An analysis of basis value determination for live beef cattle

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An analysis of basis value
determination for live beef cattle

by

William James Vollink

A Thesis Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
MASTER OF SCIENCE

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Signatures have been redacted for privacy

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CHAPTER I. INTRODUCTION, OBJECTIVES AND PROCEDURE

Introduction

The emphasis of this study is to explain the determination of daily values of par-delivery-point intertemporal price spreads for live beef cattle. A par-delivery-point intertemporal price spread is the difference between the futures price and the cash price on a given day for a commodity which meets all futures contract specifications except time. Hereafter, the daily value of the par-delivery-point intertemporal price spread will be referred to as the "basis."

The trading period of a futures contract may be divided into two distinct periods, the predelivery and delivery periods. The delivery period is the period during which contractual obligations may be fulfilled by delivery of the cash commodity. The predelivery period is the period beginning on the first day of trading for the particular contract and ending on the trading day prior to the first day of the delivery period for the contract in question. Frequently, the last part of the predelivery period is referred to as the near-option predelivery period. More specifically, the near-option predelivery period is the period beginning on the first trading day after the delivery period of the futures contract closest to, but preceding the contract in question, and ending on the final predelivery day of the contract.

The distinction between predelivery and delivery periods is made because the ability to deliver on the contractual obligation should allow arbitrage to occur more freely between cash and futures markets. Thus, separate models of basis determination will likely be required for the
Early studies of basis values for selected commodities

The topic of basis value determination for selected commodities has received substantial attention in the futures market literature. Unfortunately, previous studies have been incomplete and have caused some confusion among students of basis value determination.

Early studies of basis values for wheat were conducted by Working [17, 18]. Working focused his attention upon only one structural relationship in the model of basis determination, i.e., the relationship between the quantity of wheat storage supplied and the basis. A major contribution of his study was the incorporation of the notion of a convenience yield. The convenience yield is a measure of the advantage to owners of stocks of holding a quantity of stocks to maintain customer goodwill or efficient processing levels. This notion of a convenience yield provided an explanation for the possibility of a negative basis, i.e., an inverse carrying charge.

Working's analysis was not for a complete model of basis determination because the demand function for storage services was excluded. Given the equilibrium level of current inventory, Working's model can be used to determine the equilibrium basis. But his model does not provide for the simultaneous determination of the basis and current inventory.

A later study by Brennan [2] expanded upon Working's study of basis values by deriving a demand function for storage services from the consumer's current consumption function for the commodity. In Brennan's model, the current basis and current inventory level are jointly determined by
the supply and demand functions for storage services. However, Brennan's model is incomplete because only the storage market is considered. The basis is strictly a market-determined price for storage services. Nowhere in Brennan's model is a current cash or current futures price determined.

In a later model of basis determination, Stein [13] developed a model for the simultaneous determination of cash and futures prices. Once these two prices are determined, the current basis can also be determined. Stein's model involved two major revisions of Brennan's earlier model. First, Stein introduced a theory to explain how owners of stocks allocate their holdings between hedged and unhedged holdings in order to maximize their expected utility. Second, a futures market was included in Stein's model to allow for owners of stocks to hedge their stocks by selling futures contracts to speculators.

Although Stein's model provides a starting point for developing a model of basis determination for live beef cattle, it has some shortcomings. First, Stein's model falls apart when no storage services for finished goods are provided in a given market period. And second, Stein does not include a demand for futures contracts by long hedgers, e.g., demand for futures contracts by processors who hedge their forward sales.

Objectives

The objectives of this study are:

1. to develop a model of predelivery period basis determination for live beef cattle,

2. to develop a model of delivery period basis determination for live beef cattle, and
3. to determine the time path of daily basis values and its properties for selected futures contracts of live beef cattle.

Procedure

The following procedure will be used to achieve these objectives:

1. a general function model of predelivery period basis determination for live beef cattle will be specified,
2. a general function model of delivery period basis determination for live beef cattle will be specified,
3. static and comparative static results for each model will be derived,
4. estimable reduced form equations for the daily basis values for the predelivery and delivery periods will be derived, and
5. daily observations of basis values and of exogenous variables in the basis determination models will be collected and used to quantify the reduced form equations.

Potential Uses of Results

The potential uses of results from this thesis can be categorized into two groups, theoretical and practical.

Theoretical uses

There are two major theoretical uses of the results obtained from this study. First, the general function models of basis determination for the predelivery and delivery periods should provide more complete models of basis determination for live beef cattle than are now available. And second, the basis determination models for live beef cattle will provide
insights helpful in formulating more general models of basis determination for other agricultural commodities traded in the futures markets.

Practical uses

Three major practical uses of the results of this study may be distinguished. First, the results from estimating the reduced form equations should help to identify variables which should be included in formulating a basis forecasting model. Second, information about the properties of the time path of daily basis values for a particular live beef cattle futures contract should help in the short-term market decision-making process of cattle producers who hedge their production. And third, the information obtained from the results of the reduced form delivery period basis determination model may indicate undue restriction on arbitrage. If arbitrage appears to be unduly restricted in the delivery period, possible regulations which will enhance arbitrage may be suggested to the futures market regulatory agency.
CHAPTER II. PREDEELIVERY AND DELIVERY PERIOD MODELS OF BASIS DETERMINATION FOR LIVE BEEF CATTLE

Introduction

The purpose of this chapter is to develop general function models of basis determination for the predelivery and delivery periods of a futures contract for live beef cattle.

A General Function Model of Basis Determination in the Predelivery Period

In this section, a general function model of daily basis determination in the predelivery period will be developed. The basis determination model involves two markets, the market for storage and transformation services and the futures market. Subsections will include: (1) a description of the supply and demand functions in the market for storage and transformation services, (2) a description of equilibrium in the market for storage and transformation services, (3) a description of supply and demand functions for futures contracts, (4) a description of equilibrium in the futures market, (5) a description of equilibrium in the live beef cattle industry, (6) static results of the model, and (7) comparative static results.

The market for storage and transformation services

The quantity of finished cattle held (i.e., stored) in feedlots and unfinished cattle on feed (i.e., unfinished cattle being transformed to finished cattle) in feedlots is determined in the market for storage and transformation services. The following sections will describe aggregate supply and demand functions for storage and transformation services for
live beef cattle. Also, equilibrium in the market for storage and transformation services will be described.

The aggregate supply of storage services

The supply of storage services for live beef cattle depends upon the expected return and variance of expected return from holding finished beef cattle hedged and the expected return and variance of expected return from holding finished beef cattle unhedged.

The expected return per pound from holding finished beef cattle unhedged \((u)\) is:

\[
(2.1) \quad u = P_e - P - NMCS
\]

where \(P_e\) is the live beef cattle price expected by suppliers of storage services, \(P\) is the current finished beef cattle price, and \(NMCS\) is the net marginal cost of storage. The net marginal cost of storage function is:

\[
(2.2) \quad NMCS = NMCS(q^u_s, D)
\]

where \(q^u_s\) is the quantity of finished beef cattle which is stored unhedged by an individual and \(D\) is the number of calendar days remaining in the predelivery period. Partial derivatives in the net marginal cost of storage function are:

\[
(\frac{\partial NMCS}{\partial q^u_s} > 0, \frac{\partial NMCS}{\partial D} > 0)
\]

The net marginal cost of storage function states that net marginal costs of storing live beef cattle in the feedlot are positively related to the quantity of finished beef cattle which are stored unhedged by an individual and the number of calendar days remaining in the predelivery period. \(u\) has a variance of \(q^u_u\) which is assumed to be given.

The expected return per pound from holding finished beef cattle hedged \((h)\) is:
(2.3) \( h = (P_e - P) - (FP_e - FP) - \text{NMCS} \)

where \( FP_e \) is the expected futures price for live beef cattle and \( FP \) is the current live beef cattle futures price. Because the basis is defined as the futures price minus the cash price:

(2.4) \( B = FP - P \),

the expected return per pound of finished beef cattle which are held hedged can be rewritten:

(2.5) \( h = B - B_e - \text{NMCS} \).

The net marginal cost of storage in (2.5) may be expressed as a function of the quantity of finished beef cattle which are held hedged, \( q^H \), by an individual and the number of calendar days, \( D \), remaining in the predelivery period:

(2.6) \( \text{NMCS} = \text{NMCS}(q^H, D) \)

The function in (2.6) has partial derivatives:

\( \frac{\partial \text{NMCS}}{\partial q^H} > 0, \frac{\partial \text{NMCS}}{\partial D} > 0 \)

which means that the net marginal cost of storage is positively related to \( q^H \) and \( D \).

The supply function for beef cattle which are stored unhedged

In this subsection, the supply function for finished beef cattle which are stored unhedged will be derived. First, consider an individual supplier of unhedged storage services. Assume that the supplier is an expected utility maximizer and has a utility function, \( U(\pi) \), with absolute risk aversion, \( R_A(\pi) \), which is positive. Positive absolute risk aversion is defined:

(2.7) \( R_A(\pi) = -\frac{U''(\pi)}{U'(\pi)} > 0 \)

According to Sandmo [12], it seems reasonable to assume that \( R_A'(\pi) < 0 \)
because this reflects the hypothesis that, as a decision maker becomes wealthier, his risk premium for any risky prospect, defined as the difference between the mathematical expectation of the return from the prospect and its certainty equivalent, should decrease.

The expected utility of the individual supplier of unhedged finished beef cattle storage is given by:

\[ (2.8) \ E\{U(q^u_s)\} = E\{U[q^u_s(P_e - P - NMCS(q^u_s, D))]\}. \]

The supplier of unhedged finished beef cattle storage will select the quantity of unhedged storage to supply, \( q^u_s \), which will maximize the expected utility function in (2.8). The individual's supply function for unhedged finished beef cattle storage may be expressed:

\[ (2.9) \ q^u_s = q^u_s(P_e, P, D). \]

From Sandmo [12], the signs of the partial derivatives in (2.9), given that \( R_A'(\pi) < 0 \), are:

\[ (\partial q^u_s/\partial P_e > 0, \partial q^u_s/\partial P < 0, \partial q^u_s/\partial D < 0). \]

The sign of the partial derivative, \( \partial q^u_s/\partial D \), is negative because \( \partial NMCS/\partial D > 0 \) from (2.2).

If one assumes that the live beef cattle industry is composed of \( n \) such individual suppliers, then the aggregate supply function, \( Q^u_s \), for unhedged finished beef cattle storage is:

\[ (2.10) \ Q^u_s = Q^u_s(P_e, P, D) \]

with partial derivatives:

\[ (\partial Q^u_s/\partial P_e > 0, \partial Q^u_s/\partial P < 0, \partial Q^u_s/\partial D < 0). \]

This supply function states that the aggregate quantity of unhedged finished beef cattle which are stored is positively related to the expected return per pound of holding unhedged finished beef cattle. The
relationship between $P_e - P$ and $Q_s^u$ with $D$ held constant is presented graphically in Figure 1.

![Graph of $P_e - P$ vs $Q_s^u$](image)

Figure 1. The relationship between $P_e - P$ and $Q_s^u$ with $D$ held constant.

Note in Figure 1 that some unhedged finished beef cattle may be stored even for a negative difference between $P_e$ and $P$. This phenomenon may be explained by the concept of the convenience yield, Working [17, 18] and Brennan [2]. The convenience yield is a measure of the advantage to owners of stocks of maintaining a quantity of stocks to insure customer good will or to stabilize production at efficient levels.

The supply function for beef cattle which are stored hedged

In this subsection, the supply function for finished beef cattle which are stored hedged will be derived. Assume an individual supplier of hedged storage services has a utility function which displays positive absolute risk aversion, has decreasing absolute risk aversion over wealth, and maximizes his expected utility which is given by:

$$E[U(q_s^H)] = E[U(q_s^H(B - B_e - NMCS(q_s^H, D)))]$$

The individual supply function for hedged finished beef cattle storage
may be expressed:

\[(2.12) \quad q_s^H = q_s^H(B, B_e, D)\]

with partial derivatives:

\[(\partial q_s^H/\partial B > 0, \partial q_s^H/\partial B_e < 0, \partial q_s^H/\partial D < 0).\]

The sign of the partial derivative, \(\partial q_s^H/\partial D\), is negative because \(\partial \text{NMCS}/\partial D > 0\) from (2.6).

Assuming that the live beef cattle industry is composed of \(n\) such individual suppliers, then the aggregate supply function, \(Q_s^H\), for hedged finished beef cattle storage is:

\[(2.13) \quad Q_s^H = Q_s^H(B, B_e, D)\]

with partial derivatives:

\[(\partial Q_s^H/\partial B > 0, \partial Q_s^H/\partial B_e < 0, \partial Q_s^H/\partial D < 0).\]

This supply function states that the aggregate quantity of hedged finished beef cattle which are stored is positively related to the expected return per pound of hedged finished beef cattle held. The relationship between \(B - B_e\) and \(Q_s^H\) with \(D\) held constant is presented graphically in Figure 2.

![Graph showing the relationship between \(B - B_e\) and \(Q_s^H\) with \(D\) held constant.]

**Figure 2.** The relationship between \(B - B_e\) and \(Q_s^H\) with \(D\) held constant.
Note in Figure 2 that some hedged finished beef cattle may be stored even for a negative difference between B and B_e. This may be explained by the concept of the convenience yield.

The aggregate supply of transformation services. The aggregate supply of transformation services depends upon the expected return and variance of expected return from supplying hedged transformation services and the expected return and variance of expected return from supplying unhedged transformation services.

The expected return per pound of finished live beef cattle from providing unhedged transformation services (t_u) is:

\[(2.14) \ t_u = \ P_e - R - G \]

where \(P_e\) is the live beef cattle price expected by beef cattle producers, \(R\) is the current cost of feeder cattle per pound of finished live beef cattle, and \(G\) is the transformation cost per pound of finished live beef cattle. According to Ehrich [4], when the feeder cattle industry and the fed cattle industry are in equilibrium, the following relation between live beef cattle futures prices, feeder cattle prices, and transformation costs exists:

\[(2.15) \ FP - R - G = 0.\]

Solving (2.15) for \(R\) and substituting the result into (2.14), the expected return per pound of finished live beef cattle from providing unhedged transformation services becomes:

\[(2.16) \ t_u = \ P_e - B - P \]

\(t_u\) has a variance of \(\sigma_{t_u}^2\) which is assumed to be given.

The expected return per pound of finished live beef cattle from providing hedged transformation services (t_h) is:
(2.17) \( t_h = FP - Be - R - G \)

where FP is the current futures price for live beef cattle and Be is the basis expected by suppliers of transformation services. Solving (2.15) for \( R \) and substituting the result into (2.17), \( (t_h) \) becomes:

(2.18) \( t_h = -Be \)

t\(_h\) has a variance of \( \sigma_{th}^2 \) which is assumed to be given.

The supply function for unhedged transformation services

In this subsection, the supply function for unhedged transformation services will be derived. Assume an individual supplier of unhedged transformation services has a utility function which displays positive absolute risk aversion, has decreasing absolute risk aversion over wealth, and maximizes his expected utility which is given by:

(2.19) \( E\{U(q^u_T)\} = E\{U[q^u_T(P_e - B - P)]\} \).

The individual's supply function for unhedged transformation services, \( q^u_T \), may be expressed as:

(2.20) \( q^u_T = q^u_T(P_e, B, P) \),

with partial derivatives:

(\( \partial q^u_T / \partial P_e > 0 \), \( \partial q^u_T / \partial B < 0 \), \( \partial q^u_T / \partial P < 0 \)).

Assuming that \( n \) such individual suppliers of unhedged transformation services exist, the aggregate supply function for unhedged transformation services, \( Q^u_T \), may be expressed:

(2.21) \( Q^u_T = Q^u_T(P_e, B, P, S_{t-1}) \)

with partial derivatives:

(\( \partial Q^u_T / \partial P_e > 0 \), \( \partial Q^u_T / \partial B < 0 \), \( \partial Q^u_T / \partial P < 0 \), \( \partial Q^u_T / \partial S_{t-1} > 0 \)).

