A numerical approach to model and predict the energy absorption and crush mechanics within a long-fiber composite crush tube

Leon Pickett Jr.
Iowa State University

Follow this and additional works at: https://lib.dr.iastate.edu/rtd
Part of the Aerospace Engineering Commons, and the Applied Mechanics Commons

Recommended Citation
Pickett, Leon Jr., 'A numerical approach to model and predict the energy absorption and crush mechanics within a long-fiber composite crush tube " (2005). Retrospective Theses and Dissertations. 1766.
https://lib.dr.iastate.edu/rtd/1766

This Dissertation is brought to you for free and open access by the Iowa State University Capstones, Theses and Dissertations at Iowa State University Digital Repository. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.
NOTE TO USERS

This reproduction is the best copy available.
A numerical approach to model and predict the energy absorption and crush mechanics within a long-fiber composite crush tube

by

Leon Pickett, Jr.

A dissertation submitted to the graduate faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY

Major: Engineering Mechanics

Program of Study Committee:
Vinay Dayal, Major Professor
Dale Chimenti
Thomas McDaniel
David Hsu
Derrick Rollins

Iowa State University
Ames, Iowa
2005

Copyright © Leon Pickett Jr., 2005. All rights reserved.
INFORMATION TO USERS

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleed-through, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.
This is to certify that the doctoral dissertation of
Leon Pickett, Jr.
has met the dissertation requirements of Iowa State University

Signature was redacted for privacy.

Major Professor

Signature was redacted for privacy.

For the Major Program
DEDICATION

This dissertation is dedicated to my entire family (no exceptions); especially my mother, Barbara; my wife, LaTasha and my children, Akilah and Hasan.
# TABLE OF CONTENTS

## CHAPTER 1: GENERAL INTRODUCTION

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental Theories</td>
<td>1</td>
</tr>
<tr>
<td>Composites</td>
<td>1</td>
</tr>
<tr>
<td>Composite Fibers</td>
<td>1</td>
</tr>
<tr>
<td>Composite Matrices</td>
<td>2</td>
</tr>
<tr>
<td>Highlights &amp; Advantages</td>
<td>2</td>
</tr>
<tr>
<td>Composite Usage</td>
<td>3</td>
</tr>
<tr>
<td>Crashworthiness</td>
<td>4</td>
</tr>
<tr>
<td>Constitutive Equations</td>
<td>5</td>
</tr>
<tr>
<td>Engineering Constants</td>
<td>5</td>
</tr>
<tr>
<td>Composite Laminate Plate Theory</td>
<td>7</td>
</tr>
<tr>
<td>Laminate Constitutive Equations</td>
<td>8</td>
</tr>
<tr>
<td>Effective Moduli (Smeared Properties)</td>
<td>11</td>
</tr>
<tr>
<td>Finite Element Analysis</td>
<td>11</td>
</tr>
<tr>
<td>Composite Failure Mechanics</td>
<td>16</td>
</tr>
<tr>
<td>Fracture Mechanics</td>
<td>17</td>
</tr>
<tr>
<td>Literature Review</td>
<td>19</td>
</tr>
<tr>
<td>Energy Absorption Crushing Mechanics</td>
<td>19</td>
</tr>
<tr>
<td>Initiation of stable composite crushing modes</td>
<td>21</td>
</tr>
<tr>
<td>Fiber Orientation, Geometry and Velocity Effects</td>
<td>23</td>
</tr>
<tr>
<td>Fiber Orientation Effects</td>
<td>23</td>
</tr>
<tr>
<td>Geometry Effects</td>
<td>24</td>
</tr>
<tr>
<td>Velocity Effects</td>
<td>25</td>
</tr>
<tr>
<td>Organization of the Thesis</td>
<td>27</td>
</tr>
<tr>
<td>References</td>
<td>28</td>
</tr>
</tbody>
</table>

## CHAPTER 2: FINITE ELEMENT MODEL OF A DYNAMIC COMPOSITE CRUSH EVENT

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>31</td>
</tr>
<tr>
<td>Introduction</td>
<td>31</td>
</tr>
<tr>
<td>Methodology</td>
<td>35</td>
</tr>
<tr>
<td>Materials</td>
<td>35</td>
</tr>
<tr>
<td>Tube Dimensions</td>
<td>37</td>
</tr>
<tr>
<td>Impact Characteristics</td>
<td>37</td>
</tr>
<tr>
<td>Specimens</td>
<td>37</td>
</tr>
<tr>
<td>Computation of Energy Absorption</td>
<td>39</td>
</tr>
<tr>
<td>Chapter 3: The Effect of Ply Angle on Energy Absorption of a Circular Glass/Epoxy Crush Tube</td>
<td></td>
</tr>
<tr>
<td>-------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Abstract</td>
<td>48</td>
</tr>
<tr>
<td>Introduction</td>
<td>48</td>
</tr>
<tr>
<td>Methodology</td>
<td>53</td>
</tr>
<tr>
<td>Numerical Solution</td>
<td>53</td>
</tr>
<tr>
<td>FEA Model</td>
<td>54</td>
</tr>
<tr>
<td>Materials</td>
<td>54</td>
</tr>
<tr>
<td>Tube Dimensions</td>
<td>55</td>
</tr>
<tr>
<td>Finite Elements</td>
<td>55</td>
</tr>
<tr>
<td>Impact Characteristics</td>
<td>55</td>
</tr>
<tr>
<td>Post-Processing</td>
<td>56</td>
</tr>
<tr>
<td>Computation of Energy Absorption</td>
<td>56</td>
</tr>
<tr>
<td>Results and Discussion</td>
<td>56</td>
</tr>
<tr>
<td>Conclusions</td>
<td>66</td>
</tr>
<tr>
<td>References</td>
<td>66</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 4: The Effect of Impact Velocity on Energy Absorption of a Circular Glass/Epoxy Crush Tube</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
</tr>
<tr>
<td>Introduction</td>
</tr>
<tr>
<td>Methodology</td>
</tr>
<tr>
<td>Finite Element Model</td>
</tr>
<tr>
<td>Materials</td>
</tr>
<tr>
<td>Tube Dimensions</td>
</tr>
<tr>
<td>Impact Characteristics</td>
</tr>
<tr>
<td>Post-Processing</td>
</tr>
<tr>
<td>Computation of Energy Absorption</td>
</tr>
<tr>
<td>Results and Discussion</td>
</tr>
<tr>
<td>Impact Velocity and Impact Energy</td>
</tr>
<tr>
<td>Energy Absorption Regions</td>
</tr>
</tbody>
</table>
CHAPTER 1: GENERAL INTRODUCTION

Fundamental Theories

Composites

Composite materials are engineered materials that consist of two or more materials that together produce desirable properties that cannot be achieved with any of the constituents alone. There is no chemical bonding between the constituents of the composites. By this general definition, it is clear that there are numerous examples of composites, many of which commonly occur in nature. Long fiber structural composites consist of high strength and high modulus fibers surrounded by a matrix material. In these composites, fibers are the principal load carrying members. The matrix material keeps the fibers together, acts as a load transfer medium between fibers, and protects fibers from being exposed to the environment. For the purposes of this dissertation, the term composite is meant to refer to a specific type of long fiber structural composite comprised of glass fibers within an epoxy matrix.

Composite Fibers

In the construction of advanced composites there are a wide variety of fibers available to suit an even wider variety of applications. As such, different fibers may have different morphology, material, size and shape. Fibers are generally stiffer and stronger than the same material in bulk form. The reason for the excellent stiffness and strength properties is due to near perfect molecular chains with no grain boundaries. These fibers have very small cross sections ranging from 3 to 147 μm, which naturally results in a very high length to diameter ratio [1].
Composite Matrices

Polymers, metals and ceramics have all been successfully employed as matrix materials in the fabrication of advanced composites. Among these materials, polymers are most commonly used in advanced composites. These polymeric materials are further subdivided into thermoplastics and thermosets. Thermoplastic polymers have the advantage of softening upon heating and can be reshaped with heat and pressure. Thermoplastics offer the potential for higher toughness and higher volume, low cost processing. Thermoplastics have a useful temperature range upwards of 225°C. Conversely, thermoset polymers become cross-linked during fabrication and do not soften upon reheating. The most common thermoset polymer matrix materials are polyesters, epoxies, and polymides. Epoxies are relatively inexpensive but have better moisture resistance and lower shrinkage on curing [1].

Highlights and Advantages

Although initially sought and developed because of their potential for lighter structures, today composites have evolved into the “chosen” material for many reasons other than the opportunity to reduce weight. Many composites are now found to exhibit both high specific stiffness and high specific strength as compared with traditional engineering materials, including: aluminum and steel. These properties are significant in that they lead to improved performance and reduce energy consumption; both of which are key aspects in the design of most engineering structures. Since composites are fabricated structures, they can be engineered to meet the specific demands of particular applications. Thus, more efficient structures can be fabricated with much less material waste. Many composites can also be fabricated to have superior fatigue life as compared to traditional
engineering materials. In fact, it is primarily for this reason that composites are finding increased application in the aircraft industry. The directional thermal expansion coefficient of composites allows us to design composites, which exhibit very low cyclic thermal expansion properties. This is extremely advantageous in applications where thermal expansion is a consideration. Composites, depending on the selection of the matrix materials, can be fabricated to be entirely resistant to moisture and chemical corrosion. This leads to a substantial reduction in maintenance costs and an increase in useful life. With the obvious exception of metal matrix composites, composites are usually electrically non-conducting. However, on the other hand, copper matrix composites are now under consideration for high temperature applications because of their high thermal conductivity. Today with advanced manufacturing techniques, composites can be tailored to exact standards and components can be fabricated with low material waste. Depending on the application, these efficiencies in many cases directly lead to substantial cost savings. Pound for pound, composites are generally more expensive than traditional materials; however upon evaluating the cumulative benefits mentioned over the lifetime of the application, composites usage may lead to an overall costs savings [1].

**Composite Usage**

Composite materials have seen extensive use in a number of select fields. Composites are very attractive to the aircraft industry because of their specific stiffness, specific strength, design tailorablity and fatigue resistance. In the athletic and recreational equipment field, composites are being used in golf clubs, hockey sticks, helmets, fishing rods, boats and racecars. In the military and law enforcement, composites have long been used to manufacture bulletproof vests. The medical, construction, electronic and automotive industries have seen increased applications for composites.
Of these, the automotive industry represents a tremendous opportunity to incorporate the use of composites.

**Crashworthiness**

A structural material’s ability to absorb impact energy is generally referred to as the “crashworthiness” of the structure. In passenger vehicles, current legislation [2] requires that vehicles be designed such that, in the event of an impact at speeds up to 15.5 m/s (35 mph) with a solid, immovable object, the occupants of the passenger compartment should not experience a resulting force that produces a net deceleration greater than 20g. Additionally, it is imperative that crashworthy structures are designed to crush in a fairly predictable and controlled manner. Traditionally, these crash structures have been constructed from structural steel. Although clearly useful as structural members, steel and other metals also come with relatively higher weight trade-offs and can ultimately lead to undesirable inertial effects as it pertains to crash events.

In an attempt to overcome these negative inertial effects, lighter weight structural materials are beginning to be evaluated for crashworthiness. Foremost among these materials are long-fiber structural composites. Composite materials represent superior specific energy absorption when compared to most isotropic materials. In metals, all the energy is absorbed in plastic deformation, while in composites the fracture energy is dominant. A closer look at the corresponding stress-strain curves shows that for all high-performance composites, this relationship is essentially linear in nature. In comparison, metals exhibit a distinctly linear stress-strain relationship followed by a large range where the metal experiences plastic deformation before failure. The lesson from the comparisons of the stress strain behavior is that materials, which are essentially elastic to failure (composites, ceramics), might be considered to have no capacity for energy absorption since no plastic deformation energy is available to satisfy such requirements. However, various experiments have found that
elastic, brittle materials can be very effective energy absorbers. Nevertheless, the full extent to which this is possible is still being researched.

**Constitutive Equations**

To evaluate the capacity for a composite to manage impact energy, a closer look at its mechanics is warranted. The generalized Hooke’s law relates stresses to strains in an orthogonal coordinate system [1]. For composites, the relationships between stress and strain are linear with the most general form being

\[
\sigma_{ij} = C_{ijkl} \varepsilon_{kl}
\]

where \( \sigma_{ij} \) is the stress tensor, \( \varepsilon_{ij} \) is the strain tensor, and \( C_{ijkl} \) is the stiffness tensor, a 4th order tensor with 81 elastic constants. This linear elastic stress-strain constitutive relationship is called the Generalized Hooke’s law. For an orthotropic material, these 81 constants can be reduced to 9 independent constants;

\[
[C] = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\]

**Engineering Constants**

The simplest form of the constitutive equations is obtained when they are written in terms of stiffness and compliance coefficients, \( C_{ij} \) and \( S_{ij} \), respectively [1]. However, properties that are actually measured are called the engineering constants which relate measured stress to strains, which can be easily related to the stiffness and compliance coefficients. We now define the material
coordinates 1-2 which are aligned with the fiber directions and x-y, any arbitrary coordinate system as shown in Fig. 1-1

![Fig. 1-1 Material coordinate (1-2) and general coordinate (x-y) systems.](image)

Finally, we then arrive at the constitutive equations in principal material coordinates for an orthotropic material

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_1 \\
\gamma_2 \\
\gamma_3
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{21} & S_{31} & 0 & 0 & 0 \\
S_{12} & S_{22} & S_{32} & 0 & 0 & 0 \\
S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & S_{11} & 0 & 0 \\
0 & 0 & 0 & 0 & S_{12} & 0 \\
0 & 0 & 0 & 0 & 0 & S_{13}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_1 \\
\tau_2 \\
\tau_3
\end{bmatrix}
\]

1-3

Compliance coefficients come from the stress-strain relationships, i.e.

\[
S_{ii} = \frac{1}{E_i}; \quad S_{ij} = \frac{-E_j}{E_i}; \quad S_{21} = \frac{-E_1}{E_2}; \quad S_{22} = \frac{1}{E_2}; \quad S_{33} = \frac{1}{G_{12}}
\]

1-4

In compact form, the stress-strain relation can be written as

\[
\{\varepsilon\} = [S]\{\sigma\}
\]

1-5

Furthermore, the inverse of [S] yields [C] such that

\[
\{\sigma\} = [C]\{\varepsilon\}
\]

1-6

where \([C] = [S]^{-1}\).
Composite Laminate Plate Theory

Most long-fiber composites are actually formed by stacking thin layers of individual composite lamina plies. Variation in material properties result based upon constituent properties and fiber orientation. To better understand the behavior of these laminated plates, a number of theories have been developed. In equivalent single-layer (ESL) laminate theory, a composite is considered a 2-D plate. What results is a 2-D continuum problem as opposed to a 3-D continuum problem [3]. In order for the theory to be valid several restrictions must be made. First, the material of each layer is linearly elastic and is orthotropic. Furthermore, to function properly, each layer must be of uniform thickness. Strains and displacements must be small. The transverse shear stresses on the top and bottom surfaces of the laminate must be zero. Finally, the layers should be perfectly bonded with one another.

Of the ESL theories, the simplest theory governing the behavior of composites is the Classical Laminated Plate Theory (CLPT). This theory begins with the assumption that the Kirchhoff classical plate theory also applies to laminated plates. The Kirchhoff hypothesis states that: (1) straight lines perpendicular to the mid-surface before deformation remain straight after deformation; (2) the transverse normals do not experience elongation; and (3) the transverse normals rotate such that they remain perpendicular to the mid-surface after deformation [3]. Within a long-fiber composite, this would suggest that: (1) the transverse displacement is independent of the transverse (or thickness) coordinate; (2) the transverse normal strain $\varepsilon_{zz}$ is zero and (3) the transverse shear strains, $\varepsilon_{xz}$, $\varepsilon_{yz}$ are equal to zero.

