INTRODUCTION

The combination of a ductile metal and a high strength reinforcement leads to a metal-matrix composite which has desirable mechanical properties of high strength and toughness. These properties are usually governed by the properties of each phase, in addition to the shape, orientation and the volume fraction of the reinforcing phase. Since many of these composite properties are characteristic of the bulk, ultrasonics have been shown to provide promising NDE-methods for the characterization of metal-matrix composites ([1]). In general the desired isotropic mechanical behaviour of these composites is difficult to achieve. During manufacturing textures are developed by deformation processes and heat treatments, generating considerable anisotropy in the material’s properties. This affects the elastic as well as the plastic behaviour. Additionally the presence of textures influences the ultrasound velocity, which is also affected by stress. Therefore stress-analysis in NDE is restricted to untextured components or those with known texture influence. In this work measurements of ultrasonic velocities are used to determine the texture of metal-matrix composites and the influence of the reinforcing phase on the texture.

ORIENTATION DISTRIBUTION FUNCTION

Texture is the orientation distribution of the single crystals in the polycrystalline aggregate. A textured polycrystal is elastically anisotropic because the elastic properties of a single crystal are directionally dependent. Because of the texture the single crystal anisotropies do not vanish when averaged, thus the polycrystal looses its quasi-isotropy. Texture is mathematically described by the orientation distribution function (ODF). This function determines the possibility of finding a single crystal in the polycrystalline sample with a certain orientation with respect to the sample orientations, given by the axes of the sample fixed coordinate system. After Bunge [2] the ODF can be written as a series expansion into symmetrical generalized spherical harmonics as

\[
f(g) = \sum_{l=0}^{\infty} \sum_{\mu=1}^{M(l)} \sum_{\nu=1}^{N(l)} C_{\nu}^{\mu \nu} \hat{T}_{\nu}^{\mu \nu}(g)
\]  (1)

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where \( g \) is the orientation, the \( T_1^{HV} \) are symmetrical generalized spherical harmonics and the \( C_1^{HV} \) are the expansion coefficients. The upper limits \( M(l) \) and \( N(l) \) depend on \( l \) and on the crystal and the sample symmetry, respectively. The orthonormal function system \( T_1^{HV} \) is invariant towards all rotations of the sample symmetry (indicated by the right dot) and the crystal symmetry (indicated by the left two dots). In first approximation only the three forth-order expansion coefficients \( C_{41}^{11}, C_{41}^{12} \) and \( C_{41}^{13} \) need to be considered for texture evaluation in cubic materials with orthorhombic sample symmetry [3,4]. These three coefficients can be used to characterize both the elastic and the plastic behaviour of those materials. The forth-order coefficients in the Roe-notation [5] are called \( W_{400}, W_{420} \) and \( W_{440} \) and are related to the Bunge-coefficients by the following expressions:

\[
W_{400} = \frac{1}{24\pi^2} \sqrt{\frac{7}{6}} C_4^{11}
\]
\[
W_{420} = \frac{1}{24\pi^2} \sqrt{\frac{7}{12}} C_4^{12}
\]
\[
W_{440} = \frac{1}{24\pi^2} \sqrt{\frac{7}{12}} C_4^{13}
\]

ULTRASOUND VELOCITY RELATIONSHIPS

Inserting the elastic constants of the textured polycrystal, given by Bunge [4] for cubic crystal structure, into Christoffel’s equation for the orthorhombic sample, the following relations between the velocities of ultrasonic waves \( V_{ij} \) and the expansion coefficients \( C_4^{1V} \) result as

