A study of the effects of self-regulation on the global properties of disk galaxies

Daniel Carlton Smith

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A study of the effects of self-regulation on the global properties of disk galaxies

by

Daniel Carlton Smith

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Astrophysics

Major Professor: Curtis J. Struck

Iowa State University
Ames, Iowa
2001

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has met the dissertation requirements of Iowa State University

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For the Major Program

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For the Graduate College
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Late-type spiral galaxies share many universal properties. One school of thought proposes that this universality arises from common initial conditions at the formation of these galaxies. The initial conditions are frozen out once the disks have formed, thus accounting for common structure among disk galaxies. However, observations over the last decade suggest that most galaxies have had one or more encounters with other galaxies and that these interactions disturb their structures significantly. One solution to this paradox suggests that disks have a preferred hydrodynamic state and that some processes regulate these disks to that state. This self-regulation may occur during the initial disk formation and carry on through the life of the disk bringing it back to its preferred state even after an interaction has pushed it far from equilibrium.

The interstellar medium of disk galaxies experience broad spectrum heating from supernovae, stellar winds and intense UV fluxes from young star clusters that drive turbulent flows and produce multiple thermal phases. Star formation processes from which these young stars arise are regulated by the heat and exchange of phases that they produce. While these star formation processes are effective locally, the overall thermohydrodynamic self-regulation must act globally to account for the large scale universal structure observed in disks. Study of these global regulatory processes is an important step to understanding the formation and evolution of large scale structure in disk galaxies.

This dissertation describes our analytic and computational thermohydrodynamic models of gas disks with star formation feedback. The models suggest a number of results that are in accord with observation, as well as some novel predictions. The analytic model suggests the existence of opposing radial flows and a difference in rotational velocity between cold clouds in the midplane and warm and hot gas above and below the midplane. The heating and cooling balance in the analytic model also requires a star formation rate that is similar to the Schmidt Law. The computational models produce steady states with spiral structures that depend on the amount and location of the star formation.
CHAPTER 1 INTRODUCTION

1.1 Properties of Disk Galaxies

1.1.1 Global Properties

Galaxies are generally categorized by their shape or morphology. The Hubble classification scheme (Hubble 1926, Sandage 1961) is widely used to label galaxies by their morphology. Elliptical galaxies range in shape from spherical to oblate and contain mainly old Population II stars. Lenticular or S0 galaxies are spheroidal with a flat component at their outer rims. Flat or disk-shaped galaxies, called spiral galaxies because of the spiral pattern observed on their faces, are labeled Sa, Sb, or Sc, depending on the size of their central bulge and how tightly wound their spiral arms appear. Figure 1.1 shows an example of a typical face-on Sc galaxy. Barred spirals are labeled SBa through SBc and contain a bar shape at the center with the spiral arms starting at either end of the bar. Spiral galaxies, barred or not, are generally gas-rich and have both newer Population I and older Population II stars. Luminous spiral galaxies continue to form stars in their disks. The gross features of a late-type spiral galaxy, called “late-type” because of its position on the right side of the Hubble tuning fork (see Figure 1.2), include a stellar disk containing mainly Population I stars, a gas disk, a spheroidal bulge containing both young Population I and older Population II stars, a spherical stellar halo comprised mainly of globular clusters (also Population II), and a larger spherical dark halo, the constituents of which is the topic of much speculation and research. Figure 1.3 shows an edge-on schematic of a disk galaxy. The Milky Way, home sweet home, is a typical spiral galaxy, probably an Sbc (Binney & Merrifield 1998 pg. 171). This study is concerned with the structure of late-type disk galaxies like our own Milky Way. We will discuss disk galaxies exclusively through the rest of this dissertation unless otherwise noted.

The Milky Way contains on the order of $10^{12}$ stars, most of them existing in the disk and central bulge, while some are contained in the globular clusters which orbit the Galactic Center in the halo (see Kroupa, Tout, & Gilmore 1993). The majority of these stars have masses much smaller than that of our own Sun while very few have masses much larger. Despite this, the Galaxy weighs in with a mass
of approximately $7 \times 10^{11} \, M_\odot$ ($M_\odot$ means solar mass) (Kulessa & Lynden-Bell 1992). While some of this mass is due to the stars, gas and dust, the bulk of the mass is not visible to us through direct observation. The total mass of the Galaxy is determined by studying the kinematics of the material we can see, yet the mass calculated in this way is much larger than the amount of light we detect implies. Similar determinations can be made from observations of external galaxies as well, with similar results (e.g. Burstein & Rubin 1985). This mysterious invisible mass has been called dark matter, and it is believed that the Galaxy and all external galaxies sit in the centers of huge, dark halos. While the gravity of the dark halo will dominate the bulk motion of material within the disk, the disk's self-gravity and thermal and turbulent pressures play significant roles in the stability and structure of the disk.

The visible light we see from an external galaxy is a combination of all the light generated from the multitude of stars contained within it. Since the spatial extent of a galaxy can be resolved (unlike stars which appear as point sources) we measure a surface brightness which is expressed in magnitudes per square arcsecond which corresponds to luminosity per unit area. A measure of the total luminosity of a galaxy is the surface brightness integrated over the visible area of the galaxy. The absolute blue magnitudes of disk galaxies range from $-15$ to $-23$ and masses range from $10^8 \, M_\odot$ to $10^{12} \, M_\odot$ (Carroll & Ostlie 1996, p. 1010).
Figure 1.2 The Hubble Tuning Fork: Elliptical galaxies are called early-type galaxies because they are placed on the left while spirals are called late-type galaxies because they appear on the right of the diagram. There is no evidence for an evolutionary progression from left to right. It is more likely that the progression is in the opposite direction. Figure is from Hubble (1958).

1.1.2 Universal Properties

The surface density of nearly all disks vary with radius in the same way (references). This profile is traditionally approximated by a negative exponential; however, gas surface densities also fit a $1/r$ form as well (Struck-Marcell 1991). Theoretical studies by Kennicutt (1989, 1990, 1989a) suggest that the surface density of gaseous disks are regulated to a $1/r$ profile in the case of constant circular velocities, which is common and is discussed below. (The central bulges of spirals typically follow the $R^{1/4}$ law observed in ellipticals.) Determination of the volume density profile depends then on the thickness of the disk. Generally, the scale height of gas disks increase slowly with radius, an effect known as flaring.

Kinematic studies of disks show other universal features. The circular velocity of gas and stars remains roughly constant over a large range of distances from the center (see Figure 1.4) (Rubin, Thonnard, & Ford 1978, Rubin, Burstein, Ford, & Thonnard 1985). These so-called flat rotation curves are consistent with a mass distribution that increases linearly with radius. This is not consummate with the observation that most disks have mass densities that fall off quickly with radius. The solution to this discrepancy is the postulation of the dark matter halo.

Another universal (and useful) property of disks is the relationship found by Tully & Fisher (1977) between the maximum circular speed and the total luminosity of a galaxy. This relationship, while differing between Hubble types, has a small amount of scatter and holds for well nearly all known
luminous disk galaxies. At its core, this relationship is a link between total mass and rotation speed, which seems intuitive at first glance. However, considering that about 90% of the mass in any given galaxy is not luminous, but rather is totally undetectable except by its gravitational effects on visible matter, this relationship becomes mysterious. It requires the mass to light ratio of all disk galaxies to be the same, even though most of the mass exists in a spherical halo, not in the spinning disk, and radiates no light at all.

Tully & Fisher (1977) also noted a correlation between the maximum circular speed and the linear size of late-type galaxies and Freeman (1970) demonstrated a correlation between linear size and luminosity of spiral galaxies. Koda, Sofue & Wada (2000) have shown that spiral galaxies are distributed in an elongated plane in the three-dimensional parameter space of size, circular speed, and luminosity and that two dimensional projections of this plane account for a good deal of the scatter in the pair-wise correlations discussed above. This fundamental plane of spirals suggests that two physical parameters account for the differences between late-type galaxies. While it is generally accepted that the two parameters are the mass and angular momentum of the disks, simple virial considerations do not produce
Figure 1.4 Flat rotation curves of several spiral galaxies. Figure is from Rubin, Thonnard, & Ford (1978)

the correct slope or zero-point for the pair-wise correlations. Elizondo et al. (1999) have demonstrated that star formation feedback affects the slope of the Tully-Fisher relationship and that models with feedback considerations produce the correct slope while non-feedback models do not.

1.2 The Interstellar Medium (ISM)

1.2.1 Composition

The space between the stars in our Galaxy and external galaxies is filled with a rarefied gas and dust called the interstellar medium or ISM. The ISM is mainly composed of hydrogen and helium with a small percentage made up of "metals", anything heavier than helium. Molecular hydrogen, H₂, and other molecules exist in dense clouds where the temperature is low enough that they do not dissociate (see Spitzer 1978). H₂ is hard to detect because it has no dipole moment, so CO is often used to trace the position and abundance of H₂. Silicate dust grains and PAHs are formed in the cool outer atmospheres of red giant stars and are spread throughout the disk. Dust is detected by its IR emission and by its extinction and reddening effects.
1.2.2 Multiple Phases

The existence of multiple components in the ISM has been observed for some time. Temperatures ranging over six orders of magnitude are found in the ISM, and densities range over just as many orders of magnitude (see references in van der Hulst 1997). McKee & Ostriker (1977) suggested that the ISM is well described by three different thermal phases with varying densities and temperatures that exist in pressure balance. The three components of the McKee & Ostriker model are: a cold condensed phase, a warm diffuse phase, and a hot intercloud phase. While there are more than three phases observed, this model is a useful basis on which to interpret the general properties observed in the ISM.

Giant molecular clouds exist mainly in a very thin disk with a scale height of a few hundred parsecs. The warm neutral medium, WNM, (the Lockman layer of our Galaxy) contains small HI clouds and diffuse material and has a scale height that is significantly thicker than the colder component (Dicky & Lockman 1990. Malhotra 1995, and Haynes & Broeils 1997). A warmer component still is observed in Hα and other emission lines (see Reynolds 1996, Rand 1997a). This is known as the warm ionized medium (WIM) or diffuse ionized medium (DIM) also called the Reynolds layer of our own Galaxy. This component overlaps with the WNM but extends to greater scale heights above the disk. HII regions are observed around young OB stars where the strong UV flux has ionized a region, called the Strömgren sphere (see Spitzer 1978), around the hot stars. These HII regions are embedded in the molecular gas and dust that served as stellar birthing places for these newly formed stars. They indicate regions of recent star formation. Another phase of hot gas with a temperature of about $10^5$ K has been observed in the X-ray and radio continuum. Only recently has a hot corona of gas with a temperature of $2\times10^6$ K and a height of 8 kpc above the midplane been directly verified by Chandra observations of the external galaxy NGC 4631 (Wang et al. 2001).

1.3 Star Formation

Stars form out of the ISM when gravity overpowers thermal and turbulent pressure in overdense regions causing their collapse. The births, lives, and deaths of stars generate energy in the form of radiation and mechanical work and inject this energy back into the ISM from whence they came. The main contribution of this energy injection comes from the most massive stars which are also the shortest lived and least abundant. All elements heavier than lithium are formed by nuclear processes that occur in stars or supernovae and are fed back into the ISM, as well. Younger stars tend to have higher metallicities since the ISM from which they formed has been enriched with metals by previous
generations of stars.

1.3.1 Gravitational Instabilities

Star formation is generally confined to the disk in the overdense regions of the spiral arms. Stars are formed in the cores of molecular clouds where densities can become quite large. Jeans seminal work on homogenous, isotropic collapse due to gravitational instabilities is still the basis of the theory of when and how gas collapses into clouds and when and how these clouds collapse to form stars (see Spitzer 1978). Toomre (1964) and others have studied gravitational instabilities in the geometry of a flat rotating disk (Binney & Tremaine 1994). The so-called Q criterion for instabilities in a rotating disk is a measure of balance between thermal pressure, shear and gravity.

Schmidt (1959) proposed an empirical relationship between the star formation rate (SFR) and the gas density which has been borne out and is known as the Schmidt Law. This law seems to hold for most star forming galaxies. The Schmidt Law states that the SFR is proportional to the gas density to some power, n, where n is about 2. More recently Kennicutt (1998a) has measured a value for n of 1.4 ± 0.15 from a sample of 61 normal spirals and 36 star-burst galaxies. An earlier value of 1.3 from Kennicutt (1989) has been used to incorporate star formation in previous numerical models of disk galaxy formation (e.g. Mihos and Hernquist 1994). Any prescription for including star formation in analytic or numerical models should at least, on average, reproduce the Schmidt law.

1.3.2 Regulatory Feedback Processes

The energy fed back into the ISM from star formation comes mainly from the massive stars in the form of UV flux, strong stellar winds, and supernovae explosions which send shock waves through the ISM. Turbulent dissipation of mechanical energy from winds and supernovae contributes to the feedback process. This energy feedback can in turn cease or instigate more star formation in nearby regions of space, thus regulating the amount of star formation that occurs. This is believed to be how self-regulation operates and many examples of local models of this behavior in the ISM appear in the literature.

Many mechanisms of self-regulation based on the intrinsic stability properties of local star-cloud systems in disks have been suggested. In particular, models with cloud buildup by collisional agglomeration and disruption by star formation activity have been popular, since they often yield (Schmidt-type) SFRs with power-law dependence on local gas density (e.g., Scalo & Struck-Marcell 1984, Struck-Marcell & Scalo 1987, Dopita 1990, Dopita & Ryder 1994). Paravano has extensively investigated how star for-
mation can be regulated by the thermal conversion processes that operate between warm diffuse phases and the small cool clouds (1988, 1989, also see Franco & Shore 1984, Diaz-Miller, Franco, & Shore 1998, and Bertoldi & McKee 1997 for related models). With reasonable approximations he finds that these processes also produce a Schmidt Law SFR.

Kennicutt has pointed out that gas densities in star forming disks are close to their critical density such that Q is nearly unity through a large fraction of the disk. He suggests that the surface density is regulated to keep it near critical. If Q becomes too small the disk becomes more unstable and star formation increases until the feedback energy decreases the cool gas surface density and Q increases. If Q is too large star formation slows or halts and gas is allowed to cool and condense until the surface density increases again. (1989, 1998a)

1.4 Objectives

The objectives of this study are as follows:

1. To develop dynamic, quasi-steady state analytic and numerical models of late-type disks that include turbulent energy sources from star formation to test the hypothesis that the global structure of galaxy disks are self-regulated through star formation feedback.

2. To test the hypothesis that a preferred steady state exists as an explanation of the universal properties of disk galaxies.

3. To determine the effects of different star formation feedback implementations on the global properties of our numerical models.

4. To compare the properties of our numerical models with properties of our analytic model and to help determine the relationship between some parameters within the analytic model.

1.5 Dissertation Organization

This dissertation is organized into six chapters. Chapter 1 is the introduction and Chapter 2 is a paper published in the Astrophysical Journal describing our analytic model. Chapter 3 is a description of the numerical method used to produce our numerical models. Chapter 4 presents the results from our numerical model simulations and Chapter 5 is a discussion of those results. Chapter 6 is a general summary. Appendix A is an appendix to the paper presented in Chapter 2. Appendix B contains the FORTRAN code added to Hydra to implement the star formation feedback.
CHAPTER 2 SIMPLE MODELS FOR TURBULENT SELF-REGULATION IN GALAXY DISKS

A paper published in the Astrophysical Journal

Curtis Struck ¹, and Daniel C. Smith ²

Abstract

Supernova explosions, and winds and energetic photon fluxes from young star clusters drive outflows and supersonic turbulence in the interstellar medium in galaxy disks, and provide broad spectrum heating which generates a wide range of thermal phases in the gas. Star formation, the source of the energy inputs, is itself regulated by heating and phase exchanges in the gas. However, thermohydrodynamic self-regulation cannot be a strictly local process in the interstellar gas, since galaxy disks also have a nearly universal structure on large scales.

We propose that turbulent heating, wave pressure and gas exchanges between different regions of disks play a dominant role in determining the preferred, quasi-equilibrium, self-similar states of gas disks on large-scales. In this paper we present simple families of analytic, thermohydrodynamic models for these global states, which include terms for turbulent pressure and Reynolds stresses. In these model disks star formation rates, phase balances, and hydrodynamic forces are all tightly coupled and balanced. The models have stratified radial flows, with the cold gas slowly flowing inward in the midplane of the disk, and with the warm/hot phases that surround the midplane flowing outward.

The models suggest a number of results that are in accord with observation, as well as some novel predictions, including the following. 1) The large-scale gas density and thermal phase distributions in

¹Dr. Struck wrote the bulk of the text for this publication.
²Dr. Struck and I jointly developed this analytic model. I experimented with different forms of the equations of motion and equation of state to include turbulent viscosity and found the solutions to the model.
galaxy disks can be explained as the result of turbulent heating and spatial couplings. 2) The turbulent pressures and stresses that drive radial outflows in the warm gas above and below the disk midplane also allow a reduced circular velocity there. This effect was observed by Swaters, Sancisi & van der Hulst in NGC 891, a particularly turbulent edge-on disk. The models predict that the effect should be universal in such disks. 3) Since dissipative processes generally depend on the square of the gas density, the heating and cooling balance in these models requires a star formation rate like that of the Schmidt Law. Conversely, they suggest that the Schmidt Law is the natural result of global thermohydrodynamical balance, and may not obtain in disks far from equilibrium.

2.1 Introduction: Interstellar Gas in Galaxy Disks

2.1.1 Phase Structure and Turbulence

The recognition that supernovae, massive star winds, and other impulsive energy inputs are important heat sources for the interstellar medium (ISM) in galaxies, and that they generate the warm and hot phases, was a turning point for the theory of the ISM in galaxies (e.g., McKee & Ostriker 1977). Studies of giant expanding gas shells in our galaxy (e.g., Heiles 1984), and other nearby disk galaxies showed that the energies required to produce these structures are much greater than those produced by a single supernova or the winds of its stellar progenitor (see reviews of Brinks 1990, van der Hulst 1996). However, young clusters of hot stars and their multiple supernova do produce sufficient energy to make these supershells (e.g., Mac Low, McCray & Norman 1989, Norman & Ikeuchi 1989, Tenorio-Tagle, Rozyczka, & Bodenheimer 1990, and Tomisaka 1992). With this realization the theory of superbubbles, breakout, chimneys and large-scale outflows developed quickly, and joined the older theory of galactic fountains (see Shapiro & Field 1976, Bregman 1980, Cox 1981, Shapiro & Benjamin 1991, and Schulman, Bregman, & Roberts 1994) in contributing to our understanding of turbulent ISM heating and the disk-halo connection.

At about the same time, observational discoveries on the nature of several components of the ISM led to a improved understanding of its overall phase structure (see the reviews in the book of van der Hulst 1997). One important component is the warm neutral medium, WNM, or the Lockman layer in our galaxy. This component consists mostly of small HI clouds, and diffuse (cirrus) material distributed in a substantially thicker disk than the cold component (e.g., Dickey & Lockman 1990, Malhotra 1995, and Haynes & Broeils 1997). A second component is the warm ionized medium (WIM), or diffuse ionized medium (DIM), or the Reynolds layer in our galaxy, which is observed in Hα and other emission lines
(e.g., Reynolds 1996, Rand 1997a). These two media are continuous and overlapping, though the WIM extends to greater scale heights above the disk than the WNM.

Both components have been studied in detail in our galaxy (see the review of Dickey & Lockman 1990, and the recent HI study of Malhotra 1995), and are probably common constituents of late-type galaxies (see van der Hulst 1997). The line emission of the WIM makes it the easier component to study in other galaxies; it is observed in: 1) dwarf irregular galaxies (e.g., Hunter & Gallagher 1997), 2) edge-on disk galaxies, especially the vigorously star-forming object NGC 891 (Howk & Savage 1997, Rand 1996, 1997a, 1998, Swaters, Sancisi & van der Hulst 1997), 3) large-scale outflows from nuclear starburst galaxies (e.g., Lehnert & Heckman 1996), and 4) nearby disk galaxies at arbitrary inclinations (Wang, Heckman, & Lehnert 1997). These studies also confirm the association of the various warm-to-hot components (henceforth, collectively WHM) with star formation regions, superbubbles, and large-scale outflows. This association, in turn, suggests connections with the still hotter ISM components observed in X-rays and radio continuum.

The question of how the extensive WHM is heated (and how the WIM is ionized) in galaxies that are not experiencing extensive starbursts is not entirely answered. Strong impulsive energy sources are quite localized, while the WHM is not, in either our galaxy or others (Rand 1996, Wang, et al. 1997). UV photofluxes can transfer energy over long distances, and cosmic rays and magnetic fields can also contribute to the pressure support. However, a variety of evidence indicates that other sources are needed to heat the WHM over most scales. The evidence includes the following: 1) fountain and superbubble models, with a good deal of mechanical energy injection, are very successful in accounting for the characteristics of the hot halo gas (Shapiro & Field 1976, Bregman 1980, Li & Ikeuchi 1992, McKee 1993), 2) an extra source of support beyond that associated with random motions of typical clouds is needed to support the Galactic HI (WNM) layer (Malhotra 1995), 3) the “disturbed” WIM in external galaxies seems to require additional energy source (Rand 1997a, 1998, Wang et al. 1997), 4) observations of Faraday rotation in our galaxy (see summary in Minter & Spangler 1996) demonstrate the existence of turbulence on small scales in the diffuse ionized gas, and models suggest that turbulent heating is important on those scales (Minter & Spangler 1997, Minter & Balser 1997). Turbulence, generated by the impulsive sources and propagated by (magneto)acoustic waves and mass flows may be the missing ingredient on the intermediate scales, as well as the large and small scales.

