APPLICATION OF SCANNING ACOUSTIC MICROSCOPY TO RESIDUAL STRESS ANALYSIS: THEORY VS. EXPERIMENT

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INTRODUCTION

In this project, a new technique for the nondestructive evaluation of residual stress in manufactured components is proposed. Where most approaches to residual stress analysis have been based on monitoring the stress induced changes in sound velocity, these techniques are limited in their utility due to use of contact, shear transducers (effectively precluding scanning large areas), the inability to resolve the dependence of residual stresses on depth (important for surface treatments) and the difficulty in making accurate time delay measurements due to the very small acoustoelastic effect observed for most practical materials. Here, we propose to develop a nondestructive test technique suitable for scanning plate structures. An aspherically focused, immersion transducer is used in a scan mode to generate an axially symmetric pulse. We utilize interference phenomena between two shear waves polarized in the directions of the in-plane principal stress axes to increase resolution of the small differences in transit time between the two waves. This technique may become a powerful tool to study actual residual stress distributions in practical engineering materials.

BACKGROUND

The earliest treatment of the residual stress problem is attributed to Hughes and Kelly [1], who developed an analytical solution to the problem of wave propagation in a stressed isotropic solid, based on the Murnaghan theory of finite deformations. They included third-order elastic constants $\ell$, $m$, and $n$ (usually referred to as Murnaghan's constants) as well as the second-order Lamé constants $\lambda$ and $\mu$ for isotropic materials, in their development.

Whereas the previous analytical works were confined to the case of homogeneous strain states, Tokuoka and Iwashimizu [5] investigated the case of acoustical birefringence of ultrasonic waves in a nonhomogeneously-deformed isotropic elastic material. Subsequently, Tokuoka and Saito [6] extended the treatment to acoustical birefringence of transverse waves in stressed crystals with an arbitrary symmetry. Here, the deformation was assumed to be homogeneous.

One important problem associated with experimental residual-stress measurements which has received a great deal of attention is the difficulty of determining the direction and sense of the stress, as these are not generally known. Shear-wave birefringence has been successfully employed to determine the direction of principal stresses by a number of investigators including Benson and Raelson [7], Bergman and Shahbender [8], Hsu [9], Noronha and Wert [10], and Blinka and Sachse [11].

Most of these experimental approaches to date have used normal incidence, contact transducers. Hence, they were limited to single site measurements. However, as the stress distribution in most practical situations may be highly nonhomogeneous, scan techniques are highly desirable.

Kino et al. [12] used a water-bath coupling technique to scan samples of aluminum and pressure-vessel steel in determining the stress profiles. They employed longitudinal waves at normal incidence to determine the third-order acoustoelastic constants. Clearly, this approach is limited as to obtaining shear information. However, useful results can be obtained for some simple situations. Rolled plate samples with either a central hole or a double-edged notch, were tested under uniaxial tension along the rolling direction. They reported that the results were in good agreement with those measured destructively by strain gauge methods. A similar approach was used by Hsu, Proctor and Blessing [13] to overcome the problem of transducer-specimen couplant variations in acoustoelastic measurements. The agreement between the ultrasonic velocity measurements of stress distribution with that predicted by elasticity theory encouraged the authors to extend their work to shear waves. A contact shear transducer, as well as a noncontacting electromagnetic transducer (EMAT) was used to scan the same samples of an aluminum ring-plug assembly. The shrink-fit residual stress with a known distribution was calculated using elasticity theory. A comparison of the acoustoelastic stress measurements with the known theoretical stress distribution showed good agreement. The non-contacting EMAT yielded dramatically improved results over the contacting transducer. It was noticed, however, that the EMAT has an intrinsically low signal-to-noise ratio, as well as an initial receiver saturation at resonance. To overcome this intrinsic sensitivity problem, Egle and Koshti [14] developed an immersion scan technique, with mode conversion used to generate the required shear waves.

