 Essays on vertical product differentiation

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Essays on vertical product differentiation

by

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in partial fulfillment of the requirements for the degree of

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ABSTRACT

This dissertation explores models of heterogeneous product markets that rely on the "vertical product differentiation" formulation, where products differ in their quality levels and consumers differ in their willingness to pay for quality. The demand structure applied here is the covered-market configuration under the vertical product differentiation. With this specification, product market equilibria of the monopoly and duopoly market are derived. In particular, parameter restrictions on the degree of relative consumer heterogeneity associated with the covered-market setting are identified and used to interpret analytical results. Based on the specified demand structure, I revisit two industrial organization topics from the perspectives of vertical product differentiation.

The first essay analyzes the entry of a new product into a vertically differentiated market where an entrant and an incumbent compete in prices. Many models on strategic entry deterrence deal with "limit quantities" as the established firm's strategic tool to deter or accommodate entry. Here, however, the entry-deterrence strategies of the incumbent firm rely on "limit qualities". With a sequential choice of quality, quality-dependent marginal production cost, and a fixed entry cost, I relate the entry-quality decision and the entry-deterrence strategies to the level of an entry cost and the degree of consumer heterogeneity. In particular, the incumbent influences the quality choice of the entrant by choosing its quality level before the entrant. This allows the incumbent to "limit" the entrant's entry decision and quality levels. Quality-dependent marginal production costs in the model entail the possibility of inferior-quality entry as well as the incumbent's aggressive entry-deterrence strategies by increasing its quality level towards potential entry. Welfare evaluation confirms that social welfare is not necessarily improved when entry is encouraged rather than deterred.

The second essay is motivated by some specific economic questions that have arisen with the introduction of 'genetically modified' (GM) agricultural products. A duopoly market-entry model associated with the vertical product differentiation is developed to show how the existence of segregation costs biases the firm's quality choice behavior. Thus, the key factor of the model is the cost of segregation activities that are necessary to distinguish
GM products from non-GM products. With an increasing and convex cost of quality, the model predicts that the entrant firm has an increased incentive to enter the market with a low-quality good to reduce production costs if segregation costs are sufficiently high. When consumers are homogeneous enough, however, entry may occur with the high-quality good.
CHAPTER 1. GENERAL INTRODUCTION

Product differentiation is an important subject in the field of industrial organization since Chamberlain’s (1933) model of monopolistic competition. The reason is that most products sold by firms are not identical but truly differentiated, i.e., consumers do not view such goods as perfect substitutes. Differentiation models intend to explain which goods will be produced in a specified economy. In Chamberlain-type monopolistic competition models, each consumer is allowed to buy all varieties. In a discrete choice model, however, each consumer is allowed to buy only one variety. Families of such discrete-choice product differentiation models are of two fundamental types: one is the “horizontal product differentiation” (HPD) model of Hotelling (1929), where product varieties are characterized by the different consumers’ opinions, and the other is the “vertical product differentiation” (VPD) model where product qualities are ranked in the same way by consumers.\(^1\) Thus, while all products can have positive market share at the same price with the HPD structure, only one product will be bought if products are offered at the same price with the VPD structure. This dissertation focuses on the VPD approach, and develops a set of models to address some economic issues in this area.

1.1 An Overview of VPD Models

VPD is defined as the case in which all consumers prefer the higher quality when all varieties are offered at the same price (Shaked and Sutton, 1983). In the standard VPD model of Mussa and Rosen (1978) and Gabszewicz and Thisse (1979),

\(^1\) An interesting paper by Cremer and Thisse (1991), however, showed that the horizontal product differentiation is actually a special case of a VPD model if the marginal cost is a quadratic function of the quality.
consumers have heterogeneous preferences and each consumer buys only one variant. Mussa and Rosen-type models introduce a continuous distribution of consumers with differing preferences for qualities. In this framework it is therefore possible to express explicitly how demands are affected by quality differences.²

Mussa and Rosen-type models have dominated VPD studies of firms' quality-choice behavior, due to its yielding explicit form of demand for differentiated good. The extensive use of this model, in part, also is due to the convenient tools suggested by Tirole (1988), which is a simplified version of Shaked and Sutton (1982, 1983). Previous quality-choice models were mostly carried out for a duopoly purely due to the complexity of dealing with the multiple discontinuities in profit functions. The basic VPD framework of duopoly entails two periods: quality choice followed by simultaneous product market competition, where each firm is allowed to offer only one quality.³

Alternative studies recognized a sequential process of product innovation (e.g., Beath, Katsoulacos, and Ulph (1987) and Aoki and Prusa (1996)) or the timing of the introduction of new products incorporating learning-by-doing (e.g., Gruber (1992)) at the firm's stage of quality choice.⁴ Other important applications of the VPD model include minimum quality standards (e.g., Ronnen (1991) and Maxwell (1998)).

Most models with heterogeneous consumer preferences use a linear indirect

² In contrast to this formulation, in Dixit (1979) – type representative consumer models such as Singh and Vives (1984), Bester and Petrakis (1993), and Lin and Saggi (2002), each consumer is allowed to buy all varieties. They used a quasi-linear utility function to derive linear demands and to eliminate income effects. In particular, because only the degree of product differentiation matters with this type of model, we cannot say that a good is superior to the others.

³ As an exception, in a two-firm, two-stage game setting, Champsaur and Rochet (1989) examined a case where each firm is allowed to offer a whole range of qualities.

⁴ For example, Aoki and Prusa (1996) analyzed how the timing of investment decisions affects the levels of quality chosen by firms. In their analysis, they showed that sequential quality choice induces duopolists to make smaller quality investments than they would in a game with simultaneous quality choice.
utility function for each type of consumer and a uniform distribution on consumers' tastes to obtain an explicit solution of the game, with attention restricted to the assumptions of an uncovered market (e.g., Ronnen (1991), Choi and Shin (1992), Motta (1993), Aoki and Prusa (1996), Lehman-Grube (1997), and Bonanno and Haworth (1998)) and covered market (e.g., Tirole (1988: 296-298), Rosenkranz (1995), and Pepall (1997)). The implicit assumption is that each consumer purchases at most one unit of goods or services. However, the representation of the firms' marginal production costs is different depending on the purpose of the study. Very simple quality-choice games are established in the absence of production costs, and by assuming that quality choice is costless (e.g., Choi and Shin (1992), Lehman-Grube (1997), and Tirole (1988)). In this case, however, qualities demanded are independent of qualities. In the model of Mussa and Rosen (1978) and Bonanno and Haworth (1998), to avoid equilibria in which only the highest quality, yet the cheapest product is produced, quality-dependent constant marginal production cost is introduced, such that the higher quality good is assumed to be more expensive to manufacture. For example, quality improvement requires more skilled labor or expensive inputs. In this case, the quality-dependent marginal cost enters directly into the competitor's pricing strategy. Meanwhile, conventionally it is assumed that the R&D costs to bring about product innovation are sunk, convex, and strictly increasing in the quality level (e.g., part II of Motta (1993)).

---

5 A duopoly market is said to be covered if all consumers purchase one unit of either good. In an uncovered market setting, some consumers are allowed not to purchase at all.

6 In particular, the assumption of unit purchase seems realistic when dealing with professional services such as doctoral services and lawyer services. Also, when people would like to buy pianos, cars, and so on, the problem is not how many to buy but rather whether to buy and, if yes, what variety (Gabszewicz and Thisse, 1979).

7 However, results are not the same for this type of cost formulation. For example, assuming an uncovered market, Choi and Shin (1992) showed that the lower quality firm will choose a quality level which is a fixed proportion of the higher quality firm's choice. By contrast, Tirole (1988) shows that firms maximize product differentiation over the available range of qualities, with the covered-market assumption.
For duopoly VPD models, there are two important outcomes. First is the "maximal product differentiation" result that attains under the covered market setting. In a very simple quality-choice game model, Tirole (1988), by using the modified version of Shaked and Sutton (1982), showed that firms maximize product differentiation over the available range of qualities. Even though the model displays the absence of quality-choice costs, because price competition is more intensified the less differentiated are the goods, price competition gives firms the incentive to differentiate their products. Thus, the optimal solution for the first stage problem is the maximal product differentiation where one firm chooses the minimum possible quality and the other firm chooses the maximum possible quality. The second result is the "high-quality advantage" where the firm choosing to produce the high-quality good earns a higher profit in equilibrium than does the low-quality firm under the assumption of quality-independent marginal production costs. Tirole (1988) and Choi and Shin (1992) showed this result with the simultaneous quality choice game. The high-quality advantage has been found to hold in a sequential quality choice game by Aoki and Prusa (1996) and Lehmann-Grube (1997).

However, as we will show through proceeding chapters, both the "maximal product differentiation" and the "high-quality advantage" do not necessarily hold with the specification of quality-dependent variable costs. If one of the firms entered first (sequential choice of quality), that firm may not choose the high quality because no particular variety guarantees higher profits. In fact, there is a possibility of inferior quality entry. Also, with this cost specification, although firms want to differentiate products for strategic purpose (e.g., to soften price competition), qualities can be internally determined rather than maximally differentiated in the feasible quality interval.

In many economic models associated with the homogeneous good market
analysis, there is a scope for extensions to the VPD setting associated with heterogeneity properties of the good and consumer preferences. For example, previous R&D models where product heterogeneity was not considered could be reformulated with the introduction of a VPD model (e.g., Greenstein and Ramey, 1998). In this context, this dissertation also introduces two familiar topics in homogeneous good market models, and provides more analyses from the VPD perspectives.

### 1.2 Dissertation Organization

The remaining parts of dissertation are organized in four chapters focusing on a static model of a market for differentiated products, where a set of products are heterogeneous. In chapter 2, the question how the vertically differentiated product market can be segmented is explicitly investigated. In particular, the monopoly and duopoly demand systems identified here are applied to the proceeding chapters. In chapter 3, I revisit the entry-deterrence strategies of an existing firm in the context of "limit qualities", which is an extension of the analysis of many models on strategic entry deterrence that deal with "limit quantities" as the established firm's strategic tool to deter or accommodate entry. In chapter 4, by introducing the market entry game associated with product R&D rather than process R&D, I analyze a specific question about how private decisions by an innovator bring inferior or superior technologies in the presence of segregation costs. The last chapter summarizes the dissertation and briefly outlines opportunities for additional research.

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8 Greenstein and Ramey (1998) reassessed Arrow's (1962) well-known result concerning the effect of market structure on the returns from process innovation, by using the framework of product innovations that are vertically differentiated from older products.
CHAPTER 2. DEMAND SYSTEMS AND PRODUCT MARKET EQUILIBRIUM

In this dissertation, we follow a Mussa and Rosen (1978) or Shaked and Sutton (1982)-type VPD model to investigate firms' quality choices in the context of a non-cooperative two-stage game of duopolists, where each firm is allowed to offer only one quality, and where investments in quality are made in the first stage and then product market competition occurs in the second stage. At present, we ignore which firm produces and sells which good. Following standard practice for this type of a model, we assume a linear indirect utility function for each consumer and also assume a uniform distribution of consumers' tastes. We restrict our attention to the covered market configuration for analytical tractability.

2.1 Demand Structure under the Configuration of a Covered Market

Suppose that, on the demand side of the market, a continuum of potential consumers is differentiated by the non-negative, one-dimensional taste parameter $\theta$. The parameter $\theta$ is assumed to be distributed uniformly with density $\delta > 0$ over an interval $[\overline{\theta}, \overline{\theta}]$, with $\overline{\theta} > \overline{\theta} > 0$. We normalize the indices as $\overline{\theta} = 1$ and $\overline{\theta} - \overline{\theta} = 1$. When entry takes place, we have a situation with two goods differentiated by a quality index $X_i \in (0, \infty), i = 1, 2$, that is observable to all. It is assumed that, as proposed by Mussa and Rosen (1978), the indirect utility function of a consumer $\theta$ patronizing good $i$ is:

$$V_i(P_i, X_i, \theta) = \theta X_i - P_i,$$

where $P_i$ and $X_i$ for $i=\{1, 2\}$ are, respectively, the price and quality variables. Thus, consumers have identical preferences but differ in their taste parameter, $\theta$. Consumer $\theta$
is willing to pay up to $\theta X_i$ dollars for one unit of the product $i$. Hence his or her surplus is expressed as $V_i = \theta X_i - P_i$. In this setting, the consumer buys the good that provides the highest surplus or buys nothing if $V_i < 0$ for both goods.

Three market outcomes arise from this type of demand structure. First is the case of a partially covered market. Because a consumer buys if and only if his or her net surplus $V_i = \theta X_i - P_i$ is positive, the marginal consumer $\theta_{0i} = \frac{P_i}{X_i}$ is indifferent between buying a low-quality good 1 and not buying at all. Then consumers located at the low end where $\theta < \theta_{0i}$ do not buy either good. By denoting as $\theta_{12} = \frac{P_2 - P_1}{X_2 - X_1}$ the marginal consumer who is indifferent between buying good 1 and buying good 2, consumers between $\theta_{0i}$ and $\theta_{12}$ buy the low-quality good, while consumers with $\theta \geq \theta_{12}$ buy the high-quality good. Therefore, when the market is not covered, the demands for good 1 and good 2 are, respectively, given by $Q_1 = \theta_{12} - \theta_{0i}$ and $Q_2 = \bar{\theta} - \theta_{12}$. Second is the case where a market is fully covered at the market equilibrium. For a market to be covered, the least value consumer for quality should have non-negative surplus by purchasing the (low-quality) good. That is, we need a parameter restriction $\theta \geq \theta_{0i}$ for this type of a market configuration. Third, both covered and uncovered market configurations have the possibility of the preempted market. If there does not exist sufficient heterogeneity among consumers, then only the firm offering the lowest quality good or the firm offering the highest quality good may get a positive market share at the market equilibrium. In other words, there may be a case where an inferior quality covers all the

---

1 We follow the convention that a consumer indifferent between two products ($\theta = \theta_{12}$) is assumed to choose the product of higher quality.
product market while a superior quality is unmarketable, or vice versa.

In this dissertation, we focus only on the covered market where all consumers purchase positive quantities of the good. Then, for given prices \((P_1, P_2)\), the covered market demand systems incorporating the possibility of the preempted market case are:

\[
Q_1 = \max \left\{ 0, \min \{\theta, \theta_{12}\} - \theta \right\} \\
Q_2 = \max \left\{ 0, \theta - \max \{\theta_{12}, \theta\} \right\}, \text{ where } \theta_{12} = \frac{P_2 - P_1}{X_2 - X_1}.
\]

Therefore, covered-market equilibrium can be characterized by the cases in which only the high-quality good is sold, only the low-quality good is sold, or both types of goods are present in the market. In particular, for the cases where both goods are present, the aggregate demand functions reflect a net substitution pattern (i.e., the cross price effect is positive). The market segmentation for each type of product is described in Figure 2.1. The consumer surplus for each type of product is on the vertical axis. The slope of the line is the quality level of each product. If there is an increase of product quality, the market interval for that good is enlarged while the market interval for its neighbor product shrinks. Then the following Remark 1 states the necessary condition for both goods to be transacted in the market equilibrium (e.g., Figure 2.1 (a)).

**Remark 1.** Suppose that the market is covered. For both goods to be transacted in equilibrium, it is necessary that the quality-deflated price of the high quality good is greater than that of the low quality good.

**Proof:** The necessary condition for a duopoly market can be proven by contradiction. Suppose that the quality-deflated price of the high quality good is less than or equal to that of the low quality good (i.e., \(\frac{P_2}{X_2} \leq \frac{P_1}{X_1}\)). Then \(\frac{X_2}{P_2} \geq \frac{X_1}{P_1}\), and by using the individual rational constraint \((\theta X_i \geq P_i)\) and the self-selection constraint \((P_2 \geq P_1)\), it
follows that \((\theta X_2 - P_2) - (\theta X_1 - P_1) = P_2 \left( \frac{\theta X_2}{P_2} - 1 \right) - P_1 \left( \frac{\theta X_1}{P_1} - 1 \right) \geq (P_2 - P_1) \left( \frac{\theta X_1}{P_1} - 1 \right) \geq 0\).

Hence, in that case, all consumers would always prefer good 2 to good 1, contradicting the condition for both goods to be transacted in equilibrium. ■

\[
\begin{align*}
V_2 &= X_2 \theta - P_2 \\
V_1 &= X_1 \theta - P_1
\end{align*}
\]

(a) When \(\frac{P_1}{X_1} < \frac{P_2}{X_2}\)

\[
\begin{align*}
\theta_{\alpha} &= \frac{P_1}{X_1} = \frac{P_2}{X_2}
\end{align*}
\]

(b) When \(\frac{P_1}{X_1} = \frac{P_2}{X_2}\)
In the analysis that follows, given qualities $X_1$ and $X_2$, we focus on interior solutions in which both goods are consumed in the market and all consumers are served in equilibrium. Then, when consumers differ in their tastes $\theta$, the duopoly demand functions are defined by the following equations:

\begin{align*}
Q_1 &= \delta \int_{\theta_{12}}^\theta f(\theta) d\theta = \delta \left\{ F(\theta_{12}) - F(\theta) \right\} \\
Q_2 &= \delta \int_{\theta_{12}}^{\bar{\theta}} f(\theta) d\theta = \delta \left\{ F(\bar{\theta}) - F(\theta_{12}) \right\}
\end{align*}

where $f(\bullet)$ is the probability density function of $\theta$ and $F(\bullet)$ is the corresponding cumulative density function. Assuming uniform distribution of consumers, $f(\theta) = \frac{1}{\bar{\theta} - \theta}$, and normalizing indices as $\delta = 1$ and $\bar{\theta} = \delta$, we have demand functions:

\begin{align*}
Q_1 &= \frac{\delta}{\bar{\theta} - \theta}(\theta_{12} - \bar{\theta}) = \theta_{12} - \bar{\theta}
\end{align*}
\[ Q_2 = \frac{\delta}{\theta - \theta_1} (\theta - \theta_2) = \bar{\theta} - \theta_1 \]

where, again, \( \theta_1 = \frac{P_2 - P_1}{X_2 - X_1} \).

### 2.2 Product Market Equilibrium

#### 2.2.1 Monopoly Market Equilibrium

Consider first the monopoly market equilibrium. Because consumers are passive about the market coverage, a monopolist can determine endogenously a covered or uncovered market. Thus, to invoke the assumption of full market coverage, we need to find the parameter restriction where the monopolist would cover the market.

![Figure 2.2 Monopoly Market Segmentation](image)

Let us denote \( \hat{\theta} = \frac{P_M}{X_M} \) (with a subscript ‘M’ standing for the “monopoly”) as the marginal consumer who is indifferent between buying a good and not buying at all. Figure 2.2 describes a market segmentation of the monopoly market. If there were only one quality available, then the monopolist’s demand function would imply a linear market demand curve where the fraction of consumers who are willing to buy a good of quality \( X_M \) at any price \( P_M \) would be equal to \( Q_M = \bar{\theta} - \hat{\theta} \). Assuming that the monopolist’s unit production cost is \( C_M \), the monopolist in the market solves the following maximization problem with respect to price for a given quality.
Max \( \pi_M = (P_M - C_M)Q_M = (P_M - C_M)\left(\bar{\theta} - \frac{P_M}{X_M}\right) \)

From the first order condition for this maximization problem, we know that

\[
\frac{\partial \pi_M}{\partial P_M} = \bar{\theta} + 1 + \frac{C_M}{X_M} - \frac{2P_M}{X_M} \leq 0.
\]

If we have an interior solution such that \( \frac{P_M}{X_M} \), the market is uncovered. But in the case of a corner solution where \( \frac{P_M}{X_M} \), the market is covered for a given quality. For the corner solution, it is necessary that

\[
\bar{\theta} + 1 + \frac{C_M}{X_M} - \frac{2\theta}{X_M} \leq 0 \implies \theta \geq \frac{C_M + X_M}{X_M}
\]

In this covered-market case, the monopolist’s price is at the level at which the least value consumer (\( \theta \)) gives up all her surplus to purchase the good (i.e., \( P_M = \theta X_M \)).

Thus, the monopolist’s product market equilibrium profit is:

\[
\pi_M^* = \theta X_M - C_M
\]

In the special case of strictly convex variable costs in quality where \( C_M = X_M^2 \), as used in this dissertation, for instance, the monopoly market will be covered if \( \theta \geq 1 + X_M \). Then the corresponding product market equilibrium profit of the monopolist is

\[
\pi_M^* = \theta X_M - X_M^2.
\]

### 2.2.2 Duopoly Market Equilibrium

Now, consider the duopoly covered market equilibrium where duopoly firms
move simultaneously in the production stage with Bertrand competition. In this stage of
the game, qualities are exogenous. The market segmentation for each type of a product
is described in Figure 2.3.

![Figure 2.3 Duopoly Covered-Market Segmentation](image)

Given the firms' quality levels, $X_1$ and $X_2$, and their prices, $P_1$ and $P_2$, the
marginal consumer $\theta_{01} = \frac{P_1}{X_1}$ is indifferent between buying a low-quality good 1 and not
buying at all, and the marginal consumer $\theta_{12} = \frac{P_2-P_1}{X_2-X_1}$ is indifferent between buying a
low-quality good 1 and buying a high-quality good 2. Therefore, the demands for good 1
and good 2 are, respectively, given by $Q_{1} = \theta_{12} - \theta$ and $Q_{2} = \theta - \theta_{12}$.

The profit function of the low-quality firm is given by

$$\pi_1 = (P_1 - C_1)Q_1 = (P_1 - C_1)(\theta_{12} - \theta),$$

and that of the high-quality firm is

$$\pi_2 = (P_2 - C_2)Q_2 = (P_2 - C_2)(\theta + 1 - \theta_{12}),$$

where $C_i$, $i = 1, 2$ denotes the unit production costs for each firm $i$. The first order conditions for interior solutions yield the following
two best response functions:

$$P_1 = R_1(P_2) = \frac{1}{2}\left\{P_2 + C_1 - \theta(X_2 - X_1)\right\}$$

---

2 Under the covered-market configuration, total demand is not a function of prices, so demand
functions cannot be inverted. For Cournot competition to be meaningful, the market should be
uncovered by allowing some consumers not to buy differentiated goods (e.g., Motta (1993)).
Note that, for both maximization problems, the second order sufficient conditions associated with a concave objective function is satisfied.