This function states that the aggregate supply of unhedged transformation services is positively related to the expected return per pound of
supplying unhedged transformation services, \( t_u \). The variable \( S_{t-1} \), the total quantity of live beef cattle on feed on the previous market day, is included to show that \( Q^u_T \) is positively related to \( S_{t-1} \). The reason for this relationship is that frictions or rigidities in the market may limit period-to-period adjustment by suppliers of transformation services to the optimal level.

The supply function for hedged transformation services

In this subsection, the supply function for hedged transformation services will be derived. Assume that an individual supplier of hedged transformation services has a utility function which displays positive absolute risk aversion, has decreasing absolute risk aversion over wealth, and maximizes his expected utility which is given by:

\[
E[U(q^H_T)] = E[U(q^H_T(-B_e))].
\]

The individual's supply function for hedged transformation services may be expressed:

\[
q^H_T = q^H_T(B_e)
\]

with a partial derivative:

\[
(\partial q^H_T/\partial B_e < 0).
\]

Assuming that \( n \) such individual suppliers of hedged transformation services exist, the aggregate supply function for hedged transformation services, \( Q^H_T \), may be expressed:

\[
Q^H_T = Q^H_T(B_e, S_{t-1})
\]

with partial derivatives:

\[
(\partial Q^H_T/\partial B_e < 0, \partial Q^H_T/\partial S_{t-1} > 0).
\]

This function states that the aggregate supply of hedged transformation services is positively related to the expected return per pound of
providing hedged transformation services, \( t_h \). Again, the variable \( S_{t-1} \) is included to show that \( Q^H_T \) is positively related to the total quantity of live beef cattle on feed on the previous market day because of the frictions and rigidities previously mentioned.

The demand for storage services may be derived in three steps. First, the total quantity of finished live beef cattle available for slaughter on any given market day equals the quantity of finished live beef cattle carried in from the previous market day plus the current production of finished live beef cattle. The available quantity of finished live beef cattle, \( S^{TAQ} \), is given by:

\[
(2.25) \quad S^{TAQ} = S^{TAQ}(P, S_{t-1})
\]

with partial derivatives:

\[
(\partial S^{TAQ}/\partial P > 0, \partial S^{TAQ}/\partial S_{t-1} > 0)
\]

The positive relationship between \( S^{TAQ} \) and \( P \) is explained by fed cattle producers' current production response to the current cash price. \( S^{TAQ} \) is assumed to be positively related to \( S_{t-1} \) because it is likely that a fairly constant proportion of \( S_{t-1} \) is finished beef cattle storage.

Second, the demand for current consumption of finished beef cattle, \( Q_{cc} \), is negatively related to the current cash price and positively related to the current basis because consumers will tend to increase current consumption if the current cash price is low relative to a future price. The demand for current consumption may be expressed:

\[
(2.26) \quad Q_{cc} = Q_{cc}(P, B, D, b)
\]

with partial derivatives:

\[
(\partial Q_{cc}/\partial P < 0, \partial Q_{cc}/\partial B > 0, \partial Q_{cc}/\partial D < 0, \partial Q_{cc}/\partial b > 0).
\]

The variable, \( D \), is the number of days remaining in the predelivery period.
D is included because the demand for a given amount of storage is positively related to the length of the storage period. $b$ is a shift parameter which reflects seasonal demands of consumers.

Third, the demand for storage services, $Q_s^D$, equals the total quantity of finished beef cattle available for sale, $STAQ$, minus the quantity demanded for current consumption, $Q_{cc}$:

$$Q_s^D = STAQ(P, S_{t-1}) - Q_{cc}(P, B, D, b)$$

In general function form, the aggregate demand for storage services may be expressed:

$$Q_s^D = Q_s^D(P, B, D, S_{t-1}, b)$$

with partial derivatives:

$$\frac{\partial Q_s^D}{\partial STAQ} > 0; \frac{\partial Q_s^D}{\partial P} < 0; \frac{\partial Q_s^D}{\partial B} < 0; \frac{\partial Q_s^D}{\partial S_{t-1}} > 0; \frac{\partial Q_s^D}{\partial Q_{cc}} > 0; \frac{\partial Q_s^D}{\partial D} < 0; \frac{\partial Q_s^D}{\partial b} < 0.$$

The demand for transformation services are demanded by beef cattle processors in anticipation of future consumer demand for finished beef cattle. Cattle processors can either hedge their anticipated future needs, i.e., simultaneously buy a futures contract and sell a forward contract for beef cattle based on the current cash price for fed cattle or can satisfy their anticipated future needs through a spot purchase at a later date. The total quantity of transformation services demanded by cattle processors is a function of the expected return per pound and variance of the expected return per pound of hedging their future needs and the expected return per pound and variance of the expected return per pound of not hedging their future needs.
The expected return per pound of finished beef cattle by processors who hedge their future needs \( (l_h) \) is:

\[
(2.29) \quad l_h = B_{eP} - B
\]

where \( B_{eP} \) is the basis expected by processors. \( l_h \) has a variance of \( \sigma_{l_h}^2 \) which is assumed to be given.

The expected return per pound of finished beef cattle by processors who do not hedge their future needs \( (l_u) \) is:

\[
(2.30) \quad l_u = P - P_{eP}
\]

where \( P_{eP} \) is the cash price expected by packers. \( l_u \) has a variance of \( \sigma_{l_u}^2 \) which is assumed to be given.

The demand function for transformation services which are not hedged

In this subsection, the demand function for transformation services which are not hedged by processors will be derived. Assume an individual processor who does not hedge his demand for transformation services has a utility function which displays positive absolute risk aversion, has decreasing absolute risk aversion over wealth, and maximizes his expected utility which is given by:

\[
(2.31) \quad E\{U(q^D_{T})\} = E\{U[q^D_{T}(P - P_{eP})]\}.
\]

The processor's demand function for transformation services which are not hedged, \( q^D_{T} \), may be expressed:

\[
(2.32) \quad q^D_{T} = q^D_{T}(P, P_{eP})
\]

with partial derivatives:

\[
(\partial q^D_{T}/\partial P > 0, \partial q^D_{T}/\partial P_{eP} < 0).
\]

Assuming that \( n \) such processors, who demand transformation services which are not hedged, exist, the aggregate demand function for transformation services which are not hedged, \( Q^D_{T} \), may be expressed as:
\(Q_T^{DU} = Q_T^{DU}(P, P_P)\)

with partial derivatives:
\([\partial Q_T^{DU}/\partial P > 0, \partial Q_T^{DU}/\partial P_P < 0]\).

The demand function for transformation services which are hedged

In this subsection, the demand function for transformation services which are hedged by processors will be derived. Assume the individual processor has a utility function which displays positive absolute risk aversion, has decreasing absolute risk aversion over wealth, and maximizes his expected utility which is given by:
\[(2.34) \ E[U(q_T^{DH})] = E[U[q_T^{DH}(B_P - B)]]\].

The processor's demand function for transformation services which are hedged, \(q_T^{DH}\), may be expressed:
\[(2.35) q_T^{DH} = q_T^{DH}(B_P, B)\]

with partial derivatives:
\([\partial q_T^{DH}/\partial B_P > 0, \partial q_T^{DH}/\partial B < 0]\).

Assuming that \(n\) such processors, who demand transformation services which are hedged, exist, the aggregate demand function for transformation services which are hedged, \(Q_T^{DH}\), may be expressed:
\[(2.36) Q_T^{DH} = Q_T^{DH}(B_P, B)\]

with partial derivatives:
\([\partial Q_T^{DH}/\partial B_P > 0, \partial Q_T^{DH}/\partial B < 0]\).

This function states that the quantity of transformation services which processors hedge is positively related to the expected return per pound for hedging their future needs of finished beef cattle.
Equilibrium in the market for storage and transformation services

Equilibrium in the market for storage and transformation services for live beef cattle occurs when the total quantity of storage and transformation services supplied equals the total quantity of storage and transformation services demanded.

\[
Q^U_S(P_e, P, D) + Q^H_S(B_e, D) + Q^U_T(P_e, B, P, S_{t-1}) + Q^H_T(B_e, S_{t-1}) = Q^D_S(P, B, D, S_{t-1}, b) + Q^D_T(P, P_e) + Q^D_H(B_e, P)
\]

The endogenous variables in the market for storage and transformation services are \(Q^U_S, Q^H_S, Q^U_T, Q^H_T, Q^D_S, Q^D_T, Q^D_H, B, \) and \(P.\) The exogenous variables and shift parameters are \(P_e, B_e, P_e, B_e, P_e, D, S_{t-1}, b.\) Equilibrium in the market for storage and transformation services reduces the market into one equation and two endogenous variables, \(B\) and \(P.\) In order to determine unique values for \(B\) and \(P,\) another equation which contains \(B\) and \(P\) as endogenous variables will be developed to represent equilibrium in the futures market.

The futures market

The futures market is a competitive market where futures contracts for a commodity are traded. This section will describe the demand and supply functions for futures contracts and will define equilibrium in the futures market.

The supply of futures contracts

The supply of futures contracts equals the hedged supply of storage services, \(Q^H_S,\) and transformation services, \(Q^H_T.\) For each quantity of storage and transformation services which is hedged by suppliers of these services, an equal quantity of futures contracts is sold. Thus, the total quantity of futures contracts supplied
The demand for futures contracts The quantity of futures contracts demanded equals the quantity of contracts demanded by speculators, $Q^F_{DSP}$, plus the quantity of futures contracts demanded by processors who are long hedgers, $Q^H_T$. The quantity of futures contracts demanded by speculators is a function of the expected return per futures contract. The expected return per futures contract to speculators ($f$) is:

$$f = F_e - FP$$

where $F_e$ is the futures price expected by speculators. Solving equation (2.4) for $FP$ and substituting the result into (2.39), the expected return per unit of futures contract becomes:

$$f = F_e - B - P$$

The demand function for futures contracts by speculators In this subsection, the demand function for futures contracts by speculators will be derived. Assume an individual speculator who demands futures contracts has a utility function which displays positive absolute risk aversion, has decreasing absolute risk aversion over wealth, and maximizes his expected utility which is given by:

$$E\{U(q^\text{DSP}_F)\} = E\{U[q^\text{DSP}_F (F_e - B - P)]\}.$$ 

The speculator's demand function for futures contracts, $q^\text{DSP}_F$, may be expressed as:

$$q^\text{DSP}_F = q^\text{DSP}_F (F_e, B, P)$$

with partial derivatives:

$$\frac{\partial q^\text{DSP}_F}{\partial F_e} > 0, \frac{\partial q^\text{DSP}_F}{\partial B} < 0, \frac{\partial q^\text{DSP}_F}{\partial P} < 0.$$ 

Assuming that $n$ such speculators exist, the aggregate demand for
futures contracts by speculators, $Q_{DSP}^D$, may be expressed as:

$Q_{DSP}^D = Q_{F}^D(F_e, B, P)$

with partial derivatives:

$\frac{\partial Q_{DSP}^D}{\partial F_e} > 0, \frac{\partial Q_{DSP}^D}{\partial B} < 0, \frac{\partial Q_{DSP}^D}{\partial P} < 0$.

The quantity of futures contracts demanded by speculators may be positive or negative. When the quantity of futures contracts demanded is positive, speculators anticipate a "bull" market, i.e., they are buying futures contracts. When the quantity of futures contracts demanded by speculators is negative, speculators anticipate a "bear" market, i.e., they are selling futures contracts.

The aggregate quantity of futures contracts demanded ($Q_{DSP}^D$) is:

$Q_{DSP}^D = Q_{F}^D(F_e, B, P) + Q_{DH}^D(B_e, B)$.  

**Equilibrium in the futures market**

Equilibrium in the futures market occurs when the total quantity of futures contracts supplied equals the total quantity of futures contracts demanded.

$Q_{DSP}^H(B, B_e, D) + Q_{DH}^H(B_e, S_{t-1}) = Q_{DSP}^D(F_e, B, P) + Q_{DH}^D(B_e, B)$.  

The endogenous variables in the futures market are $Q_{DSP}^H$, $Q_{DH}^H$, $Q_{F}^D$, $Q_{DH}$, $B$, and $P$. The exogenous variables are $B_e$, $D$, $F_e$, $B_e$, and $S_{t-1}$.

**Equilibrium in the live beef cattle industry**

Equilibrium in the live beef cattle industry occurs when the market for storage and transformation services and the futures market are simultaneously in equilibrium. Industry equilibrium is described by the following two equations:
The endogenous variables in this two market system of equations is $B$, the current basis, and $P$, the current cash price for finished live beef cattle. Provided that the two equations, (2.46) and (2.47), have continuous partial derivatives and that the endogenous variable Jacobian is non-zero, equilibrium values for $B$ and $P$, i.e., the static results for industry equilibrium, exist and are given by:

\[
(2.48) \quad B = B(P_e, B_e, P_e', B_{eP}, B_{eP'}, S_{t-1}, D, b)
\]

\[
(2.49) \quad P = P(P_e, B_e, P_e', B_{eP}, B_{eP'}, S_{t-1}, D, b)
\]

Given values for the exogenous variables, the shift parameter, and the explicit functional forms of the supply and demand equations, one unique set of values for $B$ and $P$ can be determined which will create equilibrium in the live beef cattle industry.

**Comparative static results**

Comparative static results show the qualitative impacts on endogenous variables caused by changes in exogenous variables or shift parameters. The comparative static results for the predelivery basis determination model will now be derived.

First, express the two industry equilibrium equations in implicit function form:

\[
(2.50) \quad F_1 = F_1(B, P, P_e, B_e, P_{eP}, B_{eP}, F_e, D, S_{t-1}, b) = 0
\]

\[
(2.51) \quad F_2 = F_2(B, P, P_e, B_e, P_{eP}, B_{eP}, F_e, D, S_{t-1}, b) = 0
\]

Assume (1) that $F_1$ and $F_2$ possess continuous derivatives and (2) that
\( \frac{\partial F_1}{\partial B}, \frac{\partial F_1}{\partial P}, \frac{\partial F_2}{\partial B}, \frac{\partial F_2}{\partial P} \) are non-zero, regardless of where they are evaluated.

