Prediction of laminar flows over a rearward-facing step using the partially-parabolized Navier-Stokes equations

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Prediction of laminar flows over a rearward-facing step using the partially-parabolized Navier-Stokes equations

by

Ing-Tsau Chiu

A Thesis Submitted to the
Graduate Faculty in Partial Fulfillment of the
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Signatures have been redacted for privacy

Iowa State University
Ames, Iowa
1984
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NOMENCLATURE

A
A^u_j, A^v_j, A^\phi_j, A^T_j
A
A^u_j, A^v_j, A^\phi_j, A^T_j

B
B^u_j, B^v_j, B^\phi_j, B^T_j
B
B^u_j, B^v_j, B^\phi_j, B^T_j

C_f
C^u_j, C^v_j, C^\phi_j, C^T_j

D
D^u_j, D^v_j, D^\phi_j, D^T_j
D
D^u_j, D^v_j, D^\phi_j, D^T_j

E^u_j, E^v_j

F_1, F_2

G_1, G_2

H_{in}

H_{out}

h

I

J

J_{STEP}

K_{FNS}

k

M_{CSTEP}

\dot{m}

N_{J}

Constant in hybrid finite-difference scheme
Coefficients in finite-difference expressions
Constant in hybrid finite-difference scheme
Coefficients in finite-difference expressions
Skin-friction coefficient (= 2\tau_w/\rho u_{in}^2)
Coefficients in finite-difference expressions
Channel inlet hydraulic diameter
Coefficients in finite-difference expressions
Coefficients in finite-difference expressions
Dimensionless pressure gradient in x and y directions, respectively
Pressure gradient in x and y directions, respectively
Channel inlet height
Channel outlet height
Heat transfer coefficient (= \dot{q}_w/(T_w - T_{w,in}))
Gridpoint index in x direction
Gridpoint index in y direction
Designates the point just below the step
Designates the last streamwise station of the computation domain
Thermal conductivity
Designates the first streamwise station downstream of the step
Mass flow rate
Outer edge of the flow computation domain
\( \text{Nu} \)  
Nusselt number \( (= \frac{h H_{in}}{k}) \)

\( p \)  
Static pressure

\( P \)  
Dimensionless pressure \( (= \frac{p}{\rho u_{ref}^2}) \)

\( \text{Pr} \)  
Prandtl number \( (= \frac{c_p \mu}{k}) \)

\( \dot{q} \)  
Heat flux

\( \text{Re} \)  
Reynolds number based on the channel inlet height 
\( (= \frac{U_{in} H_{in}}{v}) \)

\( \text{Re}_D \)  
Reynolds number based on the channel inlet hydraulic diameter 
\( (= \frac{U_{in} D}{v}) \)

\( \text{Re}_s \)  
Reynolds number based on the step height 
\( (= \frac{u_s s}{v}) \)

\( \text{Re}_{\delta_s}^{*} \)  
Reynolds number based on the displacement thickness at the step 
\( (= \frac{u_{s} \delta_{s}^{*}}{v}) \)

\( R_c \)  
Critical mesh Reynolds number

\( R_m^+, R_m^- \)  
Mesh Reynolds number

\( \text{Sp} \)  
Dimensionless source term in pressure Poisson equation

\( S_\phi \)  
Dimensionless source term for velocity correction potential

\( \sum |S_\phi| \)  
Absolute sum of mass sources along one streamwise station

\( \text{St} \)  
Stanton number

\( \overline{\text{St}} \)  
Average Stanton number in the separated region

\( T \)  
Temperature

\( U \)  
Dimensionless streamwise velocity \( (= \frac{u}{u_{ref}}) \)

\( U_c \)  
Dimensionless streamwise velocity corrections

\( U_e \)  
Dimensionless exact streamwise velocity

\( u \)  
Streamwise velocity

\( u_{in} \)  
Average channel inlet velocity
\( u_{ref} \)  
Reference velocity

\( u_s \)  
Average velocity for the fully-developed flow or the free stream velocity for the thin boundary layer flow at the step

\( V \)  
Dimensionless transverse velocity \((= v/u_{ref})\)

\( V_c \)  
Dimensionless transverse velocity corrections

\( V_e \)  
Dimensionless exact transverse velocity

\( v \)  
Transverse velocity

\( W \)  
Weighting factor used in hybrid finite-difference scheme

\( X \)  
Dimensionless \( x \) coordinate \((= \rho u_{ref} x/\mu)\)

\( x \)  
Cartesian coordinate in streamwise direction

\( x_r \)  
Reattachment length

\( x_s \)  
distance from the leading edge to the step

\( \Delta X, \Delta X^-, \Delta X^+, \Delta X^{++} \)  
Grid spacing

\( \Delta X_u, \Delta X_u^-, \Delta X_u^+, \Delta X_u^{++} \)  
Grid spacing

\( Y \)  
Dimensionless \( y \) coordinate \((= \rho u_{ref} y/\mu)\)

\( y \)  
Cartesian coordinate in transverse direction

\( \Delta Y, \Delta Y^-, \Delta Y^+, \Delta Y^- \)  
Grid spacing

Greek symbols:

\( \delta \)  
Boundary layer thickness

\( \delta_T \)  
Thermal boundary layer thickness

\( = \int_0^\infty [(T - T_w)/(T_w - T_\infty)] \, dy \)

\( \delta^* \)  
Displacement thickness

\( \delta_s \)  
Displacement thickness at the step

\( \theta \)  
Momentum thickness

\( \theta_s \)  
Momentum thickness at the step

\( \mu \)  
Dynamic viscosity
\( \nu \)  
Kinematic viscosity

\( \rho \)  
Density

\( \tau_w \)  
Wall shear stress

\( \phi \)  
Dimensionless velocity correction potential

Subscripts:

AVG  
Averaged condition

CL  
Denotes centerline

i  
Gridpoint index in \( x \) direction

in  
Channel inlet value

j  
Gridpoint index in \( y \) direction

max  
Maximum condition

out  
Channel outlet value

ref  
Denotes reference value

s  
Step value

T  
Thermal value

w  
Wall value

\( \infty \)  
Free stream value

Superscripts:

n  
Iteration level

\( \bar{\ } \)  
Denotes average value

\( (\cdot) \)  
Time rate value
ABSTRACT

The steady partially-parabolized Navier-Stokes equations, including the energy equation, are used to analyze steady, incompressible, two-dimensional, laminar flows in symmetric and asymmetric sudden expansions. The equations are solved using primitive variables. The solution procedure involves determining velocities from the momentum equations by iteratively space marching in the main stream direction. The velocities are then corrected to satisfy continuity by assuming the corrections to be locally irrotational. After a complete sweep through the computation domain is made, the pressure field is updated by solving a Poisson equation which is obtained from the momentum equations. The energy equation is solved after a convergent solution for velocities is obtained. Type-dependent differencing is used to enable space-marching even in separated regions.

Solutions were obtained for developing and fully-developed flows in sudden expansion channels of different expansion ratios. Predicted velocity and temperature distributions, reattachment lengths, skin-friction coefficients, and Nusselt numbers are presented. For the heat transfer study, results were obtained using both the elliptic and the boundary-layer forms of the energy equation. Comparisons with experimental data and the numerical solutions reported by other researchers show that the present solution procedure performs reasonably well in the testcases studied.
I. INTRODUCTION

A. The Problem and the Scope of the Present Study

Flow separation in internal flows caused by sudden expansions has been an area of interest for many years. This type of separation occurs in many engineering applications. In several instances, this phenomenon causes unwanted pressure losses thereby decreasing the operating efficiency. On the other hand, the flow separation may be advantageous due to induced mixing effects which enhance mass and heat transfer rates and result in the development of smaller, more efficient heat exchangers. In addition to the engineering applications, the study of internal expansion flows may also be useful in the modeling of some biological phenomena like aortic stenosis.

In the present study, the partially-parabolized Navier-Stokes (PPNS) equations were solved numerically with and without heat transfer for the two-dimensional, laminar flow over a rearward-facing step. The PPNS equations are derived from the complete Navier-Stokes equations by neglecting the effects of diffusion in the streamwise direction. This offers a savings in computer storage as compared with the complete Navier-Stokes equations since all variables except the pressure are stored only for a few marching stations, depending on the numerical scheme used for the streamwise convective terms. This advantage becomes particularly significant for three-dimensional flows. Nonetheless, in separated regions, full-dimensional storage is required for all
variables. The primitive variables for velocity and pressure are used in the present numerical scheme, rather than stream function and vorticity. For this reason, it should be possible to extend the present scheme to three-dimensional flows in a straight-forward manner.

The computer code used in the present study has evolved from the codes developed earlier by Chilukuri [1] and Madavan [2]. Several features were added to the scheme described by Madavan [2] during the course of this study to improve the accuracy, speed, and range of applicability of the numerical procedure. First, the evaluation of the coefficients of convective terms was made second-order accurate to be consistent with other second-order accurate finite-difference representations in the scheme. This made the present computer code a truly second-order accurate scheme in space except for the limited flow situations where the first-order upwind scheme was used. Second, a non-iterative method was used instead of the secant method to calculate the block pressure adjustment. Third, a solution procedure for the energy equation was added extending the applicability of the computer code to flows with heat transfer. Fourth, the computer code was modified to permit starting the calculation upstream of steps. This enables the prediction of developing flows in sudden expansions since, unlike fully-developed flows, the velocity profile at the step for developing flows is frequently not available and could be complex due to upstream influences. Thus, it is generally necessary to start calculation far enough upstream of the step that conventional boundary-layer solutions can be used as inlet boundary conditions.
In this study, two-dimensional, laminar separated flow over a rearward-facing step is examined for various geometries and Reynolds numbers with and without heat transfer. All cases studied were computed starting upstream of the step. The present solution procedure is verified by comparisons with the available experimental data and numerical solutions reported by other investigators.

For the case of a fully-developed flow through a sudden expansion, three expansion ratios – 3:1, 1.5:1.0, and 1.94:1.00 – were examined and generally, a fully-developed velocity profile was used as the inlet boundary condition. Results will be compared with experimental data reported by Durst et al. [3], Denham and Patrick [4], and Armaly et al. [5,6]. Solutions of the complete Navier-Stokes equations and the boundary-layer equations reported by several investigators will also be used for comparisons.

Heat transfer results will be compared with the experimental data of Aung [7] and numerical solutions of a viscous-inviscid interaction method reported by Hall [8] for a thin boundary layer flow over an asymmetric sudden expansion. Both the fully-elliptic and the boundary-layer forms of the energy equations were solved in order to study the importance of the axial conduction term in rearward-facing step flows. The FLARE approximation [9] was also evaluated for the energy equation. Another thin boundary layer step flow without heat transfer was computed; the results will be compared with the experimental data of Eriksen [10] (also reported by Goldstein et al. [11]) and the numerical
solutions reported by Kwon and Fletcher [12] using a viscous-inviscid interaction method.

B. Literature Review

In this section, previous studies of flows over a rearward-facing step will be reviewed. The previous studies are divided into two categories – experimental and analytical (including numerical) for purpose of discussion. In general, the literature review will be restricted to laminar flows in a sudden expansion. For a description of the studies of turbulent flows, the reader is referred to the work of Kwon and Fletcher [12].

1. Experimental

Eriksen [10] (also reported in Goldstein et al. [11]) conducted a wind tunnel study of a thin boundary layer flow over a rearward-facing step. A large amount of data for the reattachment lengths and velocity profiles were reported for step heights ranging from 0.36 to 1.02 cm in the Reynolds number range ($Re_s$) 73 – 649. They found that the laminar reattachment length is not a fixed number of step heights as for turbulent flow, but could be correlated by an equation similar to the one proposed by Cramer [13],

$$\frac{x_r}{\delta_s} = 0.01325 \frac{Re_s}{\delta_s} \left[ \left( \frac{s}{\delta_s} \right)^2 + 2 \frac{s}{\delta_s} \right]$$

(1.1)

or to a fair degree of approximation,

$$\frac{x_r}{s} = 2.13 + 0.021 Re_s$$

(1.2)
subject to:
\[
\frac{\delta^*}{s} > 0.4 \quad \text{and} \quad Re_s < 520 \quad (1.3)
\]

The linear relationship, Eq.(1.2), between the reattachment length and the Reynolds number was also reported by Macagno and Hung [14] for flows in axisymmetric sudden pipe expansion. Pollard [15] in his predictions of the laminar flow (Re \(< 300\)) in an axisymmetric sudden expansion also found a linear relationship between the reattachment length and the Reynolds number. However, it should be noted that other parameters such as expansion ratio and inlet velocity profile also influence the reattachment length as pointed out in [4,5,15,16], and the linear behavior may not exist in other flow configurations as shown in [4,5].

Durst et al. [3] measured the flow over a symmetric sudden expansion. The flow was found to be strongly dependent on Reynolds number (based on the step height and the maximum upstream velocity) and three-dimensional even far from the channel side walls except at the lowest Reynolds number tested. At Reynolds numbers above 100, the flow became asymmetric because the separated regions on both walls were not of equal length. This asymmetry was more pronounced with the increase of Reynolds number. At a Reynolds number of 252, a third separation zone was found downstream of the major separated region on the wall with a smaller separated region. The asymmetric behavior disappeared very far downstream of the step.

Denham and Patrick [4] studied a two-dimensional, laminar flow over a single rearward-facing step using a directional-sensitive laser
anemometer. Velocity profiles were presented for Reynolds numbers from 73 to 229 (based on the average velocity at the step and the step height). Their results indicated a constant maximum recirculated mass flow rate - 2.3% of the total mass flow rate over the step - in the range of Reynolds numbers tested. The general flow field was found to be similar to other recirculating flows like the axisymmetric and the two-dimensional duct sudden expansions, but the reattachment lengths and the recirculated mass flow rates were smaller at a given value of Reynolds number. However, at Reynolds numbers greater than 140, Denham and Patrick [4] measured longer separated regions than did Goldstein et al. [11] who also studied the flows over a rearward-facing expansion but with a smaller step height and a thin boundary layer at the step.

More recently, Armaly et al. [5,6] conducted an extensive study for flows over a single rearward-facing step. Results were reported for the laminar, transitional, and turbulent regimes. They observed that a second separated region (see Fig. 1.1) developed in the laminar regime for $Re_D \geq 400$ and remained in existence throughout the entire transitional regime. A third separated region (see Fig. 1.1) was also observed downstream of the primary separated region at the early stage of transition. It was also found that the flow was three-dimensional with the existence of secondary separated regions. For laminar flows, the reattachment length increased with the Reynolds number. However, the increase was not linear as suggested by Macagno and Hung [14] for axisymmetric pipe expansions. This fact had also been demonstrated by
Figure 1.1. Schematic of separated regions in an asymmetric expansion channel (a) first (primary) (b) second (c) third
Denham and Patrick [4] in a similar study with a different expansion ratio. For turbulent flow, their results indicated a constant reattachment length as was reported by Abbott and Kline [17].

In another paper, Armaly et al. [18] included heat and mass transfer measurements in the flow just described above. The mass transfer rates were estimated with an ammonia-manganese chloride reaction technique. The heat transfer measurements were made using a constant temperature heated wall on the side of the channel having the step. It was concluded that the Reynolds analogy relating the local wall shear stress to the local heat transfer is not applicable in the separated region.

Aung [7] conducted heat transfer measurements for laminar and transitional flows over a rearward-facing step. The heat transfer data were taken using a Mach-Zehnder interferometer for three step heights. Reattachment lengths were measured by the smoke injection technique. It was noted that the heat transfer upstream of the step was strongly enhanced by streamline curvature. For the largest step, the average heat transfer in the separation region was shown to be 56% of the flat plate result, in agreement with the theory developed by Chapman [19] for open cavities. For all step heights investigated, the average heat transfer in the separated region could be correlated by:

\[
\overline{St} = 0.787(Re_s)^{-0.55}(s/s_x)^{0.72}
\]  

Macagno and Hung [14] studied laminar flows in an axisymmetric pipe expansion. By using oil as the working fluid, they were able to
maintain the laminar flow up to a Reynolds number of 4500 (based on pipe inlet diameter). However, at a Reynolds number of 4500, the flow was not axisymmetric; a cellular secondary flow resulted in a helicoidal motion of the flow. The reattachment lengths were measured by flow visualization and noted to be linearly dependent on the Reynolds number for Reynolds numbers (Re) below 200.

2. Analytical

Most of the solutions reported by earlier investigators for flows over a rearward-facing step were obtained by solving the complete Navier-Stokes equations using finite-difference methods. The variables generally used were stream function and vorticity rather than the primitive variables (velocity and pressure).

Hung [20] calculated flows in an axisymmetric pipe expansion using both a steady and unsteady explicit scheme. The unsteady approach was found to be more stable than the steady approach at high Reynolds numbers. This was also noted by Ghia et al. [21] in solving flows in a channel with an asymmetric constriction using the partially-parabolized Navier-Stokes equations. The results of Hung's calculation [20] showed good agreement with the measured data and was also reported in Macagno and Hung [14].

Durst et al. [3] also obtained good agreement between their numerical predictions and experimental results. The numerical scheme used was essentially the one developed by Gosman et al. [22] in the form of stream function and vorticity. Other investigators using this same
form of the Navier-Stokes equations include O'Leary and Mueller [23], Giaquinta [24], Atkins et al. [16] and Agarwal [25].

Atkins et al. [16] evaluated the performance of upwind and central differencing schemes for laminar and turbulent flows over a step in a two-dimensional channel. At low Reynolds numbers, the upwind scheme predicted longer reattachment lengths as compared with the central differencing scheme. However, away from the recirculation zone, the two solutions were virtually identical. It was also noted that use of the experimental inlet profiles provided better predictions than did the fully-developed inlet profiles. Overall, the predicted reattachment lengths were longer than the measurements, especially at high Reynolds numbers. The predicted velocity profiles agreed well with the experimental results using experimental inlet profiles.

Kumar and Yajnik [26] employed a different approach in solving the flows over a backward-facing step. Employing eigenfunction expansions of the Poiseuille flow development, the problem is reduced to nonlinear first-order ordinary differential equations which have a tendency to decouple rapidly in the downstream direction.

Leschziner [27] examined the performance of three steady-state finite-difference formulations, namely (1) the hybrid central/upwind differencing scheme, (2) the hybrid central/skew-upwind differencing scheme, and (3) the quadratic, upstream-weighted differencing scheme. The skewed plane jet, backward-facing step and sudden pipe expansion were used as test problems. It was found that the second and the third
schemes showed better solution accuracy in the cases tested. The effects of artificial diffusion induced in the hybrid central/upwind differencing scheme was noted to be very small in laminar, recirculating flows.

Atias et al. [28] also evaluated the performance of several numerical schemes. They concluded that the central differencing is the most efficient scheme (based on accuracy, stability, and economy) whenever it is stable; while the second-order upwind scheme appears to have the potential of yielding sufficient accuracy as well as stability. It was suggested by the author to apply the ADI technique in the second-order upwind scheme to improve its stability.

Recently, Armaly et al. [5] used the TEACH computer code developed by Gosman et al. [22] to predict their experimentally measured flows in the laminar regime. The predictions showed good agreement with the measurements up to a Reynolds number of 400 (based on inlet hydraulic diameter and the average inlet velocity). The same flows were also calculated by Ghia et al. [29] using the unsteady Navier-Stokes equations with stream function and vorticity as dependent variables. Their solutions were in good agreement with the experimental data for Reynolds numbers below 400 and began to deviate from the measurements at this point. However, their solutions showed the existence of secondary separated regions as was observed in the experiment. Ghia et al. [29] also predicted the flows reported by Denham and Patrick [4] and showed longer reattachment lengths as compared with the experimental data.
Kwon and Fletcher [12] also predicted the flows in Denham and Patrick [4] with boundary-layer equations. The solutions showed unexpectedly good agreement with the experimental results and suggested a very efficient method for predicting this type of recirculating flow.

The boundary-layer equations were also used by Hall [8] in predicting one of the measured flows in Armaly et al. [6]. Good agreement with the experimental data was obtained. However, attempts to predict additional separated regions were unsuccessful as was the case in the present study. Hall [8] also predicted the flow reported by Aung [7] using a viscous-inviscid interaction method. The predicted temperature profiles and the Nusselt numbers were generally in good agreement with the measured data. It was noted that the errors introduced by use of the FLARE approximation and the neglect of axial conduction terms in the boundary-layer type energy equation were small in the separated flows considered.

Pollard [15] calculated flows in axisymmetric expansions using a solution procedure based upon the SIMPLE algorithm of Patankar and Spalding [30]. It was concluded that the reattachment length varied linearly with the Reynolds number based on step height and nonlinearly with the expansion ratio.
II. ANALYSIS

A. Geometry and Coordinate System

The flows considered are assumed to be laminar and confined in a two-dimensional channel with either a symmetric or asymmetric expansion over a rearward-facing step as shown in Fig. 2.1. A Cartesian coordinate system is chosen for the present analysis and is shown along with the flow geometry. Generally, the origin of the coordinate system is placed at the step and the numerical calculation starts from upstream of the step.

B. Governing Equations

The Navier-Stokes equations represent the most complete model for viscous flows. For the case of a steady, two-dimensional, constant property, laminar flow, the Navier-Stokes equations can be written in primitive variables as [31]:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  \hspace{1cm} (2.3)

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \]  \hspace{1cm} (2.1)

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \]  \hspace{1cm} (2.2)
Figure 2.1. Schematic of computation domain for internal flows
(a) asymmetric expansion (b) symmetric expansion
Mathematically, the above equations represent a set of elliptic partial differential equations. In terms of actual flows, this means that the flow at any location is influenced by the rest of the flow field. This effect is termed as an "elliptic effect" and is transmitted by three mechanisms:

(a) convection of fluid
(b) viscous diffusion
(c) pressure forces.

However, for flows with a predominant flow direction and without recirculation, it is generally possible to neglect the viscous diffusion in that direction [1,32]. This leads to a set of simplified equations called partially-parabolized Navier-Stokes (PPNS) equations:

**x-momentum**

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} \]  

(2.5)

**y-momentum**

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} \]  

(2.6)

**continuity**

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

(2.7)
energy (fully-elliptic)

\[
\frac{\partial T}{\partial x} u + \frac{\partial T}{\partial y} v = \frac{k}{\rho C_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{2.8}
\]

Note that the boundary-layer form of the energy equation (Eq.(2.9)) was also used in this study to evaluate the importance of \( \frac{\partial^2 T}{\partial x^2} \).

energy (boundary-layer)

\[
\frac{\partial T}{\partial x} u + \frac{\partial T}{\partial y} v = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} \tag{2.9}
\]

Recently, the PPNS equations, with the help of the type-dependent differencing for the momentum convective terms, have been used to predict flows with confined separated regions by Ghia et al. [21] and Madavan [2].

The elliptic influence transmitted by pressure is determined by solving a Poisson equation which can be derived from the momentum equations [1]. Rearranging Eqs.(2.5) and (2.6), pressure gradients can be written in the form:

\[
\frac{\partial p}{\partial x} = -\rho \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} - \mu \frac{\partial^2 u}{\partial y^2} \right) = G_1 \tag{2.10}
\]

\[
\frac{\partial p}{\partial y} = -\rho \left( \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} - \mu \frac{\partial^2 v}{\partial x^2} \right) = G_2 \tag{2.11}
\]

which leads to

\[
\nu^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{\partial G_1}{\partial x} + \frac{\partial G_2}{\partial y} \tag{2.12}
\]

This Poisson equation, along with the Neumann boundary conditions, must be satisfied by the pressure.
C. Boundary Conditions

1. Asymmetric expansion

Along both solid boundaries, as shown in Fig. 2.1(a), the velocities and temperature are specified as:

\[ u = v = 0 \]  
\[ T = T_w \]  

(2.13)  
(2.14)

At the upstream boundary, the velocity and temperature profiles need be specified. As for the downstream boundary, an outflow boundary condition,

\[ \frac{\partial^2 T}{\partial x^2} = 0 \]  

(2.15)

is used for the elliptic form of the energy equation and no downstream boundary condition is necessary for the momentum equations since streamwise diffusion terms are dropped.

The boundary conditions for the pressure Poisson equation will be discussed in the next chapter.

2. Symmetric expansion

In this geometry, the upper boundary is placed along the line of symmetry, as shown in Fig. 2.1(b), where boundary conditions are specified as:

\[ v = 0 \]  
\[ \frac{\partial u}{\partial y} = 0 \]  
\[ \frac{\partial T}{\partial y} = 0 \]  

\[ \frac{\partial^2 T}{\partial y^2} = 0 \]  

(2.16)  
(2.17)  
(2.18)
Along the lower solid boundary, the boundary conditions are the same as those given above for the asymmetric case.

D. Nondimensionalization

For convenience, all the variables except temperature are nondimensionalized as follows:

- \( U \): dimensionless streamwise velocity = \( u/u_{\text{ref}} \)
- \( V \): dimensionless transverse velocity = \( v/u_{\text{ref}} \)
- \( X \): dimensionless \( x \) distance = \( \rho u_{\text{ref}} x/u \)
- \( Y \): dimensionless \( y \) distance = \( \rho u_{\text{ref}} y/u \)
- \( P \): dimensionless pressure = \( p/\rho u_{\text{ref}}^2 \)

Thus, the partially-parabolized Navier-Stokes equations (2.5) – (2.9) can also be written as:

\[
\begin{align*}
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} &= -\frac{\partial P}{\partial x} + \frac{\partial^2 U}{\partial y^2} \\
\frac{\partial V}{\partial x} + \frac{\partial U}{\partial y} &= -\frac{\partial P}{\partial y} + \frac{\partial^2 V}{\partial y^2} \\
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} &= 0 \\
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} &= 0 \\
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} &= \frac{1}{Pr} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\end{align*}
\]

where Prandtl number \( Pr = \frac{\mu c}{\kappa} \).

