APPLICATION OF VOLUME-INTEGRAL MODELS TO STEAM GENERATOR TUBING

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INTRODUCTION

The nuclear power industry faces the serious challenge of convincing a skeptical public and regulatory agencies that it can operate safely and efficiently. Nondestructive evaluation (NDE) plays a significant role in this task, and computer modeling is playing a significant role in NDE. The industry now realizes the value of using such modeling to replace expensive experimental tests, as well as to design equipment, and interpret results. Eddy-currents have a traditional place in the inspection of steam generator tubing, and the industry seeks improved tools for such inspections. In this paper, we describe progress in developing a general axisymmetric model that will be part of the volume-integral code, VIC-3D\(^1\). This model will be capable of analyzing tubes with tube supports and rolled-expansion transition zones. Features such as magnetite, sludge, etc., will be included, and materials may be either ferromagnetic or non-magnetic. The model described in this paper will include only differential (or absolute) bobbin coils. Flaws can be of three types: (1) axisymmetric (such as circumferential rings), (2) the usual thin, axially-oriented, crack that is part of VIC-3D's present library, and (3) user-defined flaws, such as inter-granular attack (IGA).

In the next generation of VIC-3D, which is planned for 1995, most of these restrictions will be relaxed, and the model will include non-axisymmetric tube configurations.

SOME TYPICAL PROBLEMS IN STEAM GENERATOR TUBING

There are a number of rather complicated geometries that appear in the inspection of steam generator tubing by means of eddy currents. Figures 1 to 3 illustrate several of them. Figure 4 illustrates a number of different flaws that must be modeled.

The model must not only contend with these geometries, but it must deal with ferromagnetic bodies as well.

\(^1\)VIC-3D is a registered trademark of Sabbagh Associates, Inc.

We start with Maxwell’s equations

\[ \nabla \times \mathbf{E} = -j\omega \mathbf{B} \]
\[ \nabla \times \mathbf{H} = j\omega \mathbf{D} + J^{(e)} . \]

Now \( \mathbf{H} = \mathbf{B}/\mu(r) = \mathbf{B}/\mu_h + \mathbf{B}/\mu(r) - \mathbf{B}/\mu_h = \mathbf{B}/\mu_h - \mathbf{M}_a \), where \( \mu_h \) is the host permeability, and \( \mathbf{M}_a \) is the anomalous magnetization vector. Thus the second of Maxwell’s equations may be written

\[ \nabla \times \mathbf{H} = j\omega \mathbf{D} + J^{(e)} + \nabla \times \mathbf{M}_a , \]

which makes clear that the Amperian current, \( \nabla \times \mathbf{M}_a \), is an equivalent anomalous electric current that arises because of the departures of the magnetic permeability of the workpiece from the host permeability, \( \mu_h \). \( J^{(e)} \), on the other hand, is an electric current that includes the anomalous current
that arises due to differences in electrical conductivity; \( J^{(e)} = \sigma_h E + (\sigma(r) - \sigma_h)J \).

The problem is axisymmetric, which means that \( E, J, J_a, \) and \( \nabla \times M_a \), have only a nonzero \( \phi \)-component, whereas \( M_a \) has \( z \)- and \( r \)-components. Hence, we have as the solutions of (1) and (2)

\[
E(r, z) = E^{(i)}(r, z) + a_\phi 2\pi \int_{\text{flaw}} G^{(ee)}_{\phi\phi}(r, z; r', z') J_a(r', z') r' dr' dz' + a_\phi 2\pi \int_{\text{flaw}} G^{(ee)}_{\phi\phi}(r, z; r', z')(\nabla \times M_a)_\phi
\]

\[
B(r, z) = -\frac{1}{j\omega} \nabla \times E
\]

\[
= B^{(i)}(r, z) - \frac{1}{j\omega} \nabla \times a_\phi 2\pi \int_{\text{flaw}} G^{(ee)}_{\phi\phi}(r, z; r', z') J_a(r', z') r' dr' dz' - \frac{1}{j\omega} \nabla \times a_\phi 2\pi \int_{\text{flaw}} G^{(ee)}_{\phi\phi}(r, z; r', z')(\nabla \times M_a)_\phi r' dr' dz' .
\]

\( E^{(i)} \) and \( B^{(i)} \) are the incident fields produced by the exciting coil. The curl operation divided by \( -j\omega \) in (3) defines a magnetic-electric operator.