CLPT is primarily based on the displacement field

$$u(x, y, z,t) = u_0(x, y, t) - z \frac{\partial w_0}{\partial x}$$  
$$v(x, y, z,t) = v_0(x, y, t) - z \frac{\partial w_0}{\partial y}$$

1-7

1-8
where \((u_0, v_0, w_0)\) are the displacement components along the \((x,y,z)\) coordinate directions, respectively, of a point on the mid-plane \((z = 0)\). The displacement field (equations 1-7 and 1-8) implies that straight lines normal to the \(xy\)-plane before deformation remain straight and normal to the mid-surface after deformation. The Kirchhoff assumption amounts to neglecting both transverse shear and transverse normal effects; thus, deformation is entirely due to bending and in-plane stretching [3].

Although they exist as a simplified form of the 3-D problem, the ESL models often provide a sufficiently accurate description of global response for thin to moderately thick laminates (gross deflections, critical buckling loads, and fundamental vibration frequencies and associated mode shapes). However, it should be noted that there are some distinct disadvantages when compared to 3-D formulations. The plate assumption only works for thin shells and as the composite laminate becomes thicker, the accuracy of the global response predicted by the ESL models decreases. Additionally, when approaching regions of intense loading or geometric and material discontinuities, the ESL models are often incapable of accurately describing the states of stress and strain. In both cases, 3-D theories are preferred [3].

**Laminate Constitutive Equations**

With the introduction of CLPT, we clearly see there is a much more efficient means of expressing the constitutive equations. The underlying assumptions of 2-D plate theory allow us to apply the general constitutive relations in determining the specific laminate constitutive relations that govern long-fiber structural composites [1]. It is often the case in the analysis of composites that a condition of plane stress, Fig. 1-2, actually exists or is a very good approximation. Thus the need to develop constitutive equations for plane stress. We start with the 3-D constitutive equation (equation
1-2) for a single layer (lamina) of a unidirectional composite with a fiber orientation, $\theta$, relative to the global coordinates. Furthermore, 2-D CLPT requires that $\sigma_{33}=\tau_{33}=0$.

![Figure 1-2: (a) 3-D and (b) 2-D states of stress](image)

Thus, in principal material coordinates, the 3-D constitutive equations become:

$$
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{21} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix}
$$

such that the stiffness terms may again be represented using engineering constants:

$$
Q_{11} = \frac{E_1}{1-v_{21}v_{21}}
$$

$$
Q_{12} = Q_{21} = \frac{v_{12}E_2}{1-v_{12}v_{21}} = \frac{v_{21}E_1}{1-v_{12}v_{21}}
$$

$$
Q_{22} = \frac{E_2}{1-v_{12}v_{12}}
$$

$$
Q_{66} = G_{12}
$$

Having clearly defined the plane stress constitutive equation in principal material coordinates, we can now define the lamina stress-strain relations by performing 2-D transformations about the $z$ (out of plane) axis, such that
where the corresponding transformation matrices for stress and stain respectively are

\[
[T] = \begin{bmatrix}
m^2 & n^2 & 2mn \\
n^2 & m^2 & -2mn \\
-2mn & -mn & m^2 - n^2
\end{bmatrix}
\]

where for plane stress problems \( m = \cos \theta \) and \( n = \sin \theta \), and \( \theta \) is the direction of the fiber from the x-axis.

Here we now may introduce the plane stress transformed reduced stiffness matrix: \( \bar{Q} = [T]^{-1}[Q][T] \)

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\
\bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\
\bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

We may now write the equations for the in plane components of stress in terms of the transformed stiffness coefficients. The laminate constitutive relation can be written as,

\[
\begin{bmatrix}
N \\
M
\end{bmatrix} =
\begin{bmatrix}
A & B \\
B & D
\end{bmatrix}
\begin{bmatrix}
\varepsilon'' \\
\kappa
\end{bmatrix}
\]

where

\[
A_j = \sum_{i=1}^{N} \bar{Q}_{ij} \left( z_i - z_{i-1} \right)
\]

\[
B_j = \frac{1}{2} \sum_{i=1}^{N} \bar{Q}_{ij} \left( z_i^2 - z_{i-1}^2 \right)
\]

\[
D_j = \frac{1}{3} \sum_{i=1}^{N} \bar{Q}_{ij} \left( z_i^3 - z_{i-1}^3 \right)
\]

Thus, the relationship between the applied stress resultants (force/unit length, [N], and moment/unit length, [M]) and the mid-surface strains and curvatures can be written as;

\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & A_{16} \\
A_{12} & A_{22} & A_{26} \\
A_{16} & A_{26} & A_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x'' \\
\varepsilon_y'' \\
\gamma_{xy}''
\end{bmatrix} +
\begin{bmatrix}
B_{11} & B_{12} & B_{16} \\
B_{12} & B_{22} & B_{26} \\
B_{16} & B_{26} & B_{66}
\end{bmatrix}
\begin{bmatrix}
\kappa_x \\
\kappa_y \\
\kappa_{xy}
\end{bmatrix}
\]
Effective Moduli (Smeared Properties)

In the preceding sections, the topics of laminated composites, composite laminate plate theory and laminate constitutive equations have been introduced and discussed. When combined, these theories provide a very firm basis for the understanding of the macroscopic behavior of composite materials; thus a non-homogeneous, anisotropic material may be represented as a homogeneous, anisotropic material. Again, starting with unidirectional layers, it is clear that each layer has distinct material properties as discussed previously. These lamina, or layer properties are influenced by the mechanical properties of both the fiber and the matrix, the volume fraction of fibers within the laminate, and the number of layers and orientation of individual lamina. These properties directly influence the effective properties of the resulting composite laminate plate. In the most general case, the material properties of stacked unidirectional lamina are orthotropic with different properties in the in-plane, longitudinal and transverse directions. As a result, unidirectional fibrous composite laminates may exhibit a nearly infinite combination of material properties as a direct result of the myriad of constituent materials and lamina variations.

Finite Element Analysis

With the increased use of fiber-reinforced composites in structural components, studies involving the behavior of components made of composites are receiving considerable attention. Functional requirements and economic considerations of design are forcing engineers to seek reliable and accurate, yet economical methods of determining static and dynamic characteristics of the
structural components. The analytical study and design of composite materials requires knowledge of
anisotropic elasticity, structural theories and failure/damage criteria. Unlike isotropic materials,
anisotropic materials exhibit complicated mechanical behavior. The partial differential equations
governing composite laminates of arbitrary geometries and boundary conditions cannot be solved in
closed form; therefore, the use of numerical methods facilitates the solution. Among the numerical
methods available for the solution of differential equations defined over arbitrary domains, the finite
element method (FEM) is the most effective method. Finite element analysis of a structural problem
is a numerical analysis of the mathematical model used to represent the behavior of the structure [4].

In FEM, the solution domain is divided into a number of discrete elements. The
displacements within an element are generally the unknown field variables that are expressed in terms
of unknown nodal values. The governing load displacement relations for each element are written.
These equations are then assembled maintaining continuity between the elements and equilibrium at
the nodes. Finally, the boundary conditions are applied and the solution obtained for the load
displacement relationship for the entire structure. This may be achieved by first expressing each
displacement component in terms of trial coordinate functions, usually expressed as polynomials, the
number of unknown coefficients depending on the number of nodal degrees of freedom of the
element as shown:

\[ u = Ac \]  \hspace{1cm} (1-24)

in which \( u = [u_x, u_y, u_z]^T \), \( c \) is a vector whose scalars are element spatial coordinates and \( A \) is a
matrix of unknown coefficients. These coefficients are determined from element boundary conditions
yielding the relationship that expresses displacements within the element to their unknown nodal
values:

\[ u = Nu' \]  \hspace{1cm} (1-25)

where the superscript, \( e \), refers to element-wise values and \( N \) are the shape functions. For a three-
dimensional element, the shape function matrix \( N \) has 3 rows and its number of columns is equal to
the total degrees of freedom of the element. This matrix may also be obtained directly by employing suitable interpolation functions [4].

The strain displacement relationships from the strain matrix, $\varepsilon$, are next generated using equation 1-25

$$\varepsilon = Bu'$$  \hspace{1cm} (1-26)

by differentiating the appropriate displacement components. The matrix $B$ is the strain displacement matrix which relates strain to displacement. It has six rows and its number of columns is equal to the number of degrees of freedom of the element [4].

The principle of stationary total potential energy, $V$, can be simply stated as that of all displacement states satisfying compatibility and boundary conditions; those that also satisfy equilibrium make the total potential energy assume a stationary value. For a stable structure, the value of $V$ is always a minimum, which is also expressed as

$$V = U - W$$  \hspace{1cm} (1-27)

in which $U$ is the internal strain energy and $W$ is the potential of the external forces. Assuming that the body is subjected to time-varying external forces, the displacements, strains, and stresses within a finite element will all be functions of time. Then the strain energy of an element is given as

$$U' = \frac{1}{2} \int \sigma^T \varepsilon_d V$$  \hspace{1cm} (1-28)

which reduces to

$$U' = \frac{1}{2} \left[ \int \varepsilon^T C \varepsilon_v dV - \int \varepsilon^T D \varepsilon_v dV - \int \varepsilon^T C \varepsilon_v dV \right]$$  \hspace{1cm} (1-29)

neglecting terms independent of elastic displacements and noting that $\varepsilon^T D \varepsilon_v = \varepsilon^T D \varepsilon$. Equation 1-29 may finally be written in terms of element nodal displacements by beginning with equation 1-26:

$$U' = \frac{1}{2} \left[ \int \varepsilon^T B^T C B u' dV - \int \varepsilon^T B^T C \varepsilon_v dV - \int \varepsilon^T B^T C \varepsilon_v dV \right]$$  \hspace{1cm} (1-31)
where \( \varepsilon \) = strain, \( \varepsilon_1 \) = thermal strain, and \( \varepsilon_i \) = initial strain

\[
U' = \frac{1}{2} \left[ \int u^T K' u' - u'^T \int B^T C \varepsilon_1 dV - u'^T \int B^T C \varepsilon_i dV \right]
\]

where \( K' \) is the element stiffness matrix, defined as

\[
K' = \int \! \! B^T C B dV
\]

The work done by concentrated loads \( p \), body force \( p_B \), and surfaces \( p_s \) in an element may be expressed as

\[
W' = \int \! \! u'^T \left( p(t) + \int \! \! N^T p_B(t) dV + \int \! \! N^T p_s(t) dS - M' \ddot{u}' \right)
\]

where \( \ddot{u}' = \dot{N} u'^T \) and \( M' \) is the element mass matrix, defined as

\[
M' = \int \! \! N^T \rho N dV
\]

The expression for the total potential energy of an element may now be written as

\[
V' = U' - W'
\]

For the entire structure, defining \( q \) as the nodal unknowns,

\[
V = \sum V' = \sum U' - \sum W'
\]

\[
V = \frac{1}{2} q^T K q - q^T p(t) - q^T \dot{p}(t) - q^T \left( p(t) + p_B(t) + p_s(t) - M\ddot{q} \right)
\]

Then the requirement of minimum total potential energy

\[
\frac{dV}{dq} = 0
\]

yields the equation of motion

\[
K q + M\ddot{q} = p_B(t) + p(t) + p_B(t) + p_s(t)
\]

\[
K q + M\ddot{q} = f(t)
\]
where the matrices and vectors refer to the entire structure and furthermore, $K = \text{stiffness matrix}$, $P_r = \text{thermal load}$, and $P_t = \text{inertia load}$. Most structures are characterized by the presence of structural as well as viscous damping, in which case equation 1-41 takes the following form:

$$K(1 + ig)q + C_D \dot{q} + M \ddot{q} = f(t)$$  \hspace{1cm} \text{(1-42)}$$

in which $C_D$ is viscous damping and equals $\alpha K + \beta M$, if the damping is proportional, and $\alpha$ and $\beta$ are the proportionality constants. For spinning structures with viscous damping the dynamic equations of motion can be written as

$$Kq + C_D \dot{q} + M \ddot{q} = f(t)$$  \hspace{1cm} \text{(1-43)}$$

or

$$(K_E + K_G + K' \dot{q} + (C_c + C_D)\dot{q} + M \ddot{q} = f(t)$$  \hspace{1cm} \text{(1-44)}$$

where $K_E = \text{elastic stiffness matrix}$, $K_G = \text{geometric stiffness matrix}$, incorporating the effect of in-plane stretching on out-of-plane motion and $K' = \text{centrifugal stiffness matrix}$. All matrices in the preceding formulation except $C_c$ are symmetric and usually highly banded. The matrix $C_c$ is skew symmetric being similarly banded. The associated matrix equation of free vibration may be written in the general form as

$$Kq + C \dot{q} + M \ddot{q} = 0$$  \hspace{1cm} \text{(1-45)}$$

where the definitions for $K$, $C$, and $M$ depend on the problem type. Similarly, the un-damped equation of free vibration takes the form

$$Kq + M \ddot{q} = 0$$  \hspace{1cm} \text{(1-46)}$$

and similar equations are encountered for structural instability or buckling problems. For static problems, the matrix equation reduces simply to

$$Kq = f$$  \hspace{1cm} \text{(1-47)}$$

Clearly, the numerical analysis of a structural system consists of two distinct yet related solution procedures. First, a finite element model of the system yields a set of algebraic equations that are then
solved by employing a suitable numerical procedure. Because of the very nature of finite element
discretization, the resulting equations tend to be rather large in size, as well as highly banded for
many practical problems. An economical solution of such problems poses as much a challenge to an
analyst as the process of discretization itself [4].

**Composite Failure Mechanics**

As it pertains to composite materials, the topic of material failure is not as absolute as it is for
isotropic materials. Since composites are comprised of individual composite lamina, each layer has its
own failure events associated with it. In regards to characterizing composite failure, it is noteworthy
to point out that composites experience local failures and final failure; thus “first failure” does not
necessarily correspond to “final failure.” The local failures are referred to as “damage,” and the
development of additional local failures with increasing load or time is called “damage accumulation”
[1].

Fibrous composite materials fail in a variety of mechanisms at the fiber/matrix (micro) level. Micro-level failure mechanisms include: fiber fracture, fiber buckling, fiber splitting, fiber pullout, fiber/matrix debonding, matrix cracking, and radial cracks. At the laminate level, micro-level mechanisms occur as lamina failures in the form of transverse cracks in planes parallel to the fibers, fiber-dominated failures in planes perpendicular to the fibers, and delamination between layers of the laminate.

Transverse fiber fracture, or the breaking of a continuous fiber into two or more distinct segments, is the most catastrophic of failure mechanisms as the fibers are typically the primary load-carrying component. Fiber failure may be the result of tensile or compressive stresses. Fiber fracture occurs under tensile load when the maximum allowable axial tensile stress (or strain) of the fiber is exceeded. Fiber pullout occurs when the fiber fractures and is accompanied by fiber/matrix
debonding. Matrix cracking occurs when the strength of the matrix is exceeded. Fiber kinking occurs when the axial compressive stress causes the fiber to buckle. The critical buckling stress for a fiber embedded in a matrix is a function of the properties of the fiber and the matrix (which provides lateral support to the fiber). Fiber splitting and radial interface cracks occur when the transverse or hoop stresses in the fiber or inter-phase region between the fiber and the matrix reaches its ultimate value [1].

There is no single theory that accurately predicts failure at all levels of analysis, for all loading conditions, and for all types of composite materials. While some failure theories have a physical basis, most theories represent attempts to provide mathematical expressions which give a "best fit" of the available experimental data, recognizing the practical limits of data collections and the limits of mathematical representations that are practical from a designer's point of view. From the standpoint of a structural designer, it is desirable to have failure criteria which are applicable at the level of the lamina, the laminate, and the structural component. Failure at these levels is often the consequence of an accumulation of various micro-level failures which coalesce and result in the final failure [1].

These micro failure events are critical, essential elements which comprise the macroscopic failure theories. There are many macroscopic failure theories (some general, others highly specialized) that have been proposed for composites. The most notable of the general theories include the maximum stress, maximum strain, Tsai-Hill, tensor polynomial and Tsai-Wu failure criteria.