Rearranging Eqs. (2.19) and (2.20), pressure gradients can be written as follows:

\[
\frac{\partial P}{\partial x} = - \left( \frac{\partial U}{\partial x} + \frac{\partial^2 U}{\partial y^2} \right) = F1
\]
\[ \frac{\partial P}{\partial Y} = - \left( \frac{\partial V}{\partial X} + \frac{\partial V}{\partial Y} - \frac{\partial^2 V}{\partial Y^2} \right) = F2 \] (2.25)

which leads to the pressure Poisson equation:

\[ \nabla^2 P = \frac{\partial^2 P}{\partial X^2} + \frac{\partial^2 P}{\partial Y^2} = \frac{\partial F1}{\partial X} + \frac{\partial F2}{\partial Y} \] (2.26)

The nondimensional boundary conditions are

(a) \( U = U_0 \)  prescribed initial conditions

\[ V = V_0 \]

(b) \( V = 0 \)  symmetry boundary conditions

\[ \frac{\partial U}{\partial Y} = 0 \]

\[ \frac{\partial T}{\partial Y} = 0 \]

(c) \( U = V = 0 \)  no-slip boundary conditions at the wall

\[ T = T_w \]

Again, the boundary conditions for the pressure Poisson equation will be discussed in the next chapter.
III. METHOD OF SOLUTION

A. The Finite-Difference Grid

A staggered grid is used, as suggested by Patankar [33], to prevent the development of a wavy pressure and velocity field. As shown in Fig. 3.1, the variables enclosed by the boomerang at three distinct locations are designated with the same set of subscripts. This is possible as none of the variables are evaluated at more than one of these three locations. Also shown in the same figure are grid spacings which are non-uniform in both the x and y directions. Figure 3.2 shows the control volumes for different governing equations.

In the present study, the computation domain starts from upstream of the step as shown in Fig. 3.3. Also shown in the same figure are the locations of the boundaries. Since the horizontal boundaries are placed along the V-velocity locations, the no-slip boundary conditions become

\[
U(I, J) = -U(I, J-1) \quad , J=2 \text{ or } JSTEP+1
\]  
\[
V(I, J) = 0. \quad , J=2 \text{ or } JSTEP+1
\]  
\[
\frac{T(I, J) + T(I, J-1)}{2} = T_w \quad , J=2 \text{ or } JSTEP+1
\]

At the outer boundary, either no-slip or symmetric boundary conditions are prescribed depending on the flow geometry. For asymmetric expansion internal flows, no-slip boundary conditions are imposed.

\[
U(I, NJ) = -U(I, NJ+1)
\]  
\[
V(I, NJ+1) = 0.
\]  
\[
\frac{T(I, NJ) + T(I, NJ+1)}{2} = T_w
\]
Figure 3.1. Staggered grid and variable locations
Figure 3.2. Control volumes for (a) x-momentum (b) y-momentum and (c) continuity, energy, and pressure Poisson equations.
Figure 3.3. Computation domain and locations of boundaries
For symmetric expansion internal flows, the boundary conditions become

\[ U(I,NJ) = U(I,NJ+1) \]  \hspace{1cm} (3.7)
\[ V(I,NJ+1) = 0. \]  \hspace{1cm} (3.8)
\[ T(I,NJ) = T(I,NJ+1) \]  \hspace{1cm} (3.9)

Notice that the boundary conditions described above for \( U \) and \( T \) use fictitious points outside the boundaries to satisfy the constraints.

At the initial boundary, velocity and temperature profiles are required and need to be prescribed at slightly different locations due to the nature of the staggered grid.

B. Finite-Difference Formulation of Momentum Equations

1. Coefficients in convective terms

In order to avoid having to solve a system of nonlinear algebraic equations, the convective terms of the momentum equations are linearized. This is achieved by lagging or extrapolating the coefficients for the convective terms.

In regions where values of velocity components are stored (generally, these include the whole recirculation region), the coefficients are evaluated at the previous iteration level.

\[ U_{x}^{n+1}_{i+1,j} = U_{x}^{n}_{i+1,j} \]  \hspace{1cm} (3.10)
\[ U_{y}^{n+1}_{i+1,j} = \left( \frac{U_{x}^{n}_{i,j} + U_{x}^{n}_{i,j-1} \Delta X_{x}}{2} \right) + \left( \frac{U_{x}^{n}_{i+1,j} + U_{x}^{n}_{i+1,j-1} \Delta X_{x}}{2} \right) \left( \frac{1}{\Delta X_{x}} \right) \]  \hspace{1cm} (3.11)
\[ v_{x_{i+1,j}}^{n+1} = \frac{v_{i+1,j}^{n} + v_{i+2,j}^{n}}{2} \]  
(3.12)

\[ v_{y_{i+1,j}}^{n+1} = v_{i+1,j}^{n} \]  
(3.13)

where the subscripts \( x \) and \( y \) designate quantities in the \( x \)-momentum and the \( y \)-momentum equations respectively.

For other regions, the coefficients are extrapolated from the adjacent upstream stations.

\[ u_{x_{i+1,j}}^{n+1} = u_{i+1,j}^{n} + (u_{i,j}^{n+1} - u_{i-1,j}^{n+1}) \cdot \frac{\Delta X_{u}}{\Delta X_{u}} \]  
(3.14)

\[ u_{y_{i+1,j}}^{n+1} = \left[ \frac{u_{i+1,j}^{n+1} + u_{i,j}^{n+1}}{2} \right] + \frac{u_{x_{i+1,j}}^{n+1} + u_{x_{i+1,j-1}}^{n+1} \cdot \Delta X^{+}}{2} \]  
(3.15)

\[ v_{x_{i+1,j}}^{n+1} = v_{i,j}^{n} + (v_{i,j}^{n+1} - v_{i-1,j}^{n+1}) \cdot \frac{\Delta X^{+} + (\Delta X^{+}/2)}{\Delta X_{-}} \]  
(3.16)

\[ v_{y_{i+1,j}}^{n+1} = v_{i,j}^{n+1} + (v_{i,j}^{n+1} - v_{i-1,j}^{n+1}) \cdot \frac{\Delta X_{-}}{\Delta X_{-}} \]  
(3.17)

However, for regions with only one upstream station such as the second station and the first station downstream of the step, extrapolation is impossible since values of velocities for only one station are available for these regions. Thus, the coefficients are simply evaluated from the previous station.

\[ u_{x_{i+1,j}}^{n+1} = u_{i,j}^{n+1} \]  
(3.18)
\[
U_{i+1,j}^{n+1} = \frac{U_{i,j}^{n+1} + U_{i,j-1}^{n+1}}{2} \tag{3.19}
\]
\[
v_{i+1,j}^{n+1} = v_{i,j}^{n+1} \tag{3.20}
\]
\[
v_{i+1,j}^{n+1} = v_{i,j}^{n+1} \tag{3.21}
\]

Notice that the second or the third method of evaluating the coefficients is always used in the first iteration even for the separated region since no values of velocities are available at a previous iteration level.

2. **Convective terms in x-direction** \( \frac{\partial u}{\partial x} , \frac{\partial v}{\partial x} \)

A three-point, second-order accurate, upwind scheme is used for the entire flow field considered except in regions with only one station in the upwind direction, such as the second station and some points at the station immediately after the step. Note that the direction of the "wind" is determined by the sign of the coefficient \( U \).

For forward-going flow with three-point differencing, the convective terms become,

\[
\left( U \frac{\partial u}{\partial x} \right)_{i+1,j} = U x_{i+1,j} \left[ \frac{U_{i+1,j}^{n+1} - U_{i,j}^{n+1}}{\Delta x} + \frac{U_{i+1,j}^{n+1} - U_{i-1,j}^{n+1}}{\Delta x} \right] \tag{3.22}
\]

for the x-momentum equation and
\[
(U_{\partial V/\partial x})_{i+1,j}^{n+1} = U_{y}^{n+1} i+1,j \left( \frac{v_{i+1,j}^{n+1} - v_{i,j}^{n+1}}{\Delta X} \right)
\]

\[
 - \frac{v_{i,j}^{n+1} - v_{i-1,j}^{n+1}}{\Delta X^-} + \frac{v_{i+1,j}^{n+1} - v_{i-1,j}^{n+1}}{\Delta X^+ + \Delta X^-} \right) \]  

(3.23)

for the \( y \)-momentum equation.

For regions with only one station available in the upwind direction, a first-order accurate, upwind differencing scheme is used.

For the \( x \)-momentum equation,

\[
(U_{\partial U/\partial x})_{i+1,j}^{n+1} = U_{x}^{n+1} i+1,j \cdot \left( \frac{u_{i+1,j}^{n+1} - u_{i,j}^{n+1}}{\Delta X u} \right) \]  

(3.24)

and for the \( y \)-momentum equation,

\[
(U_{\partial V/\partial x})_{i+1,j}^{n+1} = U_{y}^{n+1} i+1,j \cdot \left( \frac{v_{i+1,j}^{n+1} - v_{i,j}^{n+1}}{\Delta X} \right) \]  

(3.25)

For reversed flow, a three-point differencing scheme is still used, but the differencing direction is switched to the negative \( x \)-direction according to the sign of \( U \), which is negative in this region. Thus, for the \( x \)-momentum equation,

\[
(U_{\partial U/\partial x})_{i+1,j}^{n+1} = U_{x}^{n+1} i+1,j \left( \frac{u_{i+2,j}^{n} - u_{i+1,j}^{n+1}}{\Delta X u^+} \right)
\]

\[
 + \frac{u_{i+3,j}^{n} - u_{i+2,j}^{n}}{\Delta X u^+} + \frac{u_{i+3,j}^{n} - u_{i+1,j}^{n+1}}{\Delta X u^+ + \Delta X u^+} \right) \]  

(3.26)

and for the \( y \)-momentum equation,

\[
(U_{\partial V/\partial x})_{i+1,j}^{n+1} = U_{y}^{n+1} i+1,j \left( \frac{v_{i+2,j}^{n} - v_{i+1,j}^{n+1}}{\Delta X^+} \right)
\]

\[
 - \frac{v_{i+3,j}^{n} - v_{i+2,j}^{n}}{\Delta X^+} + \frac{v_{i+3,j}^{n} - v_{i+1,j}^{n+1}}{\Delta X^+ + \Delta X^+} \]  

(3.27)
It should be noted that $V_{i+2}$ and $V_{i+3,j}$ are evaluated at the previous iteration level $n$ and the logic used in deciding the differencing direction is based on the sign of $U$ [2]. However, for cases where a reversed flow region is encountered and no values are available from a previous iteration level, the FLARE approximation [9] is used. These cases occur in the first iteration for the entire reversed flow region and in later iterations for those points in the reversed flow region for which values of velocities were not stored in a previous iteration.

3. Convective terms in $y$-direction ($\frac{\partial U}{\partial Y}, \frac{\partial V}{\partial Y}$)

A hybrid differencing scheme described in [32] is used to avoid the numerical instability introduced by pure central differencing when the mesh Reynolds number $|R_m| = |V| \times \Delta Y$ exceeds a certain critical value. Although a pure upwind scheme also provides a remedy for the numerical instability mentioned above, the hybrid scheme is used to avoid a sudden change from central to upwind differencing. The present hybrid scheme uses a weighted average of the central and upwind differencing schemes for large mesh Reynolds numbers and degenerates to pure central differencing for small mesh Reynolds numbers.

For the $x$-momentum equation, the complete convective term in $y$-direction is,

$$
(V_{\frac{\partial U}{\partial Y}})_{i+1,j+1}^{n+1} = \left[ Vx_{i+1,j+1}^{n+1} \left( \frac{U_{i+1,j+1}^{n+1} - U_{i+1,j}^{n+1}}{\Delta Y^+} \right) \frac{\Delta Y^-}{\Delta Y^+ + \Delta Y^-} + Vx_{i+1,j}^{n+1} \left( \frac{U_{i+1,j}^{n+1} - U_{i+1,j-1}^{n+1}}{\Delta Y^-} \right) \frac{\Delta Y^+}{\Delta Y^+ + \Delta Y^-} \right] W
$$
\[ u_{i+1,j}^{n+1} - u_{i+1,j}^{n+1} = \frac{v_{i+1,j}^{n+1} - v_{i+1,j}^{n+1}}{\Delta Y^-} (1-W) \cdot A \]
\[ + v_{i+1,j}^{n+1} \cdot \frac{u_{i+1,j+1}^{n+1} - u_{i+1,j}^{n+1}}{\Delta Y^+} (1-W) \cdot B \]  

(3.28)

where \( W, A, \) and \( B \) are determined as follows:

\[ R_m^+ = v_{i+1,j}^{n+1} \cdot \Delta Y^- \]  

(3.29)

\[ R_m^- = v_{i+1,j}^{n+1} \cdot \Delta Y^+ \]  

(3.30)

\[ R_c^+ = \text{Critical mesh Reynolds number} = 1.9 \]

If \( R_m^+ > R_c^+ \), then \( A=1, B=0, \) and \( W = R_c^+ / R_m^+ \)

If \( R_m^- < -R_c^- \), then \( A=0, B=1, \) and \( W = -R_c^- / R_m^- \)

If \( R_m^+ \leq R_c^- \) and \( R_m^- \geq -R_c^- \), then \( A=0, B=0, \) and \( W=1. \)

and for the \( y \)-momentum equation,

\[ (V_{i+1,j}^{n+1})_{i+1,j} = v_{i+1,j}^{n+1} \left[ \frac{v_{i+1,j+1}^{n+1} - v_{i+1,j}^{n+1}}{\Delta Y^+} \cdot \frac{\Delta Y v^+ + \Delta Y v^-}{\Delta Y v^-} + \frac{\Delta Y v^+}{\Delta Y v^-} \right] \cdot \frac{1}{W} \]

\[ + v_{i+1,j}^{n+1} \cdot \frac{v_{i+1,j+1}^{n+1} - v_{i+1,j}^{n+1}}{\Delta Y^-} (1-W) \cdot A \]

\[ + v_{i+1,j}^{n+1} \cdot \frac{v_{i+1,j+1}^{n+1} - v_{i+1,j}^{n+1}}{\Delta Y^-} (1-W) \cdot B \]  

(3.31)

where \( W, A, \) and \( B \) are determined as follows:

\[ R_m^+ = v_{i+1,j}^{n+1} \cdot \Delta Y v^- \]  

(3.32)

\[ R_m^- = v_{i+1,j}^{n+1} \cdot \Delta Y v^+ \]  

(3.33)
Critical mesh Reynolds number = 1.9

If $R_m^+ > R_c^-$, then $A=1$, $B=0$, and $W = R_c^- / R_m^-$. 

If $R_m^- < R_c^-$, then $A=0$, $B=1$, and $W = -R_c^- / R_m^-$. 

If $R_m^+ \leq R_c^-$ and $R_m^- \geq R_c^-$, then $A=0$, $B=0$, and $W=1$.

4. Pressure gradient and diffusion terms

Central differencing is used for these terms. Note that here "central" means central about the points, $(i+1,j)$, at which velocities are evaluated (see Fig. 3.2(a),(b)). In the $x$-momentum equation,

$$\frac{(\partial P_{i+1,j+1})_{n+1}}{\partial x} + \Delta x^+ (\frac{u_{i+1,j+1}^{n+1} - u_{i+1,j}^{n+1}}{\Delta y^+}) = \frac{p_{i+1,j}^n - p_{i+1,j+1}^n}{\Delta y^+}$$

(3.34)

$$\frac{(\partial^2 U_{i+1,j+1})_{n+1}}{\partial y^2} = \left[ \frac{u_{i+1,j+1}^{n+1} - u_{i+1,j}^{n+1}}{\Delta y^+} - \frac{u_{i+1,j}^{n+1} - u_{i+1,j-1}^{n+1}}{\Delta y^-} \right] \frac{2}{\Delta y^+ + \Delta y^-}$$

(3.35)

and in the $y$-momentum equation,

$$\frac{(\partial P_{i+1,j})_{n+1}}{\partial y} = \frac{p_{i+1,j}^n - p_{i+1,j-1}^n}{\Delta y^-}$$

(3.36)

$$\frac{(\partial^2 V_{i+1,j+1})_{n+1}}{\partial y^2} = \left[ \frac{v_{i+1,j+1}^{n+1} - v_{i+1,j}^{n+1}}{\Delta y^+} - \frac{v_{i+1,j}^{n+1} - v_{i+1,j-1}^{n+1}}{\Delta y^-} \right] \frac{2}{\Delta y^+ + \Delta y^-}$$

(3.37)

For a point immediately inside a solid boundary, the finite-difference representation of $\partial^2 U / \partial y^2$, Eq.(3.38), which uses a point outside the boundary, can provide a poor representation for the
diffusion term. Thus, a point on the solid boundary is used instead and the finite-difference representation of $\frac{\partial^2 U}{\partial Y^2}$ becomes

$$
\left( \frac{\partial^2 U}{\partial Y^2} \right)_{i+1,j}^{n+1} = \left[ \frac{U_{i+1,j+1}^{n+1} - U_{i+1,j}^{n+1}}{\Delta Y^+} - \frac{U_{i+1,j}^{n+1} - 0}{\Delta Y^-/2} \right] \frac{2}{\Delta Y^+ + (\Delta Y^-/2)}
$$

(3.38)

for points immediately above the lower wall.

and

$$
\left( \frac{\partial^2 U}{\partial Y^2} \right)_{i+1,j}^{n+1} = \left[ \frac{0 - U_{i+1,j}^{n+1}}{\Delta Y^+/2} - \frac{U_{i+1,j}^{n+1} - U_{i+1,j-1}^{n+1}}{\Delta Y^-} \right] \frac{2}{(\Delta Y^+/2) + \Delta Y^-}
$$

(3.39)

for points immediately beneath the upper wall.

5. Complete forms of momentum equations

The finite-difference representations of the different terms in each of the momentum equations, when written for $(I+1,J)$ grid point, can be combined and rearranged as follows:

**x-momentum:**

$$
B^u_j U_{i+1,j-1}^{n+1} + D^u_j U_{i+1,j}^{n+1} + A^u_j U_{i+1,j+1}^{n+1} = C^u_j
$$

(3.40)

where

$$
B_j^u = - \frac{Vx_{i+1,j}^{n+1} \cdot \Delta Y^+ \cdot W}{\Delta Y^- (\Delta Y^+ + \Delta Y^-)} - \frac{Vx_{i+1,j}^{n+1} \cdot (1-W) \cdot A_j}{\Delta Y^-} - \frac{2}{\Delta Y^- (\Delta Y^+ + \Delta Y^-)}
$$

(3.41)

$$
A_j^u = \frac{Vx_{i+1,j}^{n+1} \cdot \Delta Y^- \cdot W}{\Delta Y^+ (\Delta Y^+ + \Delta Y^-)} + \frac{Vx_{i+1,j}^{n+1} \cdot (1-W) \cdot B_j}{\Delta Y^+} - \frac{2}{\Delta Y^+ (\Delta Y^+ + \Delta Y^-)}
$$

(3.42)
\[ D_{j}^{u} = U_{x}^{n+1} i+1,j \frac{\Delta X_{u}^{-} + 2\Delta X_{u}}{\Delta X_{u}(\Delta X_{u}^{-} + \Delta X_{u})} + E_{j}^{u} \]  
(3.43)

for forward-going flow

\[ D_{j}^{u} = - U_{x}^{n+1} i+1,j \frac{\Delta X_{u}^{++} + 2\Delta X_{u}^{+}}{\Delta X_{u}^{+}(\Delta X_{u}^{++} + \Delta X_{u}^{+})} + E_{j}^{u} \]  
(3.44)

for reversed flow

\[ E_{j}^{u} = \left[ \frac{V_{x}^{n+1} i+1,j \cdot \Delta Y^{+}}{\Delta Y^{+}(\Delta Y^{+} + \Delta Y^{-})} - \frac{V_{x}^{n+1} i+1,j+1 \cdot \Delta Y^{-}}{\Delta Y^{+}(\Delta Y^{+} + \Delta Y^{-})} \right] W \]

\[ + \frac{V_{x}^{n+1} i+1,j \cdot (1-W) \cdot A}{\Delta Y^{-}} - \frac{V_{x}^{n+1} i+1,j+1 \cdot (1-W) \cdot B}{\Delta Y^{+}} + \frac{2}{\Delta Y^{+} \cdot \Delta Y^{-}} \]  
(3.45)

\[ C_{j}^{u} = - \frac{P_{i+2,j}^{n} - P_{i+1,j}^{n}}{\Delta X^{+}} + U_{x}^{n+1} i+1,j \left[ \frac{\Delta X_{u} + \Delta X_{u}^{-}}{\Delta X_{u} \cdot \Delta X_{u}^{-}} \cdot U_{i-1,j}^{n+1} \right] \]  
(3.46)

for forward-going flow

\[ C_{j}^{u} = - \frac{P_{i+2,j}^{n} - P_{i+1,j}^{n}}{\Delta X^{+}} + U_{x}^{n+1} i+1,j \left[ - \frac{\Delta X_{u}^{+} + \Delta X_{u}^{++}}{\Delta X_{u}^{+} \cdot \Delta X_{u}^{++}} \cdot U_{i+2,j}^{n} \right. \]

\[ + \frac{\Delta X_{u}^{+}}{\Delta X_{u}^{++}(\Delta X_{u}^{+} + \Delta X_{u}^{++})} \cdot U_{i+3,j}^{n} \]  
(3.47)

for reversed flow

\[ \gamma\text{-momentum :} \]

\[ B_{j}^{v} V_{i+1,j-1}^{n+1} + D_{j}^{v} V_{i+1,j}^{n+1} + A_{j}^{v} V_{i+1,j+1}^{n+1} = C_{j}^{v} \]  
(3.48)

where

\[ B_{j}^{v} = - \frac{V_{y}^{n+1} i+1,j \cdot \Delta Y^{+} \cdot W}{\Delta Y^{+} - (\Delta Y^{+} + \Delta Y^{-})} - \frac{V_{y}^{n+1} i+1,j \cdot (1-W) \cdot A}{\Delta Y^{-} - (\Delta Y^{+} + \Delta Y^{-})} - \frac{2}{\Delta Y^{+} - (\Delta Y^{+} + \Delta Y^{-})} \]  
(3.49)
\[ V_{ij}^{n+1} = \frac{V_{y}^{n+1}/i+1,j \cdot \Delta Y_{v} - W}{\Delta Y_{v}^{+}(\Delta Y_{v}^{+} + \Delta Y_{v}^{-})} + \frac{V_{y}^{n+1}/i+1,j \cdot (1-W) \cdot B}{\Delta Y_{v}^{+}} - \frac{2}{\Delta Y_{v}^{+}(\Delta Y_{v}^{+} + \Delta Y_{v}^{-})} \] (3.50)

\[ D_{ij}^{v} = Uy_{i+1,j} \cdot \Delta X_{-} + 2\Delta X_{-} \overline{\Delta X_{+} \Delta X_{-}} + E_{ij}^{v} \] (3.51)

for forward-going flow

\[ D_{ij}^{v} = - Uy_{i+1,j} \cdot \Delta X_{-}^{+} + 2\Delta X_{+} \overline{\Delta X_{+} \Delta X_{-}} + E_{ij}^{v} \] (3.52)

for reversed flow

\[ E_{ij}^{v} = \frac{V_{y}^{n+1}/i+1,j \cdot \Delta Y_{v}^{+} - \Delta Y_{v}^{-} \cdot W + (1-W) \cdot A \cdot \Delta Y_{v}^{+} - (1-W) \cdot B}{\Delta Y_{v}^{-}} + \frac{2}{\Delta Y_{v}^{+}} \] (3.53)

\[ C_{ij}^{v} = - \frac{p_{i+1,j}^{n} - p_{i+1,j-1}^{n}}{\Delta Y_{-}} + Uy_{i+1,j} \left[ \frac{\Delta X_{-} + \Delta X_{-} \cdot V_{i+1,j}^{n+1}}{\Delta X_{-} \Delta X_{-} \cdot V_{i-1,j}^{n+1}} \right] \] (3.54)

for forward-going flow

\[ C_{ij}^{v} = - \frac{p_{i+1,j}^{n} - p_{i+1,j-1}^{n}}{\Delta Y_{-}} + Uy_{i+1,j} \left[ - \frac{\Delta X_{+}^{+} + \Delta X_{+}^{+} \cdot V_{i+2,j}^{n}}{\Delta X_{+}^{+} \Delta X_{+}^{+} \cdot V_{i+2,j}^{n}} \right] \] (3.55)

for reversed flow

The coefficients, B, A, D, and C for specific regions, such as the second station and points just beside a solid boundary, can be similarly constructed using finite-difference expressions discussed in the foregoing sections.
The equations (3.40) and (3.48), when written for each grid point at a particular station, say I+1, result in a tridiagonal coefficient matrix for each equation. The Thomas algorithm is an efficient solver for this type of matrix provided proper boundary conditions are given.

C. Finite-Difference Formulation of the Continuity Equation

If the correct pressure is used when obtaining the solution to the momentum equations, continuity would be satisfied at each point. Since the calculation procedure starts with only approximate values of the pressure, the corresponding solutions of the momentum equations will not satisfy the continuity equation. The present solution procedure adjusts the pressure field after each marching solution to the momentum equations by solving a Poisson equation for the new pressure using a source term computed from velocities which have been corrected to satisfy the continuity equation.

A detailed description of the velocity correction procedure used can be found in Chilukuri [1]; however, for completeness, a brief description is given below.

A central differencing is used for the continuity equation on a typical difference molecule shown in Fig. 3.4. For exact velocities $U_e$ and $V_e$, the continuity equation becomes,

\[
\frac{U_{ei+1,j}^{n+1} - U_{ei,j}^{n+1}}{\Delta X_u} + \frac{V_{ei+1,j+1}^{n+1} - V_{ei+1,j}^{n+1}}{\Delta Y_v} = 0
\]

(3.56)

defining

\[
U_{ei+1,j}^{n+1} = U_{ei+1,j}^{n+1} + U_{ei+1,j}^{n+1}
\]

(3.57)
Figure 3.4. The difference molecule used for the continuity equation.
\[ V_{c,i+1,j}^{n+1} = V_{i+1,j}^{n+1} + V_{c,i+1,j}^{n+1} \]  
(3.58)

where \( U \) and \( V \) are solutions of the momentum equations, and \( U_c \) and \( V_c \), velocity corrections.