Using the best response functions (7) and (8), and denoting with a superscript the production stage equilibrium values, we find that when both firms are active in the market the equilibrium prices are:

\[ P_1^* = \frac{1}{3} \{ (2C_1 + C_2) + (1 - \theta)(X_2 - X_1) \} \]

\[ P_2^* = \frac{1}{3} \{ (C_1 + 2C_2) + (2 + \theta)(X_2 - X_1) \} \]

The corresponding profits are:

\[ \pi_1^* = \frac{X_2 - X_1}{9} \left( \frac{C_2 - C_1}{X_2 - X_1} + 1 - \theta \right)^2 \]

\[ \pi_2^* = \frac{X_2 - X_1}{9} \left( - \frac{C_2 - C_1}{X_2 - X_1} + 2 + \theta \right)^2 \]

Of course, these solutions only apply when, in equilibrium, the market is in fact covered. Thus, to complete the solution, it remains to check the following two conditions. First, for exogenously given qualities, the necessary condition for both outputs to be positive in product market equilibrium as obtained from (11) and (12) is:

\[ \frac{C_2 - C_1}{X_2 - X_1} - 2 \leq \theta \leq \frac{C_2 - C_1}{X_2 - X_1} + 1 \]

This condition ensures non-negative demands at the duopoly product market equilibrium (i.e., \( Q_1^* = \frac{1}{3} \left\{ \frac{C_2 - C_1 + 1 - \theta}{X_2 - X_1} \right\} \geq 0 \) and \( Q_2^* = \frac{1}{3} \left\{ - \frac{C_2 - C_1 + 2 + \theta}{X_2 - X_1} \right\} \geq 0 \)). As illustrated
in Figure 2.4, the firm producing a low-quality good becomes a monopoly for extremely high consumer heterogeneity (such that $\theta < \frac{C_2 - C_1}{X_2 - X_1} - 2$), whereas the firm producing a high-quality good becomes a monopoly for very low consumer heterogeneity (such that $\theta > \frac{C_2 - C_1}{X_2 - X_1} + 1$). Thus, the above restriction (13) excludes these two extreme cases.

<table>
<thead>
<tr>
<th>Inferior quality preempts the market</th>
<th>Duopoly</th>
<th>Superior quality preempts the market</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>$\frac{C_2 - C_1}{X_2 - X_1} - 2$</td>
<td>$\frac{C_2 - C_1}{X_2 - X_1} + 1$</td>
</tr>
</tbody>
</table>

"Relative consumer heterogeneity" decreases

*Figure 2.4 Post- Innovative Market Structure with a Covered Market*

Second, for a market to be covered, we need to allow the consumer with the lowest marginal willingness-to-pay for quality ($\bar{\theta}$) to have non-negative surplus when she buys one unit of the low-quality product (i.e., $\bar{\theta}X_1 - P_1^* \geq 0$). Thus, for the following parameter restriction, each consumer buys one of the two varieties in non-cooperative equilibrium.

(14) 
$$\theta \geq \frac{(2C_1 + C_2) + (X_2 - X_1)}{2X_1 + X_2}$$

---

3 Heterogeneity, measured here by the ratio $\overline{\theta}/\bar{\theta}$, decreases with $\theta$ (recall that $\overline{\theta} = \theta + 1$): the greater is $\overline{\theta}$, the more homogenous are consumers. Thus, the market is likely to be preempted by the low-quality firm when consumers are relatively heterogeneous, whereas the market is likely to be preempted by the high-quality firm when consumers are relatively homogenous. For the intuitive explanation, note that $\theta$ is the marginal willingness-to-pay for quality. That is, a consumer with higher $\theta$ is willing to pay more for the higher quality good, while a consumer whose taste parameter $\theta$ is very low would not like to pay for the high quality good. Thus, the market will be preempted by the low-quality firm if $\theta$ is very low.
In the essays of this dissertation, the duopoly product market equilibrium associated with the covered market configuration is applied for the special case of a quadratic variable cost function in quality, such that $C_i = X_i^2$.

2.3 Comments on the Covered-Market Configuration

Although suggested earlier by Gabszewicz and Thisse (1979), in most VPD models, covering the market or not is not the strategic problem for firms. For example, as mentioned above, Ronnen (1991), Choi and Shin (1992), Motta (1993), Aoki and Prusa (1996), and Lehman-Grube (1997) used \textit{ex ante} uncovered market configuration in the production stage of the game. However, their models exclude the possibility of covered outcomes or a corner solution in the production stage. On the other hand, in the VPD literature, \textit{ex ante} covered market configuration about market outcomes is often used for its analytical convenience (e.g., Tirole (1988: 296-298), Rosenkranz (1995), and Pepall (1997)). The basic features of the model that we are using in this dissertation are also standard in VPD studies with the covered market configuration.

The \textit{ex ante} choice of using either a covered or an uncovered market configuration is clearly somewhat unsatisfactory. In this regard, Wauthy (1996) attempted a full characterization of quality choices without assuming \textit{ex ante} that the market is, or is not, covered in the production stage of the game. To derive the two-firm market outcomes endogenously for the degree of product differentiation and the extent of consumer heterogeneity, we need to compare three types of market equilibrium values

\footnote{Gabszewicz and Thisse (1979) showed that price competition could yield three price regions: duopoly uncovered-market outcomes where some consumers are allowed to not buying at all, duopoly covered-market outcomes where all consumers buy one of the two products, and the preempted market equilibrium.}
using two different demand systems: (i) uncovered market equilibrium with uncovered market configuration where \( Q_1 = \frac{P_2 - P_1}{X_2 - X_1} - \frac{P_1}{X_1} \) and \( Q_2 = \overline{\theta} - \frac{P_2 - P_1}{X_2 - X_1} \), (ii) covered market equilibrium associated with a corner solution under the uncovered market configuration, and (iii) covered market equilibrium with covered market configuration where \( Q_1 = \frac{P_2 - P_1}{X_2 - X_1} - \theta \) and \( Q_2 = \overline{\theta} - \frac{P_2 - P_1}{X_2 - X_1} \).

To avoid some of the analytic difficulties, however, we follow a number of previous analyses and assume ex ante that the market is characterized by a covered market configuration in the price game. Thus, the market equilibrium is defined only if we are in the covered market configuration where each consumer buys one of two goods offered. Again, this choice is motivated by a desire for which turns out to be critical if we want to keep the quality-choice problem (not yet discussed) tractable.

One last thing to discuss concerns the assumption of a uniform distribution function for \( \theta \). In an alternative approach proposed by Shaked & Sutton (1982, 1983), consumers differ by their incomes rather than by their tastes. That is, the condition \( \theta > 0 \) is equivalent to the condition that all consumers have a strictly positive income. However, our model yields similar qualitative properties to Shaked & Sutton (1982, 1983) (see Tirole (1988)). For example, a higher \( \theta \) corresponds to a lower marginal utility of income and therefore higher income. Recognizing that \( \theta \) may have the same distribution as income, the uniform distribution assumption for \( \theta \) is somewhat at odds with the reality of income distribution. Following most other studies in this area, however, we maintain the uniform distribution assumption because of the powerful significations that it provides.
CHAPTER 3. VERTICAL PRODUCT DIFFERENTIATION, ENTRY-DETERRENCE STRATEGIES, AND ENTRY QUALITIES

3.1 Introduction

The subject of 'vertical product differentiation' (VPD) in which consumers purchase at most one unit of the differentiated product has been applied extensively to explain the quality choice behavior of economic agents. To a degree, this is due to the convenient tools provided by Mussa and Rosen (1978), Gabszewicz and Thisse (1979), Shaked and Sutton (1982, 1983), and Tirole (1988: 296-298). In models of VPD, the product variants differ in their quality, and consumers differ in their willingness to pay for the quality. Although referred earlier by Bain (1956, Chapter 4) and discussed by Dixit (1979), the use of product differentiation advantages of incumbent firms as a source of the entry-deterrence strategy has not been broadly studied. Many models on strategic entry deterrence deal with "limit pricing" or "limit quantities",\(^1\) rather than "limit qualities", as the established firm's strategic tool to deter or accommodate entry. However, we recognize that, in reality, firms may compete in non-price aspects such as product differentiation.\(^2\) Thus, this paper is related to two branches of the literature: product differentiation and entry deterrence.

Dixit (1979) provides an example for the role of product differentiation in strategic entry deterrence and suggests two opposing entry conditions: a greater absolute

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\(^1\) In the model of limit quantities or limit pricing, a quantity-leader can maintain a single-supplier position by expanding output to the level at which a rival prefers to stay out of the market. But, this is different from monopoly position because the quantity-leader cannot charge the monopoly price without inducing entry, unless the entry cost is very large relative to the market.

\(^2\) By observing lack of noticeable entry of new firms over a long period in the ready-to-eat breakfast cereal industry, Schmalensee (1978) recognized that the quality choice can be used to deter rival's entry by providing substitutes to the product of the potential entrant.
advantage in demand for the established firm makes entry harder, whereas a lower cross-
price effect with the potential entrant's product leads to easier entry. However, in his
representative-consumer model where a consumer's utility is a function of all the
differentiated goods, only the degree of product differentiation matters, and no one good
is superior to the others. Some researches introduced horizontal product differentiation
to model variant choice of the firm facing new product entry. For example, Bonnano
(1987) showed that the incumbent may use a location choice (or a product specification)
in order to deter entry. Anderson and Engers (2001) also introduced horizontal product
differentiation with time in the model, to determine the number of firms and the pattern
of firm locations.

In contrast to the formulation of Dixit (1979), Hung and Schmitt (1988, 1992)
and Donnenfeld and Weber (1992, 1995) used a Shaked and Sutton (1982)-type VPD
model where goods can be directly ranked by qualities, to examine how the incumbent's
choice of product quality depends on the size of the entrant's setup costs. The original
VPD model of Shaked and Sutton (1982) showed that how quality differences relax price
competition. In their model, each firm decides simultaneously its quality level in the
stage before price competition: one firm selects the maximum product quality and the
other chooses the minimum quality to lessen price competition in the last stage of the
game, in the absence of entry threat. Although entry deterrence can only be temporary,
Hung and Schmitt (1988, 1992) altered this framework by introducing sequential entry
and subsequent threat of entry. Thus, they showed that the threat of entry induces the
incumbent firm (or the first mover) to provide a lower product quality than the
technological maximum quality. Also, with the threat of entry, they showed that quality
differentiation in duopoly equilibrium is reduced.
The idea of “limit quality”, the minimum quality of the incumbent which deters entry, is clearly suggested by Donnenfeld and Weber (1995). They investigated how product competition among duopoly incumbents (instead of a single incumbent) and a potential entrant’s fixed entry cost affect the entry-deterrence strategies and product qualities. Their result shows that rivalry among incumbents associated with simultaneous quality choice results in excessive entry deterrence while the incumbents are likely to accommodate entry if they collude. In particular, they confirmed the result of Shaked and Sutton (1982) under the assumption of sufficiently high fixed entry costs, in that entry is blockaded and incumbents choose maximally differentiated product qualities to reduce price competition. For low enough fixed entry costs, they showed that entry is accommodated and incumbents will produce extreme qualities to reduce price competition by differentiating their product than that of the entrant. In this case, the entrant chooses a quality in the middle. Finally, when fixed entry costs are moderate entry is deterred by producing less differentiated “limit qualities” which lead to intense price competition among incumbents as well as a potential entrant and low profits.

However, Shaked and Sutton (1982) – type VPD models are based on the assumption that there are fixed costs of quality improvement. Under this assumption, the marginal cost of quality itself varies, but the marginal cost of production (or the variable cost) does not change with product qualities. The results from Hung and Schmitt (1988, 1992) and Donnenfeld and Weber (1995) are also limited to the case of quality-independent costs. Thus, while these can reflect the situation where firms should engage

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3 A similar analysis associated with two incumbents who face a threat of potential entry was presented in Donnenfeld and Weber (1992), in the case of free entry where both variable and fixed costs for improving qualities are zero.

4 Peitz (2002) also introduced two incumbents and a single potential entrant in the model, as in Donnenfeld and Weber (1992, 1995) but with sequential quality choice of the incumbents, to show that higher entry costs make competition intensely as incumbents deter entry.
in R&D or advertising activities to improve qualities, these cannot reflect the variable-cost aspects of quality improvement where the higher quality good is more expensive to manufacture due to, for instance, requirements of more skilled labor or more expensive raw materials and inputs.

The fact that these results obtained may not be robust to different cost specifications has been suggested by Lutz (1996). By introducing “fixed” setup costs and “fixed” quality-dependent costs in which raising quality results primarily in an increase in fixed costs, Lutz (1996) explains how the entry-deterrence behavior of the incumbent depends on the combination of fixed costs and market sizes. Although Lutz (1996) suggested the possibility of various quality-cost specifications in the entry-deterrence model of VPD, still the result is based on the high-quality advantage as in Hung and Schmitt (1988, 1992) and Donnenfeld and Weber (1995). Meanwhile, Bergemann and Välimäki (2002) used the entry game with vertical product differentiation and uncertain demand to investigate optimal entry strategies when the quality of new variety is uncertain and is generated through purchases in the market. However, their research focuses only on the entry strategies without analyzing explicit entry-deterrence strategies.

In the product differentiation model, consumer taste is the most important dimension of the model as demands are classified according to it. However, the literature discussed earlier does not consider directly consumer taste factors to explain

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5 Under the VPD structure with quality-dependent production cost (which is “variable”), the “high-quality advantage” (where the firm choosing to produce the high-quality good earns higher profits in equilibrium than does the low-quality firm) need not hold anymore. Note that Choi and Shin (1992), Tirole (1988), Aoki and Prusa (1996), and Lehmann-Grube (1997) support the high-quality advantage by assuming quality-independent production cost structure, while Lambertini (1996) suggests that the high-quality advantage with sequential or simultaneous quality choice does not necessarily hold under the assumption of quality-dependent production cost.
quality choice behavior facing entry. The properties of the socially optimal qualities in
the presence of entry issues are also not clear. Importantly, whereas entry-deterrence
strategies are discussed, the entrant’s choice of quality levels on whether the entrant will
choose an inferior quality or a superior quality relative to the existing variety is not
highlighted. Thus, this study undertakes to fill these gaps by pursuing the following
three issues. First, to investigate the incumbent monopolist’s strategic entry deterrence
by qualities in a VPD framework, we examine how the level of a fixed entry cost and the
degree of consumer heterogeneity affect the incumbent’s choice of product quality.
Second, we suggest the reason why an innovative entrant chooses a superior or an
inferior technology compared to the existing incumbent’s variety. Thus, firms’ choice
whether to be the low-quality or the high-quality provider is endogenous. We relate this
issue to the entry-deterring strategies of the incumbent firm. Third, we explore the
welfare implications of entry. In particular, we ask how many varieties and what quality
choice of entry are socially desirable, and whether entry deterrence is disadvantageous to
the consumers, and evaluate market equilibrium values relative to socially optimal levels.

Specifically, this study constructs a model of a vertically differentiated product
market in which both prices and product qualities are endogenous, and entry is
endogenous and sequential. We restrict our attention to entry-deterrence strategies with
one incumbent–one potential entrant game. Based on a Mussa and Rosen (1978) type of
VPD framework, we provide a three-stage game: the incumbent decides her product
quality in stage 1; the potential entrant by observing the action taken by the incumbent
decides whether to enter or not, and if she decides to enter what quality will produce in
stage 2; both firms compete in prices in the last stage of the game if there is entry. Our
model is different from conventional VPD set-ups in the following ways. First, we
consider the sequential choice of qualities instead of a simultaneous choice. That is, the
incumbent firm can choose its product quality in advance of an entrant. Second, we accommodate quality-dependent marginal production cost, such that a higher quality is associated with a higher cost. Thus, unlike the assumption of Donnenfeld and Weber (1995) and Remark 1 in Hung and Schmitt (1988), where they assumed that the ranking of firms' profits is identical to the ranking of their qualities (i.e., a higher quality firm earns higher profits than does a lower quality firm), the "high-quality advantage" does not necessarily hold. In addition to the fact that no particular variety guarantees higher profits, under a quality-dependent marginal production cost, although firms want to differentiate products for strategic purpose (i.e., to soften price competition) they do not differentiate them completely but determine them in the interior of the feasible quality interval. Third, we suppose that the incumbent does not incur any entry cost, while the potential entrant must incur a fixed cost in order to enter. Therefore, entry occurs whenever strictly positive profits can be earned. Entry can only be deterred by strategic actions of the incumbent. In particular, the incumbent acts as a Stackelberg leader in determining its product quality level.

The entry-deterrence strategies that we are using for the incumbent firm facing an entry threat are from the pioneer idea of Bain (1956) as used and stated in many studies (e.g., Dixit (1979), chapter 8 of Tirole (1988), and Donnenfeld and Weber (1995)). According to this convention, if the fixed entry cost is large enough, the entrant would stay out of the market even if the incumbent firm ignores the possibility of entry. We will call this case "blockaded entry". Under the blockaded entry regime, the incumbent monopolist does not modify its strategy and still can prevent entry. If entry is not

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6 Indeed, it seems that most quality standards in manufacturing affect variable rather than fixed costs.

7 Maximal product differentiation holds under the covered-market and quality-independent marginal production cost (e.g., Tirole (1988) and Shaked and Sutton (1982)).
blockaded, the incumbent has to compare the benefit of entry prevention against the cost. According to this comparison, the incumbent may either deter or accommodate entry. In the case of a "deterred entry" strategy, the incumbent modifies its behavior by increasing or decreasing quality in order to deter entry, whereas in the case of an "accommodated entry" strategy, the incumbent chooses to allow entry. In our model, therefore, the solution of the "blockaded entry" is from the unconstrained monopolist's maximization problem, while the solution of the "deterred entry" strategy is from the constrained monopolist's maximization problem. The solution of the "accommodated entry" strategy is the Stackelberg's one. In particular, to determine the critical value of an entry cost between deterred entry and accommodated entry, we compare the incumbent's payoff associated with the deterred entry to the payoff associated with accommodated entry.

We fully characterize how fixed entry costs and consumer heterogeneity affect the threshold conditions that describe the incumbent firm's entry-deterrence strategies (blockaded, deterred, and accommodated) and the entrant's quality choice. By introducing the quality-dependent variable costs in the model, we could entail the possibility of inferior-quality entry as well as the incumbent's entry-deterrence strategies by increasing its quality level towards a potential entry.

First, when the entrant's fixed cost is sufficiently low, the entrant's choices are indifferent between entry with an inferior quality and entry with a superior quality, and the incumbent's optimal strategy is to accommodate entry. In this case, the incumbent selects a quality that is higher than the monopolist's choice. Second, if the entry cost is in a certain moderate range, the incumbent engages in entry deterrence by increasing her product quality before the entrant enters the market. Third, for a sufficiently high fixed entry cost, entry is efficiently blockaded and the incumbent chooses the monopolist's
quality level. Fourth, it is shown that while the consumer surplus is higher when the entry is accommodated than in the absence of entry, the maximum total welfare is not necessarily associated with the accommodated entry. In particular, the maximum welfare of the relatively homogenous consumers is attained at the fixed cost level, where entry is deterred. Fifth, for a certain level of fixed entry costs, there are too many varieties in the economy relative to the social optimum. We also show that Stackelberg firms associated with accommodated entry of a high quality strictly oversupply product qualities relative to the social optimum, while those associated with accommodated entry of a low quality strictly undersupply qualities. The incumbent monopolist, whether the entry is deterred or blockaded, strictly undersupplies the product quality relative to the social optimum.

The remaining parts of this chapter proceed as follows. In section 3.2, we present the model and characterize product market equilibrium. In section 3.3, we analyze entry-deterrence strategies by examining quality-stage equilibria under the threat of entry. In particular, we compare two regimes of entry: entry with a superior quality, and entry with an inferior quality. In section 3.4, we investigate the welfare properties of entry deterrence and entry accommodation. In section 3.5, we provide summary and concluding remarks.

3.2 The Model

Our analysis focuses on the entry of an innovative firm into a monopoly market. Consumers are vertically differentiated according to product qualities. Initially, there is a single established firm in an industry, the incumbent labeled ‘I’, who serves the entire market. A single potential entrant labeled ‘E’ enters the market if entry results in positive payoff, and stays out otherwise. The incumbent has a “product differentiation advantage”
relative to the entrant: whereas the entrant incurs a fixed entry cost to enter the
differentiated product market, the incumbent can change its product quality without
incurring fixed costs. We can justify this assumption by noting that only the entrant
needs entry costs for collecting target-market information, advertising a new product, and
investing in new transportation channels; thus, the entry cost is invariant with respect to
eventual quality levels.

The sequence of moves has three periods. In period 1, the incumbent I selects
its product quality $X_I$. In period 2, the potential entrant E decides to enter the market or
not, with product quality $X_E$ after observing $X_I$. Because entry incurs a fixed cost, a
potential entrant decides to enter only if profits exceed the entry cost. If an entrant enters
the market with the same quality as the existing variety, undifferentiated Bertrand
competition eliminates all profits; therefore, we will consider only the differentiated
entry, where $X_E \neq X_I$. In the last period (i.e., in the post-entry market), the firms
compete in prices (if the prospective entrant enters) given qualities. If the entrant stays
out of the market, the incumbent plays as monopoly. In the case where there is entry into
the product market, the equilibrium concept that we employ is subgame perfection with
Bertrand competition at the third stage of the game. Thus, the game is solved backwards.

3.2.1 Costs and Demand Structure

We modify the monopolist’s quality-choice model proposed by Mussa and
Rosen (1978) into the duopoly model associated with an entry game. First of all, in the

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8 The two-stage structure such that firms choose qualities “simultaneously” and compete with
prices at the last stage of the game has been used broadly since Shaked and Sutton (1982), to
capture the idea that prices can be adjusted easily, costlessly, or in the short run, but product
qualities cannot be changed as easily as price choice due to need for the modification of the
appropriate “production facilities”. Our 3-stage structure involving sequential moves in the
quality decision makes possible for the first mover (i.e., the incumbent) to deter entry by an
appropriate quality choice.
second period of the game, we suppose that the quality follower (a potential entrant) is free to choose any quality level by incurring a sunk and deterministic entry cost $F > 0$. That is, an entry cost is invariant with respect to eventual quality levels. As noted earlier, in our model, the quality leader (the incumbent) has a cost advantage relative to the entrant (the quality follower) in that it does not need to incur any fixed cost to determine its product quality.

