ULTRASONIC VELOCITIES IN TEXTURED FE-SI STEELS

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INTRODUCTION

Ultrasonic velocity and attenuation of ultrasound are usually strongly affected by grain boundaries, porosity, texture and various structural defects. This is the reason why ultrasound is often used in nondestructive testing of materials.

Texture strongly affects the anisotropy of the elastic properties of metals. That is the reason why the role of texture in the anisotropy of the ultrasonic velocities will contribute to the development of fully quantitative methods of the interpretation of ultrasonic signals. The ODF makes it possible to calculate the polycrystalline elastic constants and also ultrasonic velocities in textured polycrystalline aggregates. A method for calculating elastic constants of polycrystals for the orthorhombic physical symmetry and the cubic crystal symmetry was suggested by Morris [1]. A simpler, but less accurate method, which will be used here was proposed by Bunge [2]. First calculations of ultrasonic velocities in textured materials were made by Sayers [3] who applied Roe's texture description and obtained results for the ultrasonic velocities in three perpendicular directions in a textured specimen of austenitic stainless steel. Hinz and Szpunar [4] developed a theoretical formula for the calculation of group and phase velocities in textured materials [4] and later used the ultrasonic velocities in the prediction of the plastic strain ratio [5].

In the present paper we shall examine the correlation between elastic anisotropy and the anisotropy of ultrasonic velocity and examine velocity changes for different orientations of the polarization vector in various specimens of Fe-Si steels.

ELASTIC CONSTANTS IN POLYCRYSTALLINE TEXTURED METALS

It is well known that Hooke's law, which in one dimension assumes a linear relationship between the force applied to a material and the resulting strain, can be generalized for crystals to the form

\[ \sigma_{ij} = c_{ijkl} E_{kl} \]  

(1)
which expresses the stress tensor components \( a_{ij} \) in terms of the strain tensor components through the elastic constants \( c_{ijkl} \) of the crystal. One can as well express the strain components in terms of the stress components through the elastic compliances \( s_{ijkl} \),

\[
\varepsilon_{ij} = c_{ijkl} \sigma_{kl}
\]

which forms the inverse of \( c_{ijkl} \). Although these results are valid for single crystals, they can be adapted to polycrystalline materials by taking averages over the orientation distribution function (ODF)

\[
f(g) = \sum_{l=0}^{\infty} \sum_{\mu=1}^{M(l)} \sum_{\nu=1}^{N(l)} C_{ijkl}^{\mu\nu} T_{l}^{\mu\nu}(g)
\]

which describes the volume fraction of crystallites in the sample having an orientation specified by the rotation parameter \( g \). Assuming that the strains in all crystallites are the same, one obtains the Voigt approximation for the elastic constants, or assuming that the stresses in all crystallites are the same, one obtains the Reuss approximation for the elastic compliances.

Improvements to the restrictive assumptions of Voigt and Reuss concerning the elastic behaviour at grain boundaries can be made by following Hill's prescription of taking the average of the tensors obtained by these methods. One obtains

\[
c_{ijkl}^{H} = \frac{1}{2} (c_{ijkl}^{V} + c_{ijkl}^{R})
\]

\[
s_{ijkl}^{H} = \frac{1}{2} (s_{ijkl}^{V} + s_{ijkl}^{R})
\]

which yields a closer picture to the true elastic properties of polycrystalline materials.

PHASE AND GROUP VELOCITY

Analytically, plane wave solutions obscure the distinction between the group and the phase velocities. Experimentally, one measures the arrival time of the elastic disturbance. For this reason it is useful to consider the more realistic example of curved wavefronts to link the theoretical with the experimental. It is possible to show [4] that a curved wave surface propagates locally with the phase velocity of the plane wavefront corresponding to its surface normal i.e. \( v_{a}(n) \). This is intuitively so because a curved wave surface can always be regarded on a sufficiently small scale as approximating a plane wave. The rays that follow the phase and group velocities of the surface during its development in time, terminating on a common endpoint, can then be found to follow trajectories which both arrive at the endpoint at the same time.
Furthermore, the group velocity rays have the remarkable property of being straight lines since one can show \([4]\) that the unit normal is a fixed vector along its trajectory and \(\mathbf{c}_i = \mathbf{o}_i(n)\). One then concludes that experimental velocity measurements are really a determination of the group velocity along straight line trajectories.

