FRACTAL METHODS FOR FLAW DETECTION IN NDE IMAGERY

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INTRODUCTION

Detection and measurement of flaws play a major role in an emerging "fail-safe" philosophy of structural design and evaluation [1]. This philosophy allows the existence of flaws in parts in service but requires that the flaws be identified, measured and evaluated to determine if they could lead to catastrophic failure during the design life of the part. In this way greater use is made of the part, leading to considerable savings in materials and manufacturing costs. These savings come at the expense of the development of nondestructive inspection technologies that are required for flaw detection, identification and sizing.

A major constraint on nondestructive evaluation (NDE) techniques is the human factors associated with use of the techniques by trained operators having varying abilities and working under less than optimal conditions. Rummel [2] makes a plea that developers of NDE technology take realistic inspection environments and scenarios into account when performing research and development that will lead to new techniques. An example of a major human factors problem is the likelihood of errors due to tedious aspects of an inspection job that lead to boredom. Automation is an important element of a solution to this problem.

Many NDE techniques have been developed that detect flaws in a variety of types of imagery. For these techniques to be operated automatically with minimal human intervention, not only do the flaws need to be detected but they must also be automatically identified as flaws and distinguished from false alarms that are not flaws but appear flaw-like to the detection techniques. Successful discrimination requires good quantitative image models to uniquely characterize the flaws. Fractal models provide the type of quantitative characterization that is required.

This paper is organized as follows. The next section describes the particular flaw detection problem, fatigue crack detection and identification, that is the focus of this work. The third section provides some background on fractal image models particularly appropriate to the fatigue crack problem. The specific methods used to analyze fatigue cracks in visible imagery are described in the fourth section and include a summary of results. The fifth section describes a new method for modeling fatigue crack statistics using self-similar random processes that appears to provide significant benefits for fatigue crack identification. The final section summarizes the work.
THE FATIGUE CRACK PROBLEM

A problem of considerable importance in the non-destructive evaluation of parts in service is the monitoring of fatigue cracks, especially as large fleets of aircraft approach their "safe-life" design lifetimes. Fatigue cracks often lead to catastrophic failure of metal parts. Thus good techniques are crucial for monitoring the initiation and propagation of fatigue cracks and assessing the probability that these cracks will lead to failure of the part. Several techniques are based on detecting fatigue cracks in imagery of parts under inspection.

The general process for solving the fatigue crack problem using imagery can be divided into two basic tasks. The first, fatigue crack detection, concentrates on locating regions in an image that may possibly be fatigue cracks. At this stage a modest number of false alarms is expected. The second task, fatigue crack identification, concentrates on separating fatigue cracks from false alarms using the detected regions provided as output by the fatigue crack detection task.

Automatic methods for feature detection in digital images are often based on models of the response of the imaging sensor to the particular phenomena to be detected. For the detection of flaws in NDE imagery modeling can take the form of assumptions about the relative magnitude or geometric structure of sensor responses to various types of flaws. For example, consider the case of a bonded part to which heat is applied at one side of the bond. The emitted thermal radiation is sensed on the other side of the bond as a multi-level digital image. Debonding might be detected as regions where the emitted radiation is substantially lower than would be expected from a part with a good bond. Selective thresholding techniques frequently used in image processing could be used to locate these regions of substantially lower emitted thermal radiation. Automatic methods for feature identification in digital images are often based on models that quantitatively characterize the structure of the phenomena to be detected. For the identification of flaws in NDE imagery such models can take the form of assumptions about the ranges or trends for parameters that reflect the geometric structure of flawed and non-flawed regions of the imagery. For example, possible debonded regions located in thermal radiation imagery could be separated from false alarms if the projected area of bonds in the part are known. The area of the detected regions in the image could be measured and compared with the known projected bond areas.

Fractal image models provide the basis for solutions to the problem of automatically detecting and identifying fatigue cracks. For automatically detecting fatigue cracks in visible images, the texture of cracked and non-cracked portions of the image is characterized using fractal models for the intensity variations. For identifying the detector outputs as cracks, assumptions are made about the geometric structure of the cracks. The structure is characterized by fractal crack models that reflect the statistical properties of the physical processes that lead to fatigue crack initiation and propagation.

FRACTAL IMAGE MODELS

Mandelbrot [3] has defined a fractal as an object whose Hausdorff-Besicovitch dimension (i.e., fractal dimension) is strictly greater than its topological dimension. The topological dimension characterizes a certain type of equivalence among objects. Unfortunately, many objects that are topologically equivalent have very different appearances. The variation occurs because topological properties characterize only such aspects as connectivity and compactness but do not quantify metrical properties such as area, volume and roughness. The Hausdorff-Besicovitch dimension characterizes metrically equivalent objects. Thus it allows us to distinguish among a larger class of objects than is possible with topological dimension only.

