INTRODUCTION

There has been a renewed interest in the study of elastic properties of anisotropic materials in recent years, particularly with the wide usage of custom made advanced composites in many aspects of aerospace structures. These composites are heterogeneous and anisotropic materials. Non-destructive evaluation of composites is highly desirable, both for defects characterization and mechanical properties. Ultrasonic methods are especially well suited for determination of the elastic properties of materials.

The most common technique used to determine the elastic constants of anisotropic materials from ultrasonic wave speed measurements requires cutting the material into a number of samples such that particular symmetry directions can be accessed for normal incidence wave speed measurements [1,2]. This is a destructive technique and is not feasible for thin or inhomogeneous anisotropic materials. A truly nondestructive technique is needed. Recent work along these lines has addressed composite materials using ultrasonic immersion techniques [3-7]. However, these methods have been limited to measurements in symmetry planes. Due to this limitation, all of the elastic constants cannot be obtained by this technique alone.

In this paper, we present a general and truly nondestructive method for determining all of the elastic constants for an anisotropic material of any symmetry class. The method is based on ultrasonic wave propagation along arbitrary directions in a material using a pitch-catch immersion technique with oblique angles of incidence. Although our ultimate goal is to make measurements on composite materials, we have chosen to work initially with homogeneous, anisotropic single crystals in order to validate our mathematical and experimental analysis before embarking on heterogeneous, anisotropic composites.

DETERMINATION OF THE PHASE VELOCITY

It is well known that, in general, for an ultrasonic wave incident at an arbitrary oblique angle on an anisotropic material, three wave modes (one quasi-longitudinal and two quasi-shear) are generated in the material. The direction that the energy flows in the material for each mode is along its
Poynting (energy) vector. Associated with each of these modes is a phase vector. All three of the phase vectors lie in the plane formed by the normal to the material surface and the incident beam. Although the phase vectors are constrained to this plane, the Poynting or energy vector, to are not constrained in such a manner. The magnitude of the energy vectors, \( V_e \), are related to the magnitude of their corresponding phase vectors \( V_p \) by the relation \( V_e = V_p \cos \psi \), where \( \psi \) is the angle between the corresponding energy and phase vectors.

Using an immersion technique (i.e., where the specimen, transmitting and receiving transducers are immersed in a fluid couplant), for a parallel plate specimen, the refracted waves are transmitted from the material into the fluid from the points where the energy vectors impinge on the material-fluid interface. If one observes the transmitted waves in a plane normal to the direction of the incident beam, in general, the points of intersection of the waves with the plane are not colinear.

It is the propagation of energy that can be observed and measured. However, it is very difficult to analyze the immersion technique for anisotropic materials in terms of the energy vector. One possible way would require that the point where the energy is transmitted into the fluid be known, in addition to the travel time. This is a difficult measurement to make at small angles of incidence, due to the small spatial shift and finite size of the ultrasonic beam. It is much less difficult to analyze the immersion technique for anisotropic materials in terms of the phase vector. Figure 1 shows the reference coordinate system used for this analysis. The following analysis for the phase velocity and phase direction is valid for any symmetry class, including triclinic.

Although the analysis for the phase vector is true for all symmetry classes, the method for determining the elastic constants from the measured phase velocities ('inversion routine') being used in this work is currently restricted to orthorhombic symmetry. The material symmetry directions are assumed to be known for the materials studied and the reference axes are set to coincide with the material symmetry axes.

It has been shown in an earlier report [11] that the time of flight along the phase path is equal to the time of flight along the energy path. Consequently, one can obtain the phase velocities needed for determination of the elastic constants by measurement of the propagation time along the energy paths. For arbitrary oblique angles of incidence on anisotropic materials, the magnitude of the phase velocity for the immersion arrangement presented in Fig.2 has been shown to be [11]

\[
V_p(\theta, \phi) = \frac{\Delta t(\theta, \phi)}{d} - 2 \frac{\Delta t(\theta, \phi) \cos \theta}{d \nu_w} + \left( \frac{1}{\nu_w} \right)^2 \]

(1)

where \( d \) is the thickness of the plate, \( \theta \) and \( \phi \) are the spherical polar angles of incidence, \( \nu_w \) is the phase velocity in couplant fluid (generally water),

\[
\Delta t(\theta, \phi) = t_w - t_{w+m}(\theta, \phi)
\]

(2)

where \( t_w \) is the propagation time for the pulse from transmitter to receiver in the fluid only and \( t_{w+m}(\theta, \phi) \) is the propagation time with specimen inserted in the fluid path between the transducers. Pearson and Murri obtained the same results for the case of oblique angle of incidence but restricted their analysis to planes of symmetry.[12]

The direction cosines of the phase vector in the material \((l_x, m_x, n_x)\) can be
INCIDENT BEAM
AXES COINCIDE WITH MATERIAL SYMMETRY
\( \theta \) IS "TILT"
\( \phi \) IS "ROTATION"

Fig. 1 Coordinate system used for analysis.