THEORY RESIDUAL STRESS ANALYSIS

Sinae [15] has developed an expression for the Christoffel tensor in the presence of residual strains (or stresses) given by:

\[
\begin{align*}
\left(\lambda_{11} - \rho_o \nu^2\right)m_1 + \lambda_{12}m_2 + \lambda_{13}m_3 &= 0 \\
\lambda_{21}m_1 + \left(\lambda_{22} - \rho_o \nu^2\right)m_2 + \lambda_{23}m_3 &= 0 \\
\lambda_{31}m_1 + \lambda_{32}m_2 + \left(\lambda_{33} - \rho_o \nu^2\right)m_3 &= 0
\end{align*}
\]
where:

\[
\begin{align*}
\lambda_{11} &= \ell_1^2 \left[ \lambda + 2\mu + (4\lambda + 10\mu + 4m)\alpha_1 + (\lambda + 2\ell)\theta \right] \\
&+ \ell_1^2 \left[ \mu + 2\mu\alpha_2 - \left( 2\mu + \frac{1}{2}n \right)\alpha_3 + (\lambda + 2\mu + m)\theta \right] \\
&+ \ell_1^2 \left[ \mu - \left( 2\mu + \frac{1}{2}n \right)\alpha_2 + 2\mu\alpha_3 + (\lambda + 2\mu + m)\theta \right] \\
\lambda_{12} &= \lambda_{21} = \ell_1\ell_2 \left[ \lambda + \mu + 2(\lambda + \mu)(\alpha_1 + \alpha_2) + \left( \frac{1}{2}n - 2m \right)\alpha_3 + (2\ell + m)\theta \right] \\
\lambda_{13} &= \lambda_{31} = \ell_1\ell_3 \left[ \lambda + \mu + 2(\lambda + \mu)(\alpha_1 + \alpha_3) + \left( \frac{1}{2}n - 2m \right)\alpha_2 + (2\ell + m)\theta \right] \\
\lambda_{22} &= \ell_2^2 \left[ \mu + 2\mu\alpha_1 - \left( 2 = \mu + \frac{1}{2}n \right)\alpha_3 + (\lambda + 2\mu + m)\theta \right] \\
&+ \ell_2^2 \left[ \lambda + 2\mu + (4\lambda + 10\mu + 4m)\alpha_2 + (\lambda + 2\ell)\theta \right] \\
&+ \ell_2^2 \left[ \mu - \left( 2\mu + \frac{1}{2}n \right)\alpha_1 + 2\mu\alpha_3 + (\lambda + 2\mu + m)\theta \right] \\
\lambda_{23} &= \lambda_{32} = \ell_2\ell_3 \left[ \lambda + \mu + \left( \frac{1}{2}n - 2m \right)\alpha_1 + 2(\lambda + \mu)(\alpha_2 + \alpha_3) + (2\ell + m)\theta \right] \\
\lambda_{33} &= \ell_3^2 \left[ \mu + 2\mu\alpha_1 - \left( 2\mu + \frac{1}{2}n \right)\alpha_2 + (\lambda + 2\mu + m)\theta \right] \\
&+ \ell_3^2 \left[ \mu - \left( 2\mu + \frac{1}{2}n \right)\alpha_1 + 2\mu\alpha_2 + (\lambda + 2\mu + m)\theta \right] \\
&+ \ell_3^2 \left[ \lambda + 2\mu + (4\lambda + 10\mu + 4m)\alpha_3 + (\lambda + 2\ell)\theta \right]
\end{align*}
\]