The idea that interstellar cloud structure is turbulent is decades old (see Larson 1979, Scalo 1987 and references therein). Hydrodynamic models of local regions of the ISM with heating and cooling sources included clearly illustrate the development of turbulence (e.g., Rosen & Bregman 1995, Passot, Vazquez-
Chappell & Scalo (1997) argue that clouds themselves are multifractal manifestations of the interstellar turbulence. Elmegreen (1997) agrees that interstellar clouds have a fractal structure, and further proposes that the "holes and gaps" that make up the intercloud medium are the result of turbulent heating rather than "clearing" by supernova explosions. Norman and Ferrara (1996) calculate that the turbulent energy injection into the interstellar gas is characterized by a very broad band spectrum, and that the general turbulent pressure may exceed the thermal pressure by 1-2 orders of magnitude. Thus, there is increasing evidence that \textit{turbulence supplies a large part of the heating needed to maintain the continuous range of phases in the ISM, and much of the pressure support}. This principle is the central assumption on which the models described below are based.

2.1.2 Large-Scale Structure

Heating and cooling processes in the interstellar gas generally have characteristic length scales of less than a kiloparsec or so. These local thermal processes co-exist with global regularities in the structure of gas-rich disks like the nearly universal surface density profiles of the cold gas and stellar components. The profile forms of these components are often described as a negative exponential functions of radius in the disk, though the gas surface density is also well described as having a $1/r$ form (e.g., Struck-Marcell 1991). Kennicutt's (1989, 1990, 1998a) influential studies of star formation (henceforth SF) in galaxy disks showed that the neutral gas surface density varies with radius in such a way that it is always nearly equal to the radially dependent threshold density for gravitational instability. These results revived earlier suggestions that local gravitational instabilities are needed to assemble the massive clouds where star clusters are formed. The threshold surface density also varies as $1/r$ if the circular velocity is a constant, independent of radius, as observed in many disks.

It is widely believed that the disk gas surface density profile is a result of initial conditions, and the disk formation process. The reader is referred to the papers of Steinmetz & Müller (1995), Dalcanton, Spergel, & Summers (1997), and Mo, Mao, & White (1998) for recent discussions of disk formation and early evolution. However, as noted above, thermal and turbulent pressure forces, as well as gravity and centrifugal force, are important in the WHM, and this makes it less likely that initial profiles are "frozen out" in these media. The effects of turbulent pressure on the large-scale structure of the WHM, have not yet received much attention. In the context of the secular evolution of the cold gas, Struck-Marcell (1991) pointed out that a $1/r$ surface density profile was required to maintain the conditions of hydrostatic equilibrium in the disk with minimal transport by shear viscosity (see below). The same arguments apply to the WHM, and more strongly, because of its shorter sound-crossing timescale. This
leads us to the hypothesis that in any disk where the heating by SF activity is sufficient to maintain and cycle a large fraction of the gas through the WHM, turbulent hydrodynamic forces will regulate the gas surface density to the $1/r$ profile. Furthermore, the recent work of Martin (1998 and references therein) suggests that substantial cycling rates are quite plausible. This hypothesis provides a second pillar on which the models below are built.

Dopita (1985) described one of the first models of global self-regulation by star formation. His model was based on the assumption of equipartition between turbulent and thermal pressures. In the models derived below we will consider cases where the turbulent pressure exceeds the thermal pressure in the WHM phases, as suggested by Norman and Ferrara. This is analogous to the situation in the cold cloud ensemble, where cloud random velocities are supersonic. We will, however, assume pressure balance between different thermal phases.

Many mechanisms of self-regulation based on the intrinsic stability properties of local star-cloud systems in disks have been suggested. In particular, models with cloud buildup by collisional agglomeration and disruption by SF activity have been popular, since they often yield (Schmidt-type) SFRs with power-law dependences on local gas density (e.g., Scalo & Struck-Marcell 1984, Struck-Marcell & Scalo 1987, Dopita 1990, Dopita & Ryder 1994). Paravano has extensively investigated how star formation can be regulated by the thermal conversion processes that operate between warm diffuse phases and the small cool clouds (1988, 1989, also see Franco & Shore 1984, Diaz-Miller, Franco, & Shore 1998, and Bertoldi & McKee 1997 for related models). With reasonable approximations he finds that these processes also produce a Schmidt Law SFR.

Silk (1997, and references therein) has argued for a somewhat different star formation law, that includes a dependence on local shear, and has described self-regulation and a derivation of the Tully-Fisher relation in a model based on this law. Kennicutt (1998a,b) finds that the global properties of star formation in galaxies are consistent with both the Schmidt and Silk/Wyse phenomenological laws.

There are several difficulties with many of these approaches to self-regulation. The first is that most require some arbitrary phenomenological assumptions, i.e., about SF or cloud collision rates, equipartition, or constraints on the ambient pressure. The second is that most are local in the sense that they do not include the hydrodynamic couplings to adjacent regions. On the other hand, the so-called chemodynamical models are based on a large-scale hydrodynamical treatment. For example, Samland, Hensler, & Theis (1997 and references therein) have recently presented two-dimensional hydrodynamical models with three stellar components and two discrete gaseous phases. Inevitably, there are many uncertain parametrizations in the couplings between the components, which limits the predictive power.
of the models. Moreover, they do not include the effects of turbulent stresses in the intercloud medium, which we feel are essential to an understanding of the large-scale structure. The primary role of these stresses distinguish the models below from most previous ones.

2.2 Global Analytic Models with Multiphase Turbulence

2.2.1 Densities and Radial Velocities

A good conceptual model of gas dynamics and star formation in galaxy disks requires a quasi-static solution of multiphase hydrodynamic equations that include the important thermohydrodynamic forces of self-regulation. Ideally, the model should be simple enough to allow a clear understanding of both the structure of individual disks and the universal relations between disks. As a step towards this goal we here introduce a model of a two-component ISM described by cylindrically symmetric hydrodynamic equations, with turbulent stress terms. The two components are a cold isothermal (cloud) phase and a mean WHM described by a locally adiabatic equation of state (see below). This two-phase model is a minimal description of the multiphase interstellar gas, but it is sufficiently complex to capture many interesting behaviors, as we will see below. We will describe a simple analytic version of the model here, but the model is readily generalizable to a continuous range of phases, albeit with much increased complexity in the equations. We plan to develop that generalization in a later paper. In this paper we do not consider the effects of magnetic forces or cosmic rays separately from the thermal or turbulent pressures.

As part of the definition of a "quasi-static" disk, we will assume that the mass exchanges between phases balance locally. Then the time-independent mass continuity equations are of the form,

$$\Sigma_i v_{ri} r = \text{constant}, \quad (2.1)$$

where $\Sigma_i$ is the phase component surface density and $v_{ri}$ is the component radial velocity, and with $i = c, w$ for the two phases. For convenience, we will frequently use the approximation that $\Sigma_i = \rho_i h_i$, where $\rho_i$ is the component mass density, and $h_i$ is the component scale height in the direction perpendicular to the disk.

We assume hydrostatic equilibrium in the vertical direction (i.e., perpendicular to the disk). When the disk self-gravity dominates the vertical potential gradient, then the solutions to the component hydrostatic equations give component scale heights that increase slowly with radius (as $r^{1/2}$, see Malhotra 1995 for a discussion of the application of this approximation to the Milky Way).
The zero radial flow solution, $v_{rc} = v_{rw} = 0$, to equation (1) is often assumed. However, if $\Sigma \propto 1/r$, then there is a more general family of solutions in which $v_{ri}(r) = \text{constant}$. Especially interesting are the solutions with $v_{rc} = -(\Sigma_w/\Sigma_C)v_{rw}$, describing opposed radial flows in the two components, but with zero net radial mass transport. This allows the slow inflow of high density cold (or cooling) gas to replace gas heated into the warm phases by SF, which are on average flowing outward. The flow of individual mass elements might consist of little more than a circulation within a local fountain (like a transient convective cell), with the ensemble of fountains making up the global inflow/outflow solution. Figure 1 provides a schematic view.

Because the scale-height ratio $h_c/h_w$ is small, inflow dominates in the midplane of the disk, sandwiched between warm outflows above and below. The existence of two discrete radial velocities is a consequence of the assumption of two distinct phases, and the model can be generalized to a continuous range of phases as a function of distance from the midplane with continuously stratified radial velocities. The fact that the $1/r$ profile is a good approximation to the observed cold gas distributions in large parts of many late-type galaxy disks (Struck-Marcell 1991) provides empirical support for the constant radial velocity inflow/outflow solutions as opposed to more complicated radius-dependent flows. But why should this profile be universal?

### 2.2.2 Hydrodynamic Forces and the $1/r$ Surface Density Profile

Several decades of numerical studies suggest that when the cold gas density significantly exceeds the gravitational instability threshold in all or part of a disk, then the result is the rapid development of instabilities that generate readily observable clumps. (An example of the clumping instability in disk formation is presented in Noguchi 1998.) The massive clumps would generate strong SF and heating, which would regulate the cold gas to lower densities by the processes described above. Thus, we do not expect to find densities much in excess of the threshold, which scales as $1/r$ in a flat rotation curve disk.

Hydrodynamical stability arguments also lead to a preference for the $1/r$ surface density profile, as described by Struck-Marcell (1991). For example, if the mean random velocity of the cold cloud ensemble is a constant independent of radius (i.e., the ensemble is isothermal), then only the $1/r$ profile yields a constant net pressure force between adjacent annuli in the disk. Any deviation from that profile generates a pressure gradient proportional to the deviation, which would allow the nonlinear amplification of disturbances. This is not consistent with a hydrodynamic steady state. As noted above, this argument is even stronger when applied to the WHM.
A qualitative, but very general argument, is based on the observation that there is no obvious characteristic scale (other than the scale of the rising rotation curve in the center) over the range extending from a few kiloparsec to tens of kiloparsecs in late-type disks. This suggests that the important forces within the disk all have essentially the same radial or distance scaling. If not, we would expect there to be observational signatures associated with changes in the dominant force. To make this point more definite, we write the radial momentum equation for the cold gas component as follows,

$$v_{rc} \frac{\partial v_{rc}}{\partial r} = \left( - \frac{1}{\rho_c} \right) \frac{\partial P_c}{\partial r} - \left( \frac{GM_0}{r_o} \right) \frac{1}{r} + \frac{v_{oc}^2}{r},$$  \hspace{1cm} (2.2)

where $M_0$, $r_o$ are a gravitational scale mass and radius, respectively, and $v_{oc}$ is the azimuthal velocity in the cold component. The terms on the right-hand-side represent the pressure gradient, gravitational, the centrifugal accelerations.

In equation (2) the gravitational and centrifugal accelerations both have a $1/r$ scaling. There is no power-law solution for $v_{rc}$ that yields the desired scaling for the advection term. However, when $v_{rc}$ is constant this term vanishes. Moreover, the isothermal equation of state also yields the correct scaling in the pressure gradient acceleration for any power-law density profile, though we generally expect this term to be negligible in the cold gas. Thus, the adopted density and velocity profiles make up the only self-similar family of steady solutions to equations (1) and (2).

The remaining momentum equations do not require any changes in the density and velocity scalings. The azimuthal momentum equations are described in the next section, the radial momentum equation for the WHM is,

$$v_{rw} \frac{\partial v_{rw}}{\partial r} = \left( - \frac{1}{\rho_w} \right) \frac{\partial P_w}{\partial r} - \left( \frac{GM_0}{r_o} \right) \frac{1}{r} + \frac{v_{ow}^2}{r} + \alpha_w \frac{\Delta v_T \Delta v_0}{r},$$  \hspace{1cm} (2.3)

This equation is very similar to the previous one, and in both cases the left-hand-side is zero for the constant radial velocity models. One difference is that we expect the pressure gradient term, which contains both thermal and turbulent pressures, to be significant in this component, in contrast to the cold component. The additional last term derives from an effective turbulent shear viscosity in the azimuthal direction. This term must be included because the change in specific angular momentum due to "viscous" forces changes the effective centrifugal force from what we would expect in the absence of "viscosity". Thus, in the warm gas, the non-negligible pressure gradient, and the steady input of angular momentum allow gravity to be balanced with less centrifugal force, while maintaining a constant velocity outflow.
There is a corresponding rate of decrease of angular momentum in the cool gas, but assuming that the total mass and angular momentum of the cold gas are greater than the WHM, we have assumed that this term is negligible in equation (2). We will discuss the viscous shear terms further in the next section, but we note here that the term in equation (3) contains the quantity, \( \Delta \nu_{\phi} \), which we define as \( \max(\nu_{\phi c} - \nu_{\phi w}, \Delta \nu_T) \), and \( \Delta \nu_T \) is the turbulent velocity dispersion in the WHM.

In this simple model we will assume that the viscous coefficients \( \alpha_i \) in the two phases, the nearly circular azimuthal velocities \( \nu_{\phi i} \) and the velocity dispersions are all constant. In the case of the velocities this assumption is in accord with observation, see van der Hulst (1997). The pressure gradient is due to the random motions of the cold gas elements, and not cloud internal pressures.

### 2.2.3 Viscosity and Pressure

Many processes may contribute to the viscosity in the interstellar gas, including: viscous shear within the cloud ensemble, drag against the diffuse components, enhancements of these by bars and spiral waves, turbulent and magnetohydrodynamic couplings, etc. At the same time, turbulent pressure and turbulent angular momentum transfers to the WHM can provide support against gravity, as well as driving the radial flow. As mentioned above, the support against gravity implies that the circular velocity of this medium can be less than that of the cold gas. If the two (or multiple) phases have substantially different rotation speeds, \( \Delta \nu_{\phi} \), then we expect turbulent shear viscosity between vertical layers to be the dominant viscous term. Cold gas heated by SF will be mixed into the WHM via chimneys, fountains, and bubble shells, and will add angular momentum to the WHM. At the same time, cooling lumps or filaments of the WHM, with lower specific angular momentum than the cold gas, will rain onto the midplane, decreasing the angular momentum of the cool clouds that sweep them up. The net result will be somewhat like the friction between two thin disks forced together at different speeds.

With the assumption that this is the only significant viscous term we can write the steady azimuthal momentum equation as,

\[
\frac{v_{\phi i} \nu_{\phi i}}{r} = -\alpha_i \frac{\Delta \nu_T \Delta \nu_{\phi}}{r},
\]

where \( i=c, w \) as usual, and this equation is valid for both components, though the radial velocities have different signs. The right hand term can be understood as dimensionally similar to a Navier-Stokes kinematic viscosity term of the form \(-\nu(\partial^2 \nu_{\phi}/\partial z^2)\), with a viscosity coefficient of order \( \nu \simeq \Delta \nu_T \lambda \). In this case, the "mean free path" \( \lambda \) is of order the size of a typical chimney, fountain, or the turbulent
zone around an SF region, which we assume to be roughly constant across the disk. The vertical second derivative we approximate as of order $\Delta v_0/h_w^2$, where as noted above, the WHM scale height $h_w \propto r^{1/2}$. The coefficient $\alpha_t$ is assumed to be of order unity. Although the viscous term in the above equation may look somewhat unusual, in fact, it is a variant of the usual $\alpha$-viscosity for supersonic turbulence, $\nu \propto t_{\text{visc}} v_{\text{turb}}$, with scales appropriate to this problem.

As noted above, we can understand the last term in equation (3) as a momentum source term equivalent to a reduction in the centrifugal acceleration due to azimuthal viscosity. We assume that the radial velocities are small, and so, radial viscous accelerations are negligible. The centrifugal reduction can be treated as an acceleration, which we estimate as,

$$\frac{\partial v_{r1}}{\partial t} = \frac{v_0 v_\phi}{r} = -\alpha_t \frac{\Delta v_T \Delta v_0}{r},$$  \hspace{1cm} (2.5)

where the first equality is derived on the assumption that this acceleration must have a magnitude sufficient to reduce the azimuthal velocity to zero in a time $t/v_{r1}$. The second equality results from substituting the previous equation. It is an additional simplifying assumption that the coefficient on the right-hand-side of this equation equals that of the previous equation, though we expect them to be of the same order. This term is usually neglected in viscous disk studies because $v_{r1}$ is small, but it is important here. We will also see later that $v_{rc} << v_{rw}$, so $\alpha_c << \alpha_w$, which justifies neglecting the term in equation (2).

Because of the low densities and high temperatures that characterize most of the WHM, we generally expect radiative cooling timescales to be long, so in our simple model an adiabatic equation of state is more appropriate than an isothermal one. This raises a potential problem with the scaling of the pressure gradient acceleration. However, shock heating plays an important role in determining the temperature structure of the WHM, and the frequency and intensity of the shocks varies with radius. Since shocks generate entropy, this suggests that the mean specific entropy of the WHM varies with radius, and so, the adiabat of the warm gas must also vary with radius. In this simple model we write the equation of state for the warm gas as,

$$P_{\text{w,therm}}(r) = K(r) \rho_w^\gamma.$$ \hspace{1cm} (2.6)

where $P_{\text{w,therm}}$ is the thermal pressure, $\gamma \approx 5/3$, and the adiabatic constant $K$ varies with radius. (In reality, it probably also varies with vertical height $z$.) In particular, we obtain the desired self-similar form for the pressure gradient term with a radially dependent adiabatic constant of the form, $K(r) \propto r$. If the warm gas is flowing outward, this increase in specific entropy would be the result of the cumu-
lative effects of shocks (see Appendix A). It is interesting that while the warm phase is assumed to be locally adiabatic in this simple model, the variation of pressure (and observables that depend on thermal temperature) with radius is the same as a globally isothermal gas. (Note that in terms of surface density and surface pressure we have \( \Pi = K_S \Sigma^{5/3} \), with \( \Sigma \propto r^{-1} \), \( \Pi \propto r^{-1} \), and \( K_S \propto r^{2/3} \), and again, \( \Pi \propto \Sigma \).

At this point, the structure of a minimal model is nearly completely defined by the empirically constrained \((1/r)\) surface density profiles, the adopted equations of state for thermal pressures, the continuity equations, the radial and azimuthal momentum equations, and the condition of pressure balance between phases. Using the ideal gas law to relate (total) pressure \( P_w \) to the temperature \( T_w \), we have the following equation for the inter-phase pressure balance,

\[
R \rho_w T_w (1 + \beta) = \rho_c (\Delta v_c)^2,
\]

where the \( R \) is the gas constant, \( \beta(r) \) is a correction factor for the turbulent contribution to the total pressure in the WHM, and the right-hand-side represents the pressure in random cold cloud motions (probably also turbulence dominated). For simplicity and consistency, we assume that \( \beta \) is a constant, and so, turbulent and thermal pressures scale with radius in the same way.

2.3 Vertically-Dependent Circular Velocities and the Case of NGC 891

In this model the turbulent pressure gradient can provide some support in the WHM against gravity, so the circular velocity of this medium can be smaller than that of the cold gas. Recently, Swaters, Sancisi, & van der Hulst (1997) discovered that the rotation velocities of HI gas located in a plane parallel to, but above, the midplane of the edge-on galaxy NGC 891 are less than those in the midplane by \( 25 - 100 \text{ km/s} \). Over most of the disk the velocity difference was about \( 25 \text{ km/s} \). The highest values come from a couple of points within a radius of about 6.0 kpc. In the case of these points, the gas above the midplane may have originated in an outflow from smaller radii, and thus, considerably down the solid body part of the rotation curve. If so, the large velocity offset may be due to conservation of the small angular momenta in this gas.

Swaters et al. provide several possible explanations for the general effect, including local outflows and the angular momentum effect just mentioned. They also mention the possibility of "asymmetric drift", though they are skeptical that velocity dispersions as large as suggested by the empirical formula for asymmetric drift in our galaxy are achievable. However, since galactic asymmetric drift is a phenomenon of collisionless stars on orbits with large epicycles, or quite flattened elliptical orbits, we do not believe
it is relevant unless elements of the WHM can become isolated and collisionless for large parts of their orbits.

Instead, we believe that the general velocity offset could be the result of substantial turbulence in this vigorously star-forming object (see Rand 1996, 1997a,b, 1998, and Howk & Savage 1997). We will provide a simple numerical example below of how the model can account for this phenomenon in this section. In fact, we view this phenomenon, together with the vertical radial velocity gradient, as essential predictions of the model. Thus, detailed rotation studies, like that of Swaters et al., in other nearby, edge-on disks with known "DIG" components like NGC 3079, NGC 4631, NGC 5775, NGC 4302 (Rand 1996 and references therein), and NGC 55 (Ferguson, Wyse, & Gallagher 1996, Hoopes, Walterbos, Greenwalt 1996) should provide a strong test of the model.

2.4 Scaling the Model

We can now describe how to scale the model, and provide some concrete examples. For brevity, we will not consider boundary conditions here, but simply assume an infinite radius solution. We begin by initializing several observable quantities, and also the constant radial mass transfer rate \( \dot{N} = 2\pi r h_0 \rho_r v_r \), which is not generally observed. However, this quantity is related to the global SFR, since SF activity is responsible for the viscous and pressure forces. Numerical simulations should eventually yield a relation between them. Martin's (1998) studies of dwarf galaxies found the intriguing result that "shells lift gas out of the disk at rates comparable to, or even greater than, the current galactic star formation rates." This finding lends much credibility to the idea that both radial mass transfer and phase exchanges occur in late-type disks at interesting rates.

The remaining quantities we initialize are \( \rho_c(r_0), \Delta v_c(r_0), \Delta v_T(r_0) \), and \( \beta \). In principle, these are all evaluated at a particular radius \( r_0 \). In fact, the last three are constrained to be constants, independent of radius. The parameters determining the overall gravitational potential (essentially \( \nu_{oc} \)) can be determined from observations of stellar kinematics. Estimates for the values of \( \rho_c(r_0) \) (or \( \Sigma_c(r_0) \)) and \( \Delta v_c \) can be derived from HI observations. Like the radial mass flux, the values of \( \Delta v_c \) for the cold clouds and \( \Delta v_T \) of the WHM turbulence will be determined by the SFR. Thus, we expect that for a universal (e.g., halo) gravitational potential, this family of model star-forming disks is primarily two-dimensional, with the two dominant parameters being the cool gas density at some point and the SFR.