Upon dividing \( E \) by the anomalous conductivity, \( \sigma_a(r) = j\omega(\hat{\epsilon}(r) - \hat{\epsilon}_h) \), we get the anomalous current, \( J_a \). Similarly, \( B(r) = \frac{\mu(r)\mu_h}{\mu(r) - \mu_h} M_a(r) \). Hence, when we make these substitutions in (3), we get the coupled integral equations for axisymmetric, ferromagnetic problems:

\[
\frac{J_a(r, z)}{\sigma_a(r, z)} = E^{(i)}(r, z) + 2\pi \int_{\text{flaw}} G^{(ee)}_{\phi\phi}(r, z; r', z') J_a(r', z') r' dr' dz' + 2\pi \int_{\text{flaw}} G^{(ee)}_{\phi\phi}(r, z; r', z')(\nabla \times M_a)_\phi r' dr' dz'
\]

\[
\frac{\mu(r, z)\mu_h}{\mu(r, z) - \mu_h} M_a(r, z) = B^{(i)}(r, z) - \frac{1}{j\omega} \nabla \times a_\phi 2\pi \int_{\text{flaw}} G^{(ee)}_{\phi\phi}(r, z; r', z') J_a(r', z') r' dr' dz'
\]

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Figure 4: Illustrating a variety of anomalies ('flaws') in steam-generator tubes.

\[-\frac{1}{j\omega} \nabla \times \mathbf{a}_\phi 2\pi \int_{\text{flaw}} G^{(\text{ee})}_{\phi\phi}(r, z; r', z')(\nabla \times M_a)_{\phi} r' r'' dz' dz\]

We have dropped the vector components from the electrical variables; the scalar equation in (4) is the $\phi$-component of the electric field integral equation. The scalar Green function that appears in (4) is computed in cylindrical coordinates following the method that we described in [1], specialized to axisymmetric problems.

**DISCRETIZATION: METHOD OF MOMENTS**

Because $M(r, z) = M_r(r, z) a_\phi + M_z(r, z) a_z$, it follows that $(\nabla \times M)_\phi = \partial M_r/\partial z - \partial M_z/\partial r$. Therefore, in order for $M$ to belong to $H(\text{curl})$, the space of vector functions with bounded curls, it follows that $M_r$ must be everywhere differentiable with respect to $z$, and $M_z$ must be everywhere differentiable with respect to $r$. An expansion of the following form will satisfy these criteria:

\[M^{(r)}_a(r, z) = \sum_{in} M^{(r)}_{inm} \pi_{1m}(r) \pi_{2m}(z)\]

\[M^{(z)}_a(r, z) = \sum_{in} M^{(z)}_{inm} \pi_{1m}(r) \pi_{2m}(z)\]

where $\pi_{1m}$ is the unit pulse function, which starts at the $m$th node, and $\pi_{2m}$ is the triangular function, which starts at the $m$th node. $J_a(r, z)$, on the other hand, can be expanded in pulse functions

\[J_a(r, z) = \sum_{in} J_{inm} \pi_{11}(r) \pi_{1m}(z)\]

because there are no derivatives appearing in (4)(a), except for $\nabla \times M_a$. We proceed to discretize (4), assuming that $\nu(r, z) = \frac{\mu(r, z)\mu_h}{\mu(r, z) - \mu_h}$ and $\sigma_a(r, z)$ are constant in each cell. The result, after taking moments of (4)(a) is

\[E^{(i)}_{lm} = \frac{r^{(i)}_{z} + r^{(-)}_{z}}{2} \delta r \delta z J_{lm} - \sum_{LM} G^{(ee)}_{lm,LM} J_{LM} - \sum_{LM} G^{(em)(r)}_{lm,LM} M^{(r)}_{LM} + \sum_{LM} G^{(em)(z)}_{lm,LM} M^{(z)}_{LM}\]
where
\[
E^{(i)}_{im} = \int \int E^{(i)}(r, z) \pi_{11}(r) \pi_{1m}(z) r dr dz
\]
\[
G^{(ee)}_{im,LM} = 2\pi \int \int r dr dz \pi_{11}(r) \pi_{1m}(z) \int \int G(r, z; r', z') \pi_{11}(r') \pi_{1M}(z') r' dr' dz'
\]
\[
G^{(em)}_{im,LM}(r) = \frac{G^{(ee)}_{im,LM} - G^{(ee)}_{im,LM+1}}{\delta z}
\]
\[
G^{(em)}_{im,LM}(z) = \frac{G^{(ee)}_{im,LM} - G^{(ee)}_{im,LM+1}}{\delta r}
\]

