SCATTERING FROM AN INTERFACE DEFECT
BETWEEN FIBER AND MATRIX

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INTRODUCTION

For quantitative ultrasonic nondestructive evaluation of fiber reinforced materials, it is of great importance to understand scattering process of elastic waves by an interface defect between the fiber and matrix. In practice, the composite material is reinforced by a number of fibers embedded in the matrix. For simplicity of the mathematical modelling, however, we take a single fiber having an infinite length and a uniform cross section included in an elastic solid of infinite extent. The scattering problem of a plane time-harmonic wave by a local interface defect between the fiber and matrix is considered.

The scattering problem is divided into the following two problems; a) the scattering problem by a fiber with no interface disbond and b) the scattering problem by a local interface defect. The former problem a) is a quasi two-dimensional problem and can be solved using a boundary element method with Green’s function due to a spatially harmonic line load [1]. The wave field obtained in the problem a) is substituted into a free field for the problem b). The problem b) is solved as a local three-dimensional problem, because the free field is already known and scattering effect from the defect is confined in the vicinity of the local disbonding area. Thus, the discretization for 3-D boundary elements in the problem b) is carried out only for the small area around the interface defect.

SCATTERING BY A FIBER WITH NO INTERFACE DEFECT
— QUASI 2-D PROBLEM

We first consider the scattering problem of an incident elastic wave by a fiber with no interface disbond as shown in Fig. 1. The fiber, which is infinitely long and
has a uniform cross section, is subjected to an oblique incidence of a plane
time-harmonic wave \( \vec{u}_{I/Q2D} \) under the angle \( \psi^I \) with the longitudinal axis (say \( x_3 \)) of the fiber.

If there is no defect on the interface, the geometrical configuration is independent
of the coordinate \( x_3 \). Since the incident wave is a plane wave, furthermore, the wave
system will be translationally invariant with respect to \( x_3 \). Hence, all wave fields have
the terms \( \exp(idx_3) \) in common, where \( d = k' \cos \psi^I \) and \( k' \) is the wave number along
the propagating direction of the incident wave, and the problem can be formulated as
a quasi two-dimensional one. For example, the displacement \( \vec{u}^{Q2D}(\vec{X}) \) in the matrix
region can be represented by \( \vec{u}^{Q2D}(\vec{X}) = \vec{u}_{0}^{D}(x) \exp(idx_3) \), where \( \vec{X} \) is a
two-dimensional position vector defined by \( \vec{X} = (x_1, x_2) \).

Let \( U^{Q2D}_{ik}(\vec{X}, \vec{Y}) \) be the displacement Green’s function due to a spatially harmonic
line load. Its physical meaning is the displacement in the \( i \)-direction at the point \( \vec{X} \)
due to a line load in the \( k \)-direction applied at \( x_1 = y_1, x_2 = y_2 \) with the harmonic
intensity \( \exp(-idx_3) \). \( U^{Q2D}_{ik}(\vec{X}, \vec{Y}) \) can be expressed as follows:
\[
U^{Q2D}_{ik}(\vec{X}, \vec{Y}) = U^{D}_{ik}(\vec{X}, \vec{Y}) \exp(-idx_3)
\]  
(1)

where the explicit expression for \( U^{D}_{ik}(\vec{X}, \vec{Y}) \) is given in the paper by Li et al. [1].
Since the wave fields \( u^{D}_{ik}(\vec{X}) \) and \( U^{D}_{ik}(\vec{X}, \vec{Y}) \) are independent of the coordinate \( x_3 \),
applying the reciprocal theorem to these wave fields in the matrix domain in the
\( x_1x_2 \)-plane yields the following boundary integral equation.

