SURFACE ACOUSTIC WAVE PROBING OF CERAMIC BEARING BALLS

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INTRODUCTION

This work is a continuation of our effort to develop a nondestructive technique for the detection and characterization of surface and near surface defects in ceramic bearing balls. We reported earlier on a method for detecting and sizing submicron surface depressions using a scanning acoustic microscope[1]. Our present work deals with the detection and sizing of surface cracks in the ceramic bearing balls, a problem which requires knowledge of the surface wave reflection coefficient of the crack, either at a single frequency in the long wavelength regime or as a function of frequency in the short wavelength regime. For this purpose, we need to learn the characteristics of surface wave propagation on spherical surfaces, the scattering of the surface waves from the cracks, and we need to develop a method for exciting the surface wave. We present a detailed theory of surface wave propagation on spheres. The results indicate that an arc source focuses the surface acoustic wave in a manner similar to bulk acoustic waves focusing by spherical transducers. We will present the details of this self focusing behavior. A spherical cap transducer structure similar to a planar wedge transducer is proposed to excite the spherical surface waves. We will present the details of the design of the spherical cap transducer for efficient surface wave excitation.

FREE SPHERICAL SURFACE GREEN'S FUNCTION

In order to work out the propagation theory of waves on spherical surfaces, we have to determine the free spherical surface Green's function. For our work, we choose spherical coordinate system \((r, \theta, \phi)\) where, on the surface, \(r\) is constant and a point on the surface is noted as \(P(\theta, \phi)\).

From the wave equation:

\[ \nabla^2 \psi + k^2 \psi = 0 \]  \hspace{1cm} [1]

We write in the spherical coordinate system with \(r = \text{constant}:\)

\[ \nabla^2 \psi = \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial \psi}{\partial \theta}) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \phi^2} \]  \hspace{1cm} [2]

For a point source at the north pole:
\[ \frac{\partial^2 \psi}{\partial \phi^2} = 0 \]  

Eq. 1 becomes:
\[ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \psi}{\partial \theta} \right) + k^2 r^2 \psi = 0 \]  

The solution of this equation for \( kr \gg 1 \) is:
\[ \psi = \frac{\exp(-j(kr + \pi))}{2\sqrt{2} \pi kr \sin \theta} \]

Therefore, the potential at point \( P(\theta, \phi) \) due to a source at \( P_0(\theta_0, \phi_0) \) is:
\[ G = \frac{\exp(-j(kr + \pi))}{2\sqrt{2} \pi kr \sin \alpha} \]

where \( \alpha \) is the angle subtended by \( P \) and \( P_0 \) with respect to the center of the sphere, and:
\[ \cos \alpha = \sin \theta \sin \theta_0 \cos(\phi - \phi_0) + \cos \theta \cos \theta_0 \]

\( G \) is the so-called free spherical surface Green's function.

ACOUSTIC FIELDS GENERATED BY AN ARC SOURCE

Next, we evaluate the field due to a finite source, such as from a surface wave transducer [3]. With the spherical surface Green's function \( G \), the potential at any point can be expressed as:
\[ \psi = \int_L (\psi G - G \nabla \psi) dL \]

where \( L \) is a closed curve on the spherical surface.

We assume an arc source of \( \Psi = \Psi_0 \) on the spherical surface located at \( \theta' = \theta_0 \), and with \(-\phi_0 \leq \phi' \leq \phi_0\), as shown in Fig. 1. The boundary condition is that \( \Psi = 0 \) on the rest of the closed curve, which includes the arc source. The closed curve and the Green's function \( G \) are carefully chosen to satisfy \( G = 0 \) on the closed curve, excluding the arc source. That is, in order to calculate \( \psi \) at the point \( P(\theta, \phi) \), we have to find its image point \( P_1(\theta_1, \phi_1) \) [2], and choose:
\[ G = G_0 - G_1 \]

where \( G_0 \) and \( G_1 \) are the free spherical Green's functions of \( P \) and \( P_1 \), respectively.