\[
\rho V_{11}^2 = c_{11} - c \left( \frac{2}{5} - \frac{1}{70} \sqrt{\frac{7}{3}} \left( c_{44}^{11} - \frac{2}{3} \sqrt{\frac{7}{3}} \left( \frac{4}{3} c_{44}^{12} + \frac{7}{3} c_{44}^{13} \right) \right) \right)
\]
\[
\rho V_{22}^2 = c_{11} - c \left( \frac{2}{5} - \frac{1}{70} \sqrt{\frac{7}{3}} \left( c_{44}^{11} + \frac{2}{3} \sqrt{\frac{7}{3}} \left( \frac{4}{3} c_{44}^{12} + \frac{7}{3} c_{44}^{13} \right) \right) \right)
\]
\[
\rho V_{33}^2 = c_{11} - c \left( \frac{2}{5} - \frac{1}{70} \sqrt{\frac{7}{3}} \left( \frac{8}{3} c_{44}^{13} \right) \right)
\]
\[
\rho V_{12}^2 = c_{44} + c \left( \frac{1}{5} + \frac{1}{70} \sqrt{\frac{7}{3}} \left( \frac{1}{3} c_{44}^{11} - \frac{4}{3} \sqrt{\frac{7}{3}} c_{44}^{13} \right) \right) = \rho V_{21}^2
\]
\[
\rho V_{23}^2 = c_{44} + c \left( \frac{1}{5} - \frac{1}{70} \sqrt{\frac{7}{3}} \left( \frac{4}{3} c_{44}^{11} + \frac{2}{3} \sqrt{\frac{7}{3}} c_{44}^{12} \right) \right) = \rho V_{32}^2
\]
\[
\rho V_{31}^2 = c_{44} + c \left( \frac{1}{5} - \frac{1}{70} \sqrt{\frac{7}{3}} \left( \frac{4}{3} c_{44}^{11} - \frac{2}{3} \sqrt{\frac{7}{3}} c_{44}^{12} \right) \right) = \rho V_{13}^2
\]

where \( \rho \) is the density and \( c_{11}, c_{12} \) and \( c_{44} \) are the elastic constants of the cubic single crystal and \( c = c_{11} = c_{12} = 2c_{44} \). The velocity \( V \) is characterized
by two subscripts. The first indicates the propagation direction, while the second indicates the polarization direction of the wave. 1, 2 and 3 designate the axes in a right-handed coordinate system (Figure 1). In order to use equations (3), which are valid only for materials with cubic crystal structure, in the composites studied in this paper, the following assumptions are made. The presence of the SiC-particles will change the elastic properties of the composite as well as the texture of the matrix. The SiC-particles are randomly distributed in the specimen without any preferred orientation. This assumption is valid because the ultrasonic wavelength (frequency used is 5 MHz) is much larger than the average particle dimension (2-4 µm). Also the averaging of corresponding velocities (rotational symmetry around extrusion axis) supports the assumption. The ultrasonic velocities in the textured Al-matrix (which has a face-centered cubic crystal structure) are then computed according to the so-called equal stress condition using the velocities measured. This condition states that the Al-crystallites and the SiC-particles underlie the same stress and relates the elastic moduli of the matrix, the reinforcement and the composite. With x designating the volume percentage of SiC, the equal stress condition can be written as

\[ M_{\text{comp}} = \frac{M_{\text{Al}} M_{\text{SiC}}}{x M_{\text{Al}} + (1-x) M_{\text{SiC}}} \]  

(4)

where \( M_{\text{comp}} \), \( M_{\text{Al}} \) and \( M_{\text{SiC}} \) are the elastic moduli of the composite, the matrix and the reinforcement, respectively. Using equation (4) and the well-known relationships between the elastic moduli and ultrasonic velocity, the velocities \( v_{ij} \) in the Al-matrix can be determined.

ELASTIC ANISOTROPY

The forth-order expansion coefficients can - as it was said before - be related to the elastic anisotropy using the relationship between texture and Young's modulus \( E \), given by Bunge [4]. This relation can be written as

\[ E(\theta) = E_r + \frac{1}{4} C_{44} E_{12}^2 \cos(2\theta) + \frac{1}{4} C_{44} E_{33}^2 \cos(4\theta) \]  

(5)

where \( \theta \) is the angle to the extrusion-direction, which lies parallel to the l-direction. In equation (5), \( E_r \) is the (Voigt-) average of the Young's modulus and \( E_1 \), \( E_2 \) and \( E_3 \) are constants. From eq.(5) the following linear relationships result:

\[ E_m = \frac{1}{4} (E(0^\circ) + 2E(45^\circ) + E(90^\circ)) = \frac{1}{4} E_{11} C_{44} + E_r \]

\[ E_A = \frac{1}{2} (E(0^\circ) - E(90^\circ)) = E_{12} C_{44} \]

\[ \Delta E = \frac{1}{2} (E(0^\circ) + 2E(45^\circ) + E(90^\circ)) = 2E_{13} C_{44} \]

(6)

\( E_A \) represents the difference of Young's modulus in extrusion direction and Young's modulus perpendicular to it.
EXPERIMENTAL

The composites examined are silicon carbide (SiC)-reinforced aluminium-matrix composites with 8091 and 7064 aluminium alloys as the metal matrix. The compositions of these two alloys are shown in Table 1. The specimens contain different volume percentages of SiC, varying from 0 % up to 20 %. This is also shown in Table 1.