Upon entrance of the new firm, the resulting duopoly supplies vertically differentiated varieties with one-dimensional qualities $X_i \in (0, \infty)$, $i = 1, 2$, with larger values of $X_i$ corresponding to the higher quality ($X_2 > X_1 > 0$). To avoid the uninteresting equilibrium in which only the highest possible quality, yet cheapest product is produced, we postulate a quality-dependent constant marginal production cost, such that the higher quality good is more expensive to manufacture. Specifically, we assume that both firms employ the same technology where costs of producing $Q_i$ units of quality $X_i$ are:

(1) \[ C(X_i, Q_i) = X_i^\gamma Q_i, \]

where $Q_i$ is the quantity produced by a firm $i$. Note that this variable costs are strictly convex in quality, such that $C'(X_i) > 0$ and $C''(X_i) > 0$ hold, but for given quality we have a constant unit production cost. This specification of VPD where firms compete in prices and incur variable costs of quality is compatible with some earlier models, such as Mussa and Rosen (1978), Champsaur and Rochet (1989), Motta (1993), and Bonanno.

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\(^9\) Of course, with free entry ($F = 0$), the game degenerates to a pure Stackelberg model.
and Haworth (1998). When fixed costs are either absent or quality-independent, in our model, convexity in quality of the variable cost function insures interior solutions in the quality-choosing stage of the game.

On the demand side of the market, a continuum of potential consumers is differentiated by the non-negative, one-dimensional taste parameter \( \theta \). The parameter \( \theta \) is assumed to be distributed uniformly with density \( \delta > 0 \) over an interval \([\underline{\theta}, \overline{\theta}]\), with \( \overline{\theta} > \underline{\theta} > 0 \). We normalize the indices as \( \delta = 1 \) and \( \overline{\theta} - \underline{\theta} = 1 \). When entry takes place, we have a situation with two goods differentiated by a quality index \( X_i \in (0, \infty) \), \( i = 1, 2 \), that is observable to all. It is assumed that, as proposed by Mussa and Rosen (1978), the indirect utility function of a consumer \( \theta \) patronizing good \( i \) is:

\[
V_i(P_i, X_i, \theta) = \theta X_i - P_i,
\]

where \( P_i \) and \( X_i \) for \( i = \{1, 2\} \) are, respectively, the price and quality variables. Thus, consumers have identical preferences but differ in their taste parameter \( \theta \). Consumer \( \theta \) is willing to pay up to \( \theta X_i \) dollars for one unit of the product \( i \). Hence his or her surplus is expressed as \( V_i = \theta X_i - P_i \). In this setting, the consumer buys the good that provides highest surplus or buys nothing if \( V_i < 0 \) for two goods.

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10 With two-stage quality-price or quality-quantity choice VPD models, Bonanno and Haworth (1998) introduced a quality-dependent linear form of marginal cost; Mussa and Rosen (1978) and Part III of Motta (1993) used quality-dependent quadratic forms of marginal cost. Thus, in this case, the quality-dependent marginal cost enters directly into the competitor's pricing strategy. Importantly, although they did not explicitly indicate it, the "high-quality advantage" does not necessarily hold with the quality-dependent variable cost specifications.

11 In an alternative approach proposed by Shaked & Sutton (1982, 1983), consumers differ by their incomes rather than by their tastes. That is, \( \theta > 0 \) is equivalent to the condition that all consumers have a strictly positive income. However, our model yields similar qualitative properties to Shaked & Sutton (1982, 1983) (see, for example, Tirole (1988)). Say, a higher \( \theta \) corresponds to a lower marginal utility of income and therefore a higher income.
Three market outcomes arise from this type of demand structure. First is the case of a partially covered market. Because a consumer buys if and only if his or her net surplus $V_i = \theta X_i - P_i$ is positive, the marginal consumer $\theta_{01} = \frac{P_1}{X_1}$ is indifferent between buying a low-quality good 1 and not buying at all. Then consumers located at the low end where $\theta < \theta_{01}$ do not buy either good. By denoting as $\theta_{12} = \frac{P_2 - P_1}{X_2 - X_1}$ the marginal consumer who is indifferent between buying good 1 and buying good 2, consumers between $\theta_{01}$ and $\theta_{12}$ buy the low-quality good, while consumers with $\theta \geq \theta_{12}$ buy the high-quality good.\(^{12}\) Therefore, when the market is not covered, the demands for good 1 and good 2 are, respectively, given by $Q_1 = \theta_{12} - \theta_{01}$ and $Q_2 = \theta - \theta_{12}$. Second is the case where a market is fully covered at the market equilibrium. For a market to be covered, the least value consumer for quality should have non-negative surplus by purchasing the (low-quality) good. That is, we need a parameter restriction $\theta \geq \theta_{01}$ for this type of a market configuration. Third, both covered and uncovered market configurations have the possibility of the preempted market. If there does not exist sufficient heterogeneity among consumers, then only the firm offering the lowest quality good or the firm offering the highest quality good may get a positive market share at the market equilibrium. In other words, there may be a case where an inferior quality covers all the product market while a superior quality is unmarketable, or vice versa.

In this paper, we focus only on the covered market where all consumers purchase positive quantities of the good. Then, for given prices $(P_1, P_2)$, the covered market demand systems incorporating the possibility of the preempted market case are:

\(^{12}\) We follow the convention that a consumer indifferent between two products ($\theta = \theta_{12}$) is assumed to choose the product of higher quality.
Therefore, covered-market equilibrium can be characterized by the cases in which only the high-quality good is sold, only the low-quality good is sold, or both types of goods are present in the market. In particular, for the cases where both goods are present, the aggregate demand functions reflect a net substitution pattern (i.e., the cross price effect is positive). In the analysis that follows, given qualities $X_1$ and $X_2$, we focus on interior solutions in which both goods are consumed in the market and all consumers are served in equilibrium. That is, covering the market or not is not the strategic problem of firms in our model.\textsuperscript{13} Thus, the market equilibrium that follows applies to the parameter space where each consumer buys one of the two goods offered.

![Figure 3.1 Duopoly Covered-Market Segmentation (1)](image)

\textsuperscript{13} Wauthy (1996) provides a full characterization of quality choices when the covered or uncovered nature of the market is determined endogenously. To gain analytical convenience, however, the covered market assumption is often invoked (e.g., Tirole (1988: 296-298), Rosenkranz (1995), and Pepall (1997)).
The duopoly covered-market segmentation for each type of product is described in Figure 3.1. The consumer surplus for each type of product is on the vertical axis. The slope of the line is the quality level of each product. Note that if there is an increase of product quality, the market interval for that good is enlarged while the market interval for its neighbor product shrinks.

Thus, when consumers differ in their taste $\theta$, the duopoly demand functions are defined by the following equations:

\begin{align*}
Q_1 &= \delta \int_{\theta_{12}}^{\theta_{11}} f(\theta) d\theta = \delta \left[ F(\theta_{11}) - F(\theta) \right] \\
Q_2 &= \delta \int_{\theta_{12}}^{\theta_{22}} f(\theta) d\theta = \delta \left[ F(\theta) - F(\theta_{12}) \right]
\end{align*}

where $f(*)$ is the probability density function of $\theta$ and $F(*)$ is the corresponding cumulative density function. Assuming uniform distribution of consumers, $f(\theta) = \frac{1}{\bar{\theta} - \underline{\theta}}$, and normalizing indices as $\delta = 1$ and $\bar{\theta} - \underline{\theta} = 1$, we have demand functions:

\begin{align*}
Q_1 &= \theta_{12} - \theta \\
Q_2 &= \theta - \theta_{12}
\end{align*}

where, again, $\theta_{12} = \frac{P_2 - P_1}{X_2 - X_1}$.

### 3.2.2 Product Market Equilibrium

#### 3.2.2.1 Monopoly Market Equilibrium

In what follows, we characterize the product market equilibrium. Consider first the monopoly market equilibrium. If entry does not occur, the incumbent is a monopoly. Recall that our scenario is starting from a single established firm who, initially, may or
may not serve the entire market. Because consumers are passive about the market coverage, a monopolist determines endogenously a covered or uncovered market. Thus, to invoke the assumption of full market coverage, we need to find the parameter restriction where the monopolist would cover the market.

\[ \text{Demand for Monopoly Good (} Q_{\text{IM}} \text{)} \]

\[ 0 \quad \hat{\theta} \quad \bar{\theta} \]

\[ \theta \]

**Figure 3.2 Monopoly Market Segmentation**

Let us denote \( \hat{\theta} = \frac{P_{\text{IM}}}{X_{\text{IM}}} \) (with a subscript ‘IM’ standing for the “incumbent monopoly”) as the marginal consumer who is indifferent between buying a good and not buying at all. Figure 3.2 describes a market segmentation of the monopoly market. If there were only one quality available, then the monopolist’s demand function would imply a linear market demand curve where the fraction of consumers who are willing to buy a good of quality \( X_{\text{IM}} \) at any price \( P_{\text{IM}} \) would be equal to \( Q_{\text{IM}} = \bar{\theta} - \hat{\theta} \). Using the monopolist’s unit cost \( X_{\text{IM}}^2 \), the incumbent as a monopolist in the market solves the following maximization problem with respect to price for a given quality.

\[
\text{(5)} \quad \text{Max } \pi_{\text{IM}} = \left( P_{\text{IM}} - X_{\text{IM}}^2 \right) Q_{\text{IM}} = \left( P_{\text{IM}} - X_{\text{IM}}^2 \right) \left( \bar{\theta} - \frac{P_{\text{IM}}}{X_{\text{IM}}} \right)
\]

From the first order condition for this maximization problem, we know that

\[
\frac{\partial \pi_{\text{IM}}}{\partial P_{\text{IM}}} = \bar{\theta} + 1 + X_{\text{IM}} - \frac{2P_{\text{IM}}}{X_{\text{IM}}} \leq 0. \quad \text{If we have an interior solution such that } \frac{P_{\text{IM}}^*}{X_{\text{IM}}} > \bar{\theta} \text{ the} 
\]
market is uncovered. But in the case of a corner solution where \( \frac{P_{IM}}{X_{IM}} = \theta \) the market is covered. For the corner solution, it is necessary that

\[
\left. \frac{\partial \pi_{IM}}{\partial P_{IM}} \right|_{P_{IM} = \theta X_{IM}} = \theta + 1 + X_{IM} - 2\theta \leq 0 \iff \theta \geq 1 + X_{IM}
\]

In this covered-market case, the monopolist’s price is at the level at which the least value consumer ( \( \theta \) ) gives up all her surplus to purchase the good (i.e., \( P^*_{IM} = \theta X_{IM} \)). Thus, the monopolist’s product market equilibrium profit is:

\[
\pi_{IM}^* = \theta X_{IM} - X_{IM}^2
\]

### 3.2.2.2 Duopoly Market Equilibrium

Now, consider the duopoly covered market equilibrium where duopoly firms move simultaneously in the production stage with Bertrand competition.\(^{14}\) In this stage of the game, qualities are exogenous. The market segmentation for each type of a product is described in Figure 3.3. Given the firms’ quality levels, \( X_1 \) and \( X_2 \), and their prices, \( P_1 \) and \( P_2 \), the marginal consumer \( \theta_{01} = \frac{P_1}{X_1} \) is indifferent between buying a low-quality good 1 and not buying at all, and the marginal consumer \( \theta_{12} = \frac{P_2 - P_1}{X_2 - X_1} \) is indifferent between buying a low-quality good 1 and buying a high-quality good 2. Therefore, the demands for good 1 and good 2 are, respectively, given by \( Q_1 = \theta_{12} - \theta \) and \( Q_2 = \theta - \theta_{12} \). For market to be covered, it is necessary that \( \theta \geq \theta_{01} \).

\(^{14}\) Under the covered-market configuration, total demand is not a function of prices, so demand functions cannot be inverted. For Cournot competition to be meaningful, the market should be uncovered by allowing some consumers not to buy differentiated goods (e.g., Motta (1993)).
Then each firm maximizes its profit with respect to its own price for any given quality choice and an opponent firm’s price. With unit costs, $C_i = X_i^2$, $i = 1, 2$, the profit function of the low-quality firm is given by

$$\pi_1 = (P_1 - X_1^2)Q_1 = (P_1 - X_1^2)\left(\frac{P_2 - P_1}{X_2 - X_1} - \theta\right),$$

and that of the high-quality firm is

$$\pi_2 = (P_2 - X_2^2)Q_2 = (P_2 - X_2^2)\left(\theta + 1 - \frac{P_2 - P_1}{X_2 - X_1}\right).$$

The first order conditions for interior solutions yield the following two best response functions:

$$P_1 = R_1(P_2) = \frac{1}{2}\left(P_2 + X_1^2 - \theta(X_2 - X_1)\right)$$

$$P_2 = R_2(P_1) = \frac{1}{2}\left(P_1 + X_2^2 + (\theta + 1)(X_2 - X_1)\right)$$

Using the best response functions (8) and (9), and denoting with a superscript "∗" the production stage equilibrium values, we find that when both firms are active in the market the equilibrium prices are:

$$P_1^* = \frac{1}{3}\left((2X_1^2 + X_2^2) + (1 - \theta)(X_2 - X_1)\right)$$

$$P_2^* = \frac{1}{3}\left((X_1^2 + 2X_2^2) + (2 + \theta)(X_2 - X_1)\right)$$

Substituting these expressions into the profit functions yield the payoffs for the quality
game: \(^{15}\)

\[
\pi_1^* = (X_2 - X_1) \frac{\left( X_2 + X_1 + (1 - \theta) \right)^2}{9}
\]

\[
\pi_2^* = (X_2 - X_1) \frac{\left( -\left( X_2 + X_1 + (2 + \theta) \right)^2 \right)}{9}
\]

Of course, these solutions only apply when, in equilibrium, the market is in fact covered. Thus, to complete the solution, it remains to check the following two conditions. First, for exogenously given qualities, the necessary condition for both outputs to be positive in product market equilibrium as obtained from (12) and (13) is:

\[
X_1 + X_2 - 2 \leq \theta \leq X_1 + X_2 + 1
\]

This condition ensures non-negative demands at the duopoly product market equilibrium (i.e., \(Q_1^* \geq 0\) and \(Q_2^* \geq 0\)). As illustrated in Figure 3.4, the firm producing a low-quality good would become a monopoly for extremely high consumer heterogeneity (such that \(\theta < X_1 + X_2 - 2\)), whereas the firm producing a high-quality good would become a monopoly for very low consumer heterogeneity (such that \(\theta > X_1 + X_2 + 1\)). \(^{16}\) Thus, the restriction in (14) excludes these two extreme cases.

\(^{15}\) Note that \(\pi_1^*\) is the incumbent’s payoff and \(\pi_2^*\) is the entrant’s one when entry occurs with a superior-quality good compared to the incumbent’s quality. If the entrant chooses an inferior quality then the entrant’s payoff is \(\pi_2^*\) and the incumbent’s payoff is \(\pi_1^*\).

\(^{16}\) Heterogeneity, measured here by the ratio \(\bar{\theta}/\underline{\theta}\), decreases with \(\theta\) (recall that \(\bar{\theta} = \underline{\theta} + 1\)): the greater is \(\theta\), the more homogenous are consumers. Thus, the market is likely to be preempted by the low-quality firm when consumers are relatively heterogeneous, whereas the market is likely to be preempted by the high-quality firm when consumers are relatively homogenous. For the intuitive explanation, note that \(\theta\) is the marginal willingness-to-pay for quality. That is, a consumer with higher \(\theta\) is willing to pay more for the higher quality good, while a consumer whose taste parameter \(\theta\) is very low would not like to pay for the high quality good. Thus, the market will be preempted by the low-quality firm if \(\theta\) is very low.
Inferior quality preempts the market | Duopoly | Superior quality preempts the market

<p>| | | |</p>
<table>
<thead>
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<tbody>
<tr>
<td>0</td>
<td>$X_1 + X_2 - 2$</td>
<td>$X_1 + X_2 + 1$</td>
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"Relative consumer heterogeneity" decreases

**Figure 3.4 Post-innovative Market Structure with a Covered Market**

Second, for a market to be covered, it must be the case that the consumer with the lowest marginal willingness-to-pay for quality ($\tilde{\theta}$) has a non-negative surplus when she buys one unit of the low-quality product (i.e., $\tilde{\theta}X_1 - P^*_1 \geq 0$). It is verified that the following parameter restriction guarantees that each consumer buys one of the two varieties in the non-cooperative equilibrium:

\[
\tilde{\theta} \geq \frac{2X_1^2 + X_2^2}{2X_1 + X_2} + (X_2 - X_1)
\]

### 3.3 Quality-Choosing Equilibrium

In this section, we solve the quality-stage of the game (period 1 and 2) for given Bertrand-competition solutions at the production stage. We endogenize the entrant’s choice whether to be the low-quality or the high-quality provider, relative to the existing variety produced by the incumbent. This is a Stackelberg model of quality choices in which the leader is the incumbent firm (I) and the follower is the entrant firm (E).

The resulting game has the structure of the two-stage tree with a continuum of branches shown in Figure 3.5. At the initial node, firm I chooses a branch (a quality
Note that the entrant’s payoff function is discontinuous because the choice of a high or low quality compared to the incumbent’s one is endogenous. Likewise, the incumbent’s payoff function is discontinuous, consisting of a segment where the incumbent provides low quality, she provides high quality, and she is a monopolist. Thus, for an analytical purpose, we separate the low-quality segment and the high-quality segment, connected at $X_i = X_E$, to find the local maxima. Then we compare two local maxima to find a global maximum.

Figure 3.5 Quality-Choosing Game (when entry is accommodated)

3.3.1 Best Response Function of the Entrant

Consider first the case of entry with a superior quality. The entrant’s reduced-form payoff function from price competition in the production stage of the game is given by equation (13), and the incumbent’s payoff is given by equation (12). In period 2, a firm $E$ (the Stackelberg follower) chooses $X_E$ to maximize $\pi_E^*(X_i, X_E) - F$ for given

\[\pi_{IS}^{AEH}, \pi_{ES}^{AEH}\]

\[\pi_{IS}^{AEL}, \pi_{ES}^{AEL}\]

---

17 The subscript ‘IS’ and ‘ES’ stand for the “incumbent’s Stackelberg value” and the “entrant’s Stackelberg value”, respectively. The superscript ‘AEH’ and ‘AEL’ stand for the “accommodated entry of a high-quality good” and the “accommodated entry of a low-quality good”, respectively.
If firm $E$ enters, its best response in terms of incumbent’s quality is given by

$$X_E = \frac{X_I + \frac{\theta + 2}{3}}{3}.$$ \hspace{1cm} (18)

Then the entrant’s payoff conditional on choosing superior-quality entry is given by:

$$\pi^*_E (X_I, F) = \pi^*_E \left( X_I, \frac{X_I + \frac{\theta + 2}{3}}{3} \right) - F = \frac{4}{9} \left( \frac{\theta + 2 - 2X_I}{3} \right)^3 - F$$

The potential entrant enters the market only if this leads to a strictly positive payoff,\(^{19}\) and this holds for:

$$X_I < \lambda_H,$$

where $\lambda_H = 1 + \frac{\theta}{2} - \left( \frac{3}{2} \right)^{5/3} F^{1/3}$

We now consider the case of entry with an inferior quality. The entrant’s payoff function from price competition in the third stage of the game is given by equation (12), and the incumbent’s payoff is given by equation (13). In this case, if firm $E$ enters, its best response in terms of incumbent’s quality is given by

$$X_E = \frac{-X_I + \frac{\theta - 1}{3}}{3}.$$ \hspace{1cm} (19)

Then the entrant’s payoff, conditional on choosing inferior-quality entry, is:

\(\text{Two best response functions, } X_E = \frac{X_I + \frac{\theta + 2}{3}}{3} \text{ and } X_E = -X_I + (\theta + 2), \text{ are derived from the first order condition, } \frac{\partial \pi^*_E (X_I, X_E)}{\partial X_E} = 0, \text{ of the entrant’s maximization problem. However, } X_E = -X_I + (\theta + 2) \text{ does not satisfy a duopoly output condition (14). Then the second order condition at the equilibrium requires } \frac{\partial^2 \pi^*_E}{\partial X^2} = \frac{2}{9} \left( X_I + 3X_E - 2(2 + \theta) \right) < 0 \Leftrightarrow X_I < \frac{\theta}{2} + 1. \)

\(^{19}\) Actually, when profits are zero, the prospective entrant’s choices are indifferent between entry and no entry. However, we follow the convention that the entrant enters the market only if she can make positive payoffs (e.g., Dixit, 1980).