From the above equations (3.56), (3.57), and (3.58), the continuity equation can be rewritten in the form:

\[
\begin{align*}
\frac{U_{c,i+1,j}^{n+1} - U_{c,i,j}^{n+1}}{\Delta Xu} &+ \frac{V_{c,i+1,j+1}^{n+1} - V_{c,i+1,j}^{n+1}}{\Delta Yv^+} \\
&= - \left[ \frac{U_{i+1,j}^{n+1} - U_{i,j}^{n+1}}{\Delta Xu} + \frac{V_{i+1,j+1}^{n+1} - V_{i+1,j}^{n+1}}{\Delta Yv^+} \right] \\
&= S_{\phi,i+1,j}^{n+1} \tag{3.59}
\end{align*}
\]

where \( S_{\phi,i+1,j}^{n+1} \) is known since \( U \) and \( V \) have already been solved from the momentum equations. The equation above has four unknowns; thus further assumptions are needed to solve this equation.

(a) \( U_{c,i,j}^{n+1} = 0 \) It's obvious that the continuity equation has been satisfied at the previous station, therefore no correction is needed.

(b) Irrotational velocity corrections This assumption permits the use of a velocity correction potential to relate \( U_c \) and \( V_c \), such that

\[
\begin{align*}
U_{c,i+1,j}^{n+1} &= (\phi_{x,i+1,j}^{n+1} = \frac{\phi_{i+2,j}^{n+1} - \phi_{i+1,j}^{n+1}}{\Delta x^+} \tag{3.60} \\
V_{c,i+1,j}^{n+1} &= (\phi_{y,i+1,j}^{n+1} = \frac{\phi_{i+1,j}^{n+1} - \phi_{i+1,j-1}^{n+1}}{\Delta y^-} \tag{3.61}
\end{align*}
\]
(c) $\phi_{i+2,j}^{n+1} = 0$  This implies zero $Vc_{i+2,j}^{n+1}$ which must be the case when convergence is achieved.

By these three assumptions, Eq. (3.59) can be written in the form:

$$B_j \phi_{i+1,j-1}^{n+1} + D_j \phi_{i+1,j}^{n+1} + A_j \phi_{i+2,j}^{n+1} = C_j$$

(3.62)

and with the no-slip boundary conditions on solid walls and/or the symmetric boundary condition at the centerline,

$$Vc_{i+1,j}^{n+1} = \frac{\phi_{i+1,j}^{n+1} - \phi_{i+1,j-1}^{n+1}}{\Delta Y} = 0, \text{ j}=2, \text{ JSTEP+1}, \text{ or } NJ+1$$

(3.63)

the Thomas algorithm is used to solve this system of equations.

D. Finite-Difference Formulation of the Pressure Poisson Equation

Central differencing is used for the Poisson equation.

$$\nabla^2 p = \left[ \frac{p_{i+2,j}^{n} - p_{i+1,j}^{n+1}}{\Delta X^+} - \frac{p_{i+1,j}^{n+1} - p_{i,j}^{n+1}}{\Delta X^-} \right] \frac{1}{\Delta X u}
+ \left[ \frac{p_{i+1,j+1}^{n+1} - p_{i+1,j}^{n+1}}{\Delta Y^+} - \frac{p_{i+1,j}^{n+1} - p_{i+1,j-1}^{n+1}}{\Delta Y^-} \right] \frac{1}{\Delta Y v^+}
= F1_{i+1,j}^{n+1} + \frac{F2_{i+1,j+1}^{n+1} - F2_{i+1,j}^{n+1}}{\Delta Y v^+}
$$

(3.64)

where $F1$ and $F2$ are the pressure gradient terms calculated from the finite-difference forms of the momentum equations (2.24) and (2.25) using corrected velocities. The Neumann boundary conditions, described
below, are used on all boundaries to eliminate the need to impose a pressure gradient on boundaries [34].

\[
F^1_{i,j} = \frac{p_{i+1,j}^{n+1} - p_{i,j}^{n+1}}{\Delta x}, \quad i=1, \text{MCSTEP} - 1, \text{or KPNS} \quad (3.65)
\]

\[
F^2_{i+1,j} = \frac{p_{i+1,j+1}^{n+1} - p_{i+1,j}^{n+1}}{\Delta y v^+}, \quad i=2, \text{JSTEP} + 1, \text{or NJ} + 1 \quad (3.66)
\]

where MCSTEP, KPNS, JSTEP, and NJ are explained in Fig. 3.3.

The resulting set of equations for pressure is solved by the method of successive over-relaxation (SOR) by points [34]. An over-relaxation factor of 1.5 was used for all the calculations.

It was found necessary to under-relax the pressure source term, \( S_p \), otherwise the solution tended to stabilize with large mass sources. Generally, a small under-relaxation factor was needed at the beginning of the calculation procedure and gradually increased as the solution approaches convergence. No attempt was made to optimize the relaxation factors.

E. Finite-Difference Formulation of the Energy Equation

In the present study, the energy equation is solved after the momentum equations; the coefficients of convective terms are then taken from best available values. The finite-difference expressions for the energy equation are derived following the same procedure employed for the momentum equations - three point upwind scheme for \( \frac{\partial T}{\partial x} \), hybrid scheme for \( \frac{\partial T}{\partial y} \), and central differencing for \( \frac{\partial^2 T}{\partial x^2} \) and \( \frac{\partial^2 T}{\partial y^2} \).
Following are the finite-difference expressions for these terms on a typical difference molecule.

\[
\frac{\partial T}{\partial x}_{i+1,j} = \frac{u_{i+1,j}^{n+1} \Delta X + u_{i,j}^{n+1} \Delta X^+}{\Delta X + \Delta X^+} \left[ \frac{T_{i+1,j}^{n+1} - T_{i,j}^{n+1}}{\Delta X} \right.
\]

\[
- \frac{T_{i+1,j}^{n+1} - T_{i-1,j}^{n+1}}{\Delta X^-} + \frac{T_{i+1,j}^{n+1} - T_{i-1,j}^{n+1}}{\Delta X + \Delta X^-} \right]
\]

(3.67)

for forward-going flow

\[
\frac{\partial T}{\partial x}_{i+1,j} = \frac{u_{i+1,j}^{n+1} \Delta X + u_{i,j}^{n+1} \Delta X^+}{\Delta X + \Delta X^+} \left[ \frac{T_{i+2,j}^{n} - T_{i+1,j}^{n+1}}{\Delta X^+} \right.
\]

\[
- \frac{T_{i+3,j}^{n} - T_{i+2,j}^{n+1}}{\Delta X^{++}} + \frac{T_{i+3,j}^{n} - T_{i+2,j}^{n+1}}{\Delta X^+ + \Delta X^{++}} \right]
\]

(3.68)

for reversed flow

\[
\frac{\partial T}{\partial y}_{i+1,j} = \left[ \frac{v_{i+1,j}^{n+1} \Delta Y^+ + \Delta Y^-}{\Delta Y^+ + \Delta Y^-} \right]
\]

\[
+ \frac{v_{i+1,j}^{n+1} \Delta Y^- + \Delta Y^+}{\Delta Y^- + \Delta Y^+} \left[ W \right]
\]

\[
+ \frac{v_{i+1,j}^{n+1} \Delta Y^- + \Delta Y^+}{\Delta Y^- + \Delta Y^+} \left[ (1-W) \cdot A \right]
\]

\[
+ \frac{v_{i+1,j}^{n+1} \Delta Y^- + \Delta Y^+}{\Delta Y^- + \Delta Y^+} \left[ (1-W) \cdot B \right]
\]

(3.69)

\[
\text{where } W, A, \text{ and } B \text{ are determined as follows:}
\]

\[
R_m^+ = v_{i+1,j+1}^{n+1} \cdot \Delta Y^-
\]

(3.70)

\[
R_m^- = v_{i+1,j}^{n+1} \cdot \Delta Y^+
\]

(3.71)

\[
R_c = \text{Critical mesh Reynolds number} = 1.9
\]
If \( R_m^+ > R_c \), then \( A=1, B=0, \) and \( W = R_c/R_m^+ \)
If \( R_m^- < -R_c \), then \( A=0, B=1, \) and \( W = -R_c/R_m^- \)
If \( R_m^+ \leq R_c \) and \( R_m^- \geq -R_c \), then \( A=0, B=0, \) and \( W=1. \)

\[
\frac{\partial^2 T_{i+1,j}^{n+1}}{\partial x^2} = \left[ \frac{T_{i+2,j}^{n} - 2T_{i+1,j}^{n+1}}{\Delta x^+} - \frac{T_{i+1,j}^{n+1} - T_{i,j}^{n+1}}{\Delta x^-} \right] \frac{2}{\Delta x^+ + \Delta x^-} \tag{3.72}
\]

\[
\frac{\partial^2 T_{i+1,j}^{n+1}}{\partial y^2} = \left[ \frac{T_{i+1,j+1}^{n+1} - 2T_{i+1,j}^{n+1} + T_{i+1,j-1}^{n+1}}{\Delta y^+} - \frac{T_{i+1,j}^{n+1} - T_{i+1,j-1}^{n+1}}{\Delta y^-} \right] \frac{2}{\Delta y^+ + \Delta y^-} \tag{3.73}
\]

Combining the above equations, the energy equation can be rearranged in the form:

\[
B_j T_{i+1,j-1}^{n+1} + D_j T_{i+1,j}^{n+1} + A_j T_{i+1,j+1}^{n+1} = C_j \tag{3.74}
\]

Again, this forms a tridiagonal coefficient matrix at a particular station. The Thomas algorithm is used to solve this set of equations.

The streamwise viscous term, \( \frac{\partial^2 T}{\partial x^2} \), is dropped if the boundary-layer assumption is made. For the first iteration in solving the energy equation, the FLARE approximation is used when a reversed flow region is encountered.

Care should be taken in the finite-difference expressions for some regions in the flow field, such as the second station where only the first-order upwind scheme is possible and those points adjacent to the
boundaries, a more accurate differencing scheme for diffusion terms is preferred.

F. Solution Procedure

The present solution procedure shares many features in common with those used by Chilukuri [1], Madavan [2], and Jorgenson [35]. However, some modifications have been made to broaden its applicability and improve the calculation speed and accuracy. The major modifications include a faster procedure for the block pressure adjustment and the implementation of an energy equation solver.

The solution procedure involves a cyclic space-marching procedure from the upstream end of the flow field to the downstream end; a complete cycle of this procedure is called a "global iteration". At each streamwise station, the governing equations are solved before marching to the next station. This is repeated until the downstream boundary is reached. The pressure field is updated at each streamwise station and/or at the end of a complete cycle. This marching procedure is possible because, for a given pressure field, the momentum equations are parabolic. Following is an outline of the solution procedure.

(a) All the variables and counters are initialized. The downstream marching procedure starts from a prescribed inflow profile (for both U and V) at the initial boundary.

(b) Marching to the next streamwise station, the momentum equations are solved for the tentative U and V velocity components using
the best-known values for pressure and the Thomas algorithm. The solution of the x-momentum equation involves a simultaneous procedure of block pressure adjustments to satisfy the overall mass flow constraint. For details of this procedure, see Appendix A.

(c) The tentative velocities from step (b) are then corrected to satisfy continuity by assuming velocity corrections to be locally irrotational.

(d) Using the corrected velocities, the pressure source term $S_p$ in Eq.(3.64) is evaluated. This pressure source term must generally be under-relaxed in order to obtain a stable solution.

(e) The pressure at the current station is then updated by solving the pressure Poisson equation, Eq.(3.64), using one or more passes of SOR by points. However, except at the early stage of this study, this step was bypassed for reasons that will be explained later.

(f) Steps (b) through (e) are repeated for successive streamwise stations until the downstream boundary is reached.

(g) Before proceeding to the next global iteration, the pressure solution is further updated by repeated solutions of the pressure Poisson equation throughout the computation domain using SOR by points. A completely converged solution is not necessary at the initial stage of the calculation procedure;
however, the number of sweeps through the pressure field is increased as the solution approaches convergence.

This procedure requires two-dimensional storage for the pressure as well as the source term, $S_p$. However, if the pressure is only updated along with the marching integration sweep of the momentum equations (at step (e)), there would be no need to store $S_p$, but the calculation would take longer to converge than the present method [2,4].

(h) Steps (a) through (g) are repeated until the specified convergence criterion is met.

Figure 3.5 is the flow chart of the present solution procedure. Note that step (e) is generally bypassed since the present computer code declares 2-D "dimension" like $A(n,m)$, rather than 1-D "dimension" like $A(m)$, for pressure and the pressure source term. This eliminates the need for disk I/O between the main core storage and the external disk storage at step (g), thereby eliminating the need for step (e). This is because the only objective of step (e) was to reduce the number of disk I/O's by updating the pressure several times at each streamwise stations along with the marching integration sweep of the momentum equations and reducing the number of sweeps throughout the pressure field at step (g).
Figure 3.5. Flow chart of the solution procedure
IV. RESULTS AND DISCUSSION

The results of the present study are presented in five sections. First, a flow in a symmetric expansion channel is discussed. Then, four flow configurations are examined for fully-developed flows in asymmetric expansion channels of two expansion ratios — 1.5:1.0 and 1.94:1.00. Next, the results of thin boundary-layer flows over backsteps of various step heights, with and without heat transfer are presented. Then, a section is devoted to the discussion of the reattachment lengths of all the cases studied. Finally, the convergence characteristics of the present solution procedure are analyzed. Table 4.1 is a list of flow parameters for the cases examined in this study. The results presented in the following sections are compared with the available experimental data as well as other numerical solutions.

A. Fully-Developed Flow in a Symmetric Expansion Channel

To the author's knowledge, there is only one set of detailed experimental data available for laminar, two-dimensional, symmetric expansion flows at this time. The data were measured by Durst et al. [3] for a Reynolds number of 56 (based on step height and the maximum inlet velocity) using a laser anemometer in a channel with 3:1 symmetric expansion. The same flow was calculated numerically by Durst et al. using the Navier-Stokes equations in the form of stream function and vorticity. Madavan [2] also calculated this flow starting from the step and using a procedure similar to the present one; good agreement with the measured data was observed.
TABLE 4.1. Parameters for various flow configurations

<table>
<thead>
<tr>
<th>Configurations</th>
<th>( H_{in} ) (mm)</th>
<th>s (mm)</th>
<th>Re</th>
<th>Re_D</th>
<th>Re_s</th>
<th>( x_{s}/s )</th>
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<td>1</td>
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<td>146</td>
<td>292</td>
<td>73</td>
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</tr>
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<td>916</td>
<td>229</td>
<td>12.314</td>
<td></td>
<td></td>
<td>Armaly et al. [5,6]</td>
</tr>
<tr>
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</table>

This flow was considered to be an excellent test case for verification of the present calculation procedure due to the availability of both experimental measurements and predictions obtained by other numerical schemes. It should be noted that with the present scheme it has been possible to start the calculation upstream of the step. The convergence criterion used in this study was one-twentieth of that employed by Madavan [2].

In the calculation by the present scheme, the computation domain ranged from \( x/s = -0.25 \) to \( x/s = 10 \) and a \( 42 \times 34 \) (X by Y) grid was used. Comparisons of velocity profiles in Fig. 4.1 show that slightly stronger separations were predicted by the present method as compared with the measured data. Although not shown here, the present predictions are in
good agreement with Madavan's [2] results. Figure 4.2 shows that the pressure distributions along the wall and the center line are almost identical except near the step. This clearly indicates that the boundary-layer equations provide a fairly good approximation to this flow as already demonstrated by Kwon and Pletcher [12].

B. Fully-Developed Flow in an Asymmetric Expansion Channel

Four flow configurations are examined; the corresponding experimental data can be found in Denham and Patrick [4] or Armaly et al. [5,6]. All flows computed were assumed fully-developed at the inlet and a 52x45 (X by Y) grid, which is non-uniform in both the X and Y directions, was used for all cases. A 68x45 grid was also used in configuration 4F to study the effects of grid refinement.

1. Denham and Patrick's testcases

Denham and Patrick measured velocities by means of a laser anemometer in an asymmetric expansion channel with an inlet height of 30 mm and an area expansion of 1.5:1.0. The velocity measurements were made at several stations upstream and downstream of the step over a range of $Re_s$ from 73 to 229.

Two Reynolds numbers, $Re_s = 73$ and 229, were computed in the present study. Figures 4.3 - 4.6 show the predicted skin-friction coefficient distributions and velocity profiles. The predicted velocity profiles show good agreement with the experimental data for $Re_s = 73$, but not for $Re_s = 229$. This discrepancy may be attributed to the
Figure 4.1. Velocity profiles for the fully-developed flow in a symmetric expansion channel \((\text{Re}_s = 37.9, s = 4 \text{ mm})\)
Figure 4.2. Pressure variation across the flow in a symmetric expansion channel
(Reₘ = 37.9, s = 4 mm)
Figure 4.3. Velocity profiles for the fully-developed flow in an asymmetric expansion channel ($Re_s = 73, s = 15$ mm)
Figure 4.4. Velocity profiles for the fully-developed flow in an asymmetric expansion channel (Re_s = 229, s = 15 mm)
Figure 4.5. Skin-friction coefficient for the fully-developed flow in an asymmetric expansion channel ($Re_s = 73, s = 15$ mm)
Figure 4.6. Skin-friction coefficient for the fully-developed flow in an asymmetric expansion channel ($Re_s = 229, s = 15$ mm)
differences in the flow conditions upstream of the step where the experimental data are not those of fully-developed profiles. Comparisons of velocity profiles with those obtained by Kwon and Pletcher [12] from numerical solutions of the boundary-layer equations are also presented in the same figures. The agreement is better for the larger Reynolds number. This is reasonable since the boundary-layer equations are derived by assuming a very large Reynolds number.

Figures 4.5 and 4.6 show comparisons of skin-friction coefficient with those obtained from the boundary-layer solutions [12]. A secondary separated region on the wall opposite the step was predicted by Kwon and Pletcher [12] for both Reynolds numbers; however, in the present study, the secondary separated region was not found. Ghia et al. [29] also calculated the same flow by solving the unsteady Navier-Stokes equations and did not detect a secondary separated region either.

2. Armaly et al. testcases

Armaly et al. [5,6] reported laser-Doppler measurements of velocity distributions and reattachment lengths in a two-dimensional asymmetric expansion channel with an inlet height of 5.2 mm and an area expansion ratio of 1.94:1.00. The experimental results were presented for laminar ($Re_D < 1200$), transitional ($1200 < Re_D < 6600$), and turbulent flow ($6600 < Re_D < 8800$).

An additional region of separation was observed on the wall opposite the step near the reattachment point of the primary separated region for Reynolds number ($Re_D$) between 400 and 6600. A third
separated region was also found in the experiments on the lower wall, downstream of the primary separated region and existed in the early stage of the transitional regime.

Numerical predictions done with the TEACH [22] computer code were also presented by Armaly et al. for laminar flow. These predictions were essentially the solutions to the two-dimensional Navier-Stokes equations. Good agreement with the experimental data was obtained up to \( \text{Re}_D = 400 \). Beyond this point, the numerical predictions began to deviate from the experimental data due to the existence of additional separated regions which caused the flow in the experiment to become three-dimensional.

In the present study, flows of \( \text{Re}_D = 216 \) and 389 were calculated using a fully-developed velocity profile at the inlet. The computation domain extended from about five step heights upstream of the step to about five times the experimentally measured reattachment length of the primary separation downstream of the step.

Comparisons of velocity profiles at various streamwise locations are in good agreement with the experimental data as can be seen in Figs. 4.7 and 4.8. The distributions of skin-friction coefficient are presented in Figs. 4.9 and 4.10; however, no measurements are available for comparison. It is obvious from these figures that the distributions of skin-friction coefficient on both walls collapse into one curve sufficiently far downstream of the step. This is what is expected for fully-developed flows.
Figure 4.7. Velocity profiles for the fully-developed flow in an asymmetric expansion channel ($Re_D = 216$, $s = 4.9$ mm)
Figure 4.8. Velocity profiles for the fully-developed flow in an asymmetric expansion channel ($Re_D = 389, s = 4.9 \text{ mm}$)
Figure 4.9. Skin-friction coefficient for the fully-developed flow in an asymmetric expansion channel ($Re_D = 216, s = 4.9$ mm)
Figure 4.10. Skin-friction coefficient for the fully-developed flow in an asymmetric expansion channel ($Re_D = 389$, $s = 4.9$ mm)
In this calculation, a 52x45 grid was used for both Reynolds numbers; however, a refined grid - 68x45 - was used for Re_D = 216 to study the effects of grid refinement. As can be seen from Fig. 4.7, there are almost no observable differences between solutions from the two grids except at the station x/s = 5.10 which is about the location of the reattachment point. The predictions of the reattachment length for the two different grids are also quite close - 5.31 step heights for the coarse grid and 5.20 step heights for the fine grid. Further discussion on the reattachment length will be given later.

Attempts to obtain solutions for flows with secondary separated regions failed due to the oscillation of mass sources near the reattachment point. Perhaps the unsteadiness of the secondary separated regions prevented the present steady flow solution procedure from converging.

C. Thin Boundary Layer Flow over a Backstep

This type of flow is believed to be highly elliptic in nature, which means that the flow at a given location is influenced by the rest of the flow field. Thus, the flows both upstream and downstream of the step play an important role in determining the characteristics of the separated region.

In order to apply the present solution procedure to this type of flow, it is usually necessary to start the calculation upstream of the step due to the lack of detailed U and V inflow profiles at the step.
In setting up each case, the mass flow rate was obtained from the specified free stream velocity and displacement thickness at the step in the respective experimental results. This mass flow rate was then used in a boundary-layer solution for internal flow with uniform inlet velocity in an identical channel without a step. By comparing the displacement thickness distribution from this solution with the specified displacement thickness at the step, an apparent leading edge for the boundary layer was determined. This apparent leading edge could then be used as the starting point for the present solution procedure. However, instead of this leading edge, an arbitrary location downstream of the leading edge and far enough upstream of the step, where the conventional boundary-layer solution could be used as the inlet boundary condition, was chosen as the starting point for the present computation procedure. This would reduce the cost of computation with little compromise of the solution accuracy.

Flows with and without heat transfer were studied for this configuration. The isothermal flows will be discussed first.

1. Thin boundary layer flow over a backstep without heat transfer

Comparisons are first made with the measurements of Eriksen [10] (also reported by Goldstein et al. [11]). The measurements were performed in a small wind tunnel with a rectangular test section of 0.102 m in width and 0.153 m in height upstream of the step. The step was located on the flat top wall. The test section was 0.203 m long downstream of the step with a movable top to provide for various step
heights. Data were taken with a hot wire anemometer for flows over a range of 0.00356 – 0.01016 m in step height, 0.6096 – 2.4384 m/sec in free stream velocity at the step, and 0.00163 – 0.005 m in boundary layer thickness at the step. The reattachment location was determined by a smoke injection technique.

In the present predictions, the step height was set at 0.01016 m and the free stream velocity at the step was 0.636 m/sec. The domain of computation was chosen from 14.7 step heights upstream of the step to 32.3 step heights downstream of the step. A 52x47 grid (X by Y) was used.

The solutions obtained after 27 global iterations are shown in Figs. 4.11 to 4.16. The velocity profiles predicted by the present method (Figs. 4.11 and 4.12) are in good agreement with both the experimental data [10] and numerical solutions obtained by a viscous-inviscid interaction procedure [12]. In the separated region, smaller velocities were predicted than those obtained by Kwon and Pletcher [12] using a viscous-inviscid interaction procedure. In Fig. 4.13, the skin-friction coefficient predicted by the present method is seen to be somewhat smaller along the lower wall and larger along the upper wall than those obtained by the viscous-inviscid interaction procedure [12]. The reattachment velocity profile of the present prediction is seen to differ slightly from the Karman-Pohlhausen separation profile. The present predictions of displacement thickness are in better agreement with the measured data than those predicted by Kwon and Pletcher [12].
Figure 4.11. Velocity profiles in the separated region for a thin B.L. flow over an asymmetric expansion ($Re_s = 406.4$, $s = 10.16$ mm)
Figure 4.12. Velocity profiles in the redeveloping boundary layer for a thin B.L. flow over an asymmetric expansion ($Re_s = 406.4$, $s = 10.16$ mm)
Figure 4.13. Skin-friction coefficient for a thin B.L. flow over an asymmetric expansion ($Re_s = 406.4, s = 10.16$ mm)
Figure 4.14. Displacement thickness in the redeveloping boundary layer for a thin B.L. flow over an asymmetric expansion ($Re_s = 406.4$, $s = 10.16$ mm)
Figure 4.15. Momentum thickness for a thin boundary layer flow over an asymmetric expansion (Re_s = 406.4, s = 10.16 mm)
Figure 4.16. Shape factor in the redeveloping boundary layer for a thin B.L. flow over an asymmetric expansion ($Re_s = 406.4$, $s = 10.16$ mm)
For the momentum thickness in the separated region, both the PPNS and the viscous-inviscid interaction procedures predicted smaller values than the measured data; however, the measured data are questionable since the negative mean velocities were not measured as can be seen in Fig. 4.11. The same explanation can be given for the over-prediction of shape factor in the separated region. It is interesting to note that the predicted shape factor far downstream of the step approaches the value for a flat plate.

2. Thin boundary layer flow over a backstep with heat transfer

Though many experimental studies have been reported on hydrodynamic aspects of flow over a rearward-facing step, there appears to be very few papers concerning the corresponding heat transfer problem in laminar flow. The only available experimental data for laminar heat transfer in an asymmetric expansion channel was reported by Aung [7]. The experiments were conducted in a low speed wind tunnel with a test section of 20 cm height and 15 cm width over three different step heights – 0.38, 0.64 and 1.27 cm. The heat transfer data were obtained by means of a Mach-Zehnder interferometer; the walls were heated to a constant temperature, generally 322°K, and the free stream temperature was approximately 301°K. Only a limited amount of flow data were reported. The reattachment lengths were determined by the smoke injection technique.