Fracture Mechanics

Although each individual micro failure event is well known, their mechanics are not trivial. Fracture mechanics in particular governs the formation of interlaminar and intralaminar cracking and
how these ensuing cracks propagate [5]. Kannien states the basic equation of linear elastic fracture mechanics common in work on composites is

$$K_i(a,b,\sigma_w) = K_{ic}(T,\sigma)$$

1-48

where $K_i$ is a material independent function of the crack size, $a$, the component dimensions, $b$, and the applied stress $\sigma_w$, while $K_{ic}$ is a material property that can depend upon temperature, $T$, and loading rate, $\sigma$.

For metals, the crack length is required to be large in comparison to the value of $\left(\frac{K_{ic}}{\sigma_y}\right)^2$ where $\sigma_y$ is the corresponding yield stress; however this is not so in a composite. As a result, actual fracture in a composite routinely violates this requirement. In developing more appropriate fracture mechanics techniques for applications to fiber reinforced composite materials, several basic facts must be kept in mind. These include preparation defects, laminates defects, and fabrication defects. Nevertheless, even after controlling all of these requirements, crack propagation in a composite is still quite difficult to model. After a crack initiates it can grow and progressively lower the residual strength of a structure [6] to the point where it can no longer support design loads, making global failure imminent. A second key fact involved in the application of fracture mechanics to composite materials is the basic heterogeneous nature of fiber-reinforced composites. Within a ply, cracking can be both discontinuous and non-collinear crack growth. On the laminate level, cracking can proceed in a distinctly different manner in different plies and, in addition, inter-ply delamination can occur. Thus, equation 1-48 is ill equipped to cope with these complexities. Consequently, many researchers have pursued an energy balance approach to the problem. This does not really present a significant improvement over the fundamental difficulties associated with the stress intensity factor point view, however. As a result, composite finite element solutions have yet to account for the mechanics of composite fracture. The extent to which fracture mechanics may govern the energy absorption capacity in a composite is still unknown.
Literature Review

Energy Absorption Crushing Mechanics

The energy absorption characteristics of both metals and composites have been the subject of extensive research [7-19]. Most of the work has been experimental in nature. A very convenient shape for the crush studies on composites is the circular tube. The large moment of inertia of this shape is able to prevent the buckling and the crushing mechanism is used for the absorption of large impact loads. Using composite crush tubes, Farley [7] was able to identify the primary and secondary crushing initiators involved in a composite crush event and how they relate to energy absorption. In tubes composed of brittle fiber reinforcement, the catastrophic failure mode occurs when the lamina bundles do not bend or fracture due to the formation of very short (less than 1 ply thickness) interlaminar cracks. This leads to a very high peak load/post failure load, which is then followed by a low post failure load/peak load. As a result, the actual magnitude of energy absorbed is much less and the peak load is too high with respect to the resulting sustained crushing load value. As would be imagined, catastrophic failure modes are not of interest to the design of crash worthy structures. Structures designed to react to loads produced by catastrophically failing energy absorbers are heavier than structures designed to react to loads produced by progressively failing energy absorbers.

Composite material energy absorption mechanisms have only recently become well known. The ideal crushing behavior for a tube would be for the crushing to start at the point of application of the load and then progressively travel along the tube. The important thing is that the entire tube material crushes for the maximum energy absorption. Farley theorized that the main crushing initiators that occur within a crush structure are; transverse shearing, lamina bending, local buckling, and a combination of these. Brittle fracturing results when both the transverse shearing and lamina bending modes occur during the same crush event.
Transverse shearing, also known as fragmentation, is typically seen within brittle fiber reinforcement tubes. This mode is characterized by the creation of partial lamina bundles. These bundles result when short (less than a lamina’s thickness) interlaminar cracks form. Mechanisms like interlaminar crack growth and fracturing of lamina bundles control the crushing process for fragmentation. The main energy absorption occurs as a result of the fracturing of these lamina bundles.

Lamina bending, otherwise known as the splaying mode, is typically seen within brittle fiber reinforcement tubes. This mode is characterized by the formation of very long lamina bundles. These bundles result when interlaminar and intralaminar cracks reach a length of more than 10 lamina thicknesses. Within this mode, very long interlaminar, intralaminar, and parallel-to-fiber cracks characterize the splaying mode. The lamina bundles do not fracture. In this case, energy absorption is due to crack propagation, bending and bundle friction. Specifically, the main energy absorbing mechanism is matrix crack growth. Two secondary energy absorption mechanisms related to friction occur in tubes that exhibit splaying mode.

Local buckling, or progressive folding, is characterized by folding of the tube walls. This is the primary means by which metals absorb energy. The progressive folding mode is characterized by the formation of local buckles. This mode is exhibited by both brittle and ductile fiber reinforced composite material. Mechanisms such as plastic yielding of the fiber and/or matrix control the crushing process for progressive folding.

Brittle Fracturing is in fact a hybrid initiator, which combines both the transverse shearing and lamina bending modes. This is the mode of primary interest because this is the means by which long fiber composite materials absorb energy. It is characterized by the formation of lamina bundles of moderate length. These bundles result from the formation of interlaminar cracks whose lengths are between 1 and 10 laminate thickness. In this case energy absorption is due to fracture, friction and bending of the bundles.
Initiation of Stable Composite Crushing Modes

One of the most difficult aspects of the energy absorption of composite crush tubes is controlling the crush initiation [7]. There are two main types of loading surface geometries for axially loaded tubes. These crush tubes are either chamfered or un-chamfered (flat-ended). Flat-ended tubes made from brittle materials are likely to fail by catastrophic brittle fracture. Provided buckling modes are avoided, the brittle fracture strength of a tube, $\sigma_c$, coincides with the strength of the material and is an upper limit to the strength of the tube by failure in any mode. In composite tubes, complete separation across the fracture plane may not occur at failure. This results in interpenetration of the two halves of the tube and some residual load bearing capacity. In other words, the ensuing failure mode and behavior of flat-ended tubes impacted in an axial crush is completely catastrophic and thus results in a highly inefficient and highly unpredictable failure event. The resulting energy absorption is relatively low. Clearly this type of failure is of little value in structures that are required to collapse in a controlled way and absorb large amounts of energy.

To maximize the amount of energy absorption, it is necessary to have a means to control the crushing process. This is accomplished by initiating the ensuing crash by forcing the crushing process to begin at a desired location in a desired mode that facilitates a stable crushing event. Progressive crushing can often be induced in tubes made from brittle material by initiating, or 'triggering' fracture at one end of the tube at stresses below $\sigma_c$. This is accomplished by creating a 30°, 45° or 60° chamfer at the top load surface. This chamfer forces the crushing to initiate at this surface and leads to a much more stable sustained crushing load which propagate through the tube. A stable zone of microfracture then propagates down the tube. The most straightforward method of triggering is to chamfer one end of the tube. Crushing then initiates in the highly stressed region at the tip of the chamfer due to stress concentration and this develops into a stable crush zone located at the top of the tube. The ensuing
sequence of events depends on the chamfer angle. Figure 1-3 illustrates that local fracture occurs at the crush front and at $P_{\text{max}}$, a sharp load relaxation occurs which is followed by the formation of the crush zone.

![Figure 1-3: Typical load-deflection curve of a composite crush tube](image)

The size of the load drop depends on the chamfer angle and is reduced to zero at some angles. Further crushing occurs at approximately constant load, $\bar{P}$, and the appearance of the crush zone remains unchanged apart for small details.
Fiber Orientation, Geometry and Velocity Effects

To better understand the customization of composites, three main variables were considered in this research: fiber orientation, tube geometry and impact velocity. These variables were chosen for a couple different reasons. The primary reason was that there already exists a wealth of experimental data relating the effects of each variable to specific energy absorption. The second reason is that these variables in combination with one another may very well provide a fundamental understanding in how to best fabricate a crush tube that performs at or near an optimum value.

Fiber Orientation

Work by Farley [7] on glass/epoxy, carbon/epoxy and Kevlar/epoxy composite tubes with fiber architecture of $[0^\circ/\pm 0^\circ]_t$, where $\theta$ varied from $0^\circ$ to $90^\circ$, showed significant differences in the energy absorption trends for these materials. The specific energy of the glass/epoxy and Kevlar/epoxy tubes remained constant with increasing $\theta$ up to $45^\circ$ and above this value it increased. Whereas, the specific energy of the graphite/epoxy tubes decreased as theta increased and remained constant from $45^\circ$ to $90^\circ$. Furthermore, the graphite/epoxy, glass/epoxy and Kevlar/epoxy specimens crushed in brittle fracturing, lamina bending and local buckling modes, respectively. Within the graphite/epoxy tubes, it has been theorized that the decrease in energy absorption with respect to $\theta$ is due to a decrease in axially aligned fibers. Similarly, it has been theorized that within the glass/epoxy tubes, the energy absorption increase with respect to $\theta$ is due to an increase in the number of laterally aligned fibers. Farley and Jones [8] quasi-statically crushed carbon/epoxy and glass/epoxy tube specimens with fiber architecture $[0^\circ/\pm 0^\circ]_t$, to determine the influence of ply orientation on the energy absorption capability. They found that the energy absorption capability of the glass/epoxy tube increased with increasing $\theta$. 
Based upon this sampling of findings, it would appear that the literature search generally revealed that the fiber orientations that enhance specific energy absorption of the composite materials requires them to either: increase the number of fractured fibers; increase the material deformation; increase the axial stiffness; or increase the lateral support to the axial fibers.

**Geometry**

It was found that model geometry, plays a significant role in the absorption of energy. Farley [10] investigated the geometrical scalability of graphite/epoxy and Kevlar/epoxy, \([\pm 45^\circ]\), tubes by quasi-statically crush testing them. In this study, all circular cross section graphite/epoxy tubes exhibited a progressive brittle fracturing mode. The diameter to thickness (D/t) ratio was determined to affect the energy absorption capability of the composite materials. As the D/t ratio increased, the specific energy absorption decreased. This increase was attributed to a reduction in interlaminar cracking in the crush region of the tube. Thornton and Harwood [14] studied the effect of tube dimensions. It was found that carbon/epoxy tubes exhibited large changes in their energy absorption characteristics as tube diameter, D, wall thickness, t, and D/t ratio varied. Furthermore, it was discovered that depending on the relative density, defined as the ratio of the volume of the tube to that of a solid of the same external dimensions, the tube crushing became unstable when the relative density registered below a critical value. This relative density value was 0.025 for carbon/epoxy and 0.045 for glass/epoxy tubes. Furthermore, the specific energy was found to be essentially independent of tube dimensions for the tubes that crushed in a stable manner. Fairfull [15] and Fairfull and Hull [16] studied the effects of specimen dimensions on the specific energy of glass cloth/epoxy tubes. Here, it was discovered that the specific energy decreased with increasing diameter. The specific energy, for a given diameter, initially increased with decreasing D/t ratio up to 5. Below this value, it decreased. Based upon their findings, it was concluded that there could not be a universal
relationship to predict energy absorption capability. For graphite/epoxy and Kevlar/epoxy tubes, Farley[11] found that tube width to wall thickness ratio (w/t) was a factor that influenced the energy absorption capability of composite materials. The findings of this study indicates that energy absorption generally increased with decreasing w/t ratio: For graphite/epoxy tubes having w/t ratios in the range of 20 to 50, changes in crushing mode occurred, resulting in a decrease in energy absorption capability as w/t ratio decreased. Both graphite/epoxy tubes and Kevlar/epoxy tubes crushed in a progressive and stable manner. All graphite/epoxy tubes exhibited a lamina bending crushing mode while Kevlar/epoxy tubes exhibited a local buckling crushing mode.

In short, most of the literature generally revealed that the crush zone fracture mechanisms are influenced by the tube dimensions. Specifically it is suggested that by changing the tube dimensions, the crush mechanisms can be changed. This would imply that tube dimensions play a significant role in specific energy absorption. However, to date, the full extent of this relationship has not been quantified. Furthermore, it should be noted that some disagreement within the field exists. Additionally, it can be concluded from the experimental literature that when holding everything else constant, hollow tubes with circular cross-sections have the highest overall specific energy absorption capability followed by square and rectangular cross-sections.

**Velocity**

Upon initial review of the pertinent literature, it is noted that some distinction should be made when comparing the results of quasi-static loading to dynamic impact loading. In quasi-static testing, the tube specimen is crushed at constant speed. Here the energy absorbed is the area under the load-displacement curve. However, load in this case is just the specimen's reaction to it being crushed. It does not have a deceleration term because the crushing process is taking place at a constant speed. The measurement of the time quantity is not worthwhile because one actually controls the rate of
energy absorption rather than it being a material property as in the case of dynamic impact testing. Hence it is inferred that quasi-static testing is not a true simulation of the actual crash conditions. It can however, be used to study the failure mechanisms that take place during the crushing process.

Based upon the findings of this literature survey, it was found that velocity plays a significant role in composite tube energy absorption. Thornton [17] reported very little change in the specific energy absorption of graphite/epoxy, Kevlar/epoxy and glass/epoxy composite tubes over a wide range of quasi-static compression rates (0.01 to 0.0002 in/min). Thornton [18] also investigated the energy absorption behavior of pultruded glass/polyester and glass/vinyl ester tubes in the crushing speed range from 0.00021 to 15 m/s. He reported a 10% decrease with increasing test speed in the case of glass/vinyl ester tubes and a 20% increase in energy absorption in the glass/polyester tubes. This was attributed to the higher tensile strength and modulus of the vinylester. In stark contrast however, Farley [11] found specific energy absorption to be independent of crushing speed (up to 7.6 m/s) within kevlar/epoxy, carbon/epoxy and glass/epoxy composite tubes with fiber architecture of 

\[ [0^\circ/\pm 0^\circ]_4 \]

In dynamic crush testing, Schmueser and Wickliffe [19] reported a decrease of up to 30% in energy absorption of impacted carbon/epoxy, glass/epoxy and Kevlar/epoxy tubes with fiber architecture of 

\[ [0_2^\circ/\pm 45^\circ]_3 \]
as compared to static test results.

Upon reviewing this literature there seems to be a lack of consensus about the influence of test speed on the energy absorption. Past experimental investigation has found that impact speed plays a very important role in the energy absorption in a crush tubes in some cases and none at all in others. However, it is known that energy absorption capability is a function of testing speed when the mechanical response of the crushing mechanism is a function of strain rate. The rate at which the structure is loaded has an effect on both the material's behavior and also the structural response of the target. The strain energy absorbing capabilities of the fibers and the geometrical configuration of the target are very important to the impact resistance of composites at low rates of strain. However, the strain energy absorbing capabilities of the fibers and the geometrical configuration of the structure is
less important at very high rates of strain since the structure responds in a local buckling mode. What is important is the magnitude of energy dissipated in delamination, debonding and fiber pullout.

Organization of the Thesis

This research is aimed at the computational modeling of the crush of a circular tube. The crush has been performed on the software LS-DYNA. Effect of various parameters such as the element size, fiber angle, tube geometry and the impact velocity on the specific energy absorption have been studied and the results are presented.

The thesis has been written in the paper format. Brief description of the contents of each chapter is as follows.

Chapter 1 is an introduction to the topic and to the fundamental theories used in the work. It also contains the relevant literature search.

Chapter 2 deals with the sensitivity analysis of the Finite Element models. The element size has a strong impact on the results and this study was performed to arrive at the element size where the element size is no more a factor in the calculation of energy absorption.

Chapter 3 reports the work where the fiber angle in the tubes is changed, keeping other factors such as tube diameter and thickness constant.

Chapter 4 deals with the effect of the impact velocity on the energy absorption of a circular tube.

Chapter 5 is a paper on the effects of the tube geometry on the energy absorption of a circular tube. Here the tube diameters and the tube thickness are changed and energy absorption calculated.