The values of the mean WHM turbulence parameters \( \Delta v_T \) and \( \beta \) are the most difficult to evaluate. However, \( \beta \) primarily enters the equations in the combination \( (1 + \beta) / \beta \), and so, whenever \( \beta > a few \), turbulent wave pressure dominates in the WHM and the exact value of \( \beta \) does not greatly affect the
other quantities. Moreover, \( \beta \) is not independent of \( \Delta v_T \) (both are functions of the SFR). The variable \( \Delta v_T \), or a dimensionless combination like \( \Delta v_T / v_{oc} \), can be viewed as a scaling parameter of the model, like the Mach number in shock hydrodynamics. Different mean WHM values, appropriate for galaxies with different SFRs, are obtained by changing the value of this parameter, as we will demonstrate.

Given our initial parameter values, we can now derive the scaling equations for the remaining variables. First, from the condition of vertical hydrostatic balance, we get the cool gas scale height,

\[
h_c = \left( \frac{\Delta v_c^2}{\pi G (\rho_* + \rho_c)} \right)^{1/2},
\]

where \( \rho_* \) is the local star density. (The stars are assumed to have a larger scale height than the cool gas, so \( \rho_* \) is essentially constant.)

Next, we use the continuity equation to derive \( v_{rc} \), the cool gas inflow, and from equation (2) we derive the value of \( v_{oc} \)

\[
v_{oc} = \sqrt{\frac{GM_0}{r_o}}.
\]

This determines the last of the cool gas parameters, and we proceed to the remaining WHM quantities. Using equation (7) and the definition of \( \beta \), we note that,

\[
\beta RT_w = \Delta v_T^2.
\]

This equation can be solved directly for \( T_w \), but it also allows us to solve for the ratio of component scale heights. The scale height ratio equals the square root of the ratio of effective temperatures (see expression for \( h_c \) above), which includes both a thermal and turbulent part in the case of the WHM. These parts are related by the previous equation, and so, we can write,

\[
\frac{h_w}{h_c} = \left( \frac{1 + \beta}{\beta} \right)^{1/2} \frac{\Delta v_T}{\Delta v_c}.
\]

Using the temperature expression above in the pressure equation (eq. (7)), we get the related result for the component density ratio,

\[
\frac{\rho_c}{\rho_w} = \left( \frac{1 + \beta}{\beta} \right) \frac{\Delta v_T^2}{\Delta v_c^2}.
\]

The previous two equations can now be combined to give the component surface density ratio,

\[
\frac{\Sigma_c}{\Sigma_w} = \left( \frac{1 + \beta}{\beta} \right)^{1/2} \frac{\Delta v_T}{\Delta v_c},
\]
Finally, we return to the radial momentum equation (eq. 3) to determine \( v_{\text{ow}} \). The left-hand side of this equation is zero, assuming constant mean radial velocities, and for the pressure gradient term the radial dependence of the pressure in this model \( (P \propto \rho \propto r^{-3/2}) \) allows us to write,

\[
\frac{r}{\rho_w} \frac{\partial P_w}{\partial r} = -\frac{3 P_w}{2 \rho_w} = -\frac{3}{2} \left( \frac{1 + \frac{2}{3}}{\frac{2}{3}} \right) \Delta v_T^2.
\]

Then substituting from equation (4) for the \( \alpha \) term, multiplying by \( r \), and dividing by \( v_{\text{oc}}^2 \), the radial momentum equation can be written,

\[
\left( \frac{v_{\text{ow}}}{v_{\text{oc}}} \right)^2 + \left( \frac{v_{\text{rw}}}{v_{\text{oc}}} \right) \left( \frac{v_{\text{ow}}}{v_{\text{oc}}} \right) - \left[ 1 - \frac{3}{2} \left( \frac{1 + \frac{2}{3}}{\frac{2}{3}} \right) \left( \frac{\Delta v_T^2}{v_{\text{oc}}^2} \right) \right] = 0. \tag{2.15}
\]

where we have also used the approximation that \( GM_\odot/(r_0 v_{\text{oc}}^2) = 1 \). This is a Bondi-Parker, accretion/wind equation, and is quadratic in the variable \( (v_{\text{ow}}/v_{\text{oc}}) \). It is a central result of the model.

To evaluate that equation numerically, we use equation (13) for \( v_{\text{rw}} \), and the continuity equation, with a given mass flux, for \( v_{\text{rc}} \). As an example, let us assign the following representative values:

\[
M_o = 2 \times 10^{11} M_\odot, \quad r_o = 10 \text{kpc}, \quad \rho_c(r_o) = 3.0 \text{amu/cm}^3, \quad \Delta v_c = 6.0 \text{km/s}, \quad \Delta v_T = 30 \text{km/s},
\]

\( \beta = 3 \), a mass flow of \( M = 2.0 M_\odot \text{yr}^{-1} \), and a stellar density of \( \rho_* = \rho_c \)

(used for computing scale heights).

Then we derive values of

\[
h_e = 130 \text{pc}, \quad h_w = 770 \text{pc}, \quad \rho_w/\rho_c = 0.030, \quad v_{\text{rc}} = -3.4 \text{km/s}, \quad v_{\text{rw}} = 20 \text{km/s},
\]

\[
v_{\text{ow}} = 290 \text{km/s}, \quad v_{\text{ow}} = 277 \text{km/s}, \quad \text{and } T_w = 21,000 \text{K}.
\]

Note that the radial velocities are very small compared to the azimuthal velocities, and also less than the turbulent velocity dispersions. Thus, we expect radial velocities to be quite difficult to observe.

If instead of the above value for \( \Delta v_T \), we substitute a higher value of \( \Delta v_T = 50 \text{km/s} \), we obtain values of \( h_w = 1300 \text{pc}, \quad v_{\text{rw}} = 33 \text{km/s}, \quad \text{and } \Delta v_{\text{ow}} = 25 \text{km/s} \). The thicker WHM layer and the larger azimuthal velocity difference in this case are both in accord with the observations of NGC 891. The surface density of the WHM relative to that of the cool gas increases linearly with \( \Delta v_T \), consistent with the idea that the strong WHM emission in NGC 891 is the result of strong turbulence driven by the vigorous SF.

On the other hand, we noted above that Swaters et al. (1997) did not see such high velocity dispersions in their HI observations of NGC 891. However, these authors note that in the more nearly face-on galaxy NGC 6946, vertical velocities of up to 100 km/s were detected by Kamphuis & Sancisi (1993). These, and related observations, can be readily understood if the turbulent motions of HI gas in the thick disk and halo are primarily vertical. This situation is very natural if the most of the high
dispersion HI gas is either entrained in local fountains and outflows, or is in the form of "high velocity clouds" consisting of cooled halo material falling back onto the disk (e.g., Benjamin 1999, Benjamin & Danly 1997).

The impressive recent study of Thilker, Braun & Walterbos (1998) on large HI shells in NGC 2403 also provides input on this question. These authors find mean in-plane shell expansion velocities of 26 km/s, and individual cases extending up to 56 km/s. We expect that much of the turbulent energy has already been vented in these large bubbles, and that the shells are observed in a deceleration stage.

Recent optical observations also provide evidence for the vigorous turbulence required by our model. Wang, Heckman & Lehnert's (1997) spectroscopic study of the DIM in half a dozen nearby disks led them to suggest the existence of two components. The first is a 'quiescent DIM' with low ionization states and line widths of 20 – 50 km/s, versus the high ionization state, 'disturbed DIM' with line widths of 70 – 150 km/s. They suggest that the former is photoionized by diffuse O star radiation, while the latter is mechanically heated by supernovae and winds. In sum, turbulence with values of $\Delta v_T \approx 50$ km/s or greater seems in accord with recent observations, and seems to yield very reasonable model values for actively star-forming disks.

### 2.5 Star Formation Properties and Other Regularities in the Family of Models

In constructing the model above, we have implicitly assumed that the SF law above threshold is constrained by a self-regulated heating and cooling balance. In the simplest case, we assume that all the important heating processes are directly proportional to the SFR (e.g., O star photoheating and turbulent wave heating). The important cooling processes in the WHM include: 1) adiabatic cooling of high pressure gas elements, 2) radiative cooling in the mean WHM with a rate proportional to the WHM density squared, and 3) turbulent shock dissipation in the WHM, which depends on the SFR and gas density squared. The last two cooling rates generally scale with mean WHM gas density squared, and thus, require a similar density dependence in the SFR. That is, a Schmidt Law on average, albeit in density, rather than surface density. However, the processes involved are sufficiently complex that deviations from an $m=2$ density power would not be surprising.

Thus, we write the following schematic equation for the heating and cooling balance for regions above the SF threshold,
The left-hand-side of this equation represents the Schmidt Law heating. The right-hand-side is a schematic collisional dissipation term, with "cross section" $\sigma$ and "sound speed" $c$. For example, this term could represent radiative cooling in the WHM via collisional excitation, with the temperature dependence contained within the factor $\sigma c^3$. However, if we make the approximation that $c \propto \Delta v_T$, and assume that $f_{SF}$ is universal, then the equation provides a scaling for the net dissipation cross section $\sigma$ in terms of $\Delta v_T$ and $\rho_c/\rho_w$. (See Appendix A for further discussion of the constrained SFR.)

There is a substantial literature on individual interstellar heating and cooling terms on many scales (e.g., see the recent discussions of Norman & Ferrara 1996, and Ferriere 1998). Nonetheless, we still have a long way to go to fully understand the broad-band heating and cooling terms in the ISM. Any more specific formulation of the balance equation would probably require the introduction of insecure parametrizations. It would also require additional physics, beyond that included in the simple model considered here. We will not pursue these topics in this paper, but merely point out that they may be easier to study in the context of the well-defined global structure provided by the model.

Equations (8)-(15) show that the model is a simple similarity solution to the hydrodynamic equations. In the limit of small radial velocities and small turbulence (and thus little WHM component) the model must be essentially the same as the self-similar, viscous, Mestel disks studied recently by Bertin (1997) and Mineshige & Umemura (1996). (Our model also has some similarities to the one-component convective model of Waxman (1978).) The self-similar structure of the model helps to understand the universal properties of gas-rich galaxy disks, like the Tully-Fisher relation between maximum circular velocities and the total luminosities of disk galaxies (e.g., Courteau 1997 and references therein), and universal gas density profiles (see e.g., the extensive HI study of Broeils & Rhee 1997).

Eisenstein and Loeb (1996) point out that the small dispersion in the observational Tully-Fisher relation suggests the operation of a "strong feedback process" that "regularize(s) SF and gas dynamics," like the model presented here. On the other hand, Mo et al. (1998) believe that the small Tully-Fisher scatter could in fact come out of early galaxy formation processes. Even so, Eisenstein and Loeb are probably also correct if a large fraction of galaxies experience merger events subsequent to their formation. That is, while we expect galaxy collisions and mergers to disrupt the "universal" disk structure, turbulent self-regulation will re-establish it.

Some of these questions are answered by the new numerical hydrodynamical models of galaxy formation in several cosmologies by Elizondo et al. (1999). These models included multiple gas phases
and supernova feedback. The authors found that the Tully-Fisher scatter of their model galaxies was within the acceptable both with and without the feedback effects included. However, only the feedback models reproduced the correct slope of the Tully-Fisher relation, and the slope was quite sensitive to the feedback amplitude.

Another regularity, Freeman's Law, states that high surface brightness galaxies all have about the same central surface brightness, or that disk galaxies have a maximum central surface brightness (see Courteau 1996 and references therein). This too would seem to be a natural consequence of large-scale SF regulation, albeit in the central regions of the disk. In a number of nearby starburst galaxies the gas surface density continues to follow a power-law as far into the center as it can be resolved (Struck-Marcell 1991, Young et al. 1995). In many cases this may be the result of gas inflow driven by a bar component or other disturbance. In many other late-type disks the gradient in the surface density flattens to a value of about 10 solar masses per square parsec in the central, rising rotation curve region (though often with a central, molecular gas spike, see Young et al. 1995). We speculate that this latter case represents a normal quiescent state. Disks of both profile types are observed to have a comparable value of $\Sigma_c$ at the radius where the rotation curve flattens (Broeils & Rhee 1997). We conjecture that Freeman's Law may be the result of the fact that most stars form in the centers of bright, late-type disks at a rate appropriate to this gas density, and over comparable timescales.

2.6 Conclusions

The following list summarizes the properties and some probable consequences of the models presented in this paper.

1) A $1/r$ surface density profile is assumed in both thermal phases (on the basis of the Least Dissipation Principle and other arguments, see section 2.2). It is also assumed that vertical scale heights are determined by local self-gravity, and so, increase slowly with radius (as $\propto r^{1/2}$). This implies that mean volume densities scale as, $\rho_i \propto r^{-3/2}$.

2) Thus, the ratio of gas phase densities are constant across the star-forming region of the galaxy disk. The model predicts that the value of this ratio depends on the amount of turbulence, and specifically, on the parameter $\Delta v_T/\Delta v_c$.

3) We assume that the circular velocities of each gas phase are constants independent of radius. The equations of state and the assumption that all momentum equation terms have the same radial scaling implies that the remaining velocities, $v_{rc}, v_{rt}, \Delta v_T, \Delta v_c$, are also constant with radius.

4) In general, the model allows all of the velocities in each phase - azimuthal, radial, and dispersive
to have different (non-zero) values. The NGC 891 effect of different rotational velocities as a function of height above the disk (Swaters et al. 1997) is predicted to be generic in turbulent disks.

5) The model predicts a hierarchy in velocity magnitudes in each phase, i.e., azimuthal velocities $>$ velocity dispersions $>$ radial velocities. The low value of the latter will make them difficult to observe. This hierarchy is the result of the similarity equations and the simple wind/accretion equation (eqs. (8) - (15)).

6) However, even such low velocity radial flows are consistent with mass fluxes comparable to typical SFRs in late-type disks. In the absence of radial flows, we would expect gas consumption at smaller disk radii, to modify the gas density profile. Radial replenishment can prevent this and effectively distribute the consumption across the disk. The radial flow may also draw on reservoirs of gas in the non-star-forming outer disk, further increasing the global consumption time. Evolutionary effects will be considered in a later paper.

7) Similarly, galactic abundance gradients will be smoothed by the large-scale radial flows. The simple closed-box model of chemical evolution within isolated disk annuli is not appropriate in the context of these radial circulation models. However, the quantitative effects of radial flow are complicated by the fact that the flows are slow. E.g., with a radial flow velocity of order $3km/s$, the timescale for a gas element to cross a disk of radius $10kpc$ is a few billion years. Moreover, motion of a gas element will generally be partly advective and partly diffusive in this turbulent environment. Thus, the typical smoothing time may be only a little less than the typical disk age.

8) The model requires a balance between heating and cooling. Heating is primarily the result of SF activity, and most cooling terms depend on the second power of the gas density (assuming the constant phase balances of the model). Thus, a Schmidt Law SFR is the natural result of the thermohydrodynamical balance.

9) If the Schmidt Law (or a related parametrization, such as that of Silk and Wyse, see Silk 1997) is in fact a consequence of global hydrodynamic self-regulation, then there are some immediate corollaries. Perhaps, the most important is that transient, burst modes of SF are possible when disturbances take galaxies far from the regulated state. Thus, SF phenomenologies may be very different in highly disturbed galaxies (e.g., Struck-Marcell & Scalo 1987), or during galaxy formation. On the other hand, the Schmidt parametrization may be marginally valid in environments where, $r/\Delta v_T = \tau_{relax} < \tau$, with $\tau$ defined as an appropriate “age”. For example, the Schmidt Law may work in waves in both grand design and collisional galaxies if the wave crossing time is longer than $\tau_{relax}$. The same argument may be valid in the centers of major merger remnants.
10) Global regularities in star-forming disks, such as the Tully-Fisher relations and the Freeman Law, may also be the result of global self-regulation, of the type inherent in the present models. They may also be the result of formation processes, including turbulent self-regulation during formation. Continuing self-regulation is important for maintaining the global regularities and restoring them following a disturbance.

11) The model equations can readily be generalized beyond the two-phase version described here to include a continuous range of phases. More sophisticated treatments of viscosity, turbulence, heating and cooling processes can be included, much as detailed nuclear rates and opacities are included in stellar evolution models.

In sum, the hydrodynamic similarity model presented above is an attempt to bring together the essential thermohydrodynamical processes needed for a coherent conceptual picture of actively star-forming galaxy disks as self-regulated, multiphase, "dissipative structures." The basic hydrodynamic structure of each phase is much like that of an isothermal polytrope, but these are not quasi-static, equilibrium states. The model assumes that there are turbulent flows on many scales, and the WHM is more accurately viewed as a set of locally adiabatic states, with specific entropy gradients. The disk structure described by this model, with gas profiles regulated in accord with minimal dissipation and transport, and SF simultaneously sustained and moderated by slow radial flows, probably describes most late-type disk galaxies.

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Figure 2.1  Schematic illustrating local fountain flows, cooling filaments, and the global radial flows in a model disk (courtesy Julia K. Burzon)
CHAPTER 3 NUMERICAL MODEL DESCRIPTION

This chapter gives a description of the computational methods used to simulate the stellar and gas dynamics of a late-type galaxy. The computer program we used is called Hydra, an N-body adaptive particle-particle particle-mesh (AP3M) with smoothed particle hydrodynamics (SPH) code, created by the Hydra consortium (Couchman, Thomas, & Pearce 1995). Section 3.1 provides a brief overview of what is meant by N-body, AP3M, and SPH. Section 3.2 describes the creation of an initial model, including the composition, distribution, velocity dispersion and, where applicable, the density and temperature of the matter that makes up our model galaxy. Section 3.3 discusses radiative heating. The last two sections, 3.4 and 3.5, will deal with the physics and code implementation of radiative cooling and of heating due to star formation in the gaseous component of the model. It should be noted that Hydra was written and made public by Couchman, Thomas, & Pearce (1995) and does not contain any implementation of star formation heating in its public form. I developed the additional code to implement star formation heating in Hydra for the purpose of this study. The additions to the code are presented in Appendix B of this dissertation.

3.1 Techniques

3.1.1 Particle-Particle N-body

N-body simulations are used to study systems where the interaction between constituents are too complicated to use two- or three-body approximations. A planetary system like our own solar system can be modeled with fair accuracy by considering only two-body interactions between the sun and each planet and ignoring the planet-planet interactions. A better model may be to consider three-body interactions between the Sun, Jupiter and each other planet in turn. A full N-body treatment of the problem would not increase accuracy significantly. On the other hand, modeling a galactic system having many more than nine constituents whose interactions are roughly of equal strength between any two stars or gas clouds may benefit greatly from a full N-body simulation. The properties and evolution of a system that can be treated as a collection of collisionless bodies, for instance a gas-free galaxy, are
often studied using computational techniques falling under the category of the N-body simulation. For a thorough review of N-body techniques, see "The Art of N-Body Building" by J. A. Sellwood (1987). Calculation time for direct summing of forces from each particle scales as $O(N(N - 1))$ where $N$ is the number of particles, because each of the $N$ particles must have $N - 1$ forces calculated and summed. It does not take many particles before this method becomes unwieldy. Particle-particle force calculations between many particles are possible on today's fast computers, but is only convenient up to about $10^4$ particles (Aarseth 2001). To do simulations with $N > 10^4$ we look for alternatives to particle-particle summing of forces. Two of these alternatives are the tree method (e.g., Barnes & Hut 1989) and the particle-mesh (PM) method (Hockney & Eastwood, 1981). Calculation times for tree codes scale as $O(N \log N)$, but are not discussed here.

3.1.2 Particle-Mesh

Rather than treating the total force on a particle as the sum of forces from every other particle, it is convenient to consider each particle embedded in a scalar potential field, $\Phi(r)$. The acceleration, $a(r)$, of a particle at position, $r$, is then given by

$$a(r) = -\nabla \Phi(r), \quad (3.1)$$

and $\Phi(r)$ satisfies the Poisson equation.

$$\nabla^2 \Phi(r) = 4\pi G \rho(r), \quad (3.2)$$

where $G$ is the gravitational constant.

The particle-mesh technique solves the Poisson equation on a 3D grid of cells to calculate the gravitational potential which is then used to calculate the forces at the grid points. Forces on particles are then interpolated from the values of nearby grid points. The Greens Function method is used to solve the Poisson equation. The Fourier transform of the density and the Greens function are calculated and used to determine the transform of the potential, which is then transformed back to obtain the physical potential at any grid point. Generally, the Greens function is determined from the boundary condition that $\Phi$ goes to zero at infinity; however, it becomes prudent for the sake of the numerical calculation to chose additional boundary conditions such that the gravitational force is "softened" on some small scale length, $r_s$. This is equivalent to assigning a finite size to the particles. The softening sets a limit on the spatial resolution of the force calculation. See Hockney & Eastwood (1981) for a very thorough discussion of softening and of the PM technique, in general.
Employing efficient Fast Fourier Transform (FFT) algorithms makes the potential calculation at each grid point very fast. The time it takes to do the potential calculation on the mesh depends on the number of grid points. Once the potential is solved on the grid, the force at any point on the grid, and, thus on any particle, can be determined by differencing. Therefore, calculation time for the force on each particle calculated in this way scales linearly with $N$ (Hockney & Eastwood 1981, Sellwood 1987). This allows simulations with $N > 10^4$ to be run in a reasonable amount of time.

3.1.3 Particle-Particle Particle-Mesh (or P3M)

The Hydra code uses a technique called particle-particle particle-mesh, first developed by Hockney & Eastwood (1981), for solving the forces on each particle. The particle-mesh technique, discussed above, is used to solve the long range component of the gravitational force. The mesh error on the force becomes large at distances less than the softening, $r_s$, which is typically about two grid cells. The short range gravitational force is calculated by directly summing over each particle-particle interaction for particles within $r < r_s$ of the particle for which the force is being solved. In this way, the errors on the calculated force on small length scales can be kept low while minimizing the calculation time as well. Couchman (1991) claims this method takes only "2–5 times" longer than PM, and scales as $N$, under certain conditions (one particle per mesh cell).

