Anxiety and Attitude

The 22-item Mathematics Self-Concept Scale developed by Holly (1971) was used to measure anxiety and attitude toward mathematics (Appendix B). The test used a 7-point Likert scale, and the score was reported as a numerical total. High scores indicated a better attitude about mathematics than a low score. The possible range of scores was from a low of
22 to a high of 154. The test was administered three weeks before the treatments began and again four days after the treatment ended. The reliability of this test, when given to high school algebra students by Holly, was found to be $r=.73$. This instrument will be called the attitude pretest or posttest in the study.

Research Design

This was an experimental design with a randomly assigned control group and a randomly assigned experimental group (Campbell & Stanley, 1963). A pretest and posttest were given to determine the change in mathematical problem solving ability, and a math anxiety test was given both before and after the treatment.

Research Procedures

This research proposal was reviewed and approved by Iowa State University Human Subjects Committee. Permission was obtained from the principal and superintendent of the school where the study occurred. Parental permission was received for students to participate in the study, and to use the Iowa Test of Basic Skills scores from 1991-92, math anxiety scores, and mathematical problem solving scores in the study. The instructor of the class and the researcher chose the computer lessons and mathematical problems so that the goals of the study and of the class would be met. Procedures and lesson assignments were reviewed by the researcher and instructor before beginning the study. Students were encouraged to use calculators during the treatment.

The Collis-Romberg pretest and attitude pretest were given three weeks prior to the treatment. Students were informed of the number correct on their problem solving pretest, but were not given any feedback about specific problems. After taking the problem solving pretest, students were told that part of their grade for the unit would be based on improvement from
the pretest to the posttest. An introduction was given to the students describing the importance of problem solving skills and how the study would be conducted.

Students from the two mathematics sections were randomly assigned to the experimental group (n = 20) or control group (n=20). In one of the sections, the researcher worked with the experimental group while the teacher worked with the control group. In the other section, the researcher worked with the control group while the teacher worked with the experimental group. The general format was the same for both the control group and experimental group: A five minute discussion about the previous day's homework was followed by twenty minutes of modeling and practicing new problems. For the remainder of the class period, students worked with the computer-based lesson. Students from both the experimental and the control groups saw the same modeled problems in class and did the same problems on homework assignments.

The differences between the two treatment groups occurred during the twenty minute instructional treatment. In the control group, the problem was modeled and alternative ways for solving the problem were discussed. There were no specific steps for solving problems presented to students in this group. In the experimental group, the first problem was modeled by the teacher who demonstrated one of the heuristics of Krulik and Rudnick (1980). Students were encouraged to use the new heuristic method, as well as previously discussed heuristic strategies, to solve the other problems presented in class. The facilitator asked questions about the story problems to encourage the use of these strategies.

To clarify the differences between the two treatment groups, a sample problem done in class with the control group would consist of reading the problem out loud, having the students solve the problem individually, and then calling on a student to explain their answer. The same problem done with the experimental group would begin with reading the problem. Then the facilitator would ask the students, "What are the key words in the problem?", "What is
being asked in the problem?", "How would you explore the problem?", and "What strategies would you use to solve the problem?". The facilitator would seek responses from students. Then the students would solve the problem individually and the facilitator would call on a student to discuss their solution with the class. Daily lesson plans for the experimental group, including software packages used, are explained in Appendix E.

The computer laboratory contained 25 computers that could use Apple software and 22 computers that could use Macintosh software. Enough computers were available so each student worked with their own computer. In addition to the researcher and the teacher, a computer lab aide was available during instruction to assist students with their questions regarding the computer lessons.

The overall goal of the study was to increase mathematical problem solving skills. Both the researcher and the teacher employed some general techniques to help facilitate this goal. Each lesson began with a simple problem that could be solved by all students to help foster an atmosphere of success. Problems that were used in class and as part of homework assignments were chosen based on the background knowledge about the students; problems that incorporated skills the students had already learned were used. All of the problems came from the Tops Problem Decks AA, BB, and D (1980). These decks are geared toward students in grades 6 to 8. The computer-based lessons were chosen in the same manner. Two additional middle school mathematics teachers reviewed the problems and computer-based lessons for appropriateness and perceived student interest level.

The mathematics attitude test and the Collis-Romberg posttest were administered after the two-week unit concluded. As they took the C-R posttest, students were asked to explain in writing how they solved the problems. These responses, along with the written work for each problem, were used to analyze whether the heuristic strategies were being employed.
Analysis of the Data

National percentile ranks from the Total Math section of the 1991-92 ITBS were used to determine that the treatment groups were equivalent. Means and standard deviations were computed for the results of the Collis-Romberg tests. The number of correct responses and the SOLO level were recorded on the pretest and posttest for each student. The change in the number of correct responses from the pretest to the posttest and the change in the SOLO level were also recorded. These data were examined with respect to treatment and to gender. It was recognized that the use of change scores has been criticized in the literature recently; however, in this study it was important to note differences in mathematical problem solving ability from the pretest to the posttest. Thus, the change scores were included in the results.

The overall means and standard deviations on the attitude tests were calculated for the experimental and control groups, and with respect to gender and treatment. Significance was set at the .05 level. One t-test was used to determine the difference between treatment groups, and another was used to determine differences between genders in the study. Attitude was examined before the treatment and after the treatment; the change scores were also recorded. An analysis of covariance was performed to control for differences in ITBS scores between the control group and the experimental group and between the control group females and the experimental group females.

Qualitative data were collected from written responses to questions as the students completed the C-R posttest. These data were compared with the actual work collected for each problem. These data were used to determine whether students actually used heuristic methods during mathematical problem solving.
Summary

This study examined the effect of using computer-based learning, along with heuristic strategies and cognitive monitoring skills to learn mathematical problem solving skills. Problems and computer lessons judged to be appropriate for the seventh grade level were used. Several tests were administered to record the changes in mathematical attitude and mathematical problem solving skills. Both qualitative and quantitative data were gathered during the study.
CHAPTER IV. RESULTS

In this chapter, the results of the study will be presented and discussed as they relate to the research questions presented in chapter one. Statistics will be presented with a discussion about how they relate to the research questions. The results of the study were based on data obtained from 40 students: 21 males and 19 females. The experimental and control groups each contained 20 students.

The computer program used to analyze the data was Statview, a program designed for the Macintosh computer. The t-test was used to determine differences based on treatment or gender. The analysis of covariance was also used to control for initial differences in mathematics ability between the treatment and the gender groups, as based on the results of the 1991-92 Iowa Test of Basic Skills Total Math scores. Although the differences between groups were not statistically significant, there were some observed differences between the groups. In addition to the quantitative data, qualitative data were obtained to determine whether or not the students in the experimental group actually used heuristic strategies when completing the Collis-Romberg posttest.

Question 1

Question 1: Does the combination of heuristic strategies, cognitive monitoring activities, and computer-based learning increase mathematical problem solving ability more than general problem solving instruction combined with computer-based learning?

Scores on the C-R posttest ranged from 4 to 15 for the experimental group and from 6 to 18 for the control group. The mean score for the experimental group was 9.0 and was 9.9 for the control group. This difference was not a statistically significant result \((p = .14)\). Both treatment groups answered 1.2 more questions correctly on the C-R posttest than the pretest. The experimental group was at a mean SOLO level of 1.6 on the C-R pretest, and improved to a
mean level of 2.1 on the posttest. The control group had a mean level of 2.1 on the pretest and improved only .1 to 2.2 on the posttest. This level change was a significant difference between the experimental and control group (p = .01).

The experimental group scored 59.6 on the Total Math section of the ITBS, while the control group scored 64.6. This was not a statistically significant difference (p = .24). However, to control for any initial differences between the two groups based on these scores, an ANCOVA was performed. For the change in the number of correct responses from the pretest to the posttest, the differences between the control group and the experimental group were not significant (p = .96). However, the ANCOVA showed that the change in C-R level for experimental group students was significant (p = .05). These results are summarized in Table 1.

Homework scores for the experimental group averaged 33.3 out of 40 possible points, while the control group mean was 35.7 points. Scores on a teacher-given quiz ranged from 25 to 57 out of a possible 50 points for the experimental group. (Points beyond 50 could be earned by doing more than 10 problems correctly.) The mean was 46.2. Scores ranged from 5 to 57 for the control group, with a mean of 45.1. The means were not statistically different. When an ANCOVA was performed to control for initial differences in mathematical ability, the results showed that the experimental group performed significantly better on the quiz than the control group members (p = .01). The means, standard deviations, and t-test results for the experimental and control groups are shown in Table 1.
Question 2

Question 2: Does the teaching strategy effect problem solving skills equally for girls and boys?