\[
\frac{1}{2} \vec{u}_{i}^{D}(\vec{X}) = u^{D}_{i}(\vec{X}) + \int_{\partial S} \Gamma^{2D}_{ik}(\vec{X}, \vec{Y}) \vec{T}_{k}^{D}(\vec{Y}) d\vec{Y} - p.v. \int_{\partial S} T^{2D}_{ik}(\vec{X}, \vec{Y}) \vec{u}_{k}^{D}(\vec{Y}) d\vec{Y}
\]  
(2)

where \( \partial S \) denotes the counter line of the fiber-matrix interface \( S \) in the \( x_1x_2 \)-plane.
Here, \( \vec{T}^{D}_{ik}(\vec{Y}) \) and \( T^{2D}_{ik}(\vec{X}, \vec{Y}) \) are traction components corresponding to the
displacement fields \( \vec{u}^{D}_{k}(\vec{X}) \) and \( U^{D}_{ik}(\vec{X}, \vec{Y}) \), respectively, and \( u^{D}_{i}(\vec{X}) \) is the
displacement of the incident wave in the \( x_1x_2 \)-plane.

A similar boundary integral equation can be obtained for the wave fields in the
fiber domain as follows:

\[
\frac{1}{2} \vec{u}_{i}^{D}(\vec{X}) = \int_{\partial S} \vec{U}^{2D}_{ik}(\vec{X}, \vec{Y}) \vec{T}_{k}^{D}(\vec{Y}) d\vec{Y} - p.v. \int_{\partial S} T^{2D}_{ik}(\vec{X}, \vec{Y}) \vec{u}_{k}^{D}(\vec{Y}) d\vec{Y}
\]  
(3)

where the bar represents the wave fields in the fiber domain.
Since the interphases layer between the fiber and matrix is usually very thin, it is reasonable to approximate the interphase by distributed springs with normal and tangential stiffness, $K_N$, $K_T$ and $K_H$. Here, the subscripts $N$, $T$ and $H$ indicate the components in the normal direction and two orthogonal tangential directions on the interface, respectively. We have the interface conditions as follows:

$$t_i^{2D} = -t_i^{2D} = K_{ij} \cdot (u_j^{2D} - u_j^{D})$$

where $K_{ij}$ are the components of a tensor with spring constants given by

$$\vec{K} = \sum_{\alpha=N,T,H} \vec{\varepsilon}_\alpha \otimes \vec{\varepsilon}_\alpha K_\alpha.$$  

Here, $\vec{\varepsilon}_\alpha$ is the unit vector in the $\alpha$-direction. Eqs. (2) and (3) are combined by using the interface conditions (4). After discretization, the system of the boundary integral equations may be obtained in the following matrix form [2]:

$$\begin{bmatrix} \frac{1}{2}I + \vec{T}^{2D} + \vec{U}^{2D} \cdot \vec{K} & -\vec{U}^{2D} \cdot \vec{K} \\ -\vec{U}^{2D} \cdot \vec{K} & \frac{1}{2}I + \vec{U}^{2D} + \vec{U}^{2D} \cdot \vec{K} \end{bmatrix} \begin{bmatrix} \vec{u}^{2D} \\ \vec{u}^{D} \end{bmatrix} = \begin{bmatrix} \vec{u}^{1/2D} \\ \vec{0} \end{bmatrix}$$

where $I$ is the identity matrix.

**SCATTERING BY A LOCAL INTERFACE DEFECT — 3-D PROBLEM**

The second problem is the scattering problem by a local interface defect as shown in Fig. 2. The total displacement $u_i^{3D}(\vec{x})$ and $u_i^{3D}(\vec{x})$ in the matrix and fiber may be expressed as the sum of the quasi 2-D solutions and the scattered waves, i.e.,

$$u_i^{3D}(\vec{x}) = u_i^{Q2D}(\vec{x}) + u_i^{S/3D}(\vec{x}),$$

$$u_i^{3D}(\vec{x}) = u_i^{Q2D}(\vec{x}) + u_i^{S/3D}(\vec{x}).$$

Since the scattering effect by a fiber is included in the quasi 2-D solutions $u_i^{Q2D}$ and $u_i^{Q2D}$, $u_i^{S/3D}$ and $u_i^{S/3D}$ are related to the scattering by a local interface defect only.

