SPECULAR REFLECTION BY CONTACTING CRACK FACES

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ABSTRACT

Non-linear relations are postulated between the transmitted tractions and the crack opening displacements across contacting rough crack faces. For the opening mode these relations take into account that a crack easily opens, but that the resistance to crack closure increases rapidly as the rough crack faces come in contact. For the sliding mode they account for initial elastic resistance as well as for frictional sliding resistance, with a maximum transmissible sliding traction whose magnitude strongly depends on the averaged contact pressure on the crack faces. For an incident pulse the nonlinear ordinary differential equations for crack-opening-displacements have been solved numerically. A Kirchhoff-type approximation to the representation integral has been used to compute the specular reflection. Results of parametrical studies are presented.

INTRODUCTION

Scattering of elastic waves by an infinitesimally thin flat crack with smooth non-interacting crack faces (a perfect mathematical crack) is now well understood. Various analytical and numerical methods are available to compute the scattered fields. In most materials the faces of cracks are, however, not smooth, and the interaction of the crack faces due to wave incidence may not be negligible, unless the crack has been opened by prestressing the body. In general terms it can be stated that the applicability of results for the perfect mathematical crack depends on the relative magnitudes of a number of length parameters. Let $\Delta$ and $\varepsilon$
be characteristic values for the crack opening and the crack-face roughness. Then, the perfect mathematical crack results apply if the amplitude of the incident wave is much smaller than $\Delta$, and if the wavelength is much larger than $\varepsilon$.

In the present study, which is based on Ref.[1], the crack is represented by a flaw plane of traction continuity (the tractions may vanish) but possible displacement discontinuity. The interaction between the crack faces is described by non-linear relations across the flaw plane between averaged tractions and averaged displacement discontinuities and their derivatives.

The effects of crack-face roughness and crack-face interactions on the scattering of ultrasonic waves have been of concern to several investigators. Analytical studies have been presented by Haines [2], Thompson et al [3], and Buck et al [4]. The approach of the present paper is comparable to the one of Ref.[3], except that we take into account both normal and shear tractions across the interface. In addition the relations between the interface tractions and the interface separations are taken as non-linear. The postulated interface relations contain a number of parameters which must be determined experimentally. Experimental investigations have been carried out by Woolridge [5]-[6] and Golan [7], as well as by Thompson, et al [3].

CONDITIONS AT THE FLAW PLANE

A schematical depiction of a flaw surface is shown in Fig. 1a. It is assumed that for the purpose of computing the fields of stress and deformation elsewhere in the body, the interaction between the upper and lower faces of the flaw plane can be described by appropriate relations between the tractions and displacement across a perfectly flat surface. This surface, which is shown in Fig. 1b may be considered as the median plane of the actual flaw surface.

In the analytical model we consider averaged tractions and averaged displacement discontinuities per unit area, with respect to coordinates in the flaw plane. The averaged tractions are continuous, which implies that at $y = 0$:

$$\sigma^+_y = \sigma^-_y = \sigma^*_{y}, \quad \sigma^+_y = \sigma^-_y = \sigma^*_{yx}, \quad (2.1a,b)$$

where the $+$ and $-$ signs refer to the upper and lower sides of the flaw plane as shown in Fig. 1b. It should be noted that (2.1a,b) include the conditions for a perfect mathematical crack, which are

$$\sigma^+_y = \sigma^-_y = 0, \quad \sigma^+_y = \sigma^-_y = 0. \quad (2.2a,b)$$
In the present model for a crack with rough crack faces, \( \sigma_{yx}^*, \sigma_{yy}^* \) will be supplemented by relations between the stresses \( \sigma_{yx}^*, \sigma_{yy}^* \) and the displacement discontinuities \( [v] \) and \( [u] \), respectively.

