RECONSTRUCTION OF ELASTIC CONSTANTS FROM ULTRASONIC REFLECTIVITY DATA IN A FLUID COUPLED COMPOSITE PLATE

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INTRODUCTION

One of the important requirements for advanced composite materials is satisfaction of structure stiffness to the levels preset by the design. Such a requirement underlines the importance of nondestructive evaluation as a quality control tool to establish equivalence of the elastic properties of a manufactured composite to that of the designed composite. Several recent papers have been devoted to development of such nondestructive techniques, mainly based on application of bulk ultrasonic waves [1-5] and Lamb waves [6-8].

In this paper we introduce a nonlinear inversion scheme for reconstruction of the full matrix of elastic constants from ultrasonic experimental data on reflectivity from an orthotropic composite plate immersed in a fluid. In the reconstruction algorithm we used analytical expressions for reflection coefficients from arbitrary oriented orthotropic plate received recently in paper [9]. A different approximate inversion scheme for reflectivity data has been recently used in paper [10] and reconstruction of elastic constants from ultrasonic transmission data has recently been utilized in paper [11].

THEORETICAL BACKGROUND AND STATEMENT OF THE PROBLEM

Reflection coefficient of ultrasonic waves from a solid plate immersed in a fluid has oscillatory irregular behavior as a function of incident angle and frequency. The maxima of the reflection coefficient is alternated by its minima which correspond to real or complex zeroes. The reflection coefficient minima are usually represented in the form of a family of dispersion curves with incident angle $\theta$ (or projection of the wave velocity on plate plane $V_0/\sin \theta$) as y ordinate and parameter $fh$ as x ($f$ is frequency and $h$ is plate thickness). Experimentally these curves may be obtained by changing frequency at a given angle of incidence and finding reflection coefficient zeroes or by changing the angle of incidence.
at a given frequency. The zeroes of the reflection coefficient have been
for a long time closely associated with Lamb's wave spectrum in the plate
which is not always correct. Their actual relation has been studied in
detail only recently [12].

Intensive theoretical and experimental studies of the zeroes of the
reflection coefficient spectrum for composite material have been reported
recently [9,13,14]. In this work we will use an analytical representation
of Nayfeh and Chimenti [9] for the reflection coefficient of an ultrasonic
wave from an orthotropic plate immersed in fluid. The reflection coeffi-
cient \( R \) may be written in the following general form:

\[
R = \frac{A S - Y^2}{(S+iY)(A-iY)}
\]  

(1)

where \( A \) and \( S \) are the symmetric (S) and antisymmetric (A) Lamb wave dis-

persion functions for an orthotropic plate in a vacuum, \((S+iY)\) and \((A-iY)\)

are that for a plate in fluid (dispersion functions of leaky Lamb waves)
(see Ref. 9 for details). Function \( Y \) appears due to fluid loading and

depends on fluid parameters. Amplitude of the reflection coefficient \( R \) is a function of incident angle \( \theta \), parameter \( f_h \), fluid to composite den-
sities \( C_{ik} \), angle \( \phi \) of fiber orientation relative to the incident plane
and composite elastic constants;

\[
R = R(C_{in}, \rho, f_h, \theta, \phi)
\]  

(2)

In particular the location of \( R \) minima in the \( \theta - f_h \) plane will also be
dependent on this set of parameters.

In this work we will address the solution of the inverse problem: to
find material properties of composite \( C_{in}, \rho \), from experimental data on
\( R \) minima (in the \( f_h - \theta \) plane each experimental point "i" of the \( R \) minima
is determined by measured values of parameters \( \theta_i \) and \( (f_h)_i \)).