The discretized version of (4)(b) is
\[
B^{(i)(r)}_{lm} = \frac{\delta r r^{(+)\text{r}} - r^{(-)\text{r}}}{2} \sum_{M} Q^{(r)}_{mM} M^{(r)}_{LM} + \sum_{LM} G^{(me)(r)}_{im,LM} J_{LM} + \sum_{LM} G^{(mm)(rr)}_{im,LM} M^{(r)}_{LM} - \sum_{LM} G^{(mm)(rz)}_{im,LM} M^{(z)}_{LM} + \sum_{LM} G^{(mm)(zz)}_{im,LM} M^{(z)}_{LM},
\]
\[
B^{(i)(z)}_{lm} = \delta z \sum_{L} Q^{(z)}_{iL M} M^{(z)}_{LM} - \sum_{LM} G^{(me)(z)}_{im,LM} J_{LM} - \sum_{LM} G^{(mm)(zr)}_{im,LM} M^{(r)}_{LM} + \sum_{LM} G^{(mm)(zz)}_{im,LM} M^{(z)}_{LM} ,
\]

where the tri-diagonal matrices are symmetric, and have the following non-zero entries:
\[
Q^{(r)}_{mM} = \int \nu(r^{(-)}, z) \pi_{2M}(z) \pi_{2m}(z) dz
\]
\[
= \frac{\delta z}{6} \begin{cases} 
\nu_{im} & \text{if } M = m - 1 \\
2(\nu_{im} + \nu_{im+1}) & \text{if } M = m \\
\nu_{im+1} & \text{if } M = m + 1
\end{cases}
\]
\[
Q^{(z)}_{iL} = \int \nu(r, m \delta z) \pi_{2L}(r) \pi_{2z}(z) r dr
\]
\[
= \delta r^{2} \begin{cases} 
\nu_{im} \left( \frac{1}{12} + \frac{l}{6} \right) & \text{if } L = l - 1 \\
\nu_{im} \left( \frac{1}{4} + \frac{l}{3} \right) + \nu_{i+1,m} \left( \frac{10}{24} + \frac{l}{3} \right) & \text{if } L = l \\
\nu_{i+1,m} \left( \frac{1}{4} + \frac{l}{6} \right) & \text{if } L = l + 1
\end{cases}
\]

The first entry in each of these matrices is the lower diagonal, the second the main diagonal, and the third the upper diagonal.

The other matrices are given by
\[
B^{(i)(r)}_{lm} = \frac{-1}{j \omega} \frac{E^{(i)}_{lm} - E^{(i)}_{lm+1}}{\delta z}
\]
\[
B^{(i)(z)}_{lm} = \frac{1}{j \omega} \frac{E^{(i)}_{lm} - E^{(i)}_{lm+1}}{\delta r}
\]
\[
G^{(me)(r)}_{im,LM} = \frac{1}{j \omega} \frac{G^{(ee)}_{im,LM} - G^{(ee)}_{im,LM+1}}{\delta z}
\]
\[
G^{(mm)(rr)}_{im,LM} = \frac{G^{(me)(r)}_{im,LM} - G^{(me)(r)}_{im,LM+1}}{\delta z}
\]

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\[ G^{(mm)}(rz) = \frac{G^{(me)}_{lm,LM} - G^{(me)}_{lm,L+1M}}{\delta r} \]
\[ G^{(me)}_{lm,LM} = \frac{1}{j\omega} \frac{G^{(ee)}_{lm,LM} - G^{(ee)}_{l+1m,LM}}{\delta r} \]
\[ G^{(mm)}(zr) = \frac{G^{(me)}_{lm,LM} - G^{(me)}_{lm,LM+1}}{\delta z} \]
\[ G^{(mm)}(xz) = \frac{G^{(me)}_{lm,LM} - G^{(me)}_{lm,L+1M}}{\delta r} . \]  

(11)

SOME RESULTS WITH VIC-3D

The result of a typical VIC-3D calculation is shown in Figure 5. In the top half of the figure, we illustrate the inspection of a steam generator tube.

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Figure 5: Response of a flaw in the outer wall of an Inconel 600 tube.
made of Inconel 600 (a nonmagnetic metal). The flaw is a narrow axial notch that is located on the outer part of the tube. We assume that a tube support exists, and that the flaw lies in the vicinity of the support.

In the bottom half of the figure, we show the computed impedance-plane display of the response of the flaw to a differential bobbin probe, when the flaw is well away from the support, and when the flaw is in the middle of the support region, well away from its edge. Clearly, the coupling of the flaw to the probe is diminished by the presence of the support. Modeling, such as this, allows one to gain insight into the inspection process, and to determine a priori detectability of a flaw to a probe.

REFERENCES