A 3-D FINITE ELEMENT FORMULATION FOR ULTRASONIC NDT PHENOMENA

D. Moore and R. Ludwig

Department of Electrical Engineering
Worcester Polytechnic Institute
Worcester MA 01609

W. Lord

Department of Electrical Engineering
Colorado State University
Colorado, CO 80523

INTRODUCTION

Past research into the numerical modeling of ultrasonic wave propagation and scattering in 2-D has resulted in an implicit Newmark-type finite element algorithm [1]. However, by applying this formulation to analytical line source problems[2], it was found that the resulting matrix sizes and computer execution times, even on powerful supercomputers such as the CYBER 205, become prohibitively large. New algorithmic approaches are therefore required to study realistic 2-D NDT suitable geometries and, more importantly, to extend the numerical model to full 3-D capabilities.

In this paper an explicit element-by-element time stepping alogrithm is introduced. The approach is capable of handling very large 2-D grid sizes in excess of 64,000 quadrilateral elements; making a 3-D model feasible. In the following the basic finite element alogrithm is presented in conjunction with the principal feature of the novel elemental multiplication strategy.

EXPLICIT FINITE ELEMENT ALGORITHM

As discussed in earlier papers [1,3,4] the elastodynamic finite element matrix equation can be expressed in terms of a nodal displacement vector $\{U_j\}$ and acceleration vector $\{\ddot{U}_j\}$ as

$$[K_{IJ}]\,\{U_J\} + [M_{IJ}]\,\{\ddot{U}_J\} = \{R_I\} \qquad (1)$$

where the stiffness and mass matrix $[K_{IJ}]$, $[M_{IJ}]$ as well as surface traction vector $\{R_I\}$ are given in [1]. The indices I,J denote the nodal values of the elements. By replacing the acceleration in (1) with a central difference time approximation [5] one arrives at an explicit algorithm

$$\{U_I\}^{k+1} = \left(2[I] - \frac{\Delta t^2}{M_{II}}[K_{IJ}]\right)\{U_J\}^k - \{U_J\}^{k-1} + \frac{\Delta t^2}{M_{II}}\{F_I\}^k \qquad (2)$$

In (2) Δt signifies the time step size, k denotes the discrete time $t^k = k\Delta t$, and [I] is the identity matrix. As opposed to implicit finite element algorithms, matrix inversion is avoided by using a mass lumping procedure which replaces the consistent mass terms m_{JK} with its scaled nodal values (P= number of element nodes)

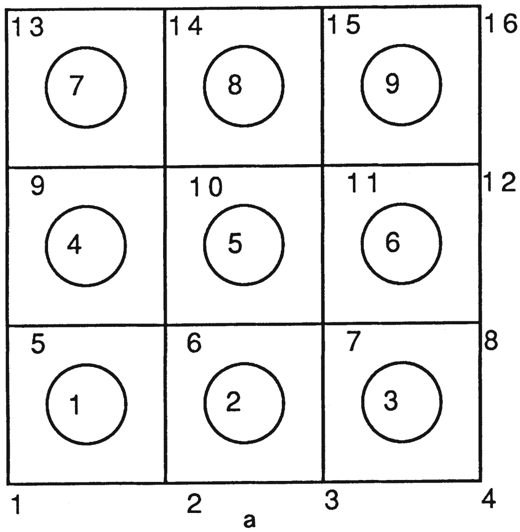
$$M_{II} = m_{II} \frac{\sum_{J=1}^{P}\sum_{K=1}^{P} m_{JK}}{\sum_{J=1}^{P} m_{JJ}} \tag{3}$$

such that $[M_{IJ}]^{-1} = [1/M_{II}]$. Rather than assembling (2) into banded global matrices, an element-by-element multiplication is conducted. The basic strategy is explained briefly on a hypothetical mesh with a total of 16 degrees of freedom as shown in Fig. 1 a). After forming global vectors (Fig. 1 b.) for the displacement vector, a mapping or gathering into a local node numbering is performed. The multiplication, as required in (2), with the associate element yields a local resultant vector which is subsequently projected or scattered into a resulting global vector. By scanning through the mesh, the program compares each locally integrated element with its predecessor and replaces the current local element with the new element type only if its elemental values differ. Different elements are obtained when defects, changes in geometry or inhomogeneities are encountered. Based on this element-by-element strategy the formulation of large global matrices can be avoided and, as a result, significant memory savings can be achieved.