Fig. 2 Schematic diagram of wave at oblique incidence angle through a parallel plate. E and P' are the energy and phase vectors.

written in terms of the incident direction cosines \((l_1, m_1, n_1)\) and an unknown angle \(\theta_r\) as

\[
\begin{align*}
1_r &= l_1 \left\{ (1 - \cos^2 \theta_r) / (l_1^2 + m_1^2) \right\}^{1/2} \\
m_r &= m_1 \left\{ (1 - \cos^2 \theta_r) / (l_1^2 + m_1^2) \right\}^{1/2} \\
n_r &= \cos \theta_r
\end{align*}
\]

(3)

where

\[
\begin{align*}
l_1 &= \cos \phi_1 \sin \theta_i \\
m_1 &= \sin \phi_1 \sin \theta_i \\
n_1 &= \cos \theta_i
\end{align*}
\]

(4)

Since this is an analysis based on the phase vector, one can determine \(\cos \theta_r\) from Snell's Law,

\[
\cos \theta_r = \sqrt{1 - \left( \frac{V \omega}{V P_\omega \phi_i} \right)^2 \sin^2 \theta_i}.
\]

(5)

All of the information about the phase vector in the material for a particular set of incidence angles is now known, Eq. 1 gives the magnitude and Eqs.3-5 give the direction. For every angle of incidence, in general, three separate phase velocities and corresponding directions can be determined. However, it is usually not possible to observe all three wave modes because of low signal amplitude (due to mode conversion and the magnitudes of the reflection and transmission coefficients) and the ability to spatially and temporally resolve the wave modes.
Once the phase velocity and direction cosines have been obtained, the elastic constants can be determined from an 'inversion routine', such as described in [9], or a similar method. The 'inversion routine', presented in detail in [9], is based on an over-determined Newton-Rhapson method using the characteristic polynomial obtained from Christoffels' equation. This inversion routine does not require that the identification of the wave mode (quasi-longitudinal, or which of the two quasi-shear) the measured phase velocities corresponds to, as would be required if the routine were based on the general solution for the phase velocities. The inversion routine requires a minimum number (27 in our case) of data points (phase velocity vectors), the only other information needed is the density of the material.

EXPERIMENTAL SYSTEM AND MEASUREMENTS

Shown schematically in Fig. 3 is the experimental arrangement used in this work. Placed on either side of the specimen were two matched transducers (center frequency of 20 MHz), one is the transmitter and the other a receiver. The position of the transmitter was fixed along the center of the specimen while the receiver coordinates (x,y,z) were controlled by computer. The angle of incidence \( \phi_i \) was varied manually and \( \theta_i \) was computer controlled, respectively. The heart of the measurement system was a versatile (2 GHz) digital signal analyzer, interfaced with the computer.

The fluid velocity was measured by first aligning the transmitting and receiving transducers and placing them at an arbitrary distance apart. The pitch-catch time of flight in this position was measured (through the fluid only). The transducers were then moved towards or apart by a known amount. The time of flight was again measured. From the change in time of flight and known path difference, an accurate fluid velocity was determined \( (V_w) \).

At this point the transducers were either left in place or moved to new positions and a reference time of flight \( (t_{w}) \) measured. The specimen was then placed on a goniometer stage at a point in-between the transducers. The specimen axes and goniometer axes were aligned and set at known angles \( (\theta_i', \phi_i') \), with respect to the incident beam. For each such orientation, the receiving transducer was scanned in a plane normal to the emergent beam to maximize the received signal. The time-of-flight \( (t_{m+w}(\theta_i', \phi_i')) \) for each set of incidence angles \( (\theta_i', \phi_i') \) was then measured. Random errors due to vibrations were minimized by averaging. The difference in the travel time

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**Fig. 3** Experimental arrangement for determination of time-of-flight.
\( \Delta t(\theta_i, \phi_i) \) is determined from Eq. 2. Using the measured specimen thickness, \( \Delta t(\theta_i, \phi_i) \) and \( (\theta_i, \phi_i) \), the phase velocity and directions cosines for each measurable wave mode were determined for each set of angles of incidence.

CHOICE OF MATERIAL

As stated earlier, homogeneous anisotropic materials were chosen over heterogeneous anisotropic materials to check both the analysis and measurement method. The inversion routine used in this work, in its current form, is applicable to materials of cubic, tetragonal or orthorhombic symmetry. Further restrictions to our choice of materials were the physical size and geometry of available single crystals and the solubility of these materials in the immersion fluid. One of the easiest fluids to work with is water, but it is a solvent for many materials. With these constraints, we selected a single crystal of spinel (aluminum magnesium oxide), which has cubic symmetry. The specimen geometry was a parallel plate, approximately 1 cm thick, with the thickness direction of the crystal along the [110] direction. This particular orientation was chosen for a number of reasons. All three of the elastic constants of the material could be determined from the one specimen by contact methods. Because of the particular orientation of the specimen, it would 'appear' to the ultrasonic waves as if it were a tetragonal material. Thus, the inversion routine would be tested with a material of tetragonal symmetry. Due to large differences in shear wave velocities (6.57 km/s and 4.23 km/s) for propagation along the [110] direction, this was an ideal anisotropic material to study.