\(\text{Two best response functions, } X_E = \frac{X_I + \frac{\theta - 1}{3}}{3} \text{ and } X_E = -X_I + (\theta - 1), \text{ are derived from the first order condition, } \frac{\partial \pi^*_E (X_I, X_E)}{\partial X_E} = 0, \text{ of the entrant’s maximization problem. However, } X_E = -X_I + (\theta - 1) \text{ does not satisfy a duopoly output condition (14). Then the second order condition at the equilibrium requires } \frac{\partial^2 \pi^*_E}{\partial X^2} = \frac{-2}{9} \left( 3X_E + X_I - 2(1 - \theta) \right) < 0 \Leftrightarrow X_I > \frac{\theta - 1}{2}. \)
The potential entrant enters the market only if this leads to a positive payoff, and this holds for:

\[ X_J > \lambda_L, \text{ where } \lambda_L = \frac{\theta - 1}{2} + \left( \frac{3}{2} \right)^{5/3} F^{1/3} \]

The 'best response function of the prospective entrant' (BRE) relevant to the quality game describes the strategy of the entrant firm as a function of the incumbent's strategy. Based on the above two conditional responses, we can characterize an actual BRE on the ranges of fixed costs. Let us define the critical value \( \hat{\lambda}_L = \frac{\theta + 1}{2} + \left( \frac{3}{2} \right)^{5/3} F^{1/3} \) such that the following equality is satisfied: \( \pi_L^E(\hat{\lambda}_L, F) = \pi_L^H(\hat{\lambda}_H, F) \). If \( X_J < \hat{\lambda}_L \), then there can be superior-quality entry because \( \pi_L^H \geq \pi_L^L \). Likewise, if \( X_J \geq \hat{\lambda}_H \), then there can be inferior-quality entry because \( \pi_L^L \geq \pi_L^H \). Now, to define completely the BRE, let us check the ranges of fixed costs. If \( \lambda_L < \hat{\lambda}_J < \lambda_H \), then the entrant's positive-profit conditions (17) and (19) are not binding. This is the case when \( F < \frac{18}{18} \). Whereas, if \( \lambda_H < \hat{\lambda}_J < \lambda_L \), then equations (17) and (19) are binding conditions. This holds for \( F > \frac{18}{18} \). For \( F = \frac{18}{18} \) and \( X_J = \hat{\lambda}_J \), entry does not occur because entrant cannot make positive payoffs. Therefore, there is a discontinuity in the BRE, and we can define it on the ranges of fixed costs as:

\[
(20) \quad \text{For } F < \frac{18}{18}, X_e = \begin{cases} 
\frac{X_J + \theta + 2}{3}, & \text{if } X_J \leq \hat{\lambda}_J \\
\frac{X_J + \theta - 1}{3}, & \text{if } X_J \geq \hat{\lambda}_J 
\end{cases}
\]
For $F > \frac{1}{18}$, $X_E = \begin{cases} \frac{X_l}{3} + \frac{\theta + 2}{3}, & \text{if } X_l < \lambda_H \\ \frac{X_l}{3} + \frac{\theta - 1}{3}, & \text{if } X_l > \lambda_L \end{cases}$

For $F = \frac{1}{18}$, $X_E = \begin{cases} \frac{X_l}{3} + \frac{\theta + 2}{3}, & \text{if } X_l < \hat{X}_l \\ \text{No entry, if } X_l = \hat{X}_l \\ \frac{X_l}{3} + \frac{\theta - 1}{3}, & \text{if } X_l > \hat{X}_l \end{cases}$

where $\hat{X}_l = \frac{\theta}{2} + \frac{1}{4}$, $\lambda_H = 1 + \frac{\theta}{2} - \left(\frac{3}{2}\right)^{5/3} F^{1/3}$, and $\lambda_L = \frac{\theta - 1}{2} + \left(\frac{3}{2}\right)^{5/3} F^{1/3}$.

Figure 3.6 shows BRE when the entrant's positive-payoff conditions (17) and (19) are not binding because fixed costs are small such that $F < \frac{1}{18}$. Note that, in the quality-choice games, payoffs are zero when qualities are identical (i.e., payoffs are zero on the 45° line). Hence the BRE is necessarily discontinuous. Conditional on choosing superior-quality entry, the best response of the entrant is $ac$ because $\pi_E^H(X_l, F) > 0$ if $X_l < \frac{\theta}{2} + 1$. Likewise, conditional on choosing inferior-quality entry, the best response of the entrant is $df$ because $\pi_E^L(X_l, F) > 0$ if $X_l > \frac{\theta - 1}{2}$. Now, we know that $\pi_E^L(X_l, F) > \pi_E^H(X_l, F)$ if $X_l = \hat{X}_l$. Therefore, the actual BRE when $F < \frac{1}{18}$ is $abef$ with discontinuity at $X_l = \hat{X}_l$.

The BRE when $F = \frac{1}{18}$ is presented in Figure 3.7. In this case, BRE jumps down at $\hat{X}_l$ because the entrant's payoff $\pi_E^H = \pi_E^L = 0$ at this level of incumbent's quality.
Figure 3.6 The Best Response Function of the Entrant (when $F < \frac{1}{18}$)

$X_E = \frac{X_I}{3} + \frac{\theta + 2}{3}$

Figure 3.7 The Best Response Function of the Entrant (when $F = \frac{1}{18}$)

$X_E = \frac{X_I}{3} + \frac{\theta - 1}{3}$
The BRE associated with high fixed costs such that $F > \frac{1}{18}$ is depicted in Figure 3.8. In this case, the positive-payoff conditions (17) and (19) are binding. The location of $\lambda_H$ and $\lambda_L$ depends on the size of the entrant’s fixed cost. In fact, the distance between $\lambda_H$ and $\lambda_L$ increases as $F$ increases. The model thus allows the possibility of incumbent’s strategic behavior. The quality leader (the incumbent), by choosing limit qualities at which the potential entrant prefers to stay out of the market, can deter entry.

Figure 3.8 The Best Response Function of the Entrant (when $F > \frac{1}{18}$)
3.3.2 Quality Leadership and Limit Qualities

In this section, we analyze the strategic behavior of the incumbent at its quality-stage of the game. We classify the outcomes of the incumbent’s quality as means of limiting prospective entrant’s choices. Due to discontinuity in the prospective entrant’s best response function, it is the size of a fixed cost that determines whether or not an entry-deterrence strategy is preferred.

3.3.2.1 Parameter Restrictions on Market Outcomes

Prior to proceeding with the analysis, it is important to note that the analysis applies only to the range of the parameter \( \theta \) which ensures that the duopoly actually covers the market. Let us first confine our attention to the post-entry duopoly (say, the case of \( F < \frac{1}{18} \)). When entry occurs with a superior quality, BRE is given by \( X_E = X_L + \frac{\theta + 2}{3} \). The incumbent’s (i.e., Stackelberg leader’s) quality choice is given by solving \( \frac{\partial \pi_i}{\partial X_i} \left( X_i, X_L + \frac{\theta + 2}{3} \right) = 0 \). Accordingly, the Stackelberg solution is characterized by \( X_{IS}^{AEH} = \frac{\theta}{2} + \frac{1}{4} \), \( X_{ES}^{AEH} = \frac{\theta}{2} + \frac{3}{4} \), \( \pi_{IS}^{AEH} = \frac{2}{9} \), and \( \pi_{ES}^{AEH} = \frac{1}{18} - F \). In order for the duopoly market to be fully covered at the Stackelberg equilibrium, one must check whether these solutions satisfy constraints (14) and (15). Straightforward calculation shows that when entry occurs with a superior quality the constraint \( \theta \geq \sqrt{\frac{19}{12}} \approx 1.2583 \) must be satisfied for both qualities to be positive and the market to be covered at equilibrium.\(^{21}\)

\(^{21}\) The duopoly condition (14) is obvious because \( X_I + X_E - 2 \leq \theta \leq X_I + X_E + 1 \iff \theta - 1 \leq \theta \leq \theta + 2 \).

The covered-market restriction (15) is \( \theta \geq \frac{2X_I^2 + X_E^2 + (X_E - X_I)}{2X_I + X_E} \iff \theta^2 \geq \frac{19}{12} \iff \theta \geq \frac{19}{\sqrt{12}} \).
Next consider the case of entry with an inferior quality. In this case, BRE is given by \( X_E = \frac{X_I}{3} + \frac{\theta - 1}{3} \). The incumbent’s quality choice is then given by solving \( \frac{\partial \pi_I}{\partial X} = 0 \). Accordingly, the Stackelberg solution is characterized by \( X_{IS} = \frac{\theta}{2} + \frac{1}{4}, X_{AE} = \frac{\theta}{2} - \frac{1}{4}, \pi_{IS} = \frac{2}{9}, \) and \( \pi_{AE} = \frac{1}{18} - F \). Again, in order for the duopoly market to be covered in the Stackelberg equilibrium qualities, one must check whether these solutions satisfy constraints (14) and (15). Straightforward calculation shows that when entry occurs with an inferior quality the constraint \( \theta \geq \frac{11}{12} \approx 0.95743 \) must be satisfied for both qualities to be positive and the market to be covered at equilibrium.\(^{22}\)

Consider now the pure monopoly market equilibrium, where entry does not occur. Because consumers are passive about the market coverage, a monopolist could determine endogenously a covered or uncovered market without \textit{ex ante} assumptions. Thus, for the specific market outcomes, we need to find the parameter restriction where the monopolist would cover or uncover the market. As we discussed earlier, maximizing equation (7) with respect to its quality level yields a pure monopoly solution under the covered-market configuration: \( X_{IM}^* = \frac{\theta}{2} \) and \( \pi_{IM}^* = \frac{\theta^2}{4} \). For this monopoly market to be covered, we need to check whether this solution satisfies the monopolist’s covered-market restriction (6). Straightforward calculation shows that the constraint \( \theta \geq 1 + X_{IM}^* \Leftrightarrow \theta \geq 2 \) must be satisfied for monopolist’s equilibrium to be covered fully in the market. Thus, for \( \theta \leq 2 \), the uncovered monopoly maximizes

\(^{22}\) The duopoly condition (14) is obvious because \( X_E + X_I - 2 \leq \theta \leq X_E + X_I + 1 \Leftrightarrow \theta - 2 \leq \theta \leq \theta + 1 \).

The covered-market restriction (15) is \( \theta \geq \frac{2X_E^2 + X_I^2 + (X_E - X_I)}{2X_E + X_I} \Leftrightarrow \theta \geq \frac{11}{12} \Rightarrow \theta \geq \frac{11}{12} \).
\[ \pi_{IM}^* = \left( P_{IM} - X_{IM}^* \right) \left( \theta + 1 - \frac{P_{IM}}{X_{IM}^*} \right) \] with respect to \( P_{IM} \) and \( X_{IM}^* \). Then the resulting equilibrium values are \( P_{IM}^* = \frac{2(1+\theta)^2}{9}, \ X_{IM}^* = \frac{1+\theta}{3}, \) and \( \pi_{IM}^* = \left( \frac{1+\theta}{3} \right)^2 \).

In conclusion, our analysis (which is confined to the duopoly covered market case) pertains to markets with \( \theta \in \left[ \frac{\sqrt{19}}{12}, \infty \right) \). Then, as illustrated in Figure 3.9, the incumbent's outcomes can be specified for two different levels of consumer heterogeneity. One is associated with the uncovered pure monopoly equilibrium where there are relatively heterogeneous consumers such that \( \theta \in \left[ \frac{\sqrt{19}}{12}, 2 \right] \). The other is associated with the covered pure monopoly equilibrium where there are relatively homogenous consumers such that \( \theta \geq 2 \).

![Figure 3.9 Equilibrium Market Segmentation](image)

**3.3.2.2 Case 1: Low Fixed Costs and Accommodated Entry**

When the entry cost is sufficiently low such that \( F < \frac{1}{18} \) entry deterrence is not
possible, so that the solutions for the entry accommodation are Stackelberg duopoly equilibria. Note that if entry takes place, the duopoly firms' Stackelberg payoffs are the same regardless which of the two possible equilibria applies. Specifically, the entrant is indifferent between entry with an inferior quality and entry with a superior quality. That is, points 'b' and 'e' in Figure 3.6 are both Stackelberg equilibria.

3.3.2.3 Case 2: High Fixed Costs and Blockaded Entry

If $F$ is so large that $\pi^H_E < 0$ and $\pi^L_E < 0$, the entrant cannot cover a fixed cost. That is, entry does not occur if the quality leader chooses its quality level between $\lambda^H_2$ and $\lambda^L_1$ in Figure 3.8. Consider first the range of relatively homogenous consumers where $\theta \geq 2$. When the entry cost is sufficiently large to satisfy the covered monopolist's quality level $X^*_M = \frac{\theta}{2} \geq \lambda_H$, or equivalently $F \geq \left(\frac{2}{3}\right)^3 \approx 0.13169$, the unconstrained monopoly optimum can be achieved. Thus, the entrant will not enter the market even when the incumbent plays its pure monopoly quality level. In this case, we say that the entry is “blockaded”.

Now, for the range of relatively heterogeneous consumers where $\theta \in \left[\frac{19}{12}, 2\right]$, entry is blockaded if the uncovered monopolist’s quality satisfies $X^*_M = \frac{1 + \theta}{3} \geq \lambda_H$, or equivalently $F \geq \hat{F}$ where $\hat{F} = \left(\frac{2}{81}\right)^{(4 + \theta)}$.

3.3.2.4 Case 3: Moderate Fixed Costs and Deterred Entry

If $F$ falls below the boundary given by $\left(\frac{2}{3}\right)^3 \approx 0.13169$ for $\theta \geq 2$, or $\hat{F}$ for
the fixed cost of entry is insufficient to deter entry when the incumbent produces the pure monopoly quality. Then the incumbent has two choices: she could expand its quality level above the unconstrained profit-maximizing level to deter entry; or she could invite entry by choosing its quality level at which point less than $\lambda_H$ or greater than $\lambda_L$, so that entry occurs immediately and the entrant's quality level rises instantaneously to the duopoly level. To analyze the entry-deterrence strategy of the incumbent, we define $X^B_t$ as the quality level that discourages entry, where the superscript B stands for "barrier". Then $X^B_t$ is given by $\max_{X_E} \{ X^*_E, X^B_t \} - F = 0$. Thus, the incumbent can choose any quality levels in $X^B_t \in [X_H, X_L]$ to deter entry.

First, consider the case where $F = \frac{1}{18}$. If entry is accommodated, $X_t \to \hat{X}_t$ is the profit-maximizing level of quality, so that the maximum payoff that the incumbent can get from the accommodation of entry is $\lim_{X_t \to \hat{X}_t} \pi^*_L (X_t, X_E (X_t)) = \frac{2}{9}$. Assuming that the market is covered, the incumbent's profit associated with the deterred entry is $\pi^*_M (X^B_t = \hat{X}_t) = \theta \hat{X}^2_t - \hat{X}^2_t = \frac{(2\theta - 1)(2\theta + 1)}{16}$. Again, one must check whether this solution satisfies the monopolist's covered-market restriction (6). Straightforward calculation shows that the constraint $\theta \geq 1 + X^B_t \iff \theta \geq \frac{5}{2}$ must be satisfied for this constrained monopolist's equilibrium to be covered fully in the market. Then we know that, when $F = \frac{1}{18}$, the incumbent finds it most profitable to deter entry if $\pi^*_M (X^B_t = \hat{X}_t) > \lim_{X_t \to \hat{X}_t} \pi^*_L (X_t, X_E (X_t))$. For $\theta \geq \frac{5}{2}$ where the constrained monopoly market is covered, we have:
(21) \[ K = \pi^*_i(X^b_{i,j} = \hat{X}_i) - \lim_{\lambda_i \to \hat{X}_i} \pi^*_i(X_j, X_E(X_j)) = \frac{36\theta^2 - 41}{144} > 0 \]

Thus, entry is deterred by the incumbent. Now, for \( \theta \in \left[ \sqrt{\frac{19}{12}}, \frac{5}{2} \right] \), the uncovered monopolist's profit is \( \pi^*_{IM}(X^b_i = \hat{X}_i) = \frac{\hat{X}_i}{4}(\theta + 1 - \hat{X}_i)^2 = \frac{(2\theta + 1)(2\theta + 3)^2}{256} \) so that:

(22) \[ T = \pi^*_i(X^b_i = \hat{X}_i) - \lim_{\lambda_i \to \hat{X}_i} \pi^*_i(X_j, X_E(X_j)) > 0 \]

because the minimum \( \pi^*_i \) at the lower bound of \( \theta \) is greater than the payoff from the accommodation, \( \pi^*_{IM}(X^b_i = \hat{X}_i, \theta = \sqrt{\frac{19}{12}}) \approx 0.418 > 0.222 \approx \frac{2}{9} \). Thus, when \( F = \frac{1}{18} \), entry is deterred by the incumbent.

Second, consider the case where \( F > \frac{1}{18} \) but the entry is not blockaded. Assuming that the market is covered, because \( \frac{\partial \pi^*_i}{\partial X_{IM}} = \theta - 2X_{IM} < 0 \) for all \( X_i > \frac{\theta}{2} \), \( X^*_{IM} = \lambda_i \) would be the incumbent's choice when she decides to deter entry. Note that this constrained monopoly choice requires a constraint \( \theta \geq 1 + X^b_{i,j} \Leftrightarrow \theta \geq 4 - 2\left(\frac{3}{2}\right)^{\frac{5}{3}} F^{\frac{1}{3}} \) to cover the market.\(^{23}\) Now, we know that if entry occurs with a high quality,

\[
\frac{\partial \pi^*_i(X_j, X_E(X_j))}{\partial X_j} = \frac{2}{81} (4X_j + 5 - 2\theta)(-4X_j + 1 + 2\theta) > 0 \quad \text{for all} \quad X_j \in \left(\frac{\theta}{2} - \frac{1}{2}, \hat{X}_i\right).
\]

Thus, \( X_i \to \lambda_i \) is the profit-maximizing level of quality if the entry is accommodated.

\(^{23}\) The minimum \( \theta \) to cover the market is located between \( 4 - 2\left(\frac{3}{2}\right)^{\frac{5}{3}} F^{\frac{1}{3}} = 2 \) when \( F = \left(\frac{2}{3}\right)^{\frac{3}{5}} \) and \( 4 - 2\left(\frac{3}{2}\right)^{\frac{5}{3}} F^{\frac{1}{3}} = 2.5 \) when \( F = \frac{1}{18} \).
so that the maximum payoff that the incumbent can get from the accommodation of entry is

$$\lim_{x_j \to x^*_j} \pi^*_i (X_i, X_e (X_i))$$. Then the incumbent finds it most profitable to deter entry if

$$\pi^*_{IM} (X^*_j = \lambda^*_j) > \lim_{x_j \to x^*_j} \pi^*_i (X_i, X_e (X_i))$$, or equivalently,

$$G = 3^5 \left( \theta \lambda^*_j - \lambda^*_j \right)^2 + (2 \lambda^*_j - 2 - \theta) (4 \lambda^*_j + 5 - 2 \theta)^2 > 0$$

Because the inequality (23) holds, the incumbent will deter entry.

Third, consider the case where \( \theta \in \left[ \frac{19}{12}, 4 - 2 \left( \frac{3}{2} \right)^3 \right] \) and \( F > \frac{1}{18} \) (but the entry is not blockaded). Then the uncovered monopoly maximizes

$$\pi^*_{IM} = \left( P - X^*_j \right) \left( \frac{1}{4} - \frac{P}{X^*_j} \right)$$

with respect to \( P \) and \( X^*_j \). Substituting the first order condition, \( \frac{\partial \pi^*_{IM}}{\partial P} = \theta + 1 - \frac{2P}{X^*_j} + X^*_j = 0 \), to make the object function in terms of a quality level yields \( \pi^*_{IM} (X^*_j) = \frac{X^*_j}{4} \left( \theta + 1 - X^*_j \right) \). Now, because

$$\frac{\partial \pi^*_{IM}}{\partial X^*_j} = \left( \frac{1}{4} - \frac{P}{X^*_j} \right) \left( \frac{1}{4} - 3 \frac{P}{X^*_j} \right) < 0$$

for all \( X_j \in \left( \frac{1}{3}, \hat{X}_j \right) \), \( X^*_j = \lambda^*_j \) would be the incumbent’s choice when she decides to deter entry. Also, we already know that if entry occurs with a high quality, \( \frac{\partial \pi^*_i (X_i, X_e (X_i))}{\partial X_i} > 0 \) for all \( X_j \in \left( \frac{1}{2}, \frac{1}{2}, \hat{X}_j \right) \).

Thus, \( X_j \to \lambda^*_j \) is the profit-maximizing level of quality when the entry is accommodated, so that the maximum payoff that the incumbent can get from the

---

24 Plugging \( \lambda^*_j = 1 + \frac{\theta}{2} - \left( \frac{3}{2} \right)^{\frac{1}{3}} + \frac{1}{F} \) into \( G \) yields \( G = A (324 - 99 A - 32 A') + 243 \left( \frac{\theta}{2} \right)^3 - 1 \), where

$$A = \left( \frac{3}{2} \right)^{\frac{1}{3}} + \frac{1}{F} \). The maximum possible \( A \) determined when \( F = \left( \frac{2}{3} \right)^{\frac{1}{3}} \) is 1. Thus, the first term of \( G \) is positive. For \( \theta \geq 2 \), the second term of \( G \) is nonnegative. Therefore, \( G \) is positive for \( \theta \geq 2 \).
accommodation of entry is \( \lim_{x_i \to \lambda_H} \pi_i^*(X_i, X_E(X_i)) \). Then the incumbent finds it most profitable to deter entry if \( \pi_{IM}^* \left( X_i^g = \lambda_H \right) > \lim_{x_i \to \lambda_H} \pi_i^*(X_i, X_E(X_i)) \), or equivalently,

\[
J = \frac{3}{4} \lambda_H \left( 2 + 1 - \lambda_H \right)^2 + \left( 2 \lambda_H - 2 - \vartheta \right) \left( 4 \lambda_H + 5 - 2 \vartheta \right)^2 > 0
\]

Note that because \( T > 0 \) at \( F = \frac{1}{18} \) and \( \frac{\partial T}{\partial F} > 0 \), the inequality (24) also holds. Thus, the incumbent will deter entry by choosing \( \lambda_H \) as its quality level.

3.3.3 Summary of Incumbent Strategies

We now characterize incumbent’s equilibrium qualities that arise in various entry-deterrence strategies faced with potential entry. Market equilibrium values for each entry-deterrence regime are summarized in Table 3-1. For entry costs such that \( F \geq \frac{1}{18} \), entry is ‘deterred’ (DE) or ‘blockaded’ (BE) so that the potential entrant cannot obtain a positive payoff. In this region of the entry cost, then the incumbent may modify its quality-choice behavior relative to the pure monopoly solution in order to prevent entry. The incumbent monopoly market is segmented as the following Figure 3.10.

