THE CHARACTERIZATION OF SURFACE DEFECTS USING RAYLEIGH WAVE
HODOGRAPHS

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BACKGROUND

The interaction of Rayleigh waves with surface inhomogeneities has received considerable attention in recent years. Though of interest to the geophysics and microelectronics communities, the subject is particularly important to those involved in nondestructive testing. This is because of industry demands for more detailed and reliable characterizations of surface-breaking defects. Material interrogation using Rayleigh waves offers an effective means of detecting such inhomogeneities because energy is confined to the near-surface region. Flaws can be detected up to depths of one or two wavelengths, corresponding to 0.31-6.1 mm over a frequency range of 0.50-10.0 MHz [1]. The associated area of coverage is ten to one hundred square feet. The material need not be conductive, as required for electromagnetic methods, and is more sensitive than bulk wave ultrasonic testing to small cracks.

An important first step in exploiting Rayleigh wave phenomena is to develop models of the associated energy-defect interactions. Because analytical modeling of phenomena associated with wavelength and defects of the same order is difficult, experimental and numerical approaches are typically used. Viktorov appears to have been the first to investigate experimentally the reflection, transmission and scattering of Rayleigh waves as functions of crack depth [2]. Reinhardt and Dally extended this work using photoelastic techniques and postulated crack depth (d): wavelength ($\lambda_0$) ratios best suited for pulse-echo and through transmission techniques [1]. Cook showed that time-of-flight measurements yield crack depth information for $d > \lambda_0$ which complements the more common method of estimating crack depth by comparing signals reflected from the top and bottom of the defect [3,4]. Following the earlier work of Viktorov, Morgan investigated the use of ultrasonic spectroscopy to classify surface inhomogeneities [4].

A finite difference model of surface wave phenomena was first introduced by Munasinghe and facilitated quantitative investigations of propagation along step geometries in isotropic and anisotropic media [5]. Bond continued this work in a number of investigations and has successfully modeled wave interactions with a variety of defect geometries [6,7]. In addition to standard reflection and transmission techniques, Bond's model has been used to consider bulk wave-to-Rayleigh wave conversion phenomena as a means of characterizing defects [8].

The defect evaluation approaches considered utilize amplitude, time-of-flight, or frequency information, but a new approach is presented in this paper that exploits the relationship between perpendicular displacement components at a selected surface observation point. As Rayleigh waves propagate, affected surface particles experience a retrograde elliptical motion [9]. A display of this movement is generated by plotting the
corresponding displacement components as a coordinate pair for the time span of interest. The resulting plot is called a Hodograph and its shape, in some measure, must characterize the way in which energy propagates along the object’s surface. The question to be answered, then, is what type of information is manifest in hodographs obtained from a defect-attenuated Rayleigh wave. A casual investigation of this question is best suited to numerical approaches because of the accompanying simplifications and test reproducibility. This has been undertaken utilizing a previously developed finite element code [10,11].

NUMERICAL DATA GENERATION

A schematic of the test geometry adopted is shown in Figure 1. The excitation signal is given by \( \cos(\omega t) [1 - \cos(\omega t/3)] \) and was input as a normal acting point source because R-waves associated with finite apertures carry only a small portion of the available signal energy. This signal approximates an L-wave transducer and is an alternative approach to the Ricker pulse for simulating Rayleigh waves [7]. A typical displacement field is shown in Figure 2.

Hodographs were obtained by windowing the displacement data — keeping only the primary hodograph features. These final plots represent approximately 265 points of 20 nsec spacing for a total signal window of 5.3\( \mu \)sec. Data was generated for varying depth and width of a test slot. A typical hodograph is shown in Figure 3.
Quantitative evaluation of the numerical Hodographs is considerably simplified by exploiting the high degree of data correlation in order to compress the characterizing information. A Fourier Descriptor approach was taken because of associated scaling and orientation insensitivity as well as the fact that it is easily implemented as a computer algorithm [12]. Figure 4 illustrates the concept that the hodograph's closed contour can be considered as a periodic signal and expanded in a Fourier series. In terms of the notation of Figure 4, the complex displacement function, $U(l)$, is thus given by

$$U(l) = \sum_{n=-\infty}^{\infty} C_n \exp \left( \frac{j2\pi nl}{L} \right)$$

where the Fourier coefficients, $C_n$, are expressed as

![Hodograph representation](image)

Fig. 3. The hodograph of a surface wave at the surface.