Topological dimension is always an integer but the Hausdorff-Besicovitch dimension can be any real number and is usually associated with
non-integral values. For example, a fractal surface in euclidean 3-space whose topological dimension is 2 could have a fractal dimension somewhere between 2 and 3. Intuitively, a fractal surface is one that is in some sense rougher than a planar surface. Pentland [4] discusses some experimental data that implies that the fractal dimension is almost twice as accurate a measure of the perceived roughness of a surface as any other measure reported to date.

An important property of many fractals, especially those used in modeling applications, is self-similarity. This property is defined by the relation

$$M(\mu X) = \mu^{f(D)} M(X)$$

where $M(X)$ represents any metric property of the fractal (e.g., area), $X$ represents scale of measurement of the metric property, $\mu$ is a scaling factor such that $0 \leq \mu \leq 1$, and $f(D)$ is a simple function of the fractal dimension $D$ that depends on the particular metric property. This relation can be viewed as a functional equation whose solution is given by

$$M(X) = K X^{f(D)}$$

where $K$ is a constant. Eq. (1) states that the metric property computed at a reduced scale of measurement is equivalent to scaling the metric property of the fractal computed at the original scale $x$. Eq. (2) states that metric properties of self-similar fractals are power-law functions of the measurement scale. These aspects of fractals agree with observed data for coastline length, stream flow volume, terrain surface area, volume of blood vessels, percolation structures and diffusion-limited aggregation [3,5]. The self-similar property embodied by Eq. (2) is believed to be more than a convenient summary of the structure of many natural phenomena. It is conjectured that physical processes acting similarly over a range of scales are often responsible for the creation of such structures.

Fractals can be either exact or statistical. An exact fractal is a mathematical construct which starts with a generator and applies a rule for scaling and transforming the generator for an infinite number of repetitions. An example is the Von Koch curve and its variants, discussed by Mandelbrot [3]. In most cases exact fractals (and even their approximations) produce structures that are too regular to be good models for natural objects. Statistical fractals are structures that exhibit the properties of exact fractals only for a large ensemble of "like" structures. In the case of self-similarity this means that the equality in Eq. (1) is replaced by a corresponding relation governing "average" behavior at different scales.

An image is a two-dimensional field of intensity values representing the digital samples of the radiation received at elements of the sensor. Various types of radiation may be recorded including visible light, x-rays, ultrasonic and thermal response. The image intensity field of visible images has been modeled as a statistical fractal. Mandelbrot [3] discusses joint work with R. Voss where applications of statistical fractal models led to the creation of realistic images of natural scenes using computer graphics. The striking natural appearance of these images implies that fractal models should make good image models. Pentland [4] gives a proof that the intensity field for an image of a spatially-isotropic fractional brownian surface, a particular statistical fractal model, is a fractional brownian surface of identical fractal dimension. From a practical perspective, the application of fractal image models to image analysis tasks such as segmentation, texture classification and object detection by Pentland [4], Ohley et al. [6], Peleg et al. [7], and Stein [8] for a variety of real images with good success motivates use of statistical fractals for image modeling.

The self-similar property of fractals is used to develop a method, referred to as the "covering method," for estimating fractal dimension (see [8] for more detail). Eq. (2) summarizes the power-law dependence of various metric properties of the image, such as the area of the image intensity.
surface or the length of curves extracted from the image, on the measure-
ment scale. Least-squares techniques are used to obtain the parameters of
Eq. (2) from local estimates of the appropriate metric property at various
scales. The estimated model parameters are stored in output images that
record at each pixel location the parameter values corresponding to neigh-
boring pixels. These output images are then used to make inferences about
various regions of the input image.

The fractal parameters must be estimated locally because most real im-
ages consist of a variety of regions with widely varying properties. When
the image data is completely contained within a homogeneous region, the
model parameters are accurate for the region. However when the data con-
tains a boundary between two regions, the model parameters depend on the
mixture of the regions contained within the estimation window. This vari-
ation can lead to incorrect inferences based on data such as the model-fit
error and the different values of model parameters. Local estimation is
usually done through the use of a sliding window which delineates a neigh-
bhood of pixels (often square) around each pixel location. Therefore the
size of the sliding window is kept small enough to limit the cases of mixed
regions within a window but large enough to obtain statistically valid sam-
pies.

FATIGUE CRACK DETECTION AND IDENTIFICATION

The following methods for fatigue crack detection and identification
were tested using a visible light photomicrograph of a fatigue crack in a
Titanium-Aluminum alloy. The image was taken using an automated system for
monitoring growth of small fatigue cracks [9]. A low-resolution overview
of the photomicrograph is presented in Fig. 1.