which yields an eigenvalue problem for the phase velocities. Historically, most practical applications of acoustoelasticity to residual stress analysis have employed contact transducers (both longitudinal and shear) for the appropriate velocity measurements and the resulting eigenvalue problem is very simple to solve. However, since shear wave propagation requires a high viscosity couplant, it is difficult to employ this approach in a scan mode. Here we explore the possibility of using oblique incidence and mode conversion to generate the required shear waves. Further, in order to increase the sensitivity of the measurement to the small changes in velocity introduced via the acoustoelastic effect, we use a spherical focus transducer, rather than a flat focus transducer, to simultaneously excite both shear polarizations. Then, due to constructive and destructive interference between the two waves in a thin plate, one measure of the extent of residual stresses will be the amplitude of the combined shear pulse. An independent measure of the residual stresses can be obtained by examining the interference between the normal incidence and oblique incidence longitudinal waves generated by the spherically focused transducer. Drescher-Krasicka [16] used the latter phenomenon to experimentally study residual stress distributions via a scanning acoustic microscopy arrangement. Here, we present a straightforward mathematical model to analyze those results quantitatively.

The approach is based on first order perturbation theory for wave propagation in anisotropic media as developed originally by Jech and Psencik [17], for geological media. Since, the acoustoelastic effect is relatively small, we treat the mathematical solutions for the stressed sample as a first order perturbation of the isotropic solution. If we begin with the Christoffel equation.
\[ (\lambda_{ik} - \rho V^2 \delta_{ik}) \alpha_k^{(m)} = 0 \] (3)

in the perturbed medium we may write

\[ (\lambda_{ik} + \Delta \lambda_{ik} - \rho (V^{(m)} + \Delta V^{(m)})^2 \delta_{ik}) (\alpha_k^{(m)} + \Delta \alpha_k^{(m)}) = 0 \] (4)

Continuing, we have

\[
\begin{align*}
(\lambda_{ik} - \rho V^{(m)} \delta_{ik}) \alpha_k^{(m)} &+ \left( \Delta \lambda_{ik} + \rho V^{(m)} \Delta V^{(m)} \delta_{ik} \right) \alpha_k^{(m)} \\
&+ \left( \Delta \lambda_{ik} + 2 \rho V^{(m)} \delta_{ik} \right) \alpha_k^{(m)} = 0
\end{align*}
\] (5)

However, the first term in this expansion must be identically zero due to equation 2. Also, we may neglect the final term as it is of higher order than the remaining two expressions. Thus, we have

\[
(\lambda_{ik} - \rho V^{(m)} \delta_{ik}) \alpha_k^{(m)} + (\Delta \lambda_{ik} + 2 \rho V^{(m)} \Delta V^{(m)} \delta_{ik}) \alpha_k^{(m)} = 0
\] (6)

If we multiply this expression by \( \alpha_i^{(m)} \), the first term vanishes due to the symmetry of the Christoffel tensor and equation [2], leaving

\[
\Delta \lambda_{ik} \alpha_k^{(m)} \alpha_i^{(m)} + 2 \rho V^{(m)} \Delta V^{(m)} = 0
\] (7)

since the eigenvectors are orthonormal.

Then,

\[
\Delta V^{(m)} = \frac{1}{2} V^{(m)-1} B_{mn}
\] (8)

where

\[
B_{mn} = \Delta \lambda_{ik} \alpha_i^{(m)} \alpha_k^{(n)}
\] (9)

The only difficulty in applying this formula arises from the degeneracy of the shear modes in the unstressed, isotropic medium as the polarization (eigen) vectors are arbitrary. We only know that they are perpendicular to the wave normal and to each other.

Taking this into account, one obtains the following result:

\[
v_{11} = v_s + \frac{1}{4v_s} \left\{ \bar{B}_{11} + \bar{B}_{22} + \sqrt{\bar{B}_{11} - \bar{B}_{22} - 4 \bar{B}_{12}^2} \right\}
\] (10)

\[
v_{22} = v_s + \frac{1}{4v_s} \left\{ \bar{B}_{11} + \bar{B}_{22} - \sqrt{\bar{B}_{11} - \bar{B}_{22} - 4 \bar{B}_{12}^2} \right\}
\]

where

\[
\bar{B}_{mn} = \Delta \lambda_{ik} e_i^m e_k^n
\] (11)

We will then have interference between two shear waves differing in phase.