RESULTS AND DISCUSSION

Since the samples used in this investigation are manufactured by extrusion, the coordinate system (Fig.1) is arranged such that the 3-direction is parallel to the extrusion direction. The ultrasonic velocities \( V_{ij} \) have been measured using the pulse-echo-overlap method. A detailed description of this method and the experimental set-up used in this investigation is given elsewhere [6]. The measured velocities are shown in Table 2 and have been found to be reproducible to within 0.3 %. Using equations (3) and (4), the expansion coefficients \( C_{41V} \) have been determined by averaging the corresponding velocities. These results are shown in Table 3. The expansion coefficients \( C_{411} \) and \( C_{413} \) are plotted versus the volume percentage of SiC in Fig. 2.

Table 1. Composition of the Al-alloys 8091 and 7064 and list of the examined metal-matrix composites.

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Si</th>
<th>Fe</th>
<th>Cu</th>
<th>Mg</th>
<th>Zr</th>
<th>Li</th>
<th>Zn</th>
<th>Cr</th>
<th>Co</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>8091</td>
<td>0.02</td>
<td>0.01</td>
<td>1.90</td>
<td>0.80</td>
<td>0.11</td>
<td>2.70</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>rem</td>
</tr>
<tr>
<td>7064</td>
<td>0.05</td>
<td>0.10</td>
<td>2.00</td>
<td>2.30</td>
<td>0.20</td>
<td>-</td>
<td>7.10</td>
<td>0.12</td>
<td>0.22</td>
<td>rem</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Specimens</th>
<th>8091 + 0 % SiC</th>
<th>8091 + 10 % SiC</th>
<th>8091 + 15 % SiC</th>
<th>7064 + 0 % SiC</th>
<th>7064 + 15 % SiC</th>
<th>7064 + 20 % SiC</th>
</tr>
</thead>
</table>
Table 2. Ultrasonic Velocities measured in the metal-matrix composites.

<table>
<thead>
<tr>
<th>$V_{ij}$</th>
<th>8091+0 %</th>
<th>8091+10 %</th>
<th>8091+15 %</th>
<th>7064+0 %</th>
<th>7064+15 %</th>
<th>7064+20 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{11}$</td>
<td>6612</td>
<td>6922</td>
<td>7050</td>
<td>6238</td>
<td>6731</td>
<td>6911</td>
</tr>
<tr>
<td>$V_{22}$</td>
<td>6606</td>
<td>6913</td>
<td>7011</td>
<td>6233</td>
<td>6734</td>
<td>6937</td>
</tr>
<tr>
<td>$V_{33}$</td>
<td>6564</td>
<td>7043</td>
<td>7168</td>
<td>6194</td>
<td>6920</td>
<td>7108</td>
</tr>
<tr>
<td>$V_{12}$</td>
<td>3484</td>
<td>3708</td>
<td>3816</td>
<td>3059</td>
<td>3435</td>
<td>3585</td>
</tr>
<tr>
<td>$V_{21}$</td>
<td>3477</td>
<td>3702</td>
<td>3805</td>
<td>3066</td>
<td>3440</td>
<td>3567</td>
</tr>
<tr>
<td>$V_{23}$</td>
<td>3467</td>
<td>3728</td>
<td>3834</td>
<td>3082</td>
<td>3479</td>
<td>3641</td>
</tr>
<tr>
<td>$V_{32}$</td>
<td>3480</td>
<td>3736</td>
<td>3842</td>
<td>3057</td>
<td>3469</td>
<td>3636</td>
</tr>
<tr>
<td>$V_{13}$</td>
<td>3472</td>
<td>3733</td>
<td>3848</td>
<td>3050</td>
<td>3470</td>
<td>3617</td>
</tr>
<tr>
<td>$V_{31}$</td>
<td>3482</td>
<td>3736</td>
<td>3844</td>
<td>3062</td>
<td>3469</td>
<td>3640</td>
</tr>
</tbody>
</table>