In the present study, flows over step heights of 0.38 and 1.27 cm were calculated. Results are compared with the experimental data of
Aung [7] and the predictions of Hall [8] obtained by a viscous-inviscid interaction method. For the smallest step ($s = 0.38$ cm), two Reynolds numbers — $Re_s = 139.8$ and $201.7$ — were calculated using a $35 \times 43$ (X by Y) grid and a computation domain ranging from $x/s = -13.6$ to $79.5$. For the largest step ($s = 1.27$ cm), flow at a Reynolds number, $Re_s = 247.4$, was calculated with a finer $40 \times 58$ (X by Y) grid and a computation domain ranging from $x/s = -13.2$ to $24.2$. Comparisons of predicted and measured temperature profiles, shown in Figs. 4.17 and 4.18, indicates that the level of agreement degenerates in the separated region with the increase of the step height. However, it should be pointed out that the measured temperature profiles in the separated region look qualitatively different for the largest step case than for the others. The measured profiles have no inflection point near the wall and look like the one measured downstream of reattachment. The predictions, on the other hand, continue to show the same type of inflection point as for the other step cases. Hall [8] also calculated the same flows using a viscous-inviscid interaction method. Interestingly, the solutions of the two numerical methods agree quite well.

Figures 4.19 to 4.21 show the distributions of Nusselt number downstream of the step. Again, the deviation from the experimental results is the greatest for the largest step case, $Re_s = 247.4$.

In the present study, both the boundary-layer (Eq.(2.9)) and the fully-elliptic (Eq.(2.8)) energy equations were used. Apparently, the streamwise diffusion term, $\frac{\partial^2 T}{\partial x^2}$, in the energy equation is negligible
Figure 4.17. Temperature profiles downstream of the step for a thin B.L. flow over an asymmetric expansion ($Re_S = 139.8$, $s = 0.38$ mm)
Figure 4.18. Temperature profiles downstream of the step for a thin B.L. flow over an asymmetric expansion ($Re_s = 247.4$, $s = 1.27$ mm)
Figure 4.19. Nusselt number downstream of the step for a thin B.L. flow over an asymmetric expansion ($Re_s = 139.8, s = 0.38$ mm)
Figure 4.20. Nusselt number downstream of the step for a thin B.L. flow over an asymmetric expansion ($Re_s = 201.7, \, s = 0.38$ mm)
Figure 4.21. Nusselt number downstream of the step for a thin B.L. flow over an asymmetric expansion ($Re_s = 247.4$, $s = 1.27$ mm)
for these flows. This can be concluded from the comparisons of temperature profiles and Nusselt numbers. In Figs. 4.17 and 4.18, no appreciable difference in temperature profiles can be observed for the two solutions; besides, for the smallest step ($Re_s = 139.8, s = 0.38$ cm), the boundary-layer energy equation was solved using the FLARE approximation. For the Nusselt number, the solutions of the boundary-layer and the elliptic forms of the energy equation agree quite well; however, the use of the FLARE approximation in the boundary-layer solution results in an under-prediction of the Nusselt number in the separated region.

D. Reattachment Length

Reattachment length is one of the major parameters which has been closely examined in this study, since it characterizes the separation bubble behind the rearward-facing step. Already several papers have been published concerning this parameter.

Comparisons of predicted and measured reattachment lengths are given for different expansion ratios. In Fig. 4.22, several predictions are compared with the experimental data for a 3:2 asymmetric expansion channel. It can be seen from the figure that both the present predictions (PPNS) and the Ghia et al. [29] predictions (unsteady N/S) give longer reattachment lengths than the experimental measurements. The discrepancy is the worst for $Re_s = 229$. This can be attributed to the difference in the flow conditions just upstream of the step where
Figure 4.22. Comparison of reattachment lengths for Denham and Patrick's testcases ($H_{out} : H_{in} = 3 : 2$)
Denham and Patrick [4] show a significant departure from the symmetric parabolic profile (see Fig. 4.4). Atkin's [16] predictions using the fully-developed inlet profile show the same trend as the present predictions; however, his predictions using the experimental inlet profile yield better agreement with the measured data. Chen et al. [36] also calculated the same flow using the finite analytic method. Although their predictions seem to have the best agreement with the measured data, they may not be any more accurate than the other predictions mentioned above because a fully-developed inlet profile was used in the calculations rather than the experimentally measured inlet profile. Thus, it is always possible that the finite analytic method would underpredict the reattachment lengths if the experimental profile was used as the boundary condition.

Figure 4.23 shows that both the present predictions and Ghia et al. [29] predictions agree well with the data measured by Armaly et al. [5] in the range of $Re_s < 200$. For $Re_s > 200$, the secondary separation zone appears and the present method does not converge. Interestingly, the Ghia et al. predictions [29] begin to deviate from the measured data for $Re_s > 200$.

For the thin boundary-layer flow, comparison is made in the form suggested by Cramer [13], which includes the effects of Reynolds number, step height, and the displacement thickness. As can be seen in Fig. 4.24., the reattachment lengths from the PPNS solutions are longer than the experimental results; however, it has been noted [37] that the flow
Figure 4.23. Comparison of reattachment lengths for Armaly et al. testcases ($H_{out} : H_{in} = 1.94 : 1.00$)
Figure 4.24. Comparison of reattachment lengths based on Cramer's correlation parameters.

\[
\frac{\frac{\bar{u}_*}{\bar{v}}}{\delta_*} = 0.01325 \frac{\bar{u}_* \max \delta_*}{\nu} \left( \frac{\delta_*}{\delta_*} \right)^2 + 2 \frac{\bar{u}_*}{\delta_*} 
\]

(CORRELATION PROPOSED BY ERIKSEN)
geometry and smoke injection technique used by Aung [7] and Eriksen [10] provide some of the shortest reattachment lengths reported. In the same figure, the PPNS solutions for the fully-developed inlet profile are also plotted. Apparently, additional parameters are needed in order to correlate the solutions with different inlet velocity profiles. Thus, the reattachment length is likely to be a function of several parameters – a Reynolds number, expansion ratio, inlet velocity profile and the displacement thickness at the step.

E. Convergence Characteristics

To study the convergence characteristics of the present solution procedure, the parameter $\Sigma|S\phi|/\dot{m}$ is examined. $\Sigma|S\phi|$ is the sum of the absolute values of the nondimensional mass sources at all the grid points along any one streamwise station. The mass source at a grid point is defined as the source density $S\phi_{i+1,j}^{n+1}$ (see Eq. (3.59)) multiplied by the control volume for which it is calculated. The solution is assumed to be converged if $\Sigma|S\phi|$ is less than 0.1% of the total mass flow rate, $\dot{m}$.

In essence, the parameter, $\Sigma|S\phi|/\dot{m}$, is an index of how well continuity is satisfied by the velocities obtained from solving the momentum equations. As shown in Figs. 4.25 and 4.26, the magnitude of the nondimensional mass sources upstream of the reattachment point is generally decreasing monotonically along with the iteration and oscillating downstream of (including) the reattachment point. The
Figure 4.25. Convergence characteristics for configuration #3
(Re_s = 229, s = 15 mm)
Figure 4.26. Convergence characteristics for configuration #5
($Re_D = 389, \ s = 4.9 \ mm$)
largest oscillation occurs near the reattachment point and generally, this oscillatory behavior propagates downstream and becomes worse with increasing Reynolds number and expansion ratio. It should be noted that the convergence characteristics presented here are for the two most difficult cases in the present study; for other cases, the number of iterations was much smaller. Table 4.2 presents the performance of the present solution procedure in terms of CPU time and number of iterations for all the cases calculated. Note that the number of iterations for SOR in solving the pressure Poisson equation (Eq. (3.64)) may influence the CPU time used. Other parameters such as the under-relaxation factor for the pressure source (see Eq. (3.64)) are also critical in determining the rate of convergence. Generally, the larger the Reynolds number (or the expansion ratio), the smaller the under-relaxation parameter. It is also found that for certain cases, smaller under-relaxation factors tend to accelerate the convergence rather than larger ones. However, for a particular case, the under-relaxation factor was gradually increased as the solution approaches convergence.
TABLE 4.2. Convergence characteristics of the testcases in this study

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<th>Configurations</th>
<th>Re</th>
<th>ReD</th>
<th>ReS</th>
<th>Grid (X by Y)</th>
<th>Global1 Iter.3</th>
<th>Max. CPU2</th>
<th>Iter.3</th>
<th>Reference</th>
</tr>
</thead>
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<td>37.9</td>
<td>75.9</td>
<td>37.9</td>
<td>42x34</td>
<td>53</td>
<td>112</td>
<td>30</td>
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</tr>
<tr>
<td>2</td>
<td>146</td>
<td>292</td>
<td>73</td>
<td>52x45</td>
<td>61</td>
<td>204.3</td>
<td>35</td>
<td>Denham and Patrick [4]</td>
</tr>
<tr>
<td>3</td>
<td>458</td>
<td>916</td>
<td>229</td>
<td>52x45</td>
<td>67</td>
<td>189.4</td>
<td>15</td>
<td>Armaly et al. [5,6]</td>
</tr>
<tr>
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<td>216</td>
<td>101.8</td>
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<td>70</td>
<td>206</td>
<td>15</td>
<td>Eriksen [10]</td>
</tr>
<tr>
<td>4F</td>
<td>108</td>
<td>216</td>
<td>101.8</td>
<td>68x45</td>
<td>70</td>
<td>240</td>
<td>15</td>
<td>Aung [7]</td>
</tr>
<tr>
<td>5</td>
<td>194.5</td>
<td>389</td>
<td>183.3</td>
<td>52x45</td>
<td>91</td>
<td>263.3</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5719</td>
<td>11438</td>
<td>406.4</td>
<td>52x47</td>
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<td>91.4</td>
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</tr>
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<td>14000</td>
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<td>16</td>
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<td>35</td>
<td>146.2</td>
<td>25</td>
<td></td>
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</tbody>
</table>

1Global iterations as defined in Section III(F).
2Fortran H compiler on NAS/AS6.
3Maximum number of sweeps throughout the pressure field at step (g) in the present solution procedure (see Section III(F)).
4Stopped at $S_k/S\|/\dot{m} < 0.15\%$ for this configuration; for the rest of the configurations, stopped at $S_k/S\|/\dot{m} < 0.1\%$.
5Fortran on DEC/VAX which is believed to be about six times slower than the NAS/AS6.
V. CONCLUSIONS AND RECOMMENDATIONS

The partially-parabolized Navier-Stokes equations have been used to analyze flows in symmetric and asymmetric sudden expansions. Good agreement was obtained with experimental data as well as solutions to the Navier-Stokes equations, boundary-layer equations, and the viscous-inviscid interaction method reported by other investigators. Thus, the neglect of the streamwise diffusion appears to be appropriate for the type of flows studied and the present solution procedure seems to be applicable to flows with separated regions. However, this conclusion should be restricted to flows with only one separated region behind the backstep. It should be noted that the present solution procedure is based on the steady partially-parabolized Navier-Stokes equations and no difficulties were encountered in computing through the reversed flow unlike the method of Ghia et al. [21] which apparently failed when a steady formulation was used.

For the reattachment length, the present predictions are generally longer than the corresponding experimental data but in good agreement with most other numerical predictions.

A non-iterative method was used instead of the secant method in the present solution procedure to calculate the block pressure adjustment. This method enhanced the rate of convergence and avoided the problem of divergence in the secant iteration loop when a small convergence criterion was specified.
The solutions obtained from solving the elliptic energy equation were in good agreement with those obtained from solving the boundary-layer energy equation. This indicates that the errors introduced by the neglect of the streamwise diffusion are very small for the type of flows considered. As for the use of the FLARE approximation, the rate of heat transfer tends to be slightly underpredicted in the separated region when this approximation is used.

The temperature profiles and Nusselt numbers predicted by the present method were in better agreement with Aung's measurements [7] in the separated region than those obtained by Hall [8] using a viscous-inviscid interaction method. This may be attributed to the fact that the PPNS solution procedure allows the pressure to vary throughout the channel while the viscous-inviscid interaction method assumes a constant pressure in the normal direction in the boundary layer region which is apparently not the case in the actual flow, especially in the separated region.

Further investigation is needed for backstep flows at high Reynolds numbers. Perhaps use of an unsteady analysis (time dependent scheme) could be a remedy for the difficulties observed in computing backstep flows at high Reynolds numbers. Attention should also be directed toward the evaluation of the performance of various methods for evaluating convective coefficients and toward the establishment of general guidelines for determining the optimum value of the under-relaxation parameter for the pressure source term. Additional
computations for backstep flows with various expansion ratios and over a wider range of Reynolds numbers would be useful in studying the mechanisms of separated flows behind the backstep.

As a ultimate goal, it would be desirable to extend the present solution procedure to handle turbulent and laminar three-dimensional flows.
VI. BIBLIOGRAPHY


VII. ACKNOWLEDGMENTS

I would like to express my sincere gratitude to Dr. Richard H. Pletcher for his encouragement and many helpful suggestions throughout the course of this study. Without these, this work would not have been accomplished.

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VIII. APPENDIX A. THE INITIAL ESTIMATED PRESSURE FIELD
AND THE BLOCK PRESSURE ADJUSTMENT

To start the present solution procedure, an initial estimate of the pressure field is required. This estimate is determined simultaneously with the solution of the x-momentum equation in the first marching integration sweep by invoking the boundary layer approximation – zero transverse and constant streamwise pressure gradient. Following is the procedure to determine the initial pressure field.

As already discussed in Section III(B), the finite-difference representation of the x-momentum equation, Eq. (3.40), written for each grid point at a certain station results in a system of tridiagonal linear equations of the form:

\[
\begin{bmatrix}
D_1 & A_1 & 0 & 0 & 0 & \cdots & 0 \\
B_2 & D_2 & A_2 & 0 & 0 & \cdots & 0 \\
0 & B_3 & D_3 & A_3 & 0 & \cdots & 0 \\
& & & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \cdots & B_{NJ-1} & D_{NJ-1} & A_{NJ-1} & 0 \\
0 & 0 & \cdots & 0 & B_N & D_N & U_N
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_2 \\
U_3 \\
\vdots \\
U_{NJ-1} \\
U_N
\end{bmatrix}
= 
\begin{bmatrix}
C_1 + F_1 \Delta P \\
C_2 + F_2 \Delta P \\
C_3 + F_3 \Delta P \\
\vdots \\
C_{NJ-1} + F_{NJ-1} \Delta P \\
C_N + F_{NJ} \Delta P
\end{bmatrix}
\] (8.1)

where the coefficients \( A_j, B_j, \) and \( D_j \) are defined in Eqs. (3.41) – (3.45), while \( C_j + F_j \Delta P \) is actually Eqs. (3.46) or (3.47) written in two parts with

\[
F_j = -\frac{1}{\Delta x^2}
\] (8.2)
\[ \Delta P = P_{i+2,j}^n - P_{i+1,j}^n \]  

(8.3)

The system of equations (8.1) can then be reduced to the following form by eliminating the lower diagonal coefficients using Gaussian elimination.

\[
\begin{bmatrix}
D_1^* A_1 & 0 & 0 & 0 & \cdots & 0 \\
0 & D_2^* A_2 & 0 & 0 & \cdots & 0 \\
0 & 0 & D_3^* A_3 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & D_{N_J-1}^* A_{N_J-1} \\
0 & 0 & \cdots & 0 & D_{N_J}^*
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_2 \\
U_3 \\
\vdots \\
U_{N_J-1} \\
U_{N_J}
\end{bmatrix}
= 
\begin{bmatrix}
C_1^* + F_1^* \Delta P \\
C_2^* + F_2^* \Delta P \\
C_3^* + F_3^* \Delta P \\
\vdots \\
C_{N_J-1}^* + F_{N_J-1}^* \Delta P \\
C_{N_J}^* + F_{N_J} \Delta P
\end{bmatrix}
\]  

(8.4)

where

\[ D_j^* = D_j - \frac{B_j}{D_{j-1}} A_{j-1} \]  

(8.5)

\[ C_j^* = C_j - \frac{B_j}{D_{j-1}} C_{j-1} \]  

(8.6)

\[ F_j^* = F_j - \frac{B_j}{D_{j-1}} F_{j-1} \]  

(8.7)

from \( j = 2 \) to \( N_J \)

Next, the above system of equations (8.4) is further reduced to a single diagonal matrix by eliminating the upper diagonal coefficients.
The velocity \( u_j \) can then be written as:

\[
U_j = \frac{C_j}{D_j} + \frac{F_j}{D_j} \Delta P
\]  

(8.11)

By integrating across the channel, the volume flow rate \( \dot{m} \) is obtained in terms of \( \Delta P \) as:

\[
\dot{m} = \sum \frac{C_j}{D_j} \Delta P + \sum \frac{F_j}{D_j}
\]  

(8.12)

With the volume flow rate already known at the initial boundary, \( \Delta P \) is solved and so are the \( U \)-velocity components.

For later marching integration sweeps, the same procedure applies in determining the block pressure adjustment except that in Eq.(8.1), \( C_j \) is defined by the entire Eq.(3.46) or (3.47) and \( \Delta P \) designates the constant pressure correction at a certain station needed in order to satisfy the mass flow constraint.
IX. APPENDIX B. LISTING OF COMPUTER PROGRAM
THIS PROGRAM SOLVES THE PARTIALLY-PARABOLIZED NAVIER-STOKES EQUATIONS FOR LAMINAR FLOWS INCLUDING REGIONS OF SEPARATION AND REATTACHMENT. THE PROGRAM IS SET UP TO HANDLE EXTERNAL FLOWS AS WELL AS INTERNAL FLOWS IN CHANNELS WITH SUDDEN EXPANSIONS. THE PRESENT PROGRAM IS A MODIFIED VERSION OF THE CODE "PAPANS" DEVELOPED BY CHILUKURI. FURTHER MODIFICATION WERE MADE BY MADAVAN.

THE FOLLOWING IS A LIST AND AN EXPLANATION OF THE INPUT PARAMETERS, IN THE ORDER IN WHICH THEY APPEAR ON THE READ STATEMENTS. ALL REAL VARIABLES ARE SPECIFIED IN THE FORMAT 7G10.4 AND INTEGER VARIABLES IN THE FORMAT 12I6.

HIGHT CHANNEL INLET HEIGHT.
HSTEP STEP HEIGHT.
XSTEP STREAMWISE LOCATION OF THE STEP.
FLARE CONSTANT FOR FLARE APPROXIMATION.
LPNS DESIGNATES THE AXIAL STATION INDEX (MCOUNT) AFTER WHICH THE GOVERNING EQUATIONS (PPNS OR NS) ARE TO BE SOLVED. FOR MOST CASES, SET TO 1.
KPNS DESIGNATES MCOUNT AFTER WHICH THE GOVERNING EQUATIONS NEED NOT BE SOLVED.
JPNS DESIGNATES MCOUNT AFTER WHICH CYCLIC ITERATION IS DESIRED. IN GENERAL, SET TO 2.
NLMT SAFETY PARAMETER, DESIGNATING THE MAXIMUM ALLOWABLE MCOUNT. SET EQUAL TO KPNS.
LPOP FOR EXTERNAL PRESSURE GRADIENT FLOWS, THIS REPRESENTS THE NUMBER OF FREESTREAM U VELOCITY INPUTS. FOR INTERNAL FLOWS, SET...
TO 0.

**NJ**
NUMBER OF GRID-POINTS IN THE Y DIRECTION.

**INV**
FOR EXTERNAL PRESSURE GRADIENT FLOWS, THIS REPRESENTS THE NUMBER OF FREESTREAM V VELOCITY INPUTS. SET TO 0 FOR INTERNAL FLOWS OR WHEN NO V VELOCITIES ARE PRESCRIBED.

**INT**
THIS REPRESENTS THE NUMBER OF TEMPERATURE INPUTS ON UPPER OR LOWER BOUNDARIES. SET TO 0 FOR ISOTHERMAL BOUNDARIES.

**ZAP1**
SET EQUAL TO 1. IF ABBREVIATED OUTPUT IS REQUIRED. FOR DETAILED OUTPUT AT EACH STREAMWISE STATION, SET TO -1.

**GLOBE**
BEGINNING CYCLE NUMBER FOR THIS RUN. SET EQUAL TO MIN.

**TOLER**
NO LONGER USED.
TOLERANCE ON STREAMWISE PRESSURE GRADIENT TO BE USED IN SECANT PROCEDURE WHEN ESTIMATING THE INITIAL PRESSURE FIELD FOR THE FIRST CYCLE.

**XE**
SAFETY PARAMETER, DESIGNATING THE MAXIMUM AXIAL DISTANCE BEYOND WHICH CALCULATION WILL STOP.

**PR**
PRANDTL NUMBER.

**CPS**
SPECIFIC HEAT.

**TEST**
VALUE USED TO CHECK FOR EDGE OF B.L.; TYPICALLY 0.9995 FOR FULLY-DEVELOPED FLOW AND 0.995 FOR DEVELOPING FLOW.

**US**
VELOCITY TO BE USED FOR NON-DIMENSIONALIZATION.

**XMUS**
ABSOLUTE VISCOSITY TO BE USED FOR NON-DIMENSIONALIZATION.

**RHOS**
FREESTREAM DENSITY TO BE USED FOR NON-DIMENSIONALIZATION.

**VW**
NORMAL VELOCITY AT WALL. SET TO 0.

**UREF**
FREESTREAM U VELOCITY. WILL VARY WITH AXIAL
DISTANCE. US HOWEVER REMAINS FIXED. SET TO THE FREESTREAM VALUE AT FIRST AXIAL STATION.

TWL TEMPERATURE FOR THE LOWER ISOTHERMAL BOUNDARY.

TWU TEMPERATURE FOR THE UPPER ISOTHERMAL BOUNDARY.

MIN BEGINNING CYCLE NUMBER FOR THIS RUN.

MAX FINAL CYCLE NUMBER FOR THIS RUN.

IUPDAT SET TO 1 IF THE BEST-KNOWN VALUES ARE USED IN EVALUATING CONVECTIVE COEFFICIENTS FOR F1 AND F2; OTHERWISE SET TO 0.

MP1 NUMBER OF MCOUNTS TO BE READ IN TO DETERMINE DETAILED PRINTOUT STATIONS.

MPC(J) J=1,MP1. MCOUNTS OF STREAMWISE STATIONS WHERE DETAILED PRINTOUT IS DESIRED.

NP1 NUMBER OF X DISTANCES TO BE READ IN TO DETERMINE DETAILED PRINTOUT LOCATIONS.

NWALL FOR ASYMMETRIC EXPANSION TESTCASES:
SET TO 2 IF ONE-SIDED DIFFERENCING IS TO BE USED TO EVALUATE F1 AND F2 NEAR WALL. SET TO -2 IF REGULAR DIFFERENCING IS TO BE USED.
FOR OTHER GEOMETRIES:
SET TO 1 FOR ONE-SIDED DIFFERENCING; SET TO -1 FOR REGULAR DIFFERENCING.

JSTEP GRID-POINT INDEX IN Y-DIRECTION FOR THE FIRST POINT BELOW THE STEP.

MCSTEP STATION INDEX (MCOUNT) FOR THE FIRST STATION DOWNSTREAM OF THE STEP.

NTEMP SET TO 1 IF ENERGY EQN IS TO BE SOLVED; OTHERWISE SET TO 0.

NWALLT SET TO 0 IF BOTH THE UPPER AND LOWER BOUNDARIES ARE ISOTHERMAL.
SET TO 1 IF THE TEMPERATURES ON BOTH BOUNDARIES ARE NOT CONSTANT.
SET TO >1 IF ONLY THE LOWER BOUNDARY IS NOT ISOTHERMAL.
SET TO <0 IF ONLY THE UPPER BOUNDARY IS NOT ISOTHERMAL.

NEXTRAP
SET TO 1 IF EXTRAPOLATION IS TO BE USED TO EVALUATE F1 AND F2 NEAR WALL; OTHERWISE SET TO 0.

XP3(J)
J=1, NP1. X DISTANCES WHERE DETAILED PRINTOUT IS DESIRED.

NPRINT
SET TO 0 IF ABBREVIATED OUTPUT IS REQUIRED WHEN SOLVING THE POISSON EQUATION FOR PRESSURE. SET TO 1 FOR DETAILED PRINTOUT. ZAP, IF SET TO 1, OVERRIDES NPRINT AND NO DETAILS ARE PRINTED.

LSOR1
IF SET TO 0, CALCULATION OF THE BEGINNING CYCLE (MIN) STARTS WITH SOLVING THE POISSON EQUATION. IN THIS CASE, THE MARCHING SWEEP FOR THIS CYCLE MUST HAVE BEEN ALREADY CALCULATED AND THE RESULTS STORED ON UNIT 9. CHOICE OF LSOR1 IS RELATED TO VALUE OF LSOR. SET TO 0 FOR FIRST CYCLE.

LSOR
IF SET TO 0, FINAL CYCLE (MAX) IS COMPLETED AND CALCULATION TERMINATES AFTER POISSON EQUATION FOR PRESSURE IS SOLVED. IF SET TO 1, THE FINAL CYCLE CALCULATION TERMINATES WITH THE MARCHING INTEGRATION SWEEP.

LEAD
SET TO 0 IF LEADING EDGE SINGULARITY IS TO BE SMOOTHED BY SPECIFYING ZERO U VELOCITY JUST AHEAD OF THE LEADING EDGE. IF SET TO 1, U VELOCITY JUST AHEAD OF THE LEADING EDGE IS TO BE SET TO UREF VALUE. LATTER OPTION NEVER USED.

FAC
OVER-RELAXATION FACTOR USED IN SOLVING THE POISSON EQUATION FOR PRESSURE BY THE METHOD OF SOR BY POINTS.

TOL
NO LONGER USED.
WHEN POSITIVE, THIS REPRESENTS TOLERANCE ON THE TOTAL MASS FLOW RATE WHEN MAKING BLOCK ADJUSTMENTS ON PRESSURE. THIS OPTION USED ONLY AFTER CONVERGENCE HAS BECOME MONOTONIC. IF BLOCK ADJUSTMENTS ARE NOT REQUIRED, SET TO ANY NEGATIVE VALUE. MAKE SURE TOL IS SMALL WHEN POSITIVE TO AVOID IMPOSING OSCILLATIONS.
NIRROT  SET TO 2.