The P3M method uses two grids. The first grid is the normal grid of the PM scheme on which the density values and eventually the potential values are calculated and stored. The second mesh is called the chaining mesh and is coarser than the potential mesh. It has a grid spacing greater than or equal to $r_s$ and is used to locate pairs of near-neighbors for the short-range PP summing (Hockney & Eastwood 1981, pp. 268-9).

3.1.4 Adaptive Refinement

Gravitational simulations often produce a large range of densities. The P3M technique becomes much slower when large density variations form in the simulation. Dense clumps cause particles in clumpy regions to have many more near-neighbors, thus the computationally intensive particle-particle interaction calculations become more numerous. To combat this effect, Hydra also employs an adaptive refinement technique. Refinement grids are placed over the densest regions. Increasing the number of cells in a region reduces the number of near-neighbors of a particle in a dense region which in turn reduces the number of particle-particle calculations to be summed.

The general recipe for placing refinements and calculating forces is presented by Couchman (1991).
Below, I describe the major steps used in calculating forces with the adaptive refinement technique used in Hydra. First, calculate the long-range forces in the current refinement region (which at the beginning of the timestep will be the whole simulation box) using PM. Next, loop through the chaining mesh cells in the current refinement region and decide which regions require further refinement and mark the chaining cells in those regions. Add these to the list of cells already marked for refinement. Finally, calculate the short-range forces on particles in all pairs of neighboring chaining mesh cells except those pairs of cells that fall in the same refinement (see Figure 3.1). Move on to the next refinement region on the list. In this way, the forces are determined on all particles in each refinement. For simplicity, each refinement region is cubic, contains an integer number of chaining mesh cells, and never overlaps other refinements, although nested refinements are allowed. Couchman claims this adaptive scheme speeds up the code by a factor of between 10 and 20 as compared to the standard P3M code when there is heavy clustering. He also measures a 50% speed increase compared to a Barnes & Hut type of tree code. (Couchman 1991)

Figure 3.1 Chaining mesh with refinements: A cell with a capital letter contains a particle for which the particle-particle sum is being done. Particles in cells with the corresponding lowercase letter are included in the sum. The shading in each box marks the refinements. (courtesy of Julia K. Burzon)
3.1.5 Smoothed Particle Hydrodynamics

The smoothed particle hydrodynamics (SPH) technique has been used for many different astro-
physical studies ranging from the formation of the moon to rotating stars to galaxy formation and
interactions to cluster formation (e.g., Gingold & Monaghan 1977, Lucy 1977, Hernquist & Katz 1989,
& Couchman 1992, Thacker & Couchman 2000). The SPH method has a Lagrangian formulation,
meaning it is not restricted to a fixed grid. It follows a distribution of fluid elements or "particles" to
sample the properties of the gas. This saves some computation time since it does not need to bother
doing computations for regions of space that are mainly empty of gas. While not as accurate as some
Eulerian hydrodynamic methods, such as PPM (Colella & Woodward 1984), it fits into N-body gravity
codes quite naturally. SPH has been put through shock tube and adiabatic collapse tests by several
investigators (e.g., Gingold & Monaghan 1982 & 1983, Hernquist & Katz 1989, Couchman, Thomas, &
Pearce 1995).

3.1.5.1 Smoothing Kernel Basics

The SPH method uses a type of weighted average over near-neighbor particles to calculate the
properties of the gas at the position of a given particle. The average value of some physical field, $f(r)$,
is estimated by using a smoothing kernel such that

$$< f(r) > = \int W(r - r', h) f(r')dr,$$  \hspace{1cm} (3.3)

where $W(r - r', h)$ is the smoothing kernel and $h$ is the smoothing length, a characteristic scale length
of the volume over which the averaging occurs. Note that $W(r - r', h)$ should be normalized such that

$$\int W(r - r', h) dr' = 1, \hspace{1cm} (3.4)$$

and $W(r - r', h)$ should approach a delta function as $h \to 0$. According to Monte-Carlo theory (Ham-
mersley & Handscomb 1964), if the value of $f(r)$ is known only at a finite number of points, $r_1, r_2, \ldots, r_N$,
and $n(r)$ is the number density of points at $r$, then

$$< f(r) > = \sum_{j=1}^{N} \frac{f_j}{< n(r_j) >} W(r - r_j, h) \hspace{1cm} (3.5)$$

or if each particle has a mass, $m_j$, associated with it so that $\rho_j = m_j < n(r_j) >$, then
Another useful property of this formalism is the easy way in which gradients of functions can be estimated. It can be shown that

$$< \nabla f(r) >= \sum_{j=1}^{N} \frac{m_j}{\rho_j} \nabla W(r - r_j, h).$$

(3.8)

The averaging occurs over particles that are within a radius of 2h of the particle whose properties are being calculated. Note that refinement meshes used in the gravity calculation allow for a more restricted search volume when SPH is searching for near-neighbors (see Thomas & Couchman 1992). The smoothing length, $h$, is variable for each particle and is determined by the density of particles such that there are always approximately 32 near-neighbors inside the smoothing length, with a minimum length equal to the gravitational softening length and the maximum length corresponding to two grid cells. See Hernquist & Katz (1989) for a detailed discussion of this implementation of SPH.

### 3.1.5.2 Hydrodynamic Equations of Motion

The point of the simulation code is to advance the position and internal state of each particle through time. The position, $r$, of a particle is advanced in time according to what Couchman et al. (1995) call a PEC (predict, evaluate, correct) scheme, which they chose for stability and memory economy. Given that $v = \dot{r}$, $a = \ddot{r}$, and $dt$ is the timestep interval, the position of a particle at timestep $n + 1$ is determined from the values of $r_n$, $v_n$, and $a$ at the previous timestep, $n$, by the following equations:

$$r_{n+1} = r_n + v_n dt + \frac{1}{2} a(r_n') dt^2,$$

(3.10)
\[ v_{n+1} = v_n + \frac{1}{2}[a(r'_n) + a(r'_{n+1})]dt, \]  

(3.11)

where

\[ r'_{n+1} = r_n + v_n dt + \frac{1}{2}a(r'_n)dt^2. \]

The trick, then, is to determine the acceleration, \( a \), for each particle at each time. The hydrodynamic equations describe the acceleration, \( a = dv/dt \), and the changes in density, \( \rho \), pressure, \( P \), and internal energy, \( e \), with time, \( t \), and space. These equations are expressed here in the Lagrangian formulation. They are, respectively, the momentum equation, the continuity equation, the energy equation, and the equation of state.

\[ \frac{dv}{dt} = -\frac{\nabla P}{\rho} - \nabla \Phi \]  

(3.12)

\[ \frac{d\rho}{dt} + \rho \nabla \cdot v = 0 \]  

(3.13)

\[ \frac{de}{dt} = -\frac{2}{3} \epsilon \nabla \cdot v + \frac{\Gamma - \Lambda}{\rho} \]  

(3.14)

\[ P = \frac{2}{3} \rho e \]  

(3.15)

where \( \epsilon = \frac{3}{2} \frac{kT}{\mu m_H} \). Here \( \Phi \) is the gravitational potential, \( v \) is the velocity of the gas, \( T \) is the temperature, \( k \) is Boltzmann's constant, \( \mu \) is the relative molecular mass, and \( m_H \) is the mass of a hydrogen atom. The continuity equation, 3.13, is trivially handled in SPH since particle masses are conserved, i.e. Equation 3.7. The equation of state, 3.15, is adiabatic. The term \( \frac{\Gamma - \Lambda}{\rho} \) in equation 3.14 represents non-adiabatic heating and cooling processes. Hydra includes a table of cooling rates as functions of temperature and metallicity (we use solar metallicity in each model) from Sutherland & Dopita (1993) used to evaluate \( \Lambda \), but does not include any heating sources. Heating and cooling are discussed in Sections 3.3 and 3.5.

We can make use of the identity

\[ \frac{\nabla P}{\rho} = \nabla \left( \frac{P}{\rho^2} \nabla \rho \right) \]  

(3.16)

to write the pressure gradient force term in the momentum equation as

\[ \frac{\nabla P_i}{\rho_i} = \frac{1}{\rho_i} \sum_{j=1}^{N} m_j \left[ \frac{P_i}{\rho_i} \nabla W(r_{ij}, h_i) + \frac{P_j}{\rho_j} \nabla W(r_{ij}, h_j) \right], \]  

(3.17)

where \( r_{ij} = r_i - r_j \). The first term in the energy equation is determined in a similar way, where \( \nabla \cdot \vec{u}_i \) is estimated in the following way:

\[ \nabla \cdot \vec{u}_i = -\frac{1}{\rho_i} \sum_{j=1}^{N} m_j \vec{v}_{ij} \cdot \frac{1}{2} \left[ \nabla W(r_{ij}, h_i) + \nabla W(r_{ij}, h_j) \right] \]  

(3.18)

where \( \vec{v}_{ij} = \vec{v}_i - \vec{v}_j \). The gradient of \( W \) is averaged over the \( i^{th} \) and \( j^{th} \) particles to account for the fact that the smoothing length, \( h \), is different for each particle.
In order for SPH to handle shocks in the gas, an artificial viscosity term is added to equation 3.12. This is implemented in Hydra by adding a term to the pressure, \( P_i \), when the divergence of the velocity is negative, \( \nabla \cdot \vec{v}_i < 0 \).

\[
P_i \rightarrow P_i + \rho_i \left[ -c_i h_i \nabla \cdot \vec{v}_i + 2(h_i \nabla \cdot \vec{v}_i)^2 \right],
\]

where \( c_i = \sqrt{5P_i/3\rho_i} \) is the adiabatic speed of sound at the \( i^{th} \) particle. The shock handling capability of Hydra has been tested by Thomas & Couchman (1992) on one dimensional shocks in three dimensional simulations and has been shown to give correct jump conditions across a shock front with a width of about 6 particles (Thomas & Couchman 1992). More detailed shock tests are described in Couchman, Thomas & Pearce (1995).

### 3.2 Initial Conditions

Our initial galaxy model was created in several steps. First, the dark halo was created by assigning positions and velocities to 30,000 collisionless (non-SPH) particles in a spherical distribution within a simulation box corresponding to a cubic region of space 100 kpc on a side. The particles were distributed according to a King model potential (see Binnev & Tremaine 1994). The halo was allowed to relax to a near equilibrium state, i.e. the velocity dispersion became constant with time. A rotating stellar disk was created in the halo by placing another 30,000 collisionless particles into a disk with a constant particle number density profile within a radius of about 5 kpc and a 1/r density profile between 5 kpc and 19 kpc. Each particle was distributed uniformly in \( \theta \) throughout an annulus and given a circular speed calculated from the total (star + dark halo) mass within the radius of that annulus. The particles were distributed in randomly in \( z \) (height from the midplane of the disk) with a scale height equal to 0.20 times the radius of the disk. The disk-halo system was relaxed again. Hydra allows two different labels for collisionless particles so that halo particles and star particles can be distinguished. Next, the gas disk was created in the same manner as the stellar disk except with a scale height of 0.15 times the disk radius. Each particle was additionally assigned a temperature of about \( 10^4 \) K, a density value of zero, and a label distinguishing them as collisional (or SPH) particles. The true density of each gas particle would be determined by Hydra during the first timestep according to the smoothing kernel (see Equation 3.7). The final disk-halo system was allowed to relax again, without radiative cooling. Cooling due to radiation is discussed in section 3.3. Figures 3.2 and 3.4 show the face-on and edge-on structure of the initial gas disk. Note that there is no spiral structure in the initial disk. Figure 3.8 shows the temperature versus density for the gas in the initial disk after sufficient relaxation.
Table 3.1 Masses of the components of the initial cooled model expressed in units of solar masses.

<table>
<thead>
<tr>
<th>Component</th>
<th>Number</th>
<th>Particle Mass</th>
<th>Total Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Star</td>
<td>30,164</td>
<td>$3.00 \times 10^5$</td>
<td>$9.05 \times 10^9$</td>
</tr>
<tr>
<td>Halo</td>
<td>29,118</td>
<td>$6.67 \times 10^6$</td>
<td>$1.94 \times 10^{11}$</td>
</tr>
<tr>
<td>Gas</td>
<td>30,238</td>
<td>$6.00 \times 10^5$</td>
<td>$1.81 \times 10^{10}$</td>
</tr>
<tr>
<td>Combined</td>
<td>89,520</td>
<td></td>
<td>$2.21 \times 10^{11}$</td>
</tr>
</tbody>
</table>

We found that once radiative cooling was enabled, the gas cooled and collapsed quickly to a thin disk. This collapse caused a great deal of ringing in the disk and the simulation had to be run for a large number of timesteps before the ringing dissipated, allowing us to study the steady state of the model. Eventually, we decided to run the initial galaxy model for 10,000 timesteps or approximately 2 billion years with cooling enabled but no star formation heating. We also extended the dark matter halo of the initial model in order to increase the flat rotation curve region (see discussion below). Once the disk became thin and quiescent we saved this model and used it as the initial condition for the rest of the simulations, thus saving calculation time. Figures 3.3 and 3.5 show the face-on and edge-on structure of this cooled gas disk. There is now a distinct spiral structure evident in the disk. The spiral structure arises from gravitational instabilities due to the increased surface density of the gas disk. Figure 3.9 shows the temperature versus density for the new initial disk. Note that there is very little gas with temperature above $10^4$ K.

Table 3.1 shows the masses of the different components of our initial cooled model. We set the masses of each particle according to how much total mass we want to have in each component and how many particles we want to use. The more particles we include, the longer it will take the simulation to run. The size of the simulation box is 100 kpc on a side. The radius of the disk is approximately 30 kpc. A total of 90,000 particles are divided evenly among star-, halo-, and gas-type particles. Each simulation is allowed to run for 9900 timesteps, corresponding to approximately 2 billion years. One average timestep is equal to about 200,000 years. The actual value of the timestep varies with time and is limited by the smoothing length over the speed of sound (Couchman, Thomas & Pearce 1995).

### 3.2.1 Running the Simulations

The simulations were run on a Digital AlphaStation 500/333 EV5 with a clock speed of 333 MHz and 160 MB of RAM. The entire program including data is contained in 754 MB. This requires that large portions of the data be swapped back and forth from the main memory to the hard disk during
the course of a simulation run. The data swapping is handled by the Digital FORTRAN compiler and is not explicitly written into the code. Running a simulation for 9900 timesteps on the our workstation takes approximately five to seven days to complete depending on the load on the workstation. A raw data file includes the position, velocity, temperature, density, smoothing length, particle type, and clock value for each particle as well as information about the total number of particles and number of grids. Each data file represents one instant in time and requires about 4.4 MB of disk space.

3.2.2 Rotation Curve of Initial Model

Figure 3.6 shows that the warm initial disk has nearly no flat rotation region, i.e. region of constant azimuthal velocity. In order to increase the size of the flat rotation region of the initial model, we expanded the size of the dark matter halo by about 10% and ran the model for 10,000 timesteps with the extended halo and with radiative cooling enabled (see above). This had the desired affect of extending the flat rotation region in the gas disk. Figure 3.7 shows the rotation curve of the initial cooled gas disk. The rotational speed, \( v_0 \), of the initial cooled gas disk increases linearly with radius from the center to a radius of about 12 kpc where it turns over and is roughly constant out to about 25 kpc. After 25 kpc the rotational speed begins to fall linearly to the end of the disk. The rotational speed in the region between 0 and 12 kpc, or rising rotation region, increases from zero to just under 200 km/s. The flat rotation region between 12 and 25 kpc keeps that speed before it slowly decreases to roughly 150 km/s between 25 kpc and the very edge of the disk, which is about 50 kpc from the center. The decreasing rotation region occurs due to the limited size of the dark matter halo. Ideally, the dark halo would extend far beyond the disk but we are limited by the size of the simulation box. Choosing a smaller disk, while solving the rotation curve problem, would reduce the spatial resolution of the model.

3.3 Cooling

As noted in section 3.2, Hydra does allow for cooling of the gas to occur due to radiative loss. The gas is cooled according to the standard cooling curve of Sutherland & Dopita (1993) for temperatures at or above 10,000 Kelvin. It is implemented by including a table of cooling rates vs. temperature and interpolating between temperatures. Other authors have either ignored cooling below 10,000 Kelvin (e.g. Thacker & Couchman 2000, from this point forward referred to as TC) or have parameterized the cooling curve to lower temperatures, even down to 10 Kelvin (e.g. Gerritsen & Icke 1997, from this
Figure 3.2 Face-on view of the initial gas disk. Axes have units of 100 kpc.

Figure 3.3 Face-on view of the cooled initial gas disk. Axes have units of 100 kpc.

Figure 3.4 Edge-on view of the initial gas disk. Axes have units of 100 kpc.

Figure 3.5 Edge-on view of the cooled initial gas disk. Axes have units of 100 kpc.
Figure 3.6 Rotation curve of the initial gas disk. R has units of 100 kpc and $v_0$ has units of km/s.

Figure 3.7 Rotation curve of the cooled initial gas disk. R has units of 100 kpc and $v_0$ has units of km/s.

Figure 3.8 log $n$ versus log $T$ of the initial gas disk. n has units of cm$^{-3}$. T has units of K.

Figure 3.9 log $n$ versus log $T$ of the cooled initial gas disk. n has units of cm$^{-3}$. T has units of K.
point forward referred to as GI) (also see Carraro et al. 1998). In both cases, we feel the cooling rate at low temperatures are under estimated.

We have experimented with ad hoc constant cooling rates for temperatures below 10,000 K. In the end, we chose a rate that is approximately 10 times the average cooling rate between 1000 and 10,000 K given in Spitzer (1978) for a medium with an ionization fraction of $10^{-4}$. This allowed a significant fraction of gas to cool to a few thousand Kelvin in simulations with the enhanced cooling. While this cooling rate is much higher than those used by GI and TC, recent work suggests that CII cooling in the WIM is stronger than once believed (Pierini et al. 2001). Without this enhanced cooling at low temperatures, the gas does not cool much below 10,000 Kelvin in our simulations.

Limited particle mass and grid resolution of the simulation makes it difficult to create high density regions where cooling is efficient enough to create cold clouds that will eventually become star forming regions. More correctly, we cannot resolve the fragmented clumps at the cores of GMCs that will actually form stars. In reality, these clumps can be as cool as 10 Kelvin. In our simulations, we lack the resolution to see clumps on such a small scale. The smallest features we resolve are of order the size of a GMC, not its core. In our simulations, we find it impossible to cool gas below a few thousand Kelvin anywhere in the disk. This poses somewhat of a dilemma. TC handle this problem by choosing a rather high temperature of 15,000 Kelvin as a threshold condition for star formation. We suffer from a similar resolution problem: however, we achieve temperatures between 1000 K and 10,000 K in a large fraction of the gas and use temperatures below 10,000 Kelvin as our threshold (see the following section for a discussion of these thresholds). In GI, the gas can drop to temperatures in the hundreds or even tens of Kelvin. It should be assumed that they achieve this by setting these low temperatures in the initial conditions, since their cooling rate is much lower than ours—below 10,000 K. Consequently, GI do not use low temperature as a criterion for star formation, rather they use a Jeans Mass criterion. Even using a cooling rate ten times as big as GI's we still do not get gas as cold as their models produce. They are not resolving clouds smaller than a typical GMC, yet they claim to have densities so large and temperatures so small as to describe the compact cores of GMCs.

3.4 Star Formation Criteria

Many studies concern the dynamics of collapsing interstellar clouds. Jeans seminal work on the matter is still the basis of the theory of when and how gas collapses into clouds and when and how these clouds collapse to form stars (Spitzer 1978). Toomre (1964) and others have studied gravitational instabilities in the geometry of a flat rotating disk (Binney & Tremaine 1994). The so-called Q criterion
for instabilities in a rotating disk is a measure of balance between thermal pressure, shear and gravity. The interaction of shear, gravity and thermal pressure is not the whole story, though. Many local feedback mechanisms have been suggested to instigate or inhibit the collapse of clouds to stars. Some suggest that compression of gas by shock waves originating from nearby supernovae may be a dominant factor in the cloud collapse (see review by Shore & Ferrini 1994). Turbulent dissipation of mechanical energy into heat from those same shocks and from strong stellar winds may shutdown other would-be star forming clouds. Turbulence may add a non-thermal component to the pressure that fights gravitational collapse in those regions. Positive and negative feedback make the process of forming stars a messy problem. So how should we pick a simple set of rules to act as criteria for star formation to occur in an SPH particle? This was especially difficult since we cannot resolve most of this behavior. I tested a variety of different criteria to see what gave the most realistic results and we briefly summarize our results in this section. I present more detailed results in the following chapter.

Early SPH simulations of galaxy formation by Mihos & Hernquist (1994) make use of the Schmidt law to determine how much star formation is occurring in each gas particle, rather than whether it is occurring. Using this method alone assumes that star formation is happening everywhere all the time, but the star formation rate is small or large depending on the local density. Thacker & Couchman (2000) follow a similar prescription to Mihos & Hernquist (1994) but use an additional set of criteria to determine whether star formation is occurring in any given gas particle. Other authors abandon an empirical "law" determination of star formation and rely only on a physical criterion or set of criteria to establish where and when star formation occurs in the simulation (Buonomo, et al. 2000, Carraro, Chiosi & Lia 1998, GI). We use the latter method because it is based on the physics rather than on a phenomenological relationship.