From this data one can see that there is first a slight decrease in the coefficients, followed by a linear increase up to 20 volume percent of SiC. These plots also show that the presence of SiC changes these coefficients indicating changes in the texture of the Al-matrix. From Table 3 it can be seen that the coefficient $C_{412}$ is zero in all specimens. This is due to the rotational symmetry around the extrusion axis. The determination of $C_{411}$ is known to be critical because it requires absolute velocity measurements. Therefore it has to be checked whether or not the determined values for $C_{411}$ are reasonable. This can be done by considering the two extreme cases of a $<111>$-fiber texture and a $<100>$-fiber texture, which are usually observed in extruded aluminium [7]. In the case of an ideal $<111>$-fiber texture, where the crystallographic direction with the highest ultrasonic velocity lies along the extrusion direction the value of $C_{411} = -4.570$ results. For an ideal $<100>$-fiber texture, where the crystallographic direction with the lowest ultrasonic velocity lies along the extrusion direction.

Table 3. Expansion coefficients $C_{4\text{ij}}$ (3-direction parallel to extrusion direction).

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$C_{411}$</th>
<th>$C_{412}$</th>
<th>$C_{413}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8091 + 0 %</td>
<td>2.205</td>
<td>0</td>
<td>2.194</td>
</tr>
<tr>
<td>8091 + 10 %</td>
<td>1.573</td>
<td>0</td>
<td>-0.431</td>
</tr>
<tr>
<td>8091 + 15 %</td>
<td>3.532</td>
<td>0</td>
<td>1.034</td>
</tr>
<tr>
<td>7064 + 0 %</td>
<td>2.358</td>
<td>0</td>
<td>1.962</td>
</tr>
<tr>
<td>7064 + 15 %</td>
<td>3.155</td>
<td>0</td>
<td>1.267</td>
</tr>
<tr>
<td>7064 + 20 %</td>
<td>5.352</td>
<td>0</td>
<td>2.175</td>
</tr>
</tbody>
</table>
direction, the value of $c_{411} = +6.770$ turns out. In both cases $c_{412}$ and $c_{413}$ are zero. These two "extreme" values constitute the upper and the lower bounds of $c_{411}$ for any texture in extruded aluminium. The values obtained (Table 3) in this work lie within these bounds. Since the results obtained seem to be reasonable, it can now be checked, whether the relationships between texture and elastic anisotropy (eq.(5), (6)) can also be applied to two phase materials such as Al-SiC composites. For this purpose the expansion coefficients are determined with the 1-direction of the coordinate system parallel to the extrusion direction. Re-naming the measured velocities leads to the $c_{41V}$-values shown in Table 4. Now the coefficients $c_{412}$ are no longer zero as in the case when the extrusion direction is parallel to the 3-direction. A plot of these values versus volume percentage of SiC (Fig.3) shows that both $c_{411}$ and $c_{413}$ increase linearly with increasing SiC-contents, whereas $c_{412}$ linearly decreases. This indicates that the presence of SiC leads to considerable changes in the texture of the aluminium matrix. The Young's moduli in 0°, 45° and 90°-directions have been determined using the relationship

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}$$

(7)

Table 4. Expansion coefficients $c_{41V}$ (1-direction parallel to extrusion direction).

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$c_{411}$</th>
<th>$c_{412}$</th>
<th>$c_{413}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8091 + 0 %</td>
<td>0.870</td>
<td>0.217</td>
<td>0.571</td>
</tr>
<tr>
<td>8091 + 10 %</td>
<td>4.766</td>
<td>-1.195</td>
<td>4.931</td>
</tr>
<tr>
<td>8091 + 15 %</td>
<td>7.080</td>
<td>-1.329</td>
<td>6.988</td>
</tr>
<tr>
<td>7064 + 0 %</td>
<td>1.433</td>
<td>-0.021</td>
<td>1.227</td>
</tr>
<tr>
<td>7064 + 15 %</td>
<td>6.458</td>
<td>-0.980</td>
<td>6.198</td>
</tr>
<tr>
<td>7064 + 20 %</td>
<td>8.115</td>
<td>-1.621</td>
<td>8.083</td>
</tr>
</tbody>
</table>
Fig. 3. Expansion coefficients a) $C_{411}$, b) $C_{412}$, c) $C_{413}$ plotted vs. volume percentage of SiC.

