SCATTERING OF IMPACT WAVE BY A CRACK IN COMPOSITE PLATE

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INTRODUCTION

Ultrasonic waves provide an efficient means of characterizing defects in structures. For this purpose it is necessary to analyze scattering by such defects. However, scattering by crack-like defects in a plate-like structure is a complicated phenomenon and the problem is made more difficult if it is a composite plate. In recent years considerable progress has been made toward understanding wave propagation in anisotropic composite plates [1-5], but not much work has been done on the scattering by cracks in a composite plate. Recently Karim and Kundu [6] and Karim et al. [7] studied scattering of elastic waves in a layered half-space and in layered fiber-reinforced composite plates by interface cracks using a boundary integral formulation. They considered antiplane motions. Although this method can be extended to plane strain motion the computational effort is considerably amplified if one considers a plate geometry. Besides, the method used by these authors is limited to planar defects. For arbitrarily shaped scatterers Sánchez-Sesma [8] reviewed various applicable methods. Most of these numerical methods require considerable computational effort to evaluate the response. Their applicability to layered and anisotropic medium is also limited.

In this study we have used a hybrid method for time domain calculations of elastic waves scattered by a delamination crack in a transversely isotropic plate. In the past, this method has been successfully applied to homogeneous or layered isotropic half-space for frequency domain calculations [9-13]. There is one advantage of this hybrid method over various boundary element methods. It is that multiple scattering and arbitrarily shaped and layered scatterers can be considered. Since the method couples the boundary integral over a surface independent of the scatterer surfaces, one can use the same Green's functions for different scatterers by proper modifications of the interior finite element mesh.

This hybrid method leads to a complex matrix equation, where the matrix is sparse and unsymmetric. A compacted data structure [14] combining the biconjugate gradient method [15] is found to be an efficient solution scheme in these wave scattering problems.

FORMULATION OF THE PROBLEM

Figure 1 shows the geometry of a delamination crack in a composite plate. As shown, this defect lies inside a fictitious contour C. We define the interior region $R_I$.
to be bounded outside by the imaginary boundary B. The exterior region $R_E$ is bounded inside by the contour C. The area between the contour C and boundary B is shared by both regions. The interior region $R_I$ is discretized with finite elements and an integral representation over C for the displacements on the boundary B is introduced to solve for the scattered field.

Let $u_i$ be the displacement component in the $i^{th}$ direction in the Cartesian frame and $\sigma_{ij}$ the second order Cauchy stress tensor $(i,j=1,2,3)$ having time harmonic behavior of the form $e^{-i\omega t}$, where $\omega$ is the circular frequency. In each region, $u_i$ and $\sigma_{ij}$ satisfy the equation of motion

$$\sigma_{ij,j} + \rho \omega^2 u_i = -f_i \quad (i, j = 1, 2, 3)$$

where $\rho$ is the mass density, $f_i$ the force per unit volume, and the factor $e^{-i\omega t}$ has been dropped. The continuity of traction and displacement at the interfaces must be satisfied.

**Exterior Region $R_E$: Boundary Integral Representation**

In this region the displacement $u_i$ is composed of two parts

$$u_i = u_i^{(f)} + u_i^{(s)}$$

where $u_i^{(f)}$ is the free-field displacement (including the incident waves and their reflections) and $u_i^{(s)}$ the scattered field. By using Betti’s reciprocity theorem, the integral representation of the total field at any point in the exterior region $R_E$ is obtained as

$$u_i(x', z') = \oint_C (u_j \Sigma_{ijk} - G_{ij} \sigma_{jk}) n_k dC + u_i^{(f)}(x', z').$$

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Fig. 1. Configuration of a composite plate with thickness H=5.08 mm, fibers are along x-direction. (dimension is not shown in the same scale)
Interior Region $R_I$: Finite Element Technique

This region encloses all the inhomogeneities. In order to get the solutions in region $R_I$, we use the finite element technique. In this approach the area of interest $R_I$ is discretized into $N$ elements. The equation of motion for region $R_I$ can be written as

$$\begin{bmatrix} S_{BB} & S_{BI} \\ S_{IB} & S_{II} \end{bmatrix} \begin{bmatrix} dB \\ d_I \end{bmatrix} = \begin{bmatrix} y_B \\ 0 \end{bmatrix}$$  \hspace{1cm} (4)

where the elemental impedance matrices $S_{ij}$ are represented by

$$S_e = \int_A \int_e (B^* C B_e - \rho \omega^2 \Phi^* \Phi_e) dx \: dz$$  \hspace{1cm} (5)

It is clear from Eq.(5) that $S_e$ is a Hermitian matrix.

Evaluating the integrals at all the nodes $N_B$ on the boundary $B$, Eq.(3) becomes

$$dB = d^{(f)}_B + \int_C (\Phi^* \Sigma - GCB) ndC dB$$

$$+ \int_C (\Phi_B^* \Sigma - GCB_B) ndC dB$$  \hspace{1cm} (6)

where $B_I = D \Phi_I$, $B_B = D \Phi_B$. Equation (6) can be written as

$$dB = d^{(f)}_B + A_{BI} dB + A_{BB} dB$$  \hspace{1cm} (7)

Combining Eq.(7) and the second of Eq.(4), we obtain

$$\begin{bmatrix} I - A_{BB} & -A_{BI} \\ S_{IB} & S_{II} \end{bmatrix} \begin{bmatrix} dB \\ d_I \end{bmatrix} = \begin{bmatrix} dB^{(f)} \\ 0 \end{bmatrix}$$  \hspace{1cm} (8)

The total field solutions at a certain frequency can be obtained by solving Eq.(8). For more details about the derivations, the reader is referred to [11].