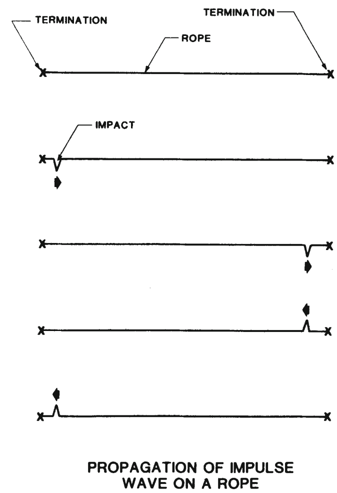
SIMULATIONS

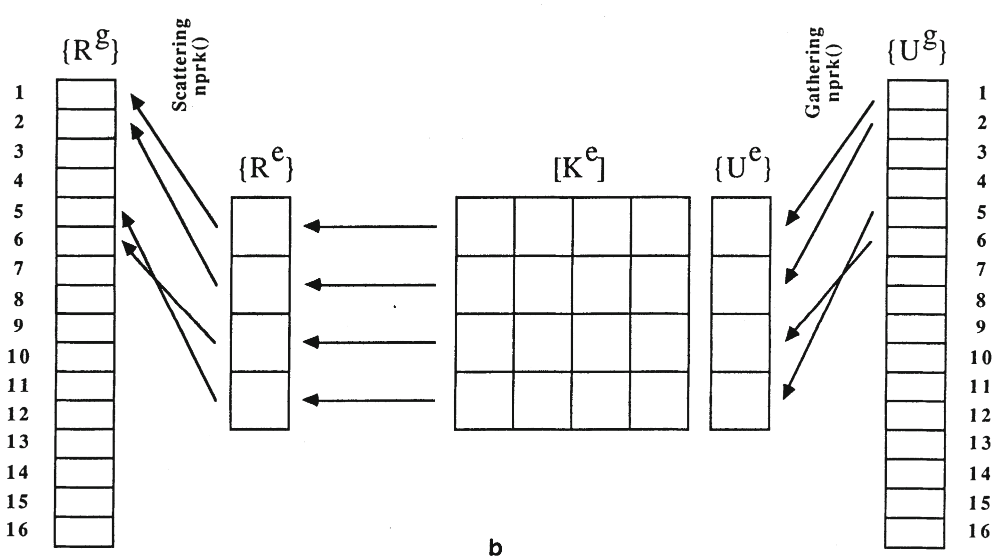
This explicit elastodynamic algorithm has been used for a variety of elastic wave propagation and scattering studies [2,4,6] The comparison to an analytical line source load (Fig. 2.) on an elastic half-space shows excellent agreement with numerical predictions, clearly indicating the potential of this new numerical model.

As an extension of the formulation beyond 2-D geometries, an initial 3-D bar analysis of a .06 cm by .06 cm by 2.0 cm parallelpiped with 900 8-nodal brick elements is considered. Fig. 3 shows the expected triangular Fourier series displacement response u_z for the front surface with the periodicity of $4L/C_L$ = 12.7 μs. Here L=2 cm is the overall length of the bar, and C_L = 6300 m/s is the longitudinal wave speed. Despite tremendous memory savings (about 300 Kbytes of total core memory for 900 brick elements), the required computer time for 100 time steps exceeds 3.5 hours on a VAX 780 class machine.



a





b

Fig. 1. Element-by-element assembly strategy. Fig. 1.a) hypothetical 16 degree-of-freedom mesh. Fig. 1.b) elemental matrix multiplication.

a)

b)

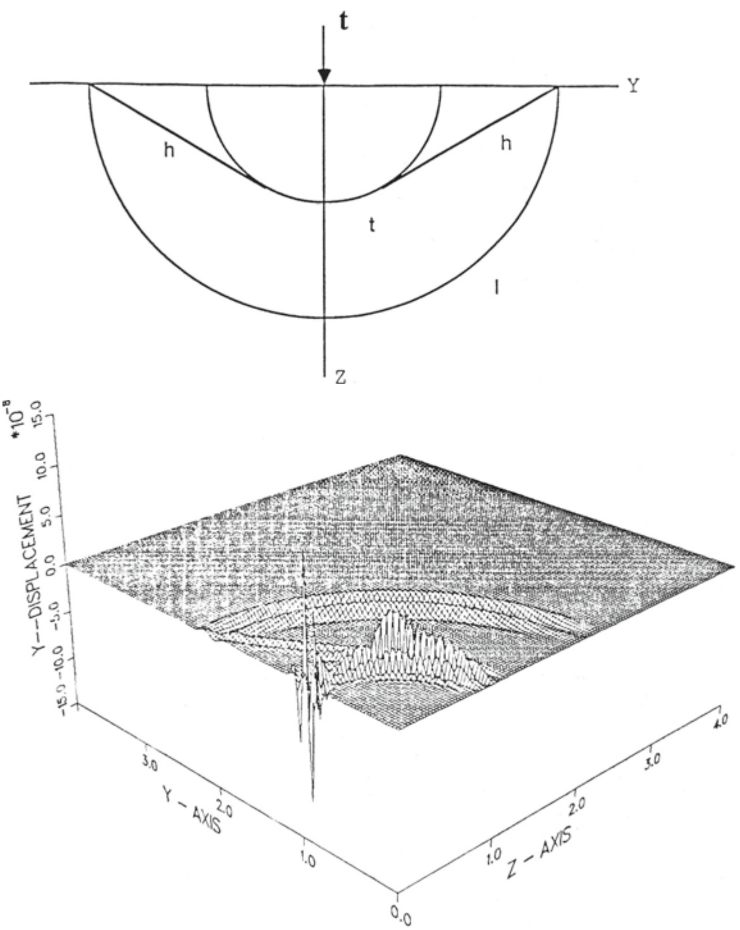


Fig. 2. Finite element line source predictions. Fig. 2.a) Expected longitudinal (l), shear (t) and head (h) wavefronts at a fixed instance in time. Fig. 2.b) Finite element displacment predictions for a 1 Mhz centerfrequency pulse at 10 μs [2].

