INCOMPLETE-DATA IMAGE RECONSTRUCTIONS
IN INDUSTRIAL X-RAY COMPUTERIZED TOMOGRAPHY

K.C. Tam, J.W. Eberhard, and K.W. Mitchell
GE Research and Development Center
P.O. Box 8
Schenectady, New York 12301

INTRODUCTION

In earlier works [1,2] it was concluded that image reconstruction from incomplete data can be achieved through an iterative transform algorithm which utilizes the a priori information on the object to compensate for the missing data. The iterative transform algorithm is schematically illustrated in Fig. 1. The image is transformed back and forth between the object space and the projection space, being corrected by the a priori information on the object in the object space, and by the known projections in the projection space. The a priori information in the object space includes a boundary enclosing the object, and an upper bound and a lower bound of the object density. Among these three pieces of a priori information, the object-enclosing boundary is the most important one; experience indicated that the upper and lower bound constraints usually make 1-2% difference in the image quality. The closer the enclosing boundary matches the actual shape of the object, the better will be the quality of the reconstructed image.
In this paper we report the results of testing the iterative transform algorithm on experimental data. X-ray sinogram data of the cross section of a F404 high-pressure turbine blade made of Ni-based superalloy were supplied to us by the Aircraft Engine Business Group of General Electric Company at Cincinnati, Ohio. Such a turbine blade is shown in Fig. 2. The fan beam data were taken in four integration periods and at a current of 4 mA. This roughly corresponds to 120000 to 160000 photon counts per channel in the air signal, and a S/N of 350 to 400. There were 750 view angles equally spaced in 360°. Fig. 3 shows the image reconstructed from the entire data set. This image serves as the reference image to evaluate the other reconstructed images in this study. It can be seen that even when the complete data set is used the image still suffers from streak artifacts. The presence of the artifacts indicates that the data set contains some systematic errors, such as scattering and beam hardening.

From the data set we simulated two kinds of incomplete data situations, incomplete projection and limited-angle scanning, and applied the iterative transform algorithm to reconstruct the images. Besides visually examining the reconstructed images to determine their quality, the wall thicknesses at various locations of the reconstructed images were also measured using a wall thickness measurement algorithm. This serves as a means to measure the quality of the images in a quantitative manner. We used an extended moment matching technique to measure the wall thicknesses.

Fig. 2. A F404 high-pressure turbine blade.

Fig. 3. The image of a cross section of the F404 blade reconstructed from complete data.
The fifteen locations where wall thickness measurements were taken and the measured wall thicknesses for the reference image in Fig. 3 are indicated in Fig. 4. These wall thickness measurements are typically 7 to 8 pixels and are assumed to be the correct values, and for each image the difference between its measurements and those of the reference image are taken to be a measure of the error in the image.

INCOMPLETE PROJECTION RECONSTRUCTIONS

The incomplete projection situation is simulated by the following procedure and is illustrated in Fig. 5: (1) set a saturation threshold equal to some fraction of the maximum value of the projection data, (2) set all the projection data above the threshold to the threshold value. This procedure simulates the incomplete projection problem caused by the overattenuation of the x rays by the long paths through the object. For this type of incomplete data, two additional constraints are available and can be easily incorporated into the iterative transform algorithm. These two constraints are (1) in the view angles where saturation occurs, only a number of detector elements contain saturated projection data while the data in the rest of the detector elements are still good; these good data can be utilized in the algorithm; (2) the saturated projection data should be bound below by the saturation threshold. When these two additional constraints are incorporated, the modified iterative transform algorithm becomes like that shown in Fig. 6.

We have constructed the exterior boundary of the blade cross section from the reference image in Fig. 3 to be used in the iterations. The constructed exterior boundary is shown in Fig. 7. It was constructed by first locating the left and right pixels in each line of the image whose values first exceed a specified threshold when proceeding from the edge to the center of the image, and then setting all pixels between and including the edge pixels to a value of 1, and the outer pixels to a value of 0. The constructed exterior boundary is what one would get if one uses a probe, either mechanical or optical, to probe the outside of the blade cross section.

![Fig. 4. The 15 locations where wall thickness measurements were taken.](image1)

![Fig. 5. Simulating incomplete projection caused by the overattenuation of x rays.](image2)
Fig. 6. Modified iterative transform algorithm for incomplete-projection image reconstruction.

Fig. 7. The region bounded by the exterior boundary constructed from the reference image in Fig. 3.

In Fig. 8a is shown the blade image reconstructed from saturated data with saturation threshold set at 60% of the maximum projection datum. The images after one and two iterations using the constructed exterior boundary as a priori information are shown in Figs. 8b and 8c, respectively. There are noticeable improvements on the concave edge of the image. This is confirmed in Fig. 8d where the wall thickness measurement errors at the various locations of the images are plotted as a function of the number of iterations.

LIMITED-ANGLE RECONSTRUCTION

In general, limited-angle scanning data is a more serious case of missing information because entire views of projection data are not available. As mentioned before two kinds of limited-angle situations were simulated in our studies. The first kind of limited-angle data situation we studied is similar to the incomplete projection situation discussed in the previous section, but instead of saturation affecting only some detector elements in a given view angle, all the data in the view angle are discarded and are assumed unknown if any one of the detector elements saturates. Therefore for the same saturation threshold the limited-angle version represents a much more serious case of missing information. In the second kind of limited-angle image reconstructions we studied, the missing data cone may orient in an arbitrary direction which in general is not related to the direction of maximum attenuation. This simulates the situations where the limited-angle scanning is caused by the inaccessibility of some view angles due to obstructions on the scanning path.
Fig. 8. Incomplete-projection image reconstructions from saturated data, saturation threshold = 0.6 * maximum projection datum. (a) Image reconstructed from saturated data; (b) Image after one iteration; (c) Image after two iterations; (d) Errors in wall thickness measurements.

Missing Data Determined by Attenuation

Fig. 9a shows the image reconstructed from limited-angle data for a saturation threshold at 90% of the maximum projection datum, and the images after one and two iterations using the iterative algorithm in Fig. 1 are shown in Figs. 9b and 9c, respectively. The image before iterations is seriously distorted. Besides the presence of streak artifacts, a portion of the wall on the concave edge is missing. After iterations the improvements are quite obvious, and the wall thickness measurements illustrated in Fig. 9d show significant error reductions at all locations.

Missing Data at Random Orientation

In the study we varied the size of the missing cone, its orientation, and whether it is a single cone or two back-to-back cones. Note that for this kind of missing data there may not be any missing information at all. In an earlier paper [3] it was concluded that a continuous fan beam scan covering \((180° + \text{fan angle})\) contains enough information to reconstruct the image uniquely, and it was shown and via the application of the iterative transform algorithm, the resulting image is basically equal to the complete-angle image in quality.

The results for the case when the missing cone is \(40°\) in magnitude and centered along the \(x\)-axis are shown in Figs. 10a-10d. After the iterations not only the artifacts outside the convex hull are reduced, but also those in the cavities are also much reduced. The iterated images look almost identical to the reference image. These results are in agreement with the above-mentioned conclusions and the fact that the \(320° = 360° - 40°\) fan beam data contains more than a continuous \((180° + \text{fan angle})\) fan beam scan. The plot in Fig. 10d shows that all the wall thickness measurements of all the images involved are quite close to that of the reference image. The set of results corresponding to a single \(100°\) missing cone oriented along the \(x\)-axis are shown in Figs. 11a-11d. Compared to the previous case, the larger missing data cone generated more artifacts in the uniterated image, but since this case also contains a continuous \((180° + \text{fan angle})\) fan beam scan, the iterations again restored the image to almost the same quality as the reference image.
Fig. 9. Limited-angle image reconstructions from saturated data, saturation threshold $= 0.9 \times$ maximum projection datum. (a) Image reconstructed from limited-angle data; (b) Image after one iteration; (c) Image after two iterations; (d) Errors in wall thickness measurements.

Fig. 10. Limited-angle image reconstructions, single missing cone $= 40^\circ$, along the $+ve$ $x$-axis. (a) Image reconstructed from limited-angle data; (b) Image after one iteration; (c) Image after two iterations; (d) Errors in wall thickness measurements.
Fig. 11. Limited-angle image reconstructions, single missing cone = 100°, along the +ve x-axis. (a) Image reconstructed from limited-angle data; (b) Image after one iteration; (c) Image after two iterations; (d) Errors in wall thickness measurements.

Fig. 12. Limited-angle image reconstructions, two back-to-back horizontal missing cones each of 40° along the x-axis. (a) Image reconstructed from limited-angle data; (b) Image after one iteration; (c) Image after two iterations; (d) Errors in wall thickness measurements.
The results corresponding to two back-to-back 40° missing cones oriented along the x-axis are shown in Figs. 12a-12d. After the iterations there are obvious improvements inside the cavities and on the wall regions, especially in locations 4, 5, and 6. But since in this case there is no continuous (180° + fan angle) fan beam scan, the image was not improved to the same extent achieved in the previous two cases. The wall thickness measurements showed agreement at location 4, but not at 5 and 6. We attribute the disagreement to the way the wall thickness measurement algorithm works: it considers only the wall profile but not its magnitude, and therefore the buildup in the magnitude at locations 5 and 6 is not reflected in the wall thickness measurements.

CONCLUSION

The results shown in this paper validated the practical value of the iterative transform algorithm in reconstructing images from incomplete x-ray data, both incomplete projections and limited-angle data. In all the cases tested there were significant improvements in the appearance of the images after iterations. The visual improvements are substantiated in a quantitative manner by the plots of errors in wall thickness measurements which in general decrease in magnitude with iterations.

ACKNOWLEDGMENTS

This work is supported by the Aircraft Engine Business Group of General Electric Company under Contract 2880-582-ICTD-E.

The author is grateful to his wife, Brenda, for preparing the manuscript.

REFERENCES