Second, take the total differentials of \( F_1 \) and \( F_2 \):

\[
\frac{\partial F_1}{\partial B} \, dB + \frac{\partial F_1}{\partial P} \, dP + \frac{\partial F_1}{\partial B} \, dB + \frac{\partial F_1}{\partial P} \, dP + \frac{\partial F_1}{\partial B} \, dB + \frac{\partial F_1}{\partial P} \, dP + \frac{\partial F_1}{\partial B} \, dB + \frac{\partial F_1}{\partial P} \, dP + \frac{\partial F_1}{\partial B} \, dB + \frac{\partial F_1}{\partial P} \, dP + \frac{\partial F_1}{\partial B} \, dB + \frac{\partial F_1}{\partial P} \, dP + \frac{\partial F_1}{\partial B} \, dB + \frac{\partial F_1}{\partial P} \, dP = 0
\]

\[
\frac{\partial F_2}{\partial B} \, dB + \frac{\partial F_2}{\partial P} \, dP + \frac{\partial F_2}{\partial B} \, dB + \frac{\partial F_2}{\partial P} \, dP + \frac{\partial F_2}{\partial B} \, dB + \frac{\partial F_2}{\partial P} \, dP + \frac{\partial F_2}{\partial B} \, dB + \frac{\partial F_2}{\partial P} \, dP + \frac{\partial F_2}{\partial B} \, dB + \frac{\partial F_2}{\partial P} \, dP + \frac{\partial F_2}{\partial B} \, dB + \frac{\partial F_2}{\partial P} \, dP + \frac{\partial F_2}{\partial B} \, dB + \frac{\partial F_2}{\partial P} \, dP = 0
\]

To derive the comparative static results for a ceteris paribus change in \( P \), set \( dB = 0 \) and divide (2.52) and (2.53) by \( dP \).

\[
\begin{align*}
(2.54) \quad & \left( \frac{\partial F_1}{\partial B} \right) \frac{dB}{dP} + \left( \frac{\partial F_1}{\partial P} \right) \frac{dP}{dP} = -\frac{\partial F_1}{\partial P} \\
(2.55) \quad & \left( \frac{\partial F_2}{\partial B} \right) \frac{dB}{dP} + \left( \frac{\partial F_2}{\partial P} \right) \frac{dP}{dP} = -\frac{\partial F_2}{\partial P}
\end{align*}
\]

This system of equations can be expressed in matrix notation in the following manner:

\[
\begin{bmatrix}
\frac{\partial F_1}{\partial B} & \frac{\partial F_1}{\partial P} \\
\frac{\partial F_2}{\partial B} & \frac{\partial F_2}{\partial P}
\end{bmatrix}
\begin{bmatrix}
\frac{dB}{dP} \\
\frac{dP}{dP}
\end{bmatrix}
= -
\begin{bmatrix}
\frac{\partial F_1}{\partial P} \\
\frac{\partial F_2}{\partial P}
\end{bmatrix}
\]

The Jacobian of this system is:
Without some assumptions about the sign of the partial derivative, \( \frac{\partial F_1}{\partial B} \), the determinant of the Jacobian cannot be signed. However, by assuming that \( \frac{\partial F_1}{\partial B} \) is positive, the determinant of the Jacobian is:

\[
\begin{vmatrix}
\frac{\partial F_1}{\partial B} & \frac{\partial F_1}{\partial P} \\
\frac{\partial F_2}{\partial B} & \frac{\partial F_2}{\partial P}
\end{vmatrix} = \begin{vmatrix}
? & - \\
+ & +
\end{vmatrix}
\]

(2.57)

\[
\frac{\partial F_1}{\partial B} < 2.58
\]

\[
\begin{vmatrix}
+ & - \\
+ & +
\end{vmatrix} > 0.
\]

The Jacobian remains the same for all comparative static results. The assumption that \( \frac{\partial F_1}{\partial B} \) is positive may be expressed:

\[
\begin{vmatrix}
\frac{\partial Q^H}{\partial B} - \frac{\partial Q^D}{\partial B} > \frac{\partial Q^U}{\partial B}
\end{vmatrix}
\]

(2.59)

This expression states that the change in the quantity of hedged storage services supplied minus the change in the quantity of hedged transformation services demanded caused by a change in the current basis is greater than the change in the quantity of unhedged transformation services caused by a change in the current basis.

Applying Cramer’s rule, the two comparative static results in (2.56) are:

\[
\frac{\partial B}{\partial e} = \begin{vmatrix}
-\frac{\partial F_1}{\partial P} & \frac{\partial F_1}{\partial P} \\
-\frac{\partial F_2}{\partial P} & \frac{\partial F_2}{\partial P}
\end{vmatrix} = \begin{vmatrix}
- & - \\
0 & +
\end{vmatrix}
\]

(2.60)
These comparative static results show that a ceteris paribus increase in the cash price expected by live beef cattle producers will cause the equilibrium basis to decrease and the equilibrium cash price to increase. These results may be verbally explained in the following manner. First, an increase in $P_e$ will cause the supply functions for storage and transformation services to shift outward. Given a current cash price, the market for storage and transformation services will be equilibrated by a decrease in the current basis. The decrease in the current basis will cause an excess demand for futures contracts in the futures market which will force the futures price to increase as suppliers of hedged storage services buy back contracts to offset their short commitments and demanders of hedged transformation services and speculators buy more contracts. Because the current basis must decrease and the current futures price will increase as a result of the increase in $P_e$, the current cash price must also increase to maintain equilibrium in both the market for storage and transformation services and the futures market.

The comparative static results for a ceteris paribus change in the basis expected by suppliers of storage and transformation services, $B_e$, are:

$$
(2.61) \frac{d\bar{P}}{dP_e} = \frac{\begin{vmatrix} \frac{\partial F_1}{\partial \bar{B}} & -\frac{\partial F_1}{\partial P_e} \\ \frac{\partial F_2}{\partial \bar{B}} & -\frac{\partial F_2}{\partial P_e} \end{vmatrix}}{|J|} = \begin{vmatrix} + & - \\ + & 0 \end{vmatrix} > 0
$$

These comparative static results show that a ceteris paribus change in the basis expected by suppliers of storage and transformation services, $B_e$, are:

$$
(2.62) \frac{d\bar{B}}{dB_e} = \frac{\begin{vmatrix} -\frac{\partial F_1}{\partial B_e} & \frac{\partial F_1}{\partial \bar{P}} \\ -\frac{\partial F_2}{\partial B_e} & \frac{\partial F_2}{\partial \bar{P}} \end{vmatrix}}{|J|} = \begin{vmatrix} + & - \\ + & + \end{vmatrix} > 0
$$
A ceteris paribus increase in $B_e$ will shift the supply functions for hedged storage and transformation services and the supply function of futures contracts inward. Given a current cash price, the current basis must increase to remove the excess demands in both markets. However, a particular increase in the basis by itself will not likely restore both markets to equilibrium simultaneously. Therefore, the cash price must change in an appropriate direction to restore equilibrium in both markets.

The comparative static results for a ceteris paribus change in the cash price expected by demanders of transformation services, $P_{eP}$, are:

$$
\begin{align*}
(2.63) \quad \frac{d\bar{P}}{dB_e} &= \begin{vmatrix}
\frac{\partial F_1}{\partial B_e} & -\frac{\partial F_1}{\partial B_e} \\
\frac{\partial F_2}{\partial B_e} & -\frac{\partial F_2}{\partial B_e}
\end{vmatrix} = \begin{vmatrix}
+ & + \\
0 & < 0
\end{vmatrix}
\end{align*}
$$

A ceteris paribus increase in $P_{eP}$ will cause the demand function for unhedged transformation services to shift inward. Given a current cash price, the market for storage and transformation services will be equilibrated by a decrease in the current basis. The decrease in the current basis will cause an excess demand for futures contracts which will force
the futures price to increase as suppliers of hedged storage services buy back contracts to offset their short commitments and demanders of futures contracts buy more contracts. Because the current basis must decrease and the current futures price will increase as a result of the increase in \( P_{eP} \), the current cash price must increase to maintain equilibrium in both the market for storage and transformation services and the futures market.

The comparative static results for a ceteris paribus change in the basis expected by demanders of transformation services, \( B_{eP} \), are:

\[
\frac{\partial B}{\partial B_{eP}} = \begin{vmatrix} -\frac{\partial F_1}{\partial B_{eP}} & \frac{\partial F_1}{\partial P} \\ -\frac{\partial F_2}{\partial B_{eP}} & \frac{\partial F_2}{\partial P} \end{vmatrix} = \frac{+}{-} + + > 0
\]

\[
\frac{\partial P}{\partial B_{eP}} = \begin{vmatrix} \frac{\partial F_1}{\partial B} & -\frac{\partial F_1}{\partial B_{eP}} \\ \frac{\partial F_2}{\partial B} & -\frac{\partial F_2}{\partial B_{eP}} \end{vmatrix} = \frac{+}{-} + + > 0
\]

A ceteris paribus increase in \( B_{eP} \) will cause the demand for hedged transformation services and the demand for futures contracts to shift outward. Given a current cash price, the current basis must increase to equilibrate the market for storage and transformation services. The increase in the current basis will also help to restore equilibrium in the futures market by removing some of the excess demand in the market. It is doubtful that an increase in the current basis will equilibrate both the market for storage and transformation services and the futures market after an exogenous outward shift of the demand functions in both markets. Therefore, the current cash price must change in the appropriate direction to restore both markets simultaneously.
The comparative static results for a ceteris paribus change in the futures price expected by speculators, $F_e$, are:

\[
(2.68) \frac{d\tilde{B}}{dF_e} = \begin{vmatrix} -\partial F_1/\partial F_e & \partial F_1/\partial \tilde{F} \\ -\partial F_2/\partial F_e & \partial F_2/\partial \tilde{F} \end{vmatrix} = \begin{vmatrix} 0 & - \\ + & + \end{vmatrix} > 0
\]

A ceteris paribus increase in $F_e$ will shift the demand for futures contracts outward. The equilibrium futures price will increase as speculators bid up the price of futures contracts. Given a value for the current cash price, the current basis will increase which will create an excess supply of storage and transformation services. Because the futures price will increase and the current basis increases, the equilibrium cash price must increase to restore equilibrium in both the market for storage and transformation services and the futures market.

The comparative static results for a ceteris paribus change in the number of calendar days till the delivery period, $D$, are:

\[
(2.70) \frac{d\tilde{B}}{dD} = \begin{vmatrix} -\partial F_1/\partial D & \partial F_1/\partial \tilde{F} \\ -\partial F_2/\partial D & \partial F_2/\partial \tilde{F} \end{vmatrix} = \begin{vmatrix} + & - \\ + & + \end{vmatrix} > 0
\]

\[
(2.71) \frac{d\tilde{F}}{dD} = \begin{vmatrix} \partial F_1/\partial \tilde{B} & -\partial F_1/\partial D \\ \partial F_2/\partial \tilde{B} & -\partial F_2/\partial D \end{vmatrix} = \begin{vmatrix} + & + \\ + & + \end{vmatrix} > 0
\]
A ceteris paribus increase in D will shift supply functions for storage and transformation services and futures contracts inward, and the demand for storage services outward. Given a value for the current cash price, the basis must increase to restore equilibrium in the market for storage and transformation services. The increase in the current basis will help to remove the excess demand for futures contracts in the futures market. However, the increase in the current basis by itself will not likely restore equilibrium in both markets. Therefore, the equilibrium cash price will change in such a manner that will restore equilibrium in both the market for storage and transformation services and the futures market.

The comparative static results for a ceteris paribus change in the total quantity of cattle on feed and in storage carried into the current market period, $S_{t-1}$, are:

$$\left(2.72\right) \frac{d\bar{B}}{dS_{t-1}} = \left| \begin{array}{cc} -\frac{aF_1}{aS_{t-1}} & \frac{\partial F_1}{\partial P} \\ -\frac{aF_2}{aS_{t-1}} & \frac{\partial F_2}{\partial P} \end{array} \right| \frac{\partial P}{dS_{t-1}} \leq 0$$

$$\left(2.73\right) \frac{d\bar{P}}{dS_{t-1}} = \left| \begin{array}{cc} \frac{\partial F_1}{\partial B} & -\frac{\partial F_1}{\partial S_{t-1}} \\ \frac{\partial F_2}{\partial B} & -\frac{\partial F_2}{\partial S_{t-1}} \end{array} \right| \frac{\partial S_{t-1}}{dS_{t-1}} \leq 0$$

As they stand, the comparative static results for $S_{t-1}$ are indeterminant because:

$$\left(2.74\right) \frac{\partial F_1}{\partial S_{t-1}} = \frac{\partial Q^U}{\partial S_{t-1}} + \frac{\partial Q^H}{\partial S_{t-1}} - \frac{\partial Q^D}{\partial S_{t-1}}$$

and is not strictly positive or negative. However, by making the assumption that:

$$\left(2.75\right) \frac{\partial F_1}{\partial S_{t-1}} = 0,$$
definite signs can be assigned to the comparative static results in (2.72) and (2.73). This assumption states that the change in supply of storage and transformation services created by a change in $S_{t-1}$ is exactly equal to the change in demand for storage and transformation services created by a change in $S_{t-1}$. The comparative static results under this assumption are:

$$\begin{vmatrix} -\frac{\partial F_1}{\partial S_{t-1}} & \frac{\partial F_1}{\partial \bar{P}} \\ -\frac{\partial F_2}{\partial S_{t-1}} & \frac{\partial F_2}{\partial \bar{P}} \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix} < 0$$

$$\begin{vmatrix} \frac{\partial F_1}{\partial \bar{B}} & -\frac{\partial F_1}{\partial S_{t-1}} \\ \frac{\partial F_2}{\partial \bar{B}} & -\frac{\partial F_2}{\partial S_{t-1}} \end{vmatrix} = \begin{vmatrix} + \\ - \end{vmatrix} < 0$$

A ceteris paribus increase in $S_{t-1}$ will cause the supply of futures contracts to shift outward. Given a value for the current cash price, the futures price, and thus, the basis, must decrease to restore equilibrium in the futures market. The decrease in the current basis will create an excess demand for storage and transformation services. To restore equilibrium in the market for storage and transformation services, the current cash price must also decrease.

The comparative static results for a ceteris paribus change in the shift parameter, $b$, are:

$$\begin{vmatrix} -\frac{\partial F_1}{\partial b} & \frac{\partial F_1}{\partial \bar{P}} \\ -\frac{\partial F_2}{\partial b} & \frac{\partial F_2}{\partial \bar{P}} \end{vmatrix} = \begin{vmatrix} - \\ 0 \end{vmatrix} < 0$$
A ceteris paribus increase in $b$ will cause the demand function for storage and transformation services to shift inward. Given a value for the current cash price, the basis must decrease to restore equilibrium in the market for storage and transformation services. The decrease in the current basis will create an excess demand for futures contracts in the futures market. The excess demand in the futures market will cause the futures price to rise which will restore equilibrium in the futures market. Because the current basis must decrease to restore equilibrium in the market for storage and transformation services and the futures price must rise to restore equilibrium in the futures market, the current cash price must also increase to restore equilibrium in both markets simultaneously.

**Summary of static and comparative static results**

The preceding description of the predelivery period model has shown that the model can be described in a two-equation system with two endogenous variables, $B$ and $P$. Assuming that the rules of the implicit function theorem are met, the static results of the model are:

\[
\begin{align*}
\bar{B} &= \bar{B}(P_e, B_e, F_e, P_e P, B_e P, S_{t-1}, D, b) \\
\bar{P} &= \bar{P}(P_e, B_e, F_e, P_e P, B_e P, S_{t-1}, D, b)
\end{align*}
\]

The signs of the sixteen comparative static derivatives derived from the predelivery period model are:

1. $\frac{d\bar{B}}{dP_e} < 0$,  
2. $\frac{d\bar{P}}{dP_e} > 0$,  
3. $\frac{d\bar{B}}{dB_e} > 0$,  
4. $\frac{d\bar{P}}{dB_e} < 0$,  
5. $\frac{d\bar{B}}{dP_{eP}} < 0$,  
6. $\frac{d\bar{P}}{dP_{eP}} > 0$,
7. $\frac{dB}{dB_eP} > 0$, 8. $\frac{dP}{dB_eP} < 0$, 9. $\frac{dB}{dF_e} > 0$,
10. $\frac{dP}{dF_e} > 0$, 11. $\frac{dB}{dD} > 0$, 12. $\frac{dP}{dD} > 0$,
13. $\frac{dB}{dS_{t-1}} < 0$, 14. $\frac{dP}{dS_{t-1}} < 0$, 15. $\frac{dB}{db} < 0$, and
16. $\frac{dP}{db} > 0$.

A General Function Model of Basis Determination in the Delivery Period

In this section, a general function model of delivery period daily basis determination will be developed. The delivery period basis determination model involves two markets, the cash commodity market and the futures market. Subsections will include: (1) a description of the supply and demand functions in the cash commodity market, (2) a description of equilibrium in the cash commodity market, (3) a description of supply and demand functions in the futures market, (4) a description of equilibrium in the futures market, (5) a description of live beef cattle industry equilibrium in the delivery period, (6) static results of the model, and (7) comparative static results of the model.

The cash commodity market

The cash commodity market is the market in which finished live beef cattle are exchanged. Sellers in the cash commodity market are producers of live beef cattle. Buyers in the cash commodity market are livestock processors.

The supply function for the cash commodity

The quantity of finished live beef cattle supplied on any given day of the delivery period is related to the current cash price and the current basis. In general function form, the supply function for finished live beef cattle, $Q_{cs}$, may
be expressed:

\[(2.80) \quad Q_{cs} = Q_{cs}(P, B, c)\]

with partial derivatives:

\(\frac{\partial Q_{cs}}{\partial P} > 0, \frac{\partial Q_{cs}}{\partial B} < 0, \frac{\partial Q_{cs}}{\partial c} > 0\).

This function states that the current quantity of finished live beef cattle supplied is positively related to the current cash price and negatively related to the current basis. The negative relationship between \(Q_{cs}\) and \(B\) exists because producers will delay their marketing actions several days when they anticipate receiving a price relatively higher at the later date. The shift parameter, \(c\), is included to allow for shifts in the supply function due to seasonal production patterns.