Chapter 6 has a collection of the compendium of observations and conclusions of this research. It also contains the limitations of this work and suggestions for further research.
References


CHAPTER 2: FINITE ELEMENT MODEL OF A DYNAMIC COMPOSITE CRUSH EVENT

Abstract

This paper investigates the ability to use commercially available numerical modeling tools to approximate the energy absorption capability of long-fiber composite crush tubes. The motivation for the work comes from the need to reduce the significant cost associated with experimental trials. This study is significant since it provides a preliminary analysis of the suitability of LS-DYNA to numerically characterize the crushing behavior of a dynamic axial impact crushing event. This paper evaluates the influence of element size on the convergence of a solution. The ultimate goal is to begin to provide deeper understanding of a composite crush event and ultimately create a successful predictive methodology. The sensitivity of the element size on the energy absorbed is studies and an element size has been achieved below which the effect of the size is very small.

Introduction

Composite materials are engineered materials that consist of two or more materials that together produce desirable properties that cannot be achieved with any of the constituents alone. Long fiber structural composites consist of high strength and high modulus fibers generally surrounded by a weak matrix material. In these composites, fibers are the principal load carrying members. Increasingly, composites are seeing more use in load bearing structural designs. In many new applications the motivation to use composites is due largely to their energy absorption capability. The subject of study of this paper is the energy absorption of long fiber structural composite comprised of glass fibers within an epoxy matrix.
As it pertains to composite materials, the topic of material failure is not as absolute as it is for isotropic materials. Each lamina in a composite laminate has its own failure events associated with it. In regards to characterizing composite failure, it is noteworthy to point out that composites experience local failures and final failure; thus “first failure” does not necessarily correspond to “final failure.” The local failures are referred to as “damage,” and the development of additional local failures with increasing load or time is called “damage accumulation” [1]. While some failure theories have a physical basis, most theories represent attempts to provide mathematical expressions which give a “best fit” of the available experimental data. From the standpoint of a structural designer, it is desirable to have failure criteria which are applicable at the level of the fiber-matrix interface, the lamina, the laminate, and the structural component. Failure at these levels is often the consequence of an accumulation of various micro-level failures which coalesce and result in the final failure [1]. These micro failure events are critical, essential elements which comprise the macroscopic failure theories. Although each individual micro failure event is well known, their mechanics are not trivial.

In addition to composite failure theory, energy is absorbed via interlaminar and intralaminar crack growth within a composite. Fracture mechanics in particular, governs the formation of interlaminar and intralaminar cracking and how these ensuing cracks propagate [2]. Kannien states the basic equation of linear elastic fracture mechanics common in work on composites to be

\[ K_1(a, b, \sigma_{\infty}) = K_{lc}(T, \dot{\sigma}) \]  

where \( K_1 \) is a material independent function of the crack size, \( a \), the component dimensions, \( b \), and the applied stress, \( \sigma_{\infty} \), while \( K_{lc} \) is a material property that can depend upon temperature, \( T \), and loading rate, \( \dot{\sigma} \). For metals, the crack length is required to be large in comparison to the value of \( \left( \frac{K_{lc}}{\sigma_y} \right)^2 \) where \( \sigma_y \) is the corresponding yield stress; however this is not so in a composite. Actual fracture in a composite routinely violates this requirement [3]. As a result, composite finite element
solutions have yet to account for the mechanics of composite fracture. The extent to which fracture mechanics may govern the energy absorption capacity in a composite is still unknown.

With the increased use of fiber-reinforced composites in structural components, studies involving the behavior of components made of composites are receiving considerable attention. Functional requirements and economic considerations of design are forcing engineers to seek reliable and accurate, yet economical methods of determining static and dynamic characteristics of the structural components. The analytical study and design of composite materials requires knowledge of anisotropic elasticity, structural theories and failure/damage criteria. Unlike isotropic materials, anisotropic materials exhibit complicated mechanical behavior. The partial differential equations governing composite laminates of arbitrary geometries and boundary conditions cannot be solved in closed form; therefore, the use of numerical methods facilitates the solution. Among the numerical methods available for the solution of differential equations defined over arbitrary domains, the finite element method (FEM) is the most effective method. Finite element analysis of a structural problem is a numerical analysis of the mathematical model used to represent the behavior of the structure [4].

In the FEM, the solution domain is divided into a number of discrete elements. The displacements within an element are generally the unknown field variables that are expressed in terms of unknown nodal values. The governing load displacement relations for each element are written. These equations are then assembled maintaining continuity between the elements and equilibrium at the nodes. Finally, the boundary conditions are applied and the solution obtained for the load displacement relationship for the entire structure. The displacement field can be expressed as:

$$u = Nu$$  \[2-2\]

where the superscript, e, refers to element-wise values and N are the shape functions, being functions of the position coordinates. The strain displacement relationships from the strain matrix, \(\varepsilon\), are given by

$$\varepsilon = Bu$$  \[2-3\]
by differentiating the appropriate displacement components. The matrix B is the strain displacement matrix which relates strain to displacement. Application of the principal of minimization of the total potential energy yields the equation of motion

\[ Kq + Cq + Mq = f(t) \]  

2-4

where the matrices and vectors refer to the entire structure and furthermore M is the mass matrix, K is the stiffness matrix, C is the damping matrix and f(t) is the load vector which is comprised of thermal, inertial, body and traction loads. The associated matrix equation of free vibration may be written in the general form as

\[ Kq + Cq + Mq = 0. \]  

2-5

The numerical analysis of a structural system consists of two distinct, yet related solution procedures. First, a finite element model of the system yields a set of algebraic equations that are then solved by employing a suitable numerical procedure. Because of the very nature of finite element discretization, the resulting equations tend to be rather large in size, as well as highly banded for most practical problems. An economical solution of such problems poses as much a challenge to an analyst as the process of discretization itself [4].

For a dynamic FEM problem, \( N \) is not unique, it is in fact a function of the entire time history of the nodal displacements \([4]\). As a result, equation 2-3 becomes:

\[ u_v = N(\sigma)u' \]

\[ u_v = N(\sigma)q'e^{\omega t} \]

\[ u_v = g(\tau)e^{\omega t} \]

2-6

and equation 2-5 becomes

\[ [K_0 - \sigma^2(M_0 - K_0) - \sigma^4(M_2 - K_1) - ...]q = 0 \]

2-7

where \( q \) is the amplitude of the nodal deformation, and \( M_0 \) and \( K_0 \) are the static mass and stiffness matrices. The other higher order terms constitute the dynamics correction. Usually, the first 3 terms are sufficient for analysis. The result is a quadratic eigenvalue problem of the form:
\[
\( (A - \lambda B - \lambda^2 C) y = 0 \)
\]

where \( \lambda = \omega^2 \). These elements and their solutions are commonly referred to as finite dynamic elements (FDE) and the dynamic element method (DEM), respectively. Their solutions usually require higher order shape functions to achieve satisfactory convergence [4]. Second or third order shape functions usually give good results in static analyses; however, higher order shape functions may be required for dynamic problems. As can be expected, use of higher order polynomial shape functions increases the computational time of the analysis. Since this model employs constant stress/strain elements, an attempt was made to approximate the effect of increasing the “effective” polynomial order of the element shape function. This was done by employing an increasing number of constant stress/strain elements in the model.

For the purposes of gaining a better general understanding of how composites absorb energy and characterizing the ability of composites to absorb energy, experimental research has been very useful. However according to the observations of Farley [5], the behavior of the lamina bundles are directly affected by the length that these cracks propagate. It is theorized that these segmented composite columns (or bundles) not only react differently based upon the length of the cracking, but also absorb differing amounts of energy. Experimentally, the energy absorption associated with fragmentation (fracture of short lamina bundles) is notably larger than the energy absorption associated with splaying, which constitutes bending of longer lamina bundles. However, the extent to which crack propagation, in addition to other energy absorption modes, specifically affects energy absorption is still unknown.

There have been some noteworthy efforts in the attempt to use finite element analysis codes to arrive at a predictive methodology to determine the energy absorption in composite crush tubes. Sigalas and Kumosa [6] have been successful in modeling the sequence of events leading to progressive crushing of composite tubes. By limiting the crush behavior to the splaying mode only, Hamada and Ramakrishna [7] were successful in crafting a finite element model for predicting the
energy absorption capability of a composite crush tube which progressively crushed in the splaying mode only.

Stacking sequence can play an important role in maximizing the specific energy absorption within a multi-layered laminate. Although the need for the development for a complete finite element model to predict composite energy absorption has been widely researched and widely accepted, it is unknown as to how much energy absorption is associated with each constituent failure event. This paper is one in a series of papers that attempts to approximate the energy absorption using classical laminated plate theory and existing finite element method capabilities. In an attempt to arrive at a predictive methodology to determine the sustained specific energy absorption in a glass/epoxy thin walled composite tube, the dynamic finite element analysis tool, LS-DYNA has been used.

**Methodology**

**Materials**

Graphite/epoxy long fiber structural composite tubes absorb more energy per unit mass than any other structural long fiber composite; however, graphite fibers are relatively expensive. As a result, e-glass fibers may prove to be a reasonable and affordable alternative in many experiments. It is for this reason that e-glass fibers were selected. The matrix material used in this study was epoxy resin. Epoxy resin is one of the most common thermoset polymer matrix materials. Epoxies are relatively inexpensive but have better moisture resistance and lower shrinkage on curing. The resulting composite employs a fiber volume fraction of 62%. This is consistent with the make up of a typical e-glass/epoxy composite used experimentally [8].
**Tube Dimensions**

The tube dimensions were chosen to be fairly comparable to existing experimental data. The tubes were nominally 4" in length, 1.5" in mean diameter, $D$, and 6 plies thick. Each ply was of 0.0125" nominal thickness. A $D/t$ ratio of 14.85 was used. Each FEA tube had a simulated chamfer to help initiate a stable crush.

**Impact Characteristics**

The actual crush event was simulated by a translating rigid body of 3 slugs moving at an initial impact velocity of 25 ft/s. This is consistent with the comparable experimental results.

**Specimens**

The specimens evaluated in this study mirrored those used in a prior experimental study conducted by Farley [8]. Each tube was 6 plies thick and was constructed of E2 glass fibers and epoxy resin. The six plies had a stacking sequence of $[0^\circ/\pm\theta/0^\circ/\pm\theta]_T$, where $\theta$ was $15^\circ$, $20^\circ$, $30^\circ$, $40^\circ$, $45^\circ$, $50^\circ$, $75^\circ$ or $90^\circ$. The $0^\circ$ refers to the axial direction of the cylinders. Each LS-DYNA input deck was pre-processed using ANSYS, such that each tube was evaluated using 256, 832, 1728, 2944, 4480 and 6336 elements. Because polynomial elements were not available, the number of elements per solution was increased. As the number of elements increases, the model is able to be refined. By increasing the number of elements, it was analogous to incrementally increasing the effective shape function of the element.
Table 2-1: Element Selection Summary

<table>
<thead>
<tr>
<th>Total number of elements in model</th>
<th>Chamfer Elements</th>
<th>Through Thickness Elements</th>
<th>Effective Shape Function (no dim)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Length (in)</td>
<td>Height (in)</td>
<td>Aspect Ratio (no dim)</td>
</tr>
<tr>
<td>256</td>
<td>0.2926</td>
<td>0.0125</td>
<td>23.41</td>
</tr>
<tr>
<td>832</td>
<td>0.1470</td>
<td>0.0125</td>
<td>11.76</td>
</tr>
<tr>
<td>1728</td>
<td>0.0981</td>
<td>0.0125</td>
<td>7.85</td>
</tr>
<tr>
<td>2944</td>
<td>0.0736</td>
<td>0.0125</td>
<td>5.89</td>
</tr>
<tr>
<td>4480</td>
<td>0.0589</td>
<td>0.0125</td>
<td>4.71</td>
</tr>
<tr>
<td>6336</td>
<td>0.0491</td>
<td>0.0125</td>
<td>3.93</td>
</tr>
<tr>
<td>8512</td>
<td>0.0421</td>
<td>0.0125</td>
<td>3.37</td>
</tr>
</tbody>
</table>

The boundary conditions were selected to simulate the experimental impact loading. The bottom of the tube was modeled as clamped and the upper chamfered edge impacted was constrained in the hoop and radial directions [8]. The axial energy load was delivered via an impact mass dropped from a prescribed height to generate the desired amount of mechanical energy. A 45° chamfer was modeled into the top surface to trigger a more stable sustained crushing event. This chamfer was approximated by a step change in the ply thickness. The schematic of the ply arrangement is shown in figure 2-1.

LS-DYNA was then used to compute the resulting deformation of a number of crush tubes.

![Figure: 2-1: The chamfer formation in finite element code, t is the ply thickness.](image)
Computation of Energy Absorption

The specific energy absorption, $E_s$, shown in equation 2-9 is directly evaluated based upon the post crush peak axial displacement, such that

$$E_s = \frac{E_T}{\rho V}$$  \hspace{1cm} 2-9

where $E_T$ is the net impact kinetic energy, $\rho$ is the material density and $V$ is the volume of material displaced during and axial tube crush. In other words the specific energy absorption can be defined as the energy absorption per unit mass of the tube.

Finite Element Model

The numerical solution was obtained using the LS-DYNA software suite. LS-DYNA [9] is a general-purpose finite element code for analyzing the large deformation dynamic response of structures. The main solution methodology is based on explicit time integration. Spatial discretization is achieved by the use of four-node quadratic elements. The elements have through the thickness integration points so that the stresses in individual plies can be calculated.

The material model used by LS-DYNA is based on Classical Laminate Plate Theory (CLPT). It allows for the “stacking” of several “layers” of plies which may have arbitrary fiber orientation. Each ply is permitted its own failure criteria by use of its own integration point. Within this model, the primary means of energy absorption occurs as a result of matrix failure, fiber failure and laminate delamination. It is important to note that although this theory has been extremely accurate within the static realm, Farley [5] theorized that there are distinct differences in the failure mechanics in dynamic failure. Most notably, he states that in addition to energy absorption via the failure modes of CLPT, dynamic crush events experience energy absorption due to fiber and matrix fracture, bundle friction, interlaminar crack formation and intralaminar crack formation. However, as stated
previously, the scope of this work is to evaluate the ability/usefulness of the LS-DYNA CLPT to numerically approximate the dynamic response observed experimentally.

**Results and Discussion**

The first objective was to evaluate the influence of element size on the convergence of a solution. This was performed by checking where the solution achieved a steady state value. The influence of the element size is an important aspect to study for several reasons. This paper is written from the vantage point of a design engineer. As such, one of the prime considerations involved in the engineering design process is design cost and accuracy. Quite simply, the longer the design-phase the higher the design, development and production cost. However, even more important is the accuracy of the solution. The use of an increasing number of elements should, in theory, increase the accuracy of the results at the cost of time.

As shown in figure 2-2, there was marked improvement in the crush modeling as the number of elements increased.

![Figure 2-2: Axial crush effect with number of elements](image)

Figure 2-2: Axial crush effect with number of element (a) pre-crush, (b) 256, (c) 832, (d) 1728, and (e) 2944 elements.
For the purposes of evaluation, a solution was considered stable when the subsequent solutions’ energy absorption does not change by more than 2%. Figures 2-3 and 2-4 show the specific energy absorption of circular tubes of various stacking sequences as a function of number of elements.

Figure 2-3: Energy absorption as a function of number of elements, \(15^\circ-40^\circ\)
Figure 2-4: Energy absorption as a function of number of elements, 45°-90°

Among the configurations that clearly achieved a steady state value, the \([0°/±20°/0°/±20°]_T\), \([0°/±40°/0°/±40°]_T\) and \([0°/±50°/0°/±50°]_T\) laminate tube solutions achieved steady state solutions the quickest. These laminate stacking sequences became stable using only 832 elements. The other five stable solutions performed admirably as well. The \([0°/±45°/0°/±45°]_T\) and \([0°/±90°/0°/±90°]_T\) laminate tube solutions became stable at 1728 elements. The \([0°/±30°/0°/±30°]_T\) became stable using 2944 elements. And the \([0°/±15°/0°/±15°]_T\) laminate tube solution became stable employing 4480 elements. Generally, many of the solutions did eventually stabilize, however a few did not. Most notably, the \([0°/±55°/0°/±55°]_T\) and \([0°/±60°/0°/±60°]_T\) laminate tubes solution did not achieve a steady state solution.