Fig. 4. Elastic anisotropy parameters plotted vs. expansion coefficients: a) $E_m$ vs. $C_{411}$, b) $E_A$ vs. $C_{412}$, c) $\Delta E$ vs. $C_{413}$.
Table 5. Young's modulus \( E \) determined in 0°, 45° and 90°-directions and elastic anisotropy parameters \( E_m, E_A \) and \( \Delta \varepsilon \).

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( E(0°) )</th>
<th>( E(45°) )</th>
<th>( E(90°) )</th>
<th>( E_m )</th>
<th>( E_A )</th>
<th>( \Delta \varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8091 + 0 %</td>
<td>79.8</td>
<td>79.9</td>
<td>80.0</td>
<td>79.9</td>
<td>0.10</td>
<td>0</td>
</tr>
<tr>
<td>8091 + 10 %</td>
<td>91.1</td>
<td>90.0</td>
<td>90.3</td>
<td>90.0</td>
<td>-0.55</td>
<td>0.55</td>
</tr>
<tr>
<td>8091 + 15 %</td>
<td>95.9</td>
<td>94.3</td>
<td>94.7</td>
<td>94.8</td>
<td>-0.60</td>
<td>1.00</td>
</tr>
<tr>
<td>7064 + 0 %</td>
<td>72.1</td>
<td>71.9</td>
<td>72.0</td>
<td>72.0</td>
<td>-0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>7064 + 15 %</td>
<td>91.9</td>
<td>90.0</td>
<td>89.6</td>
<td>90.2</td>
<td>-0.75</td>
<td>0.35</td>
</tr>
<tr>
<td>7064 + 20 %</td>
<td>99.1</td>
<td>97.2</td>
<td>96.5</td>
<td>97.5</td>
<td>-1.30</td>
<td>0.60</td>
</tr>
</tbody>
</table>

where the Lamé constants \( \lambda \) and \( \mu \) are given by

\[
\mu = \rho V_s^2 \quad \text{and} \quad \lambda + 2\mu = \rho V_l^2. \tag{8}
\]

In this relationship \( V_s \) and \( V_l \) designate the shear and the longitudinal wave velocity, respectively. Using the velocity data shown in Table 3, \( E(0°) \) and \( E(90°) \) were computed. In addition the measured shear velocities in the 45° direction are used to determine \( E(45°) \). The values for \( E(0°) \), \( E(45°) \) and \( E(90°) \) as well as the resulting values for \( E_m, E_A \) and \( \Delta \varepsilon \) are shown in Table 5. The quantities \( E_m, E_A \) and \( \Delta \varepsilon \) are plotted versus the coefficients \( C_{411}, C_{412} \) and \( C_{413} \) in Figures 4a) to c), respectively, and confirm the expected linear correlations between these parameters. It is important to note that the \( C_{41V} \) are the texture coefficients determined for the Al-matrix, whereas \( E_m, E_A \) and \( \Delta \varepsilon \) characterize the elastic anisotropy of the whole composite. From this it can be concluded that the elastic anisotropy of the composites is influenced only by the texture of the Al-matrix. Also from the linear correlation of \( E_A \) with \( C_{412} \), shown in Fig. 4b), it is expected that \( E_A \) reaches zero, when \( C_{412} \) goes to zero. Thus texture-control by means of \( C_{412} \) during the manufacturing process can lead to a considerable reduction in the elastic anisotropy of the material produced.

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EFFECTS OF INTERACTION BETWEEN STRESS AND TEMPERATURE ON ULTRASOUND VELOCITY

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INTRODUCTION

The work described here is part of a program to explore the possibility of employing a new concept for ultrasonic nondestructive evaluation of subsurface residual stress in metals. Theory indicates its potential applicability to characterization of stress to depths of 1-2 cm or more. The method employs a measurement of time variations in velocity in response to the inducement of time-varying thermal gradients within the stress region. The physical principle, as discussed below, is an interaction between stress, temperature and velocity.

Experimental results have been obtained recently for applied compressive stress in aluminum, with heat applied using an oxyacetylene torch. Ultrasonic velocity variation with time in a bar about 5 cm long is detected by demodulation of VCO frequency in a pulse phase-lock loop system. A PC-based data acquisition system is employed for frequency demodulation and data storage.

