A THEORETICAL MODEL FOR MAGNETIC FIELD MAPPING AS A MEANS OF BROKEN FIBER DETECTION IN GRAPHITE EPOXY COMPOSITES

William T. Yost

NASA Langley Research Center
MailStop 231
Hampton, VA 23665-5225

INTRODUCTION

Composite materials are gaining popularity in the aerospace industry because of their light weight and high strength, and the possibility to fabricate pieces to meet specific design needs. With the anticipated increase in usage, including the use in critical parts, it is essential to develop methods and techniques to determine the soundness of a structure fabricated from these materials.

According to fracture experts, when composite structures are under tension, they are most likely to fail because of broken fibers. At the time when we started this project, NASA was planning to build a casing for the solid rocket motor assembly. This casing was to be a graphite-epoxy system, which was to be wound on an assembly with metal flanges as end plates. It was our purpose to develop nondestructive methods to locate and assess regions where fibers may be broken. We developed a technique for location of broken fibers by imaging magnetic fields generated from currents injected in the fibers, and comparing this to the results of an image calculated from the Biot-Savart Law, using the nodes in a resistive network as the source points for the magnetic field. (1)

The purpose of this work is to give an analysis that can explain the major features of the earlier work, and to determine what other information can be obtained from their results. We do this by writing Laplace's Equation and solving it for the special case where resistive components can be considered analogous to a network of resistors.

THE EXPERIMENTAL STUDY

To test the concept, we used rectangularly shaped coupons that were constructed from 8-ply laminas of graphite-epoxy. A small (1/16 inch) hole was drilled in the center of each coupon to simulate broken fibers. Current was injected at the ends of the coupons along the length dimension, as shown in Fig. 1. The y-component of the magnetic field ($B_y$) near the surface of the composite was measured on a grid in a 2 cm x 2 cm area which was centered at the hole. Using a resistive network to model the conductive fibers, as shown in Fig. 2, and the Biot-Savart Law, we calculated $B_y$ by summing all magnetic field contributions from each node in the model. The images of the actual magnetic field plots are shown in Figs. 3 and 4. Fig. 5 shows an image, which is a plot of the magnetic field ($B_y$) calculated from the network model. Other details of the experiment and how the plots were formed from the resistive network model are contained in Ref. 1.
The first magnetic field image (Fig 3) was made from an 8-ply unidirectional T300/5208 composite, while the second image (Fig. 4) was made from a composite fabricated from the same materials, but with a fiber orientation of (0°,±45°,90°). In both figures, we see that $B_y$ drops directly above the hole. Moreover, an elongation of the dimension of the hole appears in the x-direction. There also appears substantial lobes on either side of the hole. This is especially evident in Fig. 4. From the image calculated from the model, these lobes are symmetric about a line passing through the center of the hole, and parallel to the y-axis. The lobes, however, appear to be less symmetric in the measured field images. This is probably due to a slight asymmetry in the actual fiber placement in the composite material. Otherwise, the agreement between the images from the experimentally measured fields and those calculated from the resistive network model was reasonable.

The resistive network used in the Biot-Savart calculation of the magnetic field was constructed in the following way: The network resistors lying in the x-direction were

![Damage Zone](image1)

**Fig. 1** Simplified view of typical coupon sample, with coordinate frame

![Resistive network model](image2)

**Fig. 2** Resistive network model used to calculate the y-component of the magnetic field

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assigned the same value, $R_x$. Those lying in the y-direction, were also assigned equal values, $R_y$. Resistors in the region of the 'hole' were changed to values several orders of magnitude higher than either $R_x$ or $R_y$. We found that by changing the ratio of $R_x / R_y$, we could alter the ratio of major to minor axis of the magnetic field 'image' of the hole. This ratio was adjusted until the calculated image of the hole was congruent with the measured image.

THEORETICAL TREATMENT

We started with the resistive network used in the original model, and established the coordinate system accordingly, as shown in Fig. 6. We begin the analysis with the equation of continuity,

$$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0 \ ,$$

where $J$ is the current density, and $\rho$ is the charge density. We employ steady state conditions, and hence, we can write the current density in terms of the electric field as
where $E$ is the electric field, which we write as the gradient of the potential, $\nabla V$. The conductivity varies in different directions, depending largely on the orientation direction of the fibers. To match the earlier work with its resistive network, we write Eq. 1 as

$$
\sigma_\parallel \frac{\partial^2 V}{\partial x^2} + \sigma_\perp \frac{\partial^2 V}{\partial y^2} = 0,
$$

where $\sigma_\parallel$ and $\sigma_\perp$ are the effective conductivities along the $x$ and $y$ directions respectively.