In Finite Element analysis the structure is divided into small elements and the solution is obtained. A continuous domain is divided into small discreet elements. It is well known that the
smaller the elements used, the better and more stable the solution. When the element size is reduced
the approximation of the solution is over a smaller region and hence is a better solution. The smaller
elements are able to approximate the rapidly changing stress field in this analysis. This is especially
critical in the region where the stresses change rapidly and failure is imminent. As the element size is
reduced the solution approaches convergence. This is due for a couple of reasons; both of which
contribute to artificially increasing the elastic modulus of the material in question. A closer inspection
of Table 2-1 shows that for the 256, 832 and 1728 element solutions, the aspect ratios of the chamfer
elements are 23, 12 and 8 respectively. As a result an increase in the perceived material modulus is
observed. This is compounded by the fact that a coarser mesh, independent of the effects of aspect
ratio, also results in an increase in the effective stiffness seen within the model.

**Effect of Angle, \( \theta \), on Solution Convergence**

It was observed that not all stacking sequences converged to a steady state solution. This non-
convergence is related to the angle of the plies. As the ply angles increase the convergence becomes
difficult. An explanation could be that as the angles increase the circumferential strength increases
and the axial stiffness decreases. The loading on the element is very strongly directional and we have
to resort to extremely small elements to get a good convergence. It would appear that the angle, \( \theta \),
within the stacking sequence has a noticeable effect on the convergence of a steady state solution.
Where most solutions below 45° in the \([0°/±90°/90°/±90°]_T\) stacking sequence converged, some above 45°
did not. It has been documented experimentally [8] that energy absorption in \([0°/±90°/90°/±90°]_T\)
composite tubes increases as \( \theta \) increases. This is due to the increase in circumferential stiffness and
strength of the glass/epoxy composite layers when \( \theta \) increases. As \( \theta \) increases, it is known that energy
absorption increases. This is due to an increase in the number of laterally aligned fibers in the hoop
direction. Because fibers in the hoop direction are much stronger in tension than in compression, the
increase in energy absorption is very noticeable. This translates into an increased stiffness. It is possible that as the lateral strength increases, this increase adversely affects the energy absorption in tubes of high lateral strength.

**Evaluation of Solution Time**

In regards to solution time, the relationship exhibited was fairly linear as sh. The average solution time for a 4480 element (27,360 degrees of freedom) solution was approximately 2 days. Figure 2-5 shows the effect of the number of elements on the average solution time and average percent error with respect to the converged value. The solution time increases almost linearly with the number of element but the percent error drops vary rapidly in the beginning but then becomes asymptotic and converges to a low value.
Fig. 2-5: The effect of the number of elements on the average solution time and average percent error with respect to the converged value.

**Conclusions**

The effect of the number of elements is very significant in the convergence of the solution. Initially, as the number of elements reduces to about 2000 the percent error reduces rapidly. Beyond this the increase in the number of elements does not affect the percent error. The solution convergence time is essentially linear with the increase in the number of elements. Based upon this numerical study, it has been found that for the basic geometry of this composite crush analysis, 4480
elements are sufficient in achieving a steady-state solution irrespective of stacking sequence. This corresponds to a maximum aspect ratio of 4.7 and a maximum element size of $4.62 \times 10^{-3}$ in$^2$.

References


CHAPTER 3: THE EFFECT OF PLY ANGLE ON ENERGY ABSORPTION OF A CIRCULAR GLASS/EPOXY CRUSH TUBE

Abstract

Past research has conclusively shown that long fiber structural composites possess superior specific energy absorption characteristics as compared to steel and aluminum structures. However, physical testing of composites is costly and time consuming. As a result, numerical solutions are desirable as an alternative to experimental testing. Up until this point, very little numerical work has been successful in predicting the energy absorption of composite crush structures. This research chronicles some preliminary efforts to better understand the mechanics essential in pursuit of this goal. This research is one in a series of investigations that evaluates the degree of suitability and validity of employing a numerical method to model the dynamic crushing of long fiber structural composites. This paper investigates the effect of ply orientations on the specific energy absorption in a glass/epoxy composite crush tube.

Introduction

As a whole, composites have several additional advantages over traditional materials. Primarily, they are lighter and relatively stronger than many of the traditional metals that they replace. This affords engineers the opportunity to make much stronger and safer products. There are quite a few different types of materials that are classified as composites; however, for the purposes of this research, the term composites will be used to describe long fiber-reinforced composites with an epoxy matrix. As it pertains to the analysis of composites, their material behavior differs greatly from isotropic materials. Classical laminate plate theory (CLPT) has provided a great deal of insight into
characterizing the static behavior of composite laminates. Particularly, CLPT has been very useful in providing accurate analyses in the areas of transverse particle impact damage and damage from various combinations of static loading [1].

In the most general of terms, a structural material’s ability to absorb impact energy is generally referred to as the “crashworthiness” of the structure. In passenger vehicles, current legislation [2] requires that vehicles be designed such that, in the event of an impact at speeds up to 15.5 m/s (35 mph) with a solid, immovable object, the occupants of the passenger compartment should not experience a resulting force that produces a net deceleration greater than 20 g. Additionally, it is imperative that crashworthy structures are designed to crush in a fairly predictable and controlled manner. Traditionally, these crash structures have been constructed from structural steel. Although clearly useful as structural members, steel and other metals also come with relatively higher weight trade-offs and can ultimately lead to undesirable inertial effects as it pertains to crash events.

Since composites are comprised of individual layers of composite sheets, called lamina, each layer has its own failure events associated with it. In regards to characterizing composite failure, it is noteworthy to point out that composites experience local failures and final failure; thus “first failure” does not necessarily correspond to “final failure” [3]. There is no single theory that accurately predicts failure at all levels of analysis, for all loading conditions, and for all types of composite materials. From the standpoint of a structural designer, it is desirable to have failure criteria which are applicable at the level of the lamina, the laminate, and the structural component. Failure at these levels is often the consequence of an accumulation of various micro-level failures which coalesce and result in the final failure [3]. These micro failure events are critical, essential elements which comprise the macroscopic failure theories. There are many macroscopic failure theories that have been proposed for composites. The most notable of the general theories include the maximum stress, maximum strain, Tsai-Hill, tensor polynomial and Tsai-Wu failure criteria.
Although each individual micro failure event is well known, their mechanics are not trivial. In developing more appropriate fracture mechanics techniques for applications to fiber reinforced composite materials, several basic facts must be kept in mind. These include preparation defects (e.g., resin-starved or fiber starved areas), defects in laminates (e.g., fiber breaks, ply gaps, delamination), and fabrication defects (edge delamination caused by machinery, dents, and scratches) [3]. Nevertheless, even after controlling all of these requirements, crack propagation in a composite is still quite difficult to model. After a crack initiates it can grow and progressively lower the residual strength of a structure [4] to the point where it can no longer support design loads, making global failure imminent. A second key fact involved in the application of fracture mechanics to composite materials is the basic heterogeneous nature of fiber-reinforced composites. Within a ply, cracking can be both discontinuous (e.g., fiber bridging) and non-collinear crack growth (e.g., matrix splitting). On the laminate level, cracking can proceed in a distinctly different manner in different plies and, in addition, inter-ply delamination can occur. Consequently, many researchers have pursued an energy balance approach to the problem. This does not really present a significant improvement over the fundamental difficulties associated with the stress intensity factor point of view, however. As a result, composite finite element solutions have yet to account for the mechanics of composite fracture. The extent to which fracture mechanics may govern the energy absorption capacity in a composite is still under investigation.

In an effort to better understand the specific energy absorption capabilities of composite materials, much research has been conducted. The bulk of this research has been experimental trials which have been quite successful at evaluating the sustained specific energy absorption in composite crush tubes. It has been experimentally quantified [5] that graphite/epoxy composites absorb more energy per unit mass than both 6160 Aluminum and mild steel [6]. This is primarily due to the extremely high strength or modulus (not both) exhibited within the graphite fibers. The following
research has borne out some very useful relationships regarding the effects of fiber properties and fiber ply orientations.

In the most general sense, there are four major findings in regards to the effect of the fiber properties on energy absorption. Experimental trials involving the static crushing of both glass/epoxy and graphite/epoxy composites by Farley [5] suggest a decrease in the density of a fiber causes an increase in the specific energy absorption. Additionally, he found that the higher the strain to failure of a fiber, the higher the specific energy absorption. It is also noteworthy to point out that another experimental study found that when fiber reinforced tubes crush in similar modes, energy absorption is much more sensitive to changes in the fiber failure strain than changes in the fiber stiffness. However, Schmueser and Wickliffe [7] found that both graphite and glass tubes exhibited brittle failure modes which consist of fiber splitting and ply delamination. They further theorized that with respect to aramid fibers (which exhibit ductile, progressive folding energy absorption modes), the relative lower strain to failure is the culprit. These conflicting results clearly indicate there is some disagreement in whether glass and graphite fibers actually (1) exhibit the same crushing behaviors with respect to one another; (2) fail in different crush modes with respect to one another and; (3) are influenced by ply orientations differently with respect to one another.

In an attempt to better understand how composite laminate stacking sequence influences specific energy absorption with a glass/epoxy composite, a literature search was conducted. Of the numerous findings, the most noteworthy items can be summarized as four main ideas. Energy absorption within a composite crush structure is enhanced when one of four events occur. Specific energy absorption is increased when (1) the axial stiffness of the composite is increased; (2) the lateral stiffness of the composite is increased; (3) the number of fractured fibers is increased; and (4) the composites experience an increase in material deformation. Perhaps what stand out most among these findings are the obvious contradictions of the first two and the simplicity of the latter two. For
this reason, this paper takes a closer look at the seemingly dueling premises of the effects of both axially and laterally aligned fibers.

A closer inspection of the supporting data suggests that the energy absorption capacity in glass/epoxy tubes appears to be primarily influenced by the amount of lateral support present which is directly determined by fiber orientation [5], whereas, energy absorption in graphite/epoxy tubes is primarily affected by the amount of axial stiffness in the tube. Again the axial stiffness is directly determined by fiber orientation. In the case of the filament wound tubes, for some strange reason it appears that energy absorption is influenced by both axial stiffness and lateral support. To study these aforementioned associations, this paper will focus on the influence of axial and lateral effects on energy absorption. The results of this work will hopefully lead to the next steps of creating and proving a predictive methodology for this type of energy absorption.

Because of the ability for composites to be fabricated according to specific applications, the stacking sequence plays an increasing role in maximizing the specific energy absorption within a multi-layered laminate. This layering directly influences the energy absorption capability of the composite tube. It is of great importance to discern if the experimental relationship between laminate stacking sequence and energy absorption is observed in the numerical prediction during this phase of research. Although the need for the development for a complete FEM to predict composite energy absorption has been widely researched and widely accepted, it is unknown as to how much energy absorption is associated with each constituent failure event. This paper is one in a series of papers that attempts to approximate the energy absorption using CLPT and existing FEM capabilities. In an attempt to arrive at a predictive methodology to determine the sustained specific energy absorption in a glass/epoxy thin walled composite tube, the dynamic FEA tool, LS-DYNA was used.
Methodology

Numerical Solution

The numerical solution initially begins with finite element analysis and laminate constitutive relations. Laminate constitutive equations provide an efficient, yet highly effective, means of characterizing the behavior of a 3-D composite structure as if the composite were composed of several layers of 2-D plates. When the means of characterizing the general mechanical response of a laminated composite are provided, FEA can then be performed. Numerous FEA programs exist for the numerical analysis of composites. These programs primarily use discretization and numerical approximation to simulate structural and material response. However, very few of these techniques incorporate the layered analysis resulting from the laminate constitutive equations to model composite material behavior. Even fewer consider the complex highly coupled response to structural loads exhibited by anisotropic materials.

Currently FEA composite material models are quite capable of predicting and simulating the propagation of interlaminar and intralaminar crack propagation; although these algorithms are usually performed in fracture analyses only. Although less significant during static analysis, the complex secondary effects of interlaminar and intralaminar crack propagation are notably absent in many FEA composite material models. LS-DYNA is among the FEA software suites incapable of incorporating these secondary effects of composite inter and intralaminar cracking. However the laminate constitutive equation based material models offered by LS-DYNA have demonstrated a high degree of accuracy within the realm of quasi-static analyses.
FEA Model

The numerical solution was obtained using LS-DYNA [9] which is a general-purpose finite element code for analyzing the large deformation dynamic response of structures. The main solution methodology is based on explicit time integration. In this work we have chosen four-node quadratic layered solid elements.

The material model used by LS-DYNA is based on classical laminate plate theory (CLPT). It allows for the “stacking” of several “layers” of plies which may have any arbitrary fiber orientation. Each ply is permitted its own response by use of its own integration point. Within this model, the primary means of energy absorption occurs as a result of matrix failure, fiber failure and laminate delamination. It is important to note that although this theory has been extremely accurate within the static realm, Farley [8] theorized that there are distinct differences in the failure mechanics when compared to the dynamic and realm. Most notably, he states that in addition to energy absorption via the failure modes of CLPT, energy is also absorbed via fiber and matrix fracture, bundle friction, interlaminar crack formation and intralaminar crack formation in a dynamic crush event. However, as stated previously, the scope of this work is to evaluate the ability and usefulness of the LS-DYNA’s CLPT material model to numerically calculate and compare with the dynamic response observed experimentally.

Materials

Graphite/epoxy long fiber structural composite tubes absorb more energy per unit mass than any other structural long fiber composite; however, graphite fibers are relatively expensive. As a result, E-glass fibers may prove to be a reasonable and affordable alternative in many experiments. It is for this reason that E-glass fibers were selected. The matrix material used in this study was epoxy resin. Epoxy resin is one of the most common thermoset polymer matrix materials. Epoxies are relatively
inexpensive but have better moisture resistance and lower shrinkage on curing. Maximum use
temperatures of epoxies are in the vicinity of 175°C. The resulting composite employs a fiber volume
fraction of 62%. This is consistent with the make up of a typical E-glass/epoxy composite used
experimentally. [10]

Tube Dimensions

The tube dimensions were chosen to be comparable to existing experimental data. The tubes
were nominally 4” in length, 1.5” in mean diameter, $\bar{D}$, and are 6 plies thick. Each ply was of
0.0125” nominal thickness. $\bar{D}/t$ has a value of 14.85, where $t$ is the total wall thickness. Each FEA
tube had a simulated chamfer to help initiate a stable crush. Within the FEA model the simulated
chamfer consisted of step down uniform thickness plate element in contrast to the actual variable
thickness cross-sectional geometry of a “real” chamfer. This was also successful in avoiding
numerical instabilities. Each tube contained 6 plies, with a stacking sequence of $[0^\circ/\pm\theta/0^\circ/\pm\theta]_t$,
where $\theta$ was 15°, 30°, 45°, 60°, 75° or 90°.

Finite Elements

Each tube was preprocessed using 4480 elements. It has been documented [11] that 4480
elements yield stable and reliable solution.

Impact Characteristics

The actual crush event was simulated by a translating rigid body of 3 slugs moving at an
initial impact velocity of 25 ft/s. This is consistent with the comparable experimental results.
Post-Processing

Once input into an LS-DYNA input deck, the program is able to generate a wealth of structural output data. The data was then evaluated using both the graphical user interface and individual data points. Of primary interest was the total axial deflection of each composite crush simulation as well as an individual element analysis to evaluate the effect on the material model verses specific energy absorption.