Using the fundamental solution $U^{3D}_{ik}(\vec{x}, \vec{y})$ for a 3-D full space, the following boundary integral equation can be obtained for the scattered wave field in the matrix domain.

$$\frac{1}{2} u_i^{S/3D}(\vec{x}) = \int_S U^{3D}_{ik}(\vec{x}, \vec{y}) t_k^{S/3D}(\vec{y}) dS_y - p.v. \int_S T^{3D}_{ik}(\vec{x}, \vec{y}) u_k^{S/3D}(\vec{y}) dS_y$$

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where \( t_i^{S/3D} \) and \( T_{ik}^{3D} \) are traction components corresponding to the displacement fields \( u_i^{S/3D} \) and \( U_{ik}^{3D} \), respectively. Similarly, we have the boundary integral equation for the scattered wave field in the fiber:

\[
\frac{1}{2} u_i^{S/3D}(\vec{x}) = \int_S \vec{U}_{ik}^{3D}(\vec{x},\vec{y}) t_i^{S/3D}(\vec{y}) dS_y - p.v. \int_S \vec{T}_{ik}^{3D}(\vec{x},\vec{y}) u_k^{S/3D}(\vec{y}) dS_y. \tag{10}
\]

Since the fiber has infinite length, the surface of integration in eqs.(9) and (10) also extend to infinity, which is not suitable for a numerical procedure. It is, however, clear that due to geometrical attenuation, the scattered wave fields show the asymptotic behaviors of \( \vec{u}^{S/3D}(\vec{y}) \) and \( \vec{u}^{S/3D}(\vec{y}) \to 0 \) as \( |\vec{y}| \to \infty \). Hence, in the numerical implementation, the surface \( S \) in eqs.(9) and (10) can be replaced by the finite surface \( \tilde{S} \) which covers the local interface defect and its surrounding area.

It is assumed that the traction free conditions are satisfied on the interface defect \( \tilde{S}^D \), i.e.,

\[
t_i^{3D}(\vec{x}) = 0 \quad \text{for} \quad \vec{x} \in \tilde{S}^D. \tag{11}
\]

On the other part of the interface, \( S \setminus \tilde{S}^D \), the following spring contact conditions hold true.

\[
t_i^{3D}(\vec{x}) = -t_i^{3D}(\vec{x}) = K_{ij} \cdot (u_j^{3D} - u_j^{3D}) \quad \text{for} \quad \vec{x} \in S \setminus \tilde{S}^D. \tag{12}
\]

Substituting eqs.(7) and (8) into eqs.(9) and (10) and using the interface conditions given by eqs.(11) and (12), the discretized boundary integral equations may be expressed in the matrix form:

\[
\begin{align*}
\begin{bmatrix}
\vec{y} \in S^D & \vec{y} \in \tilde{S} \setminus S^D & \vec{y} \in S^D & \vec{y} \in \tilde{S} \setminus S^D \\
\vec{x} \in S^D & \vec{x} \in \tilde{S} \setminus S^D & \vec{x} \in S^D & \vec{x} \in \tilde{S} \setminus S^D
\end{bmatrix}
& \begin{bmatrix}
\frac{1}{2} \vec{I} + \vec{T}^{3D} \\
\frac{1}{2} \vec{I} + \vec{T}^{3D} \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\vec{u}_{3D}^D \\
\vec{u}_{3D}^D \\
\vec{z}_{3D}^D \\
\vec{z}_{3D}^D
\end{bmatrix} \\
& \begin{bmatrix}
\frac{1}{2} \vec{I} + \vec{T}^{3D} \\
\frac{1}{2} \vec{I} + \vec{T}^{3D} \\
\vec{K} \\
\vec{K}
\end{bmatrix}
\begin{bmatrix}
\vec{u}^{Q2D} \\
\vec{u}^{Q2D} \\
\vec{Z}^{3D} \\
\vec{Z}^{3D}
\end{bmatrix} \\
& \begin{bmatrix}
\vec{I} + \vec{T}^{3D} \\
\vec{I} + \vec{T}^{3D} \\
\vec{I} + \vec{T}^{3D} \\
\vec{I} + \vec{T}^{3D}
\end{bmatrix}
\begin{bmatrix}
\vec{L}^{Q2D} \\
\vec{L}^{Q2D} \\
\vec{L}^{Q2D} \\
\vec{L}^{Q2D}
\end{bmatrix}
\end{align*}
\]