When making a comparison between boys and girls overall without considering differences in treatments, it was found that there was very little initial difference in mathematical ability between genders. The ITBS Total Math percentile rank for girls was 62.3, while boys ranked 62.0 nationally (p = .48). Girls had a mean of 9.3 problems correct on the C-R posttest, which was a change of 0.7 from the pretest. Boys solved 9.6 problems correctly on the C-R posttest, a change of 1.7 from the pretest. The difference in change in number of correctly solved problems was a significant difference between genders (p = .04). The change in SOLO level was not significantly different between boys and girls (p = .07). The girls' SOLO level changed a mean of 0.3 and the boys' SOLO level changed a mean of 0.5. Girls scored 35.3 out of 40 on the homework assignments and 47.5 out of 50 on the quiz. Boys had 33.8 points on the homework and 43.9 on the quiz. Neither of these differences was statistically significant (p = .26 and p = .17 respectively). These data are summarized in Table 2.

The females in the experimental group scored a mean of 56.8 on the ITBS, while the control group averaged at the 69.4 percentile (p = .15). Because of the large difference between groups, although not statistically significant, an ANCOVA was used to control for initial group differences. Scores on the C-R posttest ranged from 6 to 11 with a mean of 9.0 for the females in the experimental group. Scores ranged from 7 to 12 with a mean of 9.6 for the females in the control group. The mean change on the C-R pretest to posttest was 1.1 for the females in the experimental group and .2 for the females in the control group. This indicated that the females in the experimental group improved more from the pretest to the posttest than did the females in the control group. However, this difference was not statistically significant using a t-test (p = .12) or the ANCOVA (p = .58). Females in the experimental group were at a mean level of 1.6
on the C-R pretest and increased .5 to 2.1 on the posttest. Females in the control group actually decreased their overall level from 2.3 to 2.2. This difference in C-R levels was statistically significant (p = .02) but not when initial group differences were controlled using an ANCOVA (p = .28). This indicated that the treatment was marginally effective for females. The problem solving scores for females in the control group and in the experimental group are shown in Table 3.

The mean for the females in the experimental group was 35.4 for the homework assignments and 46.4 for the quiz. The females in the control group scored a mean of 35.1 for homework and 48.8 for the quiz. These differences were not significant (p = .46: homework; p = .32: quiz). These data are also shown in Table 3.

The two male treatment groups were much closer in mathematical ability, based on ITBS scores than were the females. The males in the experimental group scored at the 62.4 percentile nationally, while the control group males were at the 61.5 percentile (p = .46). Since the groups were not significantly different, only a t-test was used to determine whether the treatment was effective. Males in the experimental treatment had scores that ranged from 4 to 14 on the C-R posttest, with a mean of 9.0. Their change in C-R scores from the pretest to posttest was 1.3. Their C-R level on the pretest was 1.6 and increased by .5 to 2.1 on the posttest. Control group males' scores ranged from 6 to 17 on the posttest with a mean of 10.2. Their change in C-R scores was 2.0. Control group males had a mean C-R pretest level of 1.8 and increased .4 to 2.2. Neither result was significant (p = .20 for the change in the number correct and p = .07 for the C-R level change).
The mean for males in the experimental group for the homework assignments was 31.1 points, and their mean on the quiz was 45.9. Control group males had a mean homework score of 36.2 and a mean of 42.1 on the quiz. The difference in homework scores was not significant (p = .06), nor was the difference in quiz scores (p = .25). Problem solving scores for boys in the control and experimental groups are shown in Table 4.

**Question 3**

**Question 3:** How does attitude toward mathematics change with respect to treatment?

On the attitude test, high numbers indicated a positive attitude about mathematics, while low numbers indicated a negative attitude. Scores could range from a low of 22 to a high of 154. The mean attitude on the pretest for the experimental group was 104.2 with a score ranging from 44 to 134. The mean attitude increased to 108.2 for the posttest, with a low of 53 and a high of 133. This resulted in a mean change of 4.0. The control group had pretest attitude scores ranging from 58 to 141 with a mean of 99.7. Their post-treatment attitude scores decreased by .3 to 99.4, with a range of 56 to 131. Although there were differences in attitude between the control and experimental groups, they were not significant (p = .27 on the pretest, p = .11 on the posttest, and p = .20 on the change from the pretest to posttest). The results of attitude as a result of the treatment for experimental and control group students are in Table 5.

**Question 4**

**Question 4:** How does attitude toward mathematics change among genders during the treatment?

In general, females had a better attitude about mathematics than did males before the study began. The mean for females on the Mathematics Attitude pretest was 107.0, compared
with a mean of 97.3 for the males. On the posttest, the mean for females was 108.1, a positive change of 1.1. The males had a mean of 99.9 on the posttest, a positive change of 2.5 from the pretest. The differences in the posttest ($p = .14$) and in the change from the pretest to the posttest ($p = .63$) were not significant. These results are shown in Table 6.

The breakdown between gender and treatment shows some interesting interactions. The females in the control group had an attitude pretest mean of 108.0, a posttest mean of 104.2, and a net change of -3.8. That indicated that their attitude about mathematics actually decreased as a result of the control treatment. The females in the experimental group had a pretest mean of 106.1, a posttest mean of 111.6, and a mean change of 5.5. Although this difference appeared to be rather large, it was not statistically significant ($p = .23$). A summary of the attitude measures among females in the experimental and control groups is in Table 7.

The males in the control group had a pretest mean of 92.9, a posttest mean of 95.5, and a mean change of 2.5. The males in the experimental group had a pretest mean of 102.2, a posttest mean of 104.7, and a mean change of 2.5. The difference in the posttest scores was not significant ($p = .13$), nor was the change from the pretest to the posttest ($p = .67$). Since the change in the pretest to posttest scores was the same for both groups, it appeared that the treatment did not influence mathematical attitude significantly. Results of the attitude tests for males in the experimental and control groups are in Table 8.

Question 5

Question 5 was more qualitative in nature than the other four research questions: Do the students who learn heuristic strategies actually use those strategies when they are problem solving?
To explore this question, students were asked to explain how they got the solution to their answer to the fourth part of each of the five questions on the C-R posttest. Their written response was then analyzed to determine whether heuristic strategies were being used.

It was found that the students in the experimental group did use the heuristic strategies to help them solve problems. Of the students in the experimental group, 85% of them used at least one of the heuristic methods to solve problems. Diagrams were used by 80% of the students in this group to help them find a solution. Of the 80% of the students that used diagrams, 46% of them used a diagram in more than one problem. Finding a pattern, a method of looking at the information and trying to find a sequence that would help to find a solution, was used by 30% of the students in the experimental group. The guess and check method, a strategy of using trial and error until the correct response is found, was also used by 30% of the students.

Without knowing that they were using special methods, 75% of the students in the control group used at least one heuristic method to solve their problems on the C-R posttest. Similar to the results of the experimental group, diagrams were used most often by control group students. Only 10% of the control group students used a guess and check method, while 15% of the students in the control group attempted to find patterns to solve the problem. These results are shown in Figure 1.

Summary

Each of the five research questions in this study was addressed, and the results of the statistical analyses were presented. The results are as follows:

1. There was no significant difference in mean scores between the C-R pretest and posttest between the experimental and the control groups. However, there was a significant difference in the change from the C-R
pretest level to the C-R posttest level between the two treatment groups.

2. Females in the experimental group did significantly better than females in the control group on increasing their SOLO level from the pretest to the posttest. However, when initial differences in the groups were equalized, there was no significant difference in SOLO levels. There was not a significant difference between treatments for the males.

3. There was no significant difference in attitude scores or change in attitude scores between the experimental group and the control group.

4. Although females had a significantly better attitude about mathematics than males, the difference in attitude as a result of the study was not significant.

5. Students receiving the experimental treatment used heuristic strategies to help them solve mathematical problems.

In general, the treatment was somewhat effective in helping students gain mathematical problem solving skills. Attitudes about mathematics were not significantly altered as a result of the study. Males and females seemed to respond equally to the treatment.
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<td>-.25</td>
<td>.01</td>
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<td>0.5</td>
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<td></td>
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<tr>
<td>range</td>
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<td>-1 to 1</td>
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<tr>
<td><strong>Homework Scores</strong></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>average</td>
<td>33.3</td>
<td>35.7</td>
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<tr>
<td>standard deviation</td>
<td>7.3</td>
<td>6.8</td>
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<tr>
<td>range</td>
<td>21 to 46</td>
<td>17 to 44</td>
<td></td>
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<tr>
<td><strong>Quiz Scores</strong></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>average</td>
<td>46.2</td>
<td>45.1</td>
<td>-.28</td>
<td>.39</td>
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<tr>
<td>standard deviation</td>
<td>10.3</td>
<td>13.4</td>
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<tr>
<td>range</td>
<td>25 to 57</td>
<td>5 to 57</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*a* C-R number correct, posttest describes how many questions, out of the 20 possible, the student answered correctly on the Collis-Romberg posttest.

*b* C-R number correct change describes the difference in the number of correct responses out of 20 that the students had on the Collis-Romberg pretest and posttest.

*c* C-R level change describes the change in the SOLO level from the Collis-Romberg pretest to the posttest.

*d* Homework Scores describe the number of points, out of a possible 40, the students scored on daily assignments. Each assignment was worth 5 points. To earn all 5 points, students had to correctly answer 2 out of the 3 problems. An extra point could be earned daily by correctly answering all 3 problems.