We will first consider the opening mode of the crack. It is reasonable to assume that in the unloaded state, i.e., when \( \sigma_y^* = 0 \), the crack will be slightly open: \( [v] = \Delta > 0 \). In the closing mode we have \( \sigma_y^* < 0 \), and the required stress will increase rapidly as \( [v] \rightarrow 0 \). In fact we assume that an infinite interface stress \( \sigma_y^* \) is required to close the crack completely \( [v] = 0 \), i.e., to completely flatten out the roughness of the crack faces. The crack opening displacement cannot be negative since that would imply overlap of the crack faces. To open up the crack, \( [v] > \Delta \), a slight resistance has to be overcome. The behavior described here can be represented by the relation

\[
\sigma_{yy}^* = T \left( \frac{[v] - \Delta}{[v]} \right) .
\]

Here \( T \) is the maximum tensile force (very small) that can be transmitted across the crack faces. Equation (2.3) represents a non-linear spring. The relation between \( \sigma_{yy}^* \) and \( [v] \) is shown in Fig.2a.

We have assumed that opening of the crack is independent of sliding of the crack faces. The opposite however, cannot be assumed. The resistance to sliding depends very much on the
Fig. 2: Relations between averaged flaw-plane tractions, $\sigma^*_y$ and $\sigma^*_{yx}$, and averaged flaw-plane displacement separations, $[v]$ and $[u]$. 

extent of crack opening, particularly on the magnitude of $\sigma^*_y$. When $\sigma^*_y > 0$ there will be very little resistance to sliding, while for

$\sigma^*_y < 0$ there will be considerable resistance. A convenient

relation between $\sigma^*_{yx}$ and $[u]$ is one that is equivalent to the

elastic-plastic model of solid mechanics. We require

$$[u] = \frac{\delta^*_{yx}}{C} \quad \text{for} \quad |\sigma^*_{yx}| < S,$$

(2.4)

The critical value $S$ depends on $\sigma^*_y$. Here we assume the relation

$$S = S_0 \exp(-\alpha \sigma^*_y).$$

(2.5)

Equation (2.5) satisfies the condition that $S$ is small for $\sigma^*_y > 0$,

while $S$ increases rapidly as $\sigma^*_y$ becomes negative, i.e. as the

crack faces are pressed together.

At the arrival of a pulse at the flaw plane at $t = 0$, we have

$[u] = 0$. As $t$ increases (2.4) holds until $|[u]| = S/C$. If $[\dot{u}]$ is

non-zero at this point, the motion is altered abruptly to frictional sliding, in which $\sigma^*_y$ is equal to $S \, \text{sgn}[\dot{u}]$. The frictional sliding
ceases when $\dot{\mathbf{u}}$ changes sign or when $S$ exceeds $C|\mathbf{u}|$, say at $\mathbf{u} = \mathbf{u}_1$ and $\sigma_{yx}^* = S_1$. Equation (2.4) then takes over again in the form

$$
\sigma_{yx}^* = C(\mathbf{u}-\mathbf{u}_1) + S_1,
$$

(2.6)

until a later time at which $\sigma_{yx}^*$ again reaches the critical value $S$. This occurs when

$$
|\sigma_{yx}^*| = S, \quad [\dot{\mathbf{u}}] \neq 0.
$$

(2.7)

A typical displacement history is illustrated in Fig. 2b. The key point to observe is the possibility of displacement hysteresis.

The parameters $T, \Delta, C, \alpha$ and $S_0$ have to be determined from the crack geometry and from experimental data.

**SEPARATION OF FLAW-PLANE FACES**

We will first investigate the separation of the faces, $\mathbf{u}$ and $\mathbf{v}$, when a plane pulse is reflected and transmitted at an infinite flaw plane. With reference to the coordinate system shown in Fig. 1b the incident pulse is of the general form

$$
\mathbf{u}^{\text{in}} = \mathbf{d}_{\alpha} f_{\alpha}^i (\xi_{\alpha} + \eta_{\alpha}) H(\xi_{\alpha} + \eta_{\alpha}), \quad \alpha = L, T
$$

(3.1)

where $\alpha = L$ and $\alpha = T$ define an incident longitudinal and transverse wave, respectively, and $H(\ )$ is the Heaviside step function. The unit vector $\mathbf{d}_{\alpha}$ defines the displacement direction:

$$
\mathbf{d}_L = (\sin\theta_L, \cos\theta_L), \quad \mathbf{d}_T = (\cos\theta_T, -\sin\theta_T).
$$

(3.2a,b)

Also

$$
\xi_{\alpha} = t - (x/c_{\alpha})\sin\theta_{\alpha}, \quad \eta_{\alpha} = - (y/c_{\alpha})\cos\theta_{\alpha}
$$

(3.3a,b)

where

$$
c_{\alpha}^2 = (\lambda+2\mu)/\rho, \quad c_T^2 = \mu/\rho
$$

(3.4a,b)

Thus, the pulse arrives at $x = 0, y = 0$ at time $t = 0$.