When gravitational instability is the dominant factor in determining the conditions for collapse, there is a critical density criterion for the onset of star formation. We tried a density threshold that varies like $1/r^{3/2}$ corresponding to a $1/r$ surface density threshold analogous to the critical surface density as described by Kennicutt (1989). This either turned on star formation in nearly the entire disk or did nothing, depending on the normalization of the threshold profile. We then tried a constant volume density threshold, $\rho > \rho_{\text{threshold}}$. This made the central high density regions explode with star formation and left the outer disk with no star formation and no chance of condensing to high enough densities. We required that a successful model obtain star formation in the flat rotation region of the disk, but in these original density threshold models star formation only occurred in the rising rotation curve region. Since stars begin to form only in the cold, dense centers of molecular clouds, we tried...
a minimum temperature threshold. This worked a little better since we can get low temperatures to occur throughout the disk while not every particle is cold enough to fall below the threshold. Most of the gas sat at 10,000 Kelvin so using a temperature greater than this as a threshold would turn on SF in almost every particle. While using a temperature threshold instead of a density threshold allows for star formation to occur a little farther out from the center, star formation still did not occur in the flat rotation curve region. Since the temperatures and densities can vary back and forth across thresholds with timescales on the order of a few million years, particularly in the central regions of the disk, we added another criterion to avoid unrealistic effects of this rapid change. This is a time criterion which only allows particles that have maintained a low temperature (or high density) for about a free-fall time, about $10^7$ years, to form stars (see GI). This keeps the center, where the gas is quite dynamic and gas elements do not stay cold for long, from overpowering the rest of the disk.

TC use a set of criteria that includes a density threshold, convergent flow condition, temperature threshold, and a self-gravitating condition to select regions where star formation is allowed to proceed. Buonomo et al. have shown empirically that a convergent flow criterion, when used with the other criteria, has little effect on the amount of star formation in models with comparable resolution to those of TC and our own. We suggest that requiring the gas to remain in the cold, dense state for a free-fall time has a near equivalent effect to requiring a convergent flow. We also suggest that, in simulations of this resolution, limiting star formation to regions of convergent flow will mainly act to restrain star formation to the edges of spiral arms. We have found that using both temperature and density thresholds achieves this state.

Our most successful models exhibit coherent spiral structure, maintained $1/r$ like surface density profiles, and produce reasonable amounts of star formation in the flat rotation region and coincident with spiral arms (see Chapter 4). These models use the following criteria for the onset of star formation: The temperature of a particle must be below about 9000 K and stay that cold for at least $10^7$ years. Another successful model includes the above criteria and adds that the density of the particle must be above $0.14 \text{ cm}^{-3}$ and remain that high for $10^7$ years concurrent with the temperature criterion. The temperature-only model allows star formation to occur in low density regions behind spiral waves, which is unphysical. It also allows star formation to occur in the outermost part of the disk. The density-temperature model corrects both of these problems, but does not produce as much star formation (see discussion in Chapter 4).
3.5 Star Formation Heating

Star formation regions heat their surrounding medium in different ways. Young OB stars produce strong FUV radiation fields that ionize the medium nearby and heat the dusty regions outside of the ionized regions by knocking electrons off dust grains via the photoelectric effect (see Wolfire et al. 1995). OB stars also inject mechanical energy into the surrounding medium by way of strong stellar winds and supernovae. These energy inputs comprise the feedback mechanisms that affect further star formation. Modeling this feedback involves consideration of the timescale and magnitude of the energy input and the fraction that each mechanism contributes. There are several ways in which modelers have implemented energy injection into the gas in an SPH code. Radiative heating from FUV flux can be handled by adding the same amount of energy to each SPH particle equally or using a flux profile. The amount of FUV flux depends on the number of OB stars present in the disk. Mechanical heating presents another set of options. One can simply add the energy due to star formation to the star forming (SF) particle only or, distribute the energy to it and its near-neighbors via the smoothing kernel. The energy can be added directly to the internal energy value for those particles or a velocity kick can be given to the particles to simulate the mechanical aspect of the energy injection. Gerritsen & Icke (1997) discuss this at length and conclude that adding some set amount of internal energy into only the SF particle is the most stable method. We use this method with the addition of ramping up the energy input over a set amount of time until a maximum internal energy of about $10^{53}$ ergs is reached. This energy corresponds to a particle temperature of approximately $5 \times 10^5$ K. The ramp-up heating time we use is of the order of $10^7$ years. This heating begins only after the star formation criteria has been achieved for the particle. In other words, when a particle reaches a given threshold or thresholds, a clock begins counting. As long as the threshold criteria continue to stay met the clock continues to count until the clock reaches about $10^7$ years, the approximate free-fall time of a GMC. If the particle still satisfies the thresholds when the clock reached $10^7$ years, radiative cooling is turned off for the particle and the particle's internal energy is increased by 10% each timestep until the clock has counted an additional $10^7$ years or the maximum energy is reached. At this point, the particle's internal energy is held at the maximum value. Once the clock reaches the end of that time the particle's energy is no longer increased and it is allowed to cool normally. The finite heating time reflects the lifetime of the most massive stars which are responsible for the largest portion of the energy fed back into the ISM. After the last of the massive stars in a star forming SPH particle has ended its life in a supernova, the heating from that cluster of stars becomes insignificant.
CHAPTER 4 NUMERICAL MODEL RESULTS

4.1 The Models

In this section, we present the basic numerical model results and defer discussion and analysis of the models to Chapter 5. We experimented with several combinations of star formation feedback criteria resulting in over 30 different simulations. As discussed in the previous chapter, our major criteria for distinguishing acceptable models was that the star formation must occur in the flat rotation curve region and that the disk remain relatively stable. The first of couple models employed a density threshold criterion that varied as \(1/r^{3/2}\), approximately corresponding to a surface density threshold profile of \(1/r\), and energy injection corresponding to a maximum temperature of \(3 \times 10^6\) K (see Section 3.4). These models resulted in so much star formation that the disk lost a large portion of the gas to large outflows. Models with a maximum energy injection reduced to a temperature of \(5 \times 10^5\) K still produced outflows when combined with the \(1/r^{3/2}\) varying density threshold criterion. Constant density threshold models allowed for a more stable disk but only produced star formation in the central region of the disk and not in the flat rotation region. The addition of a duration criterion discussed in Section 3.4 eventually led to models with more physical behavior. We present here some of the basic results from six models that achieved star formation in the outer (flat rotation region) disk without producing unphysical outflows.

Table 4.1 lists the models and their threshold attributes. The models differ by star formation criteria or initial galaxy model. *The star formation criteria in each model included the same threshold duration timescale discussed in Section 3.4.* Feedback heating is handled the same way in all six models as described in Section 3.5. Of the six models, one model uses a temperature threshold only, one uses a density threshold only, and four models use a combination of temperature and density thresholds. Of the four temperature-density threshold models, two have different temperature threshold values, and three have the same threshold values but different gas particle masses or different star particle masses from the nominal values shown in Table 3.1. A change in particle mass corresponds to a change in initial surface density of the disk since the size of the disk is not changed. All temperature-density (TD) models have the same density threshold value as the density-only (DO) model. The temperature-only (TO)
Table 4.1 List of models.

<table>
<thead>
<tr>
<th>Model Label</th>
<th>Temperature</th>
<th>Density</th>
<th>Change in Particle Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>TO</td>
<td>9000 K</td>
<td>none</td>
<td>none</td>
</tr>
<tr>
<td>TD90</td>
<td>9000 K</td>
<td>0.14 cm$^{-3}$</td>
<td>none</td>
</tr>
<tr>
<td>TD75</td>
<td>7500 K</td>
<td>0.14 cm$^{-3}$</td>
<td>none</td>
</tr>
<tr>
<td>TD75+10</td>
<td>7500 K</td>
<td>0.14 cm$^{-3}$</td>
<td>(star) +10%</td>
</tr>
<tr>
<td>TD75-10</td>
<td>7500 K</td>
<td>0.14 cm$^{-3}$</td>
<td>(gas) -10%</td>
</tr>
<tr>
<td>DO</td>
<td>none</td>
<td>0.14 cm$^{-3}$</td>
<td>none</td>
</tr>
</tbody>
</table>

The following sections present the results from each of the six models in Table 4.1 and compare their major features. Section 4.2 describes the overall morphology and the surface density profile of the gas disks. Section 4.3 describes the location of the star forming regions within the disk and the amount of star formation produced. Section 4.4 describes temperature and density phases structure and time dependence of the phase balance. Section 4.5 presents data concerning the possibility of radial motions in the gas.

4.2 Morphology and Large-Scale Structure

In this section, we present figures showing the face-on and edge-on views of the gas disks at the end of each simulation run. As stated in Chapter 3, each model is run for 9900 timesteps corresponding to approximately 2 billion years. We show the models at the end of each run when they have had time to settle to a quasi-steady state following a period of transient ringing. All of the gas particles, and only the gas particles, are plotted in these figures. The edge-on view is not $X$ versus $Z$ but rather radius, $R$, versus $Z$, with all of the particles included so that the plot shows all possible cross-sections from the center to the edge overlaid.

The radial profiles of the gas surface density, $\Sigma_{gas}$, critical surface density for gravitational instability, $\Sigma_{crit}$, and their ratio, $Q = \Sigma_{crit}/\Sigma_{gas}$, are also presented in this section. The azimuthally averaged radial surface density profile of the gas disk is computed by averaging over 50 annuli of equal width. $\Sigma_{crit}$ and $Q$ is plotted on the same graph with the $\Sigma_{gas}$ profile for each model. A 1/r curve is also plotted to compare with the $\Sigma_{gas}$ profile. $\Sigma_{crit}$ is calculated in the same 50 annuli as $\Sigma_{gas}$. It is defined as

$$\Sigma_{crit} = \frac{\kappa \sigma_r}{\pi G}$$  \hspace{1cm} (4.1)

where $\kappa = \frac{d\Omega^2}{dr} + 4\Omega^2$ is the epicyclic frequency ($\Omega$ is the rotational velocity) and $\sigma_r$ is the radial
velocity dispersion (Binney & Tremaine 1994). According to a theoretical argument by Toomre (1964), the gas should be unstable to large-scale gravitational collapse when \( Q > 1 \), but Kennicutt (1989) has found that \( Q > 1.5 \) better describes observations of late-type galaxies. We discuss this in more detail in Section 5.3.2 of Chapter 5. The velocity dispersion profiles of each model are plotted in Figure 4.46. They are approximately isothermal in the flat rotation curve region for all of the models described here.

In Figure 4.1, the TO model is shown to have a clumpy central region and coherent spiral arms that begin about 10 kpc from the center and become open at the edge of the disk. The edge of the disk is not well defined and extends nearly to the walls of the simulation box. The edge-on view, Figure 4.2, shows a thin central region that thickens steadily as the radius increases to the very edge of the disk. Again, it is apparent that the disk extends nearly to the edge of the simulation region; however, there is an obvious drop in density at about 30 kpc. This disk increased in thickness and overall radius during the time it was allowed to be heated by star formation as can be seen by comparing it to the initial disk shown in Figure 3.5. The surface density profile for this model is shown in Figure 4.3. The \( \Sigma_{\text{gas}} \) follows a \( 1/r \) profile through most of the radius of the disk out to about 23 kpc where it drops more steeply. It remains below but close to \( \Sigma_{\text{crit}} \) in the flat rotation region which starts at a radius of about 10 kpc. \( Q \) has a relatively constant value of a few out to a radius of 23 kpc where it increases steeply as \( \Sigma_{\text{gas}} \) drops. The computed values of \( \Sigma_{\text{gas}} \) and \( \Sigma_{\text{crit}} \) become very noisy after 30 kpc due to low numbers of particles in the outermost annuli. This limitation on the surface density computation in the outer disk occurs in all of the models.

The TD90 model in Figure 4.4 shows a clumpy central region and coherent spiral structure farther out, similar to the TO model, but the spiral arms are slightly more tightly wound and the edge of the disk is more distinct. The edge-on view of TD90, Figure 4.5, shows a thin disk that thickens slowly with radius and stops thickening at a radius of about 30 kpc. Farther out, it flattens and begins to thin again. The particle farthest from the center in TD90 is not as distant as the last several particles in TO, but both disks have a steep drop in density at 30 kpc. Figure 4.6 shows the surface density profile for this model. \( \Sigma_{\text{gas}} \) follows a \( 1/r \) profile out to about 25 kpc where it drops more steeply. \( \Sigma_{\text{gas}} \) is very close to \( \Sigma_{\text{crit}} \) in the flat rotation region and they equal each other at just under 25 kpc. \( Q \) has a value equal to a few near the center and decreases to unity at a radius of 25 kpc, past which it increases steeply as \( \Sigma_{\text{gas}} \) drops.

The face-on views of the TD75, TD75+10, and TD75-10 models in Figures 4.7, 4.10 and 4.13 show the same general clumpy features in their central region common to the TO and TD90 models. Their spiral arms are thicker and not as dense as those in TD90. The edge-on views of the TD75 series,
Figures 4.8, 4.11 and 4.14, show very similar structure to TD90 but are all slightly thinner. The surface density profiles of the TD75 series follow the $1/r$ profile over a large range of the radius. All three models have surface densities that track very close to $\Sigma_{\text{crit}}$ throughout the flat rotation region of the disk, exceeding it over a range of about 10 kpc.

The DO model has a strikingly different face-on morphology from the other five models. The central region is not clumpy; rather, it is very smooth out to a radius of about 10 kpc. The tightly wound spiral arms start far from the center and are weak and indistinct. The edge-on view of the DO model, Figure 4.17, shows a thicker inner disk than the other models. There is a slight increase of thickness with radius out to 30 kpc and then a slow decrease. Overall, the disk is much smoother than the other models. $\Sigma_{\text{gas}}$ follows the $1/r$ profile very closely with little of the bumpiness that is evident in the other models. The surface density drops off more steeply than $1/r$ at radii greater than 30 kpc in a similar fashion to the other disks, though. $\Sigma_{\text{gas}}$ remains close to $\Sigma_{\text{crit}}$ through most of the radius of the disk, exceeding it at radii greater than 20 kpc.

4.3 Star Formation

The following figures show the location of the star forming (SF) particles. Each SF particle may represent some number of HII regions. Table 4.2 lists the number of SF particles in each model at timestep 9900 or a simulation time of 2.1 Gyrs. The gas consumption timescale is of the order of several billion years. The transient ringing behavior of the gas disk dissipates well within the first 5000 timesteps, so we assume the latter half of the simulation corresponds to the physical steady state of the disk. Our current study is concerned with this steady state and not the long term evolution of the disk. Consequently, we do not convert gas particles into star particles, so there is no loss of gas mass other than gas particles that leave the simulation box.

We can compare relative amounts of star formation between galaxy models here, but a comparison of the models to observed star formation rates requires calibrating the fraction of HII regions to SF particles (see discussion in Chapter 5). For now we will present the number of SF particles in each model to compare with each other. The following description concerns the amount and distribution of star formation at one instance in time. Time dependent behavior of the star formation will be discussed in Section 4.4.2.

The face-on view of the TO model, Figure 4.19, shows a significant amount of star formation throughout the disk. The SF particles occur mainly in clumps near the center, but they show up along the spiral arms and out to the edge of the simulation box. There are several SF particle at radii greater
Table 4.2  Number of star forming gas particles in each model for timestep 9900: The differences between the 3 TD75 models is not significant.

<table>
<thead>
<tr>
<th>Model Label</th>
<th>SF Particle Number</th>
<th>Fraction of SF Particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>TO</td>
<td>1223</td>
<td>0.041</td>
</tr>
<tr>
<td>TD90</td>
<td>699</td>
<td>0.023</td>
</tr>
<tr>
<td>TD75</td>
<td>408</td>
<td>0.014</td>
</tr>
<tr>
<td>TD75+10</td>
<td>528</td>
<td>0.018</td>
</tr>
<tr>
<td>TD75-10</td>
<td>444</td>
<td>0.015</td>
</tr>
<tr>
<td>DO</td>
<td>3514</td>
<td>0.116</td>
</tr>
</tbody>
</table>

than 23 kpc where the surface density drops steeply. There are also many SF particles in rarefied regions behind spiral waves. In fact, in several cases the spirals defined by the SF particles are quite distinct from the gas spirals (unlike the observations). These low density regions have cold gas particles due to adiabatic cooling through expansion which enables star formation to take place when only a temperature threshold criterion is used. This is also the case for the star formation occurring at the farthest reaches of the disk. This unwanted behavior is a prime motivation for adding a density threshold criterion to the model. Figure 4.20 shows the distribution of SF particles along the Z direction. Star formation is occurring through most of the thickness of the disk. Some SF particles even reach to the upper and lower edges, again, in contrast to observation.

Most of the SF particles in the TD90 model occur in dense clumps in the central region of the disk with some occurring in the spiral arms of the outer disk (see Figure 4.21). The star formation in the flat rotation region is confined to the dense spiral arms and does not occur in the rarefied space between them. There is also a threshold radius of about 27 kpc beyond which no star formation occurs. This corresponds to the radius at which the density of the gas drops below the chosen threshold density of $0.14 \text{ cm}^{-3}$. No SF particles occur in the region outside of this 27 kpc radius in contrast to the TO model. This threshold radius cutoff in star formation is characteristic of all of the models employing a density criterion. Figure 4.22 shows the extent of the SF particles in the vertical direction. While they still produce a relatively thick disk, they do not reach the heights of the TO model. The star formation in the outer region beyond 20 kpc seems to be confined to a thinner disk than the inner regions. This may be because of the fewer SF particles beyond 20 kpc rather than a physical feature, although the densest regions of the outer disk are confined to the midplane of the disk.

The TD75 model shows markedly less star formation on its face than the TO and TD90 models. The
star formation occurs in the clumps of the central region and follows the spiral arms in the outer region (see Figure 4.23). Again, the star formation is confined to the dense arms and absent from the spaces between. It also exhibits no SF particles outside of 27 kpc. The vertical extent of the star formation is confined to a slightly thinner disk than the TD90 model, but the thinning at the outer region that is seen in TD90 does not occur here. The edge-on view, Figure 4.24, shows the thin star forming disk and confirms the threshold radius of 27 kpc.

The TD75+10 model has slightly more SF particles than the TD75 model with the increase occurring mainly in the spiral arms as shown in Figure 4.25. The TD75+10 model exhibits the same radial threshold of 27 kpc as the previous TD models. The edge-on view of TD75+10, Figure 4.26, shows a similar vertical distribution to TD75 but with more particles.

The amount of star formation of model TD75—10 is very similar to that of TD75 (see Table 4.2. However, the face-on view shown in Figure 4.27 shows that the star formation is slightly more concentrated near the center with only a little occurring in the outer spiral arms. No star formation is seen outside a radius of 24 kpc. In the edge-on view, Figure 4.28, the star formation is confined to a thinner disk in the outer regions as occurs in TD90.

The face-on view of the DO model, Figure 4.29, shows a large amount of star formation occurring uniformly across the center of the disk to a radius of 10 kpc. Past that radius, the star formation occurs in clumps and follows along the weak spiral arm segments of the outer disk. SF particles exist out to a radius of 25 kpc beyond which the star formation stops. Star formation occurs through the entire thickness of the inner disk out to a radius of just over 10 kpc as can be seen in the edge-on view in Figure 4.30. The star forming disk has a slight pinch at 15 kpc. After the pinch widens, the vertical extent of the SF particles begins to thin again slightly until a radius of 25 kpc where the star formation ends abruptly.

4.4 Phase Balance

In this section, we describe the behavior of different thermal phases of each model. For convenience, we classify each gas particle as either hot, warm or cold. A hot particle has a temperature greater than $10^4$ K, a warm particle has a temperature of $10^4$ K, and a cold particle has a temperature less than $10^4$ K. We classify warm gas particles as those with temperature approximately equal to $10^4$ K rather than those with some range of temperatures around $10^4$ K because a large fraction of particles have a temperature equal to $10^4$ K to three decimal places. This occurs due to the sharp drop in the cooling curve at $10^4$ K. Below $10^4$ K the cooling rate is much lower than the cooling rate at or above
$10^4$ K, so $10^4$ K acts as a temporary minimum temperature. Because of the strong radiative cooling, temperatures well above $10^4$ K only occur because of the star formation feedback. Any particle with a higher temperature than this has been heated by the star formation and is analogous to the tenuous hot gas of the ISM. Particles with temperatures below $10^4$ K are cooled either by adiabatic expansion if they occur in low density regions or by radiative cooling if they occur in high density regions. The low density cool gas particles occur either between spiral arms or at the edge of the disk. The latter are unphysical and are due to the lack of particle resolution of the simulations. The former may be caused by enhanced radiative cooling after the passing of a spiral wave temporarily compresses that region. This overall classification scheme is roughly analogous to the physical ISM, but the simulations lack the resolution to accurately model the true thermal phases of the ISM. Nonetheless, we will consider the thermal behavior of the models under the simplifying assumption of these three separate phases. The implications of resolution problems will be discussed in the Chapter 5.

Table 4.3 lists the fractions of the gas mass contained in each phase for each model at timestep 9900 corresponding to the end of the simulated 2 billion years. Comparing Tables 4.2 and 4.3 we can see that the fraction of hot gas closely matches the fraction of star forming particles, as expected. The model with the most star formation has the largest hot fraction. The warm and cold gas are split fairly equally in all except the TO model. The warm gas dominates the three models with the most star formation, TO, TD90, and DO, but the cold gas dominates the low star formation models, TD75, TD75+10, and TD75−10. The fact that no model has nearly as high a fraction of gas with temperatures below $10^4$ K. as in a real late-type disk. is again the result of resolution limits, as will be discussed in the following chapter.

Table 4.3 Fraction of gas mass contained in the hot, warm, and cold phase for each model at timestep 9900.