The explicit calculation of the phase and group velocities for a textured specimen of cubic crystal symmetry and orthorhombic sample symmetry has been done in our previous paper \([4]\). The following formulae for the three phase velocity are obtained:

\[
v^l = \frac{1}{\sqrt{2\rho_0}} \sqrt{C_{66} + C_{11}n_1^2 + C_{22}n_2^2 + A^2n_1^4 + B_2n_2^4 + 2n_1^2n_2^2(2C^2 - AB)}
\]

\[
v^H = \frac{1}{\sqrt{2\rho_0}} \sqrt{C_{66} + C_{11}n_1^2 + C_{22}n_2^2 - A^2n_1^4 + B_2n_2^4 + 2n_1^2n_2^2(2C^2 - AB)}
\]

\[
v^V = \frac{1}{\rho_0} \sqrt{C_{55}n_1^2 + C_{44}n_2^2}
\]

where 4-index tensor notation of elastic constants is translated to 2-index matrix notation and we have also adopted the abbreviated forms

\[
A = C_{11} - C_{66}, \quad B = C_{22} - C_{66}, \quad C = C_{12} + C_{66}
\]

Differentiating these expressions for the phase velocities with respect to the \(n_1\), we obtain for the group velocities

\[
c^l = \frac{1}{2\rho_0} \left( \frac{1}{v^l} \right) \left( g^L_{1n_1}n_1 + g^L_{2n_2}n_2 \right)
\]

\[
c^H = \frac{1}{2\rho_0} \left( \frac{1}{v^H} \right) \left( g^H_{1n_1}n_1 + g^H_{2n_2}n_2 \right)
\]

\[
c^V = \frac{1}{\rho_0} \left( \frac{1}{v^V} \right) \left( C_{55}n_1 + C_{44}n_2 \right)
\]

where again we have the abbreviated forms

\[
g^L_{1} = C_{11} + C_{66} + [A^2n_1^2 + (2C^2 - AB)n_2^2] / \sqrt{[A^2n_1^4 + B_2n_2^4 + 2n_1^2n_2^2(2C^2 - AB)]}
\]

\[
g^L_{2} = C_{22} + C_{66} + [B^2n_2^2 + (2C^2 - AB)n_1^2] / \sqrt{[A^2n_1^4 + B_2n_2^4 + 2n_1^2n_2^2(2C^2 - AB)]}
\]
The superscripts \( L, H \) and \( V \) have significance arising from the polarization properties in untextured specimens—\( L \) corresponding to the polarization along the wave vector \( k \), \( H \) corresponding to the polarization in the sample plane but perpendicular to the wave vector \( k \), and \( V \) corresponding to the polarization perpendicular to both the wavevector \( k \) and the sample plane. In textured specimens, however, such properties are only approximate.

TEXTURE MEASUREMENTS AND ANALYSIS IN FE-SI TRANSFORMER STEELS

Theoretical predictions will be applied to analyze the anisotropy of ultrasonic velocities in Fe-Si transformer steels. Materials of three different textures are chosen, all secondary recrystallization textures, therefore the grain size in the specimens investigated is very large.

Table 1. Texture data for the Fe-Si steel specimens used for ultrasonic velocity calculations.

<table>
<thead>
<tr>
<th>( C^{11} )</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.008</td>
<td>0.67</td>
<td>-1.28</td>
<td></td>
</tr>
<tr>
<td>-6.186</td>
<td>-1.01</td>
<td>-5.74</td>
<td></td>
</tr>
<tr>
<td>3.61</td>
<td>-1.51</td>
<td>1.59</td>
<td></td>
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</table>

In order to obtain reliable statistics of the number of grains, the neutron diffraction method was used. This method is discussed in detail in a review paper on texture and neutron diffraction (Szpunar [6]). The volume of the specimen which can be investigated using neutrons is usually 10^5 times greater than that illuminated by the X-ray method and therefore even specimens with grain sizes of several millimeters can be measured.