*e* Quiz Scores describe the number of points, out of a possible 50, the students scored on the teacher-made test. To earn all 50 points, students had to solve 10 out of the 19 problems correctly. For each problem over 10 that the student solved correctly, an extra point was earned.
Table 2
Problem Solving Scores for Male and Female Students

<table>
<thead>
<tr>
<th></th>
<th>Female</th>
<th>Male</th>
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<th>Probability</th>
</tr>
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<tbody>
<tr>
<td><strong>n</strong></td>
<td>19</td>
<td>21</td>
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<tr>
<td><strong>ITBS-Total Math</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>62.3</td>
<td>62.0</td>
<td>0.05</td>
<td>0.48</td>
</tr>
<tr>
<td>standard deviation</td>
<td>23.6</td>
<td>19.4</td>
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<td></td>
</tr>
<tr>
<td>range</td>
<td>14 to 99</td>
<td>24 to 95</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>C-R number correct, post</strong></td>
<td>9.3</td>
<td>9.6</td>
<td>-0.42</td>
<td>0.34</td>
</tr>
<tr>
<td>average</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>standard deviation</td>
<td>1.6</td>
<td>3.3</td>
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<td></td>
</tr>
<tr>
<td>range</td>
<td>6 to 12</td>
<td>4 to 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>C-R number correct change</strong></td>
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<td>1.7</td>
<td>-1.80</td>
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</tr>
<tr>
<td>standard deviation</td>
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<td>1.9</td>
<td></td>
<td></td>
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<tr>
<td>range</td>
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<td>-3 to 7</td>
<td></td>
<td></td>
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<tr>
<td><strong>C-R level change</strong></td>
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<td>0.07</td>
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</tr>
<tr>
<td>standard deviation</td>
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<td>range</td>
<td>-1 to 1</td>
<td>0 to 1</td>
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<td>0.26</td>
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<tr>
<td>average</td>
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</tr>
<tr>
<td>standard deviation</td>
<td>6.7</td>
<td>7.5</td>
<td></td>
<td></td>
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<tr>
<td>range</td>
<td>17 to 46</td>
<td>21 to 45</td>
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<td><strong>Quiz Scores</strong></td>
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<td>43.9</td>
<td>0.97</td>
<td>0.17</td>
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<tr>
<td>average</td>
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<td></td>
</tr>
<tr>
<td>standard deviation</td>
<td>10.8</td>
<td>12.7</td>
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</tr>
<tr>
<td>range</td>
<td>20 to 57</td>
<td>5 to 57</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(a\) C-R number correct, post describes how many questions, out of the 20 possible, the student answered correctly on the Collis-Romberg posttest.

\(b\) C-R number correct change describes the difference in the number of correct responses out of 20 that the students had on the Collis-Romberg pretest and posttest.

\(c\) C-R level change describes the change in the SOLO level from the Collis-Romberg pretest to the posttest.

\(d\) Homework Scores describe the number of points, out of a possible 40, the students scored on daily assignments. Each assignment was worth 5 points. To earn all 5 points, students had to correctly answer 2 out of the 3 problems. An extra point could be earned daily by correctly answering all 3 problems.

\(e\) Quiz Scores describe the number of points, out of a possible 50, the students scored on the teacher-made test. To earn all 50 points, students had to solve 10 out of the 19 problems correctly. For each problem over 10 that the student solved correctly, an extra point was earned.
Table 3

Problem Solving Scores for Female Experimental and Female Control Students

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Control</th>
<th>t-value</th>
<th>Probability</th>
</tr>
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<tbody>
<tr>
<td><strong>n</strong></td>
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<td>9</td>
<td></td>
<td></td>
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<td>average</td>
<td>56.8</td>
<td>69.4</td>
<td>1.07</td>
<td>.15</td>
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<tr>
<td>standard deviation</td>
<td>21.2</td>
<td>26.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>range</td>
<td>14 to 88</td>
<td>32 to 99</td>
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<td></td>
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<tr>
<td><strong>C-R number correct, post</strong></td>
<td>9.0</td>
<td>9.6</td>
<td>.73</td>
<td>.24</td>
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<tr>
<td>standard deviation</td>
<td>1.8</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>range</td>
<td>6 to 11</td>
<td>7 to 12</td>
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<td></td>
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<tr>
<td><strong>C-R number correct change</strong></td>
<td>1.1</td>
<td>0.2</td>
<td>-1.24</td>
<td>.12</td>
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<td>1.8</td>
<td>1.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>range</td>
<td>-2 to 3</td>
<td>-1 to 2</td>
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<td>standard deviation</td>
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<td>0.5</td>
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<tr>
<td>range</td>
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<td>-1 to 1</td>
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<td></td>
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<tr>
<td><strong>Homework Scores</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
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<td>35.1</td>
<td>-.09</td>
<td>.46</td>
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<tr>
<td>standard deviation</td>
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<td>7</td>
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<td>range</td>
<td>24 to 46</td>
<td>17 to 48</td>
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<td><strong>Quiz Scores</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>46.4</td>
<td>48.8</td>
<td>.47</td>
<td>.32</td>
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<tr>
<td>standard deviation</td>
<td>10.9</td>
<td>11.2</td>
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<tr>
<td>range</td>
<td>30 to 57</td>
<td>20 to 56</td>
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<td></td>
</tr>
</tbody>
</table>

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*a C-R number correct, post describes how many questions, out of the 20 possible, the student answered correctly on the Collis-Romberg posttest.
*b C-R number correct change describes the difference in the number of correct responses out of 20 that the students had on the Collis-Romberg pretest and posttest.
*c C-R level change describes the change in the SOLO level from the Collis-Romberg pretest to the posttest.
*d Homework Scores describe the number of points, out of a possible 40, the students scored on daily assignments. Each assignment was worth 5 points. To earn all 5 points, students had to correctly answer 2 out of the 3 problems. An extra point could be earned daily by correctly answering all 3 problems.
*e Quiz Scores describe the number of points, out of a possible 50, the students scored on the teacher-made test. To earn all 50 points, students had to solve 10 out of the 19 problems correctly. For each problem over 10 that the student solved correctly, an extra point was earned.
Table 4

Problem Solving Scores for Male Experimental and Male Control Students

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Control</th>
<th>t-value</th>
<th>Probability</th>
</tr>
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<td>11</td>
<td></td>
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<tr>
<td><strong>ITBS-Total Math</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>62.4</td>
<td>61.5</td>
<td>t = -0.10</td>
<td>p = .46</td>
</tr>
<tr>
<td>standard deviation</td>
<td>18.7</td>
<td>26.8</td>
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<tr>
<td>range</td>
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<td>24 to 95</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
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<td>10.2</td>
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<td>3.1</td>
<td>3.6</td>
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<tr>
<td>range</td>
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<td>6 to 17</td>
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<tr>
<td><strong>C-R number correct change</strong></td>
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</tr>
<tr>
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<td>1.3</td>
<td>2.0</td>
<td>t = .86</td>
<td>p = .20</td>
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<td>standard deviation</td>
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<td>1.8</td>
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<td>range</td>
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<td><strong>C-R level change</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>0.4</td>
<td>t = -1.56</td>
<td>p = .07</td>
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<tr>
<td>standard deviation</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>range</td>
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<td>0 to 1</td>
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<tr>
<td><strong>Homework Scores</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>31.1</td>
<td>36.2</td>
<td>t = 1.61</td>
<td>p = .06</td>
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<tr>
<td>standard deviation</td>
<td>7.6</td>
<td>6.9</td>
<td></td>
<td></td>
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<tr>
<td>range</td>
<td>21 to 45</td>
<td>23 to 44</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
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<td>42.1</td>
<td>t = -.68</td>
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<td>standard deviation</td>
<td>10.2</td>
<td>14.9</td>
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</tr>
<tr>
<td>range</td>
<td>25 to 53</td>
<td>5 to 57</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

a C-R number correct, post describes how many questions, out of the 20 possible, the student answered correctly on the Collis-Romberg posttest.
b C-R number correct change describes the difference in the number of correct responses out of 20 that the students had on the Collis-Romberg pretest and posttest.
c C-R level change describes the change in the SOLO level from the Collis-Romberg pretest to the posttest.
d Homework Scores describe the number of points, out of a possible 40, the students scored on daily assignments. Each assignment was worth 5 points. To earn all 5 points, students had to correctly answer 2 out of the 3 problems. An extra point could be earned daily by correctly answering all 3 problems.
e Quiz Scores describe the number of points, out of a possible 50, the students scored on the teacher-made test. To earn all 50 points, students had to solve 10 out of the 19 problems correctly. For each problem over 10 that the student solved correctly, an extra point was earned.
### Table 5
Attitude Scores for Experimental and Control Group Students

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Control</th>
<th>t-value</th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>n</strong></td>
<td>20</td>
<td>20</td>
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<td></td>
</tr>
<tr>
<td><strong>Attitude Pretest Results</strong>&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>average</td>
<td>104.2</td>
<td>99.7</td>
<td>t = -.61</td>
<td>p = .27</td>
</tr>
<tr>
<td>standard deviation</td>
<td>22.5</td>
<td>23.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>range</td>
<td>44 to 134</td>
<td>58 to 141</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Attitude Posttest Results</strong>&lt;sup&gt;b&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>average</td>
<td>108.2</td>
<td>99.4</td>
<td>t = -1.12</td>
<td>p = .11</td>
</tr>
<tr>
<td>standard deviation</td>
<td>21.2</td>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>range</td>
<td>53 to 133</td>
<td>56 to 131</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Attitude Change</strong>&lt;sup&gt;c&lt;/sup&gt;</td>
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</tr>
<tr>
<td>average</td>
<td>4.0</td>
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<td>t = -.86</td>
<td>p = .20</td>
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<td>standard deviation</td>
<td>9.0</td>
<td>20.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>range</td>
<td>-18 to 22</td>
<td>-47 to 37</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup>Attitude Pretest Results are the scores received by students when they took the attitude pretest survey four weeks before the study began.