RESULTS AND DISCUSSION

To check the experimental system, measurements were made on a fused quartz specimen. The fused quartz (homogeneous and isotropic) specimen was a circular plate 0.984 cm thick. Figures 4a and 4b show the time of flight data for propagation of the longitudinal wave \( \theta_i = 5^\circ, \phi_i = 0^\circ \rightarrow 360^\circ \) (Fig. 3a) and shear wave \( \theta_i = 18^\circ, \phi_i = 0^\circ \rightarrow 360^\circ \) (Fig. 3b) through the fused quartz plate. As expected since this material is isotropic, the polar plot of the time of flight \( \Delta t(\theta_i, \phi_i) \), as a function of \( \phi_i \), is a circle for both wave modes.

Fig. 4 Time-of-flight, \( \Delta t(\theta_i, \phi_i) \), in \( \mu s \), versus \( \phi_i \) in fused quartz (isotropic). a) \( \theta_i = 5^\circ \) (longitudinal mode), b) \( \theta_i = 18^\circ \) (shear mode).

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Figure 5a shows the variation of $\Delta t(\theta, \phi)$ at $\theta = 3^\circ$, $\phi = 0^\circ \rightarrow 360^\circ$ for propagation of what appears to be a longitudinal mode for spinel. One would expect to observe two two-fold symmetry planes for propagation along [110] in a crystal of cubic symmetry. A two-fold symmetry plane can be observed in Fig. 5a corresponding to the Y-axis (the [ITO] direction). The other two-fold symmetry plane should correspond to the X-axis (the [001] direction). However, there is a readily observable lack of symmetry about the X-axis. The computer generated data in Fig. 5b shows the symmetry that was expected for the experimental data. The 'data' was calculated by solving Eq. 1 for $\Delta t(\theta, \phi)$ and using a general analytic solution for the phase velocity for $\theta = 3^\circ, \phi = 0^\circ \rightarrow 360^\circ$. The elastic constants of spinel used for this calculation were determined by contact measurements of the same specimen. These values were: $C_{11} = 28.7 \times 10^{11}$ dyne/cm$^2$, $C_{12} = 15.85 \times 10^{11}$ dyne/cm$^2$ and $C_{44} = 15.48 \times 10^{11}$ dyne/cm$^2$.

From comparison of the experimental data and 'expected' data, it was assumed that our specimen was cut and polished, such that the normal to the specimen was rotated a few degrees from the [110] direction. The data in Fig. 5a compared to the calculated values shown in Fig. 5b indicated that the specimen axis was rotated slightly about the [001] direction. The angles of incidence were recalculated using a coordinate rotation

$$\begin{pmatrix} l' \\ m' \\ n' \end{pmatrix} = R \begin{pmatrix} l \\ m \\ n \end{pmatrix}$$ (6)

where

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \Delta \beta & \sin \Delta \beta \\ 0 & -\sin \Delta \beta & \cos \Delta \beta \end{pmatrix}.$$ (7)

The rotation angle $\Delta \beta$, about the [001] direction, was found by an iteration procedure to best match the calculated values with the experimental results. Using this method it appeared that the rotation was approximately $2.7^\circ$ about the [001] direction. Figure 5c shows the recomputed 'data' using the $2.7^\circ$ rotation. Similar results were observed for other angles ($\theta$) of incidence, indicating a misorientation of the [110] direction of approximately $2.7^\circ$ rotation about the [001] direction with respect to the specimen normal.

An attempt was made to use the experimental data in the inversion routine, without success. As was stated earlier, at the present time, the inversion analysis is valid only for orthotropic or higher symmetry. The misorientation of the single crystal presents the ultrasonic waves with a sample that appears to have monoclinic symmetry, at best, requiring additional terms in the inversion analysis.

**SUMMARY**

The experimental method described here for the determination of the phase velocity, including the direction cosines, is valid for any symmetry class material. However, without 'a priori' knowledge of the orientation of the specimen, the reference coordinate system attached to the specimen may be arbitrarily oriented with respect to the symmetry of the material. In order
Fig. 5 Time-of-flight, $\Delta t(\theta_i, \phi_i)$, in $\mu$s, versus $\phi_i$ in spinel for $\theta_i=3^\circ$ ([110] direction was assumed parallel to the specimen normal).  
(a) experimental results.  (b) expected (calculated) results.  
(c) recalculated values assuming that the [110] direction was misoriented by 2.7° about the [001] direction.

to use data obtained, without some 'a priori' knowledge of the specimen orientation, would require a completely general inversion analysis.

We have demonstrated the ability to determine the phase velocity for a [110] spinal specimen. A slight amount of misorientation of the specimen was detected from the data. The phase velocity results obtained from the immersion measurements agreed well with predicted calculations of the phase velocity based on the elastic constants determined from contact measurements. Although it was not possible to determine the elastic constants from these measurements, due to a current lack of generality in the inversion routine, the sensitivity of the measurements to orientation can be seen. An aluminum [110] single crystal specimen was also used, but the results were similar to that of the spinel sample due to substantial misorientation of the aluminum sample.
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REFERENCES