\[
= \begin{bmatrix}
\frac{1}{2} \vec{I} + \vec{T}^{3D} \\
\frac{1}{2} \vec{I} + \vec{T}^{3D} \\
\vec{K} \\
\vec{K}
\end{bmatrix}
\begin{bmatrix}
\vec{a}^{Q2D} \\
\vec{a}^{Q2D} \\
\vec{Z}^{3D} \\
\vec{Z}^{3D}
\end{bmatrix} - \begin{bmatrix}
\vec{U}^{3D} \\
\vec{U}^{3D} \\
\vec{L}^{Q2D} \\
\vec{L}^{Q2D}
\end{bmatrix} \begin{bmatrix}
\vec{Q}^{2D} \\
\vec{Q}^{2D} \\
\vec{Q}^{2D} \\
\vec{Q}^{2D}
\end{bmatrix}. \tag{13}
\]

NUMERICAL RESULTS

We consider a cylindrical fiber with the radius \( a \) subjected to a plane P wave with the incident angles of \( \psi = \pi/2 \). The interface between the fiber and matrix has a disbonding area \( S^D \) of \( |x_3| < a \) and \( |\theta - \pi| < \theta^D/2 \), which is shown by a shadow zone in Fig. 3. Here, \( \theta^D \) is the central angle of the unbonding sector. In the numerical procedure, the fiber-matrix interface expanding to infinity is truncated at \( \tilde{x}_3 = 5a \). The finite cylindrical interface of \( |x_3| < 5a \) including the disbonding area is divided into 480 quadrilateral isoparametric elements with 744 nodes. The material constants are given by

\[
\frac{E}{\rho E} = 4, \quad \rho/\rho = 1, \quad \nu = \nu = 0.3 \tag{14}
\]
where $E$, $\rho$ and $\nu$ are the Young’s modulus, the mass density and the Poisson’s ratio in the matrix, respectively, and the variables with bars denote the fiber properties.

Spring constants on the bonding interface are chosen as $aK_\alpha/\mu = 1000$ ($\alpha = r, \theta$ and $z$), where $\mu$ is the shear modulus in the matrix. The spring constants used here are so large that the fiber is almost perfectly bonded with the matrix on $S \setminus S^D$.

**Displacements on the Interface**

Fig. 4 shows the absolute values of the displacements on the interface subjected to vertical incidence of a plane P wave ($\psi^I = \pi/2, \theta^I = \pi$). The incident wave number $ak^I$ is 0.535. The disbonding area $S^D$ is the cylindrical surface of $|z| < a$ and $0 < \theta \leq 2\pi$ (i.e., $\theta^D = 2\pi$). As seen in Figs. 4 (a) and (b), the displacements consist of large swells with no change along the cylindrical axis and small ripples generated near the unbonding area. The former large amplitudes represent the effect of both the incident wave and the scattering by a fiber without any disbond. The latter small disturbances are generated by the scattering from the local interface defect. Since no displacement in the $z$-direction is generated in the quasi 2-D problem for the vertical incidence of a plane P wave, only small amplitudes due to the scattering from the local disbond can be seen in Fig. 4 (c).

To make clear the distribution of the scattered waves by the interface defect, the difference between the 3-D solution and the quasi 2-D solution, namely, the scattered wave field $u^{S/2D}$, is plotted in Fig. 5. It is shown that $u^{S/2D}$ is distributed on the local area around the interface defect. This fact proves that the truncation procedure of the infinite interface $S$ in the numerical analysis is reasonable.

**Scattered Far Fields**

We here derive the far-field expression for $u^{S/2D}$. The integral expression for $u^{S/3D}$ in the matrix domain can be obtained as follows:

\[
u^{S/3D}(\vec{x}) = \int_S U^{3D}_{ik}(\vec{x}, \vec{y}) t^{S/3D}_k(\vec{y}) dS_y - \int_S T^{3D}_{ik}(\vec{x}, \vec{y}) u^{S/3D}_k(\vec{y}) dS_y
\]

\[
= \int_S U^{3D}_{ik}(\vec{x}, \vec{y}) \{t^{3D}_k(\vec{y}) - t^{Q2D}_k(\vec{y})\} dS_y
\]

