INTRODUCTION

In the design of electromagnetic NDE systems for the detection and examination of cracks and other defects in conducting materials, it is desirable to have a quantitative description of the fields in the vicinity of the defect. In previous work by this author and co-workers [1,2], the fields in the vicinity of a crack were calculated for models based on excitation by a spatially uniform applied field, as in the interior of a solenoid. The present work reports on an improved model which includes non-uniformity of the field of the exciting coil and the effects of coil size and position relative to the crack.

DESCRIPTION OF THE MODEL AND METHOD OF CALCULATION

The calculations to be presented are based on the two-dimensional model consisting of a pair of parallel wires carrying equal and opposite currents, \(I_0 e^{-i\omega t}\), oriented parallel to a v-groove crack in a slab of conducting materials. The wires, representing the coil, are infinitesimal in thickness; the wires and crack are infinite in length. This simplified model allows one to calculate the impedance per unit length of the coil, as a function of crack dimensions, coil dimensions, coil position, conductivity of the metal, and excitation frequency. By subtracting the impedance per unit length in the absence of the crack, the crack signal alone can be obtained. The model is illustrated in Fig. 1, where relevant parameters are defined.
In this two-dimensional treatment the ac electric and magnetic fields may be described by a one-component vector potential $A(x,y)$. Above the metal surface $A$ satisfies the Laplace equation; below the surface it satisfies a Helmholtz equation with propagation constant $k = (1+i)/\delta$, where $\delta = \sqrt{2/\omega \mu_0}$ is the electromagnetic skin-depth.

The Boundary Integral Equation (BIE) method may be expanded to the case of two adjoining regions. Application of the BIE method to the problem described above leads to the coupled integral equation pair,

$$
1/2A(S) - \int \left[ \frac{\partial G(S,S')}{\partial n'} A(S') - G(S,S') \frac{\partial A(S')}{\partial n'} \right] dS' = J(S) \quad (2.1a)
$$

$$
1/sA(S) + \int \left[ \frac{\partial G(S,S')}{\partial n'} A(S') - G(S,S') \frac{\partial A(S')}{\partial n'} \right] dS' = 0. \quad (2.1b)
$$

This treatment assumes that $A$ and its normal derivative, $\partial A/\partial n$, are continuous across the boundary. This boundary condition is appropriate for non-magnetic materials, as in the present application, but may be extended to include the magnetic permeability of the metal. In Eqs. (2.1), $J(S)$ is the vector potential of the source wires at the surface $S$, as if the metal were absent; $G$ is the Laplace Green's function, $-(1/2\pi) \log |S-S'|$; and $G$ is the Helmholtz Green's function, $(i/4) H_0^1(k|S-S'|)$, where $H_0^1$ is a Hankel function of the first kind. The integrations over the boundary points $S'$ are to be computed according to the Cauchy principal value recipe [3]. When the unknowns $A$ and
\( \frac{\partial A}{\partial n} \) are found on the boundary surface \( S \), the vector potential \( A \) may be constructed at any location in the metal or in the air above by further application of Green's theorem.

**COIL IMPEDANCE IN THE ABSENCE OF A CRACK**

The radiation field of an excitation coil above a half-space of finite conductivity has been obtained as a closed-form integral solution (requiring numerical evaluation) by numerous authors, only a few of whom are quoted [4,5]. The results for the coil with no crack present are also obtainable as a by-product of the BIE approach of this work. We present calculated results based on dimensions comparable to those used in the analysis of eddy current probes by Fortunko and Padget [6], and Auld [7]. The greatest sensitivity for crack detection occurs when crack and coil dimensions are approximately equal to the skin depth. Accordingly, an example for calculation was selected, as listed in Table 1. The example corresponds to a pair of No. 30 AWG enameled copper wires just touching, elevated one radius above the plane. For an operating frequency of 110 kHz, and an aluminum slab, the wire center separation and the elevation above the plane are each one skin-depth.

| TABLE I |
| Parameters for Model Calculation Based on Aluminum; Coil Frequency is 110 KHz. |

| Resistance \( \rho \) | 2.82 x 10^{-8} \( \Omega \) |
| Conductivity \( \sigma(=1/\rho) \) | 3.54 x 10^{7} \( \Omega^{-1}\text{m}^{-1} \) |
| Skin Depth \( \delta \) | 0.255 \( \text{mm} \) |
| Crack Depth \( (2\delta) \) | D | 0.51 \( \text{mm} \) |
| Crack Opening \( (.5\delta) \) | 2F | 0.128 \( \text{mm} \) |
| Wire Radius \( (.5\delta) \) | A/2 | 0.13 \( \text{mm} \) |
Although principal interest is in the impedance of the exciting coil, it is instructive to examine plots of the normal component of the Poynting vector on the surface of the metal, since they show a detailed representation of the radiation field. These plots are useful for assessing the convergence of the numerical work as well as for showing the regions of the metal where significant absorption and field penetration occur. The time average of the complex Poynting vector may be obtained from

\[
\mathbb{S} = \frac{1}{2} \text{ExH}^* \\
= -\frac{i}{2} \frac{\mu_0 \omega}{\delta} |I_0|^2 \frac{\partial A^*}{\partial n} A. \quad (3.1)
\]