---

25 We assumed that the prospective entrant enters the market only if she can make strictly positive payoffs. If, instead, we were to use a non-negative profit criterion for entry, then we need to distinguish two main cases. When \( F = \frac{1}{18} \) the non-negative profit entry criterion yields multiple equilibria (for the incumbent’s choice \( X_i = \bar{X} \) it would be an equilibrium for the entrant to choose a quality level corresponding to either point b or point e in Figure 3.6, or decides not to enter the market). When \( F > \frac{1}{18} \), on the other hand, the non-negative profit entry criterion still yields the same unique Nash equilibrium associated with entry deterrence. For the incumbent’s choice \( X_i = \lambda_H \), the entrant is now indifferent between entering or not. But if the entrant does enter, then the incumbent has a profitable deviation (by slightly increasing its quality level from \( \lambda_H \)) and so that cannot be part of a Nash equilibrium. Hence the choice \( X_i = \lambda_H \) would be part of a Nash equilibrium only if the entrant does stay out of the market.
Table 3-1. Comparison of Equilibrium Values of Entry-Deterrence Regimes

<table>
<thead>
<tr>
<th>Type of Entry</th>
<th>Variables</th>
<th>Uncovered Monopoly</th>
<th>Covered Monopoly</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Blockaded Entry</strong></td>
<td>Conditions on ((F, \theta))</td>
<td>When (F \geq \hat{F}) and (\frac{19}{12} \leq \theta \leq 2), where (\hat{F} = \left(\frac{2}{81}\right)\left(4 + \theta^3\right)^{\frac{3}{2}})</td>
<td>When (F \geq \left(\frac{2}{3}\right)) and (\theta \geq 2)</td>
</tr>
<tr>
<td>(X_i)</td>
<td>(\frac{1 + \theta}{3})</td>
<td>(\frac{\theta}{2})</td>
<td></td>
</tr>
<tr>
<td>(P_i)</td>
<td>(\frac{2(1 + \theta)^2}{9})</td>
<td>(\frac{\theta^2}{2})</td>
<td></td>
</tr>
<tr>
<td>(\pi_i)</td>
<td>(\left(\frac{1 + \theta}{3}\right)^3)</td>
<td>(\left(\frac{\theta}{2}\right)^3)</td>
<td></td>
</tr>
<tr>
<td><strong>Deterred Entry</strong></td>
<td>Conditions on ((F, \theta))</td>
<td>When (\frac{1}{18} &lt; F &lt; \hat{F}) for (\theta \in \left[\frac{19}{12}, 2\right]), and (\frac{19}{12} \leq \theta \leq 4\left(\frac{3}{2}\right)^{\frac{1}{3}}F^{\frac{1}{3}})</td>
<td>When (\frac{1}{18} &lt; F &lt; \left(\frac{2}{3}\right)) and (\theta \geq 4 - 2\left(\frac{3}{2}\right)^{\frac{1}{3}}F^{\frac{1}{3}})</td>
</tr>
<tr>
<td>(X_i)</td>
<td>(\frac{1 + \theta}{2} - \frac{3}{2}F^{\frac{1}{3}})</td>
<td>(\frac{\theta}{2} + \frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>(P_i)</td>
<td>(\frac{\lambda_\theta (1 + \theta + \lambda_\theta)}{2})</td>
<td>(\theta \lambda_\theta)</td>
<td></td>
</tr>
<tr>
<td>(\pi_i)</td>
<td>(\frac{\lambda_\theta \left(1 + \theta - \lambda_\theta\right)}{2})</td>
<td>(\theta \lambda_\theta - \lambda_\theta^2)</td>
<td></td>
</tr>
<tr>
<td><strong>Accommodated Entry</strong></td>
<td>Conditions on ((F, \theta))</td>
<td>When (F = \frac{1}{18}) and (\frac{19}{12} \leq \theta \leq \frac{5}{2})</td>
<td>When (F = \frac{1}{18}) and (\theta \geq \frac{5}{2})</td>
</tr>
<tr>
<td>(X_i)</td>
<td>(\theta + \frac{1}{4})</td>
<td>(\frac{\theta + 1}{2})</td>
<td></td>
</tr>
<tr>
<td>(P_i)</td>
<td>(\frac{(2\theta + 1)(6\theta + 5)}{32})</td>
<td>(\frac{\theta(2\theta + 1)}{4})</td>
<td></td>
</tr>
<tr>
<td>(\pi_i)</td>
<td>(\frac{(2\theta + 1)(2\theta + 3)^2}{256})</td>
<td>(\frac{(2\theta + 1)(2\theta - 1)}{16})</td>
<td></td>
</tr>
<tr>
<td><strong>Accommodated Entry</strong></td>
<td>Conditions on ((F, \theta))</td>
<td>When (0 &lt; F &lt; \frac{1}{18}) and (\theta \geq \frac{19}{12})</td>
<td></td>
</tr>
<tr>
<td>((X_i, X_e))</td>
<td>(\left(\frac{\theta + 1}{2}\right)^2 + \frac{3}{4})</td>
<td>(\left(\frac{\theta + 1}{2}\right)^2 - \frac{1}{4})</td>
<td></td>
</tr>
<tr>
<td>((P_i, P_e))</td>
<td>(\left(\frac{12\theta^2 + 12\theta + 19}{48}, \frac{12\theta^2 + 36\theta + 35}{48}\right)) when (X_e = \frac{\theta + 3}{4})</td>
<td>(\left(\frac{12\theta^2 + 12\theta + 19}{48}, \frac{12\theta^2 - 12\theta + 11}{48}\right)) when (X_e = \frac{\theta - 1}{4})</td>
<td></td>
</tr>
<tr>
<td>((\pi_i, \pi_e))</td>
<td>(\frac{2}{9} - F)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Whether to deter or accommodate entry depends on the magnitude of entry costs and on the consumer heterogeneity parameter $\theta$. First, if the entry cost is sufficiently high, the entrant will not enter even when the incumbent plays its pure monopoly quality level. That is, in this case, the incumbent firm blockades entry simply by choosing its pure monopolist's quality level. Second, for a certain moderate range of entry costs, however, the unconstrained monopoly optimum cannot be achieved because pure monopoly equilibrium level of quality is not adequate to deter entry. If the incumbent firm also cannot gain from the differentiated market, in this case, the incumbent engages in entry deterrence by increasing her product quality to prevent the prospective entrant from entering the market. Third, when the entry cost is sufficiently low such that $F < \frac{1}{18}$, entry is accommodated and the incumbent selects a quality that is strictly higher than the monopolist's choice. Note that if entry takes place, the entrant's Stackelberg profits are unchanged regardless of entry qualities.\footnote{Note also that, there is a first-mover advantage associated with quality leadership: when entry is accommodated, the incumbent (the Stackelberg leader) is in a position to obtain more profits than the entrant (the Stackelberg follower) regardless of the entrant's quality superiority or inferiority (i.e., $\pi_x > \pi_y$). In particular, the first-mover's equilibrium quality does not change whether the accommodated entry accompanies an inferior or a superior quality. Quality differences at either type of accommodated entry equilibrium are the same and equal $1/2$.} Thus, the entrant's choices are
indifferent between entry with an inferior quality and entry with a superior quality when entry is accommodated by the incumbent. The following Proposition 1 and Figure 3.11 characterize the entrant’s quality choice and the incumbent’s deterrence strategies.

**Proposition 1.** There exist cutoff levels of fixed entry costs and consumer heterogeneity such that (i) the entrant chooses either low-quality entry or high-quality entry, and the incumbent accommodates this if $F < \frac{1}{18}$ and $\vartheta \geq \frac{19}{12}$; (i) entry is deterred if $F \in \left(\frac{1}{18}, \hat{F}\right)$ for $\vartheta \in \left[\frac{19}{12}, 2\right]$, or $F \in \left(1, \left(\frac{2}{3}\right)^5\right)$ for $\vartheta \geq 2$; (iii) entry is blockaded if $F \geq \hat{F}$ for $\vartheta \in \left[\frac{19}{12}, 2\right]$, or $F \geq \left(\frac{2}{3}\right)^5$ for $\vartheta \geq 2$, where $\hat{F} = \left(\frac{2}{81}\right)^2 \left(4 + \vartheta\right)^3$.

![Figure 3.11 Zones of a Strategic Entry and Entry-Deterrence Decision](image-url)
3.4 Welfare Analysis

In this section, we consider the normative aspects of the entry problem that we have studied. First, we investigate how the market equilibrium level of consumer surplus and social welfare is affected by the changes in fixed entry costs. Second, we evaluate the entry-deterrence strategies of the incumbent in terms of social welfare criteria by solving the social planner’s maximization problem.

3.4.1 Consumer Surplus

From the demand system, where potential entrant actually enters the market, we calculate the level of aggregate consumer surplus as the sum of the surplus of consumers who buy the low-quality good and that of those who buy the high-quality good:

\[
CS = \int_{\theta_1}^{\theta_2} (\theta X_1 - P_1) d\theta + \int_{\theta_3}^{\theta_4} (\theta X_2 - P_2) d\theta
\]

(25)

\[
= \frac{(P_2 - P_1)^2}{2(x_2 - x_1)} + \frac{X_1}{2} (1 + \theta)^2 - \frac{X_1}{2} \theta^2 + P_1 \theta - P_2 (1 + \theta)
\]

In the absence of entry, consumer surplus associated with the incumbent monopolist’s market is:

(26)

\[
CS_{IM} = \begin{cases} 
\int_{\theta_1}^{\theta_2} (\theta X_{IM} - P_{IM}) d\theta = \frac{(1 + \theta)^2}{2} X_{IM} - (1 + \theta) P_{IM} + \frac{P_{IM}^2}{2X_{IM}}, & \text{for uncovered monopoly} \\
\int_{\theta_3}^{\theta_4} (\theta X_{IM} - P_{IM}) d\theta = \frac{(1 + 2\theta)^2}{2} X_{IM} - P_{IM}, & \text{for covered monopoly}
\end{cases}
\]

Substituting market equilibrium values of the quality in Table 3-1 into these definitions yields consumer surplus for each entry-deterrence regime:

---

27 As mentioned, subscript 1 and 2 denote the incumbent firm and the entrant firm, respectively, when entry occurs with a superior quality. The opposite notation applies with the entry of an inferior-quality good.

28 Regardless of whether entry is deterred or blockaded, the expression of aggregate consumer surplus is given by equation (26).
For uncovered monopoly, \( CS^u = \begin{cases} 
\frac{36\theta^2 + 36\theta - 35}{144}, & \text{if } 0 < F < \frac{1}{18} \\
\frac{8\theta^3 + 28\theta^2 + 30\theta + 9}{512}, & \text{if } F = \frac{1}{18} \\
\frac{\lambda_\mu^3 - 2(1 + \theta)\lambda_\mu^2 + (1 + \theta)^2 \lambda_\mu}{8}, & \text{if } \frac{1}{18} < F < \hat{F} \\
\frac{(1 + \theta)^3}{54}, & \text{if } F \geq \hat{F}
\end{cases} \)

For covered monopoly, \( CS^c = \begin{cases} 
\frac{36\theta^2 + 36\theta - 35}{144}, & \text{if } 0 < F < \frac{1}{18} \\
\frac{2\theta + 1}{8}, & \text{if } F = \frac{1}{18} \\
\frac{\lambda_\mu}{2}, & \text{if } \frac{1}{18} < F < \left(\frac{2}{3}\right)^5 \\
\frac{\theta}{4}, & \text{if } F \geq \left(\frac{2}{3}\right)^5
\end{cases} \)

where \( \lambda_\mu = 1 + \frac{\theta}{2} - \left(\frac{3}{2}\right)^{5/3} F^{1/3} \) and \( \hat{F} = \left(\frac{2}{81}\right)^2 (4 + \theta)^3 \).

Note that, whether the entry quality is superior or inferior compared to the incumbent’s quality level, consumer surpluses from the high- and low-quality entry are both equal to \( \frac{36\theta^2 + 36\theta - 35}{144} \) when the entry is accommodated.

Figure 3.12 depicts how consumer surplus changes as the fixed entry cost changes. The response has three distinctive phases. First, when the fixed cost is so large that the entry is blockaded, the incumbent’s quality choice and its price are not dependent on the magnitude of a fixed cost. Thus, the consumer surplus is constant in this region. Second, when the fixed entry cost decreases and so entry is not blocked, the incumbent increases its quality level to deter entry as fixed costs decrease. In this case, the consumer surplus of the relatively homogenous group (leading to the covered monopoly case) increases as fixed costs decrease, while relatively heterogeneous consumers (the...
uncovered monopoly case) become worse off. Third, when the fixed cost is so small that the incumbent cannot deter entry, the consumer surplus is independent on the level of a fixed cost because the entrant’s positive-profit conditions which are depending on $F$ are not binding. In particular, the consumer surplus from the accommodated entry is higher than that of the deterred entry and blockaded entry. The proposition below summarizes how consumer surplus varies across fixed costs.

**Proposition 2.** (i) The consumer surplus for cases with relatively homogeneous consumers is non-increasing in the fixed costs. That is, both actual entry and the potential entry associated with the deterred entry, increase consumer surplus relative to the pure monopoly situation. (ii) For cases with relatively heterogeneous consumers, the consumer surplus from the accommodated entry is higher than that of the blockaded and deterred entry. The threat of entry associated with the deterred entry, however, makes consumers worse off.
3.4.2 Equilibrium Social Welfare

In this section, we investigate how the equilibrium social welfare changes as the fixed entry cost decreases or increases. Combining measures of consumer surplus along with firm profits, in the case where the potential entrant actually enters the market, yields social welfare:

\[
W(X_1, X_2; P_1, P_2) = \frac{(P_2 - P_1)^2}{2(X_2 - X_1)} + \frac{X_2}{2}(1 + \theta)^2 - \frac{X_1}{2} \theta^2 + P_1 \theta - P_1(1 + \theta)
\]

\[
+ \left( P_1 - X_1 \right) \left( \frac{P_2 - P_1}{X_2 - X_1} - \theta \right) + \left( P_2 - X_2 \right) \left( 1 + \theta - \frac{P_2 - P_1}{X_2 - X_1} \right) - F
\]

Figure 3.12 Consumer Surplus

(b) Covered Monopoly
In the absence of entry, social welfare is defined by:

\[
W(X_M; P_M) = \begin{cases} 
\frac{(1 + \theta)^2 X_M}{2} - (1 + \theta) P_M + \frac{P_M^2}{2X_M}, & \text{for uncovered monopoly} \\
+ (P_M - X_M^2) \left( 1 + \theta - \frac{P_M}{X_M} \right), & \text{for covered monopoly} \\
\frac{(1 + 2\theta) X_M}{2} - P_M + (P_M - X_M^2), & \text{for covered monopoly}
\end{cases}
\]

Substituting market equilibrium values of the quality in Table 3-1 into these definitions yields social welfare for each entry-deterrence regime:

For \( \theta \in \left[ \frac{19}{12}, 2 \right] \), \( W^* = \begin{cases} 
\frac{36\theta^2 + 36\theta + 5}{144} - F, & \text{if } 0 < F < \frac{1}{18} \\
\frac{24\theta^3 + 84\theta^2 + 90\theta + 27}{512}, & \text{if } F = \frac{1}{18} \\
\frac{3\lambda_H^3 - 6(1+\theta)\lambda_H^2 + 3(1+\theta)^2\lambda_H}{8}, & \text{if } \frac{1}{18} < F < \hat{F} \\
\frac{(1 + \theta)^3}{18}, & \text{if } F \geq \hat{F}
\end{cases} \)
For $\theta \in \left[4 - 2\left(\frac{3}{2}\right)^{3/2}, \frac{5}{2}\right]$, $W^* =$ \[
\begin{cases} 
\frac{36\theta^2 + 36\theta + 5}{144} - F, & \text{if } 0 < F < \frac{1}{18} \\
\frac{24\theta^3 + 84\theta^2 + 90\theta + 27}{512}, & \text{if } F = \frac{1}{18} \\
\lambda_H \left(\frac{1}{2} + \theta - \lambda_H\right), & \text{if } \frac{1}{18} < F < \left(\frac{2}{3}\right)^3 \\
\frac{\theta^2 + \theta}{4}, & \text{if } F \geq \left(\frac{2}{3}\right)^3
\end{cases}
\]

For $\theta \geq \frac{5}{2}$,

$W^* =$ \[
\begin{cases} 
\frac{36\theta^2 + 36\theta + 5}{144} - F, & \text{if } 0 < F < \frac{1}{18} \\
\left(\frac{2\theta + 1}{4}\right)^2, & \text{if } F = \frac{1}{18} \\
\lambda_H \left(\frac{1}{2} + \theta - \lambda_H\right), & \text{if } \frac{1}{18} < F < \left(\frac{2}{3}\right)^3 \\
\frac{\theta^2 + \theta}{4}, & \text{if } F \geq \left(\frac{2}{3}\right)^3
\end{cases}
\]

where $\lambda_H = 1 + \frac{\theta}{2} - \left(\frac{3}{2}\right)^{5/3} F^{1/3}$ and $\hat{F} = \left(\frac{2}{81}\right) \left(4 + \theta\right)^3$.

Figure 3.13 depicts how market equilibrium level of social welfare changes as the fixed entry cost changes. The total welfare of the accommodated entry depends on the fixed entry cost. As we can see, maximum welfare is not necessarily associated with the case of accommodated entry. Although it is deterred, potential entry may be welfare-enhancing relative to the pure monopoly situation. Thus, the following proposition holds.

**Proposition 3.** Entry deterrence is not necessarily welfare decreasing. For relatively homogeneous consumers, maximum welfare is attained at $F = \frac{1}{18}$, where entry is deterred. For relatively heterogeneous consumers, maximum welfare is attained at $F = 0$, where entry is accommodated.
Figure 3.13 Equilibrium Social Welfare
3.4.3 Social Optimum

If the planner were to introduce a new variety, the planner determines the socially optimal level of qualities under marginal cost pricing. We suppose that the planner also needs a fixed entry cost to choose a new variety while she does not need it to choose existing variety. We assume that this fixed cost is same as the entry cost $F$. Thus, in the presence of entry, the planner maximizes the sum of profits and consumer surplus as:

$$\max_{x_1, x_2} W = \int_{\theta_1}^{\theta_2} (\theta x_1 - x_1^2) d\theta + \int_{\theta_3}^{\theta_4} (\theta x_2 - x_2^2) d\theta - F$$

Subject to:

$$\frac{\bar{P}_2 - \bar{P}_1}{X_2 - X_1} = \frac{x_2^2 - x_1^2}{X_2 - X_1} = x_2 + x_1$$

Solving the problem in (31) yields the efficient level of qualities as: $X_1 = \frac{\theta}{2} + \frac{1}{8}$ and $X_2 = \frac{\theta}{2} + \frac{3}{8}$. Note that, in our parameter ranges on $\theta$, the market will be fully covered with these optimal qualities because $\theta \geq \frac{\bar{P}_1}{\bar{X}_1} = \frac{\bar{X}_1^2}{\bar{X}_1} = \bar{X}_1 = \frac{\theta}{2} + \frac{1}{8} \Leftrightarrow \theta \geq \frac{1}{4}$. Meanwhile, if the planner decides not to introduce a new variety in the economy, then the optimal quality is determined by solving the following maximization problem.

$$\max_{x} W = \int_{\theta_1}^{\theta_2} (\theta x - x^2) d\theta$$

Note that the price level is not relevant in the planner’s problem. It only can make a role in redistributing surplus between consumers and producers.

Three candidate solution sets, $\{x_1 = \frac{2\theta + 1}{4}, x_2 = \frac{2\theta + 3}{4}\}$, $\{x_1 = \frac{4\theta + 1}{8}, x_2 = \frac{4\theta + 3}{8}\}$, and $\{x_1 = \frac{2\theta - 1}{4}, x_2 = \frac{2\theta + 1}{4}\}$ are derived from the first order condition of the planner's maximization problem. However, only $\{x_1 = \frac{4\theta + 1}{8}, x_2 = \frac{4\theta + 3}{8}\}$ satisfies the second order condition at the equilibrium.
Straightforward calculation yields \( \bar{X} = \frac{\theta}{2} + \frac{1}{4} \). Note that, in our parameter ranges on \( \theta \), the market will be fully covered with \( \bar{X} \) because \( \bar{P} = \frac{\bar{X}^2}{\bar{X}} = \bar{X} = \frac{\theta}{2} + \frac{1}{4} \iff \theta \geq \frac{1}{2} \).

If the planner accommodates a new variety in the economy,

\[
W(\bar{X}_1, \bar{X}_2) - F = \frac{16\theta^2 + 16\theta + 5}{64} - F.
\]

If only one variety is allowed in the economy,

\[
W(\bar{X}) = \frac{16\theta^2 + 16\theta + 4}{64}.
\]

Thus, the planner accommodates a new variety in the economy if \( W(\bar{X}_1, \bar{X}_2) - F > W(\bar{X}), \) i.e., whenever \( F < \frac{1}{64} \).

Now, let us compare the market equilibrium level of qualities to the socially optimal level of qualities. In the absence of entry, \( X^*_M = \left( \frac{1 + \theta}{3} \right) < \left( \frac{\theta}{2} + \frac{1}{4} \right) = \bar{X} \) for \( \theta \in \left[ \frac{19}{12}, 2 \right] \), \( X^*_M = \left( \frac{\theta}{2} \right) < \left( \frac{\theta}{2} + \frac{1}{4} \right) = \bar{X} \) for \( \theta \geq 2 \), and \( X^*_I = \lambda_I < \left( \frac{\theta}{2} + \frac{1}{4} \right) = \bar{X} \).

When entry is accommodated, therefore, profit maximization yields a quality difference that is too high, i.e., \( (\bar{X}_2 - \bar{X}_1) - (X^*_2 - X^*_1) = \frac{1}{4} - \frac{1}{2} < 0 \). Then the following proposition summarizes these results.

**Proposition 4.** (i) The level of entry costs that makes it socially optimal to have a new quality of good in the economy is \( F < \frac{1}{64} \). Thus, for \( F \in \left[ \frac{1}{64}, \frac{1}{18} \right] \), there are too many varieties in the economy relative to the social optimum. (ii) For a fixed entry cost with \( F < \frac{1}{64} \), Stackelberg firms provide excessive product differentiation, compared to what would be socially desirable. (iii) The incumbent monopolist, whether the entry is deterred or blockaded, strictly undersupplies product quality relative to the social optimum.