Using a formalism similar to the one used in calculating \( \nabla \psi \) for a focused bulk wave transducer [3], we find for the focused surface wave transducer:
\[ \psi = \int_{-\phi_0}^{\phi_0} (\Psi_0 \frac{\partial}{\partial \theta'} (G_0 - G_1) - (G_0 - G_1)(j kr + \frac{\cos \theta_0}{2 \sin \theta_0}) \sin \theta_0 \, d \phi' \]
ANALYTICAL EXPRESSION OF ACOUSTIC FIELDS NEAR THE SOUTH POLE

In order to work out an analytical expression for the acoustic field, we choose to start with the simplest case, i.e., we choose the source at:

\[ \theta_0 = \pi / 2 \]  \hspace{1cm} \text{[11]} 

and, we define an F-number as:

\[ F# = 1 / (2 \sin \phi_0) \]  \hspace{1cm} \text{[12]} 

In this case, the image point of \( P(\theta, \phi) \) is \( P_1(\pi - \theta, \phi) \); therefore, on the source line:

\[ G = G_0 - G_1 = 0 \]  \hspace{1cm} \text{[13]} 

and:

\[ \frac{\partial}{\partial \theta'} (G_0 - G_1) = \frac{e^{-jk\alpha}}{2\sqrt{2\pi kr}} [-2k \beta \sin \alpha \beta \cos \alpha (\sin \alpha)^{-3/2} + j \beta \cos \alpha (\sin \alpha)^{-5/2}] \]  \hspace{1cm} \text{[14]} 

where:

\[ \beta = -\cos \theta \]  \hspace{1cm} \text{[15]} 

\[ \cos \alpha = \sin \theta \cos (\phi - \phi') \]  \hspace{1cm} \text{[16]} 

Equations (10), (13), and (14) are used to calculate the field near the South Pole, where:

\[ \theta = \pi \]
\[ \beta = 1 \]
\[ \Delta \theta = \theta - \pi << 1 \]
The variation of the acoustic field along $\phi = \pi/2$ (and $\phi = -\pi/2$) is denoted as the transversal variation. In this case:

$$\cos \alpha = \sin \Delta \theta \sin \phi' = 0$$
$$\sin \alpha = 1$$
$$\alpha = \pi/2 - \Delta \theta \sin \phi'$$

and the field can be expressed as:

$$\psi = \frac{k r e^{-jk \pi/2}}{\sqrt{2 \pi kr}} \int_{-\phi_0}^{\phi_0} e^{jkL \sin \phi'} d\phi' = \sqrt{\frac{2kr}{\pi}} \sin \phi_0 \sin \left( \frac{L}{\lambda F#} \right) e^{-jk \pi/2}$$

where:

$$L = r \Delta \theta$$

We also denote the variation of the acoustic field along $\phi = 0$ (and $\phi = \pi$) as longitudinal variation, for which:

$$\cos \alpha = \sin \Delta \theta \cos \phi' = 0$$
$$\sin \alpha = 1$$
$$\alpha = \pi/2 - \Delta \theta \cos \phi'$$

The field can be written as:

$$\psi = \frac{k r e^{-jk \pi/2}}{\sqrt{2 \pi kr}} \int_{-\phi_0}^{\phi_0} e^{jkL \cos \phi'} d\phi'$$

$$= \sqrt{\frac{2kr}{\pi}} \sin \phi_0 \sin \left( \frac{L}{8\lambda F#} \right) e^{-jk \pi/2 + jkL(1+\cos \phi_0/2)}$$

Eqs. (17) and (19) are the same as those of a focused bulk wave cylindrical transducer.

From Eq. (17) we calculate that for focusing, i.e., for a beam on focus that is smaller than the transducer width, the $F#$ of the transducer must be less than $0.282(kr)^{1/2}$.

Also, from Eqs. (17) and (19) we find the transverse width $D_T$ and the depth of focus $D_L$ to be:

$$D_T(3 \text{ dB}) = 0.88 \lambda F#$$
$$D_T(\text{zero}) = 2 \lambda F#$$
$$D_L(3 \text{ dB}) = 7 \lambda F#^2$$
$$D_L(\text{zero}) = 16 \lambda F#^2$$

The above results indicate that a strong self focusing effect is present when exciting surface waves from a source of finite width. This focusing is important for the detection of a defect that is a fraction of the focal spot, which itself can be smaller than a wavelength. Thus, while operating at a frequency of 100 MHz, where the surface wave wavelength on a ceramic bearing ball is 60 $\mu$m, it is possible to detect a crack that is of the order of 6 $\mu$m in extent. More importantly, it would be possible to determine the exact reflection coefficient of the crack, which is essential for accurate sizing.
NUMERICAL RESULTS