IRR0T  SET TO 0 IF FREESTREAM U, V VELOCITIES ARE SPECIFIED. SET TO 1 IF ONLY FREESTREAM U VALUES ARE SPECIFIED AND AN IRROTATIONAL OUTER-EDGE BOUNDARY CONDITION IS TO BE USED ON THE V VELOCITIES. LATTER OPTION DOES NOT WORK TOO WELL.

NDOWN  DECIDES DOWNSTREAM BOUNDARY CONDITION. IF SET TO 0, NO DOWNSTREAM BOUNDARY CONDITION ON PRESSURE IS IMPOSED. IF SET TO 1, CONSTANT AXIAL PRESSURE GRADIENT CONDITION IS IMPOSED. IF SET TO -1, CONSTANT NORMAL PRESSURE GRADIENT CONDITION IS IMPOSED.

NOBLK  SET TO 0 IF PRESSURE BLOCK ADJUSTMENT IS USED. SET TO 1, BLOCK PRESSURE ADJUSTMENT IS BYPASSED, EVEN IN THE FIRST CYCLE. IF IRROT IS SET TO 1, THIS MUST BE SET TO 1. IN GENERAL, USE FIRST OPTION.

XBEGIN  REPRESENTS THE X COORDINATE AT WHICH THE CALCULATION BEGINS.

UB(J)  J=1,NJ+1. STARTING U VELOCITY PROFILE AT UPSTREAM BOUNDARY.

VB(J)  J=1,NJ+1. STARTING V VELOCITY PROFILE AT UPSTREAM BOUNDARY. NOTE THAT THE AXIAL LOCATION OF THIS PROFILE IS SLIGHTLY DIFFERENT THAN THAT FOR U VELOCITY PROFILE.

MSEP1  SET TO MCOUNT VALUE BEYOND WHICH THE 2D U, V VELOCITY ARRAY STORAGE IS REQUIRED. FOR SEPARATED FLOWS SET TO THE MCOUNT VALUE SLIGHTLY AHEAD OF SEPARATION POINT. FOR NON-SEPARATING FLOWS, SET EQUAL TO KPNS. ALSO SEE EXPLANATION FOR MSEP2.

MSEP2  SET TO MCOUNT VALUE BEYOND WHICH 2D U, V VELOCITY ARRAY STORAGE IS NOT REQUIRED. FOR SEPARATED FLOWS, SET TO MCOUNT VALUE SLIGHTLY BEYOND REATTACHMENT. FOR NON-SEPARATED FLOWS, SET TO KPNS. FOR FULL NAVIER-STOKES CALCULATION, THE SPECIFIED VALUES OF MSEP1 AND MSEP2 ARE OVERRIDDEN WITHIN THE PROGRAM.
MCDOWN SET EQUAL TO KPNS.

NSFULL SET TO 0 IF PARTIALLY-PARABOLIZED NAVIER-STOKES EQUATIONS ARE TO BE SOLVED. SET TO 1 IF FULL NAVIER-STOKES EQUATIONS ARE TO BE SOLVED. NOTE THAT EVEN WHEN FULL NS EQUATIONS ARE TO BE SOLVED, THIS MUST BE SET TO 0 FOR THE FIRST CYCLE.

NHYBRD IF SET TO 1, HYBRID DIFFERENCING SCHEME IS USED. IF SET TO 0, PURE UPWIND OR CENTRAL DIFFERENCING IS USED DEPENDING ON THE MESH REYNOLDS NUMBER.

NSTEP SET TO 0 FOR EXTERNAL OR INTERNAL ASYMMETRIC EXPANSION FLOWS. SET TO 1 FOR INTERNAL SYMMETRIC EXPANSION TESTCASES.

MSEP11 SET TO MCOUNT VALUE BEYOND WHICH THE 2D U, V VELOCITY ARRAYS WERE STORED IN THE PREVIOUS ITERATION; GENERALLY, MSEP11 = MSEP1.

MSEP22 SET TO MCOUNT VALUE BEYOND WHICH THE 2D U, V VELOCITY ARRAYS WERE NOT STORED IN THE PREVIOUS ITERATION; GENERALLY, MSEP22 = MSEP2.

XU(J) J=1,LPOP. X DISTANCES FOR FREESTREAM U VELOCITY INPUT. NOT REQUIRED FOR INTERNAL FLOWS.

YU(J) J=1,LPOP. FREESTREAM U VELOCITY VALUES CORRESPONDING TO XU(J) VALUES. NOT REQUIRED FOR INTERNAL FLOWS.

XV(J) J=1,INV. REPRESENTS X DISTANCE FOR FREESTREAM V VELOCITY INPUT. NOT REQUIRED FOR INTERNAL FLOWS OR IF INV IS SET TO 0.

YV(J) J=1,INV. REPRESENTS FREESTREAM V VELOCITY VALUES CORRESPONDING TO XV(J). NOT REQUIRED FOR INTERNAL FLOWS OR IF INV IS SET TO 0.

MINT BEGINNING CYCLE NUMBER FOR THE ENERGY EQN.

MAXT FINAL CYCLE NUMBER FOR THE ENERGY EQN.

NENGY SET TO 1 IF ONLY ENERGY EQN IS TO BE SOLVED;
OTHERWISE SET TO 0.

**IBL**

SET TO 1 IF B.L. ENERGY EQN IS USED; OTHERWISE, SET TO 0.

**TT(1,J)**

J=1,NJ. STARTING TEMPERATURE PROFILE AT UPSTREAM BOUNDARY.

**XTWL(J)**

J=1,INT. X DISTANCES FOR LOWER BOUNDARY TEMPERATURE INPUTS. NOT REQUIRED FOR ISOTHERMAL BOUNDARY.

**YTWL(J)**

J=1,INT. LOWER BOUNDARY TEMPERATURE VALUES CORRESPONDING TO XTWL(J) VALUES. NOT REQUIRED FOR ISOTHERMAL BOUNDARY.

**XTWU(J)**

J=1,INT. X DISTANCES FOR UPPER BOUNDARY TEMPERATURE INPUTS. NOT REQUIRED FOR ISOTHERMAL BOUNDARY.

**YTWU(J)**

J=1,INT. UPPER BOUNDARY TEMPERATURE VALUES CORRESPONDING TO XTWU(J) VALUES. NOT REQUIRED FOR ISOTHERMAL BOUNDARY.

**NG**

NUMBER OF GAUSS-SIEDEL SWEEPS TO BE CARRIED OUT WHEN SOLVING THE POISSON EQUATION FOR PRESSURE. FOR THE FIRST CYCLE, SET TO 1. FOR LATER CYCLES, GRADUALLY INCREASE THIS NUMBER.

**NT**

NO LONGER USED.

NUMBER OF TIMES THE PRESSURE IS REVISED AT EACH AXIAL STATION DURING EACH CONVENTIONAL ITERATION OF THE SOR METHOD WHEN SOLVING THE POISSON EQUATION FOR PRESSURE.

**FAC1**

UNDER-RELAXATION FACTOR TO BE USED ON THE PRESSURE GRADIENTS AFTER EACH MARCHING-INTEGRATION SWEEP.

**FAC2**

RELAXATION PARAMETER FOR VELOCITY CORRECTIONS; 1.0 WAS USED IN THIS STUDY.

NOTE THAT ONE CARD WITH NG, NT, FAC1, FAC2 VALUES IS REQUIRED FOR EACH CYCLE.

THE FOLLOWING DISK DATASETS ARE REQUIRED TO RUN THE PROGRAM.
UNIT 9
This unit contains the results of the cycle just preceding the beginning cycle.

UNIT 10
Will contain the results of the last cycle.

UNIT 16
Contains the 2D U, V velocity arrays just preceding the beginning cycle for separated flow calculations or when using the full NS equations.

UNIT 17
Contains the 2D temperature array just preceding the beginning cycle.

UNIT 18
Will contain the 2D U, V velocity arrays of the last cycle for separated flow calculations or when using the full NS equations.

UNIT 19
Will contain the 2D temperature array of the last cycle of energy eqn.
THE MAIN PROGRAM

handles input, initialization, updating, output. A do loop executes as many cycles of the calculation procedure as desired.

DIMENSION UB(100), VB(100), XP3(50), YDE1(100), YVMIN(100)
COMMON/SPARE/XU(100), YU(100), XV(100), YV(100), XUX(100), C(4, 100)
COMMON/OVER/FAC2, FAC2, FAC, NT, FLARE
COMMON/PRESUR/NG, NPRINT, LGLOBE, HPC(20)
COMMON/THOMAS/XTWL(100), YTWL(100), XTU(100), YTWU(100), DU(100), DV1(100)
COMMON/ARRAYS/F1(100), F2(100), F1(100), P(70, 100), SP(70, 100), UM(100), U1(100), USECX(100), USECY(100), USEP(70, 100), VM(100), V1(100), VSEC1X(100), VSECY(100), VSEP(70, 100), D(100), YDE2(100), YDE3(100), YDE12(100), YDE13(100), YDE23(100), VVPLU(100), VVVT(100), T(70, 100), UREF1(300), V1REF(300), TW1(70), TWU1(70)
COMMON/VAR/AFLOW, GLOBE, INV, JPNS, KPNS, LPNS, MCM, MCOUNT, MCP, NJN, NJP, NTEMP, NWALL, NWALLT, PCON, RC, PCON3, TOLER, XCONV, JS, JS3, JS4, JSTEP, MCSTEP, HIGHT, TEST
COMMON/XGRID/X(300), DELXM, DELX, DELXP, DELXPP, DELXP3, DELXUM, DELXU, DELXUP, DXUPP
COMMON/GLOBAL/ICOUNT, MIN, MAX, ZAP
COMMON/BLOC1/TOL, DELP, BLOCK
COMMON/LEAD1/LEAD, PR
COMMON/CONV/EPSJ, EPSMJ, EPS, EPSM, JMAX, JMIN
COMMON/CONV1/J1, J2, J3, PJ1, PJ2, PJ3, PNJ, JM, EPSU
COMMON/EXT/FLOW, NEXTRP
COMMON/DECIDE/NIRROT
COMMON/BDY/NDOWN
COMMON/IRR/NOBLK, JSF
COMMON/SEPER/MSEP1, MSEP2, NSFULL, MCDOWN, NHYBRD, MC1, MC2
COMMON/TCASE/NSTEP
EQUIVALENCE (YDE1(1), YDE2(2)), (VVPLU(1), YVMIN(2))

READ IN RUN INFORMATION
READ(5, 10)
READ(5, 30) HIGHT, HSTEP, XSTEP, FLARE
READ(5, 20) LPNS, KPNS, JPNS, NLMT, LPOP, NJ, INV, INT
READ(5, 30) ZAP1, GLOBE, TOLER, XE, PR, CPS, TEST
READ(5, 30) US, XMUS, RHOS, VW, UREF, TWL, TWU
10 FORMAT(80H)
20 FORMAT(12I6)
30 FORMAT(7G10.4)
C

PRINTOUT OF INPUT VARIABLES

WRITE(6,10)
WRITE(6,35) HIGHT,HSTEP,XSTEP,FLARE


WRITE(6,40) LPNS,KPNS,JPNS,NLMT,LPOP,NJ,INV,INT


WRITE(6,45) ZAP1,GLOBE,TOLER,XE,PR,CPS,TEST


WRITE(6,50) US,XMUS,RHOS,VW,UREF,TWL,TWU


C

READ(5,20) MIN,MAX,IUPDAT
WRITE(6,55) MIN,MAX,IUPDAT

55 FORMAT(5X, 'GLOBAL ITERATIONS',I3, ' TO',I3,5X, 'IUPDAT',I3)
READ(5,20) MP1,(MPC(J),J=1,MP1)
WRITE(6,60) MP1,(MPC(J),J=1,MP1)

60 FORMAT(5X, 'TOTAL NO. OF STATIONS W/ DETAIL OUTPUT = ',I3, '
(5X, 'THEY ARE --- MOUNT : ',10I6))
READ(5,20) NP1,NWALL,JSTEP,MCSTEP,NTEMP,NWALLT,NEXTRP
READ(5,30) (XP3(J),J=1,NP1)
WRITE(6,65) NP1,NWALL,JSTEP,MCSTEP,NTEMP,NWALLT,NEXTRP

65 FORMAT(5X, 'NP1 = ',I4,5X, 'NWALL = ',I4,5X, 'JSTEP = ',I4,5X, 'MCSTEP = ',I4,5X, 'NTEMP = ',I4,5X, 'NWALLT = ',I4,5X, 'NEXTRP = ',I4)
WRITE(6,70) (XP3(J),J=1,NP1)

70 FORMAT(5X, 'GLOBAL 4.6))
READ(5,20) NPRINT,LSOR1,LSOR,LEAD
READ(5,30) FAC,TOL
WRITE(6,75) NPRINT,LSOR1,LSOR,LEAD,FAC,TOL

READ(5,20) NIRROT,IRROT,NDOWN,NOBLK
NJN=NJ-1
NJ=NJ+1
READ(5,30) XBEGIN
READ(5,30) (UB(J),J=1,NJP)
READ(5,30) (VB(J),J=1,NJP)
WRITE(6,80) XBEGIN,NIRROT,IRROT,NDOWN,NOBLK
WRITE(6,85) (UB(J),J=1,NJP)
WRITE(6,90) (VB(J),J=1,NJP)


85 FORMAT(5X,'UB(J)/(5X,9G14.6))
IF(NSFULL.NE.1) GO TO 97
MSEP1=2
MSEP2=KPNS
MSEP11=2
MSEP22=KPNS

CALL STEP(XBEGIN)
KPNSM=KPNS-1
DO 100 J=1,KPNSM
XUX(J)=(X(J)+X(J+1))*0.5
100 CONTINUE
XUX(KPNS)=X(KPNS)+(X(KPNS)-X(KPNSM))*0.5

IF(IABS(NWALL).EQ.2.OR.NSTEP.EQ.1) GO TO 110
READ(5,30) (XU(J),J=1,LPOP)
READ(5,30) (YU(J),J=1,LPOP)
WRITE(6,70) (XU(J),J=1,LPOP)
WRITE(6,70) (YU(J),J=1,LPOP)
CALL SPLICO(XU,YU,LPOP,C)
CALL SFINT(XU,LPOP,XUX,KPNS,C,UREFl)
READ(5,30) (XV(J),J=1,INV)
READ(5,30) (YV(J),J=1,INV)
WRITE(6,70) (XV(J),J=1,INV)
WRITE(6,70) (YV(J),J=1,INV)
CALL SPLICO(XV,YV,INV,C)
CALL SFINT(XV,INV,X,KPNS,C,VlREF)
DO 105 J=1,KPNS
UREFl(J)=UREFl(J)/US
VlREF(J)=VlREF(J)/US
105 CONTINUE
GO TO 120
110 DO 115 J=1,KPNS
UREFl(J)=0.
VlREF(J)=0.
115 CONTINUE
120 MC1=MSEP1
MC2=MSEP2
IF(NTEMP.EQ.0) GO TO 150
XKHT=XMU*S*CPS/PR
MC1=2
MC2=KPNS
READ(5,20) MINT,MAXT,NENGY,IBL
WRITE(6,125) MINT,MAXT,NENGY,IBL
125  FORMAT(5X,'GLOBAL ITERATIONS FOR ENERGY EQN',I3,' TO',I3
      ,5X,' NERGY=',I3,5X,' IBL=',I3)
READ(5,30) (TT(1,J),J=1,NJ)
WRITE(6,130) (TT(1,J),J=1,NJ)
130  FORMAT(5X,'TT(1,J)'/ (5X,9G14.6))
DO 135 J=1,KPNS
   TWL1(J)=TWL
   TWU1(J)=TWU
135  CONTINUE
DO 137 J=1,NJ
   USEP(1,J)=UB(J)/US
137  CONTINUE
C
140  IF(NWALLT) 145,150,140
READ(5,30) (XTWL(J),J=1,INT)
READ(5,30) (YTWL(J),J=1,INT)
WRITE(6,70) (XTWL(J),J=1,INT)
WRITE(6,70) (YTWL(J),J=1,INT)
CALL SPLICO(XTWL,YTWL,INT,C)
CALL SFINT(XTWL,INT,X,KPNS,C,TWL1)
C
145  IF(NWALLT.GT.1.OR.NSTEP.EQ.1) GO TO 150
READ(5,30) (XTWU(J),J=1,INT)
READ(5,30) (YTWU(J),J=1,INT)
WRITE(6,70) (XTWU(J),J=1,INT)
WRITE(6,70) (YTWU(J),J=1,INT)
CALL SPLICO(XTWU,YTWU,INT,C)
CALL SFINT(XTWU,INT,X,KPNS,C,TWU1)
C
150  XCONV=RHOS*US/XMUS
CALL STEPY
YDE2(1)=Y(2)-Y(1)
YVMIN(1)=YDE2(1)
DO 155 J=1,NJ
   YDE1(J)=Y(J+1)-Y(J)
   YDE3(J)=YDE1(J)+YDE2(J)
   YDE12(J)=YDE1(J)*YDE2(J)
   YDE13(J)=YDE1(J)*YDE3(J)
   YDE23(J)=YDE2(J)*YDE3(J)
   YVPLU(J)=YDE3(J)*0.5
   YVT(J)=YVPLU(J)+YVMIN(J)
   DY(J)=Y(J)/XCONV
155  CONTINUE
   DY(NJP)=Y(NJP)/XCONV
   YDE1(NJP)=YDE2(NJP)
   YDE3(NJP)=YDE1(NJP)+YDE2(NJP)
   YVPLU(NJP)=YDE2(NJP)
C  NONDIMENSIONALIZE
DO 160 J=1,KPNS
   X(J)=X(J)*XCONV
160 CONTINUE
KPNSP=KPNS+1
IF (GLOBE.LE.1.) GO TO 220
IF (MINT.LE.1.) GO TO 190
DO 170 I=MC1,MC2
READ(17) (TT(I,J),J=1,NJP)
READ(16) (USEP(I,J),J=1,NJP),(VSEP(I,J),J=1,NJP)
WRITE(6,200) I,(USEP(I,J),J=1,NJP)
170 CONTINUE
REWIND 17
IF(NENGY.EQ.1) GO TO 735
GO TO 220
190 DO 195 I=MSEP11,MSEP22
READ (16) (USEP(I,J),J=1,NJP),(VSEP(I,J),J=1,NJP)
WRITE(6,200) I,(USEP(I,J),J=1,NJP)
195 CONTINUE
200 FORMAT(5X,'USEP AT MCOUNT=' ,I4,/,9X,9G14.6)
DO 215 I=2,KPNSP
READ(9) (P(I,J),J=1,NJP),(SP(I,J),J=1,NJP)
215 CONTINUE
220 DO 720 LGLOBE=MIN,MAX
C NONDIMENSIONALIZE
DO 230 J=1,NJP
U1(J)=UB(J)/US
V1(J)=VB(J)/US
F1(J)=10.
F2(J)=0.
230 CONTINUE
IF(NTEMP.EQ.0) GO TO 240
DO 235 J=1,NJ
T1(J)=TT(1,J)
235 CONTINUE
C INITIALIZING COUNTERS AND LOGIC PARAMETERS
240 JS=JSTEP
NEG=0
NPC=1
IPRT=1
MCOUNT=0
EPS=0.
C
WRITE(6,600) (DY(J),J=1,NJP)
WRITE(6,635) (U1(J),J=1,NJP)
READ(5,250) NG,NT,FAC1,FAC2
250 FORMAT(2I6,2G10.4)
WRITE(6,260) LGLOBE,NG,NT,FAC1,FAC2
260 FORMAT(/5X,'LGLOBE=' ,I4,' NG=' ,I4,' NT=' ,I4,' FAC1=' ,G14.5
      ,2X,'FAC2=' ,G14.5)
IF(LSOR1.EQ.1.AND.LGLOBE.EQ.MIN) GO TO 710
DELP= X(3)-X(2)
DELX =X(2)-X(1)
DELX  = DELXF
DELXM = DELXF
DELXUP = (X(3)-X(1))*0.5
DELXU = DELXUP
DELXUM = DELXUP

C     BEGIN COMPUTATION LOOP
C     MCOUNT = NUMBER OF STEPS IN X TAKEN.
C
270  MCOUNT = MCOUNT+1
     MCMP = MCOUNT+1
     MCMM = MCOUNT-1
     ZAP = ZAP1
     IF(MCOUNT .NE. MPC(IPRT)) GO TO 272
     ZAP = -1.
     IPRT = IPRT+1
272  JSF = JSTEP
     IF(MCOUNT .GE. MCSTEP-1) JSF = 1
     JSF = JS+1
     JS3 = JS+2
     JS4 = JS+3
     JDEL = (NJ-JSP)/4
     J1 = JSP+JDEL
     J2 = J1+JDEL
     J3 = J2+JDEL
     IF(MCOUNT .GT. KPNS-3) GO TO 275
     DELXP3 = X(MCOUNT+3)-X(MCOUNT+2)
275  DXUPP = (DELXP3+DELXPP)*0.5
     DXDIS = X(MCOUNT)/XCONV
     XUX1 = DXDIS+DELXP*0.5/XCONV
     IF(MCOUNT .NE. 1) GO TO 290
     AFLW = 0.
     DO 280 J = JSP,NJ
     AFLW = AFLW+U1(J)*YVPLU(J)
280  CONTINUE
290  WRITE(6,300) MCOUNT, DXDIS, XUX1,
     UREF1(MCOUNT), V1REF(MCOUNT), AFLW
300  FORMAT('*****MCOUNT=',I3,','***DXDIS=',G12.5
     ,3X,'XUX=',G12.5,3X,'UREF1=',G12.5,
     3X,'V1REF=',G12.5,3X,'AFLW=',G15.7)
     C     SOLVES FOR NEW U,V USING UVEL3,SOLVER,CORREC AND
     C     SOLVES FOR UPDATED PRESSURES USING POISON
C
     IF(IRROT .EQ. 0) GO TO 330
     DUDY = 2.*(UREF1(MCOUNT)-U(NJ))/YDE2(NJP)
     DELV = DUDY*DELX
     V1REF(MCOUNT) = V(NJP)+DELV
IF (MCOUNT.EQ.1) GO TO 510
CALL YHOM(IUPDAT)

C CALCULATION OF DELTASTAR(DISPLACEMENT THICKNESS)
C DST: DISPLACEMENT THICKNESS  L: LOWER WALL
C TH : MOMENTUM THICKNESS  U: UPPER WALL
C H : SHAPE FACTOR ( = DST/TH )

CALL DSTNM(1,JSP,0.,UINFL1,DSTL,THL)
HL=DSTL/THL
CFL=U1(JSP)*4./(YDE2(JSP)*UINFL1**2)
YWL=YDE2(JSP)*0.5
CFL2=(U1(JSP)/YWL-(U1(JS3)-U1(JSP))/YDE1(JSP)+
   U1(JS3)/(YWL+YDE1(JSP)))*2./(UINFL1**2)
REXL=XCONV*UINFL1*ABS(XUX1)
UINFL1=UINFL1*US
WRITE(6,340) UINFL1,DSTL,THL,HL,CFL,CFL2,REXL
340 FORMAT(3X, 'UINFL1=',G13.6,2X,'DSTL=',G12.5,2X,
   'THL=',G12.5,2X,'HL=',G12.5,2X,
   'CFL=',G12.5,2X,'CFL2=',G12.5,2X,'REXL=',G12.5)
IF(IABS(NWALL).NE.2) GO TO 460
CALL DSTNM(1,NJ,0.,UINFU1,DSTU,THU)
HU=DSTU/THU
CFU=U1(NJ)*4./(YDE1(NJ)*UINFU1**2)
YWU=YDE1(NJ)*0.5
CFU2=(U1(NJ)/YWU-(U1(NJN)-U1(NJ))/YDE2(NJ)+
   U1(NJN)/(YWU+YDE2(NJ)))*2./(UINFU1**2)
REXU=XCONV*UINFU1*ABS(XUX1)
WRITE(6,350) UINFU1,DSTU,THU,HU,CFU,CFU2,REXU
350 FORMAT(3X, 'UINFU1=',G13.6,2X,'DSTU=',G12.5,2X,
   'THU=',G12.5,2X,'HU=',G12.5,2X,
   'CFU=',G12.5,2X,'CFU2=',G12.5,2X,'REXU=',G12.5)
460 UW=U1(JSP)*US
DDXY=2.*DELX/YDE2(JSP)
WRITE(6,470) PCON,UW,U1(NJ),VW,DDXY
470 FORMAT(3X, 'PCON=',G14.6,4X,'UW=',G14.6,3X,'U1(NJ),'
   G14.6,3X,'VW=',G14.5,3X,'DX/DY=',G14.5)
IF(NTEMP.EQ.1) CALL TEMP(MINT,XKHT,IBL)