The demand function for the cash commodity

The quantity of finished live beef cattle demanded by packers on any given day of the delivery period is related to the current cash price and the current basis. In general function form, the demand function for finished live beef cattle, \(Q_{cd}\), may be expressed:

\[(2.81) \quad Q_{cd} = Q_{cd}(P, B, d)\]

with partial derivatives:

\(\frac{\partial Q_{cd}}{\partial P} < 0, \frac{\partial Q_{cd}}{\partial B} > 0, \frac{\partial Q_{cd}}{\partial d} > 0\).

This function states that the quantity of the commodity demanded is negatively related to the current cash price and positively related to the current basis. The positive relationship between \(Q_{cd}\) and \(B\) exists because cattle processors will increase their current purchases in anticipation of having to pay a relatively higher price if they delay their purchases several days. The shift parameter, \(d\), is included to allow for possible shifts in the demand function caused by seasonal consumption patterns.
Equilibrium in the cash commodity market

Equilibrium occurs in the cash commodity market when the quantity of cash commodity supplied equals the quantity of cash commodity demanded. Equilibrium in the cash commodity market may be expressed:

\[ Q_{CS}(P, B, c) = Q_{CD}(P, B, d) \]

Endogenous variables in the cash commodity market are \( Q_{CS}', Q_{CD}', B, \) and \( P \). There are no exogenous variables in the cash commodity market, however, there are two shift parameters, \( c \) and \( d \). Equilibrium in the cash commodity market reduces the number of endogenous variables to two, \( B \) and \( P \). In order to determine unique values for \( B \) and \( P \), another equation which contains \( B \) and \( P \) as endogenous variables will be developed to represent equilibrium in the futures market.

The futures market

During the delivery period of a futures contract, there is no inter-temporal difference between the cash commodity price and the futures price. According to Wildermuth and Gum [16], barring any grade, weight, or locational differences, the daily basis values in the delivery period should equal zero due to arbitrage. In the following sections, the supply and demand functions for futures contracts will be derived.

The supply of futures contracts Whenever more than one market exists for a particular commodity, price differences between the markets may arise. Normally, these differences reflect cost differences, such as transportation costs, between the markets. However, when the price differences exceed cost differences, the opportunity for arbitrage arises. Arbitrage is the process of purchasing a commodity in one market for
immediate resale in the other market in order to profit from price discrepancies between the markets. In this study, the two markets between which a price discrepancy may arise are the cash commodity and futures markets.

Consider an individual arbiter. This individual is assumed to be an expected utility maximizer, to have a utility function which displays positive absolute risk aversion, and to have decreasing absolute risk aversion over wealth. The expected return to an arbiter per pound of finished beef cattle which is arbitraged ($\bar{R}$) is:

$$\bar{R} = B - T$$

where $B$ is the current basis and $T$ is the transaction cost for that particular delivery day. If $\bar{R}$ is positive, the individual will be a short arbiter, i.e., he will sell a futures contract and buy the cash commodity to deliver on the futures contract. If $\bar{R}$ is negative, the individual will be a long arbiter, i.e., he will buy a futures contract with the intention of taking delivery on the futures contract and sell an equivalent amount of forward contracts in the cash commodity market.

$\bar{R}$ has a variance of $\sigma^2_{\bar{R}}$ which is assumed to be given. The variance, $\sigma^2_{\bar{R}}$, arises because of the risk and uncertainty involved with performing arbitrage. A short arbiter faces the risk that the cash commodity which he purchases to fulfill the contractual obligations will not meet the grade specifications of the futures contract. A long arbiter faces the uncertainty of when the seller of the futures contract will deliver the cash commodity during the delivery period. The delivery date, within the delivery period, is decided by the seller of the futures contract.

The expected utility function of the arbiter is given by:
(2.84) $E\{q_{FA}'\} = E[U[q_{FA}'(B - T)]].$

The arbiter will select the quantity of finished live beef cattle to arbitrage, $q_{FA}'$, which will maximize the expected utility function in (2.84). For purposes of explanation, the arbiter will be defined as a supplier of futures contracts. When $\bar{R}$ is positive, the arbiter sells futures contracts, i.e., supplies a positive quantity of futures contracts. When $\bar{R}$ is negative, the arbiter buys futures contracts, i.e., supplies a negative quantity of futures contracts. In general function form, the arbiter's supply function of futures contracts may be expressed:

(2.85) $q_{FA}' = q_{FA}'(B, T)$.

From Sandmo [12], the partial derivatives of (2.85) are known to be:

$\frac{\partial q_{FA}'}{\partial B} > 0, \frac{\partial q_{FA}'}{\partial T} > 0$.

Assuming that $n$ such arbiters exist, the aggregate quantity of futures contracts supplied by arbiters, $Q_{FA}'$, is:

(2.86) $Q_{FA}' = Q_{FA}'(B, T)$

with partial derivatives:

$\frac{\partial Q_{FA}'}{\partial B} > 0, \frac{\partial Q_{FA}'}{\partial T} > 0$.

This function states that the aggregate quantity of futures contracts supplied by arbiters is positively related to the expected return for arbitrage per pound of finished live beef cattle. The relationship between $\bar{R}$ and $Q_{FA}'$ is presented graphically in Figure 3. In quadrant I, the arbiters are supplying a positive quantity of futures contracts. In quadrant III, the arbiters are supplying a negative quantity of futures contracts. For simplicity, a linear relationship is presented in Figure 3. However, the relationship between $\bar{R}$ and $Q_{FA}'$ need not be linear.
Figure 3. The relationship between $\bar{R}$ and $Q_{FA}$.

The demand for futures contracts

Futures contracts in the delivery period are demanded by speculators. Consider an individual speculator in the futures market. The speculator is assumed to be an expected utility maximizer, to have a utility function which displays positive absolute risk aversion, and to have decreasing absolute risk aversion over wealth. The expected return to the speculator per pound of finished beef cattle ($f$) is:

\begin{equation}
(2.87) \quad f = F_e - FP = F_e - B - P
\end{equation}

where $F_e$ is the futures price expected by the speculator. If $f$ is positive, the individual will be a long speculator, i.e., he will buy futures contracts. If $f$ is negative, the individual will be a short speculator, i.e., he will sell futures contracts. $f$ has a variance of $\sigma_f^2$ which is assumed to be given.

The expected utility of the speculator is given by:

\begin{equation}
(2.88) \quad E\{(q_{FSP})\} = E\{U[q_{FSP}(F_e - B - P)]\}.
\end{equation}

The speculator will select the quantity of finished beef cattle futures
contracts, \( q_{FSP} \), which will maximize the expected utility function in (2.88). For purposes of explanation, the speculator will be defined as the demander of futures contracts. In general function form, the speculator's demand function for futures contracts may be expressed:

\[
(2.89) \quad q_{FSP} = q_{FSP}(F_e, B, P).
\]

From Sandmo [12], the partial derivatives of (2.89) are:

\[
(\partial q_{FSP}/\partial F_e > 0, \partial q_{FSP}/\partial B < 0, \partial q_{FSP}/\partial P < 0).
\]

Assuming that \( n \) such individual speculators exist, the aggregate quantity of futures contracts demanded by speculators, \( Q_{FSP} \), is:

\[
(2.90) \quad Q_{FSP} = Q_{FSP}(F_e, B, P)
\]

with partial derivatives:

\[
(\partial Q_{FSP}/\partial F_e > 0, \partial Q_{FSP}/\partial B < 0, \partial Q_{FSP}/\partial P < 0).
\]

This function states that the quantity of futures contracts demanded by speculators is positively related to \( f \) in (2.87). The relationship between \( Q_{FSP} \) and \( f \) is presented graphically in Figure 4.

![Figure 4](image)

**Figure 4.** The relationship between \( Q_{FSP} \) and \( f \).
4, the speculators buy futures contracts. In quadrant III, the speculators sell futures contracts. Again for simplicity, a linear relationship between $Q_{FSP}$ and $f$ is presented. However, the relationship need not be linear.

**Equilibrium in the futures market**

Equilibrium in the futures market occurs when the quantity of futures contracts supplied by arbiters equals the quantity of futures contracts demanded by speculators:

\[ (2.91) \quad Q_{FA}(B, T) = Q_{FSP}(F_e, B, P) \]

Endogenous variables in the futures market are $Q_{FA}$, $Q_{FSP}$, $B$, and $P$. The exogenous variables in the futures market are $T$ and $F_e$. Equilibrium in the futures market reduces the number of endogenous variables to two, $B$ and $P$.

**Equilibrium in the live beef cattle industry**

Equilibrium in the live beef cattle industry during the delivery period occurs when the cash commodity market and the futures market are in equilibrium simultaneously. Industry equilibrium is described in the following two-equation system:

\[ (2.92) \quad Q_{CS}(P, B, c) = Q_{CD}(P, B, d) \]

\[ (2.93) \quad Q_{FA}(B, T) = Q_{FSP}(F_e, B, P) \]

The two endogenous variables in this two market system of equations are $B$, the current basis, and $P$, the current price for finished live cattle. Provided that the two equations, (2.92) and (2.93), have continuous partial derivatives and that the endogenous variable Jacobian is non-zero, equilibrium values for $B$ and $P$, i.e., the static results for industry
equilibrium, exist and are given by:

\[(2.94) \bar{B} = \bar{B}(T, F_e, c, d)\]
\[(2.95) \bar{P} = \bar{P}(T, F_e, c, d)\]

Given values for the exogenous variables, the shift parameters, and the explicit functional forms for the supply and demand relationships in the model, one unique set of values for B and P can be determined which will create equilibrium in the live beef cattle industry.

**Comparative static results**

The comparative static results for the delivery period basis determination model will now be derived. First, express the two industry equilibrium equations in implicit function form:

\[(2.96) G_1 = G_1(B, P, T, F_e, c, d) = 0\]
\[(2.97) G_2 = G_2(B, P, T, F_e, c, d) = 0\]

Assume (1) that \(G_1\) and \(G_2\) possess continuous derivatives and (2) that \(\frac{\partial G_1}{\partial B}, \frac{\partial G_1}{\partial P}, \frac{\partial G_2}{\partial B}, \frac{\partial G_2}{\partial P}\) are non-zero, regardless of where they are evaluated.

Second, take the total differentials of \(G_1\) and \(G_2\):

\[(2.98) dG_1 = \frac{\partial G_1}{\partial B} dB + \frac{\partial G_1}{\partial P} dP + \frac{\partial G_1}{\partial T} dT + \frac{\partial G_1}{\partial F_e} dF_e + \frac{\partial G_1}{\partial c} dc + \frac{\partial G_1}{\partial d} dd = 0\]
\[(2.99) dG_2 = \frac{\partial G_2}{\partial B} dB + \frac{\partial G_2}{\partial P} dP + \frac{\partial G_2}{\partial T} dT + \frac{\partial G_2}{\partial F_e} dF_e + \frac{\partial G_2}{\partial c} dc + \frac{\partial G_2}{\partial d} dd = 0\]

To derive the comparative static results for a ceteris paribus change in \(T\), set \(dF_e, dc,\) and \(dd\) equal to zero and divide \((2.98)\) and \((2.99)\) by \(dT\):

\[(2.100) (\frac{\partial G_1}{\partial B}) dB/dT + (\frac{\partial G_1}{\partial P}) dP/dT = -\frac{\partial G_1}{\partial T}\]
\[(2.101) (\frac{\partial G_2}{\partial B}) dB/dT + (\frac{\partial G_2}{\partial P}) dP/dT = -\frac{\partial G_2}{\partial T}\]
This system of equations can be expressed in matrix notation in the following manner:

\[
\begin{bmatrix}
\frac{\partial G_1}{\partial B} & \frac{\partial G_1}{\partial P} \\
\frac{\partial G_2}{\partial B} & \frac{\partial G_2}{\partial P}
\end{bmatrix}
\begin{bmatrix}
\frac{dB}{dT} \\
\frac{dP}{dT}
\end{bmatrix} = -
\begin{bmatrix}
\frac{\partial G_1}{\partial T} \\
\frac{\partial G_2}{\partial T}
\end{bmatrix}
\]

The Jacobian of this system is:

\[
\begin{bmatrix}
\frac{\partial G_1}{\partial B} & \frac{\partial G_1}{\partial P} \\
\frac{\partial G_2}{\partial B} & \frac{\partial G_2}{\partial P}
\end{bmatrix} = \begin{bmatrix}
- & + \\
+ & +
\end{bmatrix}
\]

The determinant of the Jacobian is:

\[
(2.104) \quad J = \begin{vmatrix}
- & + \\
+ & +
\end{vmatrix} < 0,
\]

which remains the same for all comparative static results.

Third, applying Cramer's rule, the two comparative static results in (2.102) are:

\[
(2.105) \quad \frac{dB}{dT} = \begin{vmatrix}
-\frac{\partial G_1}{\partial T} & \frac{\partial G_1}{\partial P} \\
-\frac{\partial G_2}{\partial T} & \frac{\partial G_2}{\partial P}
\end{vmatrix} = \frac{?}{|J|} + \frac{?}{|J|} < 0
\]

\[
(2.106) \quad \frac{dP}{dT} = \begin{vmatrix}
\frac{\partial G_1}{\partial B} & -\frac{\partial G_1}{\partial T} \\
\frac{\partial G_2}{\partial B} & -\frac{\partial G_2}{\partial T}
\end{vmatrix} = \frac{-?}{|J|} + \frac{?}{|J|} < 0
\]

These comparative static results show that a ceteris paribus increase in
the transaction cost, $T$, has an indeterminant effect on the equilibrium basis and the equilibrium price.

The comparative static results for a ceteris paribus change in the futures price expected by speculators, $F_e$, are:

\[
(2.107) \frac{d\bar{B}}{dF_e} = \begin{vmatrix} -\partial G_1/\partial F_e & \partial G_1/\partial \bar{P} \\ -\partial G_2/\partial F_e & \partial G_2/\partial \bar{P} \end{vmatrix} = \frac{|0 + |}{|J|} > 0
\]

\[
(2.108) \frac{d\bar{P}}{dF_e} = \begin{vmatrix} \partial G_1/\partial \bar{B} & -\partial G_1/\partial F_e \\ \partial G_2/\partial \bar{B} & -\partial G_2/\partial F_e \end{vmatrix} = \frac{|-0|}{|J|} > 0
\]

These comparative static results show that a ceteris paribus increase in $F_e$ will cause both the equilibrium basis and the equilibrium cash price to increase. The increase in $F_e$ may be viewed as an outward shift of the demand function for futures contracts. Given a current cash price, the current basis must increase to restore equilibrium in the futures market. The increase in the basis will create an excess demand in the cash commodity market. In order to restore equilibrium in both the cash commodity market and the futures market, both the current basis and the current cash price must increase.

The comparative static results for a ceteris paribus change in seasonal production shift parameter, $c$, are:

\[
(2.109) \frac{d\bar{B}}{dc} = \begin{vmatrix} -\partial G_1/\partial c & \partial G_1/\partial \bar{P} \\ -\partial G_2/\partial c & \partial G_2/\partial \bar{P} \end{vmatrix} = \frac{|- + |}{|J|} > 0
\]
These comparative static results show that a ceteris paribus increase, $c$, will cause the equilibrium basis to increase and the equilibrium price to decrease. The increase in $c$ may be viewed as an outward shift of the supply function in the cash commodity. Given a current basis value, the current cash price must decrease to restore equilibrium in the cash commodity market. This decrease in the current cash price will create an excess demand in the futures market. In order to restore equilibrium in both the cash commodity market and the futures market, the equilibrium basis must increase and the equilibrium price must decrease.