<table>
<thead>
<tr>
<th>Model Label</th>
<th>Hot Fraction</th>
<th>Warm Fraction</th>
<th>Cold Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>TO</td>
<td>0.044</td>
<td>0.668</td>
<td>0.288</td>
</tr>
<tr>
<td>TD90</td>
<td>0.022</td>
<td>0.523</td>
<td>0.455</td>
</tr>
<tr>
<td>TD75</td>
<td>0.011</td>
<td>0.453</td>
<td>0.537</td>
</tr>
<tr>
<td>TD75+10</td>
<td>0.012</td>
<td>0.453</td>
<td>0.535</td>
</tr>
<tr>
<td>TD75−10</td>
<td>0.011</td>
<td>0.443</td>
<td>0.546</td>
</tr>
<tr>
<td>DO</td>
<td>0.117</td>
<td>0.477</td>
<td>0.406</td>
</tr>
</tbody>
</table>
4.4.1 Temperature-Density Structure of the Gas

Plotting temperature versus density provides an extremely valuable picture of the phase structure of the gas, and potentially a means for comparing to observation. In the figures of this section, temperature versus density is plotted on a log-log scale with temperature in units of Kelvin and density in units of cm\(^{-3}\). The density of each gas particle is computed via the smoothing kernel as described in Section 3.1.5.1. The temperature is computed from the specific energy of each particle which is determined by Equation 3.14.

Figure 4.31 shows the temperature-density structure of the TO model. The density of the gas ranges from 0.0026 cm\(^{-3}\) to 272 cm\(^{-3}\) though roughly 98% of the gas mass has a density less than about 3 cm\(^{-3}\). The minimum density corresponds to one particle with no near neighbors and is common to all of the models presented here except TD75–10 which has a minimum density of 0.0024 cm\(^{-3}\) due to the slightly reduced mass of the gas particles. Less than 2% of the total gas mass is contained in the highest density region, a very compact clump at the center of the disk. This clump has a spatial extent of about 350 pc, which is larger than the gravitational softening length and minimum smoothing length (see Section 3.1.2). The dense clump occurs in the initial cooled disk and is common to all models except the DO model. The gas temperature ranges between 1767 K and 5.5 \times 10^5 K. The hot gas exists in a range of densities from 0.0026 cm\(^{-3}\) to 3.2 cm\(^{-3}\). About 2% of the hot gas is found at the highest density. No hot gas exists at densities between roughly 3.2 cm\(^{-3}\) and 270 cm\(^{-3}\). The hot gas makes up 4.4% of the total gas mass. The warm gas spans the entire range of densities and comprises 66.8% of the total gas mass. The cold gas makes up the remaining 28.8% of the total gas mass. Roughly 4% of the cold gas is in the dense clump at the center. The majority of the cold gas particles have densities between 0.015 cm\(^{-3}\) and 3.5 cm\(^{-3}\) with the coldest gas at a density of 0.062 cm\(^{-3}\).

Figure 4.32 shows the temperature-density structure of the TD90 model. The density of the gas ranges from 0.0026 cm\(^{-3}\) to 286 cm\(^{-3}\) with most of the gas below about 4.0 cm\(^{-3}\) and the densest gas in a compact clump in the center of the disk. The gas temperature ranges between 450 K and 5.5 \times 10^5 K although only three gas particles have a temperature below 1000K. The hot gas spans a density range from 0.087 cm\(^{-3}\) to approximately 3.2 cm\(^{-3}\) with the exception of three particles, or 0.5% of the hot gas mass (0.001% of the total gas mass), which lie at a density above 100 cm\(^{-3}\). The hot gas makes up 2.2% of the total gas mass. The warm gas spans the entire range of densities and makes up 52.3% of the mass with most of it below 4.0 cm\(^{-3}\). The cold gas makes up 45.5% of the total mass, and about 99% of the cold gas mass exists at densities below 4.0 cm\(^{-3}\). Less than 0.5% of the cold gas mass has a density greater than 4.0 cm\(^{-3}\). Most of the cold gas lies between 0.015 cm\(^{-3}\) and 0.15 cm\(^{-3}\), but the
coldest gas has a density 0.005 cm\(^{-3}\).

Figure 4.33 shows the temperature-density structure of the TD75 model. The gas density ranges from 0.0026 cm\(^{-3}\) to 429 cm\(^{-3}\) with most of the gas below 8.0 cm\(^{-3}\) and the densest gas in a compact clump at the center. The highest density achieved in the TD75 model is about 50% larger than the maximum density of of TD90 or TO. The gas temperature ranges between 680 K and 5.5 \times 10^5 K. The hot gas makes up 1.1% of the total gas mass, half as much as TD90. Most of the hot gas spans a density range from 0.11 cm\(^{-3}\) to 2.8 cm\(^{-3}\). The warm gas makes up 45.3% of the total gas mass and spans the entire range of densities with most of it below 10 cm\(^{-3}\). The cold gas makes up 53.7% of the total gas mass with only about 3% of the cold gas at a density greater than 8 cm\(^{-3}\). Most of the cold gas lies between 0.03 cm\(^{-3}\) and 3.0 cm\(^{-3}\), a range that is shifted in the direction of high density compared to TD90. The coldest gas lies in the central dense clump and has a density of about 426 cm\(^{-3}\). The thermal structures of the TD75+10 and TD75−10 models, shown in Figures 4.34 and 4.35, do not differ significantly from TD75.

Figure 4.36 shows the temperature-density structure of the DO model. The gas density ranges from 0.0026 cm\(^{-3}\) to 1.79 cm\(^{-3}\). The gas temperature ranges between 1541 K and 5.5 \times 10^5 K. The dense central clump common to the previous five models is conspicuously absent from this model. The hot gas makes up 11.7% of the total gas mass and spans a density range of 0.087 cm\(^{-3}\) to 1.79 cm\(^{-3}\), though only three particles have a density less than 0.11 cm\(^{-3}\). There is a slight negative slope evident along the low-density edge of the hot particles in the log(T) versus log(\(\rho\)) plot. The warm gas makes up 47.7% of the total gas mass and spans the entire density range. The cold gas makes up the remaining 40.6% of the gas mass and spans a density range of 0.0026 cm\(^{-3}\) to 1.4 cm\(^{-3}\). Most of the cold gas has a density between roughly 0.03 cm\(^{-3}\) and 0.3 cm\(^{-3}\), while the coldest gas has a density of about 0.007 cm\(^{-3}\).

4.4.2 Time Dependence of the Thermal Phases

The graphs of thermal gas fraction versus timestep show variability of the hot, warm and cold fraction with time in each model. The gas is divided into three phases: hot, warm, and cold, as described above. A large portion of the gas sits at \(10^4\) K because of the very abrupt drop in the cooling curve at that temperature. The mass fraction of each phase is simply the number of particles in that phase divided by the total number of gas particles. The hot fraction is particularly interesting because it most closely tracks the amount of star formation occurring in a model. The only model with a large (and regular) variation in hot fraction is the DO model which will be discussed in Section 4.4.2.1. The
The rest of the models have low mean hot fraction and show small variations in hot fraction.

Each point on the plot of the fraction of hot, warm, and cold gas versus timestep is calculated from the entire set of 30,000 gas particles at a number of different timesteps. If we are to minimize computation time and storage requirements, we cannot output and save all of the model data very frequently during each simulation. The data points on these graphs are approximately 250 timesteps apart, corresponding to a sampling frequency of once every $5 \times 10^7$ years. They only span a time interval between 2000 and 7000 timesteps. A subset of 1000 gas particles is saved every 50 timesteps and used to check for periods on smaller timescales. We find no regular periods in these data except in the DO model (see discussion in 4.4.2.1).

Figures 4.37 through 4.42 show the time dependence of the thermal gas fraction for each model. The TO model has the least fluctuation in the warm and cold fractions among the models. All of the TD models exhibit a similar behavior, showing considerable stochastic fluctuations in their warm and cold fractions. The fluctuations of the hot phases are difficult to see in the TD models because the small hot fraction is plotted on the same scale as the larger warm and cold fractions. Close inspection shows moderate stochastic fluctuations in the hot fraction, as well. The DO model alone shows a large hot fraction. Also, the hot fraction of the DO model exhibits a large periodic fluctuation in contrast to the other models. Figure 4.43 shows the time dependence of the initial cooled disk when run for the same time interval as the six models, but with no star formation feedback. No gas is heated above $10^4$ K in this simulation so only the warm and cold fractions are plotted. The fluctuations of these fractions compliment each other but exhibit no obvious periodicity. They are less variable than the TD models and similar to the TO model. Table 4.4 lists the maximum, minimum, and mean values of the hot, warm, and cold fractions for each model and the simulation run with no feedback.

4.4.2.1 The DO Model Oscillations

The DO model presents unique feature of its time dependent balance of thermal phases. We detected a very regular oscillation in the fraction of hot gas with time. Figure 4.42 shows an oscillation between 10% and 15% with a period of approximately 717 timesteps corresponding to about $1.4 \times 10^8$ years. The peaks in the hot mass fraction are equally spaced and are nearly the same height, though slowly decreasing with time. There is a peak missing at timestep 4934, but the last two peaks are in phase to within an error of under 3%. We ran the same model again through an interval between timesteps 5135 and 6070 which covered one period of the oscillation, this time dumping data every $10^7$ years or about every 50 timesteps, in an attempt to resolve the shape of the peak. Figure 4.44 shows the result
Table 4.4  The maximum, minimum, and mean fraction of gas mass contained in the hot, warm, and cold phase for each model over the interval between timestep 2000 and timestep 7000.

<table>
<thead>
<tr>
<th>Model Label</th>
<th>Hot Fraction</th>
<th>Warm Fraction</th>
<th>Cold Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Maximum</td>
<td>Minimum</td>
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<tr>
<td>TO</td>
<td>0.044</td>
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<td>TD90</td>
<td>0.024</td>
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<td>0.018</td>
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<td>0.020</td>
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<td>TD75–10</td>
<td>0.011</td>
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<td>0.007</td>
</tr>
<tr>
<td>DO</td>
<td>0.130</td>
<td>0.180</td>
<td>0.110</td>
</tr>
<tr>
<td>No Feedback</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
of the better time resolution. A newly resolved period of the oscillation, 144 timesteps or $2.9 \times 10^7$ years, was disguised by aliasing. We had serendipitously chosen a sampling period almost exactly 5 times the oscillation period (to within 0.4%). Once more, we ran the DO model over an even smaller interval (timestep 5511 to 5765) that covered two peaks of the more rapid oscillation shown in Figure 4.44, outputting data about 3.3 times more frequently than the previous run. This time we were able to resolve the shape of the peaks as shown in Figure 4.45. The increase from 10% to 15% occurs over an interval of about 14 timesteps or roughly $2.8 \times 10^6$ years. The peaks have a flat top and the trailing edge drops about as fast as the leading edge rises. The peak has a height of 5.6% and a full width at half maximum of 45 timesteps or $9.0 \times 10^6$ years. This time interval corresponds to the duration of the heating due to star formation which is $10^7$ years or the lifetime of an O star. The period of the oscillation, about $2.9 \times 10^7$ years, is 50% greater than the threshold duration time of $2 \times 10^7$ years. This is consistent, however, since the time between the top of the trailing edge of the first peak and beginning of the leading edge of the second peak is almost exactly $2 \times 10^7$ years. The other 10 million years comes from combining the trailing half-width of the first peak and the leading half-width of the subsequent peak. It becomes obvious upon examining a plot of the temperature versus density, Figure 4.36, that the SF particles become hot but never decrease their density below the threshold. Therefore, all particles with sufficient density to become star forming do so at the same time once the model is allowed to advance 20 million years from its initial state. They heat to the maximum temperature over the course of 10 million years all the while maintaining their high densities. At the end of the SF lifetime the particles are allowed to cool. Cooling is rapid. Since the densities of the particles never fell below the threshold while they were being heated, they immediately begin the count to be considered star forming and begin to heat again.

The SF particles are heated by increasing their temperature by 10% every timestep until they reach the maximum temperature of $5 \times 10^5$ K. It takes a particle 41 timesteps to heat from $10^4$ K to the maximum temperature. The reason the leading edge of the oscillations in Figure 4.45 is so steep is because the hot fraction includes any particle greater than $10^4$ K. It only takes one timestep to heat from $10^4$ K to anything greater than $10^4$ K. The trailing edge is steep because radiative cooling is very rapid. The slope of the trailing edge is, in fact, a rough measure of the cooling timescale.

Why is the amount of energy added to SF particles not sufficient to cause expansion and a drop in density? The temperature is increased by a factor of 10 to 100 depending on the initial temperature of the SF particle. If the pressure is constant over the heating time then the density should decrease by the same factor. This is obviously not the case; rather, the pressure in a particle is calculated from the
density and temperature which are in turn estimated using the smoothing kernel technique described in Section 3.1.5. By adding an extra amount of energy directly to one particle, the SF particle, we are in effect increasing the pressure and doing nothing to the density at first. The density will only change once the increased pressure pushes particles apart.

The maximum temperature to which the SF particles are heated, $5 \times 10^5$ K, is near the peak of the cooling curve (see Spitzer 1978 and Sutherland & Dopita 1993). This, combined with the high density of the gas makes the cooling very efficient. The SF particle only just gets to its maximum value before cooling is re-enabled and the particle cools within a few timesteps. The added pressure only lasts a short time and the gas has little time to respond before the energy is radiated away and the pressure returns to the value of the surrounding medium. The cooling curve drops significantly past $10^6$ K so heating the SF particle to at least $10^6$ K should reduce the alacrity with which the gas cools allowing the pressure increase to reduce the density of the SF particle and its neighboring particles.

### 4.5 Radial Motion

Systematic radial motions are hard to discover from the radial velocity components of the gas particles due to the large velocity dispersion of the particles compared to the radial velocities predicted by our analytic model. The analytic model of Chapter 2 predicts opposing radial flows between hot and cold material at velocities with magnitudes of a few km/s, but the velocity dispersions in the numerical models are of the order of 10 km/s (see Figure 4.46). To overcome this we use a subset of the particles that have their velocities, positions, and other properties output more frequently than the entire disk and act as tracer particles. From this sample of 1000 particles we plot the minimum and maximum values of their radii over some time interval and look for deviations from a straight line. The scatter of the line and the shape of the plot will show whether particles tend to stay at about the same radius or whether there are large changes in radii. Plotting the initial and final values of the radii of each particle over some time interval will provide information on whether there is an inflow or outflow or both. The time interval used to produce these plots spans timestep 5000 to timestep 9900 approximately corresponding to the latter one billion years of the simulation.

Figure 4.47 shows the minimum and maximum radii of the 1000 tracer particles of the gas disk for the TO model. The particles follow a line with a slope of one between $R_{min}$ of 0 and 20 kpc. Above 20 kpc, the particles curve away from the line toward large values of $R_{max}$. Three particles have traversed nearly the entire radius of the disk during the simulation. The scatter of the line increases as $R_{min}$ increases indicating that more radial motion occurs at larger radii. Figure 4.48 shows the initial and
final radii of each tracer particle over the considered interval for the TO model. The particles follow a line with slope one throughout, but the scatter increases greatly as $R_{\text{min}}$ increases. An asymmetry of the scatter would indicate whether outflow or inflow is dominant. The relative symmetry seen in this graph suggests a balance between the two, although the two most displaced particles were flowing in toward the center.

Figure 4.49 shows the minimum and maximum radii of tracer particles for the TD90 model. The particles follow a line with a slope of one through the entire range of $R_{\text{min}}$. The scatter is similar to the TO model at radii below 20 kpc, but the scatter does not increase with increasing $R_{\text{min}}$, and there is no deviation from the line as was seen in TO. Only one particle travels any farther than 10 kpc during the time interval considered. Figure 4.50 shows the initial and final radii of each tracer particle for the TD90 model. The scatter in Figure 4.50 is similar the scatter in the TO model (Figure 4.48) for radii below 20 kpc; however, the scatter decreases at $R_{\text{min}}$ greater than 20 kpc for the TD90 model; exactly opposite behavior to the TO model. Figure 4.50 is fairly symmetric suggesting a balance between inflow and outflow. The small scatter at indicates very little radial motion in the TD90 model.

Figures 4.51 through 4.56 show the minimum versus maximum radii and the initial versus final radii plots for the TD75, TD75+10, and TD75−10 models. These three models exhibit the same general behavior. The minimum versus maximum radii plots for all three models closely follow a line of one slope over the entire range of $R_{\text{min}}$. All three exhibit less scatter than the TO or TD90 models. The initial versus final radii plots also follow a slope equals one line and show significantly less scatter than the TO and TD90 models. The gas particles exhibit almost no radial motion in these three models.

Figures 4.57 and 4.58 show the minimum versus maximum radii and the initial versus final radii plots for the DO model. Surprisingly, the DO model exhibits very similar radial motion to the TD75 series of models. The minimum versus maximum radii plot closely follows the normal slope equals one line with small scatter, as does the initial versus final radii plot. Very little radial motion is seen in the DO model. Other than some expansion of the outer disk in the TO model, these simulations do not have any obvious radial flows. Moreover, these kinematic data do not conflict with observation. Nor do they distinguish the models from each other in a significant way.
Figure 4.1 Face-on view of the TO model: Axes have units of 100 kpc.

Figure 4.2 Edge-on view of the TO model: Axes have units of 100 kpc.

Figure 4.3 Surface density profile of the TO model: The critical surface density and Q are also plotted. A $1/r$ profile is shown to compare with the gas surface density profile. The horizontal axis is plotted in units of 100 kpc and the vertical axis is plotted in units of $M_\odot/pc^2$.
Figure 4.4 Face-on view of the TD90 model: Axes have units of 100 kpc.

Figure 4.5 Edge-on view of the TD90 model: Axes have units of 100 kpc.

Figure 4.6 Surface density profile of the TD90 model: The critical surface density and $Q$ are also plotted. A $1/r$ profile is shown to compare with the gas surface density profile. The horizontal axis is plotted in units of 100 kpc and the vertical axis is plotted in units of $M_\odot/pc^2$. 
Figure 4.7  Face-on view of the TD75 model: Axes have units of 100 kpc.

Figure 4.8  Edge-on view of the TD75 model: Axes have units of 100 kpc.

Figure 4.9  Surface density profile of the TD75 model: The critical surface density and Q are also plotted. A $1/r$ profile is shown to compare with the gas surface density profile. The horizontal axis is plotted in units of 100 kpc and the vertical axis is plotted in units of $M_\odot/pc^2$. 
Figure 4.10 Face-on view of the TD75+10 model: Axes have units of 100 kpc.

Figure 4.11 Edge-on view of the TD75+10 model: Axes have units of 100 kpc.

Figure 4.12 Surface density profile of the TD75+10 model: The critical surface density and Q are also plotted. A $1/r$ profile is shown to compare with the gas surface density profile. The horizontal axis is plotted in units of 100 kpc and the vertical axis is plotted in units of $M_\odot/pc^2$. 
Figure 4.13  Face-on view of the TD75-10 model: Axes have units of 100 kpc.

Figure 4.14  Edge-on view of the TD75-10 model: Axes have units of 100 kpc.

Figure 4.15  Surface density profile of the TD75-10 model: The critical surface density and Q are also plotted. A $1/r$ profile is shown to compare with the gas surface density profile. The horizontal axis is plotted in units of 100 kpc and the vertical axis is plotted in units of $M_\odot/pc^2$. 
Figure 4.16 Face-on view of the DO model: Axes have units of 100 kpc.

Figure 4.17 Edge-on view of DO model: Axes have units of 100 kpc.

Figure 4.18 Surface density profile of the DO model: The critical surface density and $Q$ are also plotted. A $1/r$ profile is shown to compare with the gas surface density profile. The horizontal axis is plotted in units of 100 kpc and the vertical axis is plotted in units of $M_\odot/pc^2$. 
Figure 4.19  Face-on view of the TO model with SF particles marked in red: Axes have units of 100 kpc.

Figure 4.20  Edge-on view of the TO model with SF particles marked in red: Axes have units of 100 kpc.
Figure 4.21  Face-on view of the TD90 model with SF particles marked in red: Axes have units of 100 kpc.

Figure 4.22  Edge-on view of the TD90 model with SF particles marked in red: Axes have units of 100 kpc.
Figure 4.23 Face-on view of the TD75 model with SF particles marked in red: Axes have units of 100 kpc.

Figure 4.24 Edge-on view of the TD75 model with SF particles marked in red: Axes have units of 100 kpc.
Figure 4.25  Face-on view of the TD75+10 model with SF particles marked in red: Axes have units of 100 kpc.

Figure 4.26  Edge-on view of the TD75+10 model with SF particles marked in red: Axes have units of 100 kpc.
Figure 4.27  Face-on view of the TD75-10 model with SF particles marked in red: Axes have units of 100 kpc.

Figure 4.28  Edge-on view of TD75-10 model with SF particles marked in red: Axes have units of 100 kpc.
Figure 4.29  Face-on view of the DO model with SF particles marked in red: Axes have units of 100 kpc.

Figure 4.30  Edge-on view of the DO model with SF particles marked in red: Axes have units of 100 kpc.
Figure 4.31 log $n$ versus log $T$ of the TO model. $n$ has units of cm$^{-3}$. $T$ has units of K.

Figure 4.32 log $n$ versus log $T$ of the TD90 model. $n$ has units of cm$^{-3}$. $T$ has units of K.

Figure 4.33 log $n$ versus log $T$ of the TD75 model. $n$ has units of cm$^{-3}$. $T$ has units of K.
Figure 4.34 log $n$ versus log $T$ of the TD75+10 model. $n$ has units of cm$^{-3}$. $T$ has units of K.

Figure 4.35 log $n$ versus log $T$ of the TD75−10 model. $n$ has units of cm$^{-3}$. $T$ has units of K.

Figure 4.36 log $n$ versus log $T$ of the DO model. $n$ has units of cm$^{-3}$. $T$ has units of K.
Figure 4.37  Mass fraction of hot, warm, and cold gas vs. timestep for the TO model. The data points are approximately 250 timesteps apart.

Figure 4.38  Mass fraction of hot, warm, and cold gas vs. timestep for the TD90 model. The data points are approximately 250 timesteps apart.

Figure 4.39  Mass fraction of hot, warm, and cold gas vs. timestep for the TD75 model. The data points are approximately 250 timesteps apart.

Figure 4.40  Mass fraction of hot, warm, and cold gas vs. timestep for the TD75+10 model. The data points are approximately 250 timesteps apart.
Figure 4.41  Mass fraction of hot, warm, and cold gas vs. timestep for the TD75-10 model. The data points are approximately 250 timesteps apart.