C DETERMINE PRINTOUT LOCATION
IF(U1(JSP).GT.0..OR.NEG.GE.1) GO TO 510
NEG=NEG+1
IF(NEG.EQ.1) GO TO 540
510 IF(NPC.GT.NP1.OR.DXDIS.LT.XP3(NPC)) GO TO 630
NPC=NPC+1
C WRITE(6,560) US,XMUS,RHOS,MCOUNT,XCONV
560 FORMAT('/5X,'US=' ,F8.2,2X,'XMUS=' ,G14.5,2X,'RHOS=' ,
   G14.5,2X,'MCOUNT=' ,I5,2X,'/5X,'XCONV=' ,G14.5)
C DIMENSIONALIZE.
DO 580 J=1,NJP
DU1(J)=U(J)*US
DV1(J)=V(J)*US
580 CONTINUE
WRITE(6,600) (DU(J),J=1,NJP)
600 FORMAT(/2X,'Y',/,(5X,9G14.6))
WRITE(6,610) (DV1(J),J=1,NJP)
610 FORMAT(/2X,'Ul(J)-DIMENSIONAL',/,(5X,9G14.6))
WRITE(6,620) (DV1(J),J=1,NJP)
620 FORMAT(/2X,'Vl(J)-DIMENSIONAL',/,(5X,9G14.6))
IF(NTEMP.EQ.1) WRITE(6,625) (T1(J),J=1,NJP)
625 FORMAT(/2X,'T(J)',/,(5X,9G14.6))
GO TO 640
630 JS8=JS+8
WRITE(6,635) (UL(J),J=JS,JS8)
635 FORMAT(5X,'Ul1./),(5X,9G14.6))
C UPDATING ALL VARIABLES
640 PCON3=PCON
IF(MCOUNT.GT.1) AFLOW=AFLOW-V1REF(MCOUNT)*DELXU
JS=JSF
DELXM =DELX
DELX =DELXP
DELXP =DELXPP
DELXPP=DELX3
DELXUM=DELXU
DELXU =DELXUP
DELXUP=DXUPP
DO 650 J=1,NJP
UM(J)=U(J)
VM(J)=V(J)
U(J) =UL(J)
V(J) =V1(J)
F11(J)=F1(J)
650 CONTINUE
IF(NTEMP.EQ.0.OR.MCOUNT.EQ.1) GO TO 670
DO 660 J=1,NJP
TT(MCOUNT,J)=T1(J)
660 CONTINUE
C SAFETY MEASURES
670 IF(NJ.GE.200.OR.DXDIS.GE.XE.OR.MCOUNT.GE.NLMT) GO TO 680
GO TO 270
680 WRITE(6,700)
700 FORMAT(1H1)
C IF(NTEMP.EQ.1) MINT=MINT+1
IF((LSOR.EQ.0).AND.(LGLOBE.EQ.MAX))GO TO 720
GLOBE=GLOBE+1.
710 CALL SOR
WRITE(6,700)
720 CONTINUE
DO 725 I=2,KPNSP
WRITE(10) (P(I,J),J=1,NJP), (SP(I,J),J=1,NJP)
725 CONTINUE
DO 730 I=MC1,MC2
WRITE (18) (USEP(I,J),J=1,NJP), (VSEP(I,J),J=1,NJP)
730 CONTINUE
IF(NTEMP.EQ.0) STOP
IF(MAXT.LT.MINT) GO TO 740
735 CALL ELLIP(MINT,MAXT,ZAP1,XKHT,IBL)
740 DO 750 I=2,KPNS
WRITE(19) (TT(I,J),J=1,NJP)
750 CONTINUE
STOP
END
SUBROUTINE YMOM(IUPDAT)
C
CALLING PROGRAM : MAIN
C
CALLS SUBROUTINES TO SOLVE MOMENTUM EQUATIONS,
C CORRECTS VELOCITY PROFILES AND UPDATES PRESSURE.
C
COMMON/SPARE/UOLD(100),PO(100),PHI(100),SPN(100),UC(100),
VC(100),C(4,100)
COMMON/OVER/FAC1,FAC2,FAC,NT,FLARE
COMMON/ARRAYS/F1(100),F2(100),FI1(100),F(70,100),
SP(70,100),UM(100),U(100),U1(100),USECX(100),
USECY(100),USEP(70,100),VH(100),V(100),
VI(100),VSEC1X(100),VSECY(100),VSEP(70,100),
DY(100),YDE2(100),YDE3(100),YDE12(100),
YDE13(100),YDE23(100),YVPLU(100),
YVT(100),T1(100),TT(70,100),UREF1(300),
V1REF(300),TW1(70),TU1(70)
COMMON/VAR/AFLOW,GLOBE,INV,JPNS,KPNS,LPNS,MCM,MCOUNT ,
MCP,NJN,NJ,NJP,NTEMP,NWALL,NWALLT,PCON,RC,
PCON3,TOLER,NCNV,JS,JSP,JS3,JS4,JSSTEP,MCSTEP ,
HIGHT,TEST
COMMON/XGRID/X(300),DELXM,DELX,DELXP,DELXPP,DELXUP,
DELXUM,DELXU,DELXUPP
COMMON/GLOBAL/ICOUNT,MIN,MAX,ZAP
COMMON/BLOC1/TOL,DELP,BLOCK
COMMON/LEAD1/LEAD,FR
COMMON/CONV1/J1,J2,J3,PJ1,PJ2,PJ3,PNJ,JM,EPSU
COMMON/BDY/NDOWN
COMMON/SEPER/MSEP1,MSEP2,NSFULL,MCDOWN,NHYBRD,MC1,MC2
COMMON/IRR/NOBLK,JSF
JSF1=JSF+1
IF(MCOUNT.GT.2) GO TO 30
IF(LEAD.EQ.1) U(JS)=U(JSP)
DELP=0.
30 CALL COEFF
IF(GLOBE.GT.1.) GO TO 50
DO 40 J=JSF1,NJ
P(MCP, J)=P(MCOUNT,NJ)
40 CONTINUE
GO TO 140
C
50 IF(MCOUNT.NE.KPNS) GO TO 120
JCTR=(NJ+JSP)/2
IF(NDOWN) 100,120,80
80 A=P(MCP,JCTR)-P(KPNS,JCTR)
DO 90 J=JSP,NJ
P(MCP, J)=P(KPNS, J)+A
90 CONTINUE
GO TO 120
100 DO 110 J=JSP,NJ
P(MCP, J)=P(MCOUNT, J)
110 CONTINUE
CONTINUE
DO 130 J=JSFL,NJ
P(MCP,J)=P(MCP,J)+DELP
CONTINUE
CALL UVEL3
C
EPSU=0.
PJ1=(U1(J1)-USECX(J1))/U1(J1)
PJ2=(U1(J2)-USECX(J2))/U1(J2)
PJ3=(U1(J3)-USECX(J3))/U1(J3)
PNJ=(U1(NJ)-USECX(NJ))/U1(NJ)
DO 150 J=JSP,NJ
PERCJ=(U1(J)-USECX(J))/U1(J)
IF (ABS(PERCJ).LE.ABS(EPSU)) GO TO 150
EPSU=PERCJ
JM=J
CONTINUE
WRITE(6,160) EPSU,JM,PJ1,J1,PJ2,J2,PJ3,J3,PNJ
FORMAT (3X,'EPSU'G13.5,2X,'JM',I3,2X,'PJ1',G13.5,2X,'J1',I3,2X,'PJ2',G13.5,2X,'J2',I3,2X,'PJ3',G13.5,2X,'J3',I3,2X,'PNJ',G13.5/) 
C
CALL SOLVER
CALL CORREC(IUPDAT)
IF (NSFULL.NE.0) GO TO 180
IF (MCOUNT.LT.MC1.OR.MCOUNT.GT.MC2) GO TO 200
DO 190 J=1,NJP
UOLD(J)=USEP(MCOUNT,J)
USEP(MCOUNT,J)=U1(J)
VSEP(MCOUNT,J)=V1(J)
CONTINUE
C
IF(ZAP.EQ.-1.) GO TO 270
ILK=MCOUNT/5
IF(IABS(5*ILK-MCOUNT).GE.001) RETURN
WRITE(6,300) (F1(J),J=1,NJP)
WRITE(6,310) (P(MCH,J),J=1,NJP)
WRITE(6,320) (P(MCOUNT,J),J=1,NJP)
WRITE(6,330) (P(MCP,J),J=1,NJP)
FORMAT(5X,'F1(J)''/(5X,9G14.6))
FORMAT(5X,'P(J)''/(5X,9G14.6))
FORMAT(5X,'P1(J)''/(5X,9G14.6))
FORMAT(5X,'F2(J)''/(5X,9G14.6))
RETURN
END
SUBROUTINE COEFF

CALLING PROGRAM : YMOM

CALCULATES COEFFICIENTS FOR CONVfectIVE TERMS IN
MOMENTUM EQUATIONS.

COMMON/SPARE/UOLD(100), PO(100), PHI(100), SPN(100), DP(100),
VC(100), C(4,100)
COMMON/ARRAYS/F1(100), F2(100), FI1(100), P(70,100),
SP(70,100), UM(100), U(100), U1(100), USECX(100),
USECY(100), USEP(70,100), VM(100), V(100),
V1(100), VSEC1X(100), VSECY(100), VSEP(70,100),
DY(100), YDE2(100), YDE3(100), YDE12(100),
YDE13(100), YDE23(100), YVPLU(100),
YUT(100), T1(100), TT(70,100), UREF1(300),
VREF(300), TWL1(70), TWU1(70)
COMMON/VAR/AFLOW,GLOBE, INV, JPNS, KPNS, LPNS, MCM, MCOUNT,
MCP, NJN, NJP, NTEMP, NWALL, PCON, RC,
PCON3, TOLER, XCONV, JS, JSP, JS3, JS4, JSTEP, MCSTEP,
HIGHT, TEST
COMMON/XGRID/X(300), DELXM, DELX, DELXP, DELXPP, DELXP3,
DELXUM, DELXU, DELXUP, DXUPP
COMMON/XRATIO/RATIOX, RATVX, SECDX, SECVDX,
RATXR, RATVXR, SECDXR, SECVXR
COMMON/SEPER/MSEP1, MSEP2, NSFULL, MCDOWN, NYBRD, MC1, MC2
COMMON/TCASE/NSTEP
X1=DELXP/(4.*DELXU)
X2=DELX/(4.*DELXU)

IF (MCOUNT.LE.2) GO TO 10
RATIOX=DELXU/DELXUM
RATVX = DELX/DELXM
SECDX = 1./DELXU+1./(DELXU+DELXUM)
SECVDX=1./DELX+1./(DELX+DELXM)
RATXR=DELXUP/DXUPP
RATVXR = DELXP/DELXPP
SECDXR = 1./DELXUP+1./(DELXUP+DXUPP)
SECVXR=1./DELXP+1./(DELXP+DELXPP)

10 IF (GLOBE.EQ.1..OR.MCOUNT.LT.MSEP1.OR.MCOUNT.GT.MSEP2) GO TO 60
DO 20 J=JSP,NJ
USECX(J)=USEP(MCOUNT,J)
USECY(J)=(UOLD(J)+UOLD(J-1))*X1+(USEP(MCOUNT,J)+
USEP(MCOUNT,J-1)))*X2
VSEC1X(J)=(VSEP(MCOUNT,J)+VSEP(MCP,J))*0.5
VSECY(J)=VSEC1X(J)
CONTINUE

IF (MCOUNT.NE.MSEP1) GO TO 40
DO 30 J=JSP,NJ
USECY(J)=(U(J)+U(J-1))*X1+(USEP(MSEP1,J)+
USEP(MSEP1,J-1))*X2
CONTINUE

40 IF (MCOUNT.NE.MSEP2) GO TO 110
X3 = DELXP / (2. * DELX)
DO 50 J = JSP, NJ
VSEC1X(J) = (VSEP(MSEP2, J) - V(J)) * X3 + VSEP(MSEP2, J)
50 CONTINUE
GO TO 110
60 IF (MCOUNT .GT. 2) GO TO 80
DO 70 J = JSP, NJ
USECX(J) = U(J)
USECY(J) = (U(J) + U(J - 1)) * 0.5
VSEC1X(J) = V(J)
VSECY(J) = V(J)
70 CONTINUE
GO TO 110
80 X4 = RATVX + DELXP / (2. * DELXM)
DO 90 J = JSP, NJ
USECX(J) = (U(J) - UM(J)) * RATIOX + U(J)
USECY(J) = (U(J) + U(J - 1)) * X1 + (USECX(J) + USECX(J - 1)) * X2
VSEC1X(J) = (V(J) - VM(J)) * X4 + V(J)
VSECY(J) = (V(J) - VM(J)) * RATVX + V(J)
90 CONTINUE
IF (MCOUNT .NE. MCSTEP) GO TO 110
DO 100 J = JSP, JSTEP
USECX(J) = 0.
USECY(J) = 0.
VSEC1X(J) = 0.
VSECY(J) = 0.
100 CONTINUE
110 USECX(JSP) = -USECX(JSP)
USECY(JSP) = 0.
USECX(NJP) = 2.*UREF1(MCOUNT) - USECX(NJ)
VSEC1X(NJP) = (V1REF(MCOUNT) + V1REF(MCSTEP)) * 0.5
IF (MCOUNT.EQ.KPNS) VSEC1X(NJP) = V1REF(MCOUNT)
IF (NSTEP.EQ.1) USECX(NJP) = USECX(NJ)
RETURN
END
SUBROUTINE UVEL3
CALLING PROGRAM : YMOM
SOLVES X-MOMENTUM EQUATION TO OBTAIN TENTATIVE U-
VELOCITY PROFILE. DEPENDING ON INPUTS, EITHER THE
PPNS OR FULL NS EQUATIONS ARE SOLVED.
BLOCK ADJUSTMENT FOR PRESSURE W/O SECANT PROCEDURE.

DIMENSION YDE1(100),VSEC2X(100)
COMMON/OVER/FAC1,FAC2,FAC,NT,FLARE
COMMON/THOMAS/AA(100),BB(100),CC(100),DD(100),CC1(100),DV1(100)
COMMON/ARRAYS/F1(100),F2(100),FI1(100),P(70,100),
SP(70,100),UM(100),U(100),U1(100),USECX(100),
USECY(100),USEP(70,100),VM(100),V(100),
V1(100),VSEC1X(100),VSECY(100),VSEP(70,100),
DY(100),YDE2(100),YDE3(100),YDE12(100),
YDE13(100),YDE23(100),YVPLU(100),
VVT(100),T1(100),TT(70,100),UREF1(300),
V1REF(300),TWL1(70),TWU1(70)
COMMON/VAR/AFLOW,GLOBE,INV,JPNS,LPNS,MCM,MCOUNT
,MC1,NJN,NJ,NJF,NTEMP,NWALL,NWALLT,PCON,RC,
PCON3,TOLRC,XCONV,JS,JS3,JS4,JSTEP,MCSTEP
,HIGHT,TEST
COMMON/XGRID/X(300),DELXM,DELX,DELXP,DELXPP,DELXP3,
DELXUM,DELXU,DELXUP,DXUPP
COMMON/XRATIO/RATIOX,RATVX,SECDX,SECVDX,
RATXR,RATVXR,SECDXR,SECVXR
COMMON/BLOC1/TOL,DELP,BLOCK
COMMON/SEPER/MSEP1,MSEP2,NSFULL,MCDOWN,NHYBRD,MC1,MC2
COMMON/IRR/NOBLK,JSF
COMMON/TCASE/NSTEP
EQUIVALENCE (YDE1(1),YDE2(2)),(VSEC2X(1),VSEC1X(2))

C=FLARE
I0CONT=0
I2CONT=0
NLAT=0
NLATN=0
X1=(DELXUM+DELXU)/(DELXUM*DELXU)
X2=RATIOX/(DELXUM+DELXU)
DO 225 J=JSP,NJ
CC1(J)=-1.
NEGPOS=0
UTEMP=USECX(J)
IF (MCOUNT.LT.2.OR.GLOBE.EQ.1.) GO TO 90
IF (MCOUNT.LT.MSEP1.OR.MCOUNT.GT.MSEP2-2) GO TO 90
IF (USECX(J).GE.0.) GO TO 100
NEGPOS=2
I2CONT=I2CONT+1
GO TO 110
90 IF (NSFULL.EQ.1) GO TO 100
IF (USECX(J).LT.0.) USECX(J)=C*ABS(USECX(J))
100  I0CONT=I0CONT+1
110  RM1=VSEC2X(J)*YDE2(J)
       RM2=VSEC1X(J)*YDE1(J)
       IF (RM1.GT.1.9) GO TO 180
       IF (RM2.LT.-1.9) GO TO 150
       AA(J)=(RM1-2.)/YDE13(J)
       BB(J)=-(RM2+2.)/YDE23(J)
       DD(J)=2./YDE12(J)+RM2/YDE23(J)-RM1/YDE13(J)
       GO TO 183
C
150  W=-1.9/RM2
       NLATN=NLATN+1
       IF (NHYBRD.EQ.0) W=0.
       AA(J)=(VSEC2X(J)*(YDE3(J)-W*YDE1(J))-2.)/YDE13(J)
       BB(J)=-(RM2*W+2.)/YDE23(J)
       DD(J)=(2./YDE2(J)-VSEC2X(J))/YDE1(J)+
              W*(RM2+RM1)/YDE23(J)
       GO TO 183
C
180  W=1.9/RM1
       NLAT=NLAT+1
       IF (NHYBRD.EQ.0) W=0.
       AA(J)=(RM1*W-2.)/YDE13(J)
       BB(J)=-(VSEC1X(J)*(YDE3(J)-W*YDE2(J))+2.)/YDE23(J)
       DD(J)=(2./YDE1(J)+VSEC1X(J))/YDE2(J)-
              W*(RM2+RM1)/YDE13(J)
       GO TO 183
C
183  IF (MCOUNT.LE.2) GO TO 185
       IF (NEGPOS.EQ.2) GO TO 200
       IF (MCOUNT.NE.MCSTEP.OR.J.GT.JSTEP) GO TO 190
185  DD(J)=USECX(J)/DELXU+DD(J)
       CC(J)=-(P(MCP,J)-P(MCOUN, J))/DELXP+USECX(J)*U(J)/DELXU
       GO TO 220
190  DD(J)=USECX(J)*SECDX+DD(J)
       CC(J)=-(P(MCP,J)-P(MCOUN,J))/DELXP+USECX(J)*(X1*U(J)-X2*UM(J))
       GO TO 220
200  DD(J)=-USECX(J)*SECDX+DD(J)
       CC(J)=-(P(MCP,J)-P(MCOUN,J))/DELXP-USECX(J)*((DELXUP+DXUPP)
              /(DELXUP*DXUPP)*USEP(MCP,J)-
              RATXR/(DELXUP+DXUPP)*USEP(MCOUNT+2,J))
C
220  USECX(J)=UTEMP
       IF (NSFULL.NE.1.OR.MCOUN, EQ.KPNS) GO TO 225
       DD(J)=DD(J)+2./(DELXU*DELXUP)
       CC(J)=CC(J)+(USEP(MCP,J)/DELXUP+U(J)/DELXU)*
              2./(DELXU+DELXUP)
225  CONTINUE
       BB(NJP)=1.0
       IF (NSTEP.EQ.1) BB(NJP)=-1.0
       AA(NJP)=0.0
DD(NJP)=1.0
CC(NJP)=2.*UREF1(MCOUNT)
AA(JSP)=AA(JSP)-YDE2(JSP)/(YDE1(JSP)*YVPLU(JSP)*
    (YDE3(JSP)+YDE1(JSP)))
BB(JSP)=BB(JSP)+2./YDE2(JSP)
DD(JSP)=DD(JSP)+2./YDE12(JSP)-BB(JSP)
BB(JSP)=0.
IF(IABS(NWALL).LT.2) GO TO 230
AA(NJ)=AA(NJ)+2./YDE13(NJ)
BB(NJ)=BB(NJ)-YDE1(NJ)/(YDE2(NJ)*YVPLU(NJ)*
    (YDE3(NJ)+YDE2(NJ)))
DD(NJ)=DD(NJ)+2./YDE12(NJ)
230  DD(NJ)=DD(NJ)-AA(NJ)*BB(NJP)/DD(NJP)
CC(NJ)=CC(NJ)-AA(NJ)*CC(NJP)/DD(NJP)
AA(NJ)=0.
IF(NOBLK.EQ.0) GO TO 235
CALL SY(JSP,NJ,BB,DD,AA,CC,U1)
GO TO 275
235  DO 240 J=JS3,NJ
    JM=J-1
    R=BB(J)/DD(JM)
    DD(J)=DD(J)-R*AA(JM)
    CC(J)=CC(J)-R*CC(JM)
    CCl(J)=CCl(J)-R*CCl(JM)
240  CONTINUE
DO 250 J=JSP,NJN
    K=NJN-J+JSP
    KP=K+1
    R=AA(K)/DD(KP)
    CC(K)=CC(K)-R*CC(KP)
    CCl(K)=CCl(K)-R*CCl(KP)
250  CONTINUE
FLOW1=0.
FLOW2=0.
DO 260 J=JSP,NJ
    CC(J)=CC(J)/DD(J)
    CCl(J)=CCl(J)/DD(J)
    FLOW1=FLOW1+CC(J)*YVPLU(J)
    FLOW2=FLOW2+CCl(J)*YVPLU(J)
260  CONTINUE
PCON=(AFLOW-FLOW1)/FLOW2
FLOW=FLOW1+FLOW2*PCON
PBLK=PCON*DELXP
JSFl=JSF+1
DO 265 J=JSF1,NJ
    P(MCP,J)=P(MCP,J)+PBLK
265  CONTINUE
DELP=DELP+PBLK
DO 270 J=JSP,NJ
    U1(J)=CC(J)+CCl(J)*PCON
CONTINUE

U1(NJP)=CC(NJP)-BB(NJP)*U1(NJ)
U1(JS)=-U1(JSP)
IF (MCOUNT.NE.MCSTEP-1) GO TO 280
U1(I)=U1(JS)
U1(JS)=0.

WRITE(6,290) PBLK,DELP,I0CONT,I2CONT,NLAT,NLATN

FORMAT(5X,'PBLK=',G14.6,2X,'DELP=',G14.6,2X,'OCOUNT=',I3,2X,'2COUNT=',I3,'IN UVEL3 UPWIND USED',I3,' TIMES W/ +V AND',I3,' TIMES W/ -V')

RETURN
END
SUBROUTINE SOLVER
C
CALLING PROGRAM : YMOM
C
SOLVES Y-MOMENTUM EQUATION TO OBTAIN TENTATIVE V-
C
VELOCITY PROFILE.
C
DIMENSION YVMIN(100)
COMMON/OVER/FAC1,FAC2,FAC,NT,FLARE
COMMON/THOMAS/AA(100),BB(100),CC(100),DD(100),DU1(100),DV1(100)
COMMON/ARRAYS/F1(100),F2(100),F11(100),P(70,100),
SP(70,100),UM(100),U(100),U1(100),USECX(100),
USECY(100),USEP(70,100),VM(100),V(100),
V1(100),VSECX(100),VSECY(100),VSEP(70,100),
DY(100),YDE2(100),YDE3(100),YDE12(100),
YDE13(100),YDE23(100),YVPLU(100),
YVT(100),T1(100),TT(70,100),UREF1(300),
VREF(300),TWL1(70),TWU1(70)
COMMON/VAR/AFLOW,GLOBE,INV,JPNS,KPNS,LPNS,MCM,MCOUNT,
,MC,MCP,NJN,NJ,NJP,NTMP,NWALL,NWALLT,PCON,RC,
PCON3,TOLER,XCONV,JS,JSP,JS3,JS4,JSTEP,MCSTEP,
,HIG T,TEST
COMMON/XGRID/X(300), DELXM,DELX,DELXP,DELXPP,DELXPP3,
DELXUM,DELXU,DELXUP,DXUPP
COMMON/XRATIO/RATIOX,RATVX,SECDX,SECVDX,
RATXR,RATVXR,SECDXR,SECVXR
COMMON/DECIDE/NIRROT
COMMON/SEPER/MSEP1,MSEP2,NSFULL,MCDOWN,NHYBRD,MC1,MC2
EQUIVALENCE (YVPLU(1),YVMIN(2))
NLATV=0
NLATVN=0
C=FLARE
X1=(DELXM+DELX)/(DELXM*DELX)
X2=RATVX/(DELXM+DELX)
DO 240 J=JS3,NJ
NEGPOS=0
TEMP=USECY(J)
IF (MCOUNT.LE.2.OR.GLOBE.EQ.1.) GO TO 60
IF (MCOUNT.LT.MSEP1.OR.MCOUNT.GT.MSEP2-2) GO TO 60
IF (USECY(J).LT.0.) NEGPOS=2
GO TO 70
60 IF (NSFULL.EQ.1.) GO TO 70
IF (USECY(J).LT.0.) USECY(J)=C*ABS(USECY(J))
70 RM1=VSECY(J)*YVMIN(J)
RM2=VSECY(J)*YVPLU(J)
IF (RM1.GT.1.9) GO TO 180
IF (RM2.LT.-1.9) GO TO 150
AA(J)=(RM1-2.)/(YVPLU(J)+YVT(J))
BB(J)=-(RM2+2.)/(YVT(J)+YVMIN(J))
DD(J)=(VSECY(J)*(YVPLU(J)-YVMIN(J))+2.)/(YVPLU(J)*YVMIN(J))
GO TO 183
C 150 W=-1.9/RM2
NLATVN=NLATVN+1
IF (NHYBRD.EQ.0) W=0.
AA(J)=(VSECY(J)*(YVT(J)-W*YVPLU(J))-2.)/(YVT(J)*YVPLU(J))
BB(J)=-(W*RM2+2.)/(YVT(J)*YVMIN(J))
DD(J)=(VSECY(J)*(W*YVPLU(J)-YVMIN(J))+2.)/(YVPLU(J)*YVMIN(J))
GO TO 183
C 180 W=1.9/RM1
NLATV=NLATV+1
IF (NHYBRD.EQ.0) W=0.
AA(J)=(RM1*W-2.)/(YVT(J)*YVPLU(J))
BB(J)=-(VSECY(J)*(YVT(J)-W*YVMIN(J))+2.)/(YVT(J)*YVMIN(J))
DD(J)=(VSECY(J)*(YVPLU(J)-W*YVMIN(J))+2.)/(YVPLU(J)*YVMIN(J))
GO TO 183
C 183 IF (MCOUNT.LE.2) GO TO 185
IF(NEGPOS.EQ.2) GO TO 200
IF(MCOUNT.NE.MCSTEP.OR.J.GT.JSTEP+1) GO TO 190
185 DD(J)=USECY(J)/DELX+DD(J)
CC(J)=-(P(MCOUNT,J)-P(MCOUNT,J-1))/YDE2(J)+USECY(J)*V(J)/DELX
GO TO 230
190 DD(J)=USECY(J)*SECVDX/DD(J)
CC(J)=-(P(MCOUNT,J)-P(MCOUNT,J-1))/YDE2(J)+USECY(J)*
\(\text{X1} \times V(J) - X2 \times VM(J)\)
GO TO 230
200 DD(J)=-USECY(J)*SECVXR/DD(J)
CC(J)=-(P(MCOUNT,J)-P(MCOUNT,J-1))/YDE2(J)-USECY(J)*
\(((\text{DELXP}+\text{DELXPP})/(\text{DELXP} \times \text{DELXPP}) \times \text{VSEP}(MCP,J) - \\
\text{RATVXR} \times (\text{DELXP}+\text{DELXPP}) \times \text{VSEP}(MCOUNT+2,J))\)
C 230 USECY(J)=TEMP
IF (NSFULL.LT.1.OR.MCOUNT.EQ.KPNS) GO TO 240
DD(J)=DD(J)+ 2./(DELX*DELXP)
CC(J)=CC(J)+(VSEP(MCP,J)/DELXP+V(J)/DELX)/DELXU
240 CONTINUE
CC(NJP)=V1REF(MCOUNT)
IF (NIRROT.NE.2)
CC(NJP)=(UREF1(MCOUNT-1)-UREF1(MCOUNT))/DELXU*
\(\text{YVPLU(NJ)} \times 0.5 + \text{V1REF(MCOUNT)}\)
BB(NJP)=0.0
DD(NJP)=1.0
AA(NJP)=0.0
BB(JS3)=0.
CALL SY(JS3,NJP,BB,DD,AA,CC,V1)
V1(JSP)=0.
V1(JS)= -V1(JSP)
IF (NIRROT.EQ.2) V1REF(MCOUNT)=V1(NJP)
IF (NLATV.NE.0.OR.NLATVN.NE.0) WRITE (6,290) NLATV,NLATVN
290 FORMAT(5X,'IN SOLVER UPWIND USED',I3,
      ' TIMES WITH +V AND',I3,' TIMES USED WITH -V')
RETURN
END
SUBROUTINE CORREC(IUPDAT)
C       CALLING PROGRAM : YMOM
C       CORRECTS TENTATIVE VELOCITY PROFILES TO SATISFY
C       LOCAL CONTINUITY OF MASS FLOW AND ESTIMATES THE
C       NEW PRESSURE GRADIENTS.
C
DIMENSION YDE1(100),YVMIN(100),VSEC2X(100)
COMMON/SPARE/UOLD(100),Q(100),PHI(100),SPN(100),UC(100),
       VC(100),CM(4,100)
COMMON/THOMAS/AA(100),BB(100),CC(100),DD(100),DU1(100),DV1(100)
COMMON/ARRAYS/F1(100),F2(100),FI1(100),P(70,100),
       SP(70,100),UM(100),U(100),U1(100),USECX(100),
       USECY(100),USEP(70,100),VM(100),V(100),
       V1(100),VSEC1X(100),VSECY(100),VSEP(70,100),
       DY(100),YDE2(100),YDE3(100),YDE12(100),
       YDE13(100),YDE23(100),YVPLU(100),
       YVT(100),T1(100),TT(70,100),UREF1(300),
       V1REF(300),TWL1(70),TWU1(70)
COMMON/VAR/AFLOW,GLOBAL,INV,JPNS,KPNS,LPNS,MCM,MCOUNT
       ,MCP,NJN,NJP,NTIM,NEW,NEWLT,PON,RC,
       PCON3,TOLER,CONV,JS,JS3,JS4,JSTEP,MCSTEP
       ,HIGHT,TEST
COMMON/XGRID/X(300),DELXM,DELX,DELXP,DELXPP,DELXP3,
       DELXUM,DELXU,DELXUP,DXUPP
COMMON/GLOBAL/ICOUNT,MIN,MAX,ZAP
COMMON/OVER/FAC1,FAC2,FAC,NT,FLARE
COMMON/EXT/LOW,NEXTREP
COMMON/SEPER/MSEP1,MSEP2,NSFULL,MCDOWN,NHYBRD,MC1,MC2
COMMON/TCASE/NSTEP
EQUIVALENCE (YDE1(1),YDE2(2)),(YVPLU(1),YVMIN(2)),
       (VSEC2X(1),VSEC1X(2))
C
NLAT=0
NLATN=0
NLATV=0
NLATVN=0
XL=1.0/(DELXU*DELXP)
C
IF (GLOBE.GT.1.) GO TO 30
IF (MCOUNT.EQ.2) PCON3=0.0
PA=-PCON3
IF (MCOUNT.NE.KPNS) PA=PA+PCON
PA=PA/DELXU
DO 10 J=JSP,NJ
SP(MCOUNT,J)=PA
CONTINUE
IF (MCOUNT.NE.MCSTEP) GO TO 30
PA=PCON/DELXU
DO 20 J=JSP,JSTEP
SP(MCSTEP,J)=PA