RECONSTRUCTION ALGORITHM

Let \((f_h){_i}_{k}\) and \(\theta_i\) be the set of variables which give the minima of the
reflection coefficient of ultrasonic wave:

\[
R[C_{in}, \rho, (f_h){_i}_{k}, \theta_i] = R_{\text{min}}
\]  

(3)

where \( R_{\text{min}} > 0 \), \( C_{1m} \) are composite elastic constants, \( i \) is index specifying
incident angle \( \theta_i \), \( k \) is index specifying parameter \((f_h){_i}_{k}\) at a given incident
angle \( \theta_i \), \( 1 \leq i, k \leq N \), (here \( N \) is the overall number of points in the \( \theta - f_h \)
plane). Let \( E(f_h){_i}_{k} \) be the set of experimental values measured as minima
of the reflection coefficient at a given angle \( \theta_i \). The problem is stated
as a nonlinear least square minimization: Estimate a set of the parameters
\( \hat{P} = (C_{1m}, \rho) \) which minimize the norm \( \Delta \):

\[
\Delta = \frac{1}{2} \sum_{i,k}^N |(f_h){_i}_{k} - E(f_h){_i}_{k}|^2 = ||\hat{f}_h - E(\hat{f}_h)||
\]

The dimension of the unknown vector \( \hat{P} \) is equal to the number of parameters
to be estimated. The values \((f_h){_i}_{k}\) and the parameters to be found are
related by an implicit nonlinear operator \( L \). The calculations are performed
in the following steps:

1) At a step of minimization "n" the initial set of parameters is

\( C^n_{in}, \rho^n \).
2) For given \( \theta_i \) find the approximate values \((fh)_{ki}^{j}\) which give minimum values of reflection coefficient (2) by direct calculations.

3) Using \((fh)_{ki}^{j}\) as initial guesses find precise minima by iterative procedures.

4) Order found solutions by correspondence to branches of RC minima starting from low frequencies. (This step is especially important for uniqueness of the final minimization).

5) Calculate deviations \( |(fh)_{ki}^{j} - E(fh)_{kj}^{i} |^2 \)

6) Find gradient and select new set of optimization parameters \( C_{n+1}, \rho_{n+1} \).

Thickness of the plate \( h \) may also be included in optimization scheme as one of the unknown parameters.

The advantage of this least-square scheme is that it is relatively stable to the scatter of the experimental data and converges even with poor initial guesses of parameters.

![Fig. 1 Plots illustrate sensitivity of the one branch of the reflection minima to the variations of the \( C_{11} \) (left) and \( C_{33} \) (right) elastic constants.](image)

**EXPERIMENTAL TECHNIQUE**

The reflection coefficient minima have been measured on a 0.92mm unidirectional graphite-epoxy composite plate as discussed in papers [9]. Special attention has been paid to measurements at small angles of incidence which are essential for reconstruction of the elastic constant \( C_{11} \) in the fiber direction (see next section).
Fig. 2 Summary of the minima loci sensitivity to the variations of the different elastic constants at three angles of incident plane orientations to the fiber direction ($\phi = 0^\circ, 60^\circ$ and $90^\circ$).
SENSITIVITY OF THE LOCI OF THE REFLECTION COEFFICIENT MINIMA TO THE DIFFERENT ELASTIC CONSTANTS

It is clear that if the measured RC minima depend only slightly on elastic constants (estimated parameters), inverse reconstruction of the elastic constants from the experimental data will be ill posed: i.e., due to noise in the data, reconstructed values will be erroneous (in actual calculations final results will be close to the initial guesses). From the other side, if different parts of the RC minima loci depend on different elastic constants, the problem may be simplified and reconstruction of subsets of the parameters may be performed from subsets of the data.

We performed detailed analysis of the sensitivity of the RC minima to the elastic constants (nine constants for orthotropic material). An important example is shown in Fig. 1 for $C_{11}$ and $C_{33}$ for the case of plane of incidence in fiber direction $\phi = 0$. The results are shown for the second branch. The most important conclusion which follows from calculation for this and other modes is that reflection minima position are sensitive to the $C_{11}$ constant only at incident angles below first critical angle. The sensitivity to the $C_{33}$ constant is above this angle.

The results are summarized in Fig. 2, where areas of sensitivity to different elastic constants are shown by arrows. It is interesting to point out that all previously published experimental data for reflectivity minima are measured above the first critical angle and therefore completely insensitive to the longitudinal elastic modulus $C_{11}$ in the fiber direction.