It was shown in Ref.[1] that the flaw-plane tractions may be expressed as
\[- \frac{2}{\rho c_L} \sigma^*_y = \frac{D}{\cos \theta_L} [\dot{\gamma}] + 2 C_{T\alpha} \ddot{x}_\alpha (\xi_\alpha) \quad (3.5)\]

\[- \frac{2}{\rho c_T} \sigma^*_{yx} = \frac{D}{\cos \theta_T} [\dot{u}] + 2 C_{L\alpha} \ddot{x}_\alpha (\xi_\alpha) \quad (3.6)\]

where

\[C_{LL} = k^{-\frac{1}{2}} \sin 2 \theta_L \quad C_{LT} = \cos 2 \theta_T \quad (3.7a,b)\]

\[C_{TL} = C_{LT} \quad C_{TT} = -k^{-\frac{1}{2}} \sin 2 \theta_T \quad (3.7c,d)\]

\[k = c_L/c_T = [(\lambda + 2\mu)/\mu]^{\frac{1}{2}} \quad (3.8)\]

and

\[D = c_{LT}^2 - C_{LL}C_{TT} > 0 \quad (3.9)\]

The angles $\theta_L$ and $\theta_T$ are related by Snell's law

\[(1/c_L) \sin \theta_L = (1/c_T) \sin \theta_T \quad (3.10)\]

Substitution of (3.5) and (3.6) into (2.3)-(2.5) yields a set of inhomogeneous nonlinear ordinary differential equations for $[u]$ and $[v]$, which in general must be solved numerically.

Results for the displacements at the upper and lower faces of the flaw plane have been computed for the following values of the parameters in Eqs.(2.3)-(2.5).

\[\Delta = .5, \quad T = .2\mu, \quad \zeta = 3\mu/\Delta\]

\[S_\alpha = .5\mu, \quad \alpha = 10/\mu,\]

where $\Delta$ and all subsequent displacements are normalized with respect to the maximum displacement of the incident pulse, which is of magnitude unity. A Poisson's ratio of $1/3$ is taken. The shape of the incident profile, $f_{\alpha}^i$, is the same in all examples and is shown in Fig. 3a.

We consider the pulse of finite duration incident upon the flaw plane. If the incident pulse is of type $\alpha$, $\alpha = L, T$, then the displacements $u^+, v^+$ and $u^-, v^-$ on the upper and lower faces follow
Fig. 3: (a) Incident pulse profile. (b) and (c) Normal and horizontal displacements for normally incident L and T-waves, respectively.

as, see Ref.[1]:

\[
\begin{align*}
    u^{\pm} &= \pm \frac{1}{2} [u] - (v) \frac{\sin(2\theta_T - \theta_L)}{2 \cos \theta_L} + (\sin \delta_L \alpha + \cos \theta_T \delta_T) f^{1 \alpha} \\
    v^{\pm} &= \pm \frac{1}{2} [v] + [u] \frac{\sin(2\theta_T - \theta_L)}{2 \cos \theta_T} + (\cos \delta_L \alpha - \sin \theta_T \delta_T) f^{1 \alpha}.
\end{align*}
\]

The displacements \( v^{\pm} \) and \( u^{\pm} \) are shown in Figs. 3b and 3c for normally incident L and T waves, respectively. We have also plotted the displacements \( u^{-} \) and \( v^{-} \) for the stress-free interface defined by \( \sigma^*_{yy} = \sigma^*_{yx} = 0 \). For a compressive pulse, Fig. 3b shows...
that \( v^- \) is the same as for a stress-free interface until the two faces become very close together. The resistance to closing then becomes very large, causing the upper face to lift. When the peak of the pulse has passed through, and the lower face recedes, the large pressure on the two faces changes to a small tension which slowly brings the two faces back to their equilibrium positions. The decay rate in our example is exceedingly slow, leaving the flaw plane open for a long time after the passage of the pulse. For a tensile pulse there would be very little difference with reflection by a free surface. In Fig. 3c we note how the shear stress in the flaw plane causes the upper face to move in phase with the lower one, though the magnitude of \( u^+ \) is much smaller. The transfer of energy leaves \( u^- \) slightly smaller than the stress-free \( u^- \). The times at which the flaw-plane sliding is in the frictional regime (see eq. (2.6)) are indicated by the circles on the \( u^+ \) curve.