<sup>b</sup>Attitude Posttest Results are the scores received by students when they took the attitude posttest survey the week following the study.

<sup>c</sup>Attitude Change is the difference between the scores from the attitude pretest and the attitude posttest.
**Table 6**

Attitude Scores for Male and Female Students

<table>
<thead>
<tr>
<th></th>
<th>Female</th>
<th>Male</th>
<th>t-value</th>
<th>Probability</th>
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<tbody>
<tr>
<td>n</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Attitude Pretest Results(^a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>107</td>
<td>97.3</td>
<td>t = 1.35</td>
<td>p = .09</td>
</tr>
<tr>
<td>standard deviation</td>
<td>21.4</td>
<td>23.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>range</td>
<td>58 to 141</td>
<td>44 to 134</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Attitude Posttest Results(^b)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>108.1</td>
<td>99.9</td>
<td>t = 1.15</td>
<td>p = .13</td>
</tr>
<tr>
<td>standard deviation</td>
<td>24.4</td>
<td>21.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>range</td>
<td>56 to 131</td>
<td>53 to 133</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Attitude Change(^c)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>1.1</td>
<td>2.5</td>
<td>t = -.28</td>
<td>p = .39</td>
</tr>
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<td>14.8</td>
<td>16.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>range</td>
<td>-47 to 22</td>
<td>-34 to 37</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)Attitude Pretest Results are the scores received by students when they took the attitude pretest survey four weeks before the study began.

\(^b\)Attitude Posttest Results are the scores received by students when they took the attitude posttest survey the week following the study.

\(^c\)Attitude Change is the difference between the scores from the attitude pretest and the attitude posttest.
Table 7

Treatment Group Attitude Scores for Females

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Control</th>
<th>t-value</th>
<th>Probability</th>
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</thead>
<tbody>
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<td>n</td>
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<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Attitude Pretest Results</strong>&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
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<tr>
<td>average</td>
<td>106.1</td>
<td>108</td>
<td>t = .19</td>
<td>p = .43</td>
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<tr>
<td>standard deviation</td>
<td>17.9</td>
<td>25.8</td>
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<tr>
<td>range</td>
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<td>58 to 141</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Attitude Posttest Results</strong>&lt;sup&gt;b&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>111.6</td>
<td>104.2</td>
<td>t = -.65</td>
<td>p = .26</td>
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<tr>
<td>standard deviation</td>
<td>17.2</td>
<td>31.2</td>
<td></td>
<td></td>
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<tr>
<td>range</td>
<td>80 to 131</td>
<td>56 to 131</td>
<td></td>
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</tr>
<tr>
<td><strong>Attitude Change</strong>&lt;sup&gt;c&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>5.5</td>
<td>-3.8</td>
<td>t = -1.39</td>
<td>p = .09</td>
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<tr>
<td>range</td>
<td>-3 to 22</td>
<td>-47 to 20</td>
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<td></td>
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</tbody>
</table>

<sup>a</sup>Attitude Pretest Results are the scores received by students when they took the attitude pretest survey four weeks before the study began.

<sup>b</sup>Attitude Posttest Results are the scores received by students when they took the attitude posttest survey the week following the study.

<sup>c</sup>Attitude Change is the difference between the scores from the attitude pretest and the attitude posttest.
Table 8
Treatment Group Attitude Scores for Males

<table>
<thead>
<tr>
<th>Probability</th>
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<tr>
<td></td>
<td>10</td>
<td>11</td>
<td></td>
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<td>n</td>
<td>10</td>
<td>11</td>
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<td>average</td>
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<td></td>
</tr>
<tr>
<td>standard deviation</td>
<td>44 to 134</td>
<td>62 to 130</td>
<td></td>
<td></td>
</tr>
<tr>
<td>range</td>
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<td></td>
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<td></td>
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<tr>
<td>Attitude Posttest Results&lt;sup&gt;b&lt;/sup&gt;</td>
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<td>95.5</td>
<td>t = -1.00</td>
<td>p = .16</td>
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<tr>
<td>average</td>
<td>27.2</td>
<td>16.7</td>
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<td>53 to 133</td>
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</tr>
<tr>
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<td></td>
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<td>-34 to 37</td>
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</tr>
<tr>
<td>range</td>
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</tbody>
</table>

<sup>a</sup>Attitude Pretest Results are the scores received by students when they took the attitude pretest survey four weeks before the study began.

<sup>b</sup>Attitude Posttest Results are the scores received by students when they took the attitude posttest survey the week following the study.

<sup>c</sup>Attitude Change is the difference between the scores from the attitude pretest and the attitude posttest.
Figure 1

Percent of Experimental and Control Group Students Using Heuristics on the Collis-Romberg Posttest
CHAPTER V. DISCUSSION, RECOMMENDATIONS, AND CONCLUSIONS

This chapter presents a brief summary of the study. Discussion of the results is presented. Also, suggestions for further research are provided.

Summary

The purpose of this study was to investigate the effectiveness of heuristic strategies, cognitive monitoring, and computer-based learning to help students become better able to solve mathematical problems. The study was constructed in response to the suggestions made by several national organizations that identified the lack of mathematical problem solving skills among children in the United States. It also addressed the National Council of Teachers of Mathematics' standards (National Council of Teachers of Mathematics, 1991) dealing with the infusion of technology into the classrooms. These groups have stated that researchers must address effective teaching strategies using computers as instructional tools in order for this technology to be most beneficial to students.

The computer-based lesson used in the study consisted of five different software packages—one of which was used twice, and one of which was used three times (it had four separate parts). Students used these programs after instruction that included the modeling of several mathematical problems appropriate for their grade level. Students also had a daily homework assignment.

During the daily instructional time, students in the experimental group learned about heuristic strategies that gave them direction for solving mathematical problems. They also learned about cognitive monitoring and were required to respond to reflective questions on each homework assignment.

The experimental population consisted of forty seventh grade students enrolled in a middle school in Radcliffe, Iowa during the winter of 1993. Subjects were from two different
class periods taught by the same math teacher. Students were randomly assigned to the experimental group or to the control group; each group consisted of twenty students. The Iowa Test of Basic Skills Total Math scores from 1991-92 showed that the randomly assigned groups were equal or nearly equal.

The study used a pretest-posttest control-group design (Campbell & Stanley, 1963). Both the experimental group and the control group students took a paper and pencil attitude pretest and posttest. Both groups also took the Collis-Romberg test prior to and after the study. Homework was handed in and scored daily, and all students took a teacher-made quiz at the completion of the study. The quiz was composed of several of the problems that had been on homework assignments.

Comparisons were made between the treatment groups, and were summarized in Chapter IV. Comparisons were also made between genders and by treatment within genders. Data from tests were analyzed using the t-test and the analysis of covariance to determine if statistically significant differences existed between the groups. The level of significance was set at 0.05.

Five hypotheses were developed. The results were as follows:

1. Control group students and experimental group students did not show any significant difference in the number of correct responses on the C-R posttest. However, the experimental group went from a mean SOLO level of 1.6 on the pretest to 2.1 on the posttest. The control group improved only from 2.1 to 2.2. The change in the SOLO level for the experimental group was significantly greater than the change for the control group.

2. Females in the experimental group had a sizable increase in the number of correct responses on the C-R test from the pretest to the posttest, however, it was not statistically significant. Experimental group females
also showed a significantly greater increase in C-R level from the pretest to the posttest than did the control group females. In fact, the control group females actually decreased their mean SOLO level by 0.1. The changes between the C-R scores and the C-R levels were not significantly different for males in the control group and males in the experimental group.

3. Attitude between treatment groups was not significantly different. There was a wide range of scores for both treatment groups, which indicated that mathematical attitude varied from person to person.

4. Surprisingly, females had a better overall attitude about mathematics than did males prior to the study. This difference was statistically significant. However, the differences in attitude after the treatment and in the change in attitude between genders was not significant. For members of the same gender, the study failed to show a statistically significant difference between attitude about mathematics and the type of treatment.

5. Members of the experimental group used the heuristic strategies that they learned during the treatment on the C-R posttest.