THEORY

The theory of the concept has been presented previously [1,2], but a brief outline will be given here for convenience. The physical basis is a phenomenon which can be expressed as

\[ \frac{dv}{du} = -k_1 + k_2 \sigma \]  

where \( v \) is velocity, \( u \) is temperature, \( \sigma \) is stress and \( k_1 \) and \( k_2 \) are positive constants. This relationship was explored extensively by K. Salama over a range of temperatures with uniform applied and residual stress in specimens of aluminum, steel and copper [3,4,5]. The relationship holds for both compressional and transverse waves, although with different constants.

Theoretical study by Anderson [1] showed that for a one-dimensional system, if stress is considered to be a function of position (i.e., \( \sigma = \sigma(x) \)), it should be possible to infer \( \sigma(x) \) from time variations in velocity following the application of controlled transient thermal excitation to the metal. The reasoning goes as follows. The spatially averaged
acoustic velocity over a length $L$ for a one-dimensional geometry is given by

$$\langle v(t) \rangle = \frac{1}{L} \int_0^L v(x,t) \, dx,$$  \hspace{1cm} (2)

where $v(x,t)$ is the internal velocity, assumed to depend on both distance and time through a dependence on temperature and stress. Then if $u(x,t)$ is the relative temperature in the material, we can separate the effect of stress by taking the time derivative:

$$\frac{d\langle v(t) \rangle}{dt} = \frac{1}{L} \int_0^L \frac{\partial v}{\partial u} \frac{\partial u}{\partial t} \, dx.$$  \hspace{1cm} (3)

Substituting the partial derivative equivalent of Eq. (1) into (3) gives

$$\frac{d\langle v(t) \rangle}{dt} = \frac{1}{L} \int_0^L \left( \frac{-k_1 + k_2 \sigma(x)}{\partial u/\partial t} \right) \, dx.$$  \hspace{1cm} (4)

We shall assume that the "heat input" is specified by $\partial u/\partial x = -\delta(t)$ on the boundary $x = 0$, and that $\partial u/\partial x = 0$ at $x = L$. (See Fig.1) Then the homogeneous diffusion equation applies for $t > 0$, and we can set $\partial u/\partial t = c\partial^2 u/\partial x^2$ under that condition, where $c$ is the diffusivity constant. We obtain

$$\frac{d\langle v(t) \rangle}{dt} = \frac{ck_1}{L} \delta(t) + \frac{k_2}{L} \int_0^L \sigma(x) \frac{\partial^2 u}{\partial x^2} \, dx.$$  \hspace{1cm} (5)

Under the boundary conditions specified above, the temperature is given by

$$u(x,t) = \frac{1}{\sqrt{\pi ct}} [e^{-x^2/4ct} + \sum_{n=1}^{\infty} e^{-(2nL+x)^2/4ct} + e^{-(2nL-x)^2/4ct}].$$  \hspace{1cm} (6)

The first term is a good approximation for our purposes. Equation (5) can be easily solved for the rectangular stress profile $\sigma(x) = \Delta$ (a constant) between $x_1$ and $x_2$, and zero otherwise. The result is

$$\frac{d\langle v(t) \rangle}{dt} = \frac{ck_1}{L} \delta(t) + \frac{k_2 \Delta}{L \sqrt{4\pi ct}} \left[ x_1 e^{-x_1^2/4ct} - x_2 e^{-x_2^2/4ct} \right].$$  \hspace{1cm} (7)

Fig. 1. One-dimensional setup for study of ultrasonic velocity deviations due to stress and impulse heat input.
Instrumentation used in current experiments gives \( \langle v(t) \rangle \) itself, so we integrate Eq. (7), obtaining

\[
\langle v(t) \rangle = v_0 - \frac{c k_1 U(t)}{L} + \frac{k_2 A}{L \sqrt{4 \pi c}} \int_0^t \left( \frac{1}{t^{3/2}} \left( x_1 e^{-x_1^2/4ct} - x_2 e^{-x_2^2/4ct} \right) \right) dt,
\]

(8)

where \( U(t) \) is the unit step function, and \( v_0 \) is the velocity at ambient temperature. It can be noted that if the stress is zero throughout the region, the third term will be zero, and the velocity after \( t = 0 \) will be constant at \( (v_0 - c k_1 / L) \). The general appearance of Eq. (8) for non-zero stress is shown in Fig. 2 for \( c = 0.95 \) (aluminum) and \( k_2 A / k_1 = 0.6 \). Two different choices of \( (x_1, x_2) \) are shown to illustrate the effect of changing the location of the stress region.