Case 1

The first case is where the conductivities are not equal. Making the transformation,

$$
\xi = x, \quad \eta = \sqrt{\frac{\sigma_\parallel}{\sigma_\perp}} y,
$$

gives Laplace's Equation,

$$
\frac{\partial^2 V}{\partial \xi^2} + \frac{\partial^2 V}{\partial \eta^2} = 0,
$$

and transforms the circle,

$$
\xi^2 + \eta^2 = 1^2,
$$

into an ellipse,

$$
\frac{\xi^2}{a^2} + \frac{\eta^2}{b^2} = 1,
$$

with

$$
a^2 = \frac{\sigma_\parallel}{\sigma_\perp} 1^2
$$

and

$$
b^2 = 1^2.
$$

Dividing Eq. 8 by Eq. 9, and taking the positive square root gives

$$
\frac{a}{b} = \sqrt{\frac{\sigma_\parallel}{\sigma_\perp}}.
$$

Eq. 10 permits some interpretation of the samples measured in Ref. 1. We find that the "short" axis is the same as the hole diameter, while the long axis is scaled by the square root of the ratio of $\sigma_\parallel$ to $\sigma_\perp$. Measurements of the magnetic field pattern for the
unidirectional composite sample, which is shown in Fig. 3 gives a ratio of axes lengths of approximately 10, which implies that the ratio of conductivities is approximately 100. For the case of the (0°, ±45°, 90°) sample, the magnetic field scan gives an axes length ratio of approximately 3, which implies that the ratio of conductivities is approximately 9. This is a plausible value for this sample given its small dimensions and the fact that the current insertion turned out to be primarily in the 0° direction, while current insertion was less complete in the ±45° directions, due to the sample dimensions.

**Case 2. \( \sigma_\parallel = \sigma_\perp \)**

Because of the simplicity of the solution and the ease of interpretation, we choose the case, where the conductivities in the x-direction and the y-direction are equal. For this case, we have

\[
\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \ .
\]  

(11)

The solution of Eq. 11 depends on the following boundary conditions:

\[
as \ r \to \infty \ , \quad V = -E \ x = -E \ r \ \cos \ \theta , \quad (12a)
\]

\[
\text{When} \ r = a \ , \quad \frac{\partial V}{\partial n} = 0 \quad , \quad (12b)
\]

where \( n \) is the unit vector. The solution is

\[
V = -\rho \ \text{J}_0 \ r \ \left( 1 + \frac{a^2}{r^2} \right) \ \cos \ \theta \ , \quad r > a \ , \quad (13a)
\]

\[
V = -2 \ \rho \ \text{J}_0 \ r \ \cos \ \theta \ , \quad r < a \ , \quad (13b)
\]

where \( V \) is the potential, \( \rho \) is the resistivity of the medium, \( \theta \) is the angle between the x-axis and the radius vector, and \( \text{J}_0 \) is the current density at \( r = \infty \). Taking the gradient of \( V \), and using Eq. 2, we solve for the current density in the x-direction, which is given by

\[
\text{J}_x = \text{J}_0 \ \left( 1 - \frac{a^2}{r^2} \ \cos \ 2\theta \right) \ .
\]  

(14)

When \( r = a \), we have

\[
\text{J}_x = 0; \ ( \ \theta = 0, \ \pi ) \ , \quad (15a)
\]

\[
\text{J}_x = 2 \ \text{J}_0; \ ( \ \theta = \frac{\pi}{2}, \ \frac{3\pi}{2}) \ .
\]  

(15b)

This shows an increase in the current at the edges of the hole \( (\theta = \pi/2, \ 3\pi/2) \) giving a prediction of current bulges. This analysis holds for cases where conductivity in the y-direction is equal to the conductivity in the x-direction, which matches construction of a woven fiber condition where the fiber density is the same in both directions. For most composites, however, the construction is different. One would therefore expect that the shape and size of the lobes would vary from the shape as shown in the plots of Ref. 1.

**RESULTS SUMMARY**

A straightforward analysis of the current injection method by using the Biot-Savart Law, the Equation of Continuity, and Laplace's Equation yields some interesting information.
about composites constructed from graphite and epoxy. We find, for example, that the
magnetic field drops at breaks in the fibers. Because of the tendency of fibers to "bleed
across", probably at points in closest proximity, the magnetic field shows an elliptically-
shaped pattern, where the ratio of major to minor axes tells us about the conductivities. With
each break, one can also expect that side lobe patterns will form. These are caused by the
variation in current distributions around the broken fibers. Variants in these patterns could
conceivably give insights into the local disruption near the fiber breakage. Such information
could help in assessment of delaminations around the break, and repairability of the site in
the composite.

CONCLUSIONS

One can get useful information by injecting current into the fibers of composite
materials, when those fibers are conductive. On the basis of this and earlier research, it can
be established that:
(a) The magnetic field pattern drops abruptly near the axis of a hole, or a break in fibers.
(b) The magnetic "image" of the hole gives contours with the shape of an ellipse. By
measuring the semimajor and semiminor axis of the image, one can determine the ratios of
the conductivities. These ratios depend on feedthrough at proximity sites where fibers of the
different directions of the layup can make electrical contact with other fibers in adjacent
layers. The effects of delaminations should alter these values.
(c) The analysis also predicts the existence of two current bulges on each side of the hole ($\theta = \pi/2, 3\pi/2$), with the bulges being symmetric about the line defined by $\theta = \pi/2, 3\pi/2$.

Reference

1. T. N. Blaylock and W. T. Yost, in Review of Progress in Quantitative NDE, edited by