Discussion

The computer-based learning activities utilized in the study gave students the opportunity to explore mathematical problem solving in a non-threatening way. Because students worked at their own pace, learning was somewhat individualized during that portion of the instruction. Students were observed to be excited about the lessons that were used during the study and were eager to see how well they could do.
The study lasted only eight days. Of those eight days, students in the experimental group learned new heuristic strategies during five of the classes. Thus, they only had the opportunity to practice using combinations of strategies for three days. The rather small number of days involved in the study may have been a limiting factor; the experimental group students may not have had enough time to completely grasp the idea of heuristics and how those strategies might have helped them to solve mathematical problems.

The researchers felt that the time spent on instruction was not long enough for the experimental group. Although there was plenty of time during the 20-minute instructional period to discuss homework problems and model new problems for the control group, the addition of teaching a new heuristic strategy to the experimental group often meant not having enough time to go through one of the sample problems in class. While the experimental group learned about the heuristic strategies, they may not have seen enough problems modeled using those strategies for the instruction to be completely effective.

Because the experimental group C-R level change between the pretest and the posttest was significant, it seemed that the combination of the heuristic strategies, cognitive monitoring, and the computer-based learning had a positive effect. The overall change in the number of correct answers between the pretest and posttest was not statistically significant. This indicated that students were more likely to guess a correct response on the pretest than on the posttest.

Females tended to do poorer than males on problem solving according to the research discussed in Chapter II. The results of this study seemed to be consistent with the literature. Girls overall had fewer correct responses on the C-R posttest and theirs were at a lower SOLO level than boys. However, when comparing the two female treatment groups, the experimental group girls had a much higher C-R SOLO level and more correct responses on the C-R test than the control group females. This was an important finding. Both the control and experimental
treatments used the same computer-based lessons. Therefore, either the cognitive monitoring or the heuristic strategies, or the combination of the two, was an effective instructional method for females. Females in the experimental group were able to learn effectively from the computer instruction, based on the differences in problem solving scores between those students and the control group female scores.

The study did not support the idea that mathematical attitude would improve because of the use of computer-based learning. The students seemed to have a positive attitude toward instruction while working with the computers, but this did not transfer to an improved attitude about mathematics in general. The students were aware that once the two-week long study ended, instruction would return to its more routine state where computers would be used only on occasion. Two weeks of instruction may not have been long enough to change student attitudes about mathematics.

Heuristic strategies were used to complete the C-R posttest by the students that had received instruction about heuristics. The students were often not able to completely work the problem and come up with the correct solution; however, the students that had learned heuristic strategies at least were able to begin the problem. The two-week long treatment stressed the first steps in mathematical problem solving—setting up the problem correctly and finding a way to solve it. Little time was spent discussing how to follow through until a correct solution was found. The experimental group students at least had an idea about how the solution could be found, whereas many of the students in the control group did not even know what to do to begin solving a problem, unless they could draw a diagram. The heuristics gave the experimental group more options that could be used to get a problem started.
Recommendations

This study's subjects were forty students in one school. The sample size was small, and reduced the generalizability of the results. Validity of the results could be strengthened by using a larger sample. A study could also be conducted using a modified version of this treatment with students in different grade levels.

Because the study occurred within a two-week period, time may have been a significant factor in the research. It is recommended that this study be performed over a year-long period, so that a teacher could integrate mathematical problem solving and heuristic strategies into their current math content instead of a stand-alone unit. This may be more meaningful for students, and may help them complete the problem once the heuristic strategies are applied.

Students had only twenty minutes to work with the software programs since the rest of the class period was spent teaching heuristics, modeling problems, and discussing the homework assignment. Twenty minutes was probably not enough time for the students to receive maximal benefits from computer-based learning. Some students spent as much as 10 minutes learning the program and figuring out what they were required to do. If a different software program is to be used to help teach mathematical problem solving skills to students, it is suggested that at least thirty minutes of computer time be allotted. It is also recommended that the students be given complete instructions on the goals of the program and how to operate it prior to beginning the program.

Conclusions

The instructional use of computers has become a very important part of educational research. Originally, research in this area concentrated on the computer as a teaching tool, while more recent research has explored the role of the computer in learning. The computer by itself will not dramatically increase problem solving skills, so it is necessary to examine possible
treatments that use the computer as a tool along with other methods to help increase problem solving skills.

This study investigated how computers could be used as tools, along with heuristic strategies and cognitive monitoring to help students increase their mathematical problem solving skills. Students were taught heuristic strategies and problems were modeled that employed these strategies. They used computer software appropriate for their grade level to help them solve mathematical problems. Students also solved mathematical problems independently in a daily homework assignment.

The results of the study indicated that the treatment was effective in increasing students' overall level of thinking, based on the Collis-Romberg Mathematical Problem Solving Profiles. Females responded quite well to the treatment. Females that received instruction about applying heuristic strategies, cognitive monitoring, and computer-based learning had more items correct on the C-R posttest and had a higher mean SOLO level than the females that received only the computer-based learning. Both males and females that received instruction about heuristic strategies used these strategies to help them solve problems. Attitude toward mathematics was not affected by the treatment. Although the study produced several significant results, more empirical evidence is needed to support these generalizations.
BIBLIOGRAPHY


Caporrimo, R. (1990, April). Gender, confidence, math: Why aren't the girls where the boys are? Paper presented at the annual meeting of the American Psychological Association, Boston.


I would sincerely like to thank Dr. Michael Simonson for all of the suggestions, patience, and time he has offered during this study. His guidance has been a tremendous help to me. I would also like to thank Dr. Ann Thompson and Dr. Wallace Sanders for their input into the research. Because of their backgrounds, they were able to offer unique support to my project. I would also like to acknowledge the Sunburst/Wings for Learning software company. They were very gracious to send me copies of several programs for my study, and I appreciated their pleasant customer service personnel.

I am grateful to Kelly Rogers, Superintendent of the Hubbard-Radcliffe Community Schools and Ed Frangenberg, Middle School Principal for allowing their students to be used in the study.

Finally, I would like to recognize my husband, Steve, for all of his help in planning and implementing the study, and for all of his kindness, warmth, and love during the entire research period.
APPENDIX A.

THE COLLIS-ROMBERG MATHEMATICAL BATTERY
This booklet contains five mathematical problems.

They are to find out how well you can solve mathematical problems, so it is important that you answer as accurately and carefully as you can.

Each of the five problems has several parts.

Use the extra space to do your work, then write your answer on the line.

If you do not know an answer, leave the line blank.

Now do the example. Stop when you see the STOP sign.

**EXAMPLE**

This is a machine that changes numbers.

It adds the number you put in three times and then adds 2 more.

So, if you put in 4, it puts out 14.

A : If 14 is put out, what number was put in?

ANSWER: _______
B: If we put in a 5, what number will the machine put out?

ANSWER: _______

C: If we got out a 41, what number was put in?

ANSWER: _______

D: If $x$ is the number that comes out of the machine when the number $y$ is put in, write down a formula which will give us the value of $y$ whatever the value of $x$.

ANSWER: _______
Here are the answers for the example.

Read and compare them with your answers.

A: If a 14 is put out, what number was put in?

ANSWER: 14

B: If we put in a 5, what number will the machine put out?

\[ 5 + 5 + 5 + 2 = 17 \]

\[ (3 \times 5) + 2 = 17 \]

ANSWER: 17

C: If we got out a 41, what number was put in?

\[ 41 - 2 = 39 \]

\[ 39 \div 3 = 13 \]

ANSWER: 13

D: If \( x \) is the number that comes out of the machine when the number \( y \) is put in, write down a formula which will give us the value of \( y \) whatever the value of \( x \).

\[
\begin{align*}
3y + 2 & = x \\
3y & = x - 2 \\
y & = \frac{x - 2}{3}
\end{align*}
\]

ANSWER: \( y = \frac{x - 2}{3} \)
Note: The formula $x = 3y + 2$ is not a correct answer because you were asked to give a formula for $y$ in terms of $x$.

The Profile begins on the next page.

Try every part for each problem but don’t spend too much time on any one part.

If you have time, you may go back and try any part you could not do at first.
Two students set about doing addition of two numbers in different ways:

<table>
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<tr>
<th>STUDENT</th>
<th>METHOD</th>
<th>EXAMPLES</th>
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<tbody>
<tr>
<td>Mary</td>
<td>Normal addition</td>
<td>323 +437 760</td>
</tr>
<tr>
<td></td>
<td></td>
<td>136 +328 464</td>
</tr>
<tr>
<td></td>
<td></td>
<td>419 +514 933</td>
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<td>Cherry</td>
<td>Special method</td>
<td>466 +185 641</td>
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</tr>
<tr>
<td></td>
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<td>247 +366 603</td>
</tr>
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</table>

A: Write down the answer Mary would give to 19 + 28.

ANSWER: ________

B: Mary is asked to do the following problem:

\[ 237 + 484 + 159 \]

She adds together 237 and 484 and gets 621.

She then adds 159 to this answer and gets 780.

The teacher says that Mary’s answer is wrong.

Where did Mary go wrong?

ANSWER: ____________________________
C: What is the answer you would expect Cherry to give to 365 + 289?

ANSWER: __________

D: For what types of 3-digit addition problems would Cherry's method give the same answer as Mary's method?