C
20 CONTINUE
C
30 IF(MCOUNT.GT.2) GO TO 40
DO 35 J=1,NJ
UC(J)=0.
VC(J)=0.
PHI(J)=0.
Q(J)=0.
SPN(J)=0.
35 CONTINUE
40 AMASS=0.
AMR=0.
FLOW=0.
DO 50 J=JSP,NJ
AMJ=(UI(J)-U(J))/DELXU+(V1(J+1)-V1(J))/YVPLU(J)
Q(J)=AMJ*YVPLU(J)*DELXU
BB(J)=2./YDE23(J)
DD(J)=-X1-2./YDE12(J)
AA(J)=2./YDE13(J)
CC(J)=-AMJ
AMR=AMR+ABS(Q(J))
AMASS=Q(J)+AMASS
FLOW=FLOW+Ul(J)*YVPLU(J)
50 CONTINUE
FLOW=FLOW+VlREF(MCOUNT)*DELXU
AMR=AMR/DELXU
IF(ZAP.LE.0.) WRITE(6,560) (Q(J),J=1,NJP)
WRITE(6,75) AM,AMR,AMASS,FLOW
75 FORMAT(5X,'AM','G15.7',' AMR','G15.7',' AMASS','
G15.7,' FLOW','G15.7)
DD(JSP)=DD(JSP)+BB(JSP)
BB(JSP)=0.
DD(NJ)=DD(NJ)+AA(NJ)
AA(NJ)=0.
CALL SX(JSP,NJ,BB,DD,AA,CC,PHI)
PHI(JS)=PHI(JSP)
PHI(NJP)=PHI(NJ)
C
C NOW TO EVALUATE THE VELOCITY CORRECTIONS
C
DO 80 J=JSP,NJ
UC(J)=-PHI(J)/DELXP
VC(J)=-(PHI(J)-PHI(J-1))/YDE2(J)
U1(J)=U1(J)+FAC2*UC(J)
V1(J)=V1(J)+FAC2*VC(J)
80 CONTINUE
UC(JS)=-UC(JSP)
U1(JS)=-U1(JSP)
U1(NJP)=U1(NJ)
IF(NSTEP.EQ.0) U1(NJP)=2*UREF1(MCOUNT)-U1(NJ)
IF(IUPDAT.EQ.O) GO TO 105
X2=DELX/(4.*DELXU)
X3=DELXP/(4.*DELXU)
DO 82 J=JSP,NJ
USECX(J)=Ul(J)
USECY(J)=(Ul(J)+Ul(J-1))*X2+(U(J)+U(J-1))*X3
VSECY(J)=V1(J)
82 CONTINUE
USECY(JSP)=O.
IF(MCOUNT.GE.MSEP1-1.AND.MCOUNT.LE.MSEP2-1) GO TO 95
X4=0.5*DELXP/DELX
DO 85 J=JSP,NJ
VSEC1X(J)=(V1(J)-V(J))*X4+V1(J)
85 CONTINUE
IF(MCOUNT.NE.MCSTEP) GO TO 105
JSTEP=JSTEP+1
XA=1.+DELXP/DELX
DO 90 J=2,JSTEP
VSEC1X(J)=V1(J)*XA
90 CONTINUE
GO TO 105
95 DO 100 J=JSP,NJ
VSEC1X(J)=(V1(J)+VSEP(MCP,J))*0.5
100 CONTINUE
105 IF(MCOUNT.NE.MCSTEP-1) GO TO 140
Ul(1)=Ul(JS)
U1(JS)=O.
C
140 C=FLARE
DO 380 J=JSP,NJ
NEGPOX=O
NEGPOY=O
TEMP3=USECX(J)
COEFFS=USECY(J)
IF (MCOUNT.LE.2.OR.GLOBE.EQ.1.) GO TO 230
IF (MCOUNT.LT.MSEP1.OR.MCOUNT.GT.MSEP2-2) GO TO 230
IF (USECX(J).LT.0.) NEGPOX=2
IF (USECY(J).LT.0.) NEGPOY=2
GO TO 240
230 IF (NSFULL.EQ.1) GO TO 240
IF (USECX(J).LT.0.) USECX(J)=C*ABS(USECX(J))
IF (USECY(J).LT.0.) USECY(J)=C*ABS(USECY(J))
240 RM1X=VSEC2X(J)*YDE2(J)
RM2X=VSEC1X(J)*YDE1(J)
RM1Y=VSECY(J)*YMIN(J)
RM2Y=VSECY(J)*YVPLU(J)
IF (RM1X.GT.1.9) GO TO 270
IF (RM2X.LT.-1.9) GO TO 260
W=1.
A=O.
B = 0.
GO TO 278

260 W = -1.9/RM2X
A = 0.
B = 1.
NLATN = NLATN + 1
GO TO 275

270 W = 1.9/RM1X
A = 1.
B = 0.
NLAT = NLAT + 1

275 IF (NHYBRD . EQ. 0) W = 0.
C

278 IF (J . NE. JSP . AND. J . NE. NJ) . OR.
(IABS(NWALL) . LE. 1 . AND. J . EQ. NJ)) GO TO 280
IF (NWALL . LT. 1) GO TO 279
CALL SEC(1, J, FIRST, SECDER)
F1(J) = -(VSEC2X(J) + VSEC1X(J)) * 0.5 * FIRST + SECDER
GO TO 282

279 IF (J . EQ. NJ) GO TO 999

F1(JSP) = -2. * U1(J) / YDE2(J) * (RM2X * W / YDE3(J) + (1. - W) * A * VSEC1X(J)) - 8. * U1(J) / (YDE12(J) + YDE23(J)) - (U1(J + 1) - U1(J)) / YDE1(J) * (RM1X * W / YDE3(J) + (1. - W) * B * VSEC2X(J) - 4. / (YDE3(J) + YDE1(J)))

GO TO 282

999 F1(NJ) = -(U1(J) - U1(J - 1)) / YDE2(J) * (RM2X * W / YDE3(J) + (1. - W) * A * VSEC1X(J)) - 4. / (YDE3(J) + YDE2(J)) - (U1(J + 1) - U1(J)) / YDE1(J) * (RM1X * W / YDE3(J) + (1. - W) * B * VSEC2X(J) - 4. / (YDE3(J) + YDE1(J)))

GO TO 282

280 F1(J) = -(U1(J) - U1(J - 1)) / YDE2(J) * ((RM2X * W + 2.) / YDE3(J) + (1. - W) * A * VSEC1X(J)) - (U1(J + 1) - U1(J)) / YDE1(J) * ((RM1X * W - 2.) / YDE3(J) + (1. - W) * B * VSEC2X(J))

GO TO 282

282 IF (MCOUNT . LE. 2) GO TO 285
IF (NEGPOX . EQ. 2) GO TO 300
IF (MCOUNT . NE. MCSTEP . OR. J . GT. JSTEP) GO TO 290

285 F1(J) = -USECX(J) * (U1(J) - U(J)) / DELXU + F1(J)
GO TO 310

290 F1(J) = -USECX(J) * ((U1(J) - U(J)) / DELXU - (U(J) - UM(J)) / DELXUM + (U(J) - UM(J)) / (DELXUM + DELXU)) + F1(J)
GO TO 310

300 F1(J) = USECX(J) * ((U1(J) - USEP(MCP, J)) / DELXUP - (USEP(MCP, J) - USEP(MCOUNT + 2, J)) / DXUPP + (U1(J) - USEP(MCOUNT + 2, J)) / (DELXUP + DXUPP)) + F1(J)
C

310 USECX(J) = TEMP3
IF (RM1Y . GT. 1.9) GO TO 330
IF (RM2Y . LT. -1.9) GO TO 320
W = 1.
A = 0.
B=0.
GO TO 338
320 W=1.9/RM2Y
A=0.
B=1.
NLATVN=NLATVN+1
GO TO 335
330 W=1.9/RMIY
A=1.
B=0
NLATV=NLATV+1
335 IF (NHYBRD.EQ.0) W=0.
C
338 IF(NWALL.LT.1) GO TO 340
IF(J.NE.JS3.AND.(NWALL.NE.2.OR.J.NE.NJ)) GO TO 340
CALL SEC(2,J,FIRST,SECDER)
F2(J)=-VSECY(J)*FIRST+SECDER
GO TO 342
340 F2(J)=-(V1(J+1)-V1(J))/YVPLU(J)*
    ((RM1Y*W-2.)/YVT(J)+(1.-W)*B*VSECY(J))-(V1(J)-V1(J-1))/
    YVMIN(J)*((RM2Y*W+2.)/YVT(J)+(1.-W)*A*VSECY(J))
342 IF (MCOUNT.LE.2) GO TO 345
IF (NEGPOY.EQ.2) GO TO 360
IF (MCOUNT.NE.MCSTEP.OR.J.GT.JSTEP+1) GO TO 350
345 F2(J)=-USECY(J)*(V1(J)-V(J))/DELX+F2(J)
GO TO 370
350 F2(J)=-USECY(J)*((V1(J)-V(J))/DELX-(V(J)-VM(J))/DELXM+
    (V1(J)-VM(J))/(DELXM+DELX))+F2(J)
GO TO 370
360 F2(J)=USECY(J)*((V1(J)-VSEP(MCP,J))/
    DELXP-(VSEP(MCP,J)-VSEP(MCOUNT+2,J))/DELXPP+
    (V1(J)-VSEP(MCOUNT+2,J))/(DELXP+DELXPP))+F2(J)
370 USECY(J)=COEFFS
IF (NSFULL.NE.1.OR.MCOUNT.EQ.KPNS) GO TO 380
F1(J)=F1(J)+((USEP(MCP,J)-U1(J))/DELXUP-
    (U1(J)-U(J))/DELXU)*2./(DELXU+DELXUP)
F2(J)=F2(J)+((VSEP(MCP,J)-V1(J))/DELXP-
    (V1(J)-V(J))/DELX)/DELXU
380 CONTINUE
C
IF (NEXTRP.EQ.0) GO TO 420
IF (NWALL+1) 405,410,420
405 F1(NJ)=F1(NJN)+(F1(NJN)-F1(NJ-2))*YDE1(NJN)/YDE2(NJN)
F2(NJP)=F2(NJP)+(F2(NJP)-F2(NJPN))*YVPLU(NJP)/YVMIN(NJP)
410 F1(JSP)=-F1(JS3)+F1(JS3)-F1(JS4))*YDE2(JS3)/YDE1(JS3)
F2(JSP)=-F2(JS3)+F2(JS3)-F2(JS4))*YVMIN(JS3)/YVPLU(JS3)
420 IF(MCOUNT.NE.JPNS.AND.MCOUNT.NE.MCSTEP) GO TO 430
DO 425 J=JSP,NJ
SPN(J)=F1(J)/DELXU+(F2(J+1)-F2(J))/YVPLU(J)
IF(MCOUNT.EQ.MCSTEP.AND.J.GT.JSTEP)
SPN(J) = SPN(J) - FI1(J)/DELU

CONTINUE
GO TO 455

430 IF (MCOUNT .NE. KPNS) GO TO 440
DO 435 J = JSP, NJ
SPN(J) = - FI1(J)/DELU + (F2(J+1) - F2(J))/YVPLU(J)
435 CONTINUE
GO TO 455

440 DO 445 J = JSP, NJ
SPN(J) = (F1(J) - FI1(J))/DELU + (F2(J+1) - F2(J))/YVPLU(J)
445 CONTINUE
455 SPN(NJ) = SPN(NJ) - F2(NJP)/YVPLU(NJ)
SPN(JSP) = SPN(JSP) + F2(JSP)/YVPLU(JSP)

C
AMR = 0.
AMASS = 0.
FLOW = 0.
DO 460 J = JSP, NJ
AMJ = (U1(J) - U(J))/DELU + (V1(J+1) - V1(J))/YVPLU(J)
Q(J) = AMJ * YVPLU(J) * DELU
AMR = AMR + ABS(Q(J))
AMASS = Q(J) + AMASS
FLOW = FLOW + U1(J) * YVPLU(J)
SP(MCOUNT,J) = SP(MCOUNT,J) + (SPN(J) - SP(MCOUNT,J)) * FAC1
460 CONTINUE
FLOW = FLOW + V1REF(MCOUNT) * DELU
AM = AMR / DELU
WRITE(6, 475) AM, AMR, AMASS, FLOW, UC(JSP)

C
IF (ZAP .GE. 0.) RETURN
WRITE(6, 480) (UC(J), J = 1, NJP)
WRITE(6, 485) (VC(J), J = 1, NJP)
WRITE(6, 490) (PHI(J), J = 1, NJP)
WRITE(6, 500) (U1(J), J = 1, NJP)
WRITE(6, 510) (V1(J), J = 1, NJP)
WRITE(6, 520) (F1(J), J = 1, NJP)
WRITE(6, 530) (F2(J), J = 1, NJP)
WRITE(6, 540) (SPN(J), J = 1, NJP)
WRITE(6, 550) (SP(MCOUNT,J), J = 1, NJP)

C
475 FORMAT(5X, 'AM', G15.7, ' AMR', G15.7, ' AMASS', G15.7,
' FLOW', G15.7, ' UC-WALL', G15.7)
480 FORMAT(5X, 'UC(J)'/ (5X, 9G14.6))
485 FORMAT(5X, 'VC(J)'/ (5X, 9G14.6))
490 FORMAT(5X, 'PHI(J)'/ (5X, 9G14.6))
500 FORMAT(5X, 'U1(J)'/ (5X, 9G14.6))
510 FORMAT(5X, 'V1(J)'/ (5X, 9G14.6))
520 FORMAT(5X, 'F1(J)'/ (5X, 9G14.6))
530 FORMAT(5X, 'F2(J)'/ (5X, 9G14.6))
540 FORMAT(5X, 'SPN(J)'/ (5X, 9G14.6))
550 FORMAT(5X,'SP(MCOUNT,J)')/(5X,9G14.6))
560 FORMAT(5X,'Q(J)')/(5X,9G14.6))
RETURN
END
SUBROUTINE POISON
C
CALLING PROGRAM : YMOM, SOR
C
DETERMINES UPDATED PRESSURES AT ANY ONE STREAMWISE
C
LOCATION.
C
COMMON/OVER/FAC1,FAC2,FAC,NT,FLARE
COMMON/THOMAS/AA(100),BB(100),CC(100),DD(100),DU1(100),DV1(100)
COMMON/ARRAYS/FL1(100),FL2(100),FL1(100),P(70,100),
SP(70,100),UM(100),U(100),U1(100),USECX(100),
USECY(100),USEXP(70,100),VM(100),V(100),
V1(100),VSECX(100),VSECY(100),VSEP(70,100),
DY(100),YDE2(100),YDE3(100),YDE12(100),
YDE13(100),YDE23(100),YVPLU(100),
YVT(100),T1(100),TT(70,100),UREF1(300),
VREF(300),TWL1(70),TWU1(70)
COMMON/VAR/AFLOW,GLOBE,INV,JPNS,KPNS,LPNS,MCM,MCOUNT,
MCP,NJN,NJ,NJP,NTEMP,NWALL,NWGALT,PCON,RC,
PCON3,TOLENC,XCONV,JS,JSP,JS3,JSP,MCSTEP,
HIGHT,TEST
COMMON/XGRID/X(300),DELXM,DELX,DELXP,DELXPP,DELXPP3,
DELXUM,DELU,DELUX,DXUPP
COMMON/CONV/EPSJ,EPSMJ,EPS,EPSM,JMAX,JMIN
COMMON/ACCL/IJK
C
EPSJ=0.
IF((IJK.EQ.1.AND.MCOUNT.GT.JPNS).OR.(IJK.GT.1)) GO TO 15
DO 10 J=2,NJ
AA(J)=2./YDE13(J)
BB(J)=2./YDE23(J)
10 CONTINUE
AA(NJ)=0.
GO TO 20
15 IF(MCOUNT.EQ.MCSTEP) BB(JSTEP+1)=2./YDE23(JSTEP+1)
20 BB(JSP)=0.
C
NOTICE THAT AA(NJ)=0. AND BB(JSP)=0.
C
DO 90 K=JSP,NJ
J=NJ-K+JSP
IF(MCOUNT.LE.MCSTEP) GO TO 30
IF(J.LE.JSTEP) GO TO 50
GO TO 35
30 IF (MCOUNT.EQ.JPNS) GO TO 50
IF (MCOUNT.EQ.KPNS) GO TO 40
35 DD(J)=-2./((DELX*DELXP)-AA(J)-BB(J))
CC(J)=SP(MCOUNT,J)-P(MCP,J)/(DELXU*DELXP)-P(MCM,J)/(DELX*DELXU)
GO TO 80
40 DD(J)=-1./((DELXU*DELX)-AA(J)-BB(J))
CC(J)=SP(MCOUNT,J)-P(MCM,J)/(DELX*DELXU)
GO TO 80
134

50 DD(J)= -1./(DELXU*DELXP) - AA(J) - BB(J)
CC(J)=SP(MCOUNT,J) - P(MCP,J)/(DELXP*DELXU)
80 PT=(CC(J) - AA(J)*P(MCOUNT,J+1) - BB(J)*P(MCOUNT,J-1))/DD(J)
EPST=ABS(PT-P(MCOUNT,J))
P(MCOUNT,J)=(PT-P(MCOUNT,J))*FAC+P(MCOUNT,J)
C
IF(K.EQ.JSP) EPSMJ=EPST
IF(EPST.LT.EPSJ) GO TO 85
JMAX=J
EPSJ=EPST
85 IF(EPST.GT.EPSMJ) GO TO 90
JMIN=J
EPSMJ=EPST
C
90 CONTINUE
C
RETURN
END
SUBROUTINE SOR
    CALLING PROGRAM: MAIN
    SOLVING PRESSURE POISSON EQUATION USING SOR BY POINTS.

    COMMON/THOMAS/AA(100), BB(100), CC(100), DD(100), DX(100), DXU(100)
    COMMON/PRESUR/NG, NPRINT, LGLOBE, NPC(20)
    COMMON/ARRAYS/F1(100), F2(100), FI1(100), P(70, 100),
               SP(70, 100), UM(100), U(100), U1(100), USECX(100),
               USECY(100), USEP(70, 100), VM(100), V(100),
               V1(100), VSEC1X(100), VSECY(100), VSEP(70, 100),
               DV(100), YDE2(100), YDE3(100), YDE12(100),
               YDE13(100), YDE23(100), YVPLU(100),
               YVT(100), T1(100), TT(70, 100), UREF1(300),
               VREF(300), TWL(70), TWU(70)
    COMMON/VAR/AFLOW, GLOBE, INV, JPNS, KPNS, LPNS, MCM, MCOUNT
               , MCP, NJN, NJ, NJP, NTEMP, NWALL, NWALLT, PCON, RC,
               PCON3, TOLER, XCONV, JS, JSP, JS3, JS4, JSTEP, MCSTEP
               , HIGHT, TEST
    COMMON/XGRID/X(300), DELXM, DELX, DELXP, DELXPP, DELXP3,
               DELXUM, DELXU, DELXUP, DXUPP
    COMMON/GLOBAL/ICOUNT, MIN, MAX, ZAP
    COMMON/CONV/EPSJ, EPSMJ, EPS, EPSM, JMAX, JMIN
    COMMON/ACCL/IJK
    KPNSM=KPNS-1
    KPNSP=KPNS+1
    DO 10 J=JPNS, KPNSM
    DX(J)=X(J)-X(J-1)
    DXU(J)=(X(J+1)-X(J-1))*0.5
    CONTINUE
    DX(KPNS)=X(KPNS)-X(KPNSM)
    DX(KPNSP)=DX(KPNS)
    DXU(KPNS)=DX(KPNS)
    DO 270 IJK=1, NG
    EPS=0.
    IPRT=1
    DO 245 MCOUNT=JPNS, KPNS
    MCP=MCOUNT+1
    MCP=MCOUNT-1
    DELX =DX(MCOUNT)
    DELXP=DX(MCP)
    DELXU=DXU(MCOUNT)
    JSP=2
    IF(MCOUNT.LT.MCSTEP) JSP=JSTEP+1
    CALL POISON

    IF(MCOUNT.EQ.JPNS) EPSM=EPSMJ
    IF(EPSJ.LT.EPS) GO TO 50
    IMAX=MCOUNT
    EPS=EPSJ
IF (EPSMJ .GT. EPSM) GO TO 60
IMIN = MCOUNT
EPSM = EPSMJ

C
60 IF (IJK .EQ. NG) GO TO 90
IF (IJK .GT. 3.OR.MCOUNT .NE.MPC(IPRT)) GO TO 245
IPRT = IPRT + 1

90 WRITE(6,100) MCOUNT, EPSJ, JMAX, EPSMJ, JMIN, (P(MCOUNT, J), J=1, NJP)
100 FORMAT(5X, 'MCOUNT= ', I3, 5X, 'EPSJ= ', G12.5, 2X, 'AT J= ', I3,
      5X, 'EPSMJ= ', G12.5, 2X, 'AT J= ', I3/(5X, 9G14.6))

245 CONTINUE
C
WRITE(6,250) IJK, EPS, IMAX, EPSM, IMIN
250 FORMAT(5X, '***** IJK= ', I5, 5X, 'EPS= ', G14.5, 2X, 'AT MCOUNT= ', I3
      , 5X, 'EPSM= ', G14.5, 2X, 'AT MCOUNT= ', I3, '*****')
C
270 CONTINUE
DO 280 J = JSP, NJ
P(KPNSP, J) = P(KPNS, J) + DELXP*F1(J)
SP(KPNSP, J) = SP(KPNS, J)
280 CONTINUE
WRITE(6,100) KPNSP, EPSJ, JMAX, EPSMJ, JMIN, (P(KPNSP, J), J=1, NJP)
RETURN
END
SUBROUTINE ELLIP(MINT,MAXT,ZAP1,XKHT,IBL)

C CALLING PROGRAM : MAIN
C HANDLES ITERATIVE PROCEDURE FOR THE ENERGY EQN.