Fatigue Crack Detection

Previous success using fractal texture models for object detection and
the fractal appearance of cracks motivated an analysis of their potential
for crack detection in visible imagery. The texture of the surface deter-
mined by the image intensities is characterized as a fractal surface using
the model equation

\[ A(\mu) = K\mu^{2-D} \]

where \( A(\mu) \) is the estimated surface area at measurement scale \( \mu \). The pa-
rameters \( K \) and \( D \) reflect the textural characteristics of the image inten-
sity surface. These parameters relate directly to the structure of the
surface of the imaged object as discussed in the previous section. The
fractal dimension, \( D \), describes the scaling behavior of the variation of
the image intensities within the sliding window. The parameter \( K \) is a
relative measure of homogeneity of the distribution of surface area within
the sliding window.

A high-resolution section of the photomicrograph containing the fatigue
crack is shown in Fig. 2. The crack is clearly visible and can be detected
using a simple intensity threshold. To obtain a more difficult test case,
the crack region was extracted and the intrinsic noise level in the region
was raised. The noisy crack region was placed into a pure background image
which also had its intrinsic noise level raised. The resulting test image
is shown in Fig. 3. In the test image, the crack is less visible and cannot be extracted using simple thresholding techniques.

The parameters \( K \) and \( D \) were calculated by fitting the model equation to
the image intensity surface bounded by a sliding window. The value of \( K \)
was placed in an output image at the pixel location corresponding to the
center of each sliding window placement. Another output image contained
the values of \( D \). The images were used to locate possible crack regions.
The bulk of the background in the photomicrograph image is composed of a
Figure 1 Overview of photomicrograph of a fatigue crack in a titanium-aluminum alloy. Pyramid structures are micro-hardness indentations.
Figure 2  High-resolution sub-image showing section of fatigue crack in Fig. 1

Figure 3  Noise-degraded test image
two-phased alloy grain microstructure with a high degree of variation of image intensities and an inhomogeneous distribution of surface area at the range of scales used in the model fit. Therefore the background is characterized by low K values and high D values. The regions containing crack structure have a much lower variation of image intensities and more homogeneous distribution of surface area. Therefore the cracks are characterized by high K values and low D values. Thus intermediate values of K and D are appropriate to establish threshold settings to segment both the K and D images into crack and background. Fig. 4 shows only the crack region for comparison with the resulting raw detection images shown in Fig. 5 and 6. The crack region is extracted in both cases as a connected region of pixels following the crack, with better spatial delineation in the detection image using the K parameter. Most false alarms in each image are of low aspect (i.e., low perimeter to area ratio). These false alarms could be removed by an ad hoc technique based on their low aspect. However, a more systematic approach is described in the following.

**Fatigue Crack Identification**

Consider a candidate crack region detected by the technique described herein (or any other technique) and the problem of determining whether the
Figure 5  Raw detection output image, using K parameter, with crack region clearly visible
detected region is a real fatigue crack or a false alarm. Upon inspection of the fatigue crack at several resolutions, it is noticed that there exists some approximate self-similarity of the geometric structure across these resolutions. Thus a second application of a fractal model, this time a fractal curve model, is used to characterize the jagged but self-similar structure of the fatigue crack. The model equation is

\[ L(\mu) = K\mu^{1-D} \]  

where \( L(\mu) \) is the estimated length at measurement scale \( \mu \). The parameters \( K \) and \( D \) reflect the geometric structure of the crack when viewed as a one-dimensional curve. These parameters, and therefore the geometric structure of the crack, appear to relate directly to the statistics of the process of fatigue crack initiation and propagation. More detail is provided in the next section. For the purposes of characterizing the fatigue crack in initial work performed so far, only the fractal dimension parameter, \( D \), was used.

A test image was extracted from a section of the low-resolution image shown in Fig. 1. This image was used to verify that a fractal curve model potentially provides a good geometric representation of the crack. The crack curve was extracted using a simple threshold technique. This simple extraction procedure was chosen to produce a test case for crack identification to account for the usual situation in which detection procedures produce poorly delineated crack regions.

The parameter \( D \) was calculated by fitting the model equation to many segments of the extracted crack. The mean \( D \) parameter was determined to be 1.2, a value within the range of previously measured fractal dimensions reported for fracture surface profiles [10]. Even more encouraging is the observation that the coefficient of variation (i.e., the standard deviation divided by the mean) never exceeded six percent over all of the different samples of the extracted crack curve. This stability implies that the fractal curve model is a accurate representation of the geometric structure of the curve. It is hypothesized that the model will also be a stable representation for a variety of fatigue cracks in metals which have encountered different loading conditions. The intent is to use the model to quantitatively classify such cracks in automatic identification schemes.
Tests of this hypothesis against more image data are ongoing. Also, cer­
tain theoretical developments indicate that a fractal model for fatigue
cracks may be derivable from fundamental statistical descriptions of the
physical processes of crack initiation and propagation. The exciting pros­
pect of establishing a firm physical basis for the fractal models, and pro­
gress beyond their use as merely a convenient geometric characterization,
is explained in the next section.