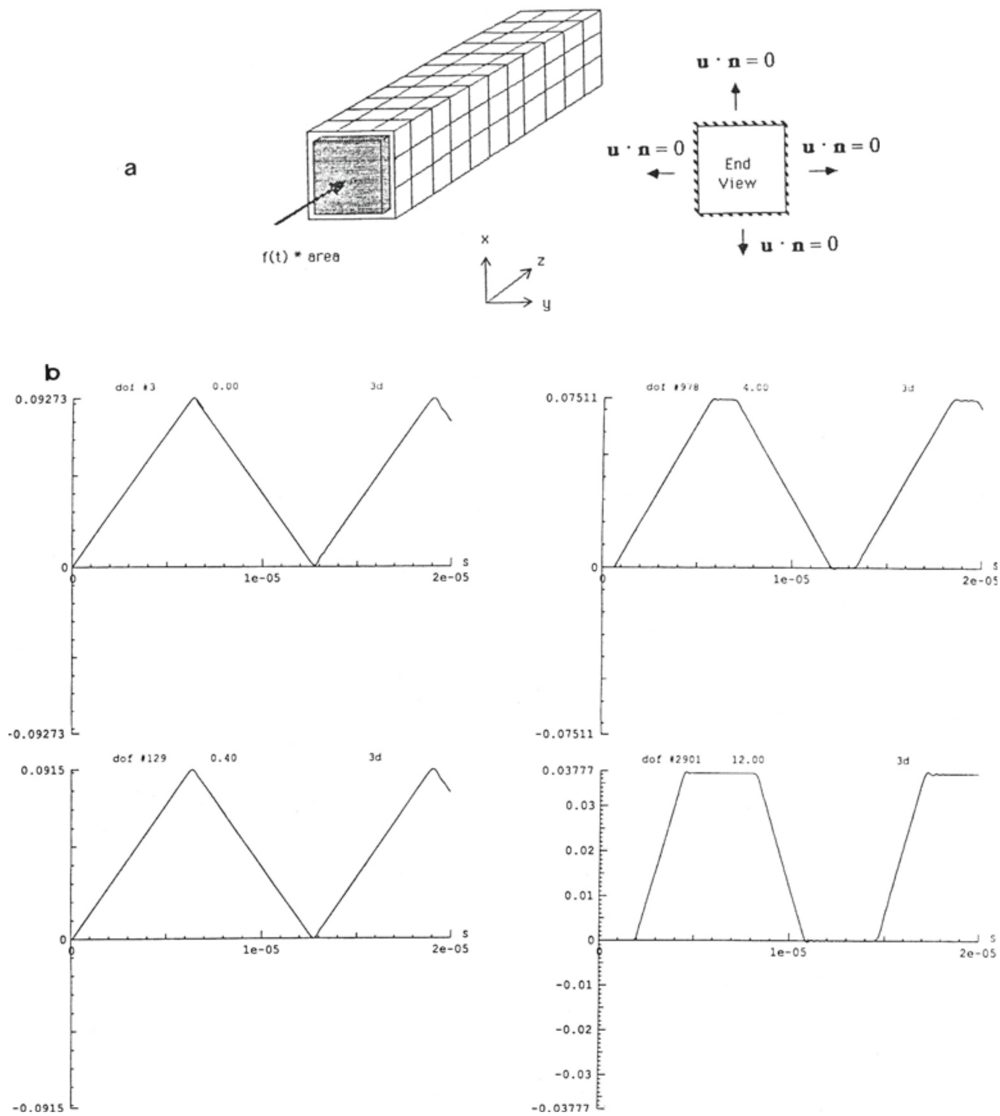


Fig. 3. Finite element 3-D bar analysis. Fig. 3 a) geometry and boundary conditions. Fig. 3.b) displacement predictions on surface.

CONCLUSIONS

Substantial work is still required to study more realistic 2-D and 3-D NDT geometries. However, the fact that the current algorithm provides exact agreement to 2-D analytical line source studies clearly reveals the potential of such a numerical time stepping algorithm in dealing with more complex NDT situations. Although the 2-D code can be executed on a medium size workstation (HP 9000 series 350 computer), the 3-D code version still requires computational resources typically found on conventional supercomputers or on the more recently introduced superminicomputers.

ACKNOWLEDGEMENTS

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