RESULTS AND DISCUSSIONS

The plate for which numerical results are given is made of graphite/epoxy with elastic constants $C_{11}=160.7$ GPa, $C_{33}=13.96$ GPa, $C_{55}=7.07$ GPa and $C_{13}=6.44$ GPa. The density is 1.8 $g/cm^3$. The geometry of the problem is shown in Fig. 1. The fibers are aligned along $x$-direction, plate thickness is 5.08 mm, the crack is located horizontally at a depth of 0.635 mm from the top surface. The line load is applied in $z$-direction at top surface at 5.08 mm horizontal distance away from the left side of mesh B and assumed to be uniform along the $y$-direction. Two different crack lengths have been investigated: 6.4 mm and 4.4 mm. The time domain incident wave signal is a Ricker wavelet.
Numerical Accuracy

The hybrid method used in this study has been tested for its accuracy for different finite element meshes by performing the zero-scatterer test. The relative error of the calculated total displacement and the incident displacement field is kept within 5% by adjusting the number of finite elements. Generally, 10 elements per wavelength is enough to accomplish the accuracy desired.

Numerical results

Figure 2(a) shows the surface response spectra of the plate without crack at points on the top surface above the crack due to impulsive load. Refering to figure 1, these points lie equally between x=-5.0 mm to x=5.0 mm. For the plotting conveniences, we have replaced amplitudes for the first two frequencies at each point to zero. This causes the fictitious peaks at $k_2H$ equal to 0.425. The maxima at $k_2H$ equal to 1.4$\pi$, 2.8$\pi$, 4.2$\pi$ and 5.6$\pi$ are seen in this figure and they are related to the cut-off frequencies of the symmetric and antisymmetric longitudinal modes, $\sqrt{\beta}(2n-1)\pi$, $\sqrt{\beta}2n\pi$, respectively. Here $\beta = \frac{C_{22}}{C_{55}}$. There are two thick curves in this figure which show the spectra at stations right above two crack tips on the top surface. Figure 2(b) is the spectra of this graphite/epoxy plate in the presence of the crack of length 6.4 mm. Comparing with figure 2(a), more peaks can be found in figure 2(b). The maxima at $k_2H$ equal to $\pi$, 2$\pi$, 3$\pi$ and 4$\pi$ are identified as cut-off frequencies of antisymmetric and symmetric shear modes respectively. They have been excited due to the existence of the crack. The peaks at $k_2H$ equal to 5.14 help determining the depth of the crack. It is because the ratio for crack length to crack depth is large ($\approx 10$). We can model this problem locally as two separate plates with thicknesses $H/8$ and $7/8H$. The cut-off frequencies for the thinner plate are eight times those for the original plate with thickness equal to $H$ and they are far beyond the frequency range we are investigating. However, the cut-off frequencies for $7/8H$ plate are expected to be found in figure 2(b). The aforementioned value 5.14 is exactly the cut-off frequency for the first symmetric mode of the $7/8H$ plate, thus indicating the depth of the crack. Another group of peaks at $k_2H$ equal to 1.2 is due to the resonance effect for the delamination crack. Keer, et al. [16] and Cawley and Theodorakopoulos [17] studied this effect in their papers. They proposed that the membrane resonance frequency for a defect might be predicted by using plate theory with length equal to crack length and thickness equal to crack depth. The natural frequency of this plate is 1.13 ($k_2H$) for simply supported ends and 2.56 ($k_2H$) for clamped ends which provide lower and upper bounds of the frequency of this peak. This result is consistent with the models that have been suggested by these authors.

Figure 3 shows the time-domain response for the same configuration. The incident signal is Ricker pulse with normalized centeral frequency at 0.75. We can first identify Rayleigh wave passing through. Then another Rayleigh wave scattered from the left tip of the crack travels back and forth between two crack tips on the crack surface. Since this is a shallow crack, this multi-reflected surface wave can be easily identified from the stations on the top surface. By measuring the travelling time of this wave between two tips, we can calculate the length of the crack. As a complementary data, the time-domain surface responses can further confirm the computed data obtained from the aforementioned membrane resonance method.

Figures 4(a) and 4(b) show the frequency spectra and time domain vertical surface displacements for a plate having a delamination crack with length 4.4 mm. Similar results can be seen, the resonance frequency complemented with crack length shown in time domain can be used to size this crack. The plate theory with simply supported ends is still a good physical model for resonance frequency prediction.
Fig. 2. Surface response spectra of a graphite/epoxy plate, (a) without crack and (b) with crack (6.4 mm), due to an impulsive load. The observation points are between $x=-5.0$ mm and $x=5.0$ mm on the top surface.
Fig. 3. Time domain surface displacements of figure 2(b) with Ricker wavelet as incident signal. The normalized central frequency is 0.75.

Fig. 4.(a) Spectra of surface displacements of a graphite/epoxy plate with crack (4.4 mm) due to an impulsive load. Receivers are at same positions as figures 2 and 3.
CONCLUSION

In this paper, a hybrid method combining the boundary integral representation and finite element technique has been used to investigate the time-domain response due to waves scattered by a delamination crack in a transversely isotropic plate. From computed results, we can quantitatively characterize the cracks, thus presenting useful data that may help in the ultrasonic nondestructive evaluation. In addition, we verify that the plate theory with simply supported ends is a good model in explaining resonance effects of crack near free surface if the crack length-to-depth ratio is large.

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