THE USE OF FINITE ELEMENT METHODOLOGY IN DESIGNING ULTRASONIC TESTS AND THE DETECTION OF WEAK BOND PLANES

A. F. Emery
University of Washington
Department of Mechanical Engineering
Seattle, Washington 98195

Consultant to Sandia National Laboratories
Livermore, California

INTRODUCTION

The ideal ultrasonic test should define unambiguously the size and location of defects, require a minimum of signal processing (i.e., operator intuition and decision) and use a minimum number of transducers. To do this it is necessary to choose the appropriate transducers and transducer locations; unfortunately this may require a time consuming series of iterative pre-tests using samples with manufactured defects which are presumed to behave similarly to real defects. The question arises as to whether numerical simulations may not provide a faster and more efficient way to design these tests. Accordingly a series of numerical tests were designed to: a) demonstrate the validity of numerical simulation; b) define the range of problems which can be treated numerically; c) describe the protocol for determining the optimal transducer position; d) determine the type of signal processing which would provide the maximum information.

This paper describes the results of part a and the application of numerical simulation to the case in which a thin section of soft material is used to bond two parts of a one-dimensional tensile specimen together.

PART I--VALIDATION OF NUMERICAL SIMULATION

Initially we used the DYNA2D [1] and HONDO [2] finite element codes to simulate several complex geometries in which the stress waves made many repeated transits of the specimen. In these tests we noticed several problems:
1. a damping of the stress waves with time
2. an apparent generation of high frequency waves
3. a gradual loss of kinetic energy

Accordingly we conducted a sequence of one-dimensional test calculations to determine the cause and cure of these difficulties. A one-dimensional rod was subdivided into 100 elements, the left end was subjected to a pressure pulse of the form (which we term a "haversine loading"). In describing the haversine load and other excitations, the wave length, $\lambda$, is defined in terms of the period or the dominant frequency as

$$\lambda = \frac{c}{f} \text{ or } = cT.$$
The right end was stress free. Figure 1 shows typical results found when using DYNA2D with default values of artificial viscosity and a time step of $0.8\delta t_c$ ($\delta t_c =$ maximum allowable time step $= c\delta x$). Here the gradual decay of the magnitude of the stress wave and the production of apparent high frequencies are seen. HONDO gave comparable results. Both of these codes use a finite element approach to define the stiffness matrix, but an explicit, centered difference time integration. Similar results were observed using NIKE2D [3], an implicit finite element code, and SHELL-SHOCK [4] which uses a spectral decomposition technique. Johnson [5] noted oscillations in displacements when using DYNA2D, but did not report stresses. Although improved results could be obtained by varying the time step size and the values of artificial viscosity, we felt that we must better understand the cause before attempting to correct the problem.

FINITE DIFFERENCE RESULTS

To provide a better means of testing the results, we analyzed the problem using a simple space and time centered finite difference scheme [6] and examined the results by computing the power spectra. Although the stress histories changed substantially with changes in the Courant number ($\tau = c\delta t/\delta x$), the power spectra remained identical, leading to the conclusion that the errors were due to phase errors. These phase errors are due to the form of the time integration. For a centered, explicit time integration, the phase error is given by

$$ P = \begin{cases} 
  P_0 \sin^2(\Omega t) & \text{for } 0 < \Omega t < \pi \\
  -P_0 \sin^2(\Omega t) & \text{for } \pi < \Omega t < 2\pi \\
  0 & \text{for } 2\pi < \Omega t 
\end{cases} $$

(1)

Figure 1 One Dimensional Stress Wave Computed using DYNA2D, $\lambda/\delta x = 6$
\[
\text{Phase Error} = -2\pi \frac{\delta x}{\lambda} + \tan^{-1} \left( \frac{2\pi S \sqrt{1 - \tau^2 S^2}}{1 - 2\tau^2 S^2} \right) \\
\tau = \frac{c\delta t}{\delta x}, \quad S = \sin \left( \frac{\pi \delta x}{\lambda} \right) \quad (2)
\]

in terms of the Courant number, \( \tau \), mesh spacing, \( \delta x \), and the wave length, \( \lambda \), of the exciting wave. Table 1 lists the errors in terms of \( \tau \) and the ratio \( \lambda/\delta x \). As expected, when the Courant number is unity, there are no phase errors for any wave length since the stresses are propagated exactly one mesh spacing per time step. As the Courant number is reduced, the errors begin to increase.

From the table it is clear that if a reduced Courant number is used, the mesh spacing must be such that \( \lambda/\delta x > 20 \) if the phase errors are to remain small for computation which require in the order of thousands of time steps. This is in agreement with results reported by Bond [7]. Unfortunately in two dimensional problems it is not possible to use a Courant number of one. For a rectangular mesh, the critical time step is given by

\[ c\delta t < \frac{\delta x \delta y}{\sqrt{(\delta x^2 + \delta y^2)}} \quad (3a) \]

and for a square mesh

\[ c\delta t < \delta x/\sqrt{2} \quad (3b) \]

Thus when choosing a maximum time step for a square mesh, if a wave travels along the x or y axis, its Courant number is equal to \( \tau = 0.707 \) and, as seen from Table 1, phase errors will accrue unless the mesh spacing is very fine. Table 2 lists the effects that were observed as functions of the Courant number and mesh size and serves as a guideline for choosing the mesh spacing.