\[
- \int_S T^{3D}_{ik}(\vec{x}, \vec{y}) \{u^{3D}_k(\vec{y}) - u^{Q2D}_k(\vec{y})\} dS_y
\]

where eq.(7) is used. Introducing the far-field approximation of $|\vec{x} - \vec{y}| \approx |\vec{x}| - \hat{\vec{x}} \cdot \vec{y}$
Figure 4. Displacement amplitudes (a) $|u_\varphi^{3D}|$, (b) $|u_\theta^{3D}|$ and (c) $|u_\phi^{3D}|$ on the cylindrical interface subjected to vertical incidence of a plane $P$ wave with the wave number $ak^f = 0.535$. The disbonding area is the cylindrical surface of $|z| < a$ and $0 < \theta \leq 2\pi$ (i.e., $\theta^{D} = 2\pi$).

\[ (\hat{x} = \frac{x}{|x|}) \] into $U_{ik}^{3D}$ and $T_{ik}^{3D}$, we have the far-field expression for $u^{S/3D}$ as follows [3]:

\[ u^{S/3D}(\hat{x}) \approx \sum_{\alpha=PSV,SH} \Omega^{S/3D}_\alpha(\hat{x}) \frac{\exp(ik_\alpha|x|)}{4\pi|x|} \quad (16) \]

where $\Omega^{S/3D}_\alpha(\hat{x})$ represents the scattered pattern of the far-field $\alpha$-wave. For example, $\Omega^{S/3D}_P$ may be written as
Figure 5. Displacement amplitudes (a) $|u_{x}^{S/3D}|$, (b) $|u_{\theta}^{S/3D}|$ and (c) $|u_{z}^{S/3D}|$ on the cylindrical interface. The other conditions are the same as in Fig. 4.

\[
\Omega_{P}^{S/3D}(\hat{x}) = \frac{1}{\mu} \left( \frac{k_P}{k_S} \right)^2 \int_{S} \hat{x}_k \{t^{3D}_{k} (\hat{y}) - t^{Q2D}_{k} (\hat{y})\} \exp(-ik_P \hat{x} \cdot \hat{y}) dS_y \\
+ ik_P \left( \frac{k_P}{k_S} \right)^2 \int_{S} \left( \sqrt{\frac{2\nu}{1-2\nu}} n_k(\hat{y}) + 2\hat{x}_k n_j(\hat{y}) \hat{x}_j \right) \\
\times \{u^{3D}_{k} (\hat{y}) - u^{Q2D}_{k} (\hat{y})\} \exp(-ik_P \hat{x} \cdot \hat{y}) dS_y
\]  

(17)

where $k_P$ and $k_S$ are the wave numbers of P and SV waves, respectively, and $n_j$ is the $j$-component of the normal vector on $S$. 

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Figure 6. Variations of $|\Omega_{PV}^{S/3D}|$ and $|\Omega_{SV}^{S/3D}|$ in the $x_1x_2$-plane, scattered by the interface disbands with the central angle $\theta^D = 2\pi$, $\pi$, $2\pi/3$ and $\pi/3$. The incident plane P wave has the incident angles of $\psi^I = \pi/2$ and $\theta^I = \pi$ and the wave number $ak^I = 0.535$.

Fig. 6 shows the variations of $|\Omega_{PV}^{S/3D}|$ and $|\Omega_{SV}^{S/3D}|$ in the $x_1x_2$-plane, scattered by the interface disbands with the central angle $\theta^D = 2\pi$, $\pi$, $2\pi/3$ and $\pi/3$. The incident angles of a plane P wave are $\psi^I = \pi/2$ and $\theta^I = \pi$ and its wave number is $ak^I = 0.535$. As a whole, a larger interface defect produces larger scattered amplitude.

Fig. 7 shows the variations of the backscattered amplitude and phase of $\Omega_{PV}^{S/3D}$ with the incident circumferential angle $\theta^I$. The incident waves are plane P waves with the wave number $ak^I = 0.535$ and 1.7, and the central angle $\theta^D$ of the unbonding sector is $\pi$.

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REFERENCES

