APPLICATION OF GAUSS-HERMITE BEAM MODEL TO THE DESIGN OF ULTRASONIC PROBES

S. J. Wormley, B. P. Newberry, M. S. Hughes
D. K. Hsu, and D. O. Thompson

Center for NDE
Iowa State University
Ames, Iowa 50011

INTRODUCTION

Work is currently underway to develop and test prototypic ultrasonic instrumentation that will produce unipolar stress pulses in a pulse-echo mode. The instrument will be used to detect and characterize flaws in materials and for the measurement of material properties important in material processing applications. In conjunction with this work, a Gauss-Hermite Beam Model [1,2,3,4] is used to predict the response of various transducer element geometries to maintain the unipolar response over a longer propagation distance.

BACKGROUND

Although many research advances have been made that have laid the way for a quantitative ultrasonic technology, most industrial use today remains qualitative despite the fact that quantitative capabilities are absolutely essential in order to use modern fracture mechanics approaches for failure prediction. One of the principal reasons for this limitation is the narrow bandwidth of commercial ultrasonic instrumentation relative to the ultrasonic bandwidth needed to properly employ the above-noted research advances over a reasonable range of flaw sizes. In an attempt to remedy the current limitation, we have recently succeeded in designing and building a new transmit/receive switch under BES Engineering that permits the generation and detection of unipolar ultrasonic pulses in a pulse-echo mode instead of the conventional bipolar pulse. It is obvious that a unipolar pulse has a large advantage in bandwidth over the current commercially-used bipolar pulse shape due to its low frequency content. This conclusion is borne out by experimental results which demonstrate that a bandwidth improvement of as much as 1000% can be achieved. This is sufficient to permit the utilization of research advances in inverse theories for flaw sizing over a reasonable range. The ultrasonic unipolar pulse may have other unique advantages. One of these is as a process control device related to the on-line control of various material processes such as casting or solidification processes.

The advantages of the unipolar response are quickly lost due to the effects of diffraction with commercial planar ultrasonic probes as propagation distance increases. The Gauss-Hermite beam model provides the tool for the investigation of probe designs that can minimize the losses due to diffraction and enhance the preservation of the unipolar response of typical ultrasonic inspection working distances.
Consider the localized source distribution shown in Fig. 1 which lies in the plane $z=0$. It has been shown [1] that the velocity potential field radiated into the fluid halfspace, $z > 0$, by this source, can be expressed, within the Fresnel approximation, as

$$\phi(x, y, z) = e^{ikz} \sum_{m,n} C_{mn} \psi_{mn}(x, y, z)$$  (1)

where the $\psi_{mn}$ are the Gauss-Hermite functions and the $C_{mn}$ are constant complex coefficients. The Gauss-Hermite functions have the form

$$\psi_{mn}(x, y, z) = a e^{-\beta x^2} e^{-\beta y^2} H_m(\gamma_x x) H_n(\gamma_y y)$$  (2)

where $H_m$ is a Hermite polynomial of order $m$ and $\alpha, \beta, \gamma$ are function of the $z$-coordinate. Their function forms are given in Ref. [1].

The coefficients of this expansion, $C_{mn}$, are found by utilizing the orthogonality property of the Gauss-Hermite functions along with the knowledge of the source distribution $\phi(x, y, 0) = \phi_0(x, y)$. For a piezoelectric transducer, what is generally known is not $\phi_0(x, y)$ but rather $V_0(x, y)$, the initial velocity distribution. However, it can be shown [1] that in the Fresnel approximation there is no distinction between the two. The $z$-D integral over the source plane necessary to calculate the $C_{mn}$ must, in general, be done numerically, and can be accomplished quite efficiently.

Using this formalism, any arbitrary localized source may be modeled as long as the Fresnel approximation is valid, $ka >> 1$, where $a$ is the characteristic dimension of the source. The coefficients for a given source must only be computed once and then can be stored. Computing the potential at field points using Eqn. (1) can be performed quite rapidly using the stored coefficients. Good convergence can generally be obtained using on the order of 25 x 25 terms, although this will depend on the specific source type.
The transducer designs modeled in this work have curved radiating surfaces rather than planar ones. This poses a minor problem since the model requires an integration over a source plane. This difference can be approximately treated by tracing rays from the actual radiating surface to a planar one, keeping up with the phase delay of each ray so that an equivalent planar source is specified.