The comparative static results of a ceteris paribus change in the seasonal consumption shift parameter, $d$, are:

\[
\begin{align*}
(2.110) \quad \frac{\partial \bar{p}}{\partial c} &= \frac{\partial G_1/\partial \bar{B} - \partial G_1/\partial \bar{d}}{\partial G_2/\partial \bar{B} - \partial G_2/\partial \bar{d}} = \frac{-}{+} < 0
\end{align*}
\]

These comparative static results show that a ceteris paribus increase in $d$ will cause the equilibrium basis to decrease and the equilibrium cash price to increase. The increase in $d$ may be viewed as an outward shift of the demand function in the cash commodity market. Given a current basis
value, equilibrium in the cash commodity market can be restored with an increase in the current cash price. This increase in the current cash price will create an excess supply in the futures market. In order to restore equilibrium in both the cash commodity market and the futures market, the current basis must decrease and the current cash price must increase.

Summary of the static and comparative static results

The preceding description of the delivery period model has shown that the model can be described in a two-equation system with two endogenous variables, B and P. Assuming that the rules of the implicit function theorem are met, the static results of the model are:

\[ \bar{B} = \bar{B}(T, F_e, c, d) \]
\[ \bar{P} = \bar{P}(T, F_e, c, d) \]

The signs of the eight comparative static derivatives derived from the delivery period model are:

1. \( \frac{d\bar{B}}{dT} < 0 \),  2. \( \frac{d\bar{P}}{dT} < 0 \),  3. \( \frac{d\bar{B}}{dF_e} > 0 \),
4. \( \frac{d\bar{P}}{dF_e} > 0 \),  5. \( \frac{d\bar{B}}{dc} > 0 \),  6. \( \frac{d\bar{P}}{dc} < 0 \),
7. \( \frac{d\bar{B}}{dd} < 0 \), and 8. \( \frac{d\bar{P}}{dd} > 0 \).
CHAPTER III. EMPIRICAL PROCEDURES,
SOLUTIONS OF LINEAR DIFFERENCE
EQUATIONS, AND DATA REQUIREMENTS

Introduction

An empirical analysis will be conducted to quantify parameters of the reduced form equations of the general predelivery and delivery period models described in Chapter II. The results of the empirical analysis will be used to determine the time path of the basis in the predelivery and delivery periods of a futures contract. Topics in this chapter include the empirical procedures to be used, the method for solving linear difference equations, and the data requirements of the study.

Empirical Procedures

The following procedures will be conducted in the empirical analysis. First, an estimable equation for quantifying the parameters of the reduced form equation of predelivery period basis determination will be developed. Second, an estimable equation for quantifying the parameters of the reduced form equation of delivery period basis determination will be developed. Third, statistical estimation problems which may arise will be discussed. And, fourth, constrained regression, i.e., a method for determining if data sets may be pooled, will be discussed.

Predelivery period equation

In this section, an estimable equation for quantifying the parameters of the reduced form equation of predelivery period basis determination will be developed. First, the reduced form equation with all the exogenous variables will be described. And, second, assumptions about the exogenous
variables will be introduced which will allow the equation to be estimated.

Reduced form equation The static results of the general function model for predelivery period basis and cash price determination show that the equilibrium basis for live beef cattle is related to the following exogenous variables:

1. $P_e$, the price expected by suppliers of storage and transformation services,
2. $B_e$, the basis expected by suppliers of storage and transformation services,
3. $P_{eP}$, the price expected by demanders of storage and transformation services,
4. $B_{eP}$, the basis expected by demanders of storage and transformation services,
5. $F_e$, the futures price expected by speculators,
6. $D$, the number of calendar days remaining in the predelivery period, and
7. $S_{t-1}$, the total quantity of live beef cattle on feed carried in from the previous market day.

Assuming that a linear relationship exists between the equilibrium basis, $B$, and each of the exogenous variables, the reduced form equation may be expressed:

$$B_t = a_0 + a_1 B_{et} + a_2 B_{et} + a_3 P_{et} + a_4 P_{et} + a_5 F_t + a_6 D + a_7 S_{t-1}$$

where $t$ denotes the particular market day in the predelivery period and the $a_i$, $i = 0, 1, \ldots, 7$; are parameters. Equation (3.1) cannot be estimated in its present form because the exogenous variables $B_{et}$, $B_{eP}$, $P_{et}$,
$P_{e_t^P}$, and $Fe_t$ are not observable. Before (3.1) can be estimated, several assumptions about the exogenous variables must be introduced.

**Simplifying assumptions** Several assumptions will be made concerning the variables $Be_t$ and $B_{eP_t}$. First, in order to reduce the number of exogenous expectations variables in the reduced form equation for the predelivery period, assume that $Be_t$ and $B_{eP_t}$ are equal. This assumption is justified because (1) the comparative static derivatives of $Be_t$ and $B_{eP_t}$ in the predelivery period basis determination model both have positive signs. Incorporating this assumption into the reduced form equation, (3.1), one obtains:

$$\text{(3.2)} \quad B_t^e = a_0 + a_1 Be_t + a_3 Pe_t + a_4 P_{eP_t} + a_5 Fe_t + a_6 D_t + a_7 S_{t-1}$$

where $a_1 = (a_1 + a_2)$.

Second, assume that suppliers and demanders of storage and transformation services expect the basis to converge toward zero as the delivery period approaches. Thus, the basis expectation can be expressed as a function of $D_t$, the number of days remaining in the predelivery period:

$$\text{(3.3)} \quad Be_t = Be_t(D_t).$$

The relationship between $Be_t$ and $D_t$ may be either positive or negative for any given near-option predelivery period. The relationship between $Be_t$ and $D_t$ will be positive if the equilibrium basis at the beginning of the near-option predelivery period is positive. The relationship will be negative if the equilibrium basis at the beginning of the near-option predelivery period is negative. Assuming that the relationship is linear, then $\partial Be_t / \partial D_t$ may be denoted by $b_1$ and (3.3) becomes:

$$\text{(3.4)} \quad Be_t = b_1 D_t$$

Substituting (3.4) into (3.2), the basis equation becomes:
Combine all the terms in (3.5) which contain $D_t$:

$$(3.6) \bar{\beta}_t = a_0 + c_1 D_t + a_3 P_t + a_4 P_{e_t} + a_5 F_t + a_7 S_{t-1}$$

where $c_1 = (a_0 b_1 + a_6)$. The sign of $c_1$ is a priori indeterminant because it depends upon the sign and magnitude of $b_1$ relative to $a_6 / a_0$.

Third, assume that $P_{et}$ equals $P_{eP_t}$. This is justified because the comparative static derivatives of $P_{et}$ and $P_{eP_t}$ in the predelivery period basis determination model both have positive signs. Incorporating this assumption into the basis equation, (3.6), one obtains:

$$(3.7) \bar{\beta}_t = a_0 + c_1 D_t + c_2 P_t + a_5 F_t + a_7 S_{t-1}$$

where $c_2 = (a_3 + a_4)$. The sign of $c_2$ is strictly positive.

Fourth, because expectations cannot be readily observed, assume that $P_{et}$ and $F_{et}$ are generated through the following Nerlove adaptive expectations functions:

$$
(3.8) P_{et} = P_{et-1} + \gamma_1 (P_{t-1} - P_{et-1}), \quad 0 < \gamma_1 < 2 \\
(3.9) F_{et} = F_{et-1} + \gamma_2 (F_{t-1} - F_{et-1}), \quad 0 < \gamma_2 < 2.
$$

The function in (3.8) states that suppliers and demanders of storage and transformation services update their price expectations daily by some proportion, $\gamma_1$, of the difference between the previous market day's cash price expectation and actual cash price. The function in (3.9) states that speculators update their futures price expectations daily by some proportion, $\gamma_2$, of the difference between the previous market day's price expectation and the actual futures price.

**Expectation function incorporation**

Several steps are involved when incorporating the expectations functions into the reduced form equation (3.7). First, the expectations functions in (3.8) and (3.9) may
be rewritten:

\[(3.10) \quad P_t = \gamma_1 \sum_{j=1}^{j-1} p_{t-j} \]
\[(3.11) \quad F_t = \gamma_2 \sum_{j=1}^{j-1} F_{t-j} \]

where \(j\) denotes the number of days over which the price expectations are generated. Second, substitute (3.10) and (3.11) into (3.7):

\[(3.12) \quad B_t = a_0 + c_1 D_t + c_2 \gamma_1 \sum_{j=1}^{j-1} p_{t-j} + a_5 \gamma_2 \sum_{j=1}^{j-1} F_{t-j} + a_7 S_{t-1} \]

All of the variables in (3.12) are observable, however, the complex parameters in (3.12) and the lack of a sufficient number of degrees of freedom will create estimation problems. The following procedure will help to eliminate these problems.

First, multiply (3.12) by \((1-\gamma_1)\) and lag one period. Then, subtract the result from (3.12):

\[(3.13) \quad B_t = [a_0 - (1-\gamma_1)a_0] + (1-\gamma_1)B_{t-1} + [c_2 \gamma_1]P_{t-1} + a_5 \gamma_2 \sum_{j=1}^{j-1} F_{t-j-1} - [c_1 \gamma_1]D_{t-1} + a_7 S_{t-2} \]

Second, multiply (3.13) by \((1-\gamma_2)\) and lag one period. Then, subtract the result from (3.13):

\[(3.14) \quad B_t = [a_0 - (1-\gamma_1)a_0 - (1-\gamma_2)a_0 + (1-\gamma_1)(1-\gamma_2)a_0] + [(1-\gamma_1) + (1-\gamma_2)]B_{t-1} - [(1-\gamma_1)(1-\gamma_2)]B_{t-2} + [c_2 \gamma_1]P_{t-1} - [(1-\gamma_1)(1-\gamma_2)]P_{t-2} + [c_1 \gamma_1]D_{t-1} - [c_1 (1-\gamma_1) + (1-\gamma_2)]D_{t-2} + [c_2 (1-\gamma_1)(1-\gamma_2)]D_{t-3} + a_7 S_{t-3} - [a_7 (1-\gamma_1) + (1-\gamma_2)]S_{t-2} + [a_7 (1-\gamma_1)(1-\gamma_2)]S_{t-3} + U_t \]
The term, $U_t$, is an error term.

Several of the variables in (3.14) must be eliminated because exact

collinearity is present, i.e., combinations of several variables are per-

fectly correlated with another variable. Perfect collinearity will cause
the $X'X$ matrix in regression analysis to be singular. Examples of perfect
collinearity are $B_{t-1} = F_{t-1} - P_{t-1}$ and $B_{t-2} = F_{t-2} - P_{t-2}$.

Thus, the

variance explained by $F_{t-1}$, $P_{t-1}$, $F_{t-2}$, and $P_{t-2}$ is explained by $B_{t-1}$ and
$B_{t-2}$. Also, the variables $D_t$, $D_{t-1}$, and $D_{t-2}$ are almost exactly collinear.

Thus, $D_{t-1}$ and $D_{t-2}$ can also be eliminated from (3.14).

The equation in (3.14) reduces from twelve explanatory variables and
an intercept term to six explanatory variables and an intercept term. The

reduced form equation is given by:

(3.15) $B_t = [a_0 - (1-\gamma_1)a_0 - (1-\gamma_2)a_0 + (1-\gamma_1)(1-\gamma_2)a_0]$

$+ [(1-\gamma_1) + (1-\gamma_2)]B_{t-1} - [(1-\gamma_1)(1-\gamma_2)B_{t-2} + [c_1]D_t$

$+ [a_7]S_{t-1} - [a_7{(1-\gamma_1) + (1-\gamma_2)}]S_{t-2}$

$+ [a_7(1-\gamma_1)(1-\gamma_2)]S_{t-3} + U_t$

In order to simplify the notation in (3.15), the reduced form equa-
tion will be expressed:

(3.16) $B_t = d_0 + d_1 B_{t-1} + d_2 B_{t-2} + d_3 D_t + d_4 S_{t-1} + d_5 S_{t-2} + d_6 S_{t-3}$

$+ U_t$

where $d_0$ and $d_i$, $i = 1, 2, ... 6$, are the reduced form equation intercept
and parameters of $B_{t-1}$, $B_{t-2}$, $D_t$, $S_{t-1}$, $S_{t-2}$, $S_{t-3}$, respectively.

By using the assumptions that $0 < \gamma_1 < 1$ and $0 < \gamma_2 < 1$, the compara-
tive static derivatives in Chapter II, and previous simplifying assump-
tions in this chapter, the following hypotheses about the signs of the
reduced form parameters in (3.16) can be derived:
(1) that $d_1$ is strictly positive because $(1-\gamma_1)$ and $(1-\gamma_2)$ are both positive,

(2) that $d_2$ is strictly negative because $(1-\gamma_1)$ and $(1-\gamma_2)$ are both positive,

(3) that the sign of $d_3$ is indeterminant because the sign of $c_1$ is indeterminant,

(4) that $d_4$ is strictly negative because $a_7$ is negative,

(5) that $d_5$ is strictly positive because $a_7\{(1-\gamma_1) + (1-\gamma_2)\}$ is negative, and

(6) that $d_6$ is strictly negative because $a_7\{(1-\gamma_1)(1-\gamma_2)\}$ is negative.

**Autocorrelation**

There is no a priori information concerning the error structure in (3.16). However, autocorrelation of the error structure may be present because time series data are to be used in this analysis. Autocorrelation means that successive error terms are correlated. Autocorrelation of the error term in an equation which contains lagged values of the dependent variable such as (3.16) creates the following estimation problems [Ladd, 10]:

(1) estimates of the parameters in the equation will be biased,

(2) estimates of the sampling variances of the parameters will be biased,

(3) estimates of the parameters will be inefficient, and

(4) the $t$ and $F$ ratios will be biased.

The Durbin-Watson $d$ statistic, which is used to detect autocorrelation in an equation which does not contain a lagged value of the dependent variable, is not appropriate for testing for autocorrelation when lagged
values of the dependent variable are among the explanatory variables because the d statistic is biased toward 2. However, for lack of a good test for autocorrelation when lagged dependent variables are among the explanatory variables, the d statistic will be used as an approximate test.

The d statistic is computed:

\[
(3.17) \quad d = \frac{\sum_{t=2}^{n} \left( e_t - e_{t-1} \right)^2}{\sum_{t=1}^{n} e_t^2}
\]

Exact significance levels for d have not been calculated by Durbin and Watson, but they have tabulated lower and upper bounds, \( d_L \) and \( d_U \), for various values of \( n \) and \( K \). \( n \) is the number of observations of the dependent variable and \( K \) is the number of explanatory variables.

For a two-tailed test for positive or negative autocorrelation:

(a) if \( d < d_L \) or \( 4 - D < d_L \), then d is significant,

(b) if d lies between \( d_L \) and \( 4 - d_U \), then d is not significant,

(c) otherwise, the test is inconclusive.

In this study, if the computed value for d falls between 1.5 and 2.5, no autocorrelation is assumed to be present.

**Delivery period equation**

In this section, an estimable reduced form equation of the delivery period basis determination model will be developed. First, the reduced form equation with all the exogenous variables of the delivery period basis determination model will be described. Second, assumptions concerning the exogenous variables will be stated which will allow estimation of the reduced form equation.
Reduced form equation  The static results of the general function model for delivery period basis and cash price determination show that the equilibrium basis is related to the following exogenous variables:

1. $T$, the transaction cost involved with arbitrage, and
2. $F_e$, the futures price expected by speculators in the delivery period.

Assuming that a linear relationship exists between the equilibrium basis, $B_t$, and each of the exogenous variables, the reduced form equation of delivery period basis determination is given by:

$$B_t = a_0 + a_1T_t + a_2F_t$$

where $t$ denotes a particular day of the delivery period and the $a_i, i = 1, 2$, are parameters. Equation (3.18) cannot be estimated in its present form because the exogenous variables, $T$ and $F_e$, are not observable. By making several assumptions about the exogenous variables, equation (3.18) can be estimated.

Simplifying assumptions  The first assumption is that $T$ is constant throughout the delivery period. By incorporating this assumption into (3.18), the reduced form equation becomes:

$$B_t = a_0 + a_2F_t$$

where $a_0 = [a_0 + a_1T]$ which is a constant.