Fig. 4. Hodograph representation as a periodic function.
In practice, a correlated contour can be reasonably approximated by only a finite number of these coefficients. Then

$$U(l) \approx \sum_{n=-m}^{m} C_n \exp \left( \frac{j2\pi nl}{L} \right)$$

The finite element algorithm is discrete and so the hodographs are, in actuality, many-sided polygons. Fu and Persoon have developed an expression for computing the Fourier coefficients of such polygons [13]:

$$C_n = \frac{L}{4\pi^2n^2} \sum_{k=1}^{m} \left( b_{k-1} - b_k \right) \exp \left( \frac{-j2\pi nk}{L} \right), \ n \neq 0$$

$$I_k = \sum_{i=1}^{k} \left| v_i - v_{i-1} \right|, \ k > 0, I_0 = 0$$

$$b_k = \frac{v_{k+1} - v_k}{|v_{k+1} - v_k|}$$

An algorithm based on this approximation has been used to compress the hodograph data from approximately 265 elements to just 21. A typical reconstruction is shown in Figure 5.

As pointed out by Udpa and Lord [12], the Fourier descriptors employed can be made invariant to starting point, scaling, translation, and orientation via the following mapping:

$$B_n = \frac{C_{1+n} \cdot C_{1-n}}{C_{1}^2}$$

For m original descriptors, this mapping results in \((m-3)/2\) invariant coefficients. Thus only eight final elements are used to characterize the hodographs completely.

Fig. 5. A comparison of a hodograph with its reconstruction.
DATA EVALUATION USING K-MEANS CLUSTERING

Data compression facilitates quantitative hodograph evaluation and motivates a consideration of various numerical techniques for categorizing defects via their invariant Fourier descriptor sets. The approach chosen is based on a K-means clustering algorithm and has five basic steps [12]:

[Step 1] Specify the number of classes \( k \) represented in the data sets to be evaluated. To categorize an unknown defect, input its descriptor set along those of known, bracketing defects.

[Step 2] Arbitrarily assign \( k \) of the data sets to be initial cluster centers.

[Step 3] Assign the remaining cluster(s) to the closest cluster center.

[Step 4] Calculate new cluster centers by computing the sample mean of all points in each cluster.

[Step 5] Repeat Steps 3 and 4 until no new assignments have been made.

This algorithm has been implemented as a simple Fortran code and used to classify defects according to depth. Figure 6 shows the five hodographs associated with rectangular slots listed in Table 1. Though there are variations in the contours with depth, a qualitative depth evaluation is not straightforward. The K-means clustering algorithm, however, makes possible quantitative categorizations, as are summarized in Table 2. Of particular significance is the last clustering which results from entering all five data sets while specifying only four clusters. If any one of the slot depths was unknown, this grouping would provide accurate dimensional information. Thus the hodographs do indeed manifest derivable information about surface inhomogeneities and represent a valid nondestructive testing technique.

<table>
<thead>
<tr>
<th>DEFECT DEPTH (d) (mm)</th>
<th>DEFECT WIDTH (w) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.8</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>1.6</td>
<td>0.8</td>
</tr>
<tr>
<td>2.0</td>
<td>0.8</td>
</tr>
<tr>
<td>3.2</td>
<td>0.8</td>
</tr>
<tr>
<td>1.6</td>
<td>1.2</td>
</tr>
<tr>
<td>1.6</td>
<td>1.6</td>
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</tbody>
</table>

Though the objective of this initial investigation was to study defects of varying depth, some consideration has been given to the classification of slots of varying width as well. The generation of only three such data sets, as listed in Table 1, precludes any clustering tests but the resulting hodographs of Figure 7 indicate that the contours are sensitive to defect width. This approach therefore shows potential as a means of characterizing defect geometries to an extent not possible using current interrogation techniques.
Fig. 6. Comparison of hodographs at a constant width (0.8 mm) with varying depth.
Fig. 7. Comparison of hodographs at a constant depth (1.6 mm) with varying width.
Table 2. The summary of groupings by defect depth.

\[
\begin{align*}
& (0.0, 0.8) + (1.6) \\
& (0.0, 0.8) + (1.6, 2.0) \\
& (0.0, 0.8, 1.6) + (3.2) \\
& (0.0, 0.8, 1.6, 2.0) + (3.2) \\
& (0.0, 0.8) + (1.6, 2.0) + (3.2)
\end{align*}
\]

CONCLUSIONS AND FUTURE WORK

Hodographs of Rayleigh wave phenomena do, to some extent, characterize surface inhomogeneities. Once windowed to discrete, closed contours, these highly correlated signals can be accurately described with approximately eight invariant Fourier descriptors. This compressed data form facilitates quantitative defect classification via clustering algorithms and provides for efficient storage of test records. Rectangular slots of varying depth have been successfully clustered and there is indication that the technique may also be useful in making width delineations.

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REFERENCES