ANSWER: ____________________________________________

______________________________________________________

______________________________________________________

______________________________________________________
The number of chirps, $\Delta$, that a cricket makes in a minute can be related to the temperature, $\Box$.

A rule that does this is $\frac{5}{36} \Delta + 4 = \Box$.

It is possible then to tell how warm it is by using a cricket as a thermometer because the warmer it gets, the faster the cricket chirps.

A: Is it true or false to say that on a normal summer’s day we would expect a cricket to chirp faster at noon than at dawn?

ANSWER: _____________

B: If you hear 120 chirps a minute, what is the temperature?

ANSWER: _____________
C: If the temperature is 19°C, approximately how many chirps per minute would a cricket be making?

ANSWER: ____________

D: According to this formula, at what temperature will crickets stop chirping?

ANSWER: ____________
A rectangle has been divided by straight lines into the smaller regions A, B, C, D as shown.

Two regions are called neighbours if their borders have a line in common.

The neighbours of A in the rectangle above are B and C.

We write this as \( n(A) = BC \).

The Table of Neighbours for the rectangle above is:

\[
\begin{align*}
  n(A) &= BC \\
  n(B) &= AC \\
  n(C) &= ABD \\
  n(D) &= C
\end{align*}
\]

A: In the map above, is D a neighbour of B?

ANSWER: ___________

B: For the drawing below, write down the neighbours of B.

ANSWER: ___________
C: Here is the Table of Neighbours for the regions marked below:

\[ n(A) = DEF \]
\[ n(B) = CDE \]
\[ n(C) = B \]
\[ n(D) = ABEF \]
\[ n(E) = ABD \]
\[ n(F) = AD \]

Which numbered region corresponds to F on the map below?

D: Information from the Table of Neighbours for the regions in this map was lost.

It is known that:

1. E has more neighbours than F.
2. F and G have two of their neighbours in common.
3. G and E have no common neighbours.

Which numbered regions correspond to E, F and G?
Jan liked building blocks from small cubes.

She made block A by putting 8 cubes like this together.

Then she went on to make blocks B and C.

A: How many cubes did Jan use to build block A?

ANSWER: __________

B: How many cubes did Jan use in building block B?

ANSWER: __________
C: If block C was made so that it looked the same on the outside but the outside cubes were stuck together so that it could be hollow on the inside, what is the smallest number of cubes that would need to be used?

ANSWER: ____________

D: A block $8 \times 5 \times 6$ was made to look solid from the outside but the outside blocks were stuck together so that the block was hollow on the inside. How many cubes would fit in the largest possible space which could be left within the block?

ANSWER: ____________
A teacher tries to guess the season and month when any child in her class was born.

If the teacher was to guess the season, she would most likely get one correct for every four guesses.

If the teacher was to guess which month any child was born, she would be likely to get one correct for every twelve.

A: If the teacher used the seasons to make her guesses, how many times do you think she would have been correct with four children's birthdays?

ANSWER: ____________

B: The teacher has twelve girls and sixteen boys in her class.

She guessed the month in which each girl was born and the season in which each boy was born.

In how many of her twenty-eight guesses was she likely to have been correct?

ANSWER: ____________
C: If the teacher guessed seven right out of sixteen for the seasons and six right out of twelve for the months, how many more correct guesses altogether has she made than you would expect?

ANSWER: __________ 

D: If the teacher wants to guess correctly an individual child's birthday season and month of birth, what is her likely success rate?

ANSWER: __________
### COLLIS-ROMBERG MATHEMATICAL PROBLEM SOLVING PROFILES

#### INDIVIDUAL DIAGNOSTIC PROFILE

**NAME:** ____________________________  **SEX:** ____________________________

**AGE:** _______ years _______ months  **DATE:** ____________________________

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|   |   |   |   |   |
|---|---|---|---|
| Q1 |   |   |   |   |
| Q2 |   |   |   |   |
| Q3 |   |   |   |   |
| Q4 |   |   |   |   |
| Q5 |   |   |   |   |

|   |   |   |   |   |
|---|---|---|---|
| Q1 | 38 | 35 | 39 | 37 | 36 |
| Q2 | 44 | 40 | 45 | 43 | 41 |
| Q3 | 50 | 47 | 51 | 49 | 48 |
| Q4 | 52 | 52 | 52 | 52 | 52 |

**SOLO Level**

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**PAGE NUMBERS FOR FUTURE LEARNING**

- Q1: 38, 35, 39, 37, 36
- Q2: 44, 40, 45, 43, 41
- Q3: 50, 47, 51, 49, 48
- Q4: 52, 52, 52, 52, 52

**Comments:**

- Q1 **NUMBER**
- Q2 **ALGEBRA**
- Q3 **SPACE**
- Q4 **MEASUREMENT**
- Q5 **CHANCE AND DATA**
- OVER-ALL
APPENDIX B.

MATHEMATICAL SELF-CONCEPT SCALE
Mathematics Self-Concept Scale

Directions: Please indicate your choice that best expresses your feelings toward mathematics.

VSA Very Strongly Agree
SA Strongly Agree
OSA Only Slightly Agree
U Undecided
OSD Only Slightly Disagree
SD Strongly Disagree
VSD Very Strongly Disagree

1. Math is an interesting and challenging course.
   VSA SA OSA U OSD SD VSD

2. Math teachers are helpful and anxious that all students achieve some degree of success.
   VSA SA OSA U OSD SD VSD

3. I have more confidence in my ability to deal with math than in my ability to deal with other academic subjects.
   VSA SA OSA U OSD SD VSD

4. The subject matter in math is too repetitious and requires too much drill.
   VSA SA OSA U OSD SD VSD

5. The amount of time devoted to math in school could be more profitably used in studying other academic subjects.
   VSA SA OSA U OSD SD VSD

6. Math classes provide the opportunity for learning values which are useful in other parts of daily living.
   VSA SA OSA U OSD SD VSD

7. When I attend a math class or hear math being discussed, I get slightly nervous.
   VSA SA OSA U OSD SD VSD

8. I feel happy when someone asks me to work a problem in math.
   VSA SA OSA U OSD SD VSD

9. I had no fear of getting poor grades or failing math at the beginning of the fall semester.
   VSA SA OSA U OSD SD VSD

10. Math makes me feel insecure.
    VSA SA OSA U OSD SD VSD

11. I am frequently bothered by feelings of inferiority in a math class.
    VSA SA OSA U OSD SD VSD

Permission granted from ETS Test Collection, Princeton, NJ
12. Because of other people in the math class, I haven't been able to achieve as much as I should.  VSA SA OSA U OSD SD VSD

13. I am usually able to ignore the feelings of others when I am attempting to complete a math assignment.  VSA SA OSA U OSD SD VSD

14. I have always liked math because it is an "exact" science.  VSA SA OSA U OSD SD VSD

15. I have always enjoyed math.  VSA SA OSA U OSD SD VSD

16. I feel ill-at-ease when I am required to solve problems mathematically.  VSA SA OSA U OSD SD VSD

17. I would hesitate to take a high school course such as chemistry, physics, or mechanical drawing if I knew math was involved.  VSA SA OSA U OSD SD VSD

18. I have negative feelings toward the teacher when difficult math problems are assigned.  VSA SA OSA U OSD SD VSD

19. I feel self-conscious when I'm with people who have superior ability in math.  VSA SA OSA U OSD SD VSD

20. When I get to high school and have a choice of electives, I would choose math over other subjects.  VSA SA OSA U OSD SD VSD

21. I always enjoy math courses because I feel I can be successful.  VSA SA OSA U OSD SD VSD

22. At the present time, grades have had very little effect on my attitude toward math.  VSA SA OSA U OSD SD VSD
APPENDIX C.

EXPERIMENTAL GROUP HOMEWORK PROBLEMS
Experimental Group Homework Assignments

Day 1

1. Kirk, Tony, Pedro, and Ray played checkers. Each boy played each of the other boys one game. How many games were played in all?  
   What are the key words?  
   What is the problem setting?  
   What is being asked?

2. What percent of the pages in a 300-page book have page numbers whose digits add to ten?  
   What are the key words?  
   What is the problem setting?  
   What is being asked?

3. Colleen earned $118. She gave $44 to Ray and half of what was left to Jen. How much money did Colleen keep?  
   What are the key words?  
   What is the problem setting?  
   What is being asked?
Day 2

1. The number of times a cricket chirps depends on the temperature. What is the temperature when the cricket chirps 48 times a minute?

<table>
<thead>
<tr>
<th>temp (F)</th>
<th># chirps</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>41</td>
<td>4</td>
</tr>
<tr>
<td>42</td>
<td>8</td>
</tr>
</tbody>
</table>

What are the key words?
What is the problem setting?
What is being asked?
How will you explore?

2. A spider is on one corner of the cube. It wants to walk to the opposite corner of the cube. It can only walk along the edges of the cube. If each possible path is along 3 edges of the cube, how many different paths can the spider take?

What are the key words?
What is the problem setting?
What is being asked?
How will you explore?

3. A driver is halfway between the mountains and the city. After she travels 40 more kilometers, she will be 100 km from the city. How far is it from the mountains to the city?