Computation of Energy Absorption

The specific energy absorption, $E_{sp}$ in equation 3-1 is directly evaluated based upon the post crush peak axial displacement, such that

$$E_{sp} = \frac{E_{i}}{\rho V}$$  \hspace{1cm} 3-1

where $E_{i}$ is the net impact kinetic energy, $\rho$ is the material density and $V$ is the volume of material displaced during and axial tube crush.

Results and Discussion

In this analysis, preliminary evaluation of the performance of the LS-DYNA program was favorable. Although physical characteristics of the experimentally observed crushing behavior were missing, the algorithms of the numerical solution executed without any instabilities. It was observed that as the angle, $\theta$, increased, the energy absorption increased and the crush deflection decreased. These preliminary observations were promising precursors to the evaluation of the effect of fiber ply orientation on specific energy absorption.

Based on previous experimental work, it is observed that in ply orientations of $[0^\circ/\pm\theta^\circ/0^\circ/\pm\theta^\circ]_t$, generally the energy absorption values are steady for $\theta$ greater than $45^\circ$ and increase
fairly linearly as $\theta$ approaches 90°. Presented in figure 3-1 is the crush distance as the ply angle $\theta$ is increased from 10° to 90°.

![Figure 3-1: Crush distance as a function of $\theta$](image)

It is notable to observe that generally as the angle increases, the column height deflection dips around 45°. It rises up again rapidly and then the crush heights decrease fairly linearly up to 90°. Although there appears to be some differences there is some significant agreement between the experimental and numerical data. We will later try to explain this anomaly in the observations. Based upon the behavior exhibited between $\theta$ and crush height, it is to be expected that the energy absorption generally increases. Figure 3-2 shows the variation of the specific energy absorption as a function of
ply angle $\theta$. Again there is a spike around $45^\circ$ which suggests a brief increase in specific energy absorption.

![Figure 3-2: Specific energy absorption as a function of $\theta$](image)

This numerical solution is consistent with the experimental observations. Although there does not seem to be an absolute correlation between the experimental and numerical results, preliminary observations are promising. It is noteworthy that certain trends are shared amongst both data sets. Figure 3-3 presents a comparison between the numerical data obtained in this research and experimental data as reported by Farley [10].
Generally, the numerical data is comparable to the experimental values. As the angle increases beyond $45^\circ$, there is a sharp increase of specific energy absorption with respect to ply angle $\theta$. For $\theta$ less than or equal to $45^\circ$, the experimental data suggests a very flat slope; in comparison, the numerical value fluctuates significantly within this range. Observe also that the experimental data suggests an increase in the absorbed at $30^\circ$.

Considering the limitations inherent in the numerical model, it was very promising that there was some notable agreement with regard to experimental results. More importantly however the discrepancy gives rise to the opportunity to explore the mechanics involved in composite energy absorption. It was experimentally suggested that both the axial and lateral moduli play significant
roles in composite energy absorption, thus we will re-visit the question of equivalent laminate properties.

When a composite laminate is used as a structural material, the individual properties of the fibers, matrix, and the lamina are not important. As a structural component, the overall behavior of the laminate is of interest. The elastic moduli and the fiber orientation of each ply contribute to the overall stiffness and strength of the laminate. Fig. 3-7 presents the axial, lateral, and the shear modulus for a \([0^\circ/\pm\theta^\circ/0^\circ/\pm\theta^\circ]\) laminate as the ply angle \(\theta\) changes.

![Figure 3-4: \([0^\circ/\pm\theta^\circ/0^\circ/\pm\theta^\circ]\) composite plate effective elastic moduli as a function of \(\theta\)](image)

Axial stiffness increases and the lateral stiffness decreases as a function of \(\theta\). The important observation is that the shear modulus reaches a maximum at \(\theta\) equal to 45° and is lowest at both 0°
and 90°. Figure 3-5 presents the axial to lateral stiffness ratios as the ply angle $\theta$ changes. This shows that the relative stiffness, axial/lateral, is maximum at $\theta$ equal to 0° and minimum at $\theta$ equal to 90°.

![Graph showing the ratio of elastic moduli as a function of $\theta$.](image)

**Figure 3-5: Ratio of elastic moduli as a function of $\theta$**

These laminate properties allow us to determine the tube's effective modulus which in turn is used to then evaluate stresses, strains and failure events. In the CLPT material model, this is important because it assumes there are only five failure mechanisms available to absorb energy.
Since isotropic materials have no preference for orientation, the determination of material strength is straightforward [12]. The determination of composite strength is based on failure criteria, analogous to the von Mises criterion, where the interaction between stresses plays an important role. This quadratic interaction criterion is the basis for evaluating failure in a composite material, and can be represented in stress space as:

\[ F_y \sigma_x \sigma_y + F_t \sigma_t = 1 \]

The coefficients \( F_y \) and \( F_t \) can be easily related to the in-plane strengths of the lamina in tension, compression and shear. Fiber orientation has a profound effect on the strength of the resultant composite in each direction. It is these theories: stiffness and strength respectively, working in concert that formulate the basis for investigating how ply orientations affect energy absorption.

Compression load in the axial direction results in tensile stress in the lateral direction. When there are no fibers in the lateral direction then the loads have to be taken by the matrix and the fibers at an angle. More fibers in the lateral direction increase its lateral strength and the tube is more resistant to crushing. This of course, is the relationship exhibited experimentally between \( \theta \) and energy absorption. It would suggest that within a glass/epoxy composite, the composite should become more resistant to axial crushing as more fibers are oriented in the lateral direction (90°). The sensitivity of energy absorption due to the composite moduli was evaluated to study this. Figures 3-6 and 3-7 present the energy absorption as a function of axial and lateral modulus, respectively; no clear relationship of observed.
Figure 3-6: Energy absorption as a function of Effective Axial Modulus
Figure 3-7: Energy absorption as a function of effective lateral modulus

Figure 3-8 shows the energy absorption as a function of the ratio of axial/lateral modulus and a trend emerges very clearly. If modulus ratio is small the energy absorption is large and as the ratio increases the energy absorption reaches a low value.
Now we will re-visit the question of the dip in the energy absorption curve at an angle of 45°. We have seen that the shear modulus is highest at an angle of 45°. This means that at this angle the laminate of has its highest torsional stiffness. This stiffness results in the reduction of crush height and an increase in the specific energy absorption. Again, the question is why this phenomenon is not observed in the actual cylinders? Our conjecture is that the epoxy has a strain rate dependent response and this visco-elastic response tends to rotate the fibers. The fibers do not fracture but rotate, thus reducing the energy absorption.
Conclusions

A study of the ply angle on the specific energy response has been presented. Beyond an angle of 45°, a very good correlation is observed between the numerical results presented here and experimental results presented elsewhere. The response of the cylinder is not only dependent on the axial and lateral stiffness but also on the shear stiffness and strength, which reach maximum at a fiber angle of 45°. The discrepancy between the experimental and numerical below 45° angle samples can be attributed to the visco-elastic behavior of the epoxy. Further work in this area has to be done to better understand and quantify this phenomenon.

References


CHAPTER 4: THE EFFECT OF IMPACT VELOCITY ON ENERGY ABSORPTION OF A CIRCULAR GLASS/EPOXY CRUSH TUBE

Abstract

Dynamic crushing behavior of composite tubes has been experimentally found to be influenced by impact velocity, tube geometry and ply orientations. This research attempts to investigate the possibility of constructing a predictive methodology to determine the energy absorption capability of a composite crush tube. The motivation for the work comes from the need to be provided a lower cost alternative to the fabrication and destructive testing of composite crush structures. This research is one in a series of investigations that evaluates the degree of suitability and validity of employing a numerical method to model the dynamic crushing of long fiber structural composites. This paper investigates the effect of impact velocity on the specific energy absorption in a glass/epoxy composite crush tube.

Introduction

Over the years, long fiber, reinforced composites have been steadily integrated into many of our everyday products. Among the industries that have seen far reaching successful integration of composites is the aerospace industry and the sporting goods. Within the aerospace industry, the use of composites have led to the significant reduction of component parts and decreased weight. Automotive industry has also started taking a keen interest in the use of composites in their structures. However, the auto industry primarily uses composites in the semi-structural or decorative parts; hood, decklids, doors and bumpers. Based largely in part to the overwhelming success of composite use in
the aerospace industry, automotive composites are now seeing increasing use as load bearing structural members. Composite’s energy absorption capability is one of the main motivators for the increased automotive applications.

The ability to absorb impact energy and be survivable for the occupants is called the “crashworthiness” of the structure. Current legislation for automobiles requires that vehicles be designed such that, in the event of an impact at speeds up to 15.5 m/s (35 mph) with a solid, immovable object, the occupants of the passenger compartment should experience a resulting force that produces a net deceleration less than 20 g. Within the aerospace industry, many rotary aircraft specify specific low impact crashworthiness requirements that are more easily facilitated by the use of composites [1].

Since composites are anisotropic and changes in stacking sequence influence macroscopic material properties, these infinite configurations lead to infinite unique energy absorption characteristics. This is compounded by the fact that there are several combinations of unique fiber and matrix materials that can be combined to form unique composites, each with unique material properties. Full understanding of these numerous combinations would involve great expense and time in any experimental research therefore a numerical solution may be an inexpensive alternative. However, it is important to note that the analytical study and design of composite materials requires knowledge of anisotropic elasticity, structural theories, failure/damage criteria and fracture mechanics.

In a crush event it has been theorized that during axial compression of composite tubes, both interlaminar and intralaminar cracking play an integral role in the energy absorption process. This is very consistent with what is observed during experimental testing. Fracture mechanics in particular governs the formation of interlaminar and intralaminar cracking and how these ensuing cracks propagate [2]. These cracks, in turn, cause the formation of lamina bundles which may fracture or bend, depending on the loads experienced by each bundle. It is of great significance to note the
absence of fracture mechanics into composite laminate plate elements in standard commercially available FEA software packages.

Compared to most popular isotropic metals, composites have a much higher specific energy absorption capacity; meaning that they absorb more energy per unit mass. The energy absorption mechanisms of composites have only recently become well known and much work is to be done to fully understand them. Farley [3] has theorized that there are three main crushing initiators involved in most crush events and the fourth is a hybrid crushing initiator which is the primary means which long fiber structural composites absorb energy. This mode is characterized by the formation of lamina bundles of moderate length during impact. As a result, composite energy absorption is due to fracture, friction and bending of the fiber bundles. Conversely metals, when subject to a compressive load, experience a buckling crush mode. This is primarily due to the high degree of plasticity exhibited. Predictable progressive folding, stable post crushing integrity and virtually unchanged material properties characterize this mode. The resulting specific energy absorption however, is noticeably lower than that exhibited by high performance structural composites.

Extensive experimental research has borne out some very useful relationships regarding the effects of fiber properties, ply orientations, impact velocity and tube geometry [1]. In most of this work there seems to be a lack of consensus about the influence of impact velocity on the energy absorption. Past experimental investigation has found that in some cases the impact velocity plays a very important role in the energy absorption in a crush tube while none at all in others. However, it is observed that energy absorption capability is a function of testing speed when the mechanical response of the crushing mechanism is a function of strain rate. Finally, it has been experimentally determined that the velocity at which the structure is loaded has an effect on the material's crushing behavior [4]. It is theorized that this is driven by a decrease in composite fracture toughness as velocity increases.
Up to now, all insights gained have been through experimental work. For the purposes of gaining a better general understanding of how composites absorb energy and characterizing the ability of composites to absorb energy, both experimental and limited numerical research has been very useful. Among the numerical methods available for the solution of differential equations defined over arbitrary domains, the finite element method (FEM) is the most effective method [5]. There have been some noteworthy efforts in the attempt to use finite element analysis codes to arrive at a predictive methodology to determine the energy absorption in composite crush tubes [6,7]. Although the need for the development for a complete finite element method to predict composite energy absorption has been widely researched and widely accepted, it is unknown as to how much energy absorption is associated with each constituent failure event. This paper is one in a series of papers that attempts to approximate the energy absorption using composite laminated plate theory and existing finite element method capabilities. In an attempt to arrive at a predictive methodology to determine the sustained specific energy absorption in a glass/epoxy thin walled composite tube, the dynamic finite element analysis tool, LS-DYNA [8] was used. Specifically, this work investigates the capability of LS-DYNA in discerning the influence of impact speed on the specific energy absorption within a glass/epoxy composite crush tube.

Methodology

Finite Element Model

The numerical solution was obtained using the LSDYNA software suite. LS-DYNA is a general-purpose finite element code for analyzing the large deformation dynamic response of structures. The main solution methodology is based on explicit time integration. Spatial discretization is achieved by the use of four-node quadrilateral elements. The material model used by LSDYNA is based on classical laminate plate theory. It allows for the “stacking” of several “layers” of plies which
may have arbitrary ply orientations. Each ply is permitted its own response by use of its own integration point. Within this model, the primary means of energy absorption occurs as a result of matrix failure, fiber failure and laminate delamination. It is important to note that although this theory has been extremely accurate within the static realm, Farley theorized that there are distinct differences in the failure mechanics when compared to the dynamic realm. Most notably, he states that in addition to energy absorption via the failure modes of CLPT, in a dynamic crush event energy is also absorbed via fiber and matrix fracture, bundle friction, interlaminar crack formation and intralaminar crack formation. However, as stated previously, the scope of this work is to evaluate the ability and usefulness of the LS-DYNA composite material model to numerically predict the dynamic response observed experimentally. Each “virtual” tube investigated in this work was preprocessed using 4480 composite shell elements. It has been documented that 4480 elements yield a very stable solution without excessive computational costs [9].

**Materials**

Graphite/epoxy long fiber structural composite tubes absorb more energy per unit mass than any other long fiber structural composite; however, graphite fibers are relatively expensive. As a result, E-glass fibers may prove to be a reasonable and affordable alternative in many experiments. It is for this reason that E-glass fibers were selected. The matrix material used in this study was epoxy resin. Epoxy resin is one of the most common thermoset polymer matrix materials. Epoxies are relatively inexpensive but have better moisture resistance and lower shrinkage on curing. Maximum use temperatures of epoxies are in the vicinity of 175°C. The resulting composite employs a fiber volume fraction of 62%. This is consistent with the make up of a typical E-glass/epoxy composite used experimentally [10].
Tube Dimensions

The tube dimensions were chosen to be comparable to existing experimental data. The tubes were nominally 4" in length, 1.5" in mean diameter and 6 plies thick. Each ply was of .0125" nominal thickness. \( \frac{D}{t} \) has a value of 14.85 where \( t \) is the wall thickness. Each finite element tube had a simulated 45° chamfer to help initiate a stable crush. This chamfer was approximated by a step change in the ply thickness in contrast to the actual variable thickness cross-sectional geometry of a "real" chamfer. This was successful in avoiding numerical instabilities.

Impact Characteristics

The actual crush event was simulated by the creation of a translating rigid body. In this study, the rigid body had a mass of nominally 3 or \( \frac{1}{2} \) slugs. For the 3-slug impact study, the velocity varied from 2.5 to 50 ft/s. For the \( \frac{1}{2} \)-slug impact study, the velocity varied from 5 to 100 ft/s. This is consistent with the comparable experimental results [4].

Post-Processing

Once input into LS-DYNA, the program is able to generate a wealth of structural output data. The data was then evaluated using both the graphical user interface and individual data points. Of primary interest was the total axial deflection of each composite crush simulation as well as an individual element analysis to evaluate the effect on the material model verses specific energy absorption.
Computation of Energy Absorption

The specific energy absorption, $E_{sp}$, is directly evaluated based upon the post crush peak axial displacement, such that

$$E_{sp} = \frac{E_i}{\rho V}$$

where $E_i$ is the net impact kinetic energy, $\rho$ is the material density and $V$ is the volume of material displaced during and axial tube crush.

Results and Discussion

Preliminary evaluation of the performance of the LSDYNA program was favorable. Although physical characteristics of the experimentally observed crushing behavior were missing, the algorithms of the numerical solution executed without any numerical instabilities. Generally, it was observed that as impact velocity increased, energy absorption values became more stable. These preliminary observations were promising precursors to the evaluation of the effect of impact velocities and impact energies on the energy absorption capacity of glass/epoxy composite crush tubes.