Note that the Stackelberg firms excessively differentiate product qualities to reduce price competition. In particular, for high-quality entry, the oversupply of product
qualities is associated with the overproduction of low-quality product; for low-quality entry, by contrast, the undersupply of product qualities is associated with the overproduction of a high-quality product. That is, from the planner’s maximization problem, socially optimal demands are determined by 

\[ \hat{Q}_1 = \hat{r}_1 \, \theta = (\tilde{x}_2 + \tilde{x}_1) - \theta = \frac{1}{2} \]

and 

\[ \hat{Q}_2 = 1 + \theta - \hat{r}_1 = 1 + \theta - (\tilde{x}_2 + \tilde{x}_1) = \frac{1}{2}. \]

Meanwhile, the Stackelberg leader’s (the incumbent’s) market share is greater than the follower’s (the entrant’s) one regardless of entry regime: for high-quality entry, equilibrium demands are 

\[ Q_2^* = \frac{P_2^* - P_1^*}{X_2^* - X_1^*} - \theta = \frac{2}{3} \]

and 

\[ Q_1^* = 1 + \theta - \frac{P_2^* - P_1^*}{X_2^* - X_1^*} = \frac{1}{3}; \] for low-quality entry, 

\[ Q_1^* = \frac{P_2^* - P_1^*}{X_2^* - X_1^*} - \theta = \frac{1}{3} \]

and 

\[ Q_2^* = 1 + \theta - \frac{P_2^* - P_1^*}{X_2^* - X_1^*} = \frac{2}{3}. \]

Therefore, for high-quality entry, there is the oversupply of product qualities \((\tilde{x}_1 = (\theta + \frac{1}{8}) < (\theta + \frac{1}{4}) = X_1^*)\) and \((\tilde{x}_2 = (\theta + \frac{3}{8}) < (\theta + \frac{3}{4}) = X_2^*)\) associated with the overproduction of a low-quality good \((\tilde{Q}_1 < Q_1^*)\) and the underproduction of a high-quality good \((\tilde{Q}_2 > Q_2^*)\). For low-quality entry, there is the undersupply of product qualities \((\tilde{x}_1 = (\theta + \frac{1}{8}) > (\theta + \frac{1}{4}) = X_1^*)\) and \((\tilde{x}_2 = (\theta + \frac{3}{8}) > (\theta + \frac{1}{4}) = X_2^*)\) associated with the underproduction of a low-quality good \((\tilde{Q}_1 > Q_1^*)\) and the overproduction of a high-quality good \((\tilde{Q}_2 < Q_2^*)\).
3.5 Summary and Conclusion

We have analyzed the strategic use of entry deterrence of an established firm, and the entrant’s quality choice, in a vertically differentiated product market. We have characterized the equilibrium properties of the three-stage game in which quality choice is sequential, price competition occurs at the last stage, production costs are quality-dependent, and a fixed entry cost is required to the potential entrant firm. With the simplest case of one incumbent firm facing one prospective entrant, we showed how the incumbent’s pre-entry decision generates various equilibrium qualities. In our Stackelberg game, the incumbent influences the quality choice of the entrant by choosing its quality level before the entrant. This allows the incumbent to limit entrant’s entry decision and quality levels. We characterized the levels of entrant’s fixed costs, and the degree of consumer homogeneity, that induce the incumbent to engage, in equilibrium, in either entry deterrence or entry accommodation. Also, we compared market equilibrium values to the socially optimal ones. The main results are as follows.

Consider first the incumbent’s behavior. For sufficiently low fixed entry costs, entry is accommodated and the incumbent chooses a quality that is strictly greater than the monopolist’s choice. For sufficiently high fixed entry cost, however, the incumbent does not gain from a differentiated duopoly market. Thus, in this case, it is never the case that entry is accommodated. The incumbent facing the potential entry of a competitor increases its quality relative to the pure monopoly level to deter entry. If the entry cost is very high the incumbent can blockade entry simply by choosing its pure monopoly quality level.

Second, the entrant firm, when fixed entry costs are sufficiently low, is indifferent between entry with a superior-quality good and entry with an inferior-quality
good. However, if the entry cost is too high, it is better for the entrant to secure zero profit by staying out of the market.

Third, on the welfare side, whether the entry occurs with a high-quality good or with a low-quality good, consumers’ surpluses are the same. We find that consumer surplus of the relatively homogenous group is non-decreasing as fixed entry costs decrease. For relatively heterogeneous consumers, however, the threat of entry associated with the deterred entry makes consumers worse off as fixed costs decrease. In terms of society’s welfare, although it is deterred, potential entry can be welfare-enhancing relative to the pure monopoly situation. In particular, the maximum welfare of the relatively homogenous consumers is attained at the fixed cost level where entry is deterred.

Fourth, we calculated the critical level of an entry cost that, below this level, the social planner would introduce a new quality of the good in the economy. It was shown that, for a region of the fixed entry cost, there are too many varieties in the market equilibrium relative to the social optimum. We also showed that Stackelberg firms associated with accommodated entry excessively differentiate product qualities to reduce price competition. It turns out that the incumbent monopolist strictly undersupplies the product quality relative to the social optimum.

We stress that our analysis on how the existence of a potential entrant influences the quality in the Shaked and Sutton (1982, 1983) type of a VPD model is based on the assumption of quality-dependent variable costs. With this quality-cost specification, unlike Hung and Schmitt (1988, 1992) and Donnenfeld and Weber (1992, 1995), the “high-quality advantage” (where the firm choosing to produce the high-quality good earns a higher profit in equilibrium than does the low-quality firm) does not necessarily
Although Lutz (1996) recognized that the quality-dependent costs could change the results of Hung and Schmitt (1988, 1992) and Donnenfeld and Weber (1992, 1995), in his case, the costs are not variable but "fixed". Also, under our quality-dependent marginal production cost, firms do not differentiate products completely (unlike Tirole (1988) and Shaked and Sutton (1982)). By introducing the quality-dependent variable costs in the model, we allow for the possibility of inferior-quality entry.

For future research, a few potential extensions of this study can be considered. First, the quality-setting model discussed in this chapter is essentially a static one-period game. In the real world, however, entry cannot occur instantaneously. It takes time to decide whether to enter, to expand facilities, and to reach long-run profits. Dynamic inferences may be worth exploring by analyzing repeated games. Second, to avoid some of the analytic difficulties, we followed a number of previous analyses and assumed ex ante that the market is characterized by a covered market configuration in the price game. Thus, the market equilibrium that we have studied applies only when we are in the covered market configuration where each consumer buys one of two goods offered. The model with partial market coverage, instead of full market coverage, may be more appealing because it allows for some potential consumers not to buy the differentiated goods. In another aspect of the model, the ex ante choice of using either a covered or an uncovered market configuration is clearly somewhat unsatisfactory. Third, we have calculated the socially optimal level of qualities. Then the next question is how to

31 Actually, we have shown that the incumbent's profit is greater than the entrant's profit, regardless of entrant's quality regime (i.e., there is a first-mover advantage).
32 As in Wauthy (1996), two-firm market outcomes (whether the market is covered or not) can be derived endogenously for the degree of product differentiation and the extent of consumer heterogeneity. In spite of attractive features of this, due to required characterization for both covered and uncovered market configuration, we cannot take anymore analytical simplicity of the covered market configuration. Beyond the difficulty of characterizing equilibrium values, also, endogenizing market outcomes seems to involve difficult problems of interpretation.
regulate differentiated firms to improve social welfare. The socially desirable intervention as regulatory remedies may involve the optimal subsidy/tax policies applied to product R&D investments, maximum price regulation, and the use of quality standards (e.g., Besanko, Donnenfeld, and White (1988), Ronnen (1991), Ecchia and Lambertini (1997), and Toshimitsu (2003)).
CHAPTER 4. INFERIOR-PRODUCT INNOVATIONS WITH EXTERNAL EFFECTS

4.1 Introduction

The motivation of this study comes from the recent introduction of ‘genetically modified’ (GM) agricultural products, where the issues of segregation between GM and conventional goods are controversial and have given rise to a number of unresolved economic questions. We view GM and non-GM goods as vertically differentiated products in terms of consumers' preferences. Although this type of product differentiation might be a polar case, in the sense that all potential buyers evaluate quality in the same way, it provides the analytical convenience. We relate the market for GM products to the apparent gap of the ‘vertical product differentiation’ (VPD) literature, where previous studies do not consider explicitly the possibility of the introduction of an "inferior product". We aim at analyzing the specific question of how private decisions by an innovator bring forth inferior or superior technologies, in a situation where consumers would be willing to pay a higher price for the information that a product is or is not genetically modified. Thus, the key factor playing a role in this analysis is the cost of segregation activities that are necessary to distinguish GM products from non-GM products.

4.1.1 An Example from the Agricultural GM Products

After the introduction of GM plants in the mid-1990s, there has been an intense debate focused largely on the relative benefits and risks of GM products.\(^1\) Due to their contribution to the reduction of production costs and improved pest control for farmers,

\(^1\) Related general economic issues are well constructed by Nelson ed. (2001), Harhoff, Régibeau,
herbicide-tolerant and insect-resistant genetically engineered crops (such as cotton, corn, and soybeans) have been cultivated increasingly in the United States and in a few other exporting countries. However, there appears to be strong consumer resistance to these products, especially in the European Union, Japan and other importing countries. A resistance is rooted in concerns about the safety of GM products with respect to human health and the environment. Thus, it seems that the GM technology can be viewed as a process innovation from a farmer’s point of view, and a product innovation from a consumer’s point of view (i.e., the new product is an imperfect substitute for an existing product). More importantly, it also seems that a fundamental feature of current GM products is to introduce to market new products that are not universally accepted as superior (Lapan and Moschini, 2004). Indeed, there is no reason, a priori, to believe that the current GM technology increases each individual’s incentive to consume if the resulting product is viewed by consumers as inferior to the old variety. The rise of GM labeling regulation, in the European Union for example, may justify this concern. However, even if the GM technology may yield inferior products, if consumers have a lower marginal valuation of quality, an inferior technology may dominate the market (e.g., Sallstrom (1999)). The important and critical point, in this setting, is the fact that


2 During 6 years (1995-2001), the global area dedicated to genetically modified crops increased more than 30 times: in 1996, only 1.7 million hectares of genetically modified crops were planted in 6 countries such as the United States, Canada, and Argentina; by the end of 2001, the total area growing genetically modified crops increased to 52.6 million hectares and the number of countries growing these crops has more than doubled (Nap et.al (2003)).

3 Ex ante regulations such as mandatory labeling and premarket approval for all foods obtained from GM products have been introduced in Europe, Japan, and elsewhere. The EU countries require that all foods containing more than 0.9% ‘genetically modified organisms’ (GMOs) must be labeled as “containing GMOs”. Japan, Australia, New Zealand, Korea have introduced or drafted labeling requirements, and other countries are considering to require GM labeling. But the United States only has a voluntarily labeling system (for more details, see USDA (2001) and Zarrilli (2000)). An additional important factor, in this setting, may be represented by concerns about the industry concentration in the seed industry (Harhoff, Régibeau, and Rockett (2001)).
keeping old and new products separated by ‘identity preservation’ (IP) may be very costly,\(^4\) so that the market outcome may lead to inefficiency (Lapan and Moschini, 2004).

Producers of high-quality products would generally like to be known as such because consumers are willing to pay more for higher quality. Thus, innovators that develop a superior product will make an effort to supply information with their new variety. In other words, in a product innovation in which the new product is ranked higher than the old product by every consumer, the cost of implementing an IP system can be expected to be internalized by the innovator. However, if there is a negative consumer reaction to a new product, innovators may have little incentive to pay for IP costs because disclosure of information about the new variety may not be beneficial to them. For example, disclosure of information by innovators may be beneficial to their potential competitors (such as GM-free producers). Thus, if consumers’ right-to-know imposes a mandatory IP system, and as long as GM innovators are not responsible for the external costs incurred by the new technology, producers and/or consumers of GM-free product are expected to pay IP costs. In this case, there will be a negative price externality in a differentiated product market of GM and non-GM products. Recognizing that the Pareto criterion requires the absence of externalities, if innovators do not internalize such external costs, it seems that there may be an inefficient level and/or type of R&D investment from a society’s point of view. The relevant point, in this case, is how much of R&D investment level is socially desirable, given the added costs involved with implementing IP system.

\(^4\) This is usually because it is necessary to keep conventional products segregated from the new variety of GMOs throughout the production and marketing system via storage, transportation, processing, and distribution (Bullock and Desquilbet, 2002).
4.1.2 VPD Models

Mussa and Rosen (1978)-type or a Tirole's (1988) simplified version of Shaked and Sutton (1982, 1983) VPD models have been extensively used to investigate the firm's R&D behavior of quality choices in the context of a non-cooperative two-stage game of identical duopolists, where each firm is allowed to offer only one quality, and where investments in quality are made simultaneously in the first stage and then product market competition occurs in the second stage. Most models with heterogeneous consumer preferences use a linear indirect utility function for each type of consumer and a uniform distribution on consumer's tastes to obtain an explicit solution of the game, with attention restricted to the case of an uncovered market (e.g., Ronnen (1991), Choi and Shin (1992), Motta (1993), Aoki and Prusa (1996), Lehman-Grube (1997), Bonanno and Haworth (1998), and Greenstein & Ramey (1998)) and covered market (e.g., Tirole (1988: 296-298), Rosenkranz (1995), and Pepall (1997)). However, as summarized in Table 4-1, the representation of the firms' marginal production costs is different depending on the purpose of the study. Very simple quality-choice models are established in the absence of production costs, and by assuming that the quality choice is costless (e.g., Tirole (1988) and Choi and Shin (1992)). In the model of Mussa and Rosen (1978) and Bonanno and Haworth (1998), to avoid equilibria in which only the maximal quality, yet the cheapest product is produced, a quality-dependent constant marginal production cost is introduced, such that the higher quality good is assumed to be more expensive to manufacture. Meanwhile, conventionally it is assumed that the R&D costs to bring about product innovation are sunk, convex, and strictly increasing in the quality level.

\[5\] Wauthy (1996) provided a full characterization of quality choice, without imposing the \textit{ex ante} restriction that the market is, or is not, covered in the second stage of the game.
Table 4-1. Assumptions on the Nature of Costs in VPD Models

<table>
<thead>
<tr>
<th>Types</th>
<th>Variable Costs</th>
<th>Fixed R&amp;D Costs</th>
</tr>
</thead>
</table>

These different quality-cost structures in a duopoly VPD model produced following two important results. First is the "maximal product differentiation" result that attains under the covered market setting. In a very simple quality-choice game model, Tirole (1988) by using the modified version of Shaked and Sutton (1982) showed that firms maximize product differentiation over the available range of qualities. Even though the model displays the absence of quality-choice costs, because price competition is more intensified the less differentiated are the goods, price competition gives firms the incentive to differentiate their products. Thus, the optimal solution for the first stage problem is the maximal product differentiation where one firm chooses the minimum possible quality and the other firm chooses the maximum possible quality. The second result is the "high-quality advantage" where the firm choosing to produce the high-quality good earns a higher profit in equilibrium than does the low-quality firm. For example, Tirole (1988), Choi and Shin (1992), Aoki and Prusa (1996), and Lehmann-Grube (1997) support the high-quality advantage result by assuming that the cost of quality choice (the R&D cost) is zero, or is born as a fixed cost in the first-stage quality.
choice while variable production costs are insubstantial.

4.1.3 Our Entry Model

In the existing literature of VPD models, it is uncommon to find studies that recognize the introduction of an “inferior product” as defined in Sallstrom (1999).\(^6\) In most previous studies, R&D expenditure allows a firm to produce only a new good where quality has been improved. But if the market consists of sufficiently many consumers whose quality preference is low, then a firm may want to serve more consumers by the introduction of a low quality good associated with a low price in the market. Examples of such lower-quality innovations may be found in the furniture industry, in the production of musical instruments, and in the food industry. Thus, it would seem restrictive to presume that a firm will carry out only a superior innovation that improves on the quality of an existing variety. In our model, we do not designate a priori the type of quality (high or low) for each firm unlike conventional duopoly models in which a firm designated as a “low type” is not allowed to choose a “high type” of product in the model, and vice versa. Thereby, in our model, whether the entrant firm chooses an inferior or a superior technology is determined endogenously.

Regarding our entry model, it should be noted that the two results associated with the conventional VPD models (i.e., maximal product differentiation and high-quality advantage) are not robust when the marginal cost of production varies with qualities. Under the VPD structure with a quality-dependent production cost, as noted by

\(^6\) In a model with variable cost of quality and heterogeneous consumers, Sallstrom (1999) showed that there is a possibility of quality-reduced innovation in the market equilibrium when new technology makes larger scale production feasible. This is due to the fact that consumers who were not buying the good will start buying it after cost reduction and so quantity increasing technological change. Then the new optimal quality may fall if the new consumers have low marginal valuation for quality.
Lambertini (1996), the high-quality advantage with a sequential or simultaneous quality choice does not necessarily hold. Also, with this cost specification, equilibrium qualities can be determined internally within the feasible quality interval (rather than being differentiated maximally). Thus, in our entry model, we accommodate a quality-dependent marginal production cost in which higher-quality entry is associated with higher costs. Indeed, it seems that most quality standards in manufacturing high-quality goods may affect variable rather than fixed costs. As a result, we can avoid the uninteresting equilibrium in which only the highest quality entry is chosen (i.e., the entrant may not earn higher profits by providing a superior quality relative to the existing variety).

In this study we ignore the possibility of drastic innovations. In a vertically differentiated product market with sufficiently wide consumer preferences, a firm never becomes a monopolist. We also consider a situation where an inferior good, although it can be produced cheaply, yields a negative externality to the producers of a superior good because it introduces the need for segregation costs. On the other hand, an entrant firm wanting to produce a new good superior to the existing one will have to internalize this external cost to produce a superior good. Thus, by developing a game-theoretic model of R&D entry, we examine how this segregation externality affects the incentives of the entrant to innovate. Also, we investigate the biases on private sector R&D and on the direction of research because of the existence of these IP costs, when an innovating firm does (or does not) internalize the externality costs brought about by the new product. Finally, we explore the welfare properties of market equilibria. In particular, we investigate how consumers are affected by the existence of externality in identity preservation costs.
In the model that follows, quality and externality parameters determine prices directly through variable costs. We show that the potential entrant firm may enter the market with a low-quality good for the high enough values of externality parameters, and vice versa. Our model takes this effect into account and characterizes the impact of externality parameters on the consumer welfare.

The remaining parts of this chapter are organized as follows. First, we specify the game-theoretic model of market entry. Second, we characterize the market equilibrium level of product innovation, and its properties with respect to interesting parameters. Third, we evaluate the market equilibrium in terms of consumers’ welfare.

### 4.2 The Model

Our analysis focuses on the entry of a single biotechnology firm into a monopoly market. Initially, the firm decides whether to invest to create a new GM product in the market where consumers differ in their willingness to pay for different observed qualities. As a simplification, it is assumed that the magnitude of the R&D investment is fixed and the innovation arises with certainty if the investment is made. Once an irreversible investment is made, we consider a two-stage game of the market for a product with heterogeneous qualities. Anticipating the product market equilibrium values, in the first stage (the development stage) the biotechnology firm chooses its entry quality; in their second stage (the production stage) both the incumbent and the entrant compete in a product market. Thus, profit maximization in the first stage does not involve strategic interactions among firms, whereas it does in the second stage. We assume that firms constitute a Bertrand price-setting duopoly in the second stage of the game. Thus, we consider only the case where the potential entrant would choose a
"high-" or "low-" quality good compared to the existing variety. This is because duopoly price competition in a homogeneous good market drives firms' profits to zero; hence the entrant is better off by differentiating its entry product from the existing variety.

4.2.1 Demand

The demand side of the market is characterized by a continuum of potential buyers differentiated by non-negative, one-dimensional taste parameter $\theta$. The parameter $\theta$ is assumed to be distributed uniformly with density $\delta > 0$ over an interval $[\theta, \overline{\theta}]$, with $\overline{\theta} > \theta > 0$. For simplicity, we normalize the indices as $\delta = 1$ and $\overline{\theta} - \theta = 1$.

Initially, it is assumed that each consumer demanded one unit of a good indexed by 0. Introducing a new good indexed by 1 in the market via successful innovation, two goods defined as labels 0 and 1 are vertically differentiated. Assuming the market is fully covered, each type of consumer $(\theta)$ demands either one unit of good 0 or one unit of good 1, and has tastes for the exogenously given one-dimensional quality index $X \in (0, \infty)$ that is observable to all, where a higher value of $X$ corresponds to a higher level of quality. It is assumed that, as proposed by Mussa and Rosen (1978), the indirect utility function of a consumer $\theta$ patronizing good $i$ is:

$$U_i (P_i, X_i, \theta) = \theta X_i - P_i,$$

where $P_i$ and $X_i$ for $i = \{0, 1\}$ are, respectively, the price and quality variables.

This utility function implies that all consumers prefer higher quality if the two qualities are offered at the same prices, but they differ in their willingness to pay:

---

7 A duopoly market is said to be "covered" if all consumers purchase one unit of either good. In an uncovered market setting, by contrast, some consumers are allowed not to purchase at all. We focus on the covered market case purely for analytical convenience.
consumer $\theta$ is willing to pay up to $\theta X_i$ dollars for one unit of the product $i$; hence his or her surplus is expressed as $U_i = \theta X_i - P_i$. It is assumed that consumers are price and quality takers: given firms’ decisions $(X_i, P_i), i = \{0, 1\}$, each consumer has to choose between purchasing one unit from the incumbent or purchasing one unit from the entrant. These decisions are based on the ‘individual rationality constraint’ (IRC) and the ‘self-selection constraint’ (SSC).\footnote{No consumer chooses a good whose price is too high or whose quality is too low. That is, if $P_i > P_0$ and $X_0 > X_i$, all consumers prefer good 0. Likewise, if $P_i < P_0$ and $X_0 < X_i$, all consumers prefer good 1. Thus, the assumption of SSC implies that $P_i \geq P_0$ if $X_0 \leq X_i$ and $P_i \leq P_0$ if $X_0 \geq X_i$.} Therefore, given the available choice set of quality and price, the market is partitioned in a straightforward manner: a consumer will buy one unit of a product if surplus is non-negative (by IRC) and greater than the surplus from consuming the other product (by SSC).

