THE EFFECTS OF CRACK CLOSURE ON ULTRASONIC SCATTERING MEASUREMENTS

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INTRODUCTION

The ultrasonic scattering from cracks is traditionally modelled by requiring that the stresses vanish on the crack faces. A violation of this assumption is found in real cracks when asperities on opposing faces are forced into contact by closure or applied stresses. Many investigators have reported substantial decreases in ultrasonic reflections, as well as partial transmission through such cracks. This paper describes a model which has been developed to take into account the effects of the contacts on the ultrasonic signals scattered from the crack.

The model is developed for the case in which there are many contacts per wavelength. It is postulated that their effect can be approximated by a change in the mechanical boundary condition applied at the crack faces. The resulting equations are equivalent to those which would be derived for the case in which the faces of an ideal crack were coupled by a massless, distributed spring. The transmission of an ultrasonic beam past the crack, and its diffraction at the crack tip, are then treated by combining these quasi-static boundary conditions, a Gaussian model for the ultrasonic beam profile, and a Kirchhoff approximation for the scattering. The theory explains the apparent shortening of the fatigue crack as deduced from low frequency transmission data and a decrease in the strength of the signal from the fatigue crack tip with respect to the signal from an EDM notch tip.
THEORY

Figure 1 illustrates the motivation for the modification of the mechanical boundary conditions to account for partial contact. Consider two half-spaces of identical material partially bonded at a planar interface. If a tensile load, \( \sigma \), is applied, two reference planes, initially separated by a distance \( l \), will increase their separation by an amount \( \Delta l \) given by

\[
\Delta l = (\sigma/E)l + \sigma/\kappa
\]  

where \( E \) is Young's Modulus and \( \kappa \) is a factor to be discussed below. The terms in Eq. (1) respectively describe the usual elastic deformation of a continuous medium plus additional elongations arising from local deformations in the vicinity of the contacting plane. Elasticity calculations provide solutions for \( \kappa \) for a variety of cases. The essential features of these solutions can be illustrated by considering the example of a periodic set of parallel slits in the interface. If one further approximates this by the response of one cell, removed from that periodic structure, then one obtains a simple analytic expression of the form

\[
\kappa_I = \frac{M}{2S\gamma} \left[ 1.071 \left( -1 - \frac{1}{\gamma} \ln (1-\gamma) \right) + 0.25\gamma - 0.357\gamma^2 + 0.121\gamma^3 + \ldots \right]^{-1},
\]

where \( M \) is Young's modulus, \( W \) is the width of the contacts, \( S \) is their separation, and \( \gamma = 1 - W/S \). As shown in Fig. 2, this can be described by a universal plot of \( S\kappa/M \) versus \( W/S \).

\[\text{Fig. 1. Motivation for modification of mechanical boundary conditions.}\]
Fig. 2. Normalized plots of interface stiffness versus fractional contact area for single-cell model of periodic array of slits. Solid curves are theoretical predictions and dashed curves are asymptotic limits.
Equation (1) predicts the excess elongation of the material under Mode-I loading. This is the case, illustrated in Fig. 1, of loading which would tend to "open" the interface. Similar results are available for Mode-II loading (forces in the plane of Fig. 1 which would tend to slide the interface) and for Mode-III loading (forces perpendicular to the plane of Fig. 1 which would tend to tear the interface). In Mode II, the same expression applies with $M$ again equal to Young's modulus. In Mode-III, the expression has the form

$$\kappa_{III} = \frac{M}{2s} \ln \left( \sec \frac{\pi y}{2} \right)^{-1},$$

(2)

where $M$ is the shear modulus. This expression is also plotted in Fig. 2.

These results have been taken as the basis for a definition of a set of extended boundary conditions to treat the effects of crack closure on ultrasonic waves. These take the form

$$\sigma_{31}^+ = \sigma_{31}^- = \kappa_{ij} (u_j^+ - u_j^-),$$

(3)

where $U$ is the mechanical displacement, the superscripts "+" and "-" refer to the two sides of the interface, and $\kappa_{ij}$ is the tensor generalization of the interface stiffness. Here the 3 axis has been taken normal to the interface and the 1 axis has been taken parallel to the interface and in the plane of Fig. 1. Use of these quasi-static boundary conditions presumes that the ultrasonic wavelength is much greater than the dimensions and separations of the contacting asperities. For the case in which the interface is modelled by the two dimensional array of stripes discussed above, $\kappa_{ij}$ is diagonal with values

$$\kappa_{11} = \kappa_{II}, \quad \kappa_{22} = \kappa_{III}, \quad \kappa_{33} = \kappa_{I},$$

(4)

It will be noted that when $\kappa=0$, Eq. (3) reduces to the stress-free conditions characterizing free surfaces. When $\kappa = \infty$, the conditions correspond to perfect contact. Variation of $\kappa$ thus allows one to continuously pass from the condition of perfect contact to that of no contact.

Solution of the Wave Equation with Effective Boundary Conditions

For a plane wave incident normally on the interface, use of the effective boundary conditions leads to a prediction of transmission ($T$) and reflection ($R$) coefficients of the form

$$R = \frac{-j\alpha}{1+j\alpha}, \quad T = \frac{1}{1+j\alpha},$$

(5)
where \( \alpha = \pi f \rho v / \kappa \), \( f \) is the frequency, \( \rho \) is the density, and \( v \) is the wave speed. Inspection of Eqs. (1)-(5) reveals that the transmission and reflection coefficients are functions of two independent parameters, \( W/S \) and \( \lambda/S \), where \( \lambda \) is the wavelength. The existence of the frequency dependence described by the dependence on the latter parameter illustrates the differences in this model and others involving just the area fraction of contact.