Impact Velocity and Impact Energy

Although there has yet to be clear consensus, there has been extensive experimental evaluation on the effect of impact velocity on specific energy absorption [4, 11-13]. As a result of this notable discrepancy the first task was to evaluate what, if any, effect velocity has on energy absorption within a numerical solution. Figure 4-1 shows the energy absorption as a function of impact velocity. It is observed that within both the 3-slug and $\frac{1}{2}$-slug data sets, high absorption takes place at low impact velocities and then the absorption reduces to a lower value with little variation.
The relationship between the impact velocity and energy absorption is not very clear from this data set; neither is the effect of increasing the impact velocity.

![Graph showing energy absorption as a function of velocity](image)

**Figure 4-1: Energy absorption as a function of velocity**

In Figure 4-2, the impact velocity is plotted as a function of crush height. Distinctly similar phenomenon is observed within the 3-slugs and $\frac{1}{2}$-slug data sets respectively. Both trend lines are clearly characterized as 2nd order behavior such that they are governed by the basic equation that

$$z = av^2$$

where $a$ has values of 0.39 s$^2$/ft and 0.056 s$^2$/ft for the $\frac{1}{2}$ slug and 3 slug trend lines, respectively.
From these line equations we clearly conclude that there are two unique relationships characterizing each trend line simultaneously. Preliminarily, this would seem to suggest that impact velocity plays some role on the net deflection of a composite crush tube. However, revisiting the general curve equation, it is immediately interesting that its form leads to a familiar and fundamental observation. The kinetic energy for a body of known mass, \( m \), and initial velocity, \( v \) is

\[
E = \frac{1}{2}mv^2. \tag{4-2}
\]

Furthermore, for every velocity, \( v \), in each data set, there exists an impact energy \( E_i \) such that both correspond to an identical crush height \( z_i \). Therefore it is clear that
\[ z_{1i} = C_1E_{1i} = a_1v_{1i}^2 \]  

and

\[ z_{2j} = C_2E_{2j} = a_2v_{2j}^2 \]

Here subscripts 1 or 2 correspond to the \( \frac{1}{2} \) or 3-slug data sets, respectively. Note that the 3-slug (nominal) mass is exactly 6.25 times larger than the \( \frac{1}{2} \) (nominal) slug mass,

\[ m_i = 6.25m_3 \]  

Finally recognizing that \( a_i = 5.75a_2 \), it is clear that the ensuing relationship between \( z \) and \( E_i \) is essentially linear, and

\[ C_1 = \frac{11.5a_2}{m_1} = 1.357 \times 10^{-4} \text{ slug/lb} \]  

and

\[ C_2 = \frac{12.5a_2}{m_1} = 1.357 \times 10^{-4} \text{ slug/lb} \]

The predicted value for both \( C_1 \) and \( C_2 \) compare favorably to the actual values taken directly from figure 4-3.

Figure 4-3: Crush height as a function of impact energy
Moreover, the actual relationship is indeed linear as predicted. Most importantly, $C_1$ and $C_2$ have an 8\% difference with respect to one another. Although not conclusive, this indeed suggests a fairly strong relationship between crush height and impact energy. As a result, this would suggest that impact velocity is much less significant in influencing energy absorption than impact energy.

Figure 4-4 shows a much clearer relationship between energy absorption and impact energy. Again, the higher initial energy values are associated with impact energies less than 2500 in-lb. Moreover for initial impact energies greater than this, energy absorption stabilizes at a constant value. Additionally, this threshold appears to be universal to both data sets. In short, specific energy absorption is far less dependent on impact velocity as is on impact kinetic energy. Although not yet completely quantified, a much clearer relationship results when evaluating the effect of impact energy on the total sustained specific energy absorption of the glass/epoxy long fiber composite crush tubes.
Energy Absorption Regions

Closer inspection of the figure 4-4 also reveals that energy absorption is independent of both impact mass and impact velocity. More importantly there are at least two distinct crush morphologies resulting in 2 distinct classes of energy absorption. In the first region, energy absorption values range from 54 to 117 kJ/kg, with an average value of 100 kJ/kg with a median value of 113 kJ/kg. In the second region, energy absorption values range from 40-63 kJ/kg with an average value of 59kJ/kg with a median value of 57 kJ/kg. Up to this point, the bulk of research on composite energy absorption primarily has been concerned with energy absorption values resulting from destructive
failure modes. However this new data suggests the existence of less destructive failure characteristics occurring at lower impact energy.

Appearance of abnormally high energy absorption values below impact energy of 2500 in-lbs clearly suggests some elastic response in the crush tube. Glass/epoxy composites fail in a brittle manner with no plastic deformation. This high energy absorption suggests that elastic strain energy is contributing to the energy absorption. The amount of spring back was evaluated to verify this conjecture. Any material loaded within its elastic region can be unloaded such that it may regain a portion of its pre-loaded length. In an attempt to evaluate the occurrence of this phenomenon, the spring back percentage was measured where spring back percentage is the measure of restored length compared to the maximum deflection during the crushing process.

**Elastic Response Energy Absorption Region**

It was observed that after the peak column deflection occurred, the amount of post crush column restoration, or spring back percentage differed greatly between the two regions. The spring back is defined as the difference between the maximum deflection in the column and the post crush equilibrium restoration distance. It was determined earlier that column deflection varies linearly with impact energy, hence the percentage of spring back with respect to the maximum deflection observed in the column in question is presented in figure 4-5. In the first region, the spring back values varied from 13.2% to 45.3%.
The results show that below an impact energy of 2,500 in-lbs there is significant spring back, while above 2,500 in-lbs the spring-back percentage practically goes to zero. This shows that the two regions represent different types of energy absorption phenomenon. This threshold, not coincidentally, corresponds to the impact energy threshold previously identified.

**Brittle Response Energy Absorption Region**

The phenomenon of elastic response is a plausible explanation in characterizing this first energy absorption region. However the next question is: What is the limit of this relationship? This
problem can be approached from two different aspects. The first is the logical extension of elastic loading; an investigation of where ultimate strength occurs. The second possible answer comes from the buckling phenomenon. The critical buckling load, $N_{Cr}$, from the eigenvalue problem of a thin walled composite tube gives, is given by:

$$
N_{Cr} = \frac{\beta^4 D_{11} + 2 \beta^2 n^2 (D_{12} + 2 D_{66}) + n^4 D_{22}}{\beta^4 r^2} \frac{\beta^2 E_\theta h}{\beta^4 + 2 \beta^2 n^2 \left( \frac{E_\theta - E_s}{2G_{xy}} \right) + n^4 \frac{E_\theta}{E_s}}
$$

where $\beta$ is the axial frequency, $D_{11}$ is the bending stiffness in the axial direction, $n$ is the circumferential full wave number, $D_{12}$ is the bending stiffness in the axial-radial plane, $D_{66}$ is the bending stiffness in the normal direction, $D_{22}$ is the bending stiffness in the radial direction, $r$ is the radius of the cylinder, $E_\theta$ is the effective radial stiffness of the composite layers, $t$ is the total shell thickness, $G_{xy}$ is the effective shear modulus of the composite layers, $\nu_{xy}$ is the effective Poisson’s ratio in the axial-radial plane, and $E_s$ is the effective axial stiffness of the composite layers [14].

Infinite numbers of buckling loads exist for a tube, each of which is associated with unique axial and circumferential wave frequencies. It is noteworthy to point out that the critical buckling load is not always associated with the lowest axial (where $m=1$) and lowest circumferential (where $n=0$) frequencies. To find the critical buckling load it is necessary to evaluate equation 4-6 among a range of $m$ and $n$. The critical buckling load is the lowest buckling load irrespective of the values of $m$ or $n$.

For a 4” column with a mean radius of $\frac{3}{4}$ “, the lowest theoretical load of 26,800 lbs occurs where $n=0$ and $m=8$. This value is then compared to the peak crushing load exhibited in each column. It is observed that $N_{Cr}$ exceeds the peak loads found in the test columns as shown in figure 4-6.
The critical buckling load is higher than the impact force exhibited in individual tubes. It should be kept in mind that the buckling load here is static while the impact force is a dynamic event. These results show that the dominant failure mode is crushing and the buckling does not occur in the results presented here. It can then be concluded that in any crushing event with tubes of this type, the first energy absorption mode would be elastic deformation and then once the material has exceeded the critical compressive failure loads crushing would occur. Now we have to be careful about these two failure modes. It is very possible that if the D/t ratio is very large then the buckling would occur before crushing. In that case the energy absorption would be elastic, then buckling and bending.
Conclusions

We have presented here the crush of a glass-epoxy composite tube failure under compressive load. The effect of the initial impact energy has been studied in detail. The initial impact energy has a profound effect on the crush failure energy absorption mode. For the tubes used in this study, the energy absorption at low impact energies is mainly due to the elastic strain energy. This has been confirmed by the elastic spring back study. Finally, for all tubes, the buckling load was higher than the crush load it can be concluded that the energy absorption was due to compressive crushing.

References


CHAPTER 5: THE EFFECT OF TUBE GEOMETRY ON ENERGY ABSORPTION OF A CIRCULAR GLASS/EPOXY CRUSH TUBE

Abstract

In experimental testing, composite crush structures have been found to experience decreasing energy absorption capability as the diameter to thickness ratio of the tube increases. Geometry, along with impact velocity and fiber stacking sequence, play important roles in influencing the overall specific energy absorption in a composite crush event. Because of the cost associated with experimental destructive testing, numerical alternatives have been investigated. This research attempts to investigate the feasibility of constructing a numerical methodology to determine the energy absorption capability of a composite crush tube, where the ultimate goal is to construct a predicative methodology based upon the validation of the engineering mechanics governing the dynamic response of composite crush tubes. The effect of the tube geometry on the specific energy absorption in a glass/epoxy composite tube has been studied here.

Introduction

As composite research and technology continues to grow, so does their use in an increasing number of applications. Composites have several advantages over traditional materials. Primarily, they are lighter and relatively stronger than many of the traditional metals that they replace. This affords engineers the opportunity to make much stronger and safer products. There are quite a few different types of materials that are classified as composites; however, for the purposes of this research, the term composites will be used to describe long fiber-reinforced composites with an epoxy matrix. As it pertains to the analysis of composites, their material behavior differs greatly from isotropic materials. Among the industries that have seen far reaching successful integration of
composites is the aerospace industry. Within the aerospace industry, the use of composites have led to
the significant reduction of component parts and decreased weight. In the auto industry, however, the
primary use of composites has been semi-structural or decorative parts; primarily, hood, decklids,
doors and bumpers. Based largely in part to the overwhelming success of composite use in the
aerospace industry, automotive composites are seeing increasing use as load bearing structural
members. This increased use of composites in many new applications is due largely to composite
energy absorption.

In passenger vehicles the ability to absorb impact energy and be survivable for the occupants
is called the “crashworthiness” of the structure. Current legislation for automobiles requires that
vehicles be designed such that, in the event of an impact at speeds up to 15.5m/s (35mph), the
occupants of the passenger compartment should not experience a resulting force that produces a net
deceleration greater than 20g. Use of composite materials in the aerospace industry is also facilitating
the crashworthiness requirements [1]. Compared to most popular isotropic metals, composites have a
much higher specific energy absorption capacity; meaning that they absorb more crush energy per
unit mass. Metals, when subject to a compressive load, experience a buckling crush mode. This is
primarily due to the high degree of plasticity exhibited. Predictable progressive folding, stable post
crushing integrity and virtually unchanged material properties characterize this isotropic crushing
mode. The resulting specific energy absorption is noticeably lower than that exhibited by high
performance structural composites.

In regards to the particulars of composite material energy absorption, their energy absorption
mechanisms have only recently become well known. Farley [2] has theorized that there are three main
crushing initiators involved in most crush events and the fourth is a hybrid crushing initiator. This 4th
initiator or brittle fracturing is the primary means which long fiber structural composites absorb
energy. This mode is characterized by the formation of lamina bundles of moderate length during
impact. As a result, composite energy absorption is due to fracture, friction and bending of the fiber
bundles. Analytically, composite laminate plate theory (CLPT) has provided a great deal of insight into characterizing the static behavior of composite laminates. Particularly, classical laminate plate theory has been very useful in the study of transverse particle impact damage and damage from various combinations of static loading [3].

The bulk of the research in this area has been experimental work, which has been quite successful at evaluating the sustained specific energy absorption in composite crush tubes. Crush tubes, due to their high moment of inertia, do not fail in buckling but absorb the impact load in the crushing mode. Using these tubes, it has been experimentally quantified [4] that graphite/epoxy composites absorb more energy per unit mass than both 6160 aluminum and mild steel. This is primarily due to the extremely high strength or modulus (not both) exhibited in the fibers. Additionally, it is of some importance that extensive experimental research has borne out some very useful relationships regarding the effects of fiber properties, stacking sequences, impact velocity and tube geometry. Most of the literature generally revealed that the tube dimensions influence the crush zone fracture mechanisms. Specifically it is suggested that by changing the tube dimensions, the crush mechanisms can be controlled [5]. This would imply that tube dimensions play a significant role in specific energy absorption. It has been experimentally determined by Farley [6] that tube geometry has an effect on energy absorption of both graphite/epoxy and Kevlar/epoxy tubes. Specifically it has been documented that as the diameter to tube-thickness-ratio increases, a decrease in energy absorption is experienced in both graphite/epoxy and Kevlar tubes. This is primarily due to an increase in the formation of interlaminar cracking. It remains to be seen if a similar response is present in glass/epoxy tubes.

For the purposes of gaining a better general understanding of how composites absorb energy and characterizing the ability of composites to absorb energy, experimental research has been very useful. However, at the same time, these research endeavors require notable capital expenditures due to a variety of reasons. Composites are anisotropic and changes in stacking sequence influence
macroscopic material properties; these infinite configurations lead to an infinite number of unique energy absorption profiles. This is compounded by the fact that there are several combinations of unique fiber and matrix materials that can be combined to form a unique composite with unique material properties. A numerical solution to this problem would greatly reduce the cost of traditional experimental endeavors. The success of the preceding experimental trials may lead to the next steps of creating and proving a predictive methodology for this type of energy absorption.

It is important to note that the analytical study and design of composite materials requires knowledge of anisotropic elasticity, structural theories and failure/damage criteria. Unlike isotropic materials, anisotropic materials exhibit complicated mechanical behavior. Upon closer investigation of the expected behavior, it has been observed that during axial compression of composite tubes both interlaminar and intralaminar cracking play an integral role in the energy absorption process. These cracks, in turn, cause the formation of lamina bundles which may fracture or bend, depending on the application of the loading experienced by each bundle. In regards to commercially available finite element analysis software packages, there are none that incorporate these fracture mechanics into composite laminate plate elements.

The use of numerical methods facilitates the solution of composite behavior, and failure mechanics equations for problems of practical importance. The finite element method (FEM) is the most effective and industry accepted method for the solution of structural analysis [7]. There have been some noteworthy efforts in the attempt to use finite element analysis codes to arrive at a predictive methodology to determine the energy absorption in composite crush tubes [8, 9]. Although the need for the development for a complete finite element model to predict composite energy absorption has been widely researched and widely accepted, it is unknown as to how much energy absorption is associated with each constituent failure event. This paper is one in a series of papers that attempts to approximate the energy absorption using CLPT and existing finite element method capabilities. In an attempt to ultimately arrive at a predictive methodology to determine the sustained
specific energy absorption in a glass/epoxy thin walled composite tube, the dynamic finite element analysis tool, LS-DYNA was used.