What are the key words?
What is the problem setting?
What is being asked?
How will you explore?
Day 3

1. What numbers should go in the O and △ in the table? Write a rule that relates A and B.
   \[
   \begin{array}{ccccccc}
   A & 2 & 3 & 5 & 7 & 9 & 10 \\
   B & 5 & 7 & 11 & 15 & O & △
   \end{array}
   \]

   What are the key words?
   What is the problem setting?
   What is being asked?
   How will you explore?
   What is your strategy?

2. A ferryboat is full when it has 10 cars on board. It is also full when it has 6 trucks on board. The ferryboat never carries cars and trucks at the same time. The ferryboat made 5 trips across the river and was full on each trip. It carried a total of 42 cars and trucks across the river. How many cars did the ferryboat carry altogether?

   What are the key words?
   What is the problem setting?
   What is being asked?
   How will you explore?
   What is your strategy?

3. Make this drawing. Put a number in each circle so that the number in the squares is the sum of the two numbers connected to the square.

   What are the key words?
   What is the problem setting?
   What is being asked?
   How will you explore?
   What is your strategy?
Day 4

1. Draw the squares. Put the numbers 1 through 9 in the squares so that no two rows, columns, or diagonals add to the same number.

What are the key words?
What is the problem setting?
What is being asked?
How will you explore?
What is your strategy?

2. How many spheres must be placed in the empty pan to balance the scale?

What are the key words?
What is the problem setting?
What is being asked?
How will you explore?
What is your strategy?

3. Suppose this pattern were folded to make a cube. Give the missing numbers.

What are the key words?
What is the problem setting?
What is being asked?
How will you explore?
What is your strategy?
Day 5

1. Paul is 12. His father is 33. How many years ago was the father 4 times as old as his son?

What are the key words?
What is the problem setting?
What is being asked?
How will you explore?
What is your strategy?

2. Odd Magic Square
Fill in the boxes with odd numbers so that the sums of the rows, columns, and two diagonals are the same.

<table>
<thead>
<tr>
<th>7</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

What are the key words?
What is the problem setting?
What is being asked?
How will you explore?
What is your strategy?

3. You have 4 boxes: red, blue, yellow, and green. The red box weighs more than the green box. The green box weighs more than the blue box. The yellow box weighs 3 grams less than the blue box. Which box weighs the least?

What are the key words?
What is the problem setting?
What is being asked?
How will you explore?
What is your strategy?
Day 6

1. Billy lost some weight. Susan lost half as much weight as Billy. Carol lost half as much weight as Susan. Carol lost 5 pounds. If Billy now weighs 110 pounds, how much did he weigh before he lost weight?

<table>
<thead>
<tr>
<th>What are the key words?</th>
<th>What is being asked?</th>
<th>How will you explore?</th>
<th>What is your strategy?</th>
</tr>
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2. You have boxes that will hold 1 cubic foot, 3 cubic feet, 9 cubic feet, and 27 cubic feet of paper. What is the fewest number of boxes you need to hold 257 cubic feet of paper?

<table>
<thead>
<tr>
<th>What are the key words?</th>
<th>What is being asked?</th>
<th>How will you explore?</th>
<th>What is your strategy?</th>
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</table>

3. Seventeen is a prime number. If you reverse the digits you get 71. 71 is also prime. How many prime numbers between 20 and 60 also become prime numbers when their digits are reversed?

<table>
<thead>
<tr>
<th>What are the key words?</th>
<th>What is being asked?</th>
<th>How will you explore?</th>
<th>What is your strategy?</th>
</tr>
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</tbody>
</table>
Day 7

1. What is the starting number? What are the key words?
   multiply by .5 What is being asked?
   multiply by 1/3 How will you explore?
   square the number What is your strategy?
   add 1
   50

2. A school has 4 entrances, 4 stairways from the first floor to the second floor and 4 stairways from the second to the third floor. If you have your math class on the third floor, how many possible paths are there for you to get to class from outside? What are the key words?
   What is being asked?
   How will you explore?
   What is your strategy?

3. A parking lot has spaces for 6 rows of cars with 20 cars in each row. There are only 15 empty spaces. How many cars are in the lot? What are the key words?
   What is being asked?
   How will you explore?
   What is your strategy?
Day 8

1. How many rectangles are in this square? Remember, squares are rectangles.

2. A 60-foot board is cut into two pieces. One piece is 12 feet longer than the other piece. How long are the two pieces?

3. What is the fewest number of coins you would need to pay a charge of any amount less than $0.26 with exact change? List the coins.

What are the key words?

What is being asked?

How will you explore?

What is your strategy?
APPENDIX D.

CLASSROOM DISCUSSION PROBLEMS
Classroom Discussion Problems

Day 1

1. What one coin should you pick up so that 1/4 of the remaining money shows heads?

2. There are 15 rows of pennies in a coin book with the same number of pennies in each row. Five of the rows are filled with Indian Head pennies. If there are 35 Indian Head pennies, how many pennies are in the coin bank?

3. What fraction of the perfect squares less than 100 are odd numbers?

Day 2

1. The second place runner in a mile was 3.5 seconds behind the first place runner. The third place runner was 6.7 seconds behind the second place runner. The fourth place runner was 22.6 seconds behind the first place runner. How many seconds behind the third place runner was the fourth place runner?

2. What row has a sum of 390?

3. The average of seven numbers is 49. If 1 is added to the first number, 2 is added to the second number, 3 is added to the third number, and so on up to the seventh number, what is the new average?

Day 3

1. A rectangular table is 3 times as long as it is wide. If it were 3 feet shorter and 3 feet wider, it would be a square. What are the dimensions of the rectangular table?
2. Tickets for the concert cost $4.50 for adults and $3.00 for children. 100 people
attended the concert and $360 was collected for the tickets. How many children
attended the concert?

3. A father is 4 times as old as his daughter is now. In 20 years he will be only
twice as old as his daughter. How old are the father and daughter now?

Day 4

1. You have 3 sticks of lengths 10 cm, 12 cm and 15 cm. How many you use these
sticks to mark off a length of 17 cm?

2. Find 2 difference ways to balance the scale by placing any five of these weights
in pan B.

3. Each pattern can be folded to form a cube. Which two cubes will look the same?

Day 5

1. Bob worked twice as long as Dan. Dan worked one hour more than Jim. Jim
worked 2 hours less than Pedro. Pedro worked three hours. How many hours
did Bob work?

2. 3 blimps = 2 blomps
4 blomps = 6 blooies
2 blimps = ? blooies

3. Find 2 two-digit prime numbers whose digits sum to 7.

Day 6

1. A clothing store bought handkerchiefs, six for $10 and sold them four for $10.
They made a $60 profit. How many handkerchiefs did they sell?
2. On a test, I had 7 times as many correct answers as incorrect answers. There were 120 items on the test. How many items did I get right?

3. How many different ways can you arrange these 4 books on the shelf?

Day 7

1. On their vacation trip, Sally, Mark, and Alice each took turns driving. Alice drove 40 more miles than Mark. Mark drove 3 times as many miles as Sally. Sally drove 25 miles. How long was the trip?

2. You and your friend each have $.80 in coins. You do not have any pennies. You both have the same number of coins. You and your friend do not have any of the same coins. You can make change for a quarter. What are your coins?

3. There are 8 teams in a basketball league. Each team plays each of the other teams twice. How many games are played?

Day 8

1. Jim's age this year is a multiple of 5. Next year Jim's age will be a multiple of 7. Jim's older brother is now 28. How old is Jim now?

2. Lisa had $1.07 in change. She had 7 coins in all but she could not make change for a half dollar, a quarter, a dime, or a nickel. What were the coins Lisa had?

3. I spent $23 for 7 items. I bought some books at $2 each, some posters at $3 each, and some records at $5 each. How many records did I buy?
APPENDIX E.

DAILY LESSON PLANS FOR THE EXPERIMENTAL GROUP
Day 1: Experimental Group

A. Discuss all of the heuristic strategies that will be used in the study:

1. Read the problem
   - Note key words
   - Get to know the problem setting
   - Recognize what is being asked
2. Explore the problem
   - Draw a diagram
   - Make a chart
   - Look for patterns
3. Select a strategy
   - Experimentation
   - Guess/Trial and Error
   - Work backwards
4. Carry out the strategy
5. Check the solution

B. Focus on heuristic 1: reading the problem. Put classroom problem one on the overhead and ask students what the key words of the problem are, what the problem setting is, and what is being asked. If students cannot come up with the correct solutions on their own, then the facilitator will prompt students. Solve the problem by asking students what they would do to get the solution. Explore alternate methods that other students may have.

C. Put classroom problem two on the overhead and ask students the same questions as on the first problem. Continue as above.

D. Put classroom problem three on the overhead. Ask students to write down the answers to the questions about heuristic number 1. Then have them write down their solution. Continue as above.

E. Distribute the homework assignment and explain that students must attempt two out of the three problems in order to get full credit for the assignment. If they try all three of the problems and get them correct, they will receive one bonus point for the assignment in addition to their regular homework grade. Stress the cues on the right hand side of the problem page as the students work through the problems. Also stress that they must complete the questions on the right hand side of the page.