The second assumption is that $F_e$, the futures price expectation of speculators, is generated through the following adaptive expectations function:

$$F_t = F_{t-1} + \gamma(F_{t-1} - F_{t-1}), \ 0 < \gamma < 2.$$  

This function states that $F_t$ is updated daily during the delivery period by some proportion of the difference between the previous day's actual
futures price and the futures price expectation. The expectations generating function is (3.20) may be rewritten:

\[ (3.21) \quad F_t = \gamma \sum_{j=1}^{n} (1-\gamma)^{j-1} F_{t-j}. \]

Substituting (3.21) into (3.19), the reduced form equation becomes:

\[ (3.22) \quad B_t = a_0 + a_2 \gamma \sum_{j=1}^{n} (1-\gamma)^{j-1} F_{t-j}. \]

Equation (3.22) is not easily estimated because of non-linearities in the parameters and a lack of degrees of freedom. However, by employing the following step, these problems are eliminated. Multiply (3.22) by (1-\gamma) and lag one period. Then, subtract the result from (3.22):

\[ (3.23) \quad \bar{B}_t = [a_0 - (1-\gamma)a_0] + [1-\gamma]B_{t-1} + [a_2 \gamma]F_{t-1}. \]

By adding an error term, \( U_t \), to (3.23), the reduced form equation of delivery period basis determination is estimable:

\[ (3.24) \quad B_t = [a_0 - (1-\gamma)a_0] + [1-\gamma]B_{t-1} + [a_2 \gamma]F_{t-1} + U_t. \]

By using the assumption that \( 0 < \gamma < 1 \), the comparative static derivatives from Chapter II, and previous simplifying assumptions in this chapter, the following hypotheses about the signs of the reduced form parameters in (3.24) can be derived:

(1) that \( [1-\gamma] \) is strictly positive, and

(2) that \( [a_2 \gamma] \) is strictly positive because \( a_2 \) and \( \gamma \) are both positive.

**Autocorrelation**  Again, due to the lack of a good test for autocorrelation when lagged dependent variables are among the explanatory variables, the Durbin-Watson d statistic will be used to test for autocorrelation in the reduced form equation, (3.24), of delivery period basis determination.
Constrained regression analysis

Constrained regression analysis is a method used to determine if data sets from separate time periods can be pooled to estimate one equation. In this study, constrained regression analysis will be used to determine (1) if data sets of the near-option predelivery period can be pooled to estimate one reduced form equation and (2) if data sets of the delivery period can be pooled to estimate one reduced form equation.

The procedure of constrained regression involves the following steps:

(1) run separate regressions of the full model, i.e., all exogenous variables are included, for each data set,

(2) pool the data sets and, using dummy variables to allow the intercept of each data set to differ but constraining the slopes of the data sets to be equal, run one regression for the full model, and

(3) use the following F-test:

\[
F = \frac{(Z_c'Z_c - Z'Z)/(G-1)p}{Z'Z/(n-Gp)}
\]

where \(Z_c'Z_c\) is the residual sum of squares from the pooled model, \(Z'Z\) is the sum of the residual sum of squares from the individual data sets, \(G\) is the number of data sets, \(p\) is the number of constraints on the slope parameters, and \(n\) is the number of observations in the pooled data set. This calculated \(F\) is compared to a tabulated \(F\). A significant value for \(F\) indicates that the slope parameters differ between data sets. Thus, the data sets should not be combined to estimate the slope parameters.

If the slope parameters do not differ, then a similar test to the one performed in (3.25) must be used to determine if the intercepts differ between the data sets. The data sets will be combined to estimate one
set of reduced form parameters if the slope parameters and intercepts do not differ between the data sets. However, if the slope parameters and intercepts differ between data sets, a regression will be run on each data set to obtain separate estimates of the slope parameters for each data set.

Solutions of Linear Difference Equations

Dynamic models reflect temporal interrelationships, i.e., past conditions affect present developments and present developments affect future conditions. In quantitative analysis, these dynamic elements of economic models usually take the form of linear difference equations. Generally, in economic studies, linear difference equations arise from rigidities or expectations. In this study, the pre-delivery and delivery period basis equations are both linear difference equations which have arisen because of price expectations of various trading groups in the live beef cattle industry.

Three important pieces of information are gathered from the solution of a linear difference equation. First, one can determine if the system described by the difference equation is stable. The system is stable if it will again attain equilibrium once it is shocked provided that a sufficient length of time is allowed for adjustment. Secondly, provided that the system is stable, one can determine the equilibrium value of the endogenous variable for any given set of exogenous variables in the system. And third, one can also determine the multipliers of the system, i.e., how a once and for all change in an exogenous variable will affect the time path of the exogenous variable.
Linear difference equations are solved to determine the time path of the endogenous variable. Information about the time path of the endogenous variable may be helpful to decision makers who pick a strategy based upon anticipated movements of the endogenous variable.

In this section, solutions of linear difference equations will be described.

Solution of a first-order linear difference equation

Consider the first-order, non-homogeneous difference equation:

\[
(3.26) \quad y_t = b_1 y_{t-1} + \sum_{i} a_i x_{it} + \varepsilon_t
\]

with the initial condition that \( y_0 = C_0 \). Also, set \( \sum_{i} a_i x_{it} + \varepsilon_t \) equal to \( X_t \) and fix \( X_t \) at the value of \( x_0 \). Then:

\[
(3.27) \quad y_1 = b_1 C_0 + x_0,
\]

\[
y_2 = b_1 y_1 + X_0 = b_1^2 C_0 + b_1 x_0 + X_0, \text{ and}
\]

\[
y_t = b_1^t C_0 + (b_1^{t-1} + b_1^{t-2} + \ldots + b_1 + 1) X_0.
\]

The geometric series multiplying \( x_0 \) may be expressed:

\[
(3.28) \quad \frac{(1-b_1^t)}{(1-b)} = \frac{b_1^t - 1 + b_1^{t-1} + b_1^{t-2} + \ldots + b_1 + 1}{1-b}.
\]

Thus, the general solution of (3.27) is:

\[
(3.29) \quad y_t = b_1^t C_0 + \left[\frac{(1-b_1^t)}{(1-b)}\right] X_0 = b_1^t [C_0 - X_0/1-b_1] + X_0/1-b_1.
\]

The time path of \( y_t \) is determined by \( b_1 \) and the sign of \( [C_0 - X_0/1-b_1] \). If \( 1 < b_1 < 0 \), \( b_1^t \) approaches zero monotonically as \( t \) increases. Hence, \( y_t \) approaches \( X_0/(1-b_1) \). If \( -1 < b_1 < 0 \), \( b_1^t \) cyclically approaches zero as \( t \) increases. Hence, \( y_t \) again approaches \( X_0/(1-b_1) \). Thus, if \( |b_1| < 1 \), the system is stable and approaches the equilibrium value of \( y_t \) denoted \( y^* \):

\[
(3.30) \quad y^* = \frac{X_0}{1-b_1}
\]
The dynamic discrepancy of period \( t \) is:

\[
(3.31) \quad d_t = b_1 t (C_0 - x_0 / 1 - b_1)
\]

which approaches zero as \( t \) increases.

If \( |b_1| > 1 \), the system is explosive and no equilibrium will be attained after the system has been shocked out of equilibrium.

Changes in equilibrium values over time can be easily computed and compared with changes in actual values of \( y_t \) if one can describe the \( x_{it} \) as linear functions of time. Assume that each of the \( x_i \) in (3.26) can be described as linear functions of time:

\[
(3.32) \quad x_{it} = p_{0i} + p_{1i} t
\]

Equation (3.32) implies that:

\[
(3.33) \quad x_{it-1} = p_{0i} + p_{1i} (t-1)
\]

By setting \( y_0 = C_0 \), equation (3.27) can be rewritten:

\[
(3.34) \quad y_t = b_1 t C_0 + \sum_{i} a_i x_{it} + b_1 \sum_{i} a_i x_{it-1} + b_1^2 \sum_{i} a_i x_{it-2} + \ldots
\]

\[
+ b_1^t \sum_{i} a_i x_{it-10}.
\]

Substitute (3.32) and (3.33) into (3.34):

\[
(3.35) \quad y_t = b_1 t C_0 + \sum_{i} a_i (p_{0i} + p_{1i} t) + b_1 \sum_{i} a_i (p_{0i} + p_{1i} (t-1))
\]

\[
+ b_1^2 \sum_{i} a_i (p_{0i} + p_{1i} (t-2)) + \ldots + b_1^t \sum_{i} a_i p_{0i}
\]

Regroup terms in (3.35):

\[
(3.36) \quad y_t = b_1 t C_0 + \sum_{i} a_i p_{0i} (1+b_1+b_1^2+\ldots+b_1^t)
\]

\[
+ \sum_{i} a_i p_{1i} (1+b_1+b_1^2+\ldots+b_1^t) - b_1 \sum_{i} a_i p_{1i} (1+2b_1+3b_1^2+\ldots)
\]

The geometric series describing \( \sum_{i} a_i p_{0i} \) and \( \sum_{i} a_i p_{1i} t \) is given by:

\[
(3.37) \quad (1+b_1+b_1^2+\ldots+b_1^t) = (1-b_1^t)(1-b_1)^{-1} = (1-b_1)^{-1}
\]

The geometric series describing \( b_1 \sum_{i} a_i p_{0i} \) is given by:
(3.38) \((1+2b_1^t+3b_1^{2t}+4b_1^{3t}+\ldots) = \frac{1}{(1-b_1)^2}\)

Substitute (3.37) and (3.38) into (3.36):

\[
y_t = b_1^tC_0 + (1-b_1)^{1-1} \sum_{i=0}^{\infty} p_{0i} - b_1^t(1-b_1)^{-1} \sum_{i=0}^{\infty} p_{0i} \\
+ (1-b_1)^{-1} \sum_{i=0}^{\infty} p_{1i} t - (1-b_1)^{-2} b_1^t \sum_{i=0}^{\infty} p_{1i}
\]

Finally, the general solution is obtained by regrouping terms in (3.39):

\[
y_t = b_1^t(C_0 - \frac{\sum_{i=0}^{\infty} p_{0i}}{(1-b_1)}) + \frac{\sum_{i=0}^{\infty} (p_{0i}+p_{1i} t)}{(1-b_1)^2} - \frac{b_1^t \sum_{i=0}^{\infty} p_{1i}}{(1-b_1)^2}
\]

The equilibrium solution to (3.40) is:

\[
y_t^* = \frac{\sum_{i=0}^{\infty} (p_{0i}+p_{1i} t)}{(1-b_1)^2}
\]

If the system is stable, \(b_1^t\) approaches zero as \(t\) increases. After a sufficient period of time, the first term on the right hand side of (3.40) becomes negligible and (3.40) may be written:

\[
y_t = \frac{\sum_{i=0}^{\infty} (p_{0i}+p_{1i} t)}{(1-b_1)^2} - \frac{b_1^t \sum_{i=0}^{\infty} p_{1i}}{(1-b_1)^2}
\]

The first term on the right hand side of (3.42) is the time path of equilibrium values for \(y_t\) and is known as the receding equilibrium. The term \(-\frac{b_1^t \sum_{i=0}^{\infty} p_{1i}}{(1-b_1)^2}\) is the dynamic discrepancy generated by the continued temporal changes in the \(x_{it}\). This discrepancy would be non-existent if \(p_{1i}\), the temporal changes of the \(x_{it}\), were zero. Note that the magnitude of this discrepancy term is positively related to the temporal changes, \(p_{1i}\), of the various \(x_{it}\).
Solution of a second-order linear difference equation

A second-order linear difference equation is a dynamic equation which contains two lagged values of the dependent variable. Consider the following second-order, non-homogeneous difference equation:

\[ y_t = b_1 y_{t-1} + b_2 y_{t-2} + \sum_{i} a_i x_{it} + \epsilon_t \]

with initial conditions that \( y_0 = C_0 \) and \( y_1 = C_1 \). The solution to (3.43) depends upon the value of \( b_1^2 + 4b_2 \) [Ladd, 10]. If \( b_1^2 + 4b_2 > 0 \), the general solution to (3.43) is:

\[
\begin{align*}
(3.44) \quad y_t &= \left[ (C_0 - X_0/1-b_1-b_2)\lambda_2 - C_1 + X_0/1-b_1-b_2 \right] \lambda_1^{t}/(\lambda_2 - \lambda_1) \\
&+ \left[ C_1 - X_0/1-b_1-b_2 - (C_0 - X_0/1-b_1-b_2)\lambda_1 \right] \lambda_2^{t}/(\lambda_2 - \lambda_1) \\
&+ X_0/1-b_1-b_2
\end{align*}
\]

where

\[
\begin{align*}
\lambda_1 &= \frac{b_1 + (b_1^2 + 4b_2)^{1/2}}{2} \\
\lambda_2 &= \frac{b_1 - (b_1^2 + 4b_2)^{1/2}}{2}
\end{align*}
\]

The \( \lambda_i \) are the roots of the quadratic equation:

\[ (3.45) \quad \lambda^2 - b_1 \lambda - b_2 = 0 \]

which is obtained from (3.43) by setting \( \sum_{i} a_i x_{it} + \epsilon_t = 0 \), replacing the \( y_{t-i} \) with \( \lambda^{(t-i)} \) for \( i = 0, 1, 2 \), and dividing the result by \( \lambda^{(t-2)} \).

The time path followed by \( y_t \) is determined by \( \lambda_1 \) and \( \lambda_2 \). If both \( \lambda_1 \) and \( \lambda_2 \) are positive and less than one, \( y_t \) approaches equilibrium, \( X_0/1-b_1-b_2 \), monotonically. If both \( \lambda_1 \) and \( \lambda_2 \) are less than one in absolute value but one is negative, \( y_t \) follows a damped cycle in its approach toward equilibrium. For each of the above two situations, the dynamic system described in (3.43) is stable. However, if either of the roots, \( \lambda_1 \) and \( \lambda_2 \), exceed one in absolute value, the dynamic system described in (3.43) will
explode and no equilibrium will exist.

Now, consider the case where \((b_1^2 + 4b_2) < 0\). In this case, \(\lambda_1\) and \(\lambda_2\) are complex numbers. For this situation, define:

\[
\begin{align*}
    c &= b_1/2 \\
    d &= [(-1)(b_1^2 + 4b_2)]^{1/2}/2 \\
    D &= (c^2 + d^2)^{1/2} \\
    \sin R &= c/D \\
    \cos R &= d/D
\end{align*}
\]

where \(D\) is the modulus, i.e., the absolute value, of the roots. The general solution to (3.43) for the case of \((b_1^2 + 4b_2) < 0\) is:

\[
(3.46) \quad y_t = D^t[(c_0 - x_0/l-b_1-b_2)\cos tR + (d)^{-1}(c_1 - (1-c)x_0/l-b_1-b_2 \\
- c_0)\sin tR] + (x_0/l-b_1-b_2)
\]

If \(D\) is less than one, \(y_t\) follows a damped sinusoidal path toward equilibrium, \(x_0/l-b_1-b_2\). If \(D\) is greater than one, \(y_t\) will explode in response to an autonomous shock to the system and no equilibrium will be attained.

Application of the difference equation solutions

Information concerning the time path of daily basis values during the predelivery and delivery periods of a futures contract may be useful to hedgers in the live beef cattle industry. For example, hedgers who know that basis values converge in a non-oscillatory path toward a particular value may decide to plan their hedge lifting strategies in order to profit from beneficial movements of the basis values.

The time path and equilibrium values for the predelivery and delivery periods of a futures contract can be obtained by solving a linear difference equation for the daily basis values. Earlier in this chapter,
estimable reduced form equations were developed for daily basis values of the predelivery and delivery periods of a futures contract. Both of these equations are linear, non-homogeneous difference equations. General solutions to equations (3.16) and (3.24) will provide information about the time path and equilibrium values of the daily basis for each contract.

Data Requirements

The description of the data requirements for this study will be divided into two parts, the near-option predelivery period and the delivery period data requirements. Only the near-option part of the predelivery period will be analyzed. This decision is made because of the assumption that hedgers are mainly interested in the near-option part of the predelivery period. Hedgers will typically choose the futures contract that matures nearest to but after the expected marketing date of the live cattle.