Let $X_1 = kX_0$, where $k > 0$, and normalize $X_0 = 1$. Then, the parameter $k$ is the relative quality chosen by the entrant firm. $k = 1$ corresponds to the homogenous product case, whereas values of $k$ other than 1, $\forall k \in (0, \infty)$ describe cases in which goods are imperfect substitutes. We will say that the innovation of good 1 is “inferior” if $k < 1$, and “superior” if $k > 1$. We denote $k_L$ as an inferior technology and $k_H$ as a superior technology. Then, for given prices, $(P_1, P_0)$, aggregating individual demand behavior into product demand functions for good 1 and good 0 yields:

$$
Q_0 = \begin{cases} 
\max \{0, \bar{\theta} - \max \{\theta_{10}, \bar{\theta}\}\}, & \text{if } k < 1 \\
\max \{0, \min \{\bar{\theta}, \theta_{10}\} - \bar{\theta}\}, & \text{if } k > 1 
\end{cases}
$$

$$
Q_i = \begin{cases} 
\max \{0, \min \{\bar{\theta}, \theta_{10}\} - \bar{\theta}\}, & \text{if } k < 1 \\
\max \{0, \bar{\theta} - \max \{\theta_{01}, \bar{\theta}\}\}, & \text{if } k > 1 
\end{cases}
$$
where \( \theta_{10} = \frac{P_0 - P_1}{1 - k_L} \) and \( \theta_{01} = \frac{P_1 - P_0}{k_H - 1} \).

In what follows, we will restrict our attention to the “non-drastic” innovation where the innovation cannot drive the existing variety out of the market, and both firms compete in the post-innovation market.\(^9\)

### 4.2.2 Costs and Firm Behavior

In the supply side of a market there is an incumbent firm producing good 0 and a prospective innovator that would enter the market with new good 1 via product innovation. The incumbent is initially endowed with a constant marginal production cost \( C_0 > 0 \) while an entrant is not initially in the market. To avoid the uninteresting equilibrium in which only the highest possible quality is chosen by the entrant, we postulate a quality-dependent constant marginal production cost for the entrant, such that the higher quality good is more expensive to manufacture. Specifically, our assumption is that if the entrant chooses quality \( k \) then its unit production cost is \( C_1 = c(k) \), where \( c'(k) > 0 \), \( c''(k) > 0 \), and \( c(1) \equiv C_0 \) for all feasible relative qualities \( k \in (0, \infty) \). This variable quality cost assumption implies that product innovation accompanies a cost-reducing or cost-increasing effect. Thus, low-quality entry will have a “cost advantage”, while high-quality entry will have a “quality advantage” at the expense of high production costs. In particular, when the following fixed R&D costs and segregation externality parameters are absent, convexity in quality of the variable cost function ensures profit maximizers in the quality-choosing stage of the entrant.

\(^9\) In the literature, “drastic” and “non-drastic” innovations are typically defined when the innovation is cost-reducing: a process innovation is drastic if the cost reduction enables the innovator to charge the monopoly price, whereas it is non-drastic if the innovator gains some cost advantage over its rivals but not one such that the firm can price like a monopoly without fear of competition (Tirole, 1988: 391-392).
We now suppose that a potential entrant is free to choose any quality $k \in (0, \infty)$ by incurring a constant fixed R&D cost. There is no competition in R&D investment. A R&D decision is a “yes” or “no” decision, and investment yields a successful innovation. A potential entrant’s decision to enter the market will be determined by the last stage payoff. Once an innovative firm chooses to enter the market, R&D costs no longer influence optimal decision-making. Also, in stage two, the previously-made R&D expenses have become sunk costs that do not directly affect the profit-maximizing output choice. Thus, without much loss of generality, we suppose that R&D costs are equal to zero.

In addition to these assumptions on costs, we make an additional assumption regarding the implementation of segregation between old and new goods. Given our VPD structure, an effective segregation system will generate a price premium for superior products whenever there are consumers who prefer them. Thus, after the innovation, it is the producer of the superior product that needs to incur a “segregation” cost to prevent co-mingling between superior and inferior products. Specifically, the assumption is that, if the entrant prefers to choose a level of quality that is below the level of the incumbent’s quality $X_0$, then it induces external segregation costs for the incumbent firm in the amount of $\gamma_L \sigma(k_L)Q_0$, where $\gamma_L > 0$, $\sigma(k_L) > 0$, and $\sigma'(k_L) \leq 0$.

On the other hand, if the entrant prefers to choose a level of quality that is above $X_0$ then it will be the party that has to incur the segregation cost $\gamma_H \sigma(k_H)Q_1$, where $\gamma_H > 0$, $\sigma(k_H) > 0$, and $\sigma'(k_H) \geq 0$. Note that we are allowing for possibly asymmetric segregation costs (i.e., $\gamma_L \neq \gamma_H$). The parameter $\gamma_j$, $j = L, H$, relates to the efficiency of the segregation process (e.g., higher $\gamma_j$ implies that segregation is more expensive or
is less effective; and \( \gamma_j = 0 \) means the absence of segregation costs). Then the second-stage total costs can be specified as follows:

\[
TC_0 = \begin{cases} 
C_0 + \gamma_s \sigma(k_h)Q_0, & \text{if } k < 1 \\
C_0 Q_0, & \text{if } k > 1
\end{cases}
\]

\[
TC_1 = \begin{cases} 
c(k_L)Q_1, & \text{if } k < 1 \\
\left\{c(k_H) + \gamma_H \sigma(k_H)\right\}Q_1, & \text{if } k > 1
\end{cases}
\]

In addition to the accommodation of asymmetric segregation costs, our formulation of segregation costs is intended to capture two different types of externality. Quality-independent segregation costs are captured by the function \( \sigma(k_j) = 1, \ j = L, H \).

On the other hand, if \( \sigma'(k_L) < 0 \) or \( \sigma'(k_H) > 0 \) the segregation costs are positively related to the quality differences. Economically, this cost specification may be used to characterize the market for GM agricultural products, which provide the potential for a differentiated marketing system and give rise to the controversial issues of segregation between GM and non-GM goods. Modern genetic engineering techniques allow scientists to manipulate genetic materials and to produce new varieties of plants and animals more quickly and easily than conventional breeding methods. To date, the first-generation biotechnology in agriculture has mainly provided agronomic benefits to producers, typically, by lowering input requirements and/or increasing yields (Nelson ed., 2001). The low-quality entry in our model, in particular, involves cost-reducing technology which does not directly affect yields (e.g., herbicide-tolerant corn and soybeans reduce herbicide use in the process of production). However, the controversial consumer responses for the safety and benefits associated with the use of first-generation GM products may make this biotechnology as an inferior one relative to the conventional
variety in the product market. Genetic modification could also be used to benefit consumers directly. Indeed, second-generation GM products such as high-oil corn and rice with enhanced Vitamin A content are considered as a quality-enhanced technological change associated with positive consumer attributes.\textsuperscript{10} The segregation and identity preservation between GM and non-GM products yields various costs, for farmers for example, during planting, harvesting, storage, transportation, and testing (Bullock and Desquilbet, 2002). Our imposition of segregation costs for the identity-preserved superior products is based on the presumption that the perfect segregation is possible.\textsuperscript{11}

4.2.3 Product Market Equilibrium

We now focus on the Nash equilibrium of the post-innovation second stage of the game in which both firms are active. Because demand functions cannot be inverted by the assumption of covered market, for the equilibrium profits in the production stage, we assume Bertrand competition.\textsuperscript{12} In this price subgame, entry quality is exogenous and in equilibrium the two firms will set prices such that the price of a higher-quality good is greater than that of a lower-quality good, because, otherwise, the low-quality firm would have no market share. The incumbent's second stage problem is to choose the profit-maximizing price for its product conditional on a given price chosen by its rival firm:

\textsuperscript{10} See Nelson ed. (2001) for various examples of GM products.

\textsuperscript{11} Technically, it is known that it is very hard to keep perfectly conventional products segregated from the new variety of GM products, as there is a possibility of contamination throughout the production and marketing system via storage, transportation, processing, and distribution (Bullock and Desquilbet, 2002). Thus, the product differentiation model involving quality uncertainty in the presence of imperfect grading and contamination problem of the GM agricultural product market can be an additional research agenda.

\textsuperscript{12} For Cournot competition to be considered, we need to assume that the market is uncovered by allowing some consumers not to buy the differentiated goods (e.g., Motta (1993)).
Let the Bertrand Nash equilibrium prices be denoted by $P_0^*$ and $P_1^*$, and their corresponding quantities $Q_0^*$ and $Q_1^*$. Solving simultaneously the two best response functions yields Bertrand Nash equilibrium prices and output levels, which are:

$$P_1^* = \begin{cases} 
MC_L + \frac{(1-k_L)}{3} \{\Delta_L + (1-\theta)\}, & \text{if } k < 1 \\
MC_H + \frac{(k_H-1)}{3} \{-\Delta_H + (2+\theta)\}, & \text{if } k > 1 
\end{cases}$$
\[ P_0^* = \begin{cases} \frac{MC^0_L + (1-k_L)}{3}\{-\Delta_L + (2+\theta)\}, & \text{if } k < 1 \\ \frac{MC^0_H + (k_H - 1)}{3}\{-\Delta_H + (1-\theta)\}, & \text{if } k > 1 \end{cases} \]

\[ Q_0^* = \begin{cases} \frac{1}{3}\{-\Delta_L + (1-\theta)\}, & \text{if } k < 1 \\ \frac{1}{3}\{-\Delta_H + (2+\theta)\}, & \text{if } k > 1 \end{cases} \]

\[ Q_1^* = \begin{cases} \frac{1}{3}\{-\Delta_L + (2+\theta)\}, & \text{if } k < 1 \\ \frac{1}{3}\{-\Delta_H + (1-\theta)\}, & \text{if } k > 1 \end{cases} \]

where \( MC^0_L = C_0 + \gamma_L \sigma(k_L) \), \( MC^1_L = c(k_L) \), \( MC^0_H = C_0 \), \( MC^1_H = c(k_H) + \gamma_H \sigma(k_H) \), \( \Delta_L = \frac{MC^0_L - MC^1_L}{1-k_L} \), and \( \Delta_H = \frac{MC^1_H - MC^0_H}{k_H - 1} \). Note that when \( k_L = k_H = 1 \), we have Bertrand’s model of price competition with an undifferentiated product in which equilibrium prices are the marginal cost of production. As noted earlier, thus, we consider only the case of strictly differentiated entry because the potential entrant can be better off (rather than have zero profits) by differentiating its entry product from the existing variety.

The corresponding equilibrium profits are:

\[ \pi_1^* = \begin{cases} (1-k_L)(Q_1^*)^2 = \frac{(1-k_L)}{9}\{-\Delta_L + (1-\theta)\}^2, & \text{if } k < 1 \\ (k_H - 1)(Q_1^*)^2 = \frac{(k_H - 1)}{9}\{-\Delta_H + (2+\theta)\}^2, & \text{if } k > 1 \end{cases} \]

\[ \pi_0^* = \begin{cases} (1-k_L)(Q_0^*)^2 = \frac{(1-k_L)}{9}\{-\Delta_L + (2+\theta)\}^2, & \text{if } k < 1 \\ (k_H - 1)(Q_0^*)^2 = \frac{(k_H - 1)}{9}\{-\Delta_H + (1-\theta)\}^2, & \text{if } k > 1 \end{cases} \]

To complete the duopoly covered-market solution, it remains to check the following two
conditions. First, for exogenously given qualities, the necessary conditions for both outputs to be positive in the product market equilibrium (i.e., \( Q_f^* > 0 \) and \( Q_l^* > 0 \)) are:

\[
\begin{align*}
\theta & \in (\Delta_L - 2, \Delta_L + 1), \quad \text{if } k < 1 \\
\theta & \in (\Delta_H - 2, \Delta_H + 1), \quad \text{if } k > 1
\end{align*}
\]

Second, for a market to be covered, it must be the case that the consumer with the lowest marginal willingness-to-pay for quality (\( \theta \)) has a non-negative surplus when she buys one unit of the low-quality product. It is verified that the following parameter restriction guarantees that each consumer buys one of the two varieties in the market equilibrium:

\[
\begin{align*}
\theta k_L - P_L^* & \geq 0 \iff \theta \geq \frac{2MC_L^0 + MC_L - 1}{2k_L + 1}, \quad \text{if } k < 1 \\
\theta - P_H^* & \geq 0 \iff \theta \geq \frac{2MC_H^0 + MC_H + k_H - 1}{2 + k_H}, \quad \text{if } k > 1
\end{align*}
\]

4.3 Market Equilibrium Levels of Product Innovation

4.3.1 Entry Qualities

In this section, we solve the first stage of the game, where the innovative monopolist undertakes product innovation. The innovative entrant’s payoff relevant for characterizing the subgame perfect Nash equilibrium in the first stage consists of the reduced form of profits from the second stage less the fixed innovation cost, i.e., \( \Pi_i^L = \pi_i^* - 0 \) associated with \( k < 1 \) and \( \Pi_i^H = \pi_i^* - 0 \) associated with \( k > 1 \). Then the equilibrium levels of product innovation \( k_j^* \), \( j = L, H \) are implicitly defined by the following first order conditions:

\[
L(k_j^*; \gamma, \theta) = \theta - 1 + \Delta_L(k_j^*) + 2\{\gamma \sigma'(k_j^*) - c'(k_j^*)\} = 0 \quad \text{for } k_j \in (0, 1)
\]
(17) \[ H(k_H^*, \gamma_H, \theta) = \theta + 2 + \Delta_H(k_H^*) - 2\left\{ c'(k_H^*) + \gamma_H\sigma'(k_H^*) \right\} = 0 \text{ for } k_H \in (1, \infty) \]

The sufficient conditions \( c'(k_L^*) - \gamma_L\sigma'(k_L^*) > \frac{1}{2} \left( \frac{\partial \Delta_L}{\partial k_L} \right) > 0 \) for an inferior innovation

and \( c'(k_H^*) + \gamma_H\sigma'(k_H^*) > \frac{1}{2} \left( \frac{\partial \Delta_H}{\partial k_H} \right) > 0 \) for a superior innovation insure that \( k_L^* \) and \( k_H^* \) are unique profit maximizers in their respective domains (see the Appendix). That is, for \( k_j^*, j = L, H \), implied by equations (16) and (17) to be profit maximizing, the sufficient condition requires that the cost differences between two firms are convex.

As noted earlier, the high-quality advantage does not necessarily hold with the specification of the quality-dependent marginal production cost. That is, as we will show later with specific examples, the entry quality is not predetermined as the higher one. The entrant has two local maxima, for the low- and high-quality segment, respectively. For the inferior-quality entry, \( \Pi_L^1 \) attains a local maximum at \( k_L^* \), where \( \frac{\partial \Pi_L^1}{\partial k_L} = 0 \), and the value \( k_L^* \) is characterized by the FOC (16). For the superior-quality entry, \( \Pi_H^1 \) attains a second local maximum at \( k_H^* \), where \( \frac{\partial \Pi_H^1}{\partial k_H} = 0 \), and the value \( k_H^* \) is characterized by the FOC (17). To determine how the direction of entrant’s quality choice and the level of quality are affected by the changes in parameter values of \( \gamma_L \), \( \gamma_H \), and \( \theta \), consider the value function \( V \) defined as the difference between two local maximum profits, or

(18) \[ V(\gamma_L, \gamma_H, \theta) = \Pi_H^1(k_H^*, \gamma_H, \theta) - \Pi_L^1(k_L^*, \gamma_L, \theta) \]

Then the function \( V \) incorporates entrant’s optimal quality-choice behavior. That is, the entry quality will be “high” relative to the existing variety if \( V > 0 \), “low” if \( V < 0 \), and
“indifferent” between low-quality and high-quality regime if $V = 0$.

4.3.2 Comparative Static Analysis

The incentive to provide qualities is related to the parameter values of $\gamma_L$, $\gamma_H$, and $\theta$. We are now interested in the derivatives of the value function $V$, namely the effects on the "direction of quality choices" in these parameters. Using the chain rule of differentiation with respect to the parameter $\gamma_L$, we have:

$$
\frac{dV}{d\gamma_L} = -\left[ \frac{\partial \Pi^L_i(k^*_L)}{\partial k^*_L} + \frac{\partial \Pi^L_i(k^*_L)}{\partial \gamma_L} \right]
$$

The first term on the right-hand side of (19) is equal to zero by the application of the Envelope Theorem. Thus, there remains only the direct effect on the direction of an entrant’s quality choice from changing $\gamma_L$:

$$
\frac{dV}{d\gamma_L} = -\frac{\partial \Pi^L_i(k^*_L)}{\partial \gamma_L} = \frac{2\sigma(k^*_L)}{9}\left[\Delta_L(k^*_L) + 1 - \theta\right] < 0
$$

(19)

Likewise,

$$
\frac{dV}{d\gamma_H} = \frac{\partial \Pi^H_i(k^*_H)}{\partial \gamma_H} = \frac{2\sigma(k^*_H)}{9}\left[\Delta_H(k^*_H) - 2 - \theta\right] < 0
$$

(20)

The inequalities above arise from the necessary conditions (14) for post-innovative duopoly market equilibrium. Unambiguously, an increase in $\gamma_j$, $j = L, H$, decreases $V$, and an increase in $\theta$ increases $V$ for all $\theta$ in the appropriate interval. Thus, the entrant
earns higher profits as a low-quality innovator for high enough \( \gamma_j \) and/or low enough \( \theta \).

Conversely, the entrant earns higher profits as a high-quality innovator for low enough \( \gamma_j \) and/or high enough \( \theta \). Thus, we can summarize the results above as follows:

**Result 1.**

(a) *When segregation is sufficiently costly, entry will occur with a low-quality good relative to the existing variety.*

(b) *When consumers are sufficiently wealthy entry will occur with a high-quality good relative to the existing variety.*

The economic intuition behind Result 1 is clear. First, because the increase in \( \gamma_j \) raises a unit cost and lowers the demand faced by the high-quality producer, the shift of consumers from high- to low-quality products raises the profits of the low-quality producer. Second, because \( \theta \) is the consumers’ marginal willingness-to-pay, a consumer with higher \( \theta \) is willing to pay more for the higher quality good, while a consumer whose taste parameter \( \theta \) is very low would not like to pay for the high-quality good. In the sense of Shaked and Sutton (1982), the parameter \( \theta \) can be interpreted as the marginal rate of substitution between income and quality, so that a higher \( \theta \) corresponds to a lower marginal utility of income and therefore a higher income. That is, wealthier consumers have a higher value of \( \theta \) and are willing to pay for a higher quality good. Under this interpretation, the entrant is more likely to enter the market with a high-quality good when consumers are wealthy, or vice versa, because \( \frac{dV}{d\theta} > 0 \).

Conditional on the chosen quality direction, let us now examine the effects of externality parameters and the degree of consumer heterogeneity\(^\text{13}\) on the level of entry.

\(^{13}\) Note that consumers’ heterogeneity, measured by the ratio \( \bar{\theta}/\theta \), decreases with \( \bar{\theta} \) (recall that \( \bar{\theta} = \theta + 1 \)): the greater is \( \bar{\theta} \), the more homogenous are consumers.
qualities. Totally differentiating equation (16) and (17) for $\gamma_L$, $\gamma_H$, and $\theta$, we obtain the effects of segregation efficiency and the degree of consumer heterogeneity on the entry quality of the biotechnology firm. The comparative static results can be obtained by the Implicit Function Theorem and by virtue of the second-order conditions as follows:

\begin{align*}
(22) & \quad \text{sign} \left( \frac{\partial k^*_L(\gamma_L, \theta)}{\partial \gamma_L} \right) = \text{sign} \left( \frac{\partial L(k^*_L; \gamma_L, \theta)}{\partial \gamma_L} \right) = \frac{\sigma(k^*_L)}{1-k^*_L} + 2\sigma'(k^*_L) \\
(23) & \quad \text{sign} \left( \frac{\partial k^*_L(\gamma_L, \theta)}{\partial \theta} \right) = \text{sign} \left( \frac{\partial L(k^*_L; \gamma_L, \theta)}{\partial \theta} \right) = 1 > 0 \\
(24) & \quad \text{sign} \left( \frac{\partial k^*_H(\gamma_H, \theta)}{\partial \gamma_H} \right) = \text{sign} \left( \frac{\partial H(k^*_H; \gamma_H, \theta)}{\partial \gamma_H} \right) = \frac{\sigma(k^*_H)}{k^*_H-1} - 2\sigma'(k^*_H) \\
(25) & \quad \text{sign} \left( \frac{\partial k^*_H(\gamma_H, \theta)}{\partial \theta} \right) = \text{sign} \left( \frac{\partial H(k^*_H; \gamma_H, \theta)}{\partial \theta} \right) = 1 > 0
\end{align*}

These results indicate that entry quality levels are positively related to the level of consumer homogeneity (Figure 4.1), while the signs of comparative static results for the externality parameters depend on the type of segregation costs. When $\sigma(k_j)$ is a constant (for example, $\sigma(k_j) = 1$), entry quality levels are positively related to the segregation inefficiency parameters (Figure 4.2). Figure 4.3, by contrast, illustrates the case $\sigma(k_j) = |1-k_j|$, such that $\sigma'(k_L) < 0$ and $\sigma'(k_H) > 0$. 
Figure 4.1 Comparative Static Analysis of $\theta$

Figure 4.2 Comparative Static Analysis of $\gamma_j$: Case of $\sigma'(k_j) = 0$
4.3.3 Examples

To derive simple reduced-form solutions of the game, we now consider the special case of $C_i = C_0 k_j^2$ with $C_0 = 1$ and $\sigma(k_j) = |1 - k_j| = \begin{cases} 1 - k_j, & \text{if } k_j < 1 \\ k_j - 1, & \text{if } k_j > 1 \end{cases}$, $j = L, H$. This specification assures that $c'(k_j) > 0$, $c''(k_j) > 0$, $\sigma'(k_j) \leq 0$, and $\sigma'(k_n) \geq 0$. Also, this specification satisfies the SOC’s in the quality-choosing equilibrium. The two local maxima and the global maximum of $\Pi_j'$, and the comparative static results are summarized in Table 4-2. In particular, in the case of "variable externality" where $\sigma(k_j) = |1 - k_j|$ and $\gamma_j > 0$, equilibrium values confirm the results that entry qualities are negatively related to the externality parameters.
Table 4-2. Examples of Entry Qualities

<table>
<thead>
<tr>
<th>Types of Externality</th>
<th>Local Maximum</th>
<th>Entry Quality</th>
<th>Comparative Static Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Variable Externality&quot;: $C_i = k_j^2$, $\sigma(k_j)=</td>
<td>1-k_j</td>
<td>$, $\gamma_j &gt; 0$</td>
<td>$k_L^* = \frac{\theta - \gamma_L}{3}$, $k_H^* = 1 + \frac{\theta - \gamma_H}{3}$</td>
</tr>
<tr>
<td></td>
<td>$\Pi_L^i(k_L^<em>) = \frac{4}{9}(1 - \frac{\theta - \gamma_L}{3})^3$, $\Pi_H^i(k_H^</em>) = \frac{4}{9}(\frac{\theta - \gamma_H}{3})^3$</td>
<td>$k^* = k_H^*$ if $\theta \geq \frac{\gamma_L + \gamma_H + 3}{2}$</td>
<td>$\frac{\partial k_L^<em>}{\partial \theta} &lt; 0$, $\frac{\partial k_H^</em>}{\partial \gamma_H} &lt; 0$</td>
</tr>
<tr>
<td>No Externality: $C_i = k_j^2$, $\gamma_j = 0$</td>
<td>$k_L^* = \frac{\theta}{3}$, $k_H^* = 1 + \frac{\theta}{3}$</td>
<td>$k^* = k_L^*$ if $\theta \leq \frac{3}{2}$</td>
<td>$\frac{\partial k_L^<em>}{\partial \theta} &gt; 0$, $\frac{\partial k_H^</em>}{\partial \theta} &gt; 0$</td>
</tr>
<tr>
<td></td>
<td>$\Pi_L^i(k_L^<em>) = \frac{4}{9}(1 - \frac{\theta}{3})^3$, $\Pi_H^i(k_H^</em>) = \frac{4}{9}(\frac{\theta}{3})^3$</td>
<td>$k^* = k_H^*$ if $\theta \geq \frac{3}{2}$</td>
<td>$\frac{\partial \Pi_L^i(k_L^<em>)}{\partial \theta} &lt; 0$, $\frac{\partial \Pi_H^i(k_H^</em>)}{\partial \theta} &gt; 0$</td>
</tr>
</tbody>
</table>

Note that the above examples apply only to the range of the parameter $\theta$ which ensures that the duopoly actually covers the market. In order for the duopoly market to be fully covered at the market equilibrium, one must check whether these solutions satisfy constraints (14) and (15). In the case of “variable externality”, if entry occurs with an inferior quality, the duopoly condition (14) is $\theta \in \left(\gamma_L - \frac{3}{2}, \gamma_L + 3\right)$ and the covered-market restriction (15) is $\theta \geq \frac{\sqrt{3\left(3\gamma_L^2 + 8\gamma_L + 16\right)} - (\gamma_L + 12)}{8}$. If entry occurs with a superior quality, the duopoly condition is $\theta \in \left(\gamma_H, \gamma_H + \frac{9}{2}\right)$ and the covered-market restriction is $\theta \geq \frac{\sqrt{3\left(5 - 2\gamma_H\right) + 2\gamma_H} - 9}{2}$. Because entry occurs with an inferior
quality when $\theta < \frac{\gamma_L + \gamma_H + 3}{2}$ or vice versa, for the special case of $\gamma_L = \gamma_H = 1$, the duopoly covered-market segmentation at the market equilibrium can be illustrated as Figure 4.4.