F. Have students use the computer-based lesson titled "Math Blaster Mystery"—Follow the Steps by Davidson, and emphasize reading the problem carefully following the same steps as discussed in class. The software will be used for the rest of the class period.

Day 2: Experimental Group

A. Review all of the heuristics from day 1. Ask students to name the problem reading strategies that were discussed on day 1. Discuss the homework assignment problems; have students explain their solutions and their answers to the questions.
B. Focus on heuristic 2: exploring the problem. Use classroom example 1 to show students how to draw a diagram when finding a solution. Example 2 is used to demonstrate making a chart, and example 3 helps students to look for a pattern.

C. Distribute the homework assignment, and again explain the grading procedures.

D. Have students work with the computer-based lesson titled "The King's Rule" by Sunburst. Emphasize making a chart and looking for patterns as students work with the software.

Day 3: Experimental Group

A. Review the exploring strategies discussed on day 2. Ask students to remember the different ways of exploring a problem: making a chart or diagram or looking for a pattern. Discuss the homework assignment problems; have students explain their solutions.

B. Focus on heuristic 3: selecting a strategy. The sample problems in this lesson will involve using trial and error to find a solution. Have students discuss what they would do in the first two problems. Then have them try the third problem on their own and discuss it as a group afterwards.

C. Distribute the homework assignment.

D. Have students work with the computer-based lesson titled "Tobbs Learns Algebra" by Sunburst. Emphasize using trial and error to find the solutions as the students work with the software.

Day 4: Experimental Group

A. Review the trial and error method discussed on day 3. Have students explain their solutions and how they found their answers.

B. Focus on heuristic 3: selecting a strategy. The sample problems in this lesson will involve experimentation in order to find a solution. Have students discuss what they would do in the first two problems. Then have them try the third problem on their own and discuss it as a group afterwards.

C. Distribute the homework assignment.

D. Have students work with the computer-based lesson titled "Math Blaster Mystery"--Decipher the Code by Davidson.

Day 5: Experimental Group

A. Review the experimentation method discussed on day 4. Have students explain their solutions and how they found their answers.
C. Distribute the homework assignment.

D. Have students work with the computer-based lesson titled "Math Shop" by Scholastic.

Day 6 through Day 8: Experimental group.

A. Discuss the homework assignment from the previous day. Students will explain their solutions and how they found their answers.

B. All of the heuristic strategies are reviewed daily. Students will try each of the three discussion problems and will verbalize their strategies to the class. The facilitator will bring up possible other strategies that could be used to solve the problem if a student does not name them.

C. Distribute the homework assignment.

D. Have students work with the computer-based lesson.
   Day 6: "Math Shop" by Scholastic
   Day 7: "Math Blaster Mystery"--Search for Clues by Davidson
   Day 8: "The Super Factory" by Sunburst
APPENDIX F.

HUMAN SUBJECTS REVIEW COMMITTEE APPROVAL
Checklist for Attachments and Time Schedule

The following are attached (please check):

12. [ ] Letter or written statement to subjects indicating clearly:
   a) purpose of the research
   b) the use of any identifier codes (names, #’s), how they will be used, and when they will be removed (see Item 17)
   c) an estimate of time needed for participation in the research and the place
   d) if applicable, location of the research activity
   e) how you will ensure confidentiality
   f) in a longitudinal study, note when and how you will contact subjects later
   g) participation is voluntary; nonparticipation will not affect evaluations of the subject

13. [ ] Consent form (if applicable)

14. [ ] Letter of approval for research from cooperating organizations or institutions (if applicable)

15. [ ] Data-gathering instruments

16. Anticipated dates for contact with subjects:

   
<table>
<thead>
<tr>
<th>First Contact</th>
<th>Last Contact</th>
</tr>
</thead>
<tbody>
<tr>
<td>December 14, 1992</td>
<td>January 20, 1993</td>
</tr>
</tbody>
</table>

17. If applicable: anticipated date that identifiers will be removed from completed survey instruments and/or audio or visual tapes will be erased:

   May 17, 1992

18. ( ) Date

   Department or Administrative Unit

   11/11/92

19. Decision of the University Human Subjects Review Committee:

   [ ] Project Approved  [ ] Project Not Approved  [ ] No Action Required

   Patricia M. Keith
   Name of Committee Chairperson

   11/19/92
   Date

   Signature of Committee Chairperson

GC: 1/90
APPENDIX G.

CONSENT OF HUBBARD-RADCLIFFE SCHOOL PERSONNEL
November 5, 1992

Kelly Rogers, Superintendent
Ed Frangenberg, Principal
Hubbard-Radcliffe Middle School
Radcliffe, IA

Dear Mr. Rogers and Mr. Frangenberg:

I am currently a graduate student at Iowa State University working on my Master's Thesis, and am interested in using the seventh grade students at Hubbard-Radcliffe Middle School as subjects in my study. I would like to explain the study and how the students will be affected.

Since my husband, Steven Poole, teaches the seventh grade students, my study will be accomplished during his classes. Students will receive instruction about mathematical problem solving, which is a topic covered regularly in his course. They will be tested and graded on the unit in the same way that they are assessed with any other unit covered in math class. Students within each of the sections will be randomly assigned into two groups: a control group and an experimental group. In one of the classes, Steve will work with the control group while I work with the experimental group. In the other section, we will switch roles. (I am certified to teach mathematics in grades 7-12 and I have had teaching experience at both the Middle School and High School levels.) All sections will have approximately twenty minutes of class discussion followed by twenty minutes of computer software use each day for two weeks (January 4 - January 15).

The control group will receive traditional instruction on problem solving (The teacher will model how the problems are solved, asking students for suggestions), followed by computer software use. The experimental group will receive training in heuristic strategies and cognitive monitoring skills followed by the same commercial computer software applications. Heuristic strategies are specific strategies that can be applied to mathematical problems, while cognitive monitoring is the process of thinking about thinking. Both groups will have the same problems modeled during class discussions as well as for homework, and both groups will use the same software programs. The software is designed to teach mathematical problem solving skills, and will be selected by Steve and I on the basis of appropriateness of level and whether problem solving skills are, in fact, addressed.

If the study is approved, students will take a problem solving pretest, as well as a math anxiety test in December. After the treatment, students would take a problem solving posttest and retake the math anxiety test. I would also like to have permission to use students’ scores from the 1991-92 Iowa Test of Basic Skills to determine if the control and experimental groups are similar in math ability. If possible, I would also like to use their scores on the 1992-93 ITBS if scores are available. Only data from the students will be used--no names will be included in the study. I have enclosed a copy of the letter that will be sent to parents requesting their permission.
In order for educators to effectively incorporate new technology and instructional strategies into their teaching, research must be done to determine how to best integrate these items. The type of activities we will use have been studied by other researchers, but the combination of the three instructional methods is somewhat unique. The results of the study should offer suggestions about how to effectively teach mathematical problem solving skills.

I am enclosing a copy of my Prospectus, which has already been approved by my Graduate Committee. It contains a broad review of the problem I will be investigating, in addition to providing a more thorough description of the methods that will be used in the study. Participation in the study will not involve any expenses for the Hubbard-Radcliffe Schools.

I would like to have your approval so that I can file the necessary forms with the Human Subjects Review Committee. If you have any questions or comments, please feel free to call me or Dr. Michael Simonson, my major professor, at 294-6840.

Thank you.

Sincerely,

Dawn M. Pooler

/Dr. Michael Simonson, Major Professor

I have read the letter and understand what the study will involve. I am willing to allow Hubbard Radcliffe students participate in the study, and I am willing to release Iowa Test of Basic Skills scores for those students with parental permission.
APPENDIX H:

PARENTAL CONSENT FORM
December 1, 1992

Dear Parent/Guardian:

As a part of your child's instruction in mathematics, he/she will be taking part in a study dealing with mathematical problem solving from January 4 - January 15. For grading purposes, students will be evaluated in the same way that they are assessed under regular instructional situations.

We are asking that you grant permission to use the results of the pretest and posttest on problem solving and math anxiety that your child will be taking for statistical analyses that are part of this study. In addition, we are asking for permission to use your child's scores from the 1991-92 and 1992-93 Iowa Test of Basic Skills. Names will not be associated with scores; scores will be used only for statistical analyses.

Please fill out and return the bottom portion of this page by Friday, December 4. If you have any questions, please contact Dawn Poole, researcher or Dr. Michael Simonson, professor at 294-6840, or one of the Hubbard-Radcliffe School personnel listed below.

Thank you.

Sincerely,

Dawn M. Poole, Researcher

Dr. Michael Simonson, Professor

Kelly Rogers, Superintendent

Ed Frangenberg, Principal

Steven Poole, Teacher

** I give permission to use the scores from the math anxiety pretest and posttest, problem solving pretest and posttest, and ITBS from 1991-92 and 1992-93 for research purposes only. I understand that no names will be used in the research.**

** I do not give permission for scores from the math anxiety pretest and posttest, problem solving pretest and posttest, and ITBS from 1991-92 and 1992-93 to be used in the research.**

Parent/Guardian Signature __________ Name of student __________ Date __________