**Methodology**

**Finite Element Model**

The numerical solution was obtained using LS-DYNA software suite. This software is chosen because of its availability, acceptance by the industry and convenience of use. The main solution methodology is based on explicit time integration. An implicit solver is currently available with somewhat limited capabilities including structural analysis [10]. The material model used by LS-DYNA is based on composite laminate plate theory. It allows for the “stacking” of several “layers” of plies with arbitrary ply orientations. Each ply is permitted its own response by use of its own integration point. Within this model, the primary means of energy absorption occurs as a result of matrix failure, fiber failure and laminate delamination. It is important to note that although this theory has been extremely accurate within the static realm, Farley theorized that there are distinct differences in the failure mechanics when compared to the dynamic and realm. Most notably, he states that in addition to energy absorption via the failure modes of CLPT, in a dynamic crush event energy is also absorption via fiber and matrix fracture, bundle friction, interlaminar crack formation and intralaminar crack formation.

The immediate objective of this research is to study the effect of the tube size on the energy absorption under a compressive load. The model of the tube is approximated by small elements. The size of the elements dictates the accuracy and stability of the solution process. The results presented here are for a tube divided into 4480 elements. For the details of arriving at this number interested reader is referred to [11].
Materials

Graphite/epoxy long fiber structural composite tubes absorb more energy per unit mass than other structural long fiber composite. The high cost of graphite fibers dictates that E-glass fibers may prove to be a reasonable and affordable alternative. It is for this reason that E-glass fibers were selected in this work. The matrix material used in this study was epoxy resin. Epoxy resin is one of the most common thermoset polymer matrix materials. Epoxies are relatively inexpensive and have better moisture resistance and lower shrinkage on curing. Maximum use temperatures of epoxies are in the vicinity of 175°C. The resulting composite employs a fiber volume fraction of 62%. This is consistent with the make up of a typical E-glass/epoxy composite used experimentally [12].

Tube Dimensions

The tube dimensions were chosen to be fairly comparable to existing experimental data; while also chosen to span a reasonable spectrum of values. The tubes were nominally 4” in length, 3¼", 1½", 2¼” and 3” in mean diameter and 3, 6, 9 or 12 plies thick. Each ply was of 0.0125” nominal thickness. Depending on the geometry of the tube in question, the diameter to thickness ratio varied from a minimum value of 3.71 to a maximum value of 59.41 (Table 5-1). Each finite element modeled tube had a simulated chamfer to help initiate a stable crush. Within the finite element model, the simulated chamfer consisted of a uniform thickness, step-wise, decreasing plate element in contrast to the actual variable thickness cross-sectional geometry of a “real” chamfer. These slight modifications were successful in avoiding numerical instabilities within the finite element analysis. Each tube contained 3n plies, with a stacking sequence of \([0°/±45°]_n\), where \(n=1, 2, 3\) or 4.
Impact Characteristics

The actual crush event was simulated by a translating rigid body of 3 slugs moving at an initial impact velocity of 25 ft/s. This is consistent with the comparable experimental results.

<table>
<thead>
<tr>
<th>Specimen Name</th>
<th>Number of Plies (no dim)</th>
<th>Mean Radius (inches)</th>
<th>D/t Ratio (no dim)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2t_2r</td>
<td>6</td>
<td>0.75000</td>
<td>14.85149</td>
</tr>
<tr>
<td>2t_3r</td>
<td>6</td>
<td>1.12500</td>
<td>22.27723</td>
</tr>
<tr>
<td>2t_4r</td>
<td>6</td>
<td>1.50000</td>
<td>29.70297</td>
</tr>
<tr>
<td>3t_1r</td>
<td>9</td>
<td>0.37500</td>
<td>4.95050</td>
</tr>
<tr>
<td>3t_2r</td>
<td>9</td>
<td>0.75000</td>
<td>9.90099</td>
</tr>
<tr>
<td>3t_3r</td>
<td>9</td>
<td>1.12500</td>
<td>14.85149</td>
</tr>
<tr>
<td>3t_4r</td>
<td>9</td>
<td>1.50000</td>
<td>19.80198</td>
</tr>
<tr>
<td>4t_1r</td>
<td>12</td>
<td>0.37500</td>
<td>3.71287</td>
</tr>
<tr>
<td>4t_2r</td>
<td>12</td>
<td>0.75000</td>
<td>7.42574</td>
</tr>
<tr>
<td>4t_3r</td>
<td>12</td>
<td>1.12500</td>
<td>11.13861</td>
</tr>
<tr>
<td>4t_4r</td>
<td>12</td>
<td>1.50000</td>
<td>14.85149</td>
</tr>
</tbody>
</table>

Table 5-1: Specimen summary

Post-Processing

Once input into LS-DYNA, the program is able to generate a wealth of structural output data. The data was then evaluated using both the graphical user interface and individual data points. Of primary interest was the total axial deflection of each composite crush simulation as well as an individual element analysis to evaluate the effect on the material model verses specific energy absorption.

Computation of Energy Absorption

The specific energy absorption in equation 5-1 is directly evaluated based upon the post crush peak axial displacement, such that
where $E_i$ is the net impact kinetic energy, $\rho$ is the material density and $V$ is the volume of material displaced during and axial tube crush.

**Results and Discussion**

Preliminary evaluation of the performance of the LS-DYNA program was favorable and the numerical algorithms of the numerical solution executed as expected. There were no numerical instabilities or other problems. Generally, it was observed that as cross-sectional area increased, the energy absorption increased and the crush deflection decreased; however, this was not entirely universal. These preliminary observations were promising precursors to the evaluation of the effect of diameter, the effect of number of plies, D/t ratio and geometric scalability.
Preliminaries

Equation 5-1, suggests that geometry will have an effect on energy absorption in a composite crush tube. Since both the impact energy and tube density is held constant, the energy absorption should decrease linearly with crush volume. Figure 5-1 shows the variation of energy absorption as a function of the crush volume and the almost linear decrease is observed. This deviation from the linear can be explained by the onset of buckling in such tubes.

Critical Buckling Load

Pickett and Dayal [13] have demonstrated the distinct classes of composites energy absorption modes. Buckling occurs when an axial load is applied on the composite tube. For a composite tube, the critical buckling load per unit length is given as
where $\beta$ is the axial frequency, $D_{11}$ is the bending stiffness in the axial direction, $n$ is the circumferential full wave number, $D_{12}$ is the bending stiffness in the axial-radial plane, $D_{66}$ is the bending stiffness in the normal direction, $D_{22}$ is the bending stiffness in the radial direction, $r$ is the radius of the cylinder, $E_\theta$ is the effective radial stiffness of the composite layers, $t$ is the total shell thickness, $G_{xy\theta}$ is the effective shear modulus of the composite layers, $v_{x,y}$ is the effective Poisson’s ratio in the axial-radial plane, and $E_t$ is the effective axial stiffness of the composite layers [14].

The critical buckling load is strongly influenced by tube diameter, radius and length. Figure 5-2 shows the variation of the critical buckling load with an increase in the $D/t$ ratio. For tubes of varying geometries, the critical buckling load decreases as the diameter to thickness ratio increases and asymptotically approaches a steady state value. The lower the $D/t$ ratio of the column, the greater is its ability to resist an axial force. Additionally it is noteworthy to point out that tubes of similar $D/t$ ratios have nearly identical theoretical critical buckling loads. This variation in buckling load may in turn influence the energy absorption capacity in a composite tube. When the $D/t$ ratio is small, buckling will not occur and energy absorption will take place mainly due to the elastic energy absorption and the compressive crush. As the $D/t$ ratio increases the crush will be preceded by buckling. The crushing will not take place and the failure will be due to the tension and compressive failure of the tube when it bends in the plane of the wall, due to buckling. In a circular tube, the buckling takes place in such a manner that the sine waves of wall bending are produced. In a bent section the main energy absorption will take place at the fold. The areas above and below the fold will bend and not crush. It is not difficult to see that this folding failure energy would take place over much less material of the tube and hence the amount of energy absorbed will be reduced.
Effect of Tube Radius

Presented in figure 5-3 is the variation in crush height as the radius of the tube is changed from 0.4 inch to 1.5 inch. The three curves are plotted for three different ply numbers. It was observed that the trendlines for 6, 9 and 12 ply tubes displayed a nonlinear inverse relationship as the radius increased.
As the number of plies increase, the tube thickness increases and the crush height decreases. It is also expected that the crush height will decrease as the radius increases. This is attributed to the increase in material mass as the cross-sectional area increases. Buckling theory of elastic hollow tubes suggests that as the mean diameter increases, the crush height decreases. This is due to the increase in the column’s ability to resist an axial load.

Figure 5-4 shows the variation of the specific energy absorbed as a function of the radius. The 6 and 9 ply tubes show a maxima and then decrease. The 12 ply tube curve keeps increasing but our conjecture is that if the radius is increased further this curve will also reach a maxima and then decrease. The reason for this is next discussed.
We base our observations on the following. Revisiting buckling theory, we see that

\[ N_{C1} = C_1 + C_2 \]

where \( C_1 \) and \( C_2 \) are unique constants. This suggests that as the radius increases, the critical buckling load asymptotically approaches \( C_2 \) or:

\[ N_{C1} = C_2 = \frac{\beta^4 E_y h}{\beta^4 + 2\beta^2 \eta^2 \left( \frac{E_y}{2G_{tu}} - \nu_k \right) + \eta^4 E_y} \]

Furthermore, as radius increases, the cross-sectional area increase is governed by a linear relationship,

\[ A = 2\pi r \]

In short, it is expected that energy absorption increases as tube radius increases. However, this relationship is not linear. In fact, these equations suggest the existence of three distinct energy
absorption regions. First, the initial elastic deformation region, second, the compressive crushing, and finally the failure associated with buckling. The order of the compressive crush and buckling will exchange depending on the tube radius and thickness.

**Effect of Number of Plies**

Based on buckling theory, as thickness increases, \( D/t \) reduces and the buckling load increases. As a result, energy absorption changes considerably as the number of layers increases. Figure 5-5 shows the relation between crush height and the number of plies for fixed diameter tubes. Most interesting is the individual behavior of each trendline as we add plies. Evaluating the influence of increasing radius (holding \( R \) constant), on crush height, an inverse relationship is observed. It suggests an asymptote relationship between crush height and number of plies such that the asymptote value of each trendline results in a non-zero minimum height.
Figure 5-5: Crush height as a function of tube thickness

Figure 5-6 presents the relation between energy absorption and number of plies.
Each trendline clearly exhibits a peak value at unique locations in their respective trendline. For example with increase in the number of plies in a 3/2 radius tube (4\textsuperscript{th} trendline), the maximum energy absorption occurs when there are 12 plies; or the 4\textsuperscript{th} data point. For a 9/8-inch tube (3\textsuperscript{rd} trendline) the maximum energy value occurs with 9 plies (3\textsuperscript{rd} data point). A subtle, yet distinct pattern emerges. Each of these tubes have the same D/t ratios such that

\[
\frac{3/2}{3\text{ plies}} = \frac{9/8}{6\text{ plies}} = \frac{9/4}{9\text{ plies}} = \frac{3}{12\text{ plies}} = 14.85\text{in/in} \quad 5-6
\]

We will now further investigate the effect of the ratio of mean tube diameter to tube wall thickness, D/t.
**D/t Ratio**

It has already been shown in Fig. 5-2 that lower D/t results in higher critical buckling load. Farley reports [6] that for both graphite/epoxy and Kevlar/epoxy tubes, a reduction in tube D/t results in an increase in energy absorption and ultimately an increase in sustained crushing load. He attributes the increase to a reduction in interlaminar cracking in the crushed region of the tube. As the length and number of interlaminar cracks decreases, the buckling load of the associated lamina bundles increases. Although Farley's study was limited to the performance of graphite/epoxy and Kevlar/epoxy tubes, it is anticipated that glass/epoxy tubes will be influenced by D/t ratio in a manner very similar to graphite/epoxy tubes. This assumption is primarily based on the findings of Schmuesser and Wickliffe [15].

It has been experimentally determined that a decrease in the density of a fiber causes an increase in the specific energy absorption. Comparing the densities of glass (sp. gr. 2.5-2.6) relative to graphite (sp. gr. 1.8), we clearly see that graphite has a lower density [16]. Furthermore, glass fiber, (3.5-4.5%) have higher strain to failure, as compared to graphite fibers (0.5-2.4%). However, the elastic modulus of glass (73-87 GPa) being lower than graphite (276-380 GPa) the total energy absorbed by the elastic part of the graphite will be higher than for the glass.
Figure 5-7: Energy absorption as a function of D/t ratio

Figure 5-7 depicts the variation of the specific energy absorption as a function of the D/t ratio. Unlike Farley’s work for graphite/epoxy [6] a distinct peak in strain energy absorbed is observed at 14.85 for glass/epoxy tubes. For lower D/t the energy absorption reduces. This can be attributed to the lower elastic strain energy of the glass fibers as compared to the graphite fibers as discussed in the previous paragraph. When the D/t is lower, the major energy absorption mechanism is elastic strain energy and this difference results in the reduction in the absorbed energy unlike Farley’s results which are for graphite/epoxy composites.
Geometric Scalability

Figure 5-8 shows the effect of the number of plies on the specific energy absorption while the D/t ratio is kept constant. This leads to the study of the geometric scalability question for glass/epoxy tubes.

![Figure 5-8: Energy absorption among tubes of identical D/t ratios](image)

Farley’s experiments suggest that energy absorption results of graphite/epoxy tubes are not readily geometrically scalable [6]. He found that graphite/epoxy tubes of identical D/t ratio do not exhibit identical energy absorption values or characteristics. He suspected that the lack of scalability is partially due to a local instability mode reducing the buckling load of the lamina bundles. For a D/t ratio of 14.85 where the peak specific energy was observed, data is plotted in figure 5-8. Each data point is associated with unique energy absorption characteristics and they do not exhibit a linear
relationship. Farley had also observed a similar relationship for graphite/epoxy tubes. Thus it is clear
that this numerical solution shows that the behavior exhibited by the glass/epoxy is not geometrically
scalable. Moreover, both the experimental and numerical data suggests that for identical D/t ratios, as
D increases, the energy absorption capability also increases.

Based upon the implication that similar D/t ratios have essentially the same theoretical critical
buckling load, it should be expected that they should not be geometrically scalable. This is due to the
fact that although each tube has the same critical buckling and D/t proportions, their radii are unique.
As a result, as both tube diameter and tube thickness increase proportionally, the cross-sectional area
increase is of course governed by equation 5-5. As a result each tube of increasing diameter and
thickness also has considerably more material volume to resist the critical buckling load. Therefore it
should be expected that as both diameter and thickness increase at the same rate, their energy
absorption is likely to increase (depending of course on the energy absorption region the tube is
instantaneously undergoing). Thus, tubes of identical D/t ratio should not be expected to be scalable.

Conclusions

We have addressed the question of the effect of the geometry on the specific energy
absorption in glass/epoxy composite tubes. The main conclusions of this work are as follows. The
specific energy absorption is a function of the tube radius and the results show that the absorption
reaches a peak and subsequently drops off. This phenomenon can be attributed to the combination of
crushing failure and the onset of in-plane buckling of the tube wall when the radius becomes large.
Number of plies or the wall thickness increase shows a unique peak for various diameters and as the
diameter increases the energy absorption increases. Thus we see the relationship of both the diameter
and the thickness and hence we have next studied the effect of the D/t ratio on the specific energy
absorption. It was observed that the specific energy absorption peaked at a value of 14.85. Above this
value the energy absorption dropped. This could be attributed to the lowering of the critical buckling
load as the D/t increases. On the other hand, when the D/t reduces the elastic deformation dominates and then the energy absorption again reduces. This has been seen to be true from the elastic bounce back study in chapter 4.

References


CHAPTER 6: GENERAL CONCLUSIONS AND FUTURE WORK

In order to establish a sound numerical methodology it was first important to verify the mechanics involved in a dynamic crush event. Experimental research has revealed the mechanics of how long fiber composite tubes crush in a dynamic event. However, the analytical methods that are currently used to solve composite crushing do not fully account for all the physical characteristics of true crushing. Thus it was necessary to build an analytical model that could more accurately reflect dynamic composite crushing mechanics. The most likely start was to build upