Quantitative prediction of the effect of vibrations on two-phase immiscible flow in porous media

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by

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For the Major Program
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CHAPTER 1. INTRODUCTION

1.1. Background

The subject of vibratory (sonic or seismic) stimulation of fluid flow in porous media has aroused increasing interest in the last decade, primarily in connection with the applications to enhanced oil recovery (EOR). While several methods of EOR, such as solvent or surfactant injection during the secondary recovery stage (using water flooding to displace the oil), are commonly used today, they rarely exceed 50% recovery threshold; at the same time, associated costs and environmental impact may be significant. Compared to these “traditional” methods, vibratory stimulation might not only result in higher recovery efficiency, but also would be ecologically clean and economical to implement, which further promotes the interest in developing such a technology.

A comprehensive review of the existing research on elastic-wave stimulation written by Beresnev and Johnson (1994) shows that documented field observations of the effect of seismic events on oil production, as well as certain laboratory studies on the effect of sound on single- and multi-phase flow in porous media, have existed for decades. However, common features to these studies were significant inconsistency in the results of field tests and, most importantly, lack of understanding of a physical mechanism that could explain this variability.

Only in recent years have well-controlled laboratory experiments provided reliable demonstration of the effect of vibration on immiscible fluid displacement in porous media (Roberts et al., 2001), and significant efforts have been devoted to theoretical studies aimed
at understanding the underlying physical mechanisms (Graham and Higdon, 2000a,b; Iassonov and Beresnev, 2003). A wide variety of mechanisms have been considered as possibly responsible for the observed effects, ranging from resonant behavior (Pan and Horne, 2000; Hilpert et al., 2000) to peristaltic motion (Aarts and Ooms, 1998), operating in various frequency and amplitude ranges. Because of stronger attenuation of high-frequency sonic waves in fluid-filled porous media, only low-frequency vibration has significant potential for stimulating the oil flow on a reservoir scale. In the lower-frequency range, the mechanism of capillary trapping and vibration-induced inertial forces, summarized by Beresnev et al. (2005), remains the most promising explanation of the entrapment and mobilization of the non-wetting phase under the effect of vibration.

Because of the large number of physical processes operating in a multi-phase (solid, liquids, gas) system, none of the aforementioned studies has been capable of explicitly quantifying the effect of vibration on fluid flow in any real application (involving natural porous media with complex pore geometry and a large number of pores). Thus, further research and development of the methods, leading to prediction of the effect of vibrations in realistic settings, is required before vibratory stimulation of oil reservoirs can become more than an abstract possibility.

1.2. Objectives

In this study, my goals are to better understand the capillary trapping and vibration-induced mobilization of a non-wetting phase ganglia in porous media, and to develop a method capable of estimating the effect of vibration on oil production in real-world
applications. In order to achieve these goals, I examined and compared three techniques of modeling the effect of vibration on two-phase immiscible displacement in porous media, differing mainly in the way a porous medium is represented in a computer model. These methods are pore-network modeling, a custom computer model for a flow in a single pore channel, and utilization of an existing commercial computational fluid-dynamics (CFD) software package. Capillary entrapment and vibration-induced inertial mobilization, discussed in more detail in Chapter 2, were the physical mechanisms incorporated into each of these computer models.

1.3. Dissertation organization

This dissertation is divided into six chapters. Chapter 2 contains a review of the previous work on the subject, as well as additional background information, emphasizing the relevant theories and experimental results. The following three chapters (Chapters 3-5) are largely independent, with each discussing the development, simulation results, and analysis of a specific computer model of the effect of vibration on two-phase immiscible displacement in porous media. In particular, Chapter 3 discusses the advantages and shortcomings of the pore-network modeling approach. Chapter 4 focuses on the creation of a custom computer algorithm for simulating the effect of vibration on capillary-trapped blobs of a non-wetting fluid in a single channel with regular converging-diverging geometry, modeled to represent more realistic pore geometry. Chapter 5 includes the results obtained using the CFD software package FLUENT™ for various two-dimensional pore structures.
Finally, Chapter 6 presents the general conclusions and provides recommendations for further research and development in this area.
CHAPTER 2. GENERAL CONCEPTS

2.1. Recent experimental studies

Documented field observations of the effect of seismic events on oil production, as well as certain laboratory studies on the effect of sound on single- and multi-phase flow in porous media, have existed for decades. A comprehensive review of these studies was provided by Beresnev and Johnson (1994). This review of the experimental work up to 1994 can be summarized as follows. While numerous observations and experimental studies appeared to demonstrate the enhancing effect of vibratory stimulation on multi-phase flow in porous media, their common feature was significant inconsistency in the results of field tests and, most importantly, lack of an underlying physical mechanism that could explain this variability.

Among the more recent well-documented advancements in experimental research of the effect of vibration on the mobilization of organic fluids in porous media, the following could be notable. Reddi and Challa (1994) examined the effect of vibration (shaking) on a flow in a test column filled with sand and saturated with water. Hexadecane was used as a non-wetting phase. The vibration frequency was 60 Hz, with various amplitudes up to 0.6 mm linear displacement (acceleration amplitude of 85 m/s\(^2\)). An increase in the removal of the non-wetting phase as a result of increasing amplitude was observed. The authors concluded that this mobilization effect was due to the changes in the pore structure (displacement of the individual grains) caused by vibration. While applicable to loose sand, this mechanism would not apply to lithified or deeply overburden porous materials.
A similar study was conducted by Reddi et al. (1998), where the sand-filled column was replaced with a plexiglass container filled with transparent glass beads with diameter of 2 mm. The observed flow enhancement was again concluded to be a result of the particles' rearrangement.

Roberts et al. (2001) conducted experiments on a water-saturated sand core sample, with an acoustic actuator used to create mechanical-stress stimulation within the core. During the experiments, a small amount of organic contaminant TCE (trichloroethylene) was added to the water. The results of the experiments showed that, when the vibration (in the frequency range from 25 Hz to 100 Hz) was applied, the concentration of TCE in the effluent of the sample increased significantly. Increasing the amplitude of vibration also increased the effluent concentration of TCE. The authors stated, however, that “the physical mechanisms responsible for the observed phenomenon are not fully understood. Further research is required before this promising new technology can be applied reliably to contaminated groundwater aquifers.”

Another set of laboratory experiments was performed by Poesio et al. (2002), who studied the effect of high-frequency (20-40 kHz) sonic stimulation on the flow of fluid through a sandstone core under a high pressure gradient. Their results also show an increase in the flow rate as a result of sonic stimulation; however, they concluded that this effect was caused by the decrease in fluid viscosity as a result of increased temperature due to the dissipation of sonic energy. Such effects are only possible at high frequencies and require high-power acoustic emitters. Due to the strong attenuation of high-frequency sonic waves in fluid-filled porous materials, such methods of stimulation cannot be used on a large scale.
Dobronravov (2002) reported on the results of the effect of weak seismic waves on oil production in several oilfields throughout the world. Utilization of one to several pairs of seismic vibrators of medium power with 10-ton force impact in cycles (relatively short durations of stimulation every few months) showed an increase in both the net fluid flow and oil cut (the fraction of oil in the effluent) in the surrounding production wells. The author claims that this technology was useful for stimulation of oil reservoirs as deep as 3000 m, and, over the course of several years, the estimated oil production increased as much as 45%. The so-called theory of the “energy level within the porous medium” invoked by the author to explain the observed results did not shed any light on the actual pore- or macroscopic-scale processes caused by weak low-frequency excitation.

Roberts et al. (2003) presented the results of using hydraulic-impact pulses, generated by a proprietary device placed in a borehole at a depth of approximately 240 meters, on oil production in surrounding wells. The data show an increase in oil production on the order of 20 %, well correlated with the start of the stimulation process. Additionally, a short-term increase was noticed as a result of a sequence of earthquakes that occurred several months prior to the start of vibratory stimulation.

While the new studies have kept gathering further evidence of the positive effect of vibration on organic-fluid production, they have provided little information useful for the construction of a general theory or a technique capable of predicting the effect of vibration in an arbitrary case. Such theory is required in order for applications of sonic or seismic stimulation to become widespread. Laboratory experiments that can provide insight into the processes occurring during vibratory stimulation at the pore scale must be able to ensure direct measurements and observations at the same scale. Because of the very small pore
sizes (on the order of 0.1 mm and less) in typical oil-bearing sandstones, designing such experiments is difficult.

Examples of a compromise solution can be found in the studies by Reddi et al. (1998) and, more recently, Li et al. (2005), where the porous medium was represented by an etched-glass micromodel. The model was quasi-two-dimensional, contained within two fused glass plates, and the pore sizes were on the order of 1 mm or larger. However, such an approach allowed direct observation of the two-phase flow, and provided a high level of control over pressures within the system and the vibration amplitude/frequency applied to the model. The results show that the application of vibration both increased the flow rate of the organic phase (TCE) through the pore structure, and reduced the residual (immobile) organic fluid saturation. It was also found that an increase in vibration amplitude resulted in lower residual TCE saturation, while an increase in the frequency led to a decrease in the mobilization effect.

2.2. Recent theoretical studies

In the past decade, new theoretical and numerical studies of the effect of vibration on flow also appeared. A variety of mechanisms explaining the increase in the flow as a result of vibratory stimulation have been proposed. For example, Aarts and Ooms (1998) theoretically investigated the flow of a compressible fluid in a tubular channel as a result of its deformation caused by a wave traveling along the wall of the channel (so-called “peristaltic motion”). They found that only ultra-sonic frequencies of very high intensities (on the order of 10 kW/m²) could produce any non-negligible net flow in the tube. Thus, this
mechanism could not be responsible for the observed effect of low-frequency seismic waves on a large scale.

Hilpert et al. (2000) analyzed the resonant behavior of an oil blob in a straight capillary, considering the oil blob trapped as a result of contact-angle hysteresis (change in the effective wetting angle as a result of contact-line motion due to mechanical imperfections (roughness) or chemical impurities of the pore wall). The applicability of their study to real-world scenarios was limited by the choice of a straight capillary tube geometry, a poor model for real porous media, characterized by wide variations in pore sizes. The significance of resonant effects in the case of a wide distribution of pores sizes is also questionable.

Another study addressing the resonance-type behavior was performed by Pan and Horne (2000), whose model of porous medium was a periodic structure of parallel two-dimensional channels with varying width. A net increase in the flow in such porous media as a result of vibratory stimulation was explained by variations of pore shape (volume) as a result of a passing wave. Again, only resonant behavior was addressed.

Iassonov and Beresnev (2003) presented a model of enhanced fluid percolation based on two assumptions: the pore-filling fluid is viscoelastic, and the capillary entrapment is caused by contact-angle hysteresis, the same mechanism as used by Hilpert et al. (2000). Vibration was modeled as an oscillating inertial body force (Biot, 1956) acting on the fluid. While this method did not rely on the resonant behavior to explain the enhancing effect of low-frequency vibration on the movement of the blob, it was plagued by an oversimplified geometry (a straight circular channel) and a certain assumptions utilized to calculate the net effect.
Graham and Higdon (2000a) studied the effect of an oscillatory body force on the flow of small droplets of a non-wetting fluid in a straight circular channel, using a custom finite-element computer model. They demonstrated the dramatic enhancement in the bulk flow rate and increase in droplet velocity, associated with increased droplet deformation in response to the oscillatory forcing. However, the use of dimensionless representation of the problem led to rather peculiar results. For example, all of their plots demonstrating the effect of oscillatory forcing were created for the value of $Re/Ca = 31.25$ (ratio of Reynolds number to capillary number). Substituting in values typical for crude oil (surface tension with water of 20 mN/m, density of 800 kg/m$^3$, and viscosity of 10 mPa·s), we can calculate the equivalent diameter of the channel. The resulting value is 78 nm, which is two orders of magnitude too small even for a very fine sandstone. Such inconsistencies make it impossible to apply their results presented in the paper to the problem of oil recovery.

In continuation of their work, Graham and Higdon (2000b) used a similar technique to model the flow of droplets in constricted channels. Unfortunately, again the dimensionless representation of the parameters of the problem make their results inapplicable to real-world situations. These limitations are discussed at more depth in Chapter 4, section 4.1.1.

2.3. Mechanism of capillary trapping

Interfacial tension is known to be a major factor constraint on multi-phase flow in porous media (Payatakes, 1982). The main reason for the entrapment of non-wetting fluid is variation in pore sizes, leading to variations in capillary pressures. This general idea has
been used in many publications, and can be illustrated with the simple case of a single blob (ganglion) of non-wetting fluid (hereafter “oil”) in a long tube filled with wetting fluid (hereafter, “water”) with a constriction (“pore throat”), schematically illustrated in Figure 2.1a. We will assume that there is a constant external pressure gradient, driving the ganglion towards the constriction in the pore channel. As the leading (right) interface enters the pore throat (Figure 2.1b), its radius must decrease, resulting in an increase in the capillary pressure created by the meniscus. According to the Laplace equation, the capillary pressure across the interface is inversely proportional to the interface curvature. Since the radius of the leading interface is smaller than that of the trailing (left) meniscus, this will create a pressure imbalance within the oil blob, resisting the external pressure gradient (shown as an opposing arrow inside the blob in Figure 2.1b). If the external pressure is not sufficiently high, the leading interface will not pass through the constriction, resulting in the entrapment of the oil ganglion.

Exact conditions for the entrapment will depend on many parameters. For simplicity, we will ignore any dynamic effects (such as inertia, or the variation in the contact angle due to menisci movement). Using the Laplace formula for capillary pressure and assuming a $180^\circ$ contact angle for the oil-water-solid contact line, we can write the entrapment criterion as

$$G < 2\sigma \frac{1}{r_{\min}} \frac{1}{r_{\max}} L,$$

where $G$ is the absolute value of the external pressure gradient, $\sigma$ is the oil/water surface tension, $r_{\max}$ and $r_{\min}$ are the maximum and the minimum radii of the pore (see Figure 2.1), and $L$ is the length of the pore channel (in case of a relatively long ganglion located in a
series of interconnected pores, $L$ can approximate the length of the oil ganglion itself. The right-hand side of [2.1] represents the critical pressure gradient ($G_{\text{crit}}$) required for the mobilization of a particular blob.

Formula [2.1] shows two important aspects of the capillary trapping. First, the higher the variation in pore sizes the higher the potential for entrapment. Second, the shorter ganglia are more difficult to mobilize by a constant external pressure gradient, and thus are more likely to remain trapped.

This mechanism of oil entrapment is implemented explicitly in all three numerical models, presented in the following chapters.

2.4. Mechanism of vibratory mobilization

While many mechanisms of mobilization have been proposed through the years, most of them are limited to high-frequency, high-intensity sonic excitation. The only mechanism capable of explaining the effect in the low-frequency, low-intensity range of seismic vibrations is inertial mobilization.

Since the elastic-wave propagation speed is significantly faster in the solid “skeleton” than in the pore-filling fluids, it is reasonable to assume that the fluid reacts to the passing wave primarily through the oscillation of the solid walls of the pore. If we consider the problem from the moving wall’s frame of reference, we find that movement of the wall will result in an inertial body force acting on the fluid (Biot, 1956). The derivation of the exact form of this body force is presented in the Appendix; as shown, for most cases of vibratory stimulation of naturally occurring porous media, components of the body force dependent on
local fluid-velocity gradients can be safely ignored. Thus, from [A.5], for harmonic oscillations of the porous medium, the vibration-induced inertial body force acting on the fluid at any given time \( t \) can be written simply as

\[
B = A \rho \sin(2\pi ft),
\]  

[2.2]

where \( A \) is the vector defining the acceleration amplitude and direction of vibration, \( \rho \) is the density of the fluid, and \( f \) is the wave frequency.

We can now consider the effect of longitudinal vibration of the pore wall to the flow of the oil blob (Figure 2.1). Since the vibration-induced body force oscillates harmonically, its net effect on the flow of a free ganglion in the pore (Figure 2.1a) should be negligibly small (zero, if we ignore contact-angle hysteresis and transitional flow effects), and so the average flow rate of the oil blob will be controlled by the pressure gradient alone. This might not be the case if the ganglion is trapped (Figure 2.1b); the combined effects of the external pressure gradient and the induced body force at some point in time might be sufficient to push the leading interface of the ganglion through. Once the leading meniscus is past the pore throat (Figure 2.1c), the capillary pressure imbalance [2.1] is decreased or gone, and the flow rate of the oil ganglion is again defined by the external gradient alone (the entrance of the trailing meniscus into the pore throat will only lead to an increase in the flow rate because the capillary-pressure imbalance [2.1] would be favorable to the forward motion of the ganglion). These simple considerations show that even low-frequency and relatively low-intensity vibrations may mobilize an entrapped oil ganglion.

Above we presented only a generic illustration of the vibratory mobilization mechanism, which cannot provide a ready-to-use formula for any situation. While we can make certain predictions (e.g., increase in the mobilization effect with increase in vibration...
amplitude), the exact parameters of the mobilization effect will depend on a multitude of factors, including pore geometry, viscosities of pore-filling fluids, vibration amplitude and frequency, and the external pressure gradient. Thus, the problem needs to be considered in a detailed quantitative manner, looking into the effect of some or all of the aforementioned factors on mobilization of the oil. This has been the goal of my research, results of which are presented in the following chapters.
Figure 2.1. Geometry of an immiscible two-phase flow through a porous channel with a constriction, under an external pressure gradient: ganglion in the open channel (A); right meniscus in the constriction, the ganglion is trapped (B); ganglion released by vibrations and driven freely again (C).
CHAPTER 3. PORE-NETWORK MODEL

3.1. Model formulation

3.1.1. Pore network approach

Understanding the physical processes responsible for trapping and mobilization of not-wetting fluids in porous media requires detailed modeling of the processes occurring in reservoir water-flooding. Field-scale experiments designed to test the effect of vibration cannot provide very useful results without detailed knowledge of the subsurface and an understanding of the physical mechanisms operating at the pore scale (Beresnev and Johnson, 1994; Beresnev et al., 2005). Laboratory experiments with realistic porous media (Reddi and Challa, 1994; Roberts et al., 2001) can provide verification of the effect of vibration on fluid flow on a macroscopic scale (e.g., a net increase in the flow rate), yet the behavior and the fate of single ganglia are difficult to track due to the relatively large scale of the model. Thus, a common compromise solution is required, such that simple observation and description of the details of the multi-phase flow on a pore scale are allowed, and yet include a sufficient number of pores to represent ganglia formation and fate are included. Such an approach in a laboratory experiment is presented in the work by Li et al. (2005), where the porous medium is represented by a pore structure etched on two glass plates; these glass plates are fused together, forming a structure of interconnected pores (Figure 3.1). This allows direct observation of the two-phase displacement process.

In the case of numerical models, the logic is somewhat different, since the major limitation is available computer speed. Single-pore detailed numerical models are possible
with today’s personal computers, and do provide insights into the mechanism of mobilization. An example of such studies can be seen in the works by Graham and Higdon (2000a,b) and W. Li (manuscript under preparation). But because interconnectivity plays an important role in ganglia dynamics (Payatakes, 1982; Dias and Payatakes 1986a,b), we must model a large number of interconnected pores. At this time, the computational resources available to an average Ph.D. student allow creation of a finite-element model of a realistic three-dimensional pore space with large number of pores and incorporating numerical simulation of fluid and solid interactions; however, the execution time for such a model would be measured in years for seconds of simulated behavior.

Therefore, just as in a laboratory experiment, certain compromises are required in order to obtain a reasonable quality representation of a real porous medium. One such compromise is a pore-network model, which operates not on the actual pore space, but on an abstract network of interconnected nodes (Payatakes, 1982; Sahimi, 1993; Celia et al., 1995). This replaces a detailed solution of the exact fluid-flow equations in the pore spaces with certain algorithms designed to simulate those processes in the abstract space of the model. This abstraction is the biggest advantage of the pore-network modeling approach, since it allows a tremendous reduction in the calculations required for the computer simulation of the flow, while retaining the complex pore interconnectivity. This is also the biggest challenge, since oversimplification of the actual physical processes can lead to results that have little to do with real porous media. Care is always needed in applying the pore-network approach.

While the pore-network approach has been used extensively for modeling ganglion behavior (Celia et al., 1995), explicit consideration of the dynamics of the complex system of
interfaces in response to forcing of a short timescale has never (to my knowledge) been attempted.

3.1.2. Static method

If the flow in porous media can be considered to be controlled by the narrower “pore throats”, the larger “pore bodies” having little impact on it, the problem is reduced to a system of linear equations defined by the conservation of volume in every “node” or pore body:

$$\sum_{j} q_{i,j} = 0.$$  \[3.1\]

Here $Z$ is the coordination number (the number of “neighbors” each pore body is directly connected to), and $q_{i,j}$ is the volumetric flow rate through the pore throat connecting pores $i$ and $j$. Assuming laminar flow, one can write

$$q_{i,j} = C_{i,j} (P_i - P_j),$$  \[3.2\]

where $C_{i,j}$ is the constant defining the hydraulic conductivity of the pore throat connecting pores $i$ and $j$, and $P_i$ and $P_j$ are the pressures in the center of each pore body.

Given boundary conditions in the form of either pressures or flow rates, and the values of $C_{i,j}$ (which can be calculated either analytically or using CFD software), we can easily solve the system [3.1]. However, when we have two immiscible fluid phases in the pore space, the problem becomes more complicated.

One of the methods of addressing this difficulty is the use of a mix of dynamic and static approximations. The non-wetting phase, occupying some of the pores, was assumed to
be in static equilibrium defined by pressure differences at the bounding menisci and pressures in the pore bodies occupied by the wetting fluid. The flow of the wetting phase was considered to be fully dynamic and calculated using equations [3.1]; equations [3.2] would be corrected by forcing a zero conductivity of the pore throats containing the interfaces or occupied by the non-wetting fluid, to account for the latter’s immobility. At every time step of the simulation, after calculating the flow field in the wetting phase, non-wetting phase ganglia could then be checked for stability, accounting for capillary pressures and the mass balance. New stable positions for the interfaces could then be calculated using an iterative scheme. If a new stable position of the interfaces could not be found within a small distance from the original, a new stable configuration could be calculated using a different algorithm, considering a possibility of the non-wetting phase invading to or retreating from one or several pore bodies. Such a transition would be considered instantaneous, equivalent to the "Haines’ jump" – a sudden jump of a meniscus during multiphase immiscible displacement, form an unstable configuration of the interfaces to a stable one (Dullien, 1992). If such an event occurs, the pressure in the water-filled pores could be recalculated using the same method as described above.

Assuming that the invasion/retreat to/from a single pore is almost instantaneous (i.e., takes significantly less time than ¼ of the vibration period), the effect of vibration can be modeled as quasi-static. Since the inertial force acting on the fluids due to vibration-induced acceleration harmonically oscillates, it should be sufficient for the purposes of checking the stability of a ganglion to test only the two extremes – the peaks of acceleration. Thus, after arriving at a stable ganglia configuration using the method described above, we could “enable” the vibration-induced acceleration and check for the stability again. If the
interfaces are still stable, the negative acceleration phase can be applied. If the ganglion moves, the next stable configuration could be found, and the process repeated in cycles. To avoid introducing a bias, we utilize the Monte-Carlo method – that is, after every successful movement, the phase of the vibration-induced acceleration is selected randomly.

Such an implementation allows a very fast calculation speed of the numerical model. However, during the implementation stage, two possible pitfalls of this method became clear: the lack of a timescale for the processes within the system (which could result in a series of events that would violate the quasi-static assumptions of the model), and possible algorithmic issues (one of which is a finite possibility of lacking a solution for the stable position of a given ganglion). Thus, the decision was made to develop a model that would treat all processes within the system dynamically.

### 3.1.3. Dynamic method

For the prototype of the model geometry, we chose the etched-glass micromodel used by Li et al. (2005). The original model consisted of “pore bodies” arranged on a square grid, connected by straight “pore channels” (or “pore throats”). The large-scale planar view of the model with the illustration of a typical part of the model geometry on a smaller scale is shown in Figure 3.1. The pore body diameters and pore-throat widths have an approximately normal distribution, with mean values of 2.30 mm and 0.76 mm, respectively. The distance between nearest neighbor pore body centers is 3.80 mm. The height (“thickness”, “depth”) of the model was assumed to be 0.20 mm.

The main reason for choosing this prototype was the possibility of an almost direct comparison between the controlled laboratory experiment and our numerical simulation.
Also, the corresponding pore-network model would be two-dimensional, which does not significantly increase the calculation speed (compared to three-dimensional representation) but makes the representation of the interfaces within the model and the presentation of the results much easier. In addition, the straight channels connecting the pore bodies were well suited for a fully dynamic implementation of the menisci movement (a similar program for channels varying in shape would require a significantly more complex and time-consuming recalculation of the hydraulic conductivities as the interfaces shift).

To implement the fully dynamic approach within the solution frame defined by equations [3.1] and [3.2], we had to accommodate the effect of capillary pressures and the changing position of the interfaces in the pore throats that would affect the form of the existing solution scheme. This was implemented using the following methods.

Let us consider the case of a single pore throat, connecting two pore bodies (marked $i$ and $j$). Neglecting the processes within the pore body itself, the total volumetric flow rate between the pore bodies would be defined by the pressure difference between the pore body centers, capillary pressures created by the interfaces within the pore throat, and hydraulic conductivity within the pore throat, which would vary since the wetting and non-wetting phases may have different viscosities; because of the horizontal orientation of the experimental model, we can ignore the effect of gravity in our numerical model.

In a thin, long channel, assuming a laminar steady-state flow, the pressure difference between any two interfaces (in the same fluid phase) $\Delta P_{n,n+1}$ will be directly related to the flow rate in the form

$$q_{i,j} = \frac{c}{\mu_{n,n+1} \cdot x_{n+1} - x_n} \Delta P_{n,n+1},$$

[3.3]
where $c$ is the *intrinsic* hydraulic conductivity of the channel, $\mu_{n,n+1}$ is the viscosity of the fluid phase between interfaces $n$ and $n+1$, and $x_n$ and $x_{n+1}$ define the linear positions of these interfaces in the pore throat. The total pressure difference between the pore-body centers can then be presented as the sum of the viscous pressure drop between the interfaces and the capillary pressures $P^n_c$ created by the interfaces in the form

$$
(P_j - P_i) = \sum_n \Delta P_{n,n+1} + \sum_n P^n_c.
$$

From [3.3], pressure differences $\Delta P_{n,n+1}$ can be obtained as functions of interfaces’ positions and the flow rate. If each interface position and curvature are known at any given time, then the only unknowns in [3.4] are the pressures in the pore bodies $P_i$ and $P_j$ and the flow rate in the pore throat $q_{i,j}$ (which, assuming incompressibility of the fluids and an unchanging shape of the interfaces, is the same throughout the pore throat). Equation [3.4] can therefore be written as

$$
q_{i,j} = A_{i,j}(P_j - P_i) + B_{i,j},
$$

where the coefficients $A_{i,j}$ and $B_{i,j}$ can be obtained from the current configuration of the interfaces in the pore throat connecting the pores $i$ and $j$ using [3.4] (the exact form of the expression will depend on the number of interfaces, their location, and shape). By substituting the expressions for the flow rate in pore throats [3.5] into the mass conservation at pore bodies [3.1], we obtain a fully defined system of linear equations that can be solved for the pressures in the pore bodies. The resulting pressures can then be substituted back into [3.5] to calculate the flow rates.
This method allows calculating the pressures and flow rates in the system at any given time, provided the location and curvatures of all interfaces were known. Knowing the flow rates, we can predict the position of the interfaces after a short time. Thus, by discretizing the time into sufficiently small time steps, we can simulate the temporal evolution of the system without imposing any external timescale while accounting for the physical mechanisms of capillary trapping and flow. To model the effect of vibration, we simply use the oscillating vibration-induced body force $B$ [2.2]. This will change [3.3] to

$$q_{i,j} = \frac{c}{\mu_{n+1}} \left( \frac{\Delta P_{n+1}}{x_{n+1} - x_n} + (B \cdot n_{i,j}) \right)$$

[3.6]

where $n_{i,j}$ is a unit vector defining the orientation and direction of the pore throat. This substitution will not affect the general form of [3.5], so the same solution methods can be used.

### 3.1.4. Testing the assumptions

The numerical method described is based on the assumptions of laminar quasi-static flow. In order to test these assumptions, we first need to estimate typical parameters of the flow in the prototype model.

In order to estimate the flow velocities, we first should know the pressure gradients that are applied to the system. In order to observe capillary trapping of a ganglion within the pores of the model, the external pressure gradient should not exceed a certain value $G_{crit}$ that can be calculated as a ratio of the maximum capillary pressure difference within a ganglion to the ganglion size. For our model, with the pore-throat channels that are wide compared to
their depth and assuming a contact angle of 0°, the capillary pressure created by an interface (meniscus) can be written in the general form of Laplace equation for non-spherical menisci,

$$\Delta P = \sigma \left( \frac{2}{h} + \frac{1}{r} \right),$$

[3.7]

where $r$ is the major radius of the interface, and $h$ is the depth of the pore channel. The maximum radius of the interface will occur in the pore bodies, and the minimum – in the pore throats. Using the average values of the pore-body radius (1.15 mm), half the average throat width (0.38 mm), and the value of surface tension between the organic fluid used in the glass-micromodel experiments by Li et al. (2005) (TCE – trichloroethylene) and water (34 mN/m), we obtain the maximum pressure difference of approximately 60 Pa. Assuming a minimal length of the ganglion to be on the order of 2 pores (7.60 mm), the external pressure gradient required to mobilize such a ganglion will be on the order of 8000 Pa/m.

The flow velocity in the pore throat can then be calculated using existing analytical formulae for the flow in a channel of rectangular cross-section. Using such formulae and tabulated data (White, 1970, p. 351), substituting the width of 0.76 mm and height (depth) of 0.2 mm, and using the viscosity of water ($10^{-3}$ Pa·s), we obtain the average flow velocity of approximately 0.13 m/s. Using the density of water (1000 kg/m$^3$), we obtain a Reynolds number $Re \approx 26$. While this value is not negligibly small for naturally occurring porous media (Fetter, 2001), it is still safe to assume the laminar flow regime (Johnson, 1998) because of regular geometry on large scale of pore in our model.

The validity of the quasi-static assumptions in modeling a time-dependant process (in our case it is the oscillating vibratory force) can be tested by estimating the parameter called the “viscous diffusion time”, which has the general form of
where \( r \) is the characteristic size of the channel, and \( \mu \) is the dynamic viscosity of the fluid. This parameter is a characteristic timescale, approximating the time required for the flow in a duct to approach steady state after a step forcing has been applied, and can be derived from simple dimensional analysis, as well as from the full analytical solution for the start flow (Johnson, 1998, p. 9-11). Since our channel is significantly wider than it is deep, it can be approximated with two-dimensional slot flow (a flow between two parallel plates), in which case a more precise formula for the viscous diffusion time is

\[
\tau_{vd} = \frac{\rho r^2}{\mu},
\]

where \( h \) is distance between the plates (the “depth” of our channel). Using the density and viscosity of water, we obtain \( \tau_{vd} = 4 \text{ ms} \). This parameter must be smaller than the period of the vibrations \( T_v \) applied to the model. Setting the criterion \( \tau_{vd} < T_v / 8 \), we obtain \( T_v > 32 \text{ ms} \), and thus the frequency of vibration \( f \) should not exceed approximately 30 Hz. This is within our range of interest, as well as within the values for the vibration frequency used by Li et al. (2005).