**EXPERIMENTAL PROCEDURE**

Scanning acoustic microscopy provides a convenient means to obtain an image of the residual stress distribution within a material. This can be obtained from the interference of the two orthogonally polarized shear wave using a spherically focused transducer to launch all the waves necessary for complete characterization of the stress state in the material. The
geometry for the experiment is shown in Figure 1. We may model the transducer as producing two mainwave components - 1) A normal incidence pulse and 2) a strong axisymmetric longitudinal lobe at oblique incidence. We are principally concerned with the shear pulses indicated by S in Figure 1. In actuality, the pulse indicated by S is the superposition of two refracted shear pulses. These signals will be slightly delayed in phase from one another, and due to the presence of residual stresses in the media, their velocities will be different. The interference may be exploited for residual stress analysis by monitoring the amplitude (not the arrival times) of the superimposed signal. Due to the constructive-destructive interference between the two signals, the amplitude of the combined signal is very sensitive to the small changes in acoustic velocity produced by the local residual stresses. Here, the shear waves are generated by the mode-converted, obliquely incident longitudinal waves. It should be noted that two distinct refracted shear waves will be observed due to the stress induced anisotropy in the piece. The axisymmetric nature of the source insures that both shear modes will be simultaneously excited. The loading apparatus is illustrated in Figure 2, along with the electronics used for load monitoring.

Figure 1. Wave propagation diagram.

Figure 2. Experimental geometry.
RESULTS AND DISCUSSION

Results from the theoretical stress distribution for an elastic solid in diametral compression were used as the basis for modeling acoustic birefringence and its effect on shear wave propagation for a thin disc under diametral compression. Several materials were considered including two steel alloys and one aluminum alloy as well as a polymer (polyurethane). The predicted image for the aluminum sample at a load of 2000 N, presented in Figure 3a. Experimental images for the same loading condition are shown in Figure 3b. At low loads the images are qualitatively similar. One major difference in the images is the interference observed experimentally in the center of the sample, but not predicted theoretically. This is attributable to an initial sample texture not removed by annealing since it also appears in the zero load image obtained with the acoustic microscope Figure 4. At higher loads, however, some important qualitative differences emerge, principally due to the assumption of a point load in the elasticity solution. The load in practice is distributed over a wide area, not applied

Figure 3. Acoustic interference patterns for Al sample at 2000 N load a) theoretical and b) experimental.

Figure 4. Acoustic interference pattern for Al sample at zero load.
at a point. When distributed loads are introduced into the stress distribution model, the edge effects seen experimentally are reflected in the stress calculation. We feel that this is the principal source of error in our treatment of the problem and are in the process of reviewing our numerical code to more accurately reflect the precise load distribution for our samples.

One of the questions that we tried to address in this study was the utility of the perturbation approach to modeling the problem. To do this we examined the predicted results for a dimetral compression experiment

a. Using first order perturbation theory; not compensating for the refracted single differences between the two shear waves.

and

b. Using first order perturbation theory but taking the refracted angle differences into account.

Results were compared with a full field solution to the problem where no such simplifying assumptions were made. These are presented in Figure 5. Clearly, viewing the medium as being slightly perturbed from the isotropic state is in error in the vicinities of the two loading sites. However, in comparing the two images, much of this error is introduced when one properly account for the differences in the refraction angles for the fast and slow shear waves.

Figure 5. Error introduced using perturbation theory
a) without refraction correction and b) with refraction correction.
CONCLUSIONS

A technique for mapping residual stress distributions based on scanning acoustic microscopy has been demonstrated.

Theoretical models were developed to predict the response of the stressed materials. First order perturbation theory was found to produce satisfactory results provided that refraction effects were properly taken into account.

Good agreement between theoretical and experimental results were observed for the case of diametric compression. One of the major sources of error in this analysis was found to be the assumption of point loading. Finite element results indicate that a distributed area load might be a more accurate model for the applied load.

REFERENCES