TRANSDUCER DESIGN

The pressure field in a fluid produced by a planar source is given in Eqns. 1 and 2, the formalism for the Gauss-Hermite beam model. These equations were derived with a planar or bicylindrically focused piston transducer in mind. However, they are more generally applicable to any localized source which can be described by a field distribution in a plane, \( p(x, y, z=0) \). Consequently, one application of the model is in the design of ultrasonic transducers, where it may be used to analyze the field patterns of prospective source types.

The most common type of focused transducer is one which is spherically or cylindrically focused by placing a lens on the face of a planar transducer. In this case, the modeling of the focusing is accomplished with the lens transformation law. For the case of an axicon, however, the probe is generally constructed, not with a lens, but rather by having a radiating surface which is conical in shape. This presents a problem for the Gauss-Hermite model, which requires a planar source distribution to utilize the orthogonality property of the functions through Eqn. 2. This is circumvented by making an approximation equivalent to the thin lens transformation. The thin lens law neglects diffraction in the lens. In a similar manner, the axicon case is treated by constructing a virtual planar source which has the appropriate phase variation, linear in this case, such that when rays are traced from the virtual source plane to the actual source surface, the correct field is obtained. This method may be used for other non-planar radiating surfaces as well, provided that the distance from any point on the radiating surface to the virtual source plane is small enough to make the thin lens law valid.

The types of focused radiators that have been modeled, along with the axicon and spherically focused piston, include a pyramidal radiator, and radiators which are elliptical, parabolic, and hyperbolic in shape. All of the above sources are axially symmetric with the exception of the pyramidal probe, which is square. The radiation fields of these probes have been computed and compared with one another with respect to beam width and depth of focus. The parameters of all of the probes were chosen to be roughly equivalent to that of a reference probe, a 0.635 cm radius spherically focused piston with a focal length of 7.62 cm.

ANALYSIS

The radiation patterns of the unfocused piston, axicon, pyramidal, hyperbolic, parabolic, and elliptical probes are shown in Figs. 2 through 7, respectively, for a frequency of 8 MHz. These plots are of beam amplitude vs. position in the x-y plane (the plane containing the beam axis and the x transverse axis). Therefore, these plots show how the transverse beam profiles evolve with propagation distance along the axis. For all of the probes except the pyramidal, this information completely describes the beam since they are axially symmetric. For the pyramidal probe, the plot depicts the beam profile in the principle planes.
Fig. 2. Beam profile pattern to 100 cm for an unfocused piston source probe at 8 MHz.

Fig. 3. Beam profile pattern to 100 cm for a conical axicon probe at 8 MHz.
Fig. 4. Beam profile pattern to 100 cm for a pyramid axicon probe at 8 MHz.

Fig. 5. Beam profile pattern to 100 cm for a hyperbolic probe at 8 MHz.
The probes which do provide some degree of focusing, behave quite similar to one another, all having a more extended depth of field than the unfocused piston probe. None of the above geometries had a significant advantage over a broad frequency range, namely 0.25 to 16 MHz.

In addition to various radiating geometries, the Gauss-Hermite beam model is suited to describing radiation patterns that are non-uniform. For example, Figs. 8 and 9 compare a standard 1/4 inch piston source probe with a piston radiator whose radiation pattern approximates a Gaussian function [6] both generated at 16 MHz. The Gaussian probe exhibits a remarkable uniformity in its beam profile over a broad range of frequencies and is well suited as a probe design to be used in conjunction with the unipolar ultrasonic instrumentation.
SUMMARY

The Gauss-Hermite beam model is a useful tool to model ultrasonic probe radiation geometries and patterns. Axicon, Pyramidal, and the conic sections, hyperbolic, parabolic, and elliptical probes provide some degree of focusing. A Gaussian design was found to produce a clean uniform focused beam over a broad bandwidth.

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REFERENCES