FRACTAL FATIGUE FAILURE MODEL

Two significant statistical interpretations of fatigue behavior in met­
als are due to Weibull [11] and Freudenthal [12]. Weibull found that the
empirical distribution of the probability of failure for a great number of
tests on notched specimens of the same material is given by the stretched
exponential law bearing his name. The Weibull distribution is currently a
popular statistical description used in probabilistic fracture mechanics
because of its relatively good success in describing most fatigue fracture
data [13]. Starting from elementary experimental facts about the submicro­
scopic aspects of the initiation and propagation ·of the fatigue process,
Freudenthal developed a probabilistic theory based on the weakest link con­
cept. He was able to derive a Weibull distribution for the probability of
failure very similar to the empirical distribution deduced by Weibull.

Early in the work described in this paper, fractal modeling of fatigue
cracks appeared to be more than an ad hoc characterization of the geometric
structure. Mandelbrot [3] describes a stable random process that is not
Gaussian and which is intimately associated with self-similar random proc­
esses (i.e., statistical fractals). He calls it the Levy flight process
which consists of a sequence of jumps connected by stopovers. The lengths
of the jumps follow the hyperbolic probability distribution

\[
\text{Prob} (L > \lambda) = \begin{cases} 
\lambda^{-D} & \lambda \geq 1 \\
1 & \lambda < 1
\end{cases}
\]  

where \( L \) is the length of the jumps. Realizations of the process are char­
acterized by clusters of short jumps such as in a random walk joined by
long jumps. This behavior is due to the probability of a long jump, ac­
cording to the hyperbolic distribution, being larger than the corresponding
long jump probability in the random walk case. For values of near 1.2,
with the stopovers connected by straight lines, patterns are produced that
bear a striking resemblance to images of fatigue cracks in metals such as
shown in Fig. 1. Thus the structural similarity between realizations of
Levy flight processes and fatigue cracks in metals is strongly suggestive
that deeper connections exist. Further evidence is provided by Schlesinger
and Klafter [14] who make the empirical observation that many diverse re­
laxation phenomena in complex random materials can be described by the same
stretched exponential law as used above for fatigue statistics. They also
show how several theories of the relaxation process based on fractal models
can be used to derive the stretched exponential law in a natural fashion.
This result encourages development of a fractal model for the fatigue proc­
ess which can be used to derive the stretched exponential law of Weibull.

Initial steps were taken to develop a fractal fatigue failure model
based on a weakest link concept similar to Freudenthal. The uniqueness of
the approach is in the assumption that fatigue failure is governed by the
growth of submicroscopic cracks to a critical length versus the usual as­
sumption that growth is based on reaching critical stress concentrations.
The probability that the length of a crack is greater than a given length
scale is assumed to be governed by a hyperbolic distribution. A major con­
sequence of this assumption is that one is able to derive a probability of
failure governed by a Weibull distribution similar to that found empiri­
cally and also derived by Freudenthal. Also, it appears that under the ad­
ditional assumption that linear elastic fracture mechanics is valid for
cracks up to the critical size, a direct relationship between the parameters of the fracture stress-based Weibull distribution and the fracture length-based Weibull distribution can be derived. From this promising relationship one hopes to be able to predict the parameter D that characterizes the geometric structure of a fatigue crack in a metal specimen under given loading conditions. An additional benefit to assuming a hyperbolic length probability distribution is that Eq. (2) is derivable as an expected value for the length of crack. This is a satisfying alternative to assuming the form of Eq. (2) based on phenomenological grounds. Thus the fractal model is quite promising as a statistical characterization of fatigue failure.

SUMMARY

Initial methods for applying fractal models to the automatic detection and identification of fatigue cracks in NDE imagery are discussed. Results of testing these methods on a visible light photomicrograph image of a fatigue crack in a titanium-aluminum alloy are presented. The crack detection method successfully distinguished between intensity surface texture parameters for cracked and non-cracked regions in a noise-degraded version of the test image. The characterization of fatigue crack geometric structure using a fractal model is shown to be stable for the crack in the test image and appears promising as a means for automatic crack identification. Finally, a new statistical fatigue failure model based on a self-similar random process called the Levy flight process is introduced. This model shows promise for application to the automatic identification of fatigue cracks in NDE imagery.

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