Figure 4.42  Mass fraction of hot, warm, and cold gas vs. timestep for the DO model. The data points are approximately 250 timesteps apart.

Figure 4.43  Mass fraction of warm and cold gas vs. timestep for the "no heating" model. The data points are approximately 250 timesteps apart.
Figure 4.44 Mass fraction of hot, warm, and cold gas vs. timestep for the DO model. The data points are approximately 50 timesteps apart. This plot covers the interval between 5100 and 6100 timesteps.

Figure 4.45 Mass fraction of hot, warm, and cold gas vs. timestep for the DO model. The data points are approximately 12 timesteps apart. This plot covers the interval between 5500 and 5800 timesteps.
Figure 4.46 The $r$, $\phi$, and $z$ components of the velocity dispersion versus radius for all six models. The models from left to right and top to bottom are: TO, TD90, TD75, TD75+10, TD75−10, DO. R has units of 100 kpc. $\sigma$ has units of km/s.
Figure 4.47  \( R_{\text{min}} \) vs \( R_{\text{max}} \) for the TO model:
Axes have units of 100 kpc.

Figure 4.48  \( R_{\text{init}} \) vs \( R_{\text{final}} \) for the TO model:
Axes have units of 100 kpc.

Figure 4.49  \( R_{\text{min}} \) vs \( R_{\text{max}} \) for the TD90 model:
Axes have units of 100 kpc.

Figure 4.50  \( R_{\text{init}} \) vs \( R_{\text{final}} \) for the TD90 model:
Axes have units of 100 kpc.
Figure 4.51 $R_{\text{min}}$ vs $R_{\text{max}}$ for the TD75 model: Axes have units of 100 kpc.

Figure 4.52 $R_{\text{init}}$ vs $R_{\text{final}}$ for the TD75 model: Axes have units of 100 kpc.

Figure 4.53 $R_{\text{min}}$ vs $R_{\text{max}}$ for the TD75+10 model: Axes have units of 100 kpc.

Figure 4.54 $R_{\text{init}}$ vs $R_{\text{final}}$ for the TD75+10 model: Axes have units of 100 kpc.
Figure 4.55 $R_{\text{min}}$ vs $R_{\text{max}}$ for the TD75–10 model: Axes have units of 100 kpc.

Figure 4.56 $R_{\text{init}}$ vs $R_{\text{final}}$ for the TD75–10 model: Axes have units of 100 kpc.

Figure 4.57 $R_{\text{min}}$ vs $R_{\text{max}}$ for the DO model: Axes have units of 100 kpc.

Figure 4.58 $R_{\text{init}}$ vs $R_{\text{final}}$ for the DO model: Axes have units of 100 kpc.
CHAPTER 5 DISCUSSION OF NUMERICAL RESULTS

In this chapter, we analyze the results of our numerical models presented in Chapter 4. Section 5.1 reviews our early attempts at feedback models, what didn’t work and why. In Section 5.2, we examine the limited particle resolution issue, discussing what we can and cannot resolve with 30,000 gas particles. In Section 5.3, we discuss the three main aspects of the formalism needed to implement star formation feedback, and we discuss some novel and unexpected results concerning the effects of star formation feedback on the gas disks. We discuss more stringent criteria for choosing the best model and how to relate the models to observation in Section 5.4. In Section 5.5, we examine whether a comparison of the numerical model and analytic model is meaningful.

5.1 Feedback Failures: What Doesn’t Work

Before concentrating on the six models detailed in Chapter 4, we tried many other possible parameters. Our earliest attempts at feedback models involved large energy input (we increased the temperature of each SF particle to $3 \times 10^6$ K) and liberal star formation criteria resulting in huge outflows in which the disk lost a large fraction of gas particles. Consequently, our first criterion for choosing reasonable models was that they must be stable and not contain unphysical outflows. After reducing the energy input and using more restrictive star formation criteria, our next challenge was to choose star formation criteria that allowed star formation to occur in the flat rotation curve region without causing so much star formation in the central region as to once again cause unphysical outflows. A constant density-only threshold criterion causes a great deal of star formation to occur in the center of the disk immediately upon the start of the simulation. This causes the density of the outer disk to fall and stay below the threshold density before it has a chance to form stars. A constant temperature-only threshold criteria exhibits similar behavior. In order to limit the star formation in the central disk and give the outer disk a chance to cool enough to reach threshold, it becomes necessary to impose a threshold duration criterion. Any particle that reaches the threshold must stay at or beyond threshold for a set amount of time approximately equal to the free-fall time of a GMC, about $10^7$ years, before
it is considered star forming. The addition of this duration criterion allows star formation to occur in the flat rotation curve region. This is a key result of this numerical study. *Delaying the heating until a particle has remained in the threshold regime for a local free-fall time is a necessary star formation criterion for allowing star formation to occur in the flat rotation curve region.*

### 5.2 Resolution Limitations: What We Can’t See

The computational method used to produce our numerical models limits the spatial and mass resolution of the models in three ways. The number of particles is limited by our computational facilities and the time we are willing to wait for the completion of each simulation. We are able to use about 90,000 particles total in each simulation; however, only a third of those particles represent the gas disk. The other 60,000 particles provide the gravitational potential produced by the stellar disk and dark halo in a self-consistent manner. The number of SPH particles we can use limits the mass resolution of the models. Assuming a gas disk mass on the order of $10^{10} \, M_\odot$, each particle gets a mass on the order of $10^6 \, M_\odot$. This means we are not able to resolve structures with masses less than $10^6 \, M_\odot$. This includes small to moderately sized GMCs and smaller gas clouds that exist in the ISM. The limited mass resolution of the models prevents us from distinguishing true cold thermal phases. Particularly, we cannot resolve small, dense, cold clouds with densities of hundreds of atoms per cubic centimeter and temperatures of hundreds or even tens of Kelvin where star formation actually occurs; rather, we must assume that SF particles with densities on the order of a few tenths of atoms per cubic centimeter and temperatures of a few thousand Kelvin trace the location of the true dense, cold clouds without actually resolving them.

The spatial resolution is imposed on the models in two ways. First, the gravitational force calculation uses a softening analogous to a standard Plummer softening of 250 pc (see Binney & Tremaine 1994). Second, the SPH method uses a smoothing kernel (see Section 3.1.5.1) with a corresponding smoothing length. The density and specific energy of the gas is calculated at the point of an SPH particle by averaging over the values of particles within a distance of approximately twice the smoothing length from the point. This smoothing length is variable with a minimum value equal to the gravitational softening and a maximum equal to two grid cells or about 1.5 kpc. The large scale spiral patterns observed in late-type galaxies have length scales much larger than the minimum spatial resolution of our models and are easily discernible in the models. Diffuse HI clouds, translucent molecular clouds, and GMCs observed in our own Galaxy have sizes on the order of 100 pc or fewer and are near or below the spatial resolution of our models. Consequently, we are too limited by mass and spatial resolution
to distinguish cold condensed clouds in our models. HII regions generated by individual O or B stars are observed to have sizes smaller than 1 pc, while large HII regions observed in external galaxies can have sizes on the order of 100 pc, still below the spatial resolution of our models. The physics of star formation and heating due to star formation is microscopic to the models; therefore, our implementation of star formation feedback as described in Chapter 3 is a very broad, macroscopic approximation to reality. Fortunately, the effects of the feedback in our models produce some macroscopic properties that correspond with observation and are discussed in Section 5.4.

5.3 Feedback Formalism: What Works and How Do We Know?

The method we used to incorporate star formation feedback into simulations of gas disks was discussed in the last three sections of Chapter 3. The three important aspects of this feedback formalism are the star formation criteria, the heating method, and the amount of heating. While we have experimented with all three aspects, the models presented in Chapter 4 focus on the differences between our choice of star formation criteria. The heating method and amount are the same in all six models. Nonetheless, we must still consider them in our analysis of the models.

5.3.1 Heating Method and Amount

Gerritsen & Icke (1997) have studied different methods of adding the feedback energy into the gas and have found that adding energy directly to the SF particles and curtailing radiative cooling on those particles for a time interval of approximately the lifetime of massive O stars gives resulting morphologies that most closely resemble observed galaxies. In particular, they compare the size and distribution of bubbles in the disk to determine the most reasonable feedback method. Other methods they tested include distributing the energy over the neighboring particles using the smoothing kernel or providing a velocity kick to neighboring particles in a direction away from the SF particle. Adding all of the energy directly to the SF particle only is equivalent to increasing the pressure in that particle. Without limiting the cooling during the heating period the extra energy is radiated away too quickly, having little to no effect on the gas. We have verified this last point with our early simulations.

While the method we use to add energy into the gas from star formation processes is virtually the same as the method preferred by Gerritsen & Icke (1997), the amount and rate at which we add it is experimental. Limited resolution requires that we increase the specific energy, and thus the temperature of a $10^6$ solar mass chunk of material to values larger than seem physically reasonable. This is essentially the complimentary problem to using overly small densities and overly large temperatures for
star formation criteria. Using small temperature thresholds \( T < 1000 \text{ K} \) and large density thresholds \( \rho > 10^2 \text{ cm}^{-3} \) seems more physical. In a similar manner, trying to use energy feedback amounts estimated from stellar wind theory and observed supernovae explosions has created problems. The amount of energy we add to each SF particle equals roughly \( 10^{53} \text{ ergs} \) which is about 100 times the energy released by a Type II supernova. This energy is not added all at once: rather, it is ramped up gradually over a time interval corresponding to the approximate lifetime of a massive O star, about \( 10^7 \text{ years} \). We chose this energy input in order to achieve a maximum temperature of about \( 5 \times 10^5 \text{ K} \) in the SF particles. This choice was based on trial and error. Our earliest experiments increased temperatures to as high as \( 3 \times 10^6 \text{ K} \) but were abandoned because they produced unphysical outflows.

The maximum temperature of \( 5 \times 10^5 \text{ K} \) is near the peak of the cooling curve, so the SF particles cool rapidly as soon as the star formation epoch is ended and radiative cooling is allowed to take effect again. The rapid cooling seems to leave insufficient time for the SF particle density to respond. The temperature-density plots of the five models with a density threshold. Figures 4.32 through 4.36, show that the hot gas does not reach densities much lower than the density threshold value of 0.14 cm\(^{-3}\). The SF particles do not push the surrounding particles very much, thus lessening the density of the star forming regions. This density response to the feedback is considered crucial for self-regulation of the star formation, but the SF particles of the models seem to show very little density response to the heating. This may cause some concern as to whether the models are truly self-regulating on a global scale since the local self-regulation process may be stymied by too little feedback energy. However, the results show that the feedback has an effect on the overall morphology and thermal structure of the gas disk. The face-on morphologies and thermal phase fractions differ from each other in an obvious manner between models. The differences most likely arise from the difference in the number of SF particles produced in each model. While we expect differences in the fractions of hot, warm, and cold gas between models, the differences in global spiral structure between models is surprising.

### 5.3.2 Star Formation Criteria

The star formation criteria for each model in Chapter 4 fall into three categories: temperature only (TO), temperature plus density (TD), or density only (DO). In the TO model, a gas particle must fall below a chosen threshold temperature and remain below that temperature for a simulation time of 20 million years before it is considered star forming and heating begins. For the TD models, a gas particle must meet the temperature condition of the TO model and also maintain a density above a chosen threshold density during the same 20 million years to be considered star forming. All four TD models
have the same threshold density, but the temperature threshold is different between one model and the other three. Finally, a gas particle in the DO model need only maintain a density above the chosen threshold for the same amount of time as the other models in order to be marked as star forming. See Table 4.1 in the previous chapter for the list of models and their specific threshold values. In Chapter 4, we compared several properties of the resulting disks, including:

- Spiral structure
- Surface density profile
- Star formation amounts and distribution
- Temperature-Density structure
- Time dependence of the thermal phases

Three of TD models, TD75, TD75+10, and TD75-10, use the same criteria with different initial surface densities and these models give essentially identical results. This suggests that the properties listed above depend only very weakly on the initial surface density of the disks. Two of the TD models, TD90 and TD75, have star formation criteria that differ only in the value of the temperature threshold criterion (they have the same initial surface density, too). The main difference between these two models is the number of SF particles they produce. The TD90 model produces 71% more SF particles than the TD75 model. Not surprisingly, the fraction of hot gas \( T > 10^4 \text{ K} \) in the TD90 model exceeds the fraction of hot gas in the TD75 model by a factor of two. We expect all of the TD models to have few SF particles than either the TO or the DO, since the TD models place two restrictions rather than one on the star formation. As expected, the TD models generate the fewest SF particles even when their threshold values are the same as the values of the TO and DO thresholds. Likewise, the hot fraction is smallest in the TD models among all of the models. The fraction of hot gas is directly determined by the amount of star formation (the number of SF particles).

The fractions of warm gas \( (T \approx 10^4 \text{ K}) \) and cold gas \( (T < 10^4 \text{ K}) \) differ among the models, as well. When the initial model is allowed to run for the standard 9900 timesteps with no star formation feedback, the resulting warm and cold fractions separate widely, with the cold gas making up nearly 75% of the disk (there is no hot gas phase since there is no star formation heating). The large cold gas fraction (> 50%) of the TD75 models is not surprising since these models have the smallest amount of star formation. The TD90 model, with twice the hot fraction of the TD75 model, has a warm fraction over 50%. The ratio of warm to cold gas in TD90 is the inverse of that in TD75. The TO model has significantly more star formation than the TD models and has a correspondingly larger hot fraction. The warm phase dominates the TO model, since any gas that falls below the temperature
threshold and remains there, no matter its density, will eventually become an SF particle and be heated. Consequently, the cold fraction of the TO model is the smallest of the models. The DO model, which produces the most star formation and, thus, the largest hot fraction, has a larger cold fraction AND a smaller warm fraction than the TO model. In fact, the warm and cold fraction are nearly equal in the DO model with the warm fraction only slightly higher than the cold. This comparison of the warm and cold fractions among the different models demonstrates an interesting result. The values of the warm and cold fractions do not depend solely on the amount of star formation, but also on how the star formation is determined.

We might expect that the DO model, with so much hot gas compared to the TO model, would have a comparable amount of warm gas. Instead, it has significantly less warm gas than TO. The DO model is heating more of the gas from the warm phase to the hot phase, while TO converts only gas from the cold phase into the hot phase. In reality, stars form from the coldest, densest regions of the ISM converting the cold condensed phase into the hot diffuse phase. From the point of view of the artificial thermal phases defined above, the TO model is more correct since it changes only cold gas to hot, while the DO model converts both warm and cold gas into the hot phase. As we shall discuss in the next section, the TO model produces a global spiral pattern, a star formation distribution, and an amount of star formation that matches observation more closely than does the DO model. However, the TO model has a different problem. It contains an unphysical distribution of SF particles in regions of low density behind spiral waves and in the outermost disk. The cold low density regions of the disk are themselves unphysical and a consequence of limited particle resolution as discussed in Section 5.2. Better particle resolution in future simulations should minimize this effect.

A striking difference among the TO, TD, and DO models is the dependence of the global spiral structure on the star formation criteria. All three types of models produce spiral structures different from the spiral structure of the initial disk. The TO and TD models exhibit more spiral wavefronts with more contrast (thinner, denser arms) than the initial disk, while the DO model exhibits very weak spiral arm segments that start farther out from the center than those of the initial disk or the TO and TD models. We might attribute the differences in spiral waves among models to differences in the gravitational stability of the disks. Equation 4.1 from Chapter 4 defines the critical surface density, $\Sigma_{\text{crit}}$, for gravitational stability of a flat, rotating disk (see Binney & Tremaine 1994, Kennicutt 1989). Figure 5.1 plots the ratio of the gas surface density, $\Sigma_{\text{gas}}$, to $\Sigma_{\text{crit}}$ versus the radius for each model. This ratio equals the inverse of the Q stability criteria of Toomre (1964), $Q = \Sigma_{\text{crit}}/\Sigma_{\text{gas}}$. Regions of large $Q$ are stable to disturbances, but waves passing through regions of small $Q$ can grow to create
large scale density fluctuations. Kennicutt (1989) has found that $Q < 1.5$ is a reasonable criteria for gravitational instability in star-forming, late-type galaxy disks since most disks in his sample obtained a value of $Q > 1.5$ beyond the radius of the HII disk, i.e. where the star formation is truncated. The horizontal line on Figure 5.1 indicates $Q \approx 1.5$. Our models tend to satisfy this instability criteria in the flat rotation curve region and fall far enough below $\Sigma_{\text{crit}}$ at radii greater than about 27 kpc to become stable again.

![Figure 5.1 $\Sigma_{\text{gas}}/\Sigma_{\text{crit}}$ versus R for the TO, TD90, TD75, TD75+10, TD75−10, DO, and no-feedback models at timestep 9900. The vertical line marks the end of the flat rotation curve. The horizontal line denotes $\Sigma_{\text{gas}}/\Sigma_{\text{crit}} = 0.67$ corresponding to $Q \approx 1.5$.](image)

This radius corresponds closely to the edge of the star-forming disk in all but the TO model. The TO model falls near or below this stability threshold throughout most of the disk, while the other models stay above it in the flat rotation region. The star formation in the TO model occurs even in low density regions making the amount of star formation less sensitive to this gravitational instability. The greater amount of star formation in the outer disk of TO generates a higher velocity dispersion there, thus increasing $\Sigma_{\text{crit}}$ in the outer disk. Consequently, $\Sigma_{\text{gas}}$ of the TO model stays just below $\Sigma_{\text{crit}}$ and $Q > 1.5$ all the way through the rising rotation curve region. Even though the TO disk is gravitationally stable, the spiral structure of TO is more pronounced than the initial model with
no feedback and $Q \ll 1.5$. Conversely, the DO model has a $Q < 1.5$ throughout the flat rotation curve region but shows extremely weak spiral structure. Apparently, when it comes to the creation of large-scale spiral structure, gravitational instability is not the whole story. The local interaction of the star formation feedback is at least partially responsible for generating global spiral structure. The idea of generating spiral waves in a late-type disk through reaction-diffusion interactions has been modeled analytically 13 years ago by Nozakura & Ikeuchi (1988), but has not been discussed in the context of full hydrodynamic simulations until now.

The differences observed among the TO, TD, and DO models indicate that our choice of star formation criteria can have profound effects on the global spiral structure and the thermal structure of the models. The general shape of the surface density profile is robust and not affected much by our choice of criteria, though the velocity dispersion and thus critical surface density is increased mildly by the greater star formation in the outer disk of the TO model. This effect is not seen in the TD and DO models since the star formation is truncated by the drop in density at large radii. Using a temperature-only threshold criteria can result in an unphysical distribution of star formation. Density thresholds keep star formation confined to the spiral arms in models with strong spiral features; however, the large amount of star formation in the DO model smoothes the disk nearly eliminating the spiral structure. The TD models are found to be very robust to differences in initial surface density and give generally consistent results between models with different temperature threshold values.

5.4 Comparison to Observation

In this section, we examine some criteria for deciding which star formation model is most realistic. The radial and azimuthal distribution of the SF particles are readily compared to Hα images that trace the distribution of HII regions in external late-type disks. The amount of star formation produced in a model can be compared to observations by counting the SF particles, calibrating the number of HII regions per SF particle and comparing the numbers of SF particles to the number of HII regions observed through radio and Hα emission. A comparison of the thermal phase balance of the models with observations of our own Galaxy and external galaxies may also be a good measure of a successful model. However, our numerical models lack the mass and spatial resolution to distinguish cold clouds from the warm medium. A large fraction of the gas mass of the ISM lie in clouds with temperatures below 1000 K, but our models do not resolve structures small and dense enough to achieve temperatures so low. Comparing the ratio of hot gas (about $10^5$ K) to the warm ($10^4$ K) and cool (less than $10^4$ K) gas may not provide a reasonable measure of the models quality in this case. We therefore rely on
comparisons of star formation distribution and amounts to distinguish among the most realistic models.

5.4.1 Distribution of Star Forming Regions

It is difficult to scale the number of SF particles in the numerical models with the amount of star formation that is observed in luminous late-type galaxies. However, assuming that each SF particle represents one or a few HII regions, the models presented in the previous chapter have a number of SF particles of the same order of magnitude as the number of HII regions observed in external galaxies (see Section 5.4.2 below). All of the models, except the DO model, exhibit spiral structure in the gas disk comparable to observed late-type galaxies. The morphology of the SF particles strongly resembles observations of HII regions in external galaxies. We can simulate a grayscale Hα map of our models for comparison with continuum-subtracted Hα images of observed late-type star forming galaxies by taking advantage of the clock variable for each particle used to measure how long a particle is heating due to star formation. The clock value corresponds to time spent being heated so it should be proportional to the luminosity of each particle. Binning the clock value of each particle in a 100 by 100 grid covering the face-on view of a model, we plot the surface intensity of each bin in grayscale and smooth the values across bin boundaries to produce a simulated Hα map. We use the clock value instead of the temperature in order to more easily pick out the SF particles and to give better contrast in the final image.