### Table 1

Phase Errors of Centered Differencing for 1-D Waves

<table>
<thead>
<tr>
<th>( \lambda/\delta x )</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>.997</td>
<td>.926</td>
<td>.823</td>
<td>.653</td>
<td>.000</td>
</tr>
<tr>
<td>4</td>
<td>.110</td>
<td>.091</td>
<td>.068</td>
<td>.038</td>
<td>.000</td>
</tr>
<tr>
<td>6</td>
<td>.032</td>
<td>.026</td>
<td>.018</td>
<td>.010</td>
<td>.000</td>
</tr>
<tr>
<td>8</td>
<td>.013</td>
<td>.011</td>
<td>.008</td>
<td>.004</td>
<td>.000</td>
</tr>
<tr>
<td>10</td>
<td>.007</td>
<td>.005</td>
<td>.004</td>
<td>.002</td>
<td>.000</td>
</tr>
<tr>
<td>20</td>
<td>.001</td>
<td>.001</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
</tr>
<tr>
<td>30</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
</tr>
</tbody>
</table>

### Table 2

Effect of reduced \( c\delta t/\delta x \) on Zero Order Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leading Edge Wave Speed</td>
<td>None</td>
</tr>
<tr>
<td>Angle of Reflection</td>
<td>None</td>
</tr>
<tr>
<td>Energy Partition</td>
<td>None</td>
</tr>
<tr>
<td>Power Spectrum</td>
<td>None</td>
</tr>
<tr>
<td>Maximum Reflected Stress</td>
<td>Unaffected if ( (\lambda/\delta x)(c\delta t/\delta x) &gt; 20 )</td>
</tr>
<tr>
<td>Wave Length, Phase Angle</td>
<td>Unaffected if ( (\lambda/\delta x)(c\delta t/\delta x) &gt; 12 )</td>
</tr>
</tbody>
</table>
FINITE ELEMENT RESULTS

Most finite element programs utilize the finite element methodology to determine the relationship between the displacements and the stresses. However a variety of time integrators are used, ranging from the explicit, centered difference to the Newmark Beta method \([8]\]. The phase errors (frequently termed "period lengthening" in the finite element lexicon) for the centered explicit method are the same as for the finite difference method. Implicit integration, such as the Houbolt and Newmark methods, display different characteristics. The phase errors of the Newmark method with a value of beta of \(1/4\) are given by

\[
\text{Phase Error} = -2\pi \frac{\delta x}{\lambda} + \tan^{-1}\left(\frac{2\tau S\sqrt{1 - \tau^2 S^2}}{1 - 2\tau^2 S^2}\right)
\]

where

\[
\tau = \frac{c\delta t}{\delta x}, \quad S = \frac{\sin\left(\frac{\pi \delta x}{\lambda}\right)}{\sqrt{1 + \tau^2 \sin^2\left(\frac{\pi \delta x}{\lambda}\right)}}
\]

and are listed in Table 3. The essential difference between the explicit and implicit methods is the divisor in the definition of \(S\), which leads to large errors as the time step is increased. Because of this divisor, there is no value of the Courant number for which the errors are zero as is the case for the explicit method. Inasmuch as implicit integration is used to achieve large time steps, the values listed for large time steps, suggest that the associated errors may be unacceptable.

Whereas Finite Difference methods can be tailored for the specific problem, the value of the Finite Element method is that it can be used in an automatic way without special coding to handle interfaces or general boundary conditions. However the method contains several deformation modes with the consequence that the critical wave speed is only 82\% of sound speed. Thus for two dimensional problems, using a square mesh, the maximum Courant number is \(0.82 \times 0.707 = 0.61\). Fortunately, the effect of this is to reduce the phase errors.

Figure 2 shows the effect of using a very fine mesh and the explicit, centered integrator in HONDO. Although the effective Courant number of 0.61 still leads to a noisy stress signal, the stress wave peaks do not show the unacceptable decay demonstrated by the coarser meshes.

| Table 3 |
| Phase Errors of Newmark Integration for 1-D Waves |
| Errors are given for a time step of \(\delta t_c = c\delta x\) |