When the heat flux is not well-represented by a \( \delta \)-function, we can characterize the experimental result as the convolution of \( \langle v(t) \rangle \) with a "pulse" function \( p(t) \), representing the actual heat flux, i.e.,

\[
w(t) = \langle v(t) \rangle * p(t) = \int \langle v(t) \rangle p(t-\lambda) d\lambda,
\]

(9)

with the integral taken over all \( \lambda \). The result of the convolution is of course a "smoother" version of \( \langle v(t) \rangle \). The initial drop in velocity due to the sudden influx of heat is no longer instantaneous, but lasts as long as the duration of \( p(t) \). Subsequent variation of \( \langle v(t) \rangle \) with time due to interaction of heat with the stress is similarly broadened.

Fig. 2. Theoretical curves representing Eq. (8) for two rectangular stress profiles, one slightly (0.25 cm) displaced from the other.
EXPERIMENTS

Electronics

Measurement of the time-varying velocity is accomplished using the system of Fig. 3. A MicroUltrasonics pulsed phase lock loop (PPLL) [6] generates and receives ultrasonic pulses of preselected length, generally amounting to around 5-15 cycles of the operating frequency (in the range 2-5 MHz). The pulses are gated on at a rate of a few hundred per second. The frequency of the oscillator (VCO) is controlled in the usual way by a feedback voltage representing phase error between pulse echoes and the continuously running VCO. Due to certain characteristics of the PPLL circuitry, this feedback voltage does not follow frequency deviation very precisely, so a PC-based data acquisition system has been designed to perform the dual functions of frequency demodulation and data storage.

Figure 4 is a block diagram of this system [7]. An adjustable local oscillator produces a voltage which is mixed with that of the VCO to produce a nominal difference frequency, usually around 10 kHz. The resultant waveform is shaped for TTL compatibility, divided by two and used to gate the 14.3 MHz clock voltage of the PC into a high speed counter. Counter output in the form of a 16-bit word for each gating interval (this interval being equal to the period of the difference frequency) is

Fig. 3. Experimental system for measurement and storage of time-varying ultrasonic velocity data.

Fig. 4. The data acquisition system.
transmitted to the PC and stored on floppy disk. Thus the VCO frequency is effectively sampled at a rate of around 5000/sec. The frequency resolution is then \( f_d^2/f_c \), where \( f_d \) is the difference frequency and \( f_c \) the clock frequency (14.3 MHz in this case). Phase jitter in the low-frequency waveform results in substantial random noise, amounting to about 20-30 Hz; however the sampling rate allows a sufficient extent of adjacent sample averaging to give a high degree of smoothing while still allowing more than enough time resolution. For example, 50-sample averaging at the 5000/sec rate gives a resolution of .01 seconds.

**Signal Processing**

Owing to the fact of its being obtained as a result of a heat diffusion process, one should not expect that the signal would have a high information content in most circumstances of interest. The signal processing method must therefore operate on the measured data as efficiently as possible. Two approaches to this problem are being studied: one is based on a generalized matrix inversion with constraints; the other is a parameter fitting technique.

For the matrix inversion, the signal (which could be either \( \Phi(t) \) or some processed version thereof) is discretized with respect to time, resulting in a signal vector \( S \) with elements \( s(t_i), i = 1, 2, 3, \ldots, N \). The stress profile, \( \sigma(x) \), is also put into discrete form with respect to values of \( x \), giving a vector \( \Sigma \) with elements \( \sigma(x_j), j = 1, 2, 3, \ldots, M \). Similarly \( \partial \sigma/\partial t \) is discretized with respect to both \( x \) and \( t \) to give an \( N \times M \) matrix \( G \) with elements \( G_{ij} \). For the simplest assumption concerning the statistics of the noise, the estimate of \( \Sigma \) then proceeds according to the formula [8]

\[
\hat{\Sigma} = (G^T G)^{-1} G^T S.
\]  

Because of the limited information content of the signal vector in this case, this inversion tends to be quite unstable unless \( N \) is relatively small [9]. Much improvement can be gained, however, if knowledge of the nature of \( S \) can be incorporated in the form of one or more constraints. One such constraint applicable for compressive stress is "positivity" i.e., \( \sigma(x) \geq 0 \) (or "negativity" for tensile stress), and trial calculations using simulated data with added Gaussian noise show that this is an extremely effective way to stabilize the inversion. However, the computational procedure is nonlinear and iterative, and considerably more complex than that for the unconstrained solutions [10].