The crystal orientation distribution functions were calculated from (110), (200) and (211) pole figures. The resulting \( C^{11} \) texture coefficients are listed in Table 1. The texture of the A specimen is very strong, and the density of the orientation distribution at maximum is 70 times random and corresponds to the Goss texture. The specimens B and C represent a more complex recrystallization texture.

\[
g_1^L = C_{11} + C_{66} + [A^2 n_1^2 + (2C^2 - AB) n_2^2] / \sqrt{[A^2 n_1^4 + B^2 n_2^4 + 2n_1^2 n_2^2 (2C^2 - AB)]} (15)
\]

\[
g_2^H = C_{22} + C_{66} + [B^2 n_2^2 + (2C^2 - AB) n_3^2] / \sqrt{[A^2 n_1^4 + B^2 n_2^4 + 2n_1^2 n_2^2 (2C^2 - AB)]} (16)
\]
CALCULATED ANISOTROPY OF VELOCITIES IN TEXTURED FE-SI STEELS

A calculation of the ultrasonic velocities were carried out for various directions of polarization. Equations (6), (7) and (8) were used to calculate these velocities: the velocity VL having polarization along the propagation direction, the VPH with polarization perpendicular to the direction of propagation and parallel to the specimen surface and the velocity VPV, with the polarization perpendicular to the propagation direction and to the specimen surface. The series expansion coefficients used in the calculation are given in Table 1, the elastic constants of Fe-Si monocrystal, C_{11}, C_{12}, C_{44} being equal to 2.41x10^{11}, 1.46x10^{11} and 1.12x10^{11} N/m², respectively.

Fig. 1 represents the velocities calculated for three different polarization directions. The VL distribution shows a minimum at angles a=90°, β=0° i.e. along the rolling direction, along which direction we also observe a high density of the <100> crystallographic directions. Another minimum (see Fig. 1) also corresponds to a maximum of density of <100> directions.

Table 2. Velocity changes for specimens A, B and C.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>VL [m/s]</th>
<th>[%]</th>
<th>VPH [m/s]</th>
<th>[%]</th>
<th>VPV [m/s]</th>
<th>[%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6120-5800</td>
<td>5.4</td>
<td>3310-2830</td>
<td>15.0</td>
<td>3700-3300</td>
<td>11.4</td>
</tr>
<tr>
<td>B</td>
<td>6020-5860</td>
<td>2.7</td>
<td>3300-3140</td>
<td>5.0</td>
<td>3360-3280</td>
<td>2.4</td>
</tr>
<tr>
<td>C</td>
<td>6160-5840</td>
<td>5.3</td>
<td>3245-2945</td>
<td>9.7</td>
<td>3710-3310</td>
<td>11.4</td>
</tr>
</tbody>
</table>

In Fig. 2 a comparison between the anisotropies of VL and the Young's modulus is made. Maxima and minima of these two properties are observed along the same directions.

All calculated velocities are strongly anisotropic. The changes in velocities as listed in Table 2 show that for a strongly textured specimen A, the velocity varies from about 5 percent of the average value in the case of VL to 15 percent for VPH. Predicted changes of velocity with direction are smaller for the specimen B, which has rather weak texture. Texture of the specimen C is stronger and therefore there is considerable anisotropy of the ultrasonic velocities (see Table 2). According to our calculation for all specimens investigated, the ultrasonic velocity with the polarization along the direction of propagation is about two times higher than the VPH and VPV velocities. It has been also demonstrated that the VL is less strongly affected by the texture. The percentage of changes of the VL is less than 5.5 percent, while the VPH may change by 15 percent.
Fig. 1 Phase velocities calculated along various directions of the specimen ($\alpha$, $\beta$); a) longitudinal velocities for specimen A; b) vertical shear velocities for specimen A; c) horizontal shear velocities for specimen A.
Fig. 2  Phase velocity (LV) for specimen B represented for various directions of the specimen on stereographic projection compared with calculated anisotropy of the Young's modulus.
In our previous paper [4] it has been demonstrated that the phase and group velocities are the same for propagation directions which coincide with the symmetry axis of the grain orientation distribution i.e. rolling direction (RD) or transverse direction (TD). For all other directions the group velocities are less than the phase velocities. Differences, for strongly textured specimens, can be as high as 20 percent.

CONCLUSION

Results presented here illustrate the influence of texture on the anisotropy of ultrasonic velocities in Fe-Si steels. The velocity of ultrasound with polarization along the direction of propagation is more strongly affected by texture changes than horizontal VPH and vertical VPV velocities. Observed differences are significant in the case of ultrasound in materials testing, for instance, the texture dependent anisotropy of various physical properties of materials can be often obtained from ultrasonic measurements.

REFERENCES