Thus, we can conclude that our assumption of the laminar quasi-steady flow used during the construction of the numerical pore-network model is justified.

3.1.5. Modeling interfaces

In general, the interfaces in the model can take a variety of shapes and sizes. However, in order to obtain a reasonable calculation speed, we chose to ignore certain types
of interfaces and flows. For example, consideration of film and crevice flow (flow in thin
films and streams of wetting fluid along the pore walls or in the small crevices in the pores
filled with the non-wetting fluid), while originally considered for implementation, was later
abandoned because of the difficulty in defining and calibrating these flows to correspond to
experimental data.

This leaves the interfaces of the same linear scale as the pore size (e. g., interfaces
across the pore throats, or interfaces in the pore bodies with the radii comparable to the pore-
body size); examples can be seen in Figure 3.2. Even for these interfaces, certain
approximations had to be made, in order to simplify the description to the level acceptable
for a network model incorporating a large number of pores. Observations of the actual
laboratory model showed that the majority of the menisci between the wetting and non-
wetting fluids kept a circular shape in the initial stages of displacement, and that the contact
angle appeared to be close to 0° for the water-TCE-glass contact line (unfortunately, no
picture is available with a sufficient resolution to show the details of the interfaces’ shape).
Thus, the interfaces in our numerical model were assumed to always keep the circular shape,
with the radius explicitly defined by the position of the fluid-fluid-solid contact line and the
contact angle.

The most important factor in the entrapment of the non-wetting fluid in porous media
is the capillary pressure (Beresnev et al., 2005). Our approximation of the interface shape
and behavior, allowing for the change in interface curvature (radius) in response to the
change in a position, was meant to provide a sufficient basis for proper modeling of the
capillary trapping within the model. As described above, we can easily calculate the
capillary pressure created by an individual meniscus in the model using [3.7]. The
interface's radius of curvature can also be easily calculated knowing the position of the interface.

Representation of the interfaces within pore bodies presents a more challenging issue. Since our goal is to model a large number of pores (the reason we chose the network-modeling approach in the first place), our approximation of the interfaces must be as simple as possible. Thus, we chose to represent all the interfaces (including those in the pore bodies) as two-dimensional circular arcs, defined by the point(s) of contact with the pore walls and a radius. Neglecting the third dimension is justified because the depth of the model is constant and small compared to the size of even the most narrow pore throats.

3.1.6. Algorithm description

This section describes the algorithm responsible for simulating the flow in our model. As will be shown shortly, the number of small details that must be accounted for during the execution of the program is significant, therefore a detailed bookkeeping description would be problematic for the scope of this text. Thus, only the general structure of the algorithm is given here. The actual simulation code was written in C++ in a modular, object-oriented form. While the program was written and tested on a Windows™ machine, the code was designed to be easily portable for potential use on different software platforms.

The main loop of the simulation consists of these main parts: the construction and solution of the flow-equation system defined by [3.1] and [3.5], movement of the interfaces, and post-processing for handling major changes in the interface configurations.

The flow-field solution stage, while the most time-consuming part, is quite straightforward in implementation. First, the data on the interfaces and the boundary
conditions (usually defined by the pressures on some boundary nodes, and no-flow condition on the others) are collected and used to build the matrix corresponding to the system of equations defined by [3.1] and [3.5]. The result of this operation is a symmetric sparse matrix, with the size equal to total number of pores in the model. The symmetry and the banded nature of the matrix enable compact storage and fast processing. The solution is then found using a customized (to enable faster execution for our specific type of data structures) implementation of the conjugate gradients iterative method. This solution provides pressures at each of the boundary nodes, which is then used to calculate the flow rates in the pore throats.

The second stage of the cycle now calculates the movement of the interfaces in response to the flow in the pore throats calculated in the previous stage. The new position of the interfaces is determined for a pre-defined time step that is the maximum period of time during which processes in the model can still be considered quasi-static. The duration of this time step is determined by several factors. One is changes in the external conditions (such as oscillations of the model; the time step must be significantly smaller than the period of vibration). Another factor is defined by the processes occurring in the model itself. Various changes to the interfaces are possible during this step: for example, an interface can enter or leave a pore body; or two interfaces might coalesce; these processes can result in a noticeable change in the local interface structure, thus resulting in a sudden change of capillary pressures. Since this violates the quasi-steady flow assumption, the basic flow field must be recalculated. The model is not allowed to advance past events that interrupt the continuity of the interface movement in a single step, and so this introduces an additional limitation on the time step, different in every cycle of simulation.
Finally, the post-processing stage analyzes the new configuration of the interfaces and takes action as needed to respond to events such as those described in the previous paragraph. For example, if the non-wetting phase invades a pore occupied by the wetting phase from one of the pore throats, at the instant the pore body is 100 % occupied by the non-wetting phase, the original interface is replaced by new interfaces, located in each wetting-phase filled pore throat; similarly, an interface must be created and destroyed during the retreat of the non-wetting phase from the pore body (these processes are illustrated in Figure 3.3). Obviously, there are many of possible scenarios that must be (and are) considered in the program. When the post-processing stage finishes, the program performs statistical calculations and any required input-output operations, if required; then the cycle is repeated.

3.2. Results and discussion

3.2.1. Model parameters

The pore-body and throat sizes used in the numerical model were made an exact match to those in the experimental model by Li et al. (2005), shown in Figure 3.1. In case a size of a grid used in the numerical model was different from the original’s 50x50 pores (e.g., 25x25), the pore sizes were selected from the same dataset, but the numerical model’s geometry was not an exact match to any part of the experimental system. Note that an exact match in the physical, not geometrical, conditions between the numerical and the experimental models was impossible due to the lack of measurements of local pressure in the
experiment, a quantity that would be required to run an exactly matching numerical simulation.

During most numerical experiments in the code-testing phase, viscosity and density of both fluids were set to the same values, equal to those of water \( (\mu = 10^{-3} \text{ Pa} \cdot \text{s}, \rho = 1000 \text{ kg/m}^3) \). The surface tension was also set to a "typical" value of \( \sigma = 20 \text{ mN/m} \).

Each series of experiments described below started with time-step optimization procedure used during simulations. In addition to the criteria described in section 3.1.6, internal testing of the model was used to determine the maximum acceptable time step. This testing consisted of running a series of simulations with identical initial and boundary conditions, but with successively smaller values of the maximum time step. A time step value was considered acceptable if the subsequent test simulations (with smaller time steps) yielded the same configuration of the interfaces within the system, with not more than 1% difference in the exact position or radius of each interface. A typical value of the maximum time step, obtained from these tests and used in most simulations described below, was one millisecond.

3.2.2. Performance

Despite the simplification of the equations describing the physical processes in the model and optimization of the code for the calculation speed, the speed-performance of the final model was disappointing. While the flow calculations were acceptably fast, the time-step limitations resulting from the choice of interface description diminished the calculation rate exponentially proportional to the number of interfaces. Certain restructuring of the algorithm handling the changes in the interface structure allowed to improve the performance
(it is now approximately linearly proportional to the number of interfaces). Even after this improvement, a model of the size of 50x50 pores would take anywhere from less than a minute to several hours (depending on the number of interfaces in the system) of runtime for every second of model time (using a personal computer with a 1.5GHz Intel Pentium-3 CPU). Thus, most subsequent simulations were run on grids of 15x15 or 25x25 pores.

3.2.3. Trapping and mobilization of a single ganglion

To determine the model’s ability to adequately represent the physical processes it was designed to simulate, a series of numerical experiments was set up. These tests consisted of the following stages: demonstration of the capillary-entrainment phenomena in the absence of vibrations, including testing for the mobilization by the pressure gradient alone; investigation of the model’s ability to simulate the long-term movement of ganglia; demonstration of the mobilization effect of vibration on entrapped isolated ganglia, and large-scale (large number of interfaces) simulations to show the dynamics of multiple ganglia.

The first testing stage was set up in a relatively small model (usually 15x15 pores), with the no-flow boundary condition at the top and the bottom of the grid (implemented by removing the corresponding pore throats), and the controlled-pressure boundary condition at the left and right sides of the model. Then a small ganglion of a non-wetting phase was introduced into the interior part of the model, otherwise filled with the wetting phase (since the model supports only two fluid phases, it will be assumed hereafter that, unless indicated otherwise, the pores in the model are filled with the wetting phase). This setup is illustrated in Figure 3.4.
A constant pressure gradient was then applied to the system, by varying the pressure on the inlet and outlet sides of the model. In several simulation runs with the same initial conditions, the pressure gradient was varied, thus finding a value of pressure gradient at which the ganglion would become immobile. The pressure gradient was then raised in small increments until the ganglion would become mobile at least for a short time (the criterion for mobilization was the ganglion’s occupying any pore previously filled with the wetting fluid). The pressure required for mobilizing the ganglion was recorded, yet due to large variations in ganglion volume and position and the short timescale of the simulations, the mobilization pressure variations did not provide much meaningful information. However, these results, together with the case-by-case analysis of the pressures within the model and interface positions and curvatures, were sufficient to demonstrate that the model properly simulated capillary trapping.

The second stage of testing used the same approach as the first one, in that the initial conditions were the same: a relatively small single ganglion of a non-wetting fluid and a constant pressure gradient across the model. The main difference was that the simulation was executed until either the ganglion became stable, or it left the model altogether. Some examples of ganglia behavior can be seen in Figure 3.5 in the form of snapshots of the model space taken at different times. Again, the model demonstrated the expected behavior in terms of both properly accounting for the capillary forces and demonstrating an appropriate timescale of the processes.

The next stage added vibration to the model in order to demonstrate the mobilization effect. The initial conditions for this model were copied from the output of the previous series of experiments, with a single immobile ganglion in the presence of a known pressure
An oscillating body force [3.6] with a set frequency (representing the vibrations applied to the model along the direction of the pressure gradient, illustrated in Figure 3.4) was then applied to the fluids; if after a preset time limit the ganglion remained stable, vibration amplitude was increased, until eventually the vibration amplitude was sufficient to mobilize the ganglion. Every ganglion that was immobile under the initial pressure gradient was mobilized by a finite vibration amplitude (usually in the range of 1 m/s\(^2\) to 10 m/s\(^2\); the vibration frequency being constant at 10 Hz), demonstrating the model’s capability to show the mobilizing effect of vibration on isolated capillary-trapped ganglia of a non-wetting fluid.

### 3.2.4. Large-scale experiments: pressure-driven flow

The final testing stage was meant to investigate the model’s capability to simulate larger-scale systems, with a large number of interfaces in various complex configurations. One of the most interesting simulation setups at this stage was the comparison of the numerical and experimental models. Since direct comparison (e.g., which fluid occupied which pore body, or where a particular interface was located) would be meaningless because of the inexact replication of the experimental geometry and conditions in the numerical model, considering the simplifications taken during the construction of the numerical model and the lack of control of the exact pressure distribution in the laboratory model, a different kind of test was needed. We chose to calculate the residual concentration of the non-wetting phase. To do so, the model was originally completely filled with the non-wetting phase, then the wetting phase was injected from one side of the model under constant predefined pressure. The amount (volume) of the non-wetting phase remaining in the system was plotted against time. Figure 3.6 shows the volume of the non-wetting fluid (normalized to
the initial, 100 % saturation volume) as a function of time for various values of the external pressure gradient in the 25x25 poremodel. In should be noted that the maximum residual concentration of the non-wetting fluid in our model did not exceed 30 %. This is higher than, yet reasonably close to, the results of the laboratory experiments by Li et al. (2005). The discrepancy can be explained by the lack of film- and crevice-flow implementation in the numerical simulation.

### 3.2.5. Large-scale experiments: effect of vibration

In order to simulate the effect of vibration on the mobilization of residual (immobile) non-wetting fluid phase, we used the following technique. Instead of manually introducing several ganglia into the system, we ran the simulation without vibrations (with only a constant external pressure gradient driving the flow), until all the remaining non-wetting fluid in the model became immobile. The resulting configuration was used as initial conditions for the simulations with vibrations enabled. The result of one of such experiments can be seen in Figure 3.7, where the relative saturation (normalized to the full void volume) of the non-wetting phase is plotted versus time for three experiments (lattice size is 15x15 pores and the pressure gradient is 630 Pa/m). The simulations differ in the amplitude of the vibrations applied to the model; the frequency of vibration was 10 Hz in both excitation cases. This clearly demonstrated the effect of vibration on the mobilization of the residual, entrapped non-wetting fluid in the pores.

For the experiments with the vibration on from the beginning, the results were not as consistent. As shown in Figure 3.8, the residual saturation was often higher if vibrations were applied; in some cases, however, such as one shown in Figure 3.9, vibrations did
produce positive results (higher mobilization), but that only happened when the vibration amplitude was relatively small, and flow rates in the system were relatively high (i.e., when the external pressure gradient was the dominant force compared to the vibration effect). This led us to believe that our numerical model was not capable of adequately simulating the effect of high-intensity oscillatory forcing (such as the vibration effect) when a large number of interfaces existed in the lattice.

Utilization of the visualization code written by Robert Ewing for this purpose allowed us to directly observe the configuration of the interfaces in the numerical model and thus helped to find the root of the problem. Figure 3.10 demonstrates a series of snapshots for various times for two simulation runs (with and without vibrations) in a 15x15 model, with the constant external pressure gradient of 630 Pa/m. When high-amplitude vibratory forcing was present, the non-wetting fluid front tended to break up, producing a large number of very small (one pore body or less in volume) ganglia, which, due to their small size, were extremely difficult to move with either type of forcing (pressure gradient or oscillatory body force) applied to the system.

3.2.6. Limitations of the model

While breakup of the displacement front or large ganglia as a result of high-intensity oscillating force might seem a normal process, such behavior (formation of ganglia of size of one pore body or less) was never observed in the laboratory experiments by Li et al. (2005). After a thorough investigation of the numerical model, we came to the conclusion that the formation of the very small isolated blobs of the non-wetting fluid seen in Figure 3.10 was mostly the result of the specifics of the interface-handling algorithm, and not of the physical
processes the model was designed to simulate. Such artifacts would only occur when many interfaces were present in the system, thus creating a highly sensitive and imbalanced system, in which every sudden change in capillary pressure (inevitably produced during certain events, such as formation of new interfaces as a response to an interface movement; for example, see Figure 3.3, part B) would significantly affect the behavior of other interfaces. The presence of high-intensity oscillatory forcing resulted in the events involving creation and destruction of the interfaces occurring much more often than they would if only a constant external pressure gradient were applied, thus increasing the undesirable effects to the extent that they become an obvious problem.

The underlying reason for this problematic behavior was the strict rules for the interface movement and transformation, resulting from the necessity to simplify the description of the interfaces’ shape, position, and dynamics. This was an inherent limitation which could not be remedied without drastic changes to the most basic principles of the model. Such a course of action was considered, yet eventually rejected, since the required complexity of the interface description would bring the model on par with finite-element fluid dynamics models in terms of the calculation speed, thus negating all the advantages of the pore-network modeling approach. Attempts to remedy the situation by introducing additional and modifying the existing rules used for modeling the interface dynamics were deemed unproductive, since such modifications, being purely artificial and not directly based on actual physics of the processes, would only make the model more abstract, with a higher potential to produce unrealistic results.
3.3. Summary

Using a pore-network simulation approach, we built a numerical model designed to simulate two-phase immiscible displacement in a pseudo-two-dimensional pore space, consisting of circular pore bodies arranged on a square grid, connected with straight channels (a.k.a. pore throats). The model included algorithms capable of calculating the pressures and flow rates in the pore space, as well as tracking and transforming the fluid-fluid interfaces (menisci) in wide range of situations.

The performance of the model in terms of the computational speed was barely acceptable, in most cases prohibiting simulation of very large number of pores in a reasonable timeframe using only personal computers available during the development of the model.

With only a small number of fluid-fluid interfaces, the model exhibited reasonable imitation of immiscible two-phase displacement processes under a variety of conditions. The final versions of the model were fully functional, and capable of demonstrating both the capillary trapping and the effect of vibrations on mobilization of residual entrapped non-wetting fluid. This provides additional proof of the possibility of vibratory simulation as means of enhancing two-phase immiscible flow in porous media, as well as a potential for numerical modeling of this process.

The model’s applicability for simulating large-scale systems with high number of interfaces was severely limited, however, due to the limitations imposed by the simplified description of the fluid-fluid interfaces’ shape and dynamics. Attempts to "fix" or calibrate
the model were found unproductive, since such actions would only further undermine the confidence in the model’s ability to provide realistic estimates.

In its current form the model appears to be sufficient as a tool for demonstrating the two-phase immiscible flow in a two-dimensional porous media and the effect of vibration on the mobilization on *isolated small ganglia*. Unfortunately, the model falls short of our original goal of large-scale simulations (involving tens of thousands of pores) in both calculation speed and adequate representation of the underlying physical process. Direct comparison of the results of the numerical and laboratory models also appears to be difficult.

We consider further development of the model unnecessary because of the inherent limitations described in the previous paragraph. Improvements to the model that would place more confidence in the results of large-scale simulations would require abandoning the pure pore-network modeling concept and replacing it with the combination of methods, including a type of finite-element model for the behavior of the interfaces. Such algorithms could be implemented but would result in a relatively slow calculation speed, probably not useful for modeling high numbers of pores.
Figure 3.1. Illustration of the etched-glass laboratory model used by Li et al. (2005).
Figure 3.2. Illustration in the types and scales of the interfaces considered in the pore-network numerical model. The grey color denotes the non-wetting fluid.
Figure 3.3. Illustration of the modeling of the process of invasion to (A) and retreat from (B) a pore body by the non-wetting fluid phase (grey).
Figure 3.4. Schematic illustration of initial and boundary conditions used in single-ganglion experiments. Non-wetting fluid is marked with the darker color.
A. Case 1

0.0 sec 0.5 sec 1.0 sec 1.5 sec

2.0 sec 2.5 sec 3.0 sec 3.5 sec

B. Case 2

0.0 sec 0.5 sec 1.0 sec 1.5 sec

Figure 3.5. Movement of single ganglia under a constant pressure gradient, obtained from two numerical simulations with different initial conditions. External pressure gradient is 630 Pa/m. The darker color denotes the non-wetting fluid.
Figure 3.6. Relative saturation of non-wetting fluid as a function of time for several numerical experiments in the 25x25 lattice with various values of external pressure gradient.
Figure 3.7. Relative saturation of non-wetting fluid as a function of time for several numerical experiments in the 15x15 lattice with various values of vibration-induced forcing. Pressure gradient is 630 Pa/m, vibration frequency $f = 10$ Hz. Vibration was enabled at approximately 20 seconds after the start of experiment(s).
Figure 3.8. Illustration of the experiment with the vibration applied from the beginning of the simulation; high-amplitude case. Grid size is 25x25 pores. Pressure gradient is 456 Pa/m; vibration frequency is $f = 10$ Hz.
Figure 3.9. Illustration of the experiment with the vibration applied from the beginning of the simulation; low-amplitude, high-pressure-gradient case. Grid size is 25x25 pores. Pressure gradient is 2280 Pa/m. Vibration frequency is $f = 10$ Hz.
A. Without vibration

B. With vibration (frequency $f = 10$ Hz, acceleration amplitude $A = 10 \text{ m/s}^2$)

Figure 3.10. Result of modeling the immiscible displacement with (A) and without (B) vibrations. External pressure gradient is the same in both cases (630 Pa/m). The darker color denotes the non-wetting fluid.
CHAPTER 4. SINGLE-CHANNEL MODEL

4.1. Model formulation

4.1.1. Rationale

Several studies have recently addressed the mobilization mechanism based on the effect of vibrations on overcoming capillary entrapment that holds the fluids in place (Graham and Higdon, 2000b; Hilpert et al., 2000; Iassonov and Beresnev, 2003), which allowed explanation of miscellaneous observations of the enhancement in non-wetting phase flow by vibrations under field and laboratory conditions. However, each of the studies listed above had one or another notable deficiency that made it difficult or impossible to directly apply toward estimating the effect of vibration on two-phase flow in real porous media. For example, in the works by Hilpert et al. (2000) and Iassonov and Beresnev (2003), the pore space was represented by straight circular channels, whereas the converging-diverging geometry of the pores is known to be the main reason for the capillary entrapment (Payatakes, 1982; Beresnev et al., 2005).

Graham and Higdon (2000b) introduced a converging-diverging pore geometry, but most of their results were based on the assumption that the material property number $a \rho \sigma / \mu^2$ was zero. Since $a$ represents the typical size of the channel, $\rho$ and $\mu$ are the density and viscosity of the fluid, and $\sigma$ is the surface tension, the material property number can only be considered exactly zero if either density or the surface tension is zero; the former would result in no effect of vibration, while the latter would be equivalent to neglecting the very process responsible for the entrapment of the non-wetting phase. In addition,
dimensionless values of vibration frequency of the vibration used in their model, when translated to SI, were above 1 kHz, which is well above the reasonable frequency range for sonic stimulation on a large scale (Beresnev and Johnson, 1994).

Thus, the objective of this study was the development of a numerical model that would meet the following criteria. First, it must incorporate the converging-diverging geometry of the pores. Second, the results obtained using this model must be directly applicable to the actual pores; this criterion imposes limits on the linear scale of the model (pore size must correspond to typical oil-bearing formations), as well as on the pressure gradient and vibration frequency and amplitude. Third, the model should be sufficiently simple (and therefore reasonably fast) to enable collection and analysis of the results over a wide range of input parameters in a reasonable timeframe. Finally, the model should reveal the basic physics of capillary entrapment and mobilization. To satisfy the last two criteria, we had to limit our model to a single pore channel instead of two- or three-dimensional pore structures.

4.1.2. Geometry of the pore channel

The model we consider is a capillary tube of circular cross-section with the radius varying with length as

$$R(z)=\frac{1}{2}\left[(r_{\text{max}}+r_{\text{min}})+(r_{\text{max}}-r_{\text{min}})\cos\left(\frac{2\pi z}{l_p}\right)\right],$$  \[4.1\]

where $z$ is the axial coordinate, $r_{\text{min}}$ and $r_{\text{max}}$ are the minimum (pore throat) and maximum (pore body) radii of the channel, and $l_p$ is the length of a single pore. This geometry is illustrated in Figure 4.1. This model represents a periodic structure of pore elements with
converging-diverging geometry, capturing the crucial factor responsible for the capillary entrapment of non-wetting fluids in porous space (Beresnev et al., 2005). The following typical values defining the channel geometry [4.1] will be used in the calculation throughout this chapter: \( r_{\text{min}} = 25 \mu m, r_{\text{max}} = 50 \mu m, \) and \( l_p = 500 \mu m. \)

4.1.3. Interfaces and capillary forces

The ganglion of non-wetting fluid (hereafter referred to as “oil”) is represented by a finite volume of oil situated within the pore channel, bound on both ends by water-oil interfaces (menisci) (Figure 4.1). The interfaces are assumed to be spherical, with the radius of curvature uniquely defined by the contact angle and the position of the contact line on the pore wall. This assumption is only valid in the quasi-static case; the validity of which is addressed in section 4.1.5.

The model was originally designed to handle an arbitrary value of the contact angle; however, a zero contact angle (complete water wetting of the wall with oil being the non-wetting fluid) is used for all the calculations hereafter. Contact-angle hysteresis is not considered (the walls of the pore are assumed to be sufficiently smooth).

Under these conditions, the capillary pressures \( P_c \) on the inner (oil) side of each meniscus at any given moment are

\[
P_c^x = \frac{2\sigma}{r_x^z},
\]

where \( r_x^z \) is the radius of curvature of the meniscus, and the superscripts refer to the left (“-” for the negative direction of the z-axis, or up-gradient) and the right (“+” for the positive
direction of the $z$-axis, or down-gradient) interfaces, respectively (Figure 4.1). The value of surface tension $\sigma = 20$ mN/m will be used in all calculations presented in this chapter.

We postulate that the length of the blob $L$ is significantly greater than the length of a single pore $l_p$. In this case, the end effects (such as the flow in the vicinity of the interfaces) can be neglected in calculation of the dynamics of the entire blob.

4.1.4. Modeling vibrations

To simulate the effect of vibrations, we assume that the entire pore channel is oscillating longitudinally with the acceleration amplitude $A$ in the same phase. This assumption is reasonable, since the wavelength of a low-frequency sonic wave in typical oil-bearing rock is much larger than the length of a single pore. For example, assuming a frequency of 50 Hz and sound velocity of 2500 m/s, we obtain the wavelength of 50 meters, which thus constitutes approximately $10^5$ pore lengths. Thus, even for a channel length as high as 10000 pores, the assumption of "rigid" vibration of the wall is still reasonable.

The vibration of the channel wall will result in an oscillating body force acting on the fluid [2.2]. Since we are only considering longitudinal vibrations of the pore wall, throughout this chapter we will refer to the vibration-induced body force as a scalar quantity $B$. For the fluid density of the pore-filling fluid, a typical for crude oil value of 800 kg/m³ will be assumed.

4.1.5. Quasi-static approach

We apply a quasi-static approach to the solution, i.e., treat the flow within the model at any time as steady-state. The limits of applicability of this assumption can be seen if we
calculate the viscous diffusion time [3.8]. Substituting the maximum radius of 50 μm and fluid viscosity of $5 \cdot 10^{-3}$ Pa·s, we obtain the viscous diffusion time of 0.4 ms. This value should be significantly smaller than the period of vibration $T_v$ (see section 3.1.4). For the highest vibratory frequency we use ($f = 40$Hz), the period of vibration is $T_v = 25$ ms. We see therefore that the condition $\tau_{vd} \ll T_v$ is always satisfied, justifying the use of the quasi-static approach.

The quasi-steady approach allows us to calculate the oil flow rate at any given moment based on the conductivity $C$ of the channel, which will generally depend on the system geometry and the current position of the interfaces. For large oil-blob length $L \gg l_p$, however, we can neglect the effects of the menisci position on the conductivity and calculate the oil flow rate $Q$ as

$$Q = \frac{C}{\mu} \left( -\frac{dP}{dz} + \frac{P^- - P^+}{L} + B \right), \quad [4.3]$$

where the conductivity depends only on the geometry of a single pore, $\frac{dP}{dz}$ is the external pressure gradient, and $B$ is the oscillatory body force.

The value of $C$ was calculated numerically using the computational fluid-dynamics software tool FLUENT™ ([www.fluent.com](http://www.fluent.com)). Utilizing a steady-state axisymmetric laminar flow solver for the geometry of a single pore [4.1], and applying a pressure difference between the inlet and the outlet of the pore, we obtained the flow rate, which in turn was used to calculate the value of $C = 4.44 \cdot 10^{-19}$ m$^4$.
To justify the usage of the constant conductivity (as calculated above), we also must ensure that the flow in the channel is laminar at any time. This can be done by calculating the Reynolds number

$$Re = \frac{D V D}{\mu},$$

[4.4]

where \( V \) is the maximum fluid velocity and \( D \) is the diameter of the channel. Assuming the parabolic (Poiseuille) velocity profile, \( V \) can be calculated as twice the average fluid velocity in the narrowest part of the pore,

$$V = 2 \frac{Q}{\pi r^2_{\text{min}}}. \quad [4.5]$$

In naturally occurring porous media, the turbulence may be observed for Reynolds numbers as low as 1 (Fetter, 2001). Using [4.5], [4.4], and [4.3], we calculate that, to achieve \( Re = 1 \), we would need to apply a pressure gradient of about \( 10^6 \) Pa/m (hereafter we refer to the absolute value of the pressure gradient for simplicity; it is obvious from [4.3] that the negative pressure gradient drives the fluid in the positive direction of the \( z \) axis in Figure 4.1). This value is extremely high and cannot be achieved in a typical oilfield environment, except in the close proximity of the well. Our assumption of the laminar flow is thus well justified for the types of applications we are considering.

4.1.6. Algorithm of flow calculation

As follows from [4.3], the flow in the model depends on three types of forces acting on the blob: the external pressure gradient \( dP/dz \), the capillary pressures on oil-water interfaces inside the blob \( P_c^z \), and the oscillating body force \( B \) due to vibration. In the
absence of vibration, the entrapment ("plugging") criterion is simply the balance between the external gradient and the capillary pressures,

\[- \frac{dP}{dz} < \frac{P_c^+ - P_c^-}{L}. \tag{4.6} \]

Considering that the right-hand side of [4.6] will vary with the menisci position, the criterion [4.6] assumes the maximum capillary-pressure difference \( P_c^+ - P_c^- \) for all possible range positions of the blob for its given volume. This allows finding the static pressure gradient needed to mobilize a particular oil blob, as well as the equilibrium position of the menisci for any pressure gradient that is not sufficient to mobilize the oil.

If the ganglion is mobile, we can calculate the flow rate using [4.3]. The calculations then proceed in time in a step-wise manner. Knowing the current flow rate, we calculate the positions and curvatures of the interfaces at the next time step. Since the cross-section of the channel [4.1], but also the shape (radius) of the menisci is changing, an iterative solution is used to obtain exact location of the interfaces based of the volumetric change on each time step. An adaptive time-step selection algorithm is employed, requiring that: a) the time step be significantly smaller than the period of vibration, and b) at any given time step, the capillary pressure not change by greater than a pre-defined threshold (typically, less than 2.5%). These limitations are employed to minimize the artifacts caused by the discretization of the flow and menisci movement, such as an interface being able to move through its equilibrium position in only one time step. In addition, actual testing with various values of the maximum time step was performed in order to avoid unintended effects of the time discretization.
The following criteria were used to determine the result of each dynamic simulation. The oil ganglion was considered immobile if its leading (right) interface had not advanced beyond its last known maximum position (along the pore axis z) for several periods of vibration (the actual number of cycles considered varied from simulation to simulation; typically, though, the majority of the entrapped ganglia are either mobilized in only one or two periods of vibration, or remain immobile). The ganglion was considered mobile if the leading interface had traveled a distance of one pore length or greater (due to periodic structure of the pore, this is a sufficient condition). Each simulation time could be limited by these criteria and provide a simple yes/no answer, saying whether the oil was mobile or not under the applied initial conditions. In the case of the average-flow calculation, the mobilization criterion was slightly modified, in that the required movement of the interface was extended to several pore lengths in order to improve the precision of the result.

4.2. Results and discussion

4.2.1. Presenting results of simulations

As seen from [4.6], in case of no vibration, the mobilization of the blob is simply a matter of creating a sufficient pressure difference

\[ \Delta P_0 = \frac{dP}{dz} L \]  [4.7]

by the external gradient. It follows that, for a given gradient, the longer ganglia will be mobile while the shorter ones will be not (Payatakes, 1982). Also, the pressure difference required for the mobilization will be an approximately periodic function of \( L \) (or the ganglion
volume), because the resisting capillary-pressure imbalance $P_c^+ - P_c^-$ is periodic with period $l_p$. This behavior can be seen in Figure 4.2, where the threshold pressure difference $\Delta P_\theta$ is plotted against the ganglion volume varying over one pore element. The fate of the ganglion will thus depend highly on very slight variations in the ganglion volume within one pore; this variability should be properly accounted for in the representation of the flow-mobilization phenomenon.

A meaningful way of characterizing the fate of the blob (and quantifying the mobilizing effect of vibration) on a macroscopic level would then be to define it not for a ganglion of any particular volume (or length) but for an ensemble of ganglia occupying the range of lengths from $L - l_p/2$ to $L + l_p/2$. In the following, we use the fraction of mobile ganglia over all ganglia in the ensemble. An example of such a calculation is presented in Figure 4.3, where the fraction of mobile blobs is plotted versus the external pressure gradient; each point on the curve is the value for an ensemble of 1000 blobs with the “macroscopic” length of $L = 10$ cm increasing in uniform increments over one pore element. All ganglia are mobile when the gradient exceeds the maximum capillary pressure-difference $P_c^+ - P_c^-$ calculated for all lengths.

4.2.2. Amplitude effect

This method and the statistical measure introduced can be used to calculate the effect of vibration on the mobilization of the ganglia in the model. Since almost any pressure gradient would be sufficient to mobilize a certain fraction of blobs of particular length (Figure 4.3), we assume in the following that, before the start of vibrations, only immobile
ganglia remain in the porous medium, representing the residual oil saturation. It then becomes convenient to calculate the fraction of mobilized ganglia, i.e., their number that have become mobilized as a result of vibratory stimulation ($M_A$) normalized by the total number of ganglia immobile under the given pressure gradient without vibrations ($M_0$),

$$M^* = \frac{M_A}{M_0}.$$  \[4.8\]

This result is shown in Figure 4.4, where $M^*$ is plotted versus vibration amplitude for several values of pressure gradient. As in Figure 4.3, each data point represents an ensemble of 1000 volumes of the blobs with $L = (10 \text{ cm} \pm 0.5 \text{ l}_p)$. The stimulation frequency is 10 Hz. It is seen that the effect grows with acceleration amplitude, in accordance with the predictions made from the basic consideration of the mobilization mechanism (section 2.4).

The curves in Figure 4.4 are informative yet insufficient to fully describe the vibratory mobilization, as the effect will also depend on the frequency and the ganglion length.

### 4.2.3. Frequency effect

From the basic description of the vibratory mobilization mechanism (see section 2.4), we would expect that the effect of vibration will depend on the frequency. Considering the limit of zero frequency (equivalent to constant inertial forcing added to the external pressure gradient), we can expect that, for a given amplitude, the lower frequencies will generally result in more effective mobilization. This expectation comes from the following reasoning. In order to mobilize the entrapped blob, the vibration must force its leading ("+") meniscus through the narrowest part of the throat over one period. Past this point, the radius of the
meniscus starts to progressively increase as the meniscus moves forward, creating a proportional decrease in the resisting capillary force; as a result, the blob accelerates upon exiting. This acceleration causes a nonlinear growth in the distance traveled with the time spent in the "unplugged" position; this nonlinearity makes the frequency effect possible.

Calculations corroborate the frequency dependence of the mobilization effect. It is demonstrated in Figure 4.5, where the fraction of mobilized ganglia is plotted against frequency for several acceleration amplitudes $A$. Recall that at the pressure gradient of 7050 Pa/m 50% of the ganglia are trapped (Figure 4.3). It is clear that the effect of vibration at a given $A$ ranges from full mobilization (100%) to no mobilization (0%), depending on the frequency.

We also observe that, in the range of frequencies presented, the acceleration amplitude required to mobilize a set fraction of ganglia appears to increase almost linearly with the frequency. This dependence is better illustrated in Figure 4.6, where the amplitude required for the mobilization of a fraction of immobile oil is plotted against frequency. The linear relationship is easily explained. As we have pointed out, the blob's leading meniscus must be carried to the narrowest point of the constriction over one period of vibration to be mobilized; for a blob of a given volume, the required travel distance constitutes a certain fraction $s$ of the length of the pore. If the blob’s velocity is $V$, this leads to the mobilization condition $V T \approx s l_p$. Since $V \sim A$, we obtain $A/f \sim s l_p = \text{const}$, which is the pattern seen in Figure 4.6.

It stands to reason that there are frequency limits within which this argument applies. At the high-frequency end, the linear dependence between the mobilizing amplitude and the frequency will hold as long as the condition of quasi-static approximation, $\tau_{nl} \ll T$, applies.
At the other extreme, at sufficiently low frequencies (including zero, or static mobilizing forcing), the distance traveled over one period will always exceed the length of the pore; as long as this is the case, the frequency effect will not be seen, and the linear dependence will “level off” at the value corresponding to the static mobilization forcing presented in Figure 4.3. This behavior is demonstrated in Figure 4.7 obtained from simulations. It shows that the frequencies at which the leveling-off occurs are extremely low (on the order of one-hundredth of a Hz; they will scale as \( I_c^{-1} \)); for practical purposes, this deviation from the \( A/f = \) const dependence may be ignored.

It follows that in practice one could conveniently characterize the effect of vibration with one parameter, \( A \) or \( f \), knowing that a constant \( A/f \) leads to the same result. It is also important that this ratio is proportional to the square root of sound intensity, a measure of the energy of a sonic wave used in acoustics,

\[
I = \frac{\rho_s c}{8\pi^2} \left( \frac{A}{f} \right)^2,
\]

where \( \rho_s \) is the density of the solid and \( c \) is the wave-propagation velocity. We then infer that another way of characterizing the effects of the amplitude and frequency would be to use the intensity measure: the mobilizing effect of the fields having the same intensity is the same. As follows from Figure 4.6, to increase the effect, the intensity must be increased.

4.2.4. Ganglion length and pressure gradient effects

The effect of the length of the entrapped ganglion on the mobilization, for a given gradient, will be to proportionally scale the required amplitude (intensity), as can be understood from equation [4.6]. Considering that the maximum \( P^+ - P^- \) is about the same
for every length, a decrease in $L$ will result in the increase in the right-hand side of [4.6] and so higher absolute values of external pressure gradient will be required to overcome the capillary forces. Consequently, a proportionally larger body-force amplitude $B$ will be needed to unplug the ganglion. Similarly, at larger pressure gradients and given $L$, the same effect will be achieved at proportionally smaller vibration intensities.

If was found, however, that there was no perceptible difference between the amplitude required for the mobilization of ganglia of various lengths as long as the total pressure difference [4.7] remained the same. This is illustrated in Figure 4.8, where the fraction of the ganglia mobilized by the given vibration amplitude is plotted against the length of the ganglia; the pressure gradient was different for every ganglion length in order to keep the pressure difference [4.7] constant. The maximum difference observed between the ganglia lengths two-orders-of-magnitude apart is only 5%, which is comparable to the variations due to changes in vibration amplitude (Figure 4.4) or frequency (Figure 4.5) of only a few percent. This result is expected, considering that, for the large mobilization effect, the value of the vibration-induced body force must be significantly higher than the values of the external gradient (see, for example, Figure 4.4).

4.2.5. Fluid viscosity effect

The results shown in previous section, in addition to the frequency effects, suggests that, for relatively large ganglia lengths, the viscosity of the pore-filling fluid should be an important factor controlling the mobilization of ganglia. Given the equation defining the flow rate in the pore channel [4.3], and considering that the amplitude of vibratory-induced forcing is significantly higher than that of the external pressure gradient or capillary pressure
difference, we expect almost linear increase in the vibration amplitude required for mobilization with the increase in fluid viscosity.

Figure 4.9 shows the vibration amplitude is plotted against the fluid viscosity for two different ganglia lengths (1 cm and 10 cm), for a constant pressure difference [4.7]. For the longer ganglion, as expected, we see an almost linear dependence between the vibration amplitude and the flow rate; for the shorter ganglion, the dependence approaches linear as the viscosity of the fluid increases.

4.2.6. Vibration effect on flow rate

We have so far discussed the vibratory effect on the mobilization of oil ganglia. From the practical standpoint, it is also important to characterize the effect of vibrations on the flow rate, which, for example, could be used to predict an absolute increase in organic-fluid production. We again present these results in terms of the mean flow rates, averaged over oil ganglia with lengths in the range $L \pm l_p/2$.

The dependence of the average flow rate on the pressure gradient (with no vibrations applied) for an oil blob of length $L = 10$ cm is shown in Figure 4.10. To emphasize the stimulating effect of vibrations, it is convenient to normalize the enhanced flow rate (with the vibrations applied) by that without vibrations, representing a relative increase in $Q$,

$$Q^* = \frac{Q_A - Q_0}{Q_0},$$

[4.10]

where $Q_A$ is the rate given vibrations with amplitude $A$, and $Q_0$ is the rate without vibrations. This result is shown in Figure 4.11, where $Q^*$ is plotted against the external
gradient for different values of $A$. Clearly, the relative mobilization effect increases with amplitude.

The effect of frequency on the flow rate will understandably be controlled by the ratio $A/f$, as was the case for the fraction of mobilized ganglia in Figure 4.6. This dependence is illustrated in Figure 4.12. The same flow-rate increase is achieved for a constant value of the ratio $A/f$.

4.2.7. Estimation of realistic seismic effects

It would be interesting to estimate the magnitude of the residual-oil mobilization effect that could be expected under realistic field conditions. According to the data available to the author and from the author’s experience, the existing non-explosive borehole seismic-energy sources are capable of creating maximum displacements at a distance of ~300 m on the order of $10^{-6}$ m at the central frequency of ~100 Hz, providing an estimate of maximum acceleration of ~0.4 m/s$^2$. According to Figure 4.5, the mobilization effect at such amplitudes and frequencies could be expected to be negligibly small, except for the volume of the reservoir immediately surrounding the borehole. We infer that much more powerful sources are needed to stimulate reservoir volumes within a few hundred meters. It is not within the purview of this writing to suggest how this could be technically achieved. It stands to reason, though, that a significant stimulation effect could be achieved with the use of surface vibrators to stimulate the near-surface flow, which has direct implications for the remediation of ground water contaminated by entrapped organic fluids (e.g., leaking gas tanks).
4.3. Summary

In this chapter, we presented a model for approximate calculation of the effect of vibration on the mobilization and average flow of oil blobs in pore channels of variable radius, representing more realistic converging-diverging geometry of the naturally occurring porous media.

The calculated effects of amplitude and frequency agree with expectations from the mobilization mechanism (see Chapter 2). For a particular entrapped blob, there is a minimum amplitude required for its mobilization. This amplitude scales linearly with the frequency such that the ratio $A/f$ is kept constant. At a given amplitude, higher frequencies reduce the mobilization effect; if the frequency becomes sufficiently high (within the limits of applicability of the quasi-static model), the mobilizing effect of vibrations stops altogether. We should mention, however, that this statement applies only within the applicability limits of our model, detailed in section 4.1.5 of this chapter. At higher frequencies, other mechanisms, such as reduction in the effective capillary force due to change in the menisci shape, or breakup of ganglia, might result in some mobilization effect.

In terms of practical applications, this model can be used to estimate absolute volumes of oil mobilized by given amplitudes and frequencies of vibration, provided sufficient information on pore-size and ganglia-size distribution in the porous reservoir is available. It also provides the minimum amplitude and maximum frequency required to initiate the stimulation effect.

Calculations show that, for the parameters of the model corresponding to typical oil-bearing formations, the vibratory amplitudes required to achieve noticeable mobilization and
increase in oil flow rate are rather high (e.g., Figures 4.5 and 4.11), on the order of 10 m/s$^2$ and much higher at the frequencies in excess of 10 Hz, which is not realistically achievable with available borehole seismic sources. The goal of the low-frequency stimulation of large volumes of oil reservoirs thus seems to be beyond the reach of the existing technical means, unless significant increases in the elastic-wave energies of seismic sources are obtained. This does not, however, apply to the possibility of vibratory stimulation of near-surface contaminated aquifers, where the necessary sonic energies can be created from the surface.

Our model deals with a single pore channel and does not incorporate the interconnectivity of real porous media. Interconnectivity can nevertheless be argued to favor the release of entrapped ganglia compared to a single channel of the same “average” geometry. Indeed, if an oil ganglion occupies a large number of pores, some of the down-gradient menisci will likely be closer to the mobilization threshold than the meniscus of the “average” channel; the mobilization will then occur through that “more susceptible” throat. In this sense, our model can be viewed as an “upper-bound” estimate for the respective mobilizing parameters of the vibratory fields. Similar treatment of multiple-pore ganglia in interconnected pore systems, which would require complete characterization of (still poorly understood) dynamic behavior of menisci as they invade a node with several pore throat, is yet to be provided; some aspects of this task are addressed in the next chapter.
Figure 4.1. Schematic illustration of the model pore channel and entrapped oil blob.
Figure 4.2. The effect of oil ganglion volume on pressure difference required for the mobilization.
Figure 4.3. The effect of external pressure gradient on the mobilization of oil ganglia of length $L = 10$ cm spanning the range of lengths within one pore element.
Figure 4.4. The effect of vibration amplitude on the mobilization of oil ganglia of length $L = 10$ cm. The frequency of vibrations is 10 Hz.
**Figure 4.5.** The effect of vibratory frequency on the mobilization of ganglia ($L = 10$ cm) for several amplitudes under the external pressure gradient of $7050$ Pa/m.
Figure 4.6. The amplitude and frequency effect on the mobilization of entrapped ganglia 

\[ L = 10 \text{ cm}; \left| \frac{dP}{dz} \right| = 7050 \text{ Pa/m}. \]
Figure 4.7. The amplitude and frequency effect on the mobilization of entrapped ganglia for very low vibration frequencies ($L = 10$ cm; $\left| \frac{dP}{dz} \right| = 7050$ Pa/m).
Figure 4.8. The effect of ganglion length on the mobilizing effect of vibration ($A = 35 \text{ m/s}^2$; $f = 10 \text{ Hz}$) under a constant pressure difference (760 Pa).
Figure 4.9. The effect of fluid viscosity on the amplitude required to mobilize 100% of the entrapped ganglia. Constant pressure difference of 705 Pa is assumed. Vibration frequency is $f = 10$ Hz.
Figure 4.10. The effect of external pressure gradient on the average flow rate for the ganglia of $L = 10$ cm spanning the range of lengths within one pore element.
Figure 4.11. The relative increase in the average flow rate as a result of application of vibration with various amplitudes. The frequency is 10 Hz.
Figure 4.12. The amplitude and frequency effect on the average flow rate of oil ganglia

\( (L = 10 \text{ cm}; \frac{dP}{dz} = 7050 \text{ Pa/m}). \)
5.1. Model formulation

5.1.1. Rationale

Because of the high complexity of the problem of dynamic immiscible two-phase flow in a system consisting of a large number of interconnected pores of variable shapes, obtaining exact analytical solutions is not a feasible option. The next best alternative to an analytical solution would be a finite-element or finite-volume model, explicitly accounting for all the physical mechanisms involved. The idea of using a computational fluid-dynamics approach for modeling the effect of vibration on the flow in porous media has already been briefly addressed in previous chapters, but was dismissed, primarily due to limits in available computer resources, in favor of simpler (and thus faster) models. However, from analysis of the obtained results, we conclude that the methods we have utilized so far are, by themselves, insufficient to achieve the goal of creating a model capable of accurately predicting the effect in realistic scenarios. For example, the pore-network model described in Chapter 3 was severely limited because of the oversimplified handling of the fluid-fluid interfaces within the pore bodies; the single-channel model of Chapter 4, while serving its purpose of accounting for the converging-diverging geometry of the pores, was, by definition, limited to one pore channel, whereas the interconnectivity of the pores is important for the formation, and thus potential mobilization, of the ganglia of non-wetting fluid.
In this chapter we therefore investigate the potential for using standard computational fluid-dynamics (CFD) methods to analyze and predict the effect of vibration on mobilization of trapped non-wetting fluid.

### 5.1.2. Software and methods

A wide variety of CFD methods for solving the problems of multi-phase fluid flow exist today. When it comes to the implementation stage, however, there is a choice to make among writing a custom code, using an existing source code or using commercial software. While the construction of a problem-specific code has the potential for significantly better performance, the complexity of our problem, as well as variations in the geometry and initial/boundary conditions, would require the ultimate implementation to be extremely flexible and robust; writing such a computer program would be a challenging and time-consuming task. Thus, a decision was made to use the commercial CFD software package FLUENT, well-known for its extensive capabilities in solving a wide variety of fluid-flow problems (including multi-phase flow) using the finite-volume approach.

To model the immiscible multi-phase flow specific to our problem, we used the “Volume of Fluid” (VOF) method implemented in FLUENT. This method relies on representing the phases in terms of their volume fraction in each finite-volume cell. The interface between the fluids is then reconstructed at each time step using geometric reconstruction, where an interface in each cell is represented by a segment of a straight line (or a plane in case of a three-dimensional mesh), based on the known volume fraction within the cell and its neighbors. The VOF method is appropriate for modeling the effects of
surface tension in problems requiring the interfaces to be sharply defined at any time during the simulation.

The equations solved by FLUENT at each step of the simulation are the momentum-conservation (including the terms accounting for surface tension), continuity (mass-conservation), and volume-fraction equations; the energy transfer is neglected. Incompressible flow is assumed, and gravitational forces are ignored. The general solution technique is based on converting the governing equations into algebraic form (using the finite-volume method); these equations are then solved iteratively. The convergence criteria are the scaled residuals for the continuity equation and each component of the fluid velocity, which are defined by the ratios of the total residuals summed over all computational cells and the corresponding total absolute values. The exact form of the equations being solved, as well as the details of the solution technique, can be found in Fluent, Inc. (2003). The typical values of the scaled residual used to determine the convergence in the simulations presented below are $10^{-4}$-$10^{-6}$.

We employed the unsteady, segregated solver (meaning that the governing equations are solved sequentially; the limitation dictated by the use of the VOF model) provided by FLUENT. Although adaptive-time step evaluation methods were available, a constant time step was used in every simulation, because this approach provided more consistent results and therefore simpler post-processing for creating dynamics animations.

As described in the Appendix, the effect of vibration can be modeled by simply introducing an oscillatory body force [A.5]. This force can be simulated in FLUENT by defining the source terms (terms added to the right-hand side of the momentum equation [A.1]). Even though only constant source terms can be defined directly, FLUENT supports a
powerful mechanism of plug-ins called “user-defined functions” (UDF), allowing execution of external code (either interpreted or compiled from a C source), with extensive access to the inner data of the model and the solver. Using this mechanism, we defined the time-dependent source terms for each component of the momentum equation. Since the utilization of UDFs has a noticeable performance hit (decrease in calculation speed), we attempted to limit the number of external functions used. This was done by using the simplified form of the vibration-induced body force [2.2].

To create the finite-volume mesh (grid) for the FLUENT simulations, we utilized GAMBIT, an advanced CFD pre-processor designed for geometry description and meshing, all within a single user interface. Since the VOF model, with geometric reconstruction of the interfaces for a problem where interfacial tension is important, works best on quadrilateral (for two-dimensional problems) meshes because of the higher accuracy of the interfaces’ representation (Fluent, Inc., 2003), only this type of grid has been used. Because grid type and resolution can significantly affect both the quality of the results and the calculation speed, a short series of tests with various grid resolutions was performed for each of the simulations presented in this chapter.

5.2. Results and discussion

5.2.1. Behavior of interfaces in a two-dimensional pore body

In order to remedy the limitations of our pore-network model, we decided to utilize FLUENT to simulate the variety of fluid-fluid interface configurations and their evolution in a single pore, originally with the hope of improving the algorithm responsible for handling
the menisci within the pore bodies (see section 3.3). Since creating an explicitly three-dimensional mesh representing the geometry described in section 3.1.3 would make the model too slow, a two-dimensional geometry was chosen. While this representation results in somewhat different flow dynamics (in our pore-network model, the flow was primarily controlled by the shallow “depth” of the model), the fluid-fluid interface configurations should be similar.

Since the general form of the pore geometry and fluid parameters (viscosity, density, surface tension, and contact angle) have already been defined (see section 3.1.3), all we had to do was choose the proper set of methods and parameters defining the numerical algorithms in FLUENT.

The general mesh structure we used was generated using automatic mesh formation in GAMBIT, with a structured quadrilateral mesh in the pore throats, and an unstructured quadrilateral mesh in the pore body. Running several simulations with progressively more refined meshes, we found the optimal mesh resolution (Figure 5.1) sufficient to resolve fine details of the interfaces, yet coarse enough to allow running a high number of simulations in a limited timeframe. Using similar technique, we found that the optimal time step for the selected mesh was $10^{-5}$ seconds.

A variety of situations were modeled with this experimental setup. We started with the invasion of a non-wetting fluid (oil) from one or more of the four adjacent throats into the pore body, under various boundary conditions (defined as pressures in the in- and outlet pore throats). For example, Figure 5.2 shows the results of three simulations of the oil invasion into a pore body (as snapshots of the fluid-phase distributions taken at different times). In the first two simulations, the oil is injected from the pore throat on the left under a constant
pressure; in the first case (Figure 5.2a), the pressures in the outlet pores are the same, while in the second case (Figure 5.2b) the pressure in the end of the “top” pore throat was set slightly higher. A somewhat different type of invasion scenario, in which the oil is injected from two pore throats simultaneously (although under slightly different pressures) can be seen in Figure 5.2c.

A comparison between the results obtained using the pore-network model and the FLUENT simulations showed that, in general, algorithms written for the pore-network model performed very well in simulating the behavior of the menisci during the events of “oil invasion” (including more complex cases of oil entering the pore body from several throats at various rates). While details of the interface dynamics were not captured in the network model (and that has never been intended), the resulting configurations of the interfaces and capillary pressures in most cases were sufficiently similar (same number and general locations of the interfaces), thus requiring only minor corrections to the algorithm.

However, when it came to modeling the “retreat” of the oil from a pore body, we knew that our pore-network model was not handling such processes well. Thus, attempting to find some sort of a general classification scheme for the variety of “retreat” scenarios arising from the FLUENT simulations was our main priority.

We soon realized, though, that there were at least two reasons that would make such classifications practically implausible. First, the number of potential “scenarios” that could arise in the network model would be directly proportional to the number of pores and the time of the simulation, and using a set number of pre-set “scenarios” would likely be inadequate (it would only extend the average simulated time until problems appeared in the network model; it would not resolve the problem completely). Second, the configurations of
the interfaces during even rather simple "retreat" scenarios simulated in FLUENT were so complex that they could not be translated into the network-model algorithms while keeping the generally simple description of the menisci. Several examples of such behavior can be seen in Figure 5.3, where the progression of several "retreat" scenarios with slightly different boundary conditions is shown. Replacing the dynamic description of the interfaces during such events with discrete steps from the initial to final configurations would not be possible either, since such an approach can be applied only if the system can be considered quasi-static, whereas our original goal was fully dynamic treatment of all the processes in the model.

Thus, we can conclude that an abstract representation of the interfaces in a pore-network model will not allow for adequate modeling of highly dynamics processes (such as vibration) in a system involving a large number of pores and/or complex configurations of the interfaces. While a similar conclusion was reached after analyzing the performance of our pore-network model (described in Chapter 3), comparison with detailed CFD modeling let us ensure that this is not just the result of a particularly bad selection of rules representing the behavior of the interfaces.

5.2.2. Validation of the single-channel model

Another use of the CFD methods is the validation of the single-channel model described in Chapter 4. Due to the explicit consideration of the model geometry, as well as the capillary and inertial forces due to vibration within the known applicability limits (see section 4.1.5), the results of the modeling in Chapter 4 should be reliable. However, certain processes that have been neglected, such as film flow, deviations of the menisci shapes from
spherical in response to the inertial forces, or non-uniform flow-velocity distribution, as well
as somewhat stretched limits of applicability (especially the quasi-static assumption), leave
accuracy of the results presented in Chapter 4 uncertain. A confirmation of the model
against accurate simulations and determination of its limits of applicability is thus needed.

Implementing the single-channel model in FLUENT is an easy task because of the
axial symmetry of the problem, which enables a two-dimensional representation of the
geometry and flow solutions. A structured quadrilateral mesh, increasing precision of the
interface representation and solution stability, can be used and easily adjusted to
accommodate a higher resolution near the pore walls for the purposes of modeling the film
flow of the wetting fluid (water).

Our choice of the geometry was somewhat different from that used in Chapter 4 (see
section 4.1.2) and was defined with the following parameters: \( r_{\text{min}} = 20 \, \mu m, r_{\text{max}} = 60 \, \mu m, \) and \( l_p = 320 \, \mu m \) (see Figure 4.1). We decided to use a shorter pore body with a higher
difference between the minimum and the maximum radii because such a geometry should
generally result in higher velocity gradients and potentially larger effects of inertial forces, as
well as larger changes in the interface shapes, thus providing a more challenging testing
ground.

One of the assumptions made during the formulation of the single-channel model in
Chapter 4 was a relatively large length of the oil blob compared to the length of a single
pore. For the model in FLUENT, the calculation speed would be inversely proportional to
the length of the channel (assuming the same mesh resolution throughout the channel); thus,
a certain compromise in the selection of the channel length had to be used. We decided to
start with the length of six pores, which could be subsequently increased if necessary. Starting with a longer channel could result in unnecessarily long calculation times.

Also, in order to enable direct comparison, the density and viscosity of both wetting and non-wetting phases FLUENT were set equal, since the single-channel model in Chapter 4 ignored the wetting fluid outside the blob, assuming that the length of the blob was very close to the length of the channel. The following parameters were used in both models: $\rho = 1000 \text{ kg/m}^3$, $\sigma = 30 \text{ mN/m}$, the wetting contact angle of $0.1^\circ$, and different values of fluid viscosities.

Before the numerical experiment, a study of spatial and temporal resolution was conducted. The parameters considered were the critical pressure difference required to mobilize a ganglion and typical rates of the movement of an interface within the pore channel. The final mesh size for each pore was eight cells in the radial direction with a slight bias towards the pore wall for better resolution of the interface curvature, and 60 cells in the longitudinal direction, spaced evenly (illustrated in Figure 5.4). The selected time step was $2.5 \cdot 10^{-6}$ seconds.

Due to significantly slower speed of the FLUENT model, we could not run thousands of simulations for slight variation in ganglion volume to obtain statistical results similar to those presented in Chapter 4. Thus, only the behavior one ganglion volume was simulated. The volume of the ganglion placed in the pore channel (illustrated in Figure 5.5) was selected so that it exhibited a maximum capillary pressure difference [4.7] (see section 4.2.1), and therefore was the most difficult to mobilize.

There was a slight discrepancy between the capillary pressures in the FLUENT model and the single-channel model (that matched the theoretical estimate [4.2]). The FLUENT
model exhibited a slightly higher (about 4.5%) capillary pressure, so the ganglion shown in Figure 5.5 was mobilized under the pressure difference of 2090 Pa, whereas the theoretical calculations showed the maximum capillary-pressure difference this ganglion could exhibit was 2000 Pa. This discrepancy was the same for all levels of mesh refinement. To enable a meaningful comparison between the models, we adjusted the value of the respective pressure difference used in the single-channel model. For example, for comparison with the FLUENT run with the 1990 Pa pressure difference, the corresponding value for the single-channel model was 1900 Pa; the pressure difference reported hereafter corresponds to the single-channel model.

For the comprehensive comparison, a series of experiments was designed, sampling the frequency range from 5 to 100 Hz. The parameter for the comparison was the amplitude required for the mobilization of the oil blob under a given pressure difference applied between the inlet and the outlet of the channel. In both cases, the amplitude was found using an iterative approach. The computer code for the single-channel model already incorporated this option, providing the required mobilization amplitude with a precision of 0.1% or better. In FLUENT, a series of simulations was run, varying the acceleration amplitude of the applied body force until the mobilization threshold was bracketed within an acceptably small interval (within 5%).

Figure 5.6 shows the results obtained for the external pressure difference of 1900 Pa (100 Pa below the mobilization threshold) and viscosity of the fluids $\mu = 5 \times 10^{-3}$ Pa·s. There is a noticeable discrepancy of the results, which can be attributed to the effect of the interfaces on the flow patterns neglected in the single-channel model. The validity of this assumption can be clearly seen when we compare the results for a longer channel (twelve pore lengths,
same mesh density) shown in Figure 5.7, where both FLUENT and the single-channel model of Chapter 4 produce remarkably similar data. Because of the long calculation time in FLUENT with the longer channel, the remaining comparative simulations were run for the shorter geometry.

The results of a series of simulations with the lower external pressure difference (1500 Pa) are shown in Figure 5.8. In this case, the results of both the single-channel model and the FLUENT simulations match almost perfectly in the lower frequency range. At the higher frequencies, FLUENT shows noticeably larger required vibration amplitudes (similar trend can be seen in all plots). This can be explained by the breaching of the applicability limits (the quasi-static assumption) in this frequency domain in the single-channel model, discussed in section 4.1.5.

Figure 5.9 shows the dependence of the mobilization amplitude on the frequency for the fluid viscosity equivalent to that of water ($\mu = 10^{-3}$ Pa·s). As expected, the frequency dependence is much reduced in both models; the single-channel model shows slightly higher mobilization amplitudes, similar to the comparison under the high external-pressure difference (Figure 5.6).

In the simulation run with a higher fluid viscosity ($\mu = 5 \times 10^{-3}$ Pa·s), we did not observe any noticeable film flow or similar effects. For the low-viscosity experiments, however, film flow leading to the snap-off of the oil ganglion in the pore throat was found, but only for relatively high vibration amplitudes (none of the simulation results shown in Figure 5.9 exhibited the snap-off or perceptible film flow). An example of this behavior is shown in Figure 5.10. While in real porous media the film and crevice flows are likely to be of higher importance, the lack of proper geometric representations for crevices and other irregularities
on the pore wall makes our model incapable of adequately predicting this aspect of realistic flow. In addition, detailed investigations of film flow and ganglion break-up are beyond the scope of this comparative study.

Overall, the performance of the approximate single-channel model compared to the accurate FLUENT simulations was better than originally thought. The formation of a sufficiently thick water film on the pore walls, resulting in the accumulation of water in the pore throats and leading to the ganglion snap-off, was observed only for low-viscosity fluids and vibration amplitudes higher than those required for the mobilization of the most resistant oil blobs (and thus resulting in the highest flow velocities once the leading meniscus was past the pore throat). This additional applicability limit that does not noticeably interfere with the predictions for most realistic scenarios of vibratory-stimulation applications.

5.2.3. Two-dimensional system of interconnected pores

In the previous sections of this chapter, we described two different uses for the CFD in relation to modeling an immiscible flow in porous media, limited to a single pore body. Since so far we have been unable to effectively model the effect of vibration on a large scale, explicitly accounting for the pore interconnectivity, and using only abstract representations of the fluid flow and the fluid-fluid interface dynamics, an application of the CFD methods to larger numbers of pore is in order.

Even though in the time frame of this study the calculation speed of personal computers increased several-fold, detailed modeling of the dynamic multi-phase flow in a large number of pores is still unacceptably slow. However, if we consider the pore network (Chapter 3), we see that we are already sacrificing many small details; for example, we have
to consider a simplified pore geometry that does not explicitly account for film flow, we
approximate the flow rate in the pore throats, and we ignore pressure gradients within pore
bodies. Thus, we can argue that a similarly low-detailed CFD model might be acceptable for
simulating the general behavior of ganglia in geometries containing large numbers of pores.
To determine if such low-detail stimulations can be effectively run through FLUENT, we
conducted the following study.

We chose to model the porous medium as a two-dimensional structure, with pore
bodies located on a square grid and connected with converging-diverging throats. In contrast
with the geometry used in the pore-network model (see section 3.1.3), the converging-
diverging throats enable greater residual saturation of the non-wetting phase (ganglia are
more easily trapped). Also, this time the fluids differed in density and viscosity: the non-
wetting phase (oil) was more viscous and less dense ($\mu = 5 \cdot 10^{-3}$ Pa·s, $\rho = 800$ kg/m$^3$) than the
wetting phase (water; $\mu = 10^{-3}$ Pa·s, $\rho = 1000$ kg/m$^3$); the surface tension between the fluid
phases was set to $\sigma = 20$ mN/m, and the contact angle for the wetting fluid was 0.1°.

The first few simulations were conducted for the pore size (distance between centers
of adjacent pore bodies) of 2.0 mm, which is quite close to the scale of the pore-network
model and thus the etched-glass micromodel used by Li et al. (2005). An unstructured
quadrilateral mesh was generated in GAMBIT (Figure 5.11); a coarser mesh would not be
appropriate, since some of the smaller pore throats would have only one mesh element (cell)
across. On this mesh, the maximum time step that did not result in the lack of convergence
or obvious differences in the simulation results was $10^{-4}$ seconds. Even with these settings,
FLUENT demonstrated a rather slow-speed performance, calculating about 1 second of
model time in 24 hours of real time. This was sufficient to demonstrate the ganglia
formation and entrapment in our relatively small model (Figure 5.12a), but not very useful for studying the long-term effects of vibration. Due to the performance hit associated with using UDF, only short mobilization tests could be performed in a reasonable amount of time (an example of a ganglion mobilized by vibration can be seen in Figure 5.12b). Another potential problem was low resolution of the mesh in some of the more narrow throats. An increase in resolution, or even an addition of a boundary layer (providing higher mesh resolution near the pore walls) resulted in a significant degradation in speed performance.

Due to the lack of a proper laboratory-experiment counterpart (an experiment involving an etched-glass micromodel implementing the same geometry has been planned but not started yet), the decision was made to scale the model down in subsequent simulations, in order to bring the pore size to the scale of $10^{-1}$ mm, which better corresponded to the typical pore size of an oil-bearing sandstone. This would provide the results that could be related to real-world problems, as well as compared with those obtained using the single-channel model (Chapter 4).

The decrease in the linear scale resulted in a corresponding decrease in the time scale (time step), thus slowing the calculation speed even further. Since direct reduction in the resolution of the unstructured mesh (Figure 5.11) was not possible, a structured mesh was created. Such a mesh would result in a lower total number of mesh elements, while keeping the same relative level of refinement in all pores. This would allow for adequate representation of the interfaces in the pore throats, while significantly decreasing the mesh resolution in pore bodies (compared to an unstructured grid). This trade-off is nevertheless beneficial, considering that the flow is controlled by the smaller pore throats (in both flow
velocity and capillary pressures, if menisci are present), and not the pore bodies. The proper level of refinement was to be determined.

Figure 5.13 shows several levels of grid refinement considered, starting with as low as 2 mesh elements across the pore. A number of FLUENT simulations were then run to determine the lowest level of grid refinement that would result in an acceptable representation of the fluid flow and, more importantly, fluid-fluid interface behavior. The following two types of tests were conducted: static (comparing the capillary pressures created by the interfaces in the model) and dynamic (comparing the configuration of the interfaces at various times during the simulation).

The static test was performed on a simple model consisting of three pores (similar to those shown in Figure 5.13) connected in one line, thus forming a pore channel with two constrictions. The exact size of the pore was 0.2 mm. The non-wetting fluid was injected from the left inlet under a constant pressure; the pressure was then raised in small increments until the model reached an equilibrium between the external and the capillary pressures. The external pressure at which the interfaces would pass through the pore throat (and accelerate towards the next constriction) was the parameter of interest. The time step chosen for all these simulations was $10^{-6}$ seconds, sufficiently small for the finest grid resolution considered. The results of these numerical experiments are shown in Table 5.1. With the exception of the lowest grid resolution level (two cells per pore throat), the static capillary pressure created by the interfaces in the constrictions of the pore channel barely changed with the grid refinement.

The dynamic testing consisted of comparing the fluid-phase distribution at various times. Several “scenarios” in different model geometries were simulated and then compared.
A variety of comparison methods could be used: for example, calculating the fraction of the model volume (area) occupied by the non-wetting fluid would provide a measure of the dissimilarity between two simulations with different mesh sizes. In most of the tests conducted, this value varied in time but never exceeded 10%, and it did not always correlate with important differences in interface configurations. Thus, a simple visual comparison of the fluid distributions in the models was conducted. Because animation sequences of the simulations cannot be presented in this text, snapshots of fluid configurations at certain times will be shown to illustrate the differences between the simulation results.

The results for flow in a simple three-pore geometry used in the static tests are presented in Figure 5.14. The interface curvature for the lowest grid resolution tested appears to be inadequate, but the capillary pressures are nevertheless calculated properly by the FLUENT (this can also be seen from Table 5.1). Also, while the rate of the oil inflow into the system varies with the grid resolution, these variations are minor (did not exceed 8%). An example of a comparison in a more complex geometry can be seen in Figure 5.15, where an “oil invasion” scenario is illustrated. Several other scenarios have also been simulated, and in all cases the results obtained with the lowest mesh resolution considered (level 3) were remarkably similar to those obtained with higher mesh resolution, with very few exceptions creating the same general interface configuration but minor (under 10%) variations in the flow rates. Based on the results of this extensive testing, we can make an educated decision on the grid resolution optimized for speed.

When modeling a large number of pores, the precision of the events on the scale of a single pore is of relatively little importance. Based on the results of grid-resolution tests presented above, we can say that, if using the mesh-resolution level 3, we could expect an
error in the representation of the interfaces in local pores to be, on average, less than 10% in terms of their positions (compared to the more detailed representation of the model domain by a finer mesh) on the timescale typical for oil invasion/retreat processes in one to several pores. If our goal were detailed modeling of the real processes in the exact geometry given, such an error would likely be unacceptable. Fortunately, our aim has been different – we are trying to represent processes in real porous media; by assuming the geometry of the pores seen in Figure 5.16, we already introduce significant errors in the results of the simulations. The only systematic error observed due to the lowering of mesh resolution to level 3 was in the formation of water film; however, because our model is two-dimensional, it will not exhibit a noticeable film flow, nor will it show the snap-off of the non-wetting phase in pore throats (the interface curvature will always correspond to the wall curvature, leading to the spreading of the wetting fluid thinner along the wall). Thus, we can conclude that using the mesh-resolution level 3 is sufficient for our purposes.

Another important parameter of our numerical simulations is the time step. Although we conducted a time-step sensitivity study for various mesh resolutions, we were most interested in the results for the coarsest acceptable mesh. An example of the comparison between the results obtained at different values of the time step is presented in Figure 5.16. Based on these results, as well as on a number of other simulations, we concluded that for the time steps below $5 \times 10^{-6}$ seconds all simulations were practically identical. For the values of time step between $5 \times 10^{-5}$ and $5 \times 10^{-6}$, certain differences in the results could be observed; however, cases of significantly different interface configurations (like that shown in Figure 5.16 for the time step of $10^{-5}$ seconds) were rare, and similar differences could originate from just a few-percent variation in the boundary conditions (inlet and outlet pressure). As time
step approached $10^{-4}$ seconds, the modeled fluid interfaces lost stability, showing oscillations and breakup without apparent cause. Following the same principles outlined above, we argue that a value of the time step of $5 \cdot 10^{-5}$ seconds is adequate for our modeling goals.

Based on the estimates for the lowest acceptable mesh resolution obtained, we constructed a model geometry illustrated in Figure 5.17, with mesh level 3 in each pore (see Figure 5.13). However, a regular arrangement of pores provoked formation of very stable displacement fronts in simulations of water injection, and, consequently, almost no residual saturation. To resolve this problem, some of the pore throats were blocked off, preventing the formation of large interfaces spanning many pores. Figure 5.18 shows the results of the simulation of oil displacement by water in the amended geometry; the higher tortuosity of the pore space resulted in a higher residual saturation and formation of several isolated oil blobs. To test the stability of these residual blobs, a series of simulations that included an oscillating body force (applied in the direction of the flow in the model, left-to-right) were conducted. The vibration amplitude required for the mobilization of these ganglia was between 250 m/s$^2$ and 350 m/s$^2$ because of the relatively small ganglion length and low external pressure gradient. Obtaining a precise estimate of the mobilization amplitude for each ganglion would require too many additional “iterations” (simulations runs with different vibration amplitudes), and thus was not attempted.

A different scenario was developed to demonstrate the effect of vibrations on the residual saturation of the non-wetting fluid. In the model, originally saturated with water, several large oil ganglia were manually inserted, and an external pressure difference of 1100 Pa applied by setting the appropriate boundary conditions. These initial and boundary conditions were then used in several simulations: one “control” run, with pressure difference
as the only driving force, and several runs with addition of an oscillatory body force. The results obtained in the “control” run and one with the vibratory stimulation enabled are illustrated in Figure 5.19. A comparison of fluid configurations in these two simulations at various times can be seen in Figure 5.20, where the relative oil saturation (the ratio of the volume occupied by oil to the total volume of the model) is plotted versus time for several simulations with various parameters of vibratory stimulation. The effect of vibration can be clearly seen in the faster removal of oil from the model, as well as in the lower final residual saturation.

Although the optimization of the numerical mesh and the time step allowed us to simulate noticeably larger numbers of pores in a shorter timeframe, the resulting calculation speed is still insufficient to obtain statistically viable results for variations in ganglion size, pore geometry, or even an estimation of the required vibration amplitude. While shorter tests for the stability of a given ganglion configuration can be performed in as little as 10-30 minutes, simulations of the ganglion movement across the model space may take from 10 hours to several days. The usefulness of such modeling can then be compared to the small-scale laboratory experiments such as conducted by Li et al. (2005); although simplified, our numerical simulations provide results in a comparable timeframe and have a significantly higher control over the exact parameters, including the easy-to-change geometric parameters of the pores.
5.3. Summary

Based on our FLUENT numerical simulations of the behavior of interfaces in a pore body resembling those used in the pore-network model in Chapter 3, we conclude that single-pore CFD modeling does not appear to be possible for the purposes of tabulating the dynamic interface configurations needed for improving the interface description in the pore-network approach. This results from the high number of possible scenarios that can arise in a large system and, importantly, from the very complex behavior of the interfaces in the pore body that cannot be formalized by a pre-determined set of “scenarios” without sacrificing the dynamic formulation.

The use of FLUENT to validate the processes modeled in the single-channel approximation (Chapter 4) proved the general validity of the method, even for the vibration frequencies bordering the quasi-static assumptions (section 4.1.5). We have also demonstrated that, even though CFD calculations can be effectively used for the purpose of modeling the mobilization of a single oil ganglion in the pore channel, obtaining extensive statistical data (e.g., mobilization parameters for various geometries of the channel, or even for a large number of different volumes of the oil blob within the same channel) would still be beyond practical feasibility. This is because of the still relatively long calculation time, which amounts to hours and days for a single simulation run, such as one required to test the mobility of a ganglion under a given frequency and amplitude, and a rather tedious process of preparation of mesh data for the simulation.

In section 5.2.3 of this chapter, I demonstrated that careful optimization of the mesh resolution and the time step allows using FLUENT for simulating a reasonable number of
interconnected pores in the timeframe of less than a day on an up-to-date personal computer. This type of model can be useful in determining certain features of the effect of vibration on isolated entrapped ganglia in a particular pore geometry (e.g., the mobilization threshold, or an effective duration of the vibratory simulation), provided someone is willing to invest a significant amount of computer time in these simulations. The usefulness of these results for the potential applications of the vibratory-stimulation technology will likely be limited because of the simplified model geometry, the small scale of the model, and the two-dimensional representation of the pore space.

Overall, I conclude that the direct use of CFD software for modeling the effect of vibration on fluid flow in large number of pores is unlikely to provide answers and estimates applicable to real-world utilization of the technology, primarily due to the slow calculation speed for even two-dimensional models of a relatively small number of pores. This conclusion is based on the assumption of using only a single personal computer for the simulations; the availability of significantly greater computational resources might bring us closer to the fulfillment of this goal.
Figure 5.1. The finite-volume two-dimensional mesh used in FLUENT simulations of the interfaces’ behavior in two-dimensional pores.
A. Symmetric oil invasion

B. Asymmetric oil invasion

C. Asymmetric oil invasion from two pore throats

Figure 5.2. Results of the simulations of several “oil invasion” scenarios; darker color indicates the non-wetting phase. Pressures in the pore throats are 0 Pa, unless otherwise indicated. Time elapsed between subsequent frames is 0.05 seconds in all three scenarios.
A. Slight pressure asymmetry

B. Moderate pressure asymmetry

C. Significant pressure asymmetry

Figure 5.3. Results of the simulations of several “oil retreat” scenarios; darker color indicates the non-wetting phase. Pressures in the pore throats are 0 Pa, unless otherwise indicated. Time elapsed between subsequent frames is 0.05 seconds in all three scenarios. Note that the capillary pressure created by the interfaces in the pore throats is on the order of 20 Pa.
Figure 5.4. Illustration of the numerical grid used in the FLEUNT simulations for comparison with the single-channel model (only one of six pores is shown).
Figure 5.5. Distribution of fluids in the FLUENT model under the external pressure difference of 1990 Pa. Oil (non-wetting fluid) is shown in dark color.
Figure 5.6. Acceleration amplitude required for mobilization of the short (6 pore lengths) ganglion under the external pressure difference of 1900 Pa. Fluid viscosity is \( \mu = 5 \cdot 10^{-3} \text{ Pa} \cdot \text{s} \).
Figure 5.7. Acceleration amplitude required for mobilization of the longer (12 pore lengths) ganglion under the external pressure difference of 1900 Pa. Fluid viscosity is $\mu = 5 \times 10^{-3}$ Pa·s.
Figure 5.8. Acceleration amplitude required for mobilization of the short (6 pore lengths) ganglion under the external pressure difference of 1500 Pa. Fluid viscosity is $\mu = 5 \times 10^{-3}$ Pa·s.
Figure 5.9. Acceleration amplitude required for mobilization of the short (6 pore lengths) ganglion under the external pressure difference of 1900 Pa. Fluid viscosity is $\mu = 10^{-3}$ Pa·s.
Figure 5.10. Illustration of film flow and snap-off of the oil blob observed in a FLUENT simulation (only part of the 6-pore channel is shown). Non-wetting phase (oil) is shown in dark color. External pressure difference is 1900 Pa, fluid viscosity is $\mu = 10^{-3}$ Pa·s, vibration amplitude and frequency are $A = 60$ m/s$^2$ and $f = 5$ Hz.
Figure 5.11. The *unstructured* quadrilateral mesh used in FLUENT simulations. The boundary conditions are defined by the pressures in the inlets (left-most pore throats) and the outlets (right-most pore throats) of the model.
Figure 5.12. Illustration of capillary entrapment and mobilization in the model with unstructured mesh. Non-wetting fluid is shown in dark color. External pressure difference is 67 Pa. A: entrapped oil ganglion, model time = 0.75055 seconds; B: mobilized ganglion ($f = 10$ Hz, $A = 5$ m/s$^2$, left-to-right), model time = 1.5321 seconds.
Figure 5.13. Illustration of the structured quadrilateral grid and levels of grid refinement.
Figure 5.14. Results of the mesh resolution study. Oil (dark) is injected from the left side, external pressure difference is 1100 Pa. The current model time is $3 \cdot 10^3$ seconds after the start of simulation.
A. Model time = $3 \cdot 10^{-3}$ seconds