In Fig. 2 the real (dissipative flux) and imaginary (reactive flux) parts of the complex Poynting vector, as calculated according to the data of Table 1, are shown for the case without a crack.

**Figure 2**  Poynting vector on the surface of a metallic slab in the absence of a crack. The real part of the Poynting vector is indicated by circles; the imaginary part by squares. Distance is in units of the skin depth and the Poynting vector is in units of \(\mu_0 \omega x 10^{-3}\). The exciting wires are located at \(x = \pm 0.5\delta\) and are at an elevation of 0.5\delta.

**COIL IMPEDANCE WITH A CRACK**

The calculations for the case with a crack present proceed in the same manner as in Sec. II, but with several geometric parameters added which describe the crack and its distance from
the coil. For this investigation a crack depth of 2.0 \( \delta \) with a half-opening of 0.25 \( \delta \) was selected. At frequency 110 kHz, and for aluminum metal, this corresponds to a crack depth of 0.51 mm and a half-opening of 0.064 mm, as listed in Table 1. This was selected as a model case for which the crack dimensions and the skin-depth are of the same order of magnitude. Figs. 3, 4, and 5 show the Poynting vector for values of \( P \), the crack-to-coil center displacement of 2.5\( \delta \), 0.8\( \delta \) and 0.0\( \delta \), respectively. The coil dimensions and elevation are the same as for the no-crack case of Sec. II. Qualitative examination of the figures shows that in the presence of a crack, a portion of the integrated Poynting flux is "stolen" from the nearer of the peaks in the field distribution of the coil. The Poynting vector at the corners of the crack is somewhat increased over the value that would occur at that position if no crack were present. Inside the crack, the Poynting vector decays to zero in approximately one skin-depth; thus this orientation of applied H-field is not sensitive to crack depth (contrary to the case when the H-field is parallel to the crack [2]).

Finally we present the results for the change of complex impedance induced in the coil wire by the crack, using the crack dimensions of Table 1. The calculations were performed for a scan of \( P \), and for frequencies 110 kHz, 1.1 MHz and 11 kHz. Fig. 6 gives the magnitude of the change vs. \( P \) and Fig. 7 gives the phase of the charge vs. \( P \). If the exciting wires are copper, No. 30 AWG, then the dc resistance of the pair is approximately 0.8 \( \Omega / \text{m} \).

**Figure 3** Poynting vector on the surface of the slab with a crack, for crack-coil displacement \( P = 2.5\delta \).
Figure 4  Poynting vector on the surface of the slab with a crack, for crack-coil displacement $P = 0.8\delta$.

Figure 5  Poynting vector on the surface of the slab with a crack, for crack-coil displacement $P = 0.08\delta$. 
Figure 6  Magnitude of impedance change due to the crack as a function of crack-coil displacement P, for several frequencies. Crack and coil dimensions are given in Table 1.
Thus the low frequency (11 KHz) detectibility will be very poor, as is seen in Fig. 6. At high frequency there is oscillation with variation in $P$, which will reduce detectibility, especially when a multi-turn coil is used. Thus, for this orientation in which coil wires are parallel to the crack, it is found that optimum detectibility occurs when crack dimension and skin-depth are of the same order of magnitude. This is to be contrasted with the case where the coil wires and crack are perpendicular for which Auld [7] reports increasing sensitivity at somewhat higher frequencies.

REFERENCES

7. B. A. Auld, private communication.

DISCUSSION

B. Bishop (Los Alamos National Laboratory): How many intervals did you take in the disposition of the field on the crack?

A.K. Kahn: Typically, I was using about 100. The typical one that I had up there had about 100, so we had 200 unknowns and so it would be 200 by 200.

B. Bishop: This was just on the surface of the crack?

A.K. Kahn: No. This is on the whole surface, whole interface.

B. Bishop: Which is infinite?

A.K. Kahn: Well, I terminated it. I would do a course run, find out where it dropped to zero. Then I would terminate the runout in the zero region, allowing a little bit for safety.