COMMON/PRESUR/NG,NPRINT,LGLOBE,MPC(20)
COMMON/ARRAYS/F1(100),F2(100),F11(100),P(70,100),
  SP(70,100),UM(100),U(100),U1(100),USECX(100),
  USECY(100),USEP(70,100),VM(100),V(100),
  V1(100),VSECX(100),VSECY(100),VSEP(70,100),
  DY(100),YDE2(100),YDE3(100),YDE12(100),
  YDE13(100),YDE23(100),YVPLU(100),
  YVT(100),T1(100),TT(70,100),UREF1(300),
  VIREF(300),TWL1(70),TWU1(70)
COMMON/VAR/AFLOW,GLOBE,INV,JPNS,KPNS,LPNS,MCM,MCOUNT,
  MCP,NJN,NJ,NJP,NTEMP,NWALL,NWALLT,PCON,RC,
  PCON3,TOLERX,XCONV,JS,JSP,JS3,JS4,JSTEP,MCSTEP,
  HIGHT,TEST
COMMON/XGRID/X(300),DELXM,DELX,DELXP,DELXPP,DELXP3,
  DELXUM,DELXU,DELXUP,DXUPP
COMMON/XRATIO/RATIOX,RATVX,SECDX,SECVDX,
  RATXR,RATVXR,SECDXR,SECVDXR
COMMON/GLOBAL/ICOUNT,MIN,MAX,ZAP
DO 40 IGLOBE=MINT,MAXT
  IPRT=1
  DELXPP=X(4)-X(3)
  DELXP =X(3)-X(2)
  DELX =X(2)-X(1)
  DELXM =DELX
  JS =JSTEP
  DO 40 MCOUNT=2,KPNS
    MCP=MCOUNT+1
    MCH=MCOUNT-1
    ZAP=ZAP1
    IF(MCOUNT.NE.MPC(IPRT).OR.ZAP.GT. -1 .) GO TO 5
    ZAP=-1.
    IPRT=IPRT+1
    WRITE(6,3) MCOUNT
    3 FORMAT(5X,'T1(J) AT MCOUNT = ',I3)
    5 IF(MCOUNT.LE.KPNS-3) DELXP3=X(MCOUNT+3)-X(MCOUNT+2)
      IF(MCOUNT.GE.MCSTEP) JS=1
      JSP=JS+1
      JS3=JS+2
      RATVX =DELX/DELXM
      SECVDX=1./(DELX+DELXM)
      CALL TEMP(IGLOBE,XKHT,IBL)
      EPST=0.
      DO 20 J=JSP,NJ
        DT=T1(J)-TT(MCOUNT,J)
        IF(ABS(DT).LE.ABS(EPST)) GO TO 10
        EPST=DT
JM = J
10 TT(MCOUNT, J) = T(J)
20 CONTINUE
TT(MCOUNT, JS) = T(1(JS))
TT(MCOUNT, NJP) = T(1(NJP))
WRITE(6, 30) MCOUNT, EPST, JM
30 FORMAT(5X, 'MCOUNT=', I3, 5X, 'EPST=', G12.5, 2X, 'AT J=', I3/)
C UPDATING GRID SPACINGS
DELXM = DELX
DELX = DELXP
DELXP = DELXPP
DELXPP = DELXP3
40 CONTINUE
RETURN
END
SUBROUTINE TEMP(MINT, XKHT, IBL)
C CALLING PROGRAM : MAIN, ELLIP
C SOLVES THE ENERGY EQUATION TO OBTAIN TEMPERATURE
C PROFILE.
C
DIMENSION YDE1(100)
COMMON/THOMAS/AA(100), BB(100), CC(100), DD(100), DU1(100), DV1(100)
COMMON/ARRAYS/F1(100), F2(100), FI1(100), P(70, 100),
  SP(70, 100), UM(100), U(100), U1(100), USECX(100),
  USECY(100), USEP(70, 100), VM(100), V(100),
  VI(100), VSEC1X(100), VSECY(100), VSEP(70, 100),
  DV(100), YDE2(100), YDE3(100), YDE12(100),
  YDE13(100), YDE23(100), YVPLU(100),
  YVT(100), T1(100), TT(70, 100), UREF1(300),
  V1REF(300), TWL1(70), TWU1(70)
COMMON/VAR/AFLOW, GLOBE, INV, JPN, KPN, LPNS, MCM, MCOUNT,
  MCP, NJN, NJ, NJP, NTEMP, NWALL, NWALLT, PCON, RC,
  PCON3, TOLER, XCONV, JS, JSP, JS3, JS4, MCSTEP,
  MCSTEP, HIGHT, TEST
COMMON/XGRID/X(300), DELXM, DELX, DELXP, DELXPP, DELXP3,
  DELXUM, DELXUP, DXUPP
COMMON/XRATIO/RATIOX, RATVX, SECDX, SECVDX,
  RATXR, RATVXR, SECDXR, SECVXR
COMMON/GLOBAL/ICOUNT, MIN, MAX, ZAP
COMMON/SEPER/MSEP1, MSEP2, NSFULL, MCDOWN, NHYBRD, MC1, MC2
COMMON/TCASE/NSTEP
COMMON/LEAD1/LEAD, PR
EQUIVALENCE (YDE1(1), YDE2(2))
C=0.
PR2=2./PR
X1=DELX/(DELX+DELXP)
X2=DELXP/(DELX+DELXP)
X3=(DELXM+DELX)/(DELXM*DELX)
X4=RATVX/(DELXM+DELX)
DO 200 J=JSP,NJ
  COEFFS=USEP(MCOUNT,J)*X1+USEP(MCM,J)*X2
  RM1=VSEP(MCOUNT,J+1)*YDE2(J)
  RM2=VSEP(MCOUNT,J)*YDE1(J)
  CC(J)=0.
  IF (RM1.GT.1.9) GO TO 20
  IF (RM2.LT.-1.9) GO TO 10
  AA(J)=(RM1-PR2)/YDE13(J)
  BB(J)=-(RM2+PR2)/YDE23(J)
  DD(J)=PR2/YDE12(J)+RM2/YDE23(J)-RM1/YDE13(J)
  GO TO 30
C
10  W=-1.9/RM2
  IF (NHYBRD.EQ.0) W=0.
  AA(J)=(VSEP(MCOUNT,J+1)*(YDE3(J)-W*YDE1(J)))-PR2)/YDE13(J)
  BB(J)=-(RM2*W+PR2)/YDE23(J)
DD(J) = (PR2 - RM1) / YDE12(J) + W*(RM2 + RM1) / YDE23(J)
GO TO 30

C

20 W = 1.9 / RM1
IF (NYBRD.EQ.0) W = 0.
AA(J) = (RM1*W - PR2) / YDE13(J)
BB(J) = -(VSEP(MCOUNT, J) * (YDE3(J) - W*YDE2(J) + PR2) / YDE23(J)
DD(J) = (PR2 + RM2) / YDE12(J) - W*(RM2 + RM1) / YDE13(J)

C

30 IF (MINT.LE.1.OR.MCOUNT.EQ.KPNS) GO TO 40
IF (IBL.EQ.1) GO TO 37
DD1 = PR2/ (DELX*DELXP)
CC(J) = PR2/ (DELX*DELXP)* (TT(MCP,J) / DELXP + TT(MCM,J) / DELX)
IF (MCOUNT.NE.MCSTEP.OR.J.GT.JSTEP) GO TO 35
DD1 = 2.*DD1
CC(J) = PR2/ (DELX*DELXP)* (TT(MCP,J) / DELXP +
* 2.*TWL1(MCM)/DELX)

35 DD(J) = DD(J) + DD1
37 IF (COEFFS.GE.0.) GO TO 170
IF (MCOUNT.EQ.KPNS-1) GO TO 39
DD(J) = DD(J) - COEFFS*(2.*DELX*DELXP)/(DELXP*(DELX*DELXP))
CC(J) = CC(J) + COEFFS*(DELX*DELXP)/(DELX*DELXP)*
* TT(MCOUNT+2, J) - (DELX*DELXP)/(DELX*DELXP)*
* TT(MCP, J))
GO TO 200

39 DD(J) = DD(J) - COEFFS/DELX
CC(J) = CC(J) + COEFFS*TT(MCM,J)/DELX
GO TO 200

40 IF (COEFFS.LT.0.) COEFFS = C*ABS(COEFFS)
170 IF (MCOUNT.LE.2) GO TO 180
DD(J) = DD(J) + COEFFS/DELX
CC(J) = CC(J) + COEFFS*TT(MCM,J)/DELX
GO TO 200

180 IF (MCOUNT.EQ.MCSTEP.AND.J.LE.JSTEP) GO TO 190
DD(J) = DD(J) + COEFFS*SECVDX
CC(J) = CC(J) + COEFFS*(X3*TT(MCM,J) - X4*TT(MCOUNT-2, J))
GO TO 200

190 DD(J) = DD(J) + COEFFS*2./DELX
CC(J) = CC(J) + COEFFS*2.*TWL1(MCM)/DELX
200 CONTINUE

BB(NJP) = 1.0
IF (NSTEP.EQ.1) BB(NJP) = -1.0
AA(NJP) = 0.0
DD(NJP) = 1.0
CC(NJP) = 2.*TWL1(MCOUNT)
AA(JSP) = AA(JSP) - PR2*YDE2(JSP)/(YDE13(JSP)*
* (YDE3(JSP) + YDE1(JSP))
BB(JSP) = BB(JSP) + PR2/YDE23(JSP)
CC(JSP) = CC(JSP) + (4.*PR2/(YDE23(JSP)+
* YDE12(JSP)) + 2.*BB(JSP))*TWL1(MCOUNT)
141

\[ DD(JSP) = DD(JSP) + PR2/YDE12(JSP) - BB(JSP) \]

\[ BB(JSP) = 0. \]

IF(IABS(NWALL).LT.2) GO TO 210

AA(NJ) = AA(NJ) + PR2/YDE13(NJ)

\[ BB(NJ) = BB(NJ) - PR2*YDE1((NJ)/YDE13(NJ) \]

\[ CC(NJ) = CC(NJ) + 4.*PR2*TWU1(MCOUNT)/YDE12(NJ)+YDE13(NJ) \]

\[ DD(NJ) = DD(NJ) + PR2/YDE12(NJ) \]

210 CALL SY(JSP,NJP,BB,DD,AA,CC,T1)

T1(JS) = 2.*TWL1(MCOUNT) - T1(JSP)

IF(MCOUNT.NE.MCSTEP-1) GO TO 220

T1(1) = T1(J)

TT(MCOUNT,1) = T1(1)

DO 215 J=2, JSTEP

T1(J) = TWL1(MCOUNT)

215 CONTINUE

220 IF(ZAP.EQ.-1.) WRITE(6,240) (T1(J),J=1,NJP)

240 FORMAT(5X,'T1(J)' ,/( SX , 9Gl4.6))

C

C CALCULATION OF THERMAL B.L. THICKNESS

C

CALL DSTNM(-1,JSP,TWL1(MCOUNT),TINFL,DTHL,XXX)

YW1 = YDE12(JSP)*0.5

HTWL = -XKHT*(T1(JSP) - TWL1(MCOUNT))*2./(DY(JSP) - DY(JS))

HTWL2 = -XKHT*((T1(JSP) - TWL1(MCOUNT))/YW1 - (T1(JSP) - T1(JSP))/YDE2(JSP) + (T1(JSP) - TWL1(MCOUNT))/(YW1+YDE1(JSP)))

* XCONV

HCOEL = HTWL/(TWL1(MCOUNT) - TINFL)

HCOEL2 = HTWL2/(TWL1(MCOUNT) - TINFL)

XNU1 = HCOEL1*HIGHT/XKHT

XNU1 = HCOEL1*HIGHT/XKHT

WRITE(6,260) TWL1(MCOUNT), TINFL, DTHL, HTWL, HCOEL, XNU1, HTWL2, HCOEL2, XNU12

260 FORMAT(5X,'TWL1'=1,G13.6,6X,'TINFL'=1,G13.6,6X,

'DTHL'=1,G13.6,7X,'XKHT'=1,G13.6,5X,'HTWL'=1,G13.6,6X,

'HCOEL1'=1,G13.6,6X,'XNU1'=1,G13.6,6X,'HTWL2'=1,G13.6,5X,'XNU12'=1,G13.6)

IF(IABS(NWALL).NE.2) RETURN

CALL DSTNM(-1,NJ,TWU1(MCOUNT),TINFU,DTHU,XXX)

YWU = YDE12(NJ)*0.5

HTWU = -XKHT*(T1(NJ) - TWU1(MCOUNT))*2./(DY(NJP) - DY(NJ))

HTWU2 = -XKHT*((T1(NJ) - TWU1(MCOUNT))/YWU - (T1(NJ) - T1(NJ))/YDE2(NJ) + (T1(NJ) - TWU1(MCOUNT))/(YWU+YDE2(NJ)))

* XCONV

HCOEU = HTWU/(TWU1(MCOUNT) - TINFU)

HCOEU2 = HTWU2/(TWU1(MCOUNT) - TINFU)

XNUU1 = HCOEU1*HIGHT/XKHT

XNUU2 = HCOEU2*HIGHT/XKHT

WRITE(6,270) TWU1(MCOUNT), TINFU, DTHU, HTWU, HCOEU, XNUU1, HTWU2, HCOEU2, XNUU2
270 FORMAT(5X,'TWU1=',G13.6,6X,'TINFU=',
G13.6,6X,'DTHU=',G13.6,/5X,'HTWU=',G13.6,
6X,'HCOEU=',G13.6,6X,'XNUUP=',G13.6,/5X,'HTWU2='
,G13.6,5X,'HCOEU2=',G13.6,5X,'XNUUP2=',G13.6)
SUBROUTINE DSTNM (NN, NB, TDW, UINF, DELTA, THETA)

C CALLING PROGRAM : MAIN
C THIS SUBROUTINE CALCULATES DISPLACEMENT THICKNESS
C AND THERMAL B.L. THICKNESS.
C FOR THERMAL B.L. THICKNESS, SET NN.LT.0
COMMON/ARRAYS/Fl(lOO), F2(100), F11(100), P(70, 100),
SP(70, 100), UM(100), U(100), Ul(100), USECX(100),
USECY(100), USEP(70, 100), VM(100), V(100),
Vl(100), VSEC1X(100), VSECY(100), VSEP(70, 100),
DY(100), YDE2(100), YDE3(100), YDE12(100),
YDE13(100), YDE23(100), YVPLU(100),
VT(100), Tl(100), TT(70,100), UREF1(300),
V1REF(300), TWL1(70), TWU1(70)
COMMON/VAR/AFLOW, GLOBE, INV, JPNs, KPNS, LPNS, MCM, NCOUNT
, MCP, NJN, NJ, NJP, NTEMP, NWALL, NWallT, PCON, RC,
PCON3, TOLER, XCONV, JS, JSP, JS3, JS4, JSTEP, MCSTEP,
, HIGHT, TEST
IF(NN.LT.0) GO TO 60
IF(NB.GT.JSP) GO TO 35
DELTA=Ul(JSP)*(YVPLU(JSP)-YDE2(JSP)/4.)
THETA=Ul(JSP)**2*(YVPLU(JSP)-YDE2(JSP)/4.)
JSP7=JSP+7
DO 20 J=JS3,NJ
DELTA=DELTA+Ul(J)*YVPLU(J)
THETA=THETA+Ul(J)**2*YVPLU(J)
IF(Ul(J-1)/Ul(J).GT.TEST.AND.Ul(J).GT.0.AND.J.GT.JSP7) GO TO 30
20 CONTINUE
30 DELTA=DELTA-Ul(J)*YDE2(J)*0.5
THETA=(Ul(J)*DELTA-THETA+Ul(J)**2*YDE2(J)*0.5)/(Ul(J)**2*XCONV)
DELTA=DY(J)-(DY(J)+DY(JSP))*0.5
-DELTA/(Ul(J)*XCONV)
UINF=Ul(J)
RETURN
35 DELTA=Ul(NJ)*(YVPLU(NJ)-YDE2(NJP)/4.)
THETA=Ul(NJ)**2*(YVPLU(NJ)-YDE2(NJP)/4.)
NJM7=NJ-7
DO 40 J=3,NJ
K=NJ-J+2
DELTA=DELTA+Ul(K)*YVPLU(K)
THETA=THETA+Ul(K)**2*YVPLU(K)
IF(Ul(K+1)/Ul(K).GT.TEST.AND.Ul(K).GT.0.AND.K.LT.NJM7) GO TO 50
40 CONTINUE
50 DELTA=DELTA-Ul(K)*YDE2(K)*0.5
THETA=(Ul(K)*DELTA-THETA+Ul(K)**2*YDE2(K)*0.5)/(Ul(K)**2*XCONV)
DELTA=(DY(NJ)+DY(NJP))*0.5-DY(K)
-DELTA/(Ul(K)*XCONV)
UINF=Ul(K)
RETURN
60 IF(NB.GT.JSP) GO TO 90
DELTA=T1(JSP)*YVPLU(JSP)+YDE2(JSP)*
(TDW-T1(JSP))/4.
DO 70 J=JS3,NJ
   DELTA=DELTA+T1(J)*YVPLU(J)
   IF((TDW-T1(J-1))/(TDW-T1(J)).GT.TEST) GO TO 80
70  CONTINUE
80  DELTA=DELTA-T1(J)*YDE2(J+1)*0.5
    DELTA=(DELTA/XCONV-T1(J)*((DY(J)+(DY(JS)+
    DY(JSP)*0.5))/(TDW-T1(J))
    UINF=T1(J)
    RETURN
90  DELTA=T1(NJ)*YVPLU(NJ)+YDE2(NJP)*
      (TDW-T1(NJP))/4.
DO 95 J=3,NJ
   K=NJ-J+2
   DELTA=DELTA+T1(K)*YVPLU(K)
   IF((TDW-T1(K+1))/(TDW-T1(K)).GT.TEST) GO TO 100
95  CONTINUE
100 DELTA=DELTA-T1(K)*YDE2(K)*0.5
    DELTA=(DELTA/XCONV-T1(K)*((DY(NJ)+(DY(NJP))*0.5-
    DY(K)))/(TDW-T1(K))
    UINF=T1(K)
    RETURN
END
SUBROUTINE SEC(N,J,FIRST,SECDER)
C    CALLING PROGRAM : CORREC
C    ESTIMATES FIRST AND SECOND DERIVATIVES USING ONE-
C    SIDED DIFFERENCING FORMULAE.
C
COMMON/ARRAYS/F1(100),F2(100),F1(100),P(70,100),
SP(70,100),UM(100),U(100),U(100),USECX(100),
USECY(100),USEP(70,100),VM(100),V(100),
V1(100),VSECX(100),VSECY(100),VSEP(70,100),
DY(100),YDE2(100),YDE3(100),YDE12(100),
YDE13(100),YDE23(100),YVPLU(100),
YVT(100),T(100),TT(70,100),UREF1(300),
V1REF(300),TWL1(70),TWU1(70)
COMMON/VAR/AFLOW,GLOBE,INV,JPNS,KPNS,LPNS,MCM,MCOUNT
 ,MCP,NJN,NJ,NJP,NTEMP,NWALL,NWALLT,PCON,RC,
PCON3,TOLER,XCONV,J5,JSP,J3,J4,JSTEP,MCSTEP
 ,HIGHT,TEST
IF(J.LT.NJ-3) GO TO 10
  J1=J-1
  J2=J-2
  J3=J-3
  GO TO 20
10  J1=J+1
  J2=J+2
  J3=J+3
  IF (N.NE.1) GO TO 30
  I=U1(J)
  S2=U1(J1)
  S3=U1(J2)
  S4=U1(J3)
  GO TO 40
30  AY1=ABS((DY(J1)-DY(J))*XCONV)
    AY2=ABS((DY(J2)-DY(J))*XCONV)
    AY3=ABS((DY(J3)-DY(J))*XCONV)
  40  P2=AY2+AY1
    AY2=AY2-AY1
    DS2=S2-S1
    DS3=S3-S2
    FIRST=PY2/(AY1*AY2)*DS2-AY1/(DY2*AY2)*DS3
    SECDER=2.*(PY2/(AY3*AY1))*(S4-S3)/(AY3-AY2)-
    DS3/DY2)+(PY2+AY3)/(AY2*AY3)*(DS3/DY2-DS2/AY1))
RETURN
END
SUBROUTINE SPLICO(X,Y,M,C)

C

THIS SUBROUTINE DETERMINES CUBIC SPLINE COEFFICIENTS
C

FOR INTERPOLATION WHICH IS DONE BY SUBROUTINE SFINT.
C

DIMENSION A(70,3),B(70),C(4,M),D(70),E(70),P(70),
W(70),X(M),Y(M),Z(70)

MM=M-1
DO 2 K=1,MM
D(K)=X(K+1)-X(K)
P(K)=D(K)/6.
2 E(K)=(Y(K+1)-Y(K))/D(K)
DO 3 K=2,MM
3 B(K)=E(K)-E(K-1)
   A(1,2)=-1.-D(1)/D(2)
   A(1,3)=D(1)/D(2)
   A(2,3)=P(2)-P(1)*A(1,3)
   A(2,2)=2.*P(1)+P(2))-(A(1,2)
   B(2)=B(2)/A(2,2)
DO 4 K=3,MM
   A(K,2)=2.*P(K-1)+P(K)-A(K,2)
   B(K)=B(K)-A(K,2)
   A(K,3)=P(K)/A(K,2)
   B(K)=B(K)/A(K,2)
4 Q=D(M-2)/D(M-1)
   A(M,1)=1.+Q+A(M-2,3)
   A(M,2)=-Q-A(M,1)*A(M-1,3)
   B(M)=B(M-2)-A(M,1)*B(M-1)
   Z(M)=B(M)/A(M,2)
MN=M-2
DO 6 I=1,MN
   Z(K)=B(K)-A(K,2)*Z(K+1)
   Z(1)=A(1,2)*Z(2)-A(1,3)*Z(3)
DO 7 K=1,MM
   Q=1./(6.*D(K))
   C(1,K)=Z(K)*Q
   C(2,K)=Z(K+1)*Q
   C(3,K)=Y(K)/D(K)-Z(K)*P(K)
   C(4,K)=Y(K+1)/D(K)-Z(K+1)*P(K)
7 W(K)=E(K)-P(K)*(Z(K+1)+2.*Z(K))
   W(M)=E(M-1)+P(M-1)*(Z(M-1)+2.*Z(M))
RETURN
END
SUBROUTINE SFINT(XX, NS, XF, KPNS, CC, Y)
C THIS SUBROUTINE INTERPOLATES USING CUBIC SPLINE COEFFICIENTS.
C
DIMENSION XX(NS), XF(KPNS), CC(4,NS), Y(KPNS)

NN = 1
DO 50 I = 1, KPNS
IF(XX(I) .GE. XF(I)) GO TO 40
IF(XX(NS) .GT. XF(I)) GO TO 10
NN = NS - 1
GO TO 40
10 NB = NN + 1
DO 20 J = NB, NS
IF(XX(J) .GE. XF(I)) GO TO 30
20 CONTINUE
30 NN = J - 1
40 A = XX(NN + 1) - XF(I)
B = XF(I) - XX(NN)
Y(I) = CC(1, NN) * A * A * A + CC(2, NN) * B * B * B + CC(3, NN) * A + CC(4, NN) * B
50 CONTINUE
RETURN
END
SUBROUTINE SY(IE, LE, BB, DD, AA, CC, SS)
C THIS SUBROUTINE SOLVES A SYSTEM OF EQUATIONS HAVING
C A TRIDIAGONAL COEFFICIENT MATRIX USING THE THOMAS
C ALGORITHM.
C
DIMENSION AA(LE), BB(LE), CC(LE), DD(LE), SS(LE)
IP = IE + 1
DO 10 I = IP, LE
  R = BB(I) / DD(I - 1)
  DD(I) = DD(I) - R * AA(I - 1)
  CC(I) = CC(I) - R * CC(I - 1)
10  SS(LE) = CC(LE) / DD(LE)
DO 20 I = IP, LE
  J = LE - I + IE
20  SS(J) = (CC(J) - AA(J) * SS(J + 1)) / DD(J)
RETURN
END
SUBROUTINE STEPY
C CALLING PROGRAM : MAIN
C DETERMINES THE GRID SPACING IN THE Y DIRECTION
C FOR AUNG 13-9.
C
COMMON/SPARE/XU(100),YU(100),XV(100),YV(100),Y(200),
XUX(200),C(4,100)
COMMON/VAR/AFLOW,GLOBE,INV,JPNS,KPNS,LPNS,MCM,MCOUNT
,MCP,NJN,NJ,NJF,NTEMP,NWALL,NWALLT,PCON,RC,
PCON3,TOLER,C,CONV,JS,JS3,JS4,JSTEP,MCSTEP
,HIGHT,TEST
Y(1)=-0.00191*XCONV
Y(2)=0.00191*XCONV
Y(3)=0.00382*XCONV
Y(4)=0.00573*XCONV
Y(5)=0.00764*XCONV
Y(6)=0.00955*XCONV
Y(7)=0.01147*XCONV
Y(8)=0.01347*XCONV
DY=0.002*XCONV
DO 10 J=9,21
DY=DY*1.15
Y(J)=Y(J-1)+DY
10 CONTINUE
Y(NJP)=0.66967*XCONV
Y(NJ)=0.66767*XCONV
DY=0.002*XCONV
DO 30 J=28,NJN
K=NJN+28-J
DY=DY*1.15
Y(K)=Y(K+1)-DY
30 CONTINUE
DY=(Y(28)-Y(21))/7.
DO 20 J=22,27
Y(J)=Y(J-1)+DY
20 CONTINUE
RETURN
END
SUBROUTINE STEP(XBEGIN)
C        CALLING PROGRAM : MAIN
C        DETERMINES THE GRID SPACING IN THE X DIRECTION
C        FOR AUNG 13-9.
C
COMMON/XGRID/X(300),DELXM,DELX,DELXP,DELXPP,DELXPP,DELXUM,DELXU,DELXUP,DXUPP
X(1)=-0.17
X(2)=-0.13
X(3)=-0.09
X(4)=-0.0475
X(5)=-0.005
DX=0.01
DO 10 J=6,14
   X(J)=X(J-1)+DX
10 CONTINUE
DO 20 J=15,35
   DX=DX*1.115
   X(J)=X(J-1)+DX
20 CONTINUE
RETURN
END