As an example, we compare the continuum-subtracted Hα image of the nearby, face-on Sc galaxy, NGC 628 (from Lelièvre & Roy 2000), with the face-on smoothed surface plots of the TO, TD90, and TD75 models in Figure 5.2. HII regions in NGC 628 are observable in its outer disk at distances greater than, \( R_{25} \), the \( B = 25 \) blue isophote (Ferguson et al. 1998, Lelièvre & Roy 2000) (\( R_{25} \) is often used as a measure of disk radius). All three models have a generally similar morphology to the example galaxy, NGC 628, though each model has unique differences from the real galaxy. The TO model exhibits HII regions in the far outer disk similar to NGC 628, but the HII regions of TO are not as well confined to the spiral arms as those in the observation. This is especially true in the outer-most regions of the disk. The TD90 model does not contain HII regions in it’s outer disk, but the HII regions are more confined to the spiral arms than the TO model. The TD75 model has too little star formation compared to the other two models, though the HII regions are confined to the spiral arms. The TD75+10 and TD75-10 models look very similar to the TD75 model and are not shown here. The DO model, also not shown (refer instead to Figure 4.29), has a smooth distribution of SF particles in the central disk with very weak spiral structure farther out. It does not resemble the late-type disk of NGC 628.
5.4.2 Scaling the Amount of Star Formation

The mass assigned to the SPH gas particles (see Table 4.1) places a lower limit on the mass of any gravitationally bound region that is resolvable by the simulation. This mass is of the order of $10^8 M_\odot$ or the mass of a very large GMC. We may scale the amount of star formation in our numerical models to compare with observation by assuming each SF particle represents a set number of HII regions. A simple estimate of the average number of HII regions per SF particle may be obtained from observations of the sizes and densities of HII regions in the Galaxy and external galaxies. By determining a typical mass of an HII region in late-type spirals from observation we estimate an upper limit of HII regions per SF particle. A typical HII region that can be detected in an external galaxy has a mass of $10^4 M_\odot$ though masses range from a few times $10^2 M_\odot$ up to $10^8 M_\odot$ (e.g. Kennicutt, Edgar, & Hodge 1989). Choosing $10^4 M_\odot$ as the mass of a typical observable HII region places an upper limit of 100 HII regions per SF particle. We must assume some fraction of HII region mass to SF particle mass in order to get a number of HII regions per SF particle between zero and 100. We suggest that this fraction is related to the efficiency of turning gas mass to star mass and should be of the order of a few percent. To get an order of magnitude estimate, we choose this fraction to be 1% so that each SF particle corresponds to
one HII region. The DO model contains about 3500 to 5000 SF particles depending on what phase of its oscillation we consider. The TO model produces about 1200 SF particles at any given timestep, while the TD90 model produces about 700 SF particles. The remaining models, the TD75 series, contain between 400 and 500 SF particles. Observations of external galaxies detect anywhere from a few tens of HII regions to just over 1000 (e.g. Kennicutt et al. 1989). Our rough scaling gives reasonable numbers of HII regions in all but the DO model.

Over all, the TD90 model seems the closest fit to observations of the models presented. All of the models have a $1/r$ surface density profile and velocity dispersions on the order of 10 km/s. However, TD90 is distinguished from the other models be a combination of three things; the right amount of star formation, the global spiral pattern typical of late-type disks, and star formation regions confined to the spiral arms. The TO and other TD models have one or more of these attributes but only the TD90 model has all three. The DO model, which lacks all three attributes, fits observation the least well.

5.5 Comparison of the Computational and Analytic Models

The solution to the equations of motion in the analytic model assume a constant circular speed; therefore, they are valid only in the flat rotation region of a disk. The rising rotation region is assumed to be small relative to the rest of the disk—and is ignored. Therefore, I can only compare the behavior of the flat rotation region of the numerical models with that of the analytic model. Unfortunately, the flat rotation region of the numerical models suffers from three limitations:

1.) It is smaller than its physical analogue.
2.) It is not entirely flat.
3.) The lack of resolution makes it difficult to produce star formation in the flat rotation region without producing too much star formation in the center.

These limitations make comparison between the behavior of the analytic and numerical models difficult. The numerical models do not produce vertical gradients of the radial or azimuthal velocities predicted by our analytic model. This does not necessarily indicate a failing of the analytic model; rather, the numerical models do not have the resolution to distinguish between the warm and cold phases on the scale necessary to observe these gradients. However, the amount of feedback in the numerical models also failed to produce velocity dispersions much larger than 10 km/s in the disk. It should be noted that the analytic model predicts very small radial and azimuthal velocity gradients for disks with velocity dispersions of only 10 km/s. The numerical model still might produce these gradients given sufficient heat to increase the turbulent motions in the gas. Since observations show
that 10 km/s is a typical dispersion for late-type disks, it may also be the case that the analytic model is an "asymptotic limit", more applicable to starburst disks.
CHAPTER 6 SUMMARY

In this chapter, I summarize the primary results and accomplishments of this dissertation. Section 6.1 restates the initial questions we wished to address and some new questions that we encountered along the way, and Section 6.2 summarizes the answers we discovered. In Section 6.3, I review the objectives stated at the end of Chapter 1 and discuss whether or not they have been met. Finally, in Section 6.4, I discuss future work and some plans on how to do it.

6.1 Questions Asked

Our analytic model addresses the question of whether a preferred global steady state can exist in such a dynamic and turbulent medium as the ISM of late-type spiral galaxies, and what the properties of that state might be. How does feedback from turbulent energy sources such as supernovae and stellar winds effect the structure and kinematics of these disks? The 1/r surface density profile observed in disks combined with the continuity equation allow for constant (or zero) opposing radial flow solutions to the hydrodynamic equations of a multi-phase system. Why do disks prefer this surface density profile? What role does the feedback play in this self-regulation?

Our original purpose for constructing numerical models of late-type galaxy gas disks was to attempt to test the predictions of the analytic model, particularly the slow radial flows that may be impossible to detect by observations of real galaxies. As is usually the case with research, we uncovered more questions than answers. The resolution of ours and others' numerical models is not good enough to include realistic star formation criteria or energy feedback mechanisms, so the different formalisms we and others have employed to include this star formation feedback are largely experimental. What are the best formalisms that have been tested and published? More importantly, how do we define what is meant by "best" or even "good enough"?

Once we have decided what methods to use to incorporate feedback in our models we look to the models for answers about the nature of the disks. What is the nature of the state into which these models relax? Does turbulence affect the global structure of the disk, as in the analytic model? Is
the feedback sufficient to produce substantial turbulence in the disk? How does the feedback affect the global structure of the disk?

6.2 Questions Answered

Our analytic model is a similarity solution to the gas dynamic equations of motion that include turbulent viscosity terms that help to support the warm gas against gravity by transferring angular momentum between warm and cold opposing radial flows. The energy produced by star formation activity is transmitted through the ISM via shocks and dissipative turbulent flows and provides a mechanism by which the global structure of the disk may regulate itself to a preferred state. We agree with Kennicutt's (1989) argument that the surface density is regulated to the critical surface density for gravitational instability which follows a 1/r profile. We suggest that turbulent dissipation of the star formation feedback provides an important vehicle to regulate the disk globally.

We have produced quasi-steady state numerical models of gas disks which incorporate star formation feedback. The first time star formation begins to heat the gas in each simulation, the initial model is perturbed and requires some time to settle to a steady state. All of our models, except the DO model, achieve a quasi-steady state in about 1000 timesteps or a few times $10^8$ years. The collapse timescale and heating timescale for one gas particle are parameters that we set in the code. Both are on the order of 50 timesteps. The cooling timescale depends heavily on the density and temperature of the particle, though for most of our models the cooling timescale for particles heated by star formation is very short, about 10 timesteps, once the heating ends. Cooling for particles with temperatures around $10^4$ K occurs much more slowly. On the other hand, the heating process acts locally on a timescale much shorter than the time it takes the models to settle to a steady state. This suggests that the timescale for relaxation to a steady state is not a numerical artifact imposed by the parameterized heating. However, it may be strongly coupled to the cooling rate we use for particles at or below $10^4$ K.

An example of an oscillatory state (as opposed to a steady state) is manifest in the DO model. This model exhibits a very regular temporal variation in the amount of hot gas. The period of oscillation corresponds directly to the heating timescales imposed by the star formation criteria and duration. This oscillation is a direct result of the implementation of the star formation criteria and feedback, and is not physical. This example demonstrates an important pitfall, and care must be taken to examine the timescales of different behaviors when some timescales are explicitly (or implicitly) imposed on the model by the implementation of feedback.

Other authors have studied different implementations of star formation feedback mainly in galaxy
formation models (e.g. Mihos & Hernquist 1994, Gerritsen & Icke 1997, Carraro, Chiosi & Lia 1998, Buonomo, Carraro, Chiosi & Lia 2000, Thacker & Couchman 2000). General criteria for star formation are common to many of these models: a density threshold criterion, a temperature threshold criterion, and a collapse time criterion. These criteria are implemented differently by different authors, but they are conceptually similar across models. We have found that delaying the heating until a particle has remained in the threshold regime is the most important criterion to allow star formation to occur throughout the disk. This criterion acts in a similar manner to the “convergent flow” criterion that is used by some authors. No matter whether we use a temperature or density or both criteria, the collapse time criterion suppresses star formation in rapidly changing regions of the disk, particularly in the center.

In order to determine which formalism gives the “best” results, we must chose a set of criteria for judging our models. Thermal phases are not sufficiently well resolved enough to distinguish the cold, condensed clouds where most of the mass of the ISM resides. Consequently, we cannot use ratios of thermal phases as a good indicator of a realistic model. In general, we find that mass and spatial resolution limit us to comparing the amount and distribution of the star forming regions produced by the models to observations as a way of determining the most realistic models. From these comparisons we determine that our TD90 model emulates observation marginally better than our other models. The combined temperature and density thresholds give the best distribution of star forming regions, as well as a reasonable amount of star formation when the temperature threshold is about 9000 K. Lower thresholds give significantly less star formation.

The choice of how feedback should be injected into the gas is based mainly on the morphology produced by different methods and has been discussed at length by Gerritsen & Icke (1997). They determined that adding the energy directly to the star forming particle while inhibiting radiative cooling over some lifetime of the star forming region, typically the lifetime of a massive O star, gives the most realistic models in terms of the morphology of the model disk. The amount of energy per SF particle is also an important parameter that has been generally estimated from supernovae luminosities and stellar wind speed considerations. However, basing this amount on these types of estimates suffers from analogous limitations to choosing temperature and density threshold values based on the temperature and density of molecular cloud cores. Poor resolution requires us to use higher temperatures and lower densities than seem physical. This energy input is another value that remains largely experimental.

The models presented in Chapter 4 heat SF particles to a maximum temperature of $5 \times 10^5$ K. Because this temperature is near the peak of the cooling curve, SF particles cool rapidly after the
heating ends. We observe from plots of temperature versus density that the SF particles are not hot long enough to decrease their density significantly. However, the feedback still has an effect on the global spiral patterns of the disks. This spiral structure of the model disks depends on the amount and location of the star formation, which is constrained by the star formation criteria. Our discovery that feedback has a strong influence on the spiral waves of the gas disk is exciting, but the mechanism of this influence is still mysterious, especially when the feedback seems to have little effect on the local density of the star forming regions. We speculate that the turbulent waves generated locally by feedback in star forming regions combine into self-organizing, large-scale disturbances that, in turn, affect the global spiral structure of the disk.

6.3 Objectives Met?

We incorporate turbulent energy sources from star formation processes in our analytic model. This feedback regulates the global disk structure to a quasi-steady state on timescales smaller than the gas consumption timescale. This model demonstrates a possible mechanism by which the universal structure of disks can be interpreted as a preferred state to which the disks may regulate themselves.

Our numerical models do not have the resolution to reproduce the behavior predicted by our analytic model, namely the slow radial flow nor the different rotational speeds of different layers. Also, the energy input we use is not sufficient to produce large velocity dispersions in the gas. While our numerical models produce general features observed in real galaxies, the resolution is not sufficient to produce the more specific features of our analytic model. Consequently, it is difficult and perhaps not possible to infer anything from our numerical models that would relate to the parameters of our analytic model.

Nonetheless, our numerical models have demonstrated that feedback has an effect on the global structure of gas disks. One surprising effect we discovered is the influence of the feedback on the spiral waves. Our studies of these models have provided several new questions to answer and avenues to explore, which is the true point of any exercise.

6.4 Future Work

The current mass and spatial resolution of our numerical models is inadequate to do a proper comparative study with the analytic model. I am very interested in parallel programming and would like to see the Hydra code adapted to run on a distributed memory parallel computing platform such as a Beowulf cluster. With a parallel version of the code and the proper hardware, a million particle
simulation should be possible. This would allow us a factor of ten better mass resolution which should be enough to resolve molecular clouds consisting of more than one SPH particle. This resolution may open up the possibility of using star formation criteria that more closely approximate reality. More importantly, it should also allow us to distinguish condensed clouds and diffuse gas providing a model that can be more directly compared with the analytic model and with observation.

In addition, the analytic model needs to be examined in the context of perturbations from the steady-state. I would like to examine perturbations of the analytic model to study how robust the predicted "preferred state" really is. I also want to add the time-dependent terms to the equations of the analytic model to study the secular evolution of the solutions. I need to generalize the analytic model to include a range of thermal phases. Finally, I would like to examine the possible behavior of the gas at the inner and outer boundaries of the disk.
APPENDIX A  A PLAUSIBILITY ARGUMENT FOR RADIAL
ENTROPY INCREASE AND THE SCHMIDT LAW

The positive radial entropy gradients in the WHM component of the model presented in Chapter 2
may seem unphysical, especially since this gas is expanding in an average outward flow. However, there
is an obvious entropy source in the nonlinear acoustic waves that partially support this flow. At the
same time, radiative cooling provides an obvious entropy sink, yet when this gas experiences significant
cooling it is generally transformed into the cool component. Thus, on average, the entropy of gas that
stays in the WHM either increases, or is balanced by adiabatic expansion.

Specifically, consider a non-cooling element of the WHM moving outward in the mean flow above
the midplane of the disk. Suppose for simplicity, that its specific entropy is significantly increased
only when it passes directly over a young star cluster, assuming such star clusters are the primary
source of shock turbulence. Then, the rate at which the entropy of that element is increased will be
proportional to the number of young clusters it passes over per unit time. (Note that because of the
reduced azimuthal velocity of the WHM, the element will pursue a spiral trajectory as viewed in a
reference frame comoving with the midplane gas.)

If the SFR is described by a Schmidt Law, and assuming most new stars are born in clusters, then
the number of clusters within a thin annulus of width \( \Delta r \), at radius \( r \) is,

\[
N_{cl} = 2\pi r \Delta r \left( \frac{\psi}{M_{cl}} \right) \tau_{cl} = c_1 \frac{\Delta r}{\tau_{cl}^{1+m}}.
\]  

(A.1)

In this equation, \( \psi \) is the usual SFR (mass of stars produced per unit area per unit time), \( M_{cl} \) is the
mean cluster mass, and \( \tau_{cl} \) is the mean lifetime of the massive stars in the cluster. The final equality
assumes a Schmidt Law of the form \( \psi \propto \Sigma_c^{2+m} \), and a surface density profile of the form \( \Sigma_c \propto 1/r \).
The constant \( c_1 \) is the combination of all constants in the previous equality.

We further assume that the gas element expands in the azimuthal direction by an amount propor­
tional to \( r \) as it moves outward. This is just the expansion that is required to maintain the assumed
surface density profile. It also guarantees that the WHM element covers a constant fraction of each thin
annulus it crosses. Thus, the gas element crosses a constant fraction of the clusters \( N_d \) in each thin annulus, and it is reasonable to assume that the rate of shock hits and entropy increase it experiences is proportional to this annular cluster fraction. Thus, the net entropy increase in traveling from radius \( r_1 \) to radius \( r \) is

\[
\Delta S \simeq c_2 \int \frac{dr}{r} \simeq c_2 \log(\tau/r_1),
\]

in the case \( m = 0 \). The constant \( c_2 \) contains the product of the earlier constant \( c_1 \), the annulus fraction covered by the gas element, and the mean entropy input per cluster.

For a perfect gas,

\[
S = \log(P/p^\gamma) = \log(K_V(r)),
\]

where the second equality makes use of equation (6), in section 2.3, (and the subscript \( V \) emphasizes that these are volume quantities). Assuming that this equals the preceding equation to within an additive constant, we have,

\[
K_V(r) \propto r^{c_2}.
\]

As noted in section 2.3 the quasi-steady model presented above requires that \( c_2 = 1 \). One factor contained within the constant \( c_2 \) is the magnitude of the SFR, e.g., the SFR at a particular radius. Thus, the self-regulating feedback processes can adjust the SFR in such a way that \( c_2 \) is driven towards unity, giving the desired entropy gradient.

Therefore, the fact that the entropy gradient is a power-law follows from the \( m = 0 \) spatial dependence of the Schmidt Law SFR, while the value of the power depends on the magnitude of the SFR. In other words, the entropy gradient required for a hydrodynamic steady state can be achieved by feedback adjustments to the amplitude and spatial dependence of the SFR.

Now let us consider the effects of these self-regulating processes from a slightly different point of view. According to equations (6) and (7) the mean WHM temperature scales as, \( T_w \propto (1 + \beta)K_Vr^{2/3} \). For a globally adiabatic gas, \( K \) is constant, and \( T_w \propto 1/r \) (for constant \( \beta \)). While, as described above, for a locally adiabatic gas, with a radial entropy gradient such that \( K_V \propto r \), we have \( T_w \) constant. If the latter alternative does not obtain, then the variation of scale height with radius will be different than the \( h \propto r^{1/2} \) form assumed above. Qualitatively, if the scale height increases less rapidly, then warm gas remains closer to the midplane and is denser, so cooling rates are increased. If more of this
gas goes into the cool phase, we expect that the SFR will increase (relative to the quasi-steady model), driving increased turbulence and heating, and increasing the scale height.

Conversely, if the scale height increases more rapidly with radius, the SFR will be less than in the steady model, eventually diminishing pressure support, and reducing the scale height. More generally, because of the temperature dependence of the cooling rates, significant temperature gradients would likely result in pressure imbalances, which would lead to time-dependent convection (as well as thermal conduction). That is, temperature gradient states do not generally satisfy the steady state equations above.

These considerations are qualitative, and have loopholes, but they do show why global states with modest entropy gradients would be preferred. They also provide some insight into how closely the Schmidt Law SFR is connected to such states.
APPENDIX B  THE STAR FORMATION FEEDBACK CODE

B.1  Heat Subroutine

I wrote the following subroutine for Hydra Version 3.0. This subroutine implements the temperature-density threshold criteria for star formation as described in Chapter 3. It can be modified to use either a temperature-only or a density-only criterion. The subroutine is called in the "updaterv" subroutine.

```
SUBROUTINE heat(i,dn,e,clk)
  c
  c This subroutine determines whether or not a gas particle
  c meets certain criteria to begin star formation. Then it
  c adds energy directly to the particle to represent
  c heating due to that star formation.
  c
  c i = particle number
  c dn = density of particle i
  c e = internal energy of particle i
  c clk = clock for particle i
  c
  c The current version of 'heat' has the following properties
  c as of 12/14/00:
  c
  c ramped up (to max) and finitely sustained heating due to SF
  c finite lifetime of heating (approx. lifetime of 0 star)
  c temperature and density SF threshold
  c time delay criterion
  c
```
include 'psize.inc'
include 'pinfo.inc'

integer cntr
real dn,e,clk
real dncrit,dncrit0,elmill,halfmill,tlife
real growl,shrink1,dnscale1,tscale1
real rcenter,elk,e10k,ecrit

save cntr
data cntr /0/

Scaling factors

growl=float(L)
growl=growl-2.*padding
shrink1=1./growl
dnscale1=shrink1**3
tscale1=growl**2
rcenter=padding+0.5*growl+1

Added by dcs Aug., Oct., and Nov. 99
Make heating due to star formation continue for
the life time of an 0 star, about 10 million years.
The gas is heated to 1/2 million degrees.
On 1/21/00 dcs softened heating by increasing e(i)
by 10% incrementally till maximum SF temp is reached.

e1mill=tscale1*2.17046437008
halfmill = e1mill/2
elk=tscale1*2.170e-3
e10k=tscale1*2.170e-2
ecrit=0.75*e10k
dncrit=200*dnscale1

c On 12/12/00 dcs set dncrit = 200 code units (0.14 cm^-3).

c On 11/13/00 dcs set ecrit = 9000K.

c On 01/26/01 dcs set ecrit = 7500K.

c Criterion for SF is temperature and density threshold (12/12/00 dcs).

c Lifetime of massive stars about 10Myr.

tlife=0.010

c

c Check for SF criteria and do SF heating.

if((e.lt.ecrit.and.dn.gt.dncrit).
 & or.clk.ge.0.0) then
 oldclk=clk
 clk=clk+dtime
 if(clk.ge.0.0.and.clk.lt.tlife) then
  if (oldclk.lt.0.0) cntr=cntr+1
  if(e.ge.halfmill) then
   e=halfmill
  else
   e=1.1*e
  endif
 endif
 endif
else
 clk=-0.020
endif

c

c Reset clock when it runs past tlife.

if (clk.ge.tlife) then
 clk=-0.020
 cntr=cntr-1
 end if

c
******************************************************************************
B.2 Updaterv Subroutine

Below is the portion of the updaterv subroutine from Hydra that calls the heat subroutine. It also calls the standard radiative cooling subroutine, cool, as well as doing the ad hoc cooling I added below $10^4$ K. This code only calls the cool subroutine or does the ad hoc cooling below $10^4$ K if the clk variable is less than zero. The clk variable counts up from $-0.020$ (corresponding to 20 million years) when the particle is in the threshold regime. It does not begin heating until the clk reaches zero. If the particle leaves the threshold regime before clk gets to zero, clk is immediately reset to $-0.020$.

```fortran
    #ifdef HEAT
        CALL heat(i,dn(i),e(i),clk(i))
    #endif

    c Scaling info needed to do extra cooling.
    growl=float(L)
    growl=growl-2.*padding
    tscalel=growl**2

    e1k=tscalel*2.170e-3
    e10k=tscalel*2.170e-2

    if (clk(i).lt.0.0) then
        if (e(i).le.e10k.and.e(i).ge.e1k) then
            c I am increasing the amount of cooling below 10,000 K by a factor of ten. Here is the old delam:
            c delam = 0.131*tscale1*dn(i)*dtime
            c Below is the new, increased delam. dcs 1/26/01
```
c

delam=1.31*tscale1*dn(i)*dt ime

e(i)=e(i)-delam

else

CALL cool(dn(i),e(i),dt,zmetc)

endif

endif

c
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