Results for other angles of incidence have been presented in Ref.[1]. For incident transverse waves the effect of interaction of the faces of the flaw plane is small if the displacement is polarized to produce separation of the faces.

SPECULAR REFLECTION AND TRANSMISSION BY A CRACK

Let the crack be located in the plane \( y = 0 \). It has been shown in Ref.[1] that for incident wave motion in the plane of \( x \) and \( y \), the scattered longitudinal and transverse fields in the specular and shadow directions are

\[
\begin{align*}
    u^L(x,t) &= \frac{-A}{4\pi Rc_L} \left\{ T \cdot -2 \sin 2\theta_L [u^+] \cos 2\theta_T \right\} (t - R/c_L) \\
    u^T(x,t) &= \frac{-A}{4\pi Rc_T} \left\{ \cos 2\theta_T [u^+] \sin 2\theta_T \right\} (t - R/c_T)
\end{align*}
\]

(4.1)

and

\[
\begin{align*}
    u^L(x,t) &= \frac{-A}{4\pi Rc_L} \left\{ T \cdot -2 \sin 2\theta_L [u^+] \cos 2\theta_T \right\} (t - R/c_L) \\
    u^T(x,t) &= \frac{-A}{4\pi Rc_T} \left\{ \cos 2\theta_T [u^+] \sin 2\theta_T \right\} (t - R/c_T)
\end{align*}
\]

(4.2)

where \( u \) and \( v \) are the displacement components in the \( x \) and \( y \) directions, and we have used the notation

\[
[u^L_T(t - R/c_\beta)] = [u^L_T](t - R/c_\beta), \quad \beta = L,T.
\]

(4.3)

Here \( R \) is the distance from the origin, which has been taken on the crack faces. Equation (4.3) shows that the discontinuity in the particle velocity across the crack faces, computed at the
origin, radiates as a spherical wave. The displacement vectors for the reflection and shadow fields (superscripts \( r \) and \( t \), respectively) are given by

\[
\begin{align*}
\mathbf{d}_L^r &= (\sin\theta_L^r, -\cos\theta_L^r), & \mathbf{d}_T^r &= (-\cos\theta_T^r, -\sin\theta_T^r) \\
\mathbf{d}_L^t &= (\sin\theta_L^t, \cos\theta_L^t), & \mathbf{d}_T^t &= (\cos\theta_T^t, -\sin\theta_T^t).
\end{align*}
\]  

(4.4a,b)

For a normally incident wave of type \( \gamma \), the specular far-field amplitude

\[ U_\gamma \equiv (R/A)^\alpha y^\gamma \mathbf{d}_\alpha^\gamma \]  

(4.6)

is the same whether \( \gamma = r \) or \( t \), and is independent of \( R \) and \( A \). In Figs. 4a and 4b we have plotted \( U_L \) and \( U_T \) for normally incident \( L \) and \( T \) waves, respectively. The discontinuity in \( U_L \) is related to the rapid closure of the crack (compare with Fig. 3b). The dashed curves in Fig. 4 are the same amplitudes for the stress-free crack. We have also plotted in Figs. 4c and 4d the magnitudes of the Fourier transforms of \( U_L \) and \( U_T \), respectively. These are given by
where $f$ is the frequency. We observe that the frequency spectra of the incident pulses are altered, indicating the non-linear nature of the system. In particular, the rapid closure due to L-incidence generates significant low and high-frequency components.

Finally, we have considered the effect of pre-stressing on the scattered waves. Fig. 5 shows the specular far-field for a T-wave normally incident on the crack. All the parameters are as before except $S_o$, which is increased, corresponding to a greater compressive stress on the faces (see eq.(2.5)). As the stress increases, the sliding of the interface is restricted and the horizontal motion is governed more by (2.4). Eventually, the specular amplitude becomes proportional to the derivative of the stress-free crack amplitude.

![Fig. 5: The effect of compressive stressing of the crack on the transverse specular amplitude.](image)

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REFERENCES