According to Eq. (10), the acoustic field at any location on the spherical surface due to an arc source can be calculated numerically. Figures 2-4 show some of these numerical calculations. Figures 2 and 3 show, respectively, the transversal and longitudinal variations of the acoustic field around the south pole for the case of $kr = 100$. For comparison, we display the fields of two arc sources with an F-number of 1, but at two different locations of $\theta = 90^\circ$ and $\theta = 67^\circ$, respectively. We note that near the focal point, the fields generated by the two sources are very similar, and agree with the analytical solution. Thus, we conclude that the analytical solution which is valid for $\theta = 90^\circ$ can be used for other angles with good accuracy.

Figure 4 shows the azimuthal variation of the acoustic fields at several different latitudes for a sphere of $kr = 100$, and with an arc source with an F-number of 1 at
Fig. 4. Azimuthal variation of acoustic fields at several different latitudes. The sphere: $kr = 100$. The sources: $F\# = 1, \theta_0 = 35^\circ, -30^\circ \leq \phi \leq 30^\circ$.

$\theta_0 = 35^\circ$. As expected, it is seen that the beam diverges as it propagates towards $\theta = 90^\circ$, then converges as it continues towards the south pole.

SPHERICAL CAP TRANSDUCER AND SURFACE WAVE CONVERSION EFFICIENCY

To generate an arc surface wave source on a bearing ball, we propose to use a spherical cap transducer. The cap transducer shown in Fig. 5 consists of an arc longitudinal transducer on the flat top of a buffer rod, while in the bottom of the rod a spherical depression is made to fit the bearing ball under test. This transducer is similar to a wedge transducer on a flat surface, where the angle of incidence $\theta_0$ is equal to the Rayleigh critical angle. Because of the sphericity of the bearing ball, this condition is only met over a small width of the longitudinal transducer.

Fig. 5. Schematic of spherical cap transducer for the generation of a surface wave on a sphere.
Using ref. [4], with some simple modifications for the spherical case, we get the surface wave conversion efficiency as:

$$\eta_0 = \frac{2(1 - e^{-\alpha L})^2}{\alpha L}$$

[24]

where $L$ is the width of the transducer on the ball surface and $\alpha$ is the surface wave leaky rate per unit length. We note that the above expression is the same as for a flat transducer, because we use a transducer with a small width $L$, which in our case corresponds to length over the bearing ball. The efficiency $\eta_0$ as a function of $\alpha L$ is shown in Fig. 6. A maximum conversion efficiency of 81% is obtained for $\alpha L = 1.28$.

Table 1 summarizes several cap transducer designs for the inspection of Si$_3$N$_4$ bearing balls, where $\lambda_R$ is the wavelength of the surface wave. Since the leaky rates for these glass-ceramic combinations is large, the necessary transducer widths for optimum efficiency are too small and lead to excessive diffraction in the buffer rod. Therefore, a compromise has to be reached to maintain a working transducer.

<table>
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<tr>
<th>Material</th>
<th>$\theta_0$</th>
<th>$\alpha \lambda_R$</th>
<th>$L/\lambda_R$</th>
<th>$\eta_0$</th>
</tr>
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<tr>
<td>Glass SK-6</td>
<td>67°</td>
<td>1.11</td>
<td>4</td>
<td>0.43</td>
</tr>
<tr>
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<td>1.07</td>
<td>4</td>
<td>0.45</td>
</tr>
<tr>
<td>Glass #2</td>
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<td>0.9</td>
<td>4</td>
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CONCLUSIONS

We developed a theory to analyze the behavior of surface acoustic waves on a spherical surface. We find that surface waves due an arc source are focused in a manner similar to focused bulk acoustic wave transducers. The spot sizes we obtain are comparable to a wavelength and are given by $0.44\lambda F^#$. This effect enhances the power of this approach for small
surface defect detection in spherical objects. We have also designed several spherical cap transducers to provide good surface wave conversion efficiency. We expect to have some experimental results in the near future.

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REFERENCES