The near-option part of the predelivery period begins on the first market day after the 20th day of the previous contract month and ends on the 5th day of the month of the contract in question. The delivery period begins on the 6th day of the month of the contract in question unless the 6th day falls on a Friday, holiday, or weekend. In that case, the delivery period begins on the first market day after the 6th day. The delivery period ends on the 20th day of the month of the contract in question.

Predelivery period data requirements

Data requirements for the predelivery period include daily basis values for live beef cattle and a proxy for the number of beef cattle on feed.
Basis values  Daily basis values for live beef cattle cannot be obtained directly from any one source. However, because the basis is defined as the difference between the futures price and the cash price on a given day for a commodity which meets all futures contract specifications, the basis can be calculated indirectly from several sources. First, the futures price used is the closing price of the live beef cattle futures contract which is obtained from the Wall Street Journal. Second, the cash price used is the average price for choice live beef steers weighing 1000-1250 pounds at the Omaha stockyards. The Omaha stockyards is chosen because it is a par delivery point for the live beef cattle futures contract.

Beef cattle on feed proxy  No data are available for the daily number of cattle on feed. The smallest time increment for which cattle on feed data are available is one month. These data are in the Cattle on Feed Report issued by the U.S.D.A. [15]. For lack of a better proxy, the monthly data for cattle on feed will be used, although it is realized that much information about the daily number of cattle on feed is lacking. The cattle on feed proxy is calculated by adding the placements for a given month to the cattle inventories at the beginning of the month and subtracting the cattle marketings during the month. All observations for any month will have the same value for the cattle on feed proxy. Table 1 shows the values of the cattle on feed proxy to be used in this study.

Delivery period data requirements

The data requirements for the delivery period include daily basis values and the futures price.
Table 1. Cattle on feed, 7 states report

<table>
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<tr>
<th>Date</th>
<th>Inventory</th>
<th>Placed</th>
<th>Marketed</th>
<th>Cattle on feed</th>
</tr>
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<td>+</td>
<td>1837</td>
<td>1558</td>
</tr>
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CHAPTER IV. RESULTS

Introduction

In this chapter, results of the statistical analysis of the reduced form equations for daily basis value determination models for the near-option predelivery period and the delivery period will be presented. Separate sections will be devoted to the results of the two reduced form equations.

Results of the predelivery period model

Data for the near-option predelivery period for the live beef cattle futures contracts were collected for all contracts beginning with the February, 1974, contract and ending with the February, 1976, contract. 1973 data were not included because of the price controls which were imposed on retail beef prices during the summer months of 1973.

The following analysis was made of the near-option predelivery basis data. First, constrained regression was used to determine if the individual data sets of a given futures contract for different years could be pooled. A data set includes all observations of the dependent and independent variables for a given futures contract. Second, ordinary least squares regression was used to estimate parameters of the reduced form equation for the near-option predelivery period. And third, the estimated parameters were used to determine the time path properties of the near-option predelivery period basis values.

Constrained regression results In this study, the February contract is the only contract for which there are three data sets, i.e., data were collected for the near-option part of the predelivery period for 1974,
1975, and 1976. Data were collected only for 1974 and 1975 for the April, June, August, October, and December contracts.

The calculated $F$-value, for determining whether the slope parameters of the yearly data sets of the February contract differ, is:

$$F = \frac{(43.6219 - 11.481 - 7.463 - 4.784)/(2)(6)}{23.728/(68-18)} = 3.4930$$

where $n = 68$ observations, $G = 3$ data sets, and $p = 6$ constraints on the slope parameters. The tabled value for the $F$ statistic at the 5 percent level of significance with 12 and 50 degrees of freedom is:

$$F_{12,50}^{12} = 2.41.$$ 

Comparing the table value for $F_{12,50}^{12}$ to the calculated $F$ value shows that the slope parameters do differ significantly between the three yearly data sets for the February contract. Thus, separate models were estimated for each yearly data set of the February contract.

Results of the constrained regression analysis for all contracts are presented in Table 2. The slope parameters differ significantly between the yearly data sets for both the February and April contracts. Thus, separate models must be estimated for each yearly data set for these contract months.

However, the slope parameters do not differ significantly between the yearly data sets for the June, August, October, and December contracts. The yearly data sets for each of these contracts can be combined to estimate one model for each contract.

Ordinary least squares results In this section, the final estimates of the parameters for the reduced form equation of the near-option part of the predelivery period for each contract will be presented. Values for the final estimated parameters for each model are presented in Table
Table 2. Results of the constrained regression analysis on the near-option part of the pre-delivery period

<table>
<thead>
<tr>
<th>(Contract Month)</th>
<th>February</th>
<th>April</th>
<th>June</th>
<th>August</th>
<th>October</th>
<th>December</th>
</tr>
</thead>
<tbody>
<tr>
<td>calculated F</td>
<td>3.4930(^a)</td>
<td>2.5072(^a)</td>
<td>2.3085</td>
<td>.8335</td>
<td>2.1524</td>
<td>2.0567</td>
</tr>
<tr>
<td>tabled F (5% level of significance)</td>
<td>2.41</td>
<td>2.36</td>
<td>2.34</td>
<td>2.37</td>
<td>2.49</td>
<td>2.38</td>
</tr>
<tr>
<td>n = no. of observations</td>
<td>68</td>
<td>49</td>
<td>51</td>
<td>46</td>
<td>37</td>
<td>45</td>
</tr>
<tr>
<td>G = no. of data sets</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>p = no. of parameter constraints</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

\(^a\)Denotes slope parameters differ between yearly data sets for the contract.
3. Also, the values for the multiple correlation coefficient, $R^2$, and the Durbin-Watson d statistic are presented.

Exogenous variables are left in the final models if the t-values of their estimated parameters differ significantly from zero at the 5 percent level of significance. Note that the models for the April, 1974, and April, 1975, contracts include exogenous variables with parameters which do not differ significantly from zero. These variables are included because their elimination causes the t-values of several other variables to become insignificant.

Referring to Table 3, one will note that separate models were estimated for each yearly data set for the February and April contracts. Yearly data sets were pooled for the June, August, October, and December contracts. A separate model was estimated for the pooled data set for each of these contracts.

The exogenous variables with coefficients which most consistently differ from zero in the final models are (1) $B_{t-1}$, the basis value from the previous market day, and (2) $D_t$, the number of calendar days remaining in the near-option part of the predelivery period. The variable, $B_{t-1}$, is present in six of the nine models. $D_t$ is present in five of the nine models.

The values of the multiple correlation coefficient for the final models indicate that the models do a reasonable job of explaining basis variation in the near-option part of the predelivery period. The values for $R^2$ range from a high of .9677 for the pooled August contract to a low of .3923 for the 1975 data set for the April contract.

The calculated Durbin-Watson d statistic ranged from 2.4470 in the
Table 3. Estimates of the coefficients for the final predelivery period models

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>.20527 (.6630)*</td>
<td>30.8400 (3.0970)</td>
<td>-.0772 (-.7820)</td>
<td>-45.1129 (-34.969)</td>
<td>.0165 (.0632)</td>
<td>.12007 (.9177)</td>
<td>.1321 (.6703)</td>
<td>10.0592 (.7001)</td>
<td>-62.297 (-2.1580)</td>
</tr>
<tr>
<td>B_t-1</td>
<td>.4348 (2.1317)</td>
<td>.8721 (16.9146)</td>
<td>.8402 (13.3979)</td>
<td>.8155 (8.0195)</td>
<td>.5034 (2.7723)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B_t-2</td>
<td>-.3331 (-2.6546)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D_t</td>
<td>.1673 (12.6061)</td>
<td>.0902 (3.4985)</td>
<td>-.0750 (-9.4379)</td>
<td>-.0408 (-3.3737)</td>
<td></td>
<td></td>
<td></td>
<td>- .0845 (-3.254)</td>
<td></td>
</tr>
<tr>
<td>S_t-1</td>
<td>-.0027 (-2.3006)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-.0108 (-2.1604)</td>
<td></td>
</tr>
<tr>
<td>S_t-2</td>
<td>.0018 (1.2209)</td>
<td>.0149 (4.6794)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.0092 (1.9805)</td>
<td></td>
</tr>
<tr>
<td>S_t-3</td>
<td>-.0027 (-2.2539)</td>
<td>-.0079 (-2.514)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.0074 (2.1455)</td>
<td></td>
</tr>
<tr>
<td>R^2</td>
<td>.8784</td>
<td>.6538</td>
<td>.8538</td>
<td>.9677</td>
<td>.7104</td>
<td>.8067</td>
<td>.7201</td>
<td>.3923</td>
<td>.4498</td>
</tr>
<tr>
<td>Durbin-Watson d</td>
<td>1.7522</td>
<td>2.4470</td>
<td>2.2850</td>
<td>2.1320</td>
<td>2.2573</td>
<td>1.8516</td>
<td>2.2769</td>
<td>1.9878</td>
<td>2.3478</td>
</tr>
</tbody>
</table>

*Parentheses denote t-values.
model for the April, 1974, contract to 1.7522 in the model for the February, 1974, contract. Because the d statistic never exceeded 2.5 or fell below 1.5, no correction was made for autocorrelation.

**Time path properties** In this section, the properties of the time path of the basis values during the near-option part of the predelivery period will be described. Time path properties which will be evaluated are:

1. the stability of the basis values, i.e., do the basis values converge toward an equilibrium value after the system has been shocked, and
2. the equilibrium value.

Also, the general solution for each final model will be presented.

**February, 1974, model** The final model for the February, 1974, contract is:

\[
B_t = 0.2053 + 0.1673D_t
\]

where \(D\) denotes the number of calendar days remaining in the near-option part of the predelivery period. Because the lagged value of the basis is not present in the final model for this contract, the model is not dynamic. One can see that the basis values decrease in a linear path as the delivery period approaches, i.e., \(D\) decreases. The estimated value of the basis for the final near-option predelivery day is the intercept of (4.3), \(0.2053\).

**April, 1974, model** The final model for the April, 1974, contract is:

\[
B_t = 30.8400 + 0.4348B_{t-1} - 0.3331B_{t-2} + 0.0902D_t - 0.0027S_{t-1} \\
+ 0.0018S_{t-2} - 0.0027S_{t-3}
\]

This model is dynamic because lagged values are among the explanatory
variables.

According to Ladd [10], changes in equilibrium values over time can be easily computed for dynamic equations by expressing the exogenous variables, i.e., $D_t$, $S_{t-1}$, $S_{t-2}$, and $S_{t-3}$, as linear functions of time. Rather than express these variables as linear functions of time, $D_t$, $S_{t-1}$, $S_{t-2}$, and $S_{t-3}$ will be expressed in terms of $D$, the number of calendar days until the delivery period. This technique will allow the predelivery period basis values to be described entirely as a function of $D$, i.e., the time path of the basis values will be determined.

For simplicity, assume that $S_{t-1} = S_{t-2} = S_{t-3}$ at time period $t=0$. Equation (4.4) then reduces to

$$B_t = 30.8400 + 0.4348B_{t-1} - 0.3331B_{t-2} + 0.0036S_{t-1}.$$  

Regressing $S_{t-1}$ on $D_t$, one obtains the linear equation:

$$S_{t-1} = 8392.05 + 22.96D_t.$$  

Substituting (4.6) into (4.5), the final reduced form equation for the April, 1974, contract becomes:

$$B_t = 0.6286 + 0.4348B_{t-1} - 0.3331B_{t-2} + 0.0075D_t.$$  

The general solution to (4.7) is:

$$B_t = (0.5771)^{T-D_t}[(C_0 - 0.6998)cos 0.125\pi(T-D_t)
+ (1.871)(C_1 - 0.5477 - 0.2174C_0)sin 0.125\pi(T-D_t)]
+ 0.6998 - 0.1925D_t,$$

where $T$ is the total number of calendar days in the near-option predelivery period, $C_0$ is the initial value for $B_{t-1}$ at period $t=0$, and $C_1$ is the initial value for $B_{t-2}$ at the period $t=0$. Note that $T - D_t = t$ increases as the delivery period approaches.

The April, 1974, model is stable because the first term on the right
hand side of (4.8) approaches zero as the delivery period approaches.

\[
(4.9) \lim_{T-Dt \to \infty} (.5771)^{T-Dt} = 0
\]

The equilibrium value for the model, described by the last two terms on the right hand side of (4.8), is a moving equilibrium which increases linearly toward a value of $0.6998 as the delivery period approaches. In summary, the time path of the near-option pre-delivery basis values of the April, 1974, contract takes a damped sinusoidal pattern toward a moving equilibrium which linearly increases toward $0.6998 as the delivery period approaches.

**June, 1974 and 1975, model**

The final model for the pooled data sets of the June, 1974 and 1975, near-option pre-delivery periods is:

\[
(4.10) B_t = -0.0772 + 0.8721B_{t-1}.
\]

This model is dynamic because \( B_{t-1} \) is in the final model.

The general solution for this model is:

\[
(4.11) B_t = (.8721)^{T-Dt}(C_0 + 0.6036) - 0.6036
\]

\( C_0 \) is the initial value of \( B_{t-1} \) at the period \( t=0 \). The June model is stable because the first term on the right hand side of (4.11) approaches zero as the delivery period nears.

\[
(4.12) \lim_{T-Dt \to \infty} (.8721)^{T-Dt} = 0
\]

The equilibrium value for the June model is a constant throughout the near-option pre-delivery period, i.e., $-0.6036.

The value of \( C_0 \) for the June, 1974 and 1975, contracts are $3.775 and $1.095, respectively. Inserting these values into (4.11), one can see that the basis values decrease in a non-oscillatory pattern toward a value of $-0.6036 for both the June, 1974 and 1975, contracts.
August, 1974 and 1975, model

The final model for the pooled data sets of the August, 1974 and 1975, near-option predelivery periods is:

\[ (4.13) B_t = -45.1129 - 0.0750D_t + 0.0149S_{t-2} - 0.0079S_{t-3}. \]

This model is not dynamic. However, by expressing \( S_{t-2} \) and \( S_{t-3} \) as linear functions of \( D \), the path of the basis values can be determined.

For simplicity, assume that \( S_{t-2} = S_{t-3} \) in period \( t=0 \). Equation (4.13) then reduces to:

\[ (4.14) B_t = -45.1129 - 0.0750D_t + 0.0070S_{t-2}. \]

Regressing \( S_{t-2} \) on \( D_t \), the following relationship between \( S_{t-2} \) and \( D_t \) is discovered:

\[ (4.15) S_{t-2} = 6489.19 + 4.31D_t + 490.33X \]

where \( X \) is a dummy variable which takes the value of 1 if the observations of \( S_{t-2} \) are from the 1975 data set.

Substituting (4.15) into (4.14), the final model for the August, 1974 and 1975, contracts becomes:

\[ (4.16) B_t = 3.7437 - 0.04484D_t \] for 1974

and

\[ (4.17) B_t = -3.1209 - 0.04484D_t \] for 1975.

The equations (4.16) and (4.17) show that the basis values increase linearly for both the 1974 and 1975 contracts. However, the intercepts for (4.16) and (4.17) differ substantially because the values for \( S_{t-2} \) in 1974 are much larger than the values for \( S_{t-2} \) in 1975. The basis values increase linearly toward $3.7437 in 1974 and -$3.1209 in 1975 as the delivery period nears.

October, 1974 and 1975, model

The final model for the October, 1974 and 1975, contracts is:
(4.18) \( B_t = 0.0165 + 0.5386B_{t-1} - 0.0408D_t \).

This model is dynamic because the variable \( B_{t-1} \) appears in the final model.

The general solution for the model is:

(4.19) \( B_t = (0.5386)^{T-D_t}(C_0 - 0.0358) + 0.1391 - 0.0884D_t \)

The October model is stable because the first term on the right hand side of (4.19) approaches zero as the delivery period nears.

(4.20) \( \lim_{T-D_t \to \infty} (0.5386)^{T-D_t} = 0 \)

The equilibrium value for (4.19), denoted by the last two terms on the right hand side of (4.19), is a moving equilibrium which increases linearly toward the value of $0.1391 as the delivery period nears.

The values for \( C_0 \) in 1974 and 1975 are -$1.430 and -$2.900, respectively. Inserting these values into (4.19), one can see that the basis values converge in a non-oscillatory pattern toward the value of the moving equilibrium which increases linearly toward $0.1391 as the delivery period nears.

**December, 1974 and 1975, model**

The final model for the December, 1974 and 1975, contracts is:

(4.21) \( B_t = 0.12007 + 0.8402B_{t-1} \)

This model is dynamic because the variable \( B_{t-1} \) appears in the final model.

The general solution for the December model is:

(4.22) \( B_t = (0.8402)^{T-D_t}(C_0 - 0.7514) + 0.7514 \)

The December model is stable because the first term on the right hand side of (4.22) approaches zero as the delivery period nears.

(4.23) \( \lim_{T-D_t \to \infty} (0.8402)^{T-D_t} = 0 \)

The equilibrium value in this model is $0.7514 which remains constant.
throughout the near-option predelivery periods for December, 1974 and 1975, contracts.

The values for $c_0$ in 1974 and 1975 are $3.995$ and $-3.630$, respectively. Inserting these values into (4.23), one can see that the basis values converge in a non-oscillatory pattern toward the value of $.7514$ as the delivery period nears. However, the basis values are positive in 1974 and decrease throughout the near-option predelivery period. In 1975, the basis values are negative and increase toward $.7514$.

February, 1975, model

The final model for the February, 1975, contract is:

$$(4.24) \quad B_t = .1321 + .8155B_{t-1}$$

This model is dynamic.

The general solution for the February, 1975, model is:

$$(4.25) \quad B_t = (.8155)^{T-D}t(c_0 - .7160) + .7160$$

This model is stable because the first term on the right hand side of (4.25) approaches zero as the delivery period nears. The equilibrium value for this model is $.7160$ and remains constant throughout the near-option predelivery period.

The value for $c_0$ in 1975 is $3.725$. Inserting this value into (4.25), one can see that the basis values converge in a non-oscillatory path toward the value of $.7160$ as the delivery period nears.

April, 1975, model

The final model for the April, 1975, contract is:

$$(4.26) \quad B_t = 10.0592 + .5034B_{t-1} - .0108S_{t-1} + .0092S_{t-2}.$$ 

This model is dynamic.

Again, in order to obtain a general solution which is strictly a
function of $D$, the variables $S_{t-1}$ and $S_{t-2}$ will be expressed as linear functions of $D$. For simplicity, assume that $S_{t-1} = S_{t-2}$ at time period $t=0$. Equation (4.26) then reduces to:

$$\text{(4.27)} \quad B_{t} = 10.0592 + 0.5034B_{t-1} - 0.0016S_{t-1}$$

Regressing $S_{t-1}$ on $D_{t}$ yields the following relationship:

$$\text{(4.28)} \quad S_{t-1} = 5737.57 - 2.06D_{t}.$$ 

Substituting (4.28) into (4.27), the April, 1975, model is:

$$\text{(4.29)} \quad B_{t} = 0.8792 + 0.5034B_{t-1} + 0.0033D_{t}.$$ 

The general solution to (4.29) is:

$$\text{(4.30)} \quad B_{t} = (0.5034)^{T-D_{t}}(C_{0} - 1.77) + 1.77 + 0.0066D_{t}.$$ 

The April, 1975, model is stable because the first term on the right hand side of (4.30) approaches zero as the delivery period nears.

$$\text{(4.31)} \quad \lim_{T-D_{t} \to \infty} (0.5034)^{T-D_{t}} = 0$$

Equilibrium in the April, 1975, model is a moving equilibrium which decreases linearly toward the value of $1.770$ as the delivery period nears.

The value for $C_{0}$ in 1975 is $1.57$. Inserting this value into (4.30), one can see that $B_{t}$ increases at a decreasing rate at the beginning of the near-option predelivery period and then continues to decrease in a decreasing fashion as the delivery period nears.

**February, 1976, model**

The final model for the February, 1976, contract is:

$$\text{(4.32)} \quad B_{t} = -62.297 - 0.0854D_{t} + 0.0074S_{t-3}.$$ 

This model is not dynamic.

By expressing $S_{t-3}$ as a linear function of $D_{t}$, one can determine the path which the basis takes during the near-option predelivery period of
the February, 1976, contract. Regressing $S_{t-3}$ on $D_t$, the relationship is:

$$\text{(4.33)} \quad S_{t-3} = 8349 + 4.41D_t$$

Substituting (4.33) into (4.32), the final model for February, 1976, is:

$$\text{(4.34)} \quad B_t = -.514 - .0519D_t$$

One can see that the basis values increase linearly toward the value of $-$.514 as the delivery period approaches.

Results of the delivery period model

Data for the delivery period for the live beef cattle futures contracts were collected for all contract months beginning with the February, 1974, contract and ending with the February, 1976, contract. The data were divided into six sets, one for the February contract, one for the April contract, etc. Then, separate reduced form equations were estimated for each of the six data sets and for selected combined data sets. F-tests were performed to determine whether and how the data sets could be combined. Ordinary least squares regression was used to obtain estimates of the parameters of the reduced form equation (3.24).

The regression results are presented in Table 4. Results of the F-test indicated that all data sets except the one for the October contracts could be combined. Results in the first row of Table 4 are for the five combined data sets (all contracts except October) with both independent variables in the reduced form equation. The second row presents results for the same combined data sets, but with the lagged value of the futures price deleted. Results for the data set of the October contract are presented in the third row.

From Table 4, one can see that the coefficient of the variable, $B_{t-1}$,
Table 4. Estimated least-squares parameters for the delivery period basis equations

<table>
<thead>
<tr>
<th>Equations</th>
<th>Regression coefficients and (t-values)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intercept</td>
</tr>
<tr>
<td>pooled data sets, full model</td>
<td>.0832</td>
</tr>
<tr>
<td>pooled data sets, reduced model</td>
<td>.1390</td>
</tr>
<tr>
<td>October data set, full model</td>
<td>14.6620</td>
</tr>
</tbody>
</table>

*P < .01.

is highly significant in the equation of the pooled data sets and the coefficient of the variable, $F_{t-1}$, is highly significant in the equation of the October data set. As indicated by the $R^2$ of the equations, the proportion of the total variance is significant but not overwhelming.

**Time path properties** Because the reduced regression equation for the pooled data set is a difference equation, properties of the time path of the basis values during the delivery period can be obtained. The general solution to the reduced equation of the pooled data set is:

$$(4.35) \ B_t = (0.6154)^t (C_0 - 0.3614) + 0.3614$$

where $C_0$ is the basis on the first day of the delivery period and $t$ denotes the day of the delivery period. This general solution shows that the time path of the basis values in the delivery period is stable and non-oscillatory, and that the basis values converge toward a value of $.36.$
CHAPTER V. SUMMARY AND CONCLUSIONS

Introduction

In this chapter, the problem statement, the objectives of the study, the model development, and the empirical results will be summarized. Also, several conclusions will be drawn from the empirical results.

Summary of problem statement and objectives

As mentioned in Chapter I, the emphasis of this study is to describe the determination of daily basis values for live beef cattle in the pre-delivery and delivery periods of the futures contract. Although the topic of basis value determination for selected commodities has received considerable attention in literature concerning the futures markets, the previous studies have been incomplete and have created some confusion among students of basis value determination.

Objectives laid out in Chapter I included the following: (1) the development of a model of basis value determination for the predelivery period, (2) the development of a model of basis value determination for the delivery period, and (3) the determination of the time path of basis values for the predelivery and delivery periods of selected futures contracts.

Potential uses of results from this thesis were categorized into two groups, theoretical and practical. Major theoretical uses include (1) a more complete model of basis determination has been developed and (2) the model may help in the development of models for other commodities. Major practical uses include (1) identification of variables which should be included in a basis forecasting model, (2) information about time path
properties of basis values which should help in short-term hedging
decisions, and (3) identification of factors influencing hedging potential
in the live cattle futures market.

Summary of predelivery period model

In Chapter II, a general function model of basis value determination
was developed for the predelivery period. The model was a two-market
equilibrium model, the two markets being the market for storage and trans-
formation services and the futures market. Each supply and demand rela-
tionship in the two markets was described in detail in general function
notation. Through simultaneous equilibrium in the two markets, i.e.,
industry equilibrium, the model reduces to two equations and two endogenous
variables, \(B\), the current basis value, and \(P\), the current cash price for
live beef cattle.

The exogenous variables in the model are:

1. \(P_e\), the price expected by suppliers of storage and transformation
   services,
2. \(B_e\), the basis expected by suppliers of storage and transformation
   services,
3. \(P_{ep}\), the price expected by demanders of storage and transformation
   services,
4. \(B_{ep}\), the basis expected by demanders of storage and transformation
   services,
5. \(F_e\), the futures price expected by speculators,
6. \(D\), the number of calendar days remaining in the predelivery part
   of the futures contract, and
(7) $S_{t-1}$, the total quantity of live beef cattle on feed carried in from the previous market day.

Employing the implicit function theorem and Cramer's rule, the qualitative impacts on $B$ of changes of the exogenous variables were determined to be: (1) that a ceteris paribus increase in $P_e$, $P_eP$, or $S_{t-1}$ would decrease the value of $B$ and (2) that a ceteris paribus increase in $B_e$, $B_eP$, $P_e$, or $D$ would increase the value of $B$. The qualitative impacts on $P$ of changes of the exogenous variables were determined to be: (1) that a ceteris paribus increase in $P_e$, $P_eP$, or $F$ would increase the value of $P$, (2) that a ceteris paribus increase in $S_{t-1}$ would decrease the value of $P$, and (3) that a ceteris paribus increase in $B_e$, $P_eP$, or $D$ would have an indeterminant impact on $P$.

In Chapter III, an estimable reduced form equation was developed for the predelivery period. In order to develop the estimable equation, several simplifying assumptions were made. First, $B_e$ equals $B_eP$. Second, the basis expectation converges toward zero as the delivery period nears. Third, $P_e$ equals $P_{eP}$. And finally, the cash price and futures price expectations are generated in the following manner:

(a) $P_{e_t} = P_{e_{t-1}} + \gamma_1(P_{t-1} - P_{e_{t-1}}), 0 < \gamma_1 < 1$

(b) $F_{e_t} = F_{e_{t-1}} + \gamma_2(F_{t-1} - F_{e_{t-1}}), 0 < \gamma_2 < 1$

With these assumptions, the estimable reduced form equation is:

(5.1) $B_t = d_0 + d_1B_{t-1} + d_2B_{t-2} + d_3D_t + d_4S_{t-1} + d_5S_{t-2} + d_6S_{t-3} + U_t$

As discussed in Chapter III, the hypotheses concerning the signs of the reduced form equation parameters, $d_i$, $i = 1, 2, \ldots, 6$, are:

(1) that $d_1$ is positive,
(2) that $d_2$ is negative,
(3) that $d_3$ is indeterminant,
(4) that $d_4$ is negative,
(5) that $d_5$ is positive, and
(6) that $d_6$ is negative,

Conclusions from empirical results

The major conclusions to be drawn from this study concerning the near-option predelivery period basis values include the following.

First, the model does a reasonable job in explaining the total variance of the near-option predelivery period basis values as indicated by the multiple correlation coefficients of the reduced form equations in Table 3, Chapter IV. The multiple correlation coefficients, $R^2$, range from a high of .9677 for the August, 1974 and 1975, model to a low of .3923 for the April, 1975, model. On the average for all models, the $R^2$ is about .70.

Second, the signs of the estimated parameters of the reduced form equations are generally consistent with the hypothesized signs of the parameters. The only estimated parameter which differs from its hypothesized sign is the parameter for the variable $S_{t-3}$ in the February, 1976, model. Under the assumptions in the development of the estimable reduced form equation, the parameter for $S_{t-3}$ is hypothesized to have a negative sign while the estimated parameter is actually positive.

Third, expectations of various traders in the live beef cattle industry appear to explain part of the total variance of the near-option predelivery period. This conclusion is indicated by the number of times that
the estimated parameter of the variable $B_{t-1}$ differs significantly from zero. In six of the nine final models for the near-option predelivery period basis values, the estimated parameter of the variable $B_{t-1}$ differs significantly from zero.

And fourth, in general, when the basis values at the beginning of the near-option predelivery period are large and positive, the basis values will decrease throughout near-option predelivery period of the futures contract. When the basis values are large and negative at the beginning of the near-option predelivery period, the basis will increase throughout the period.

**Summary of delivery period model**

In Chapter II, a general function model of basis value determination was developed for the delivery period. The model was a two-market equilibrium model, the two markets being the cash commodity market and the futures market. Each supply and demand relationship in the two markets was described in detail.

Through simultaneous equilibrium in the two markets, the model reduced to two equations and two endogenous variables, $B$, the current basis value, and $P$, the current cash commodity price. The two exogenous variables are $F_e$, the expected futures price one or more days in advance, and $T$, the transaction cost per unit arbitrated.

With the aid of the implicit function theorem, unique equilibrium values for $B$ and $P$ exist and may be expressed:

(5.2) $\bar{B} = \bar{B}(F_e, T)$, and
(5.3) $\bar{P} = \bar{P}(F_e, T)$. 
These reduced form equations are the static results of the general function model.

The qualitative impacts of changes in the exogenous variables on $B$ were determined to be: (1) that a ceteris paribus increase in $F_e$ would increase the value of $B$ and (2) that a ceteris paribus increase in $T$ would have an indeterminant impact on $B$. The qualitative impacts on $P$ were determined to be: (1) that a ceteris paribus increase in $F_e$ would increase the value of $P$ and (2) that a ceteris paribus increase in $T$ would have an indeterminant impact on $P$.

In Chapter III, an estimable reduced from equation for the delivery period of the futures contract was developed. To derive the estimable equation, the following assumptions were introduced. First, the reduced form equation was assumed to be linear. Second, the transaction cost associated with arbitrage, $T$, was assumed to be constant during a given delivery period. And third, the expected futures price, $F_{e_t}$, was assumed to be generated by the adaptive expectations generating mechanism:

$$\text{(5.4)} \quad F_{e_t} = F_{e_{t-1}} + \gamma(F_{t-1} - F_{e_{t-1}})$$

where $0 < \gamma < 2$. The estimable reduced form equation was determined to be:

$$\text{(5.5)} \quad B_t = b_0 + b_1 F_{t-1} + b_2 F_{e_{t-1}} + U_t$$

Hypotheses about the signs of the reduced form equation parameters, $b_1$ and $b_2$, are: (1) that $b_1$ is strictly positive and (2) that $b_2$ is strictly positive.

Conclusions from empirical results

The major conclusions to be drawn from this study concerning the delivery period basis values include the following.
First, the proportion of the total delivery period basis variation is significant but not overwhelming. The multiple correlation coefficient, $R^2$, is .393 for the pooled data set. One factor that may account for part of the unexplained variance is risk, i.e., returns from arbitrage may be so variable that arbitrage is not undertaken unless the expected return from arbitrage is quite large. Another factor that may account for part of the unexplained variance is the volume of cattle available to arbitragers at the Omaha terminal market on a given day may be so small that arbitrage does not affect the basis.

Second, expectations of futures prices a day or more in advance apparently affect the basis values during the delivery period. The high significance of the estimated parameter for the lagged basis value, $B_{t-1}$, of the reduced form equation adds support to this conclusion.

And third, a major conclusion is that the variability of the delivery period basis values for live beef cattle is quite large. The high variability of the basis values imparts considerable variability to hedging returns which deters from the attractiveness of the live cattle futures market contract as a hedging tool. Efforts to improve the effectiveness of arbitrage, and thus reduce delivery period basis variation, seem warranted.

Value of Study and Suggestions for Further Research

The major value of the study will likely be derived from the theoretical models developed in this study. Hopefully, the models will provide understanding of the determination of basis values for live cattle. Also, disagreement with the models may provoke others to develop alternative
models. Such an exercise may provide an even better understanding of basis value determination.

Suggestions for further research include the following. First, an attempt should be made to revise and improve the models developed in this study. Second, a basis forecasting model should be developed for hedgers. And third, a model should be developed to help hedgers in their short-run marketing decisions, i.e., should the hedger lift his hedge today, a week from today, two weeks from today, etc.
REFERENCES