Fig. 3 Calculation of the reflection minima from the set of elastic constants, reconstructed from the data with noise, where $\varepsilon$ is relative noise ($\varepsilon = 0.1\%$, 1% and 5%).

\[ \phi = 0^\circ \]

\[ \varepsilon = 0.001 \]

\[ \varepsilon = 0.01 \]

\[ \varepsilon = 0.05 \]
SENSITIVITY OF THE RECONSTRUCTION TO THE NOISE IN DATA (Simulated Results)

To check the stability of the reconstructed elastic constants to noise in the experimental data, the reconstruction was performed with simulated noise. Typical examples are shown in Fig. 3, with calculations for initial data with random noise (examples are shown for standard deviations of noise: 0.1%, 1% and 5%). The simulated experimental data are shown by stars. The solid lines are the calculated loci of the minima using reconstructed elastic constants from the data with noise. Some of the results are shown in Table 1. In the top row are the exact values of the elastic constants (at noise level $\phi = 0$). Below are the reconstructed values shown for different noise levels. The results are stable to different initial guesses. One can see that the effect of noise becomes significant for the $C_{13}$ constant. This result is not unexpected since it is known that its calculation is also sensitive to noise in data in bulk wave measurements. The overall results are satisfactory even for 5% uncertainty in the experimental data.

RESULTS OF RECONSTRUCTION FROM EXPERIMENTAL DATA

The results of reconstruction are summarized in Table II. All nine elastic constants have been considered independent. It is found that $C_{33} > C_{22}$, which is consistent with the fact that during material manufacturing pressure was applied in the direction of axis 3. Elastic constants $C_{11}$, $C_{12}$, $C_{13}$, $C_{33}$, and $C_{55}$ have been reconstructed from the data at $\phi = 0$ (incident plane along fiber direction); $C_{22}$, $C_{23}$ and $C_{44}$ from the data at $\phi = 90^\circ$ and $C_{66}$ from the data at $60^\circ$. One should point out that sensitivity of reflection minima to constants $C_{12}$ and $C_{66}$ is relatively small and reconstructed values have low confidence.

Based on the received elastic constants, loci of the reflection minima have been recalculated. The results of calculations are shown in Figure 4 together with experimental points (circles). Results for the incidence plane in the fiber direction ($\phi = 0$) are excellent, and for $90^\circ$ and $60^\circ$ the results are good. Slight degradation of the data quality for $90^\circ$ and $60^\circ$ is due to the high density of branches which complicates both experimental measurements and reconstruction.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>Summary of the reconstructed elastic constants (in GPa) from data with different noise.</th>
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<tbody>
<tr>
<td></td>
<td>$C_{11}$</td>
</tr>
<tr>
<td>GIVEN</td>
<td>115.6</td>
</tr>
<tr>
<td>$\phi = 0^\circ$</td>
<td></td>
</tr>
<tr>
<td>NOISE</td>
<td>$0.001$</td>
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<tr>
<td>RECONSTRUCTED RESULTS</td>
<td>$0.01$</td>
</tr>
<tr>
<td></td>
<td>$0.05$</td>
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</table>

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>Elastic constants reconstructed from experimental data in GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{11}$</td>
<td>$C_{22}$</td>
</tr>
<tr>
<td>133</td>
<td>12.9</td>
</tr>
</tbody>
</table>

1416
Fig. 4. Loci of the reflection minima calculated from reconstructed elastic constants listed in Table 1. Initial experimental data are shown as circles. Results are for three orientations ($\phi = 0, 60^\circ$ and $90^\circ$).
CONCLUSION

Accurate computational schemes are proposed for reconstruction of composite elastic constants from ultrasonic reflectivity data from an immersed composite plate. The inversion is based on a nonlinear least square minimization of deviations between experimental and calculated data of the RC minima in the incident angle-frequency plane. It is found that the reconstruction scheme is stable to the initial guesses of elastic constants and to noise in the experimental data. It has also been found that the positions of the minima are sensitive to the longitudinal modulus along the fiber direction at incident angles only below the first critical angle (below 9° for the graphite-epoxy composite).

REFERENCES

8. V. K. Kinra and V. Dazal, "Ultrasonic NDE of Composites Using Lamb Waves: Theory and Experiment" (this Proceedings).