**EXPERIMENTS**

Figure 3 illustrates the experimental configuration of interest. A focused longitudinal wave is injected into a modified compact tension specimen normal to its surface, and both the longitudinal wave transmitted in the forward direction past the crack and transverse waves scattered at 45° from the crack tip are detected. In order to account for the frequency dependent width of the ultrasonic beam and the position of that beam with respect to the crack tip, an extension of the plane wave analysis must be made. In this work, the paraxial theory of Gaussian beams has been used to account for the frequency dependent influences of diffraction on the beam width. The Kirchhoff approximation, coupled with the plane wave transmission coefficient of Eq. (5), then leads to the expression for the normalized crack response at transducer position \( x_1 \),

\[
\Gamma^N(x_1) = \left( \frac{\Gamma^c}{\Gamma^R} \right) = A \int_{-\infty}^{\infty} dT(x) e^{-\frac{(x-x_1)^2}{w^2}} e^{j(k(x-x_1)\sin \theta)}.
\]

Here it has been assumed that the crack response \( \Gamma^c \) has been normalized by the measured transmission through the sample in the absence of the crack \( \Gamma^R \) to remove the frequency dependency of attenuation and transducer efficiency. The normalization factor \( A \) is a function of various beam and interface transmission parameters to be discussed elsewhere, and \( w \) is a beam width parameter.

![Fig. 3. Experimental configuration for obtaining crack signal \( \Gamma^C \) and reference signal \( \Gamma^R \) (Image)](image-url)
When $\theta=0$, Eq. (6) is a simple overlap integral of the interface transmission coefficient, whose value is assumed to vary along the crack length, and the beam profiles. When $\theta\neq0$, the constant phase planes of the transmitter and receiver radiation patterns do not coincide, and this leads to the final, sinusoidally varying factor in the integral.

The fatigue crack was grown in a 7075 aluminum compact tension specimen, using load shedding techniques to maintain a constant $\Delta K_I/K_{IC}=25\%$. The geometry of this sample had been modified so that, in the plane of the measurement, only the shape shown in Fig. 3 remained. This allowed illumination of the crack with well focused ultrasonic beams. The broad band probes used had 10 MHz nominal center frequency, 1.8 cm (3/4 in.) diameter, and 10 cm (4 in.) focal length in water. At 10 MHz, the diffraction limited focal spot size was estimated to be 1.5 mm (0.060 in.), which was adjusted to be in the plane of the crack.

RESULTS

Figure 4 shows the results when the beam was transmitted directly past a saw slot, simulating a fully open crack. The left figure shows the experimental observations. The right shows the prediction of the theory when $\kappa$ was chosen to step from $\infty$ to 0 at the crack tip (T stepped from 1 to 0). Theory and experiment are in good agreement. Note, in particular, that the transition from complete transmissions (beam positioned to miss crack) to zero transmission (beam completely obscured) becomes sharper at higher frequencies. This is because of the smaller dimensions of the diffraction limited spot size of the focused beam at the higher frequencies. In addition, at all frequencies, the 50% transmission occurs when the beam is centered on the crack tip so that the focal spot is half on and half off of the crack.

![Fig. 4. Ultrasonic transmission past an open crack (saw slot) at 2, 4, 6, 10 and 14 MHz. Left, experiment; right, theory.](image-url)
Figure 5 presents the behavior observed for the fatigue crack. The experimental response is considerably different from that of the saw slot. Note that the higher frequencies do not exhibit nearly as sharp a transition as they did on the saw slot. This is taken as evidence in support of the existence of a closure region in which there is a gradual transition from the fully open to the fully closed conditions. In our model, this would correspond to a gradual change in the values of $S$ and $W$, and thus in $\kappa$, along the crack. Note, also, that the responses at different frequencies no longer cross in general. At the 50% transmission point, the crack appears 0.8 mm longer at 14 MHz than it appears at 2 MHz.

These features of the measurement are in good agreement with the predictions of the model. Trial values of $\kappa$, continuously varying from $\infty$ to 0 as a function of distance from the crack tip, were tested by comparing the predictions of Eq. (6), with $\psi=0$, to the data. Of those forms tried, the best fit was obtained using an exponential function with a 1/e decay distance of 0.6 mm. The model predictions are shown at the right of Fig. 5. The greater apparent crack length at higher frequencies arises from the frequency dependence of $T$. The lack in sharpness of the crack tip observed at higher frequencies, as compared to the saw slot, occurs because the diffraction limited focal spot size has become smaller than the characteristic dimensions of the transition region so that the latter dominates the measurement response.

Figure 6 compares the responses of the saw slot and fatigue crack when the 45° diffracted $T$ waves were measured. The responses are considerably different. The peak responses of the fatigue crack are weaker, they decrease more rapidly with frequency, and do not narrow as rapidly with frequency. These diffracted signals are produced by the sharp discontinuity in the elastic field at the crack tip. Within the above model for the slot, this
corresponds to a truncation of an integral of an oscillating function. The closure region changes this truncation to a more gradual damping, and thereby accounts for the aforementioned effects. Unexpected scattering effects reported for two planar interfaces pressed together and not described by the quasi-static model may also contribute to these observations. Quantitative comparisons of theory and experiment will be reported elsewhere.

CONCLUSIONS

The derived model describes the increase in apparent fatigue crack length at higher frequencies as deduced from transmission measurements, and the decrease of the fatigue crack tip diffracted response with respect to an EDM notch. The agreement is semi-quantitative for the transmission case, but further work is required to gain a fuller understanding of the tip diffracted case.

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