![Figure 4.4 Equilibrium Market Segmentation: Case of Variable Externality with $\gamma_L = \gamma_H = 1$](image)

In the absence of externality, if entry occurs with an inferior quality, the duopoly condition (14) is $\theta \in \left(\frac{-3}{2}, 3\right)$ and the covered-market restriction (15) is $\theta \geq \frac{3\left(\sqrt{3} - 1\right)}{2} \approx 1.0981$. If entry occurs with a superior quality, the duopoly condition is $\theta \in (0, 9)$ and the covered-market restriction is $\theta \geq \frac{3\left(\sqrt{15} - 3\right)}{2} \approx 1.3095$. Because entry occurs with an inferior quality when $\theta < \frac{3}{2}$ or vice versa, the duopoly covered-market segmentation at the market equilibrium can be illustrated as Figure 4.5.

![Figure 4.5 Equilibrium Market Segmentation: Case of No Externality](image)
As we see, entry quality will be superior relative to the existing variety if consumers are sufficiently homogeneous. Note that the range $\theta \in \left[\frac{7}{4}, \frac{5}{2}\right]$ entails superior-quality entry in the case of no externality, and inferior-quality entry when the segregation cost externality is present (see Figures 4.4 and 4.5). Therefore, the existence of a segregation cost externality "biases" entry decisions in favor of low-quality entry.

### 4.4 Welfare Evaluation

In this section, we consider how an individual consumer is affected by the changes in externality parameters. Changing $\gamma_j$ has a negative effect on the individual consumer surplus by increasing product prices. However, the direction of a quality change is ambiguous. That is, when entry occurs with a low-quality good, the change in individual consumer surplus of the low-type consumer with respect to the externality parameter becomes:

\[
\frac{d}{d\gamma_L} (\theta k_L^* - P_L^*) = \frac{1}{3} \left[ -2c'(k_L^*) - \gamma_L \sigma'(k_L^*) + 1 + 3\theta - \frac{\partial k^*_L}{\partial \gamma_L} - \sigma(k_L^*) \right]
\]

and, for the high-type consumer, it is

\[
\frac{d}{d\gamma_L} (\theta - P_0^*) = \frac{1}{3} \left[ -c'(k_L^*) - 2\gamma_L \sigma'(k_L^*) + 2 + \theta \frac{\partial k^*_L}{\partial \gamma_L} - 2\sigma(k_L^*) \right].
\]

When entry occurs with a high-quality good, the change in individual consumer surplus of the low-type consumer with respect to the externality parameter becomes:

\[
\frac{d}{d\gamma_H} (\theta - P_0^*) = \frac{1}{3} \left[ -c'(k_H^*) - \gamma_H \sigma'(k_H^*) - 1 + \theta \frac{\partial k^*_H}{\partial \gamma_H} - \sigma(k_H^*) \right]
\]

and, for the high-type consumer, it is
Now, to sign how individual consumer surplus is affected by the externality parameters, consider the special case of the previously established example of variable externality associated with convex costs. When entry occurs with the low-quality good, for all $\theta \in [\theta, 1 + \theta]$ and for all $\theta$ in the appropriate interval, we have:

\[
(27)' \quad \frac{d}{d\gamma_L} \left( \theta k^*_L - P^*_L \right) = -\frac{9\theta - 7\theta + 3 + 7\gamma_L - 1}{27} \frac{1}{3} \sigma \left( k^*_L \right) = \frac{10\theta - 9\theta - 12 - 10\gamma_L}{27} < 0
\]

\[
(28)' \quad \frac{d}{d\gamma_L} \left( \theta - P^*_0 \right) = -\frac{\theta + 6 + 8\gamma_L - 2}{27} \frac{1}{3} \sigma \left( k^*_0 \right) = \frac{5\theta - 24 - 14\gamma_L}{27} < 0
\]

Therefore, effects on the individual consumer surplus in response to the increase in $\gamma_L$ are negative. At the same time, the market share for the low-quality good 1 is enlarged as $\gamma_L$ increases, because now the marginal consumer $\theta_{l0}$ who is indifferent between buying a low-quality good 1 and buying a high-quality good 0 is located on the right-hand side of the original point on the line of a taste parameter $\theta$:

\[
(31) \quad \frac{d\theta_{l0}}{d\gamma_L} = \frac{d \left( P^*_0 - P^*_L \right)}{d\gamma_L} = -\frac{7\theta + 6 + 2\gamma_L}{9} = \frac{2}{9} > 0
\]

However, for the high-quality entry, effects on the individual consumer surplus in response to the increase in $\gamma_H$ are ambiguous:

\[
(29)' \quad \frac{d}{d\gamma_H} \left( \theta - P^*_0 \right) = \frac{9 + \gamma_H - \theta}{27} \frac{1}{3} \sigma \left( k^*_0 \right) = \frac{9 - 4\theta + 4\gamma_H}{27}
\]
because $\theta \in [\theta', 1 + \theta]$ and $\frac{9}{2} < \gamma_H + \frac{9}{2}$ from the duopoly constraint. Note that, in this case, the market share for the low-quality good 0 is enlarged as $\gamma_H$ increases:

$$
\frac{d\theta}{d\gamma_H} = \frac{d}{d\gamma_H} \left( \frac{P^*_L - P^*_0}{k^*_H - 1} \right) = \frac{d}{d\gamma_H} \left( \frac{7\theta + 9 + 2\gamma_H}{9} \right) = \frac{2}{9} > 0
$$

These are addressed in the following result.

**Result 2.** With convex quality costs and variable externality such that $C_i = k^*_j$ and

$$
\sigma(k_j) = |1 - k_j| \text{ for } j = L, K,
$$

(a) when entry occurs with the low quality, all consumers lose from the increase in segregation costs;

(b) however, when entry occurs with the high quality, some consumers may have benefits from the increase in segregation costs;

(c) in both entry regimes, the market share for the low-quality good is enlarged.

### 4.5 Concluding Remarks

A duopoly model associated with VPD is developed to show how the existence of segregation costs biases the firm’s quality choice behavior in the covered market setting. The entrant chooses the degree to which it differentiates its product from an already existing one. With an increasing and convex cost of quality, the model predicts that the entrant firm has an incentive to enter the market with a low-quality good to reduce production costs when the values of externality parameters are sufficiently high,}

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Note that equation (29)' is negative if $\theta > \frac{9}{4} + \gamma_H$, and the equation (30)' is negative if $\theta < 10.8 - 8\gamma_H$ for given $\theta = 3.2$. 

or vice versa. When consumers are homogeneous enough, entry may occur with a high-quality good. The model also explains that how consumers are affect by the increase in segregation inefficiency. In our specific example associated with the convex quality costs and variable externality, we found that all consumers lose from the increase in segregation costs when the entry occurs with the low-quality good.

In this entry model, unlike the standard duopoly VPD models where a firm designated as a low type is not allowed to choose a high type of product, we see how firms choose inferior or superior technology. In particular, quality-dependent costs occur in the second stage of the game when actual production takes place. Thus, quality and an externality parameter determine prices directly through variable costs. Literally, the result confirms that the high-quality advantage does not necessarily hold with the introduction of quality-dependent marginal production costs. Also, by the introduction of a quality-dependent marginal production cost, we have shown that equilibrium qualities could be internal to the interval of possible qualities, rather than maximally differentiated (or unbounded) as in Tirole's (1988) covered market setting. That is, if marginal costs are too high relative to product qualities, such innovations will not be undertaken. By contrast, if an entry quality is too low, such goods will not be sold in the market. Thus, there exist finite solutions of entry qualities.

Some remaining issues related to these results can be mentioned as a guide for future research. A natural question to ask is whether the equilibrium outcome can be improved from a social point of view. To answer this question, first of all, it would be interesting to evaluate market equilibrium values of entry qualities in terms of social welfare criteria. That is, we need to characterize the direction of socially optimal quality choices, and the optimal level of entry qualities conditional on the chosen quality.
direction. However, the use of corrective taxes, subsidies, or regulations to improve social welfare is feasible only if the quality is observable. Thus, the case where qualities are not verifiable or very costly to verify, especially when GM product quality is concerned due to the presence of imperfect grading and contamination problem, would be an interesting subject for the future research.
Appendix. Sufficient Conditions for Equilibrium Qualities to be Unique Profit Maximizers

Maximization of profit $\Pi_j$ with respect to $k_j$, $j = L, H$ yields the following first order conditions:

(A.1) \[ \frac{\partial \Pi_L}{\partial k_L} = D(k_L^*) \cdot L(k_L^*) = 0 \]

(A.2) \[ \frac{\partial \Pi_H}{\partial k_H} = G(k_H^*) \cdot H(k_H^*) = 0 \]

where \( D(k_L^*) = \frac{\Delta_L + 1 - \Theta}{9}, \)
\( L(k_L^*) = 2\{\gamma_L \sigma^*(k_L^*) - c'(k_L^*)\} + \Delta_L - 1 + \Theta, \)
\( G(k_H^*) = \frac{2 + \Theta - \Delta_H}{9}, \)
\( H(k_H^*) = 2 + \Theta + \Delta_H - 2\{c'(k_H^*) + \gamma_H \sigma^*(k_H^*)\}. \)

Knowing that \( D(k_L^*) > 0 \) and \( G(k_H^*) > 0 \) by the parameter restrictions of the non-drastic innovation where these restrictions guarantee positive demands of two goods in the product market equilibrium, the entry quality will be determined by \( L(k_L^*) = 0 \) if the entry occurs with an inferior quality and \( H(k_H^*) = 0 \) if the entry occurs with a superior quality. Then the SOCs require:

(A.3) \[ \frac{\partial^2 \Pi_L}{\partial k_L^2} = D'(k_L^*) \cdot L(k_L^*) + D(k_L^*) \cdot L'(k_L^*) < 0 \rightarrow \]
\( L'(k_L^*) = 2\{\gamma_L \sigma^*(k_L^*) - c'(k_L^*)\} + \frac{\Delta_L}{\partial k_L} < 0 \) (because \( L(k_L^*) = 0 \) and \( D(k_L^*) > 0 \))

(A.4) \[ \frac{\partial^2 \Pi_H}{\partial k_H^2} = G'(k_H^*) \cdot H(k_H^*) + G(k_H^*) \cdot H'(k_H^*) < 0 \rightarrow \]
\( H'(k_H^*) = \frac{\partial \Delta_H}{\partial k_H} - 2\{c^*(k_H^*) + \gamma_H \sigma^*(k_H^*)\} < 0 \) (because \( H(k_H^*) = 0 \) and \( G(k_H^*) > 0 \))

Therefore, the SOCs can be rewritten as:
To sign (A. 3)' and (A. 4)', we use \( D(k_L^*) > 0 \) and \( G(k_H^*) > 0 \). That is, \( L(k_L^*) = 0 \) implies the inequality 
\[
\Delta_L + 1 - \gamma = 2\left( \gamma \sigma'(k_L^*) - c'(k_L^*) + \Delta_L \right) > 0
\]
and hence 
\[
\frac{\partial \Delta_L}{\partial k_L} = \frac{\gamma \sigma'(k_L^*) - c'(k_L^*) + \Delta_L}{(1 - k_L^*)} > 0.
\]
Likewise, \( H(k_H^*) = 0 \) implies the inequality 
\[
2 + \gamma - \Delta_H = 2\left( c'(k_H^*) + \gamma \sigma'(k_H^*) - \Delta_H \right) > 0
\]
and hence 
\[
\frac{\partial \Delta_H}{\partial k_H} = \frac{c'(k_H^*) + \gamma \sigma'(k_H^*) - \Delta_H}{(k_H^* - 1)} > 0.
\]
Therefore, the signs of (A. 3)' and (A. 4)' are positive.
CHAPTER 5. GENERAL CONCLUSIONS

5.1 A General Discussion

This dissertation consists of two essays using game theoretic approaches in the area of industrial organization. In chapter 2, I introduced static models of a market for differentiated products to analyze quality-choice behavior of the firm under various scenarios. Chapter 3 looks at the potential for the use of quality choice as an entry deterrence strategy in a sequential entry game. Chapter 4 investigates the potential for product segregation costs to bias the firm’s quality-choice behavior. Throughout the dissertation I seek theoretical and practical contributions, by investigating partial or complete disagreements between homogeneous and heterogeneous product market analyses.

The main idea explored in this dissertation is rooted in Shaked and Sutton (1982) where firms decide whether to enter, then (if they enter) what qualities to produce, and finally what price to charge, given qualities. Their VPD model is characterized by a “finiteness” or “natural oligopoly” property where at most two firms are sustainable in non-cooperative equilibrium for the game in which firms have positive profits. One firm chooses the lowest possible quality level. The other firm chooses the highest possible quality level. This is similar to the “principle of maximum differentiation” which appears in Hotelling-type spatial models with quadratic transportation costs. In particular, in this type of equilibrium, the firm choosing to produce the high-quality good earns higher profits in equilibrium than does the low-quality firm. Thus, if there is an entrant, the new innovation would be always superior to the existing variety because the entrant firm wants to have high-quality advantage.
I extended the model of Shaked and Sutton (1982) in a few different directions. Among them, the most noticeable feature is the introduction of variable costs of production, which is quality-dependent. The common approach in Shaked and Sutton (1982) type of VPD models has been to assume that quality improvement costs are fixed. Thus, the marginal cost of quality itself may vary, but the marginal cost of production (or the variable cost) does not change with product qualities. Whereas this assumption can reflect the situation where firms should engage in R&D or advertising activities to improve qualities, this formulation cannot reflect the variable-cost aspects of quality improvement where the higher quality good is more expensive to manufacture due to, for instance, requirements of more skilled labor or more expensive raw materials and inputs (e.g., Mussa and Rosen's (1978) monopoly model). Importantly, the “high-quality advantage” need not hold with the assumption of quality-dependent marginal production costs. Therefore, by allowing quality-dependent marginal production costs, I allowed for the possibility of inferior innovation relative to the existing variety (e.g., the first generation of 'genetically modified' agricultural products, canned foods, furniture, and musical instruments). The main findings of the dissertation are as follows.

In chapter 2, I clarify the monopoly and duopoly demand structure and the associated product market equilibrium under the covered market configuration. In particular, parameter restrictions on the duopoly covered market are suggested by the degree of relative consumer heterogeneity.

The “entry-deterrence model” in chapter 3 analyzes the entry of a new product into a vertically differentiated market where an entrant and an incumbent compete in prices. I consider sequential instead of simultaneous entry to study the leader-follower aspect of the game between firms. With a sequential choice of quality, quality-
dependent marginal production costs, and a fixed entry cost, I relate the entry-quality decision and the entry-deterrence strategies to the level of entry cost and the degree of consumer heterogeneity. In particular, the incumbent influences the quality choice of the entrant by choosing its quality level before the entrant. This allows the incumbent to limit the entrant's entry decision and quality levels. Quality-dependent marginal production costs in the model allow for the possibility of inferior-quality entry as well as the incumbent's aggressive entry-deterrence strategies by increasing its quality level towards potential entry. First, for sufficiently low fixed entry costs, entry is accommodated and the entrant's choices are indifferent between entry with an inferior quality and entry with a superior quality. In this case, the incumbent selects a quality that is higher than the monopolist's choice. Second, if the entry cost is in a certain moderate range, the incumbent engages in entry deterrence by increasing her product quality before the entrant enters the market. Third, if the entry cost is very high, entry is efficiently blockaded and the incumbent chooses the pure monopolist's quality level. Fourth, it is shown that while the consumer surplus is higher when entry is accommodated than in the absence of entry, the maximum total welfare is not necessarily associated with the accommodated entry. In particular, the maximum welfare of the relatively homogenous consumers is attained at the fixed cost level, where entry is deterred. Fifth, for a certain level of fixed entry costs, there are too many varieties in the economy relative to the social optimum. We also showed that Stackelberg firms associated with accommodated entry excessively differentiate product qualities to reduce price competition.

The "externality model" in chapter 4 focuses on the potential entrant's R&D behavior rather than the entry-deterrence strategies of the incumbent, in a vertically differentiated product market. This model is motivated by some current economic
questions arising from the advent of 'genetically modified' (GM) agricultural product markets, which provides the potential for differentiated good market and gives rise to the controversial issues of segregation between GM and conventional goods. By developing a duopoly market-entry model associated with the vertical product differentiation, this essay proposes an analytical framework to examine how the existence of segregation costs biases the firm's quality choice behavior, and to study the associated welfare effects. Thus, the key factor of the model is the cost of segregation activities that are necessary to distinguish GM products from non-GM products. With an increasing and convex cost of quality, the model predicts that the entrant firm has an incentive to enter the market with a low-quality good to reduce production costs if segregation costs are sufficiently high, and vice versa. When consumers are homogeneous enough, entry will occur with a high-quality good relative to the existing variety. In the special case of the convex quality costs and variable externality, it is found that all consumers lose from the increase in segregation externality when entry occurs with the low-quality good.

5.2 Suggestions for Additional Research

Product differentiation models are used to address issues where product characteristics are not given. In reality, the assumption of homogeneous products would be the exception. It seems that specific product markets provide a wide variety of products in response to the nature of demand. In this sense, the product differentiation approach would be more realistic than the homogenous good market approach, especially if consumers do not view goods as perfect substitutes. Therefore, in many economic models associated with the homogeneous product market analysis, there is a scope for extensions to the product differentiation setting associated with heterogeneity properties of the good and consumer preferences.
In this dissertation, I used VPD models to endogenize product qualities of the firms. The consumers’ taste parameter as the determination of product varieties is an important feature that does not appear in models of horizontal product differentiation. In the following, I suggest some possible opportunities to extend the work of this dissertation.

First, although I limited the model by assumptions such as a covered market, the strategic quality-choice model with partial market coverage instead of full market coverage would be more appealing, in that it allows for some potential consumers not to buy the differentiated goods. In spite of analytical difficulties, further research also can be done with endogenized market outcomes where the firm decides whether to cover the market or not.

Second, we calculated the socially optimal level of qualities in chapter 3. Then the next question would be how the social planner regulates differentiated firms to improve social welfare. The socially desirable intervention as regulatory remedies may involve the product R&D subsidy/tax policy, maximum price regulation, and the use of minimum quality standards as mentioned in chapter 3.

Third, although this dissertation is restricted to two types of products, high-quality and low-quality products, we may incorporate more products that differ in various characteristics. That is, it may be interesting to consider more than two oligopolists involving many incumbents facing many potential entrants.

Fourth, the analyses in this dissertation are based on standard assumptions of the VPD model which, in some senses, ignores a few critical aspects of R&D activities such as uncertainty in innovation and patent races. Future research may include these issues under the VPD setting.
Fifth, the theoretical application of this dissertation can be extended to the particular question related to the advent of GM agricultural products, where the issues of segregation between GM and conventional products are controversial and have given rise to a number of unresolved economic questions. The analytical supply and demand framework to examine the economic effects of segregation in the presence of quality uncertainty (due to the presence of imperfect grading and contamination problem of the GM agricultural product market) would be the one example.

Finally, product differentiation occurs in market for services as well as for goods. For example, consumers can choose different medical, education, and banking services from a number of alternative suppliers. In this sense, we can apply the analysis extensively to the various service markets.
REFERENCES