B. Model time = $8 \cdot 10^{-3}$ seconds

Figure 5.15. Results of the mesh resolution study. Oil (indicated in dark color) is injected from the inlets (indicated with dark solid arrows) under pressure of 1150 Pa; the pressure in the outlet (indicated with white arrow) is zero.
A. Model time = $5 \cdot 10^{-3}$ seconds

B. Model time = $6 \cdot 10^{-3}$ seconds

C. Model time = $10^{-2}$ seconds

Figure 5.16. Results of the time step sensitivity study; dark color indicates oil (non-wetting fluid). Water is injected from the left and right inlets under pressure of 300 Pa; the oil flows out the top and bottom outlets (zero relative pressure).
Figure 5.17. Illustration of the model geometry and numerical grid constructed based on the mesh resolution sensitivity study.
Figure 5.18. Results of the simulation of oil (shown in dark color) displacement by water. External pressure difference is 200 Pa.
Figure 5.19. Results of the simulation of oil ganglia (shown in dark color) stability under external pressure difference of 1100 Pa. Vibration of frequency $f = 10$ Hz and amplitude $A = 100$ m/s$^2$ is applied (C, D) after the model reached equilibrium (all oil is immobile) at time $= 1.0$ sec.
Figure 5.20. Relative oil saturation in the model as a function of time for several simulations with the same initial conditions (see Figure 5.19) and various amplitudes of applied vibrations. External pressure gradient is 1100 Pa, Vibration frequency is \( f = 10 \) Hz.
Table 5.1. Results of the mesh resolution study, static case. Pressure intervals indicated bracket the exact mobilization pressure. Model geometry and mesh resolution levels are illustrated in Figure 5.13.

<table>
<thead>
<tr>
<th>Mesh resolution level</th>
<th>Maximum pressure in first constriction, Pa</th>
<th>Maximum pressure in second constriction, Pa</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>600 – 700*</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td>895 – 900</td>
<td>1375 – 1380</td>
</tr>
<tr>
<td>6</td>
<td>875 – 880</td>
<td>1365 – 1370</td>
</tr>
<tr>
<td>12</td>
<td>880 – 890</td>
<td>1340 – 1350**</td>
</tr>
<tr>
<td>24</td>
<td>890 – 900</td>
<td>1350 – 1375**</td>
</tr>
</tbody>
</table>

* Results inconsistent; no further testing conducted
** Formation of wetting fluid film observed
CHAPTER 6. CONCLUSIONS

6.1. Summary

In this work, I presented the result of three separate studies, utilizing different numerical techniques to investigate the effect of vibrations on two-phase immiscible flow in porous media. One of the most important aspects of these studies was the attempt to obtain simulation results that could be related to real-world application of the vibratory stimulation of oil reservoirs or contaminated aquifers. This has been approached by avoiding abstract dimensionless representations; a careful consideration of all real parameters involved in the models, such as the geometry, fluid viscosity, and surface tension, was employed instead.

The pore-network model, described in Chapter 3, was meant to be a compromise solution that would include the dynamic consideration of the capillary entrapment and vibratory mobilization in a form that would allow fast calculation speed, thus enabling large scale (thousands of pores) simulations of ganglion formation and fate. While the model has been capable of demonstrating both the trapping and the vibratory mobilization of the residual non-wetting phase for small isolated ganglia, the choice of the algorithms for the representation of the fluid-fluid interfaces (menisci) led to a calculation speed noticeably below our original expectations, and, more importantly, to the inability of the model to properly handle large numbers of interfaces. Because the model was based on the fully-dynamic representation of the menisci movement in the pores, attempts to calibrate certain aspects of the menisci behavior simply to “fix” the observed inconsistencies would lead to the results defined by the calibration process, and not the underlying capillary mechanism.
We attempted to remedy this situation by modeling the exact dynamic behavior of the interfaces in a single pore using the considered software package FLUENT. A high number of possible scenarios have been discovered, and the results affirmed our conclusion that using a tabulated set of parameters to define the dynamics of the interfaces is not compatible with the fully dynamic formulation; on the other hand, implementing a detailed dynamic modeling of interface behavior would result in an unacceptably low calculation speed.

The most productive outcome of this work has been the creation of the so-called single-channel model (Iassonov and Beresnev, 2005), described in Chapter 4, designed to simulate the movement of an oil blob in a long pore channel with periodic constrictions. It used a converging-diverging geometry of the channel, as opposed to the straight “capillary tubes” considered in the number of previous studies, such as those of Hilpert et al. (2000), Graham and Higdon (2000a), or Iassonov and Beresnev (2003), critical for the correct description of the phenomenon of capillary trapping. This model has also been designed to predict the behavior of a large oil ganglion, not a small droplet of the non-wetting fluid (Graham and Higdon, 2000b). Such a representation of the non-wetting phase is much more appropriate for the residual oil in the reservoirs. Most importantly, our model can estimate the effect of vibration for a variety of parameters of the channel geometry and the pore-filling fluids in a very short time. The validity of the assumptions and the accuracy of the results were validated by comparing the latter with the exact hydrodynamic simulations in FLUENT. The downside of the single-channel model is, quite obviously, its inability to model the interconnectivity of the pores.

The calculated effects of the amplitude and frequency agree with expectations from the underlying mobilization mechanism (Chapter 2). In terms of practical applications, the
model can be used to estimate the absolute volumes of oil mobilized by given amplitudes and frequencies of vibration, provided sufficient information on pore-size and ganglia-size distribution of the porous reservoir is available. Calculations show that, for the parameters of the model corresponding to typical oil-bearing formations, the vibratory amplitudes required to achieve a noticeable mobilization and increase in the oil flow rate are on the order of $10 \text{m/s}^2$. Even higher amplitudes are needed at the frequencies in excess of 10 Hz, which is not realistically achievable with the available borehole seismic sources. This does not however apply to the possibility of vibratory stimulation of near-surface contaminated aquifers, in which necessary sonic energies can be created from the surface.

In naturally occurring porous media, one should expect to find significant variations not only at the pore scale (such as variations in pore sizes), but also on the macroscopic scale: heterogeneity and fracturing in sedimentary formations can be defining factors for the flow in a porous medium, thus leading to high variations in the local pressure gradient. Such variations are not considered in the estimates presented above. Thus, pore-scale models must be used in conjunction with the analysis of the flow on a macroscopic scale in order to properly estimate the effect of vibration in any particular field or laboratory application.

An extensive analysis of the applicability of the CFD package FLUENT to modeling the processes of capillary trapping and vibratory mobilization has also been performed. Based on this work, I have concluded that, while being able to provide illuminating results on the ganglion behavior in relatively small, two-dimensional system, CFD simulations using FLUENT will not be able to give answers useful for realistic field applications of the technology, primarily due to the small scale of the models and the low calculation speed. Utilization of supercomputers or similar computational resources could enable simulations of
large number of pores in a three-dimensional environment; this possibility has not become practical yet.

6.2. Recommendations for future studies

The work presented in this dissertation helped us better understand the physical processes responsible for trapping and mobilization of non-wetting fluids in porous media, as well as defined areas where additional work is needed. The following directions for future studies can be identified.

For experimental research on a laboratory scale involving realistic porous media (e.g., sandstone core samples), better documentation and elaboration of the conditions for effective mobilization is required, primarily in terms of the parameters of the vibrations for given external pressures, in order to directly correlate the experimental results with field data.

Small-scale experiments with artificial porous media (e.g., work by Li et al., 2005) can be useful to observe the dynamics of the ganglia formation and trapping. However, since the general mobilization effect of vibration has been confirmed both theoretically and experimentally, the challenge in such experiments now shifts to correlating the observed results with the processes in real porous media.

For the numerical modeling work, the challenges lie in the simulations of a sufficiently large porous domain able to supply more precise predictions than those presented in this dissertation. Considering the heterogeneity of real oil-bearing formations, numerical models will have to be flexible and fast, in order to be able to provide estimates for a wide
variety of parameters. The utilization of CFD software is unlikely to reach this goal at the
time of this writing, since it would require unreasonably extensive computational resources
and long computational times. Thus, more abstract models have to be developed.

One of the methods for modeling a large number of pores could be in the application
of the same pore-network approach outlined in Chapter 3 but with quasi-static flow
formulation. Instead of attempting to model the interface dynamics in detail, this model
could use the criteria for an interface passing through a pore throat obtained using a more
accurate approach, such as the single-channel model presented in Chapter 4, which could be
tabulated. Combined with a somewhat simplified treatment of the fluid saturation in pores,
such a model could be extremely fast, thus enabling simulation of a large number of pores
even in three-dimensional topology. Previously, this approach was not possible, because the
dynamics of the interfaces in the pore constrictions under the effect of vibration was unclear.

Another interesting direction of investigation is the study of the effect of minor
features of pore geometry (e.g., rough walls, crevices, angular geometry of the channel, etc.)
on the mobilization potential of vibration on an interface in the pore constriction. This
modeling can be performed using the existing CFD software, and the results can be
integrated as correction factors to the single-channel model.

In addition to the pore-scale modeling, it would be beneficial to attempt to obtain
analytical formulation of the effect of vibration on a macroscopic scale. This can be done by
not only increasing the size of the numerical models, but also analysis and integration of the
results obtain for small-scale models with large-scale considerations of the flow in porous
media, including such macroscopic factors as heterogeneity and fracturing.
We assume that the pore is filled with an incompressible Newtonian fluid characterized by the density $\rho$ and viscosity $\mu$. In this case, ignoring the energy transfer, the fluid flow will be completely defined by two equations (Johnson, 1998): the momentum-conservation (Navier-Stokes) equation,

$$\rho \left( \frac{\partial U}{\partial t} + (U \cdot \nabla)U \right) = -\nabla P + \mu \nabla^2 U + \rho g,$$  \[A.1\]

where $U$ is the flow velocity and $g$ is gravity, and the mass-conservation condition

$$(\nabla \cdot U) = 0.$$  \[A.2\]

The velocity of the fluid in the moving frame of reference of the pore wall is

$$u = U - w,$$  \[A.3\]

where $w$ is the velocity of the pore wall. Substituting [A.3] into [A.1], taking into consideration that $w$ depends only on time and not on spatial location (i.e., the wall moves in a "rigid" manner; see section 4.1.4 for the justification of this assumption), we arrive at

$$\rho \left( \frac{\partial u}{\partial t} + (u \cdot \nabla)u \right) = -\nabla P + \mu \nabla^2 u + \rho g + B,$$  \[A.4\]

where

$$B = -\rho \left( \frac{\partial w}{\partial t} + (w \cdot \nabla)w \right)$$  \[A.5\]

is the effective body force, acting on the fluid in the frame of reference of the wall. Comparing [A.4] and [A.1], we see that they are identical but for the body force [A.5].
In modeling the effect of vibration throughout the dissertation, we ignored the components of the body force \([A.5]\) proportional to the local fluid-velocity gradients \((-\rho(\mathbf{w} \cdot \mathbf{V})\mathbf{u})\). In order to justify this assumption, we need to estimate the magnitude of the neglected terms relative to the other terms of the momentum equation. Since we also assume a laminar flow, the contribution of the viscous terms in equation \([A.1]\) should provide an adequate scale for the comparison. We will use the pore geometry defined in section 4.1.2. Postulating that the vibration applied to the porous medium is a harmonic oscillation, defined by its acceleration amplitude \(A\) and frequency \(f\), we can, using \([A.4]\) and \([A.5]\), write the criteria for neglecting the aforementioned terms of the body force in polar coordinates as

\[
\frac{A}{2\pi f} \frac{\partial u_z}{\partial z} << \mu \left( \frac{\partial^2 u_z}{\partial z^2} + \frac{\partial^2 u_z}{\partial r^2} \right),
\]

\[
\frac{A}{2\pi f} \frac{\partial u_r}{\partial z} << \mu \left( \frac{\partial^2 u_r}{\partial z^2} + \frac{\partial^2 u_r}{\partial r^2} \right),
\]

where \(u_z\) and \(u_r\) are the axial and radial components of the fluid velocity, respectively. The derivatives in \([A.6]\) and \([A.7]\) can be estimated by simple scaling considerations,

\[
\frac{\partial u_z}{\partial z} \sim \frac{u_{\text{max}}}{l_p},
\]

\[
\frac{\partial^2 u_z}{\partial z^2} \sim \frac{u_{\text{max}}}{l_p^2}.
\]

Here \(u_{\text{max}}\) is the highest absolute value of the fluid velocity in the pore. The radial velocity \(u_r\) can be estimated from the mass-conservation equation \([A.2]\), which will yield

\[
\frac{\partial u_z}{\partial z} \sim \frac{\partial u_r}{\partial r}.
\]
From [A.8] and [A.10], we obtain

\[ \frac{u_z}{u_x} \sim \frac{r_{\text{max}}}{l_p}. \]  

[A.11]

Using [A.8-A.11], [A.6] and [A.7] become identical and can be written in the form

\[ \frac{A}{2\pi f} \ll \mu d_p \left( \frac{1}{l_p^2} + \frac{1}{r_{\text{max}}^2} \right). \]  

[A.12]

The right-hand side of [A.12] can be calculated by substituting the parameters of the pore-geometry introduced in section 4.1.2 and oil viscosity of \( \mu = 5 \) mPa·s. The resulting value is 1010 m/s, which should then be compared to the left-hand side of [A.12], corresponding to the velocity amplitude of wall oscillation. Considering the extreme case of ultra-low frequency, we obtain that, for the frequency \( f = 0.005 \) Hz, the acceleration amplitude \( A \) must be significantly smaller than 30 m/s²; for higher frequencies, the maximum acceptable acceleration amplitude will also rise (e.g., for \( f = 1 \) Hz, \( A \ll 6300 \) m/s²). These conditions are met in all calculations presented in the dissertation; even more so, in order to sustain vibration that would approach such values of displacement velocity in just one cubic meter of a typical sandstone, a power on the order of several terawatts \( (10^{12} \) W) would be required, which is about a kiloton TNT-equivalent every second. Such energies are not likely to be used on a large scale for the purpose of improving oil production.

Thus, we can assume that using a simplified form of the vibratory-induced pressure gradient for modeling the flow of oil in the pores of a typical oil-bearing rock is justified for all imaginable applications of sonic/seismic stimulations techniques.
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