A second approach is based on representing the stress profiles in terms of standard "shapes". For the simplest shapes (e.g., Gaussian, Lorentzian, parabolic, rectangular) three parameters (say, \( A, w \) and \( c \)) are sufficient to characterize magnitude, width and center location. One then seeks to minimize the integral of the squared difference between the theoretical and measured signals:

\[
F(A, w, c) = \int_0^T [s(t, A, w, c) - s_m(t)]^2 dt,
\]

(or its equivalent in discrete form) with respect to variations in \( A, w, \) and \( c \). Preliminary results with simulated data and noise show that this approach may be quite effective, but as yet a detailed comparison between this and the matrix inversion approach has not been made.
Heat Input

It is evident that the greatest possible thermal gradients in the metal and therefore the maximum signal responses to stress will occur for a δ-function heat input. This can be achieved to a good degree of approximation in this case for a heat source consisting of a pulsed laser with pulse duration less than a millisecond or so; however, such a source is not available to us at the present time. It has therefore been necessary to employ a torch, with flame applied to the metal bar end for a brief interval. Both a "MAPP" torch and an oxyacetylene torch have been used. The results shown in the next section were obtained using the oxyacetylene torch, which is apparently about four or five times hotter than the other. At present the configuration of the specimen and means of stress application make it awkward to use mechanical shuttering for timing of heat exposure, so for the time being the torch is hand-held and passed by the end of the specimen in a smooth motion. Despite the uncertainties that might be expected for this procedure, the records show a fair amount of uniformity, with time duration for the initial velocity change (and, therefore, the exposure time itself) generally close to 1/4 second.

RESULTS

Stress was applied over a region from 1 cm to 2.25 cm from the heat input end of an aluminum bar, using the configuration of Fig. 5. Stress values were 0, 3.2, 4.8, 5.6 and 8.0 ksi, as indicated for the representative results shown in Fig. 6. A fairly reasonable degree of proportionality to stress is seen, despite a certain amount of noise in the measurements. In addition, heat application often spilled over to the sides and stress contact bars, resulting in an "input" heat flux after the initial drop of velocity. In the case of zero stress, firm contact was maintained between the specimen and the contact bars, so that heat loss through the latter would not be appreciably different than for the other stress values. Since heat input amount varied as much as 20% or so from one measurement to the next, the curves have been all been normalized with respect to initial velocity decrease. Also, the reference point for alignment in time was chosen to be the point of maximum slope during this interval of velocity change. The widths of the velocity de-

Fig. 5. Application of stress. Because of appreciable heat flux out of the bar, one-dimensional theory is not strictly applicable.
crease regions are fairly uniform at around 1/4 second, as noted previously. The amount of change in this region indicates an energy input in the neighborhood of 50 Joule/cm² during heat exposure. Because the horizontal scale is linear with respect to sampling points and since the sampling rate varies with the difference frequency, it is nonlinear with time. As a result, the time scale of the different curves is not quite the same, and so the superposition of the curves is not strictly justified. The approximate location of the 1-second point (after minimum) has been sketched in for reference.

The theoretical model for these experimental results is obviously Eq. (9), although of course the precise shape of $p(t)$ is unknown at present. A discrepancy with theory exists, however, in that the magnitude of velocity deviation (e.g., 500 cm/sec at 1 sec for 8 kpi) following the initial drop is significantly greater for experiment than predicted theoretically. At present, it is believed that this is due to the thermal gradient in the stress region being larger than predicted by the one-dimensional theory, as a result of conduction of heat through the stress contact regions. Tests are underway using less thermally conducting contacts to see if the discrepancy is due to this factor.

CONCLUSIONS

Preliminary experimental results now clearly indicate the viability of a new concept for nondestructive characterization of subsurface residual stress in metals. The method requires measurement of temporal variations in ultrasonic velocity following transient application of heat to the surface of the metal.

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REFERENCES