<table>
<thead>
<tr>
<th>(\lambda/\delta x)</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>5.0</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.287</td>
<td>1.571</td>
<td>2.034</td>
<td>2.592</td>
<td>2.847</td>
</tr>
<tr>
<td>4</td>
<td>0.211</td>
<td>0.340</td>
<td>0.615</td>
<td>1.053</td>
<td>1.285</td>
</tr>
<tr>
<td>6</td>
<td>0.067</td>
<td>0.120</td>
<td>0.262</td>
<td>0.571</td>
<td>0.773</td>
</tr>
<tr>
<td>8</td>
<td>0.029</td>
<td>0.054</td>
<td>0.132</td>
<td>0.350</td>
<td>0.522</td>
</tr>
<tr>
<td>10</td>
<td>0.015</td>
<td>0.029</td>
<td>0.075</td>
<td>0.230</td>
<td>0.377</td>
</tr>
<tr>
<td>20</td>
<td>0.002</td>
<td>0.004</td>
<td>0.011</td>
<td>0.049</td>
<td>0.114</td>
</tr>
<tr>
<td>30</td>
<td>0.001</td>
<td>0.001</td>
<td>0.003</td>
<td>0.017</td>
<td>0.048</td>
</tr>
<tr>
<td>40</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.008</td>
<td>0.024</td>
</tr>
</tbody>
</table>
PART II--BONDS

Consider a solid state diffusion bond which joins two materials and suppose that the bond is defective in that it has areas of imperfect bonding. When a tensile stress wave passes through the bond, the areas of non-adherence will open and there will be an acoustic impedance mismatch and a reflected wave. Thus the bond should be detectable by a probing ultrasonic wave. Generally one chooses an ultrasonic frequency such that the wave length is small in comparison to that of the defect in order to detect the defect. For defects in diffusion bonds, the thickness of the bond is unknown, and it is unlikely that a sufficiently short wave length signal can be utilized. The question then is what is the nature of the signal reflected from the bond and what information does it contain.

Thomas [9] describes an experiment in which two identical pieces of material are joined by a thin layer of softer, temperature dependent material, but of the same density. The entire specimen was heated to a uniform temperature. Since the thin bond loses strength when heated, the effect of heating is to reduce the sound speed and to change the acoustic impedance of the bond. An ultrasonic study of this specimen should provide a first order approximation to the response of a defective diffusion bond. Figure 3 shows a comparison between the power spectrum of the incident haversine wave and that of the numerically computed reflected wave when normalized to the same peak amplitude. Using 14 elements in the bond and a spacing to ensure that the Courant number was the same in the bond and the surrounding material, the numerical results were indistinguishable from the analytical values given by

\[ |R| = 1 - \frac{2}{\sqrt{4\cos^2(kL) + (Z + 1/Z)^2\sin^2(kL)}} \] (5)

Where \( R \) = reflection coefficient, \( L \) = thickness of the bond, \( Z \) = ratio of acoustic impedences, \( k \) = wave number.

When the mesh spacing was kept constant, the Courant number in the bond was less than that in the surrounding material and small differences in the absolute magnitude of the power spectra were noted. When the spectra were normalized, no differences could be detected. From the theory, the reflected wave displays a zero energy at bond thicknesses and wave lengths given by
Figure 3 Power Spectra of Incident and Reflected Waves

\[ kL = n\pi \quad n = 1, 2, \ldots \]  

Figure 4a shows the experimentally determined spectra for waves reflected from the bond and from a specimen in which the bond was replaced by an infinite layer of water. Using the measured sound speeds and approximate bond thicknesses reported by Thomas [9], the reflection coefficients were calculated and are shown in Figure 4b. The measured values are significantly higher than the predicted values and show an unusual dip at about 3 MHz. In addition the measured values should show a zero reflection at 9 MHz. Unfortunately, the probing signal was deficient in energy at this frequency and the attenuation cannot be observed.

One explanation of the differences may be that the bond material is dispersive. Figure 5a shows a normalized power spectra for an incident haversine wave with a nominal frequency of 5 MHz and waves reflected from a bond whose sound speed varies with frequency at a rate of 1% per 1 MHz. The results agree with those of Figure 4a in that the peak in the reflected wave is at a slightly lower frequency, but the dip at about 3 MHz is not demonstrated. The dispersive effect is small. Another possibility is that the bond thickness may be different than reported. Figure 5b illustrates the effect of doubling the thickness and a qualitative agreement is noted. However, both maxima of reflection coefficients are equal and the minimum is a sharp zero. These are at variance with the lower experimental peak near 2 MHz and a non-zero dip at about 3 MHz.

CONCLUSIONS

These results indicate that explicit numerical methods are satisfactory only if the mesh size is small enough to ensure minimum phase errors, while implicit techniques, in general, have unacceptable errors. Similar problems have been addressed in the numerical simulation of fluid flow and methods of zero-average phase errors have been proposed by Fromm [10] and compact schemes which may reduce errors have been discussed in references 11 and 12. Whether such improved algorithms can be found or whether sophisticated signal processing techniques, such as Cepstral analysis, will be needed to improve the simulation is still an open question.
With regard to diffusion bonds, although regions of reduced wave speed yield power spectra which are similar to those experimentally observed, the difference are currently unexplained. There is the possibility that some of the differences may be due to a variation of the bond thickness across the specimen width. This remains to be tested. If, on the other hand, the effects are due to frequency dependent properties, then only the frequency decomposition simulations may be employed. This would have a significant impact on how we can numerically simulate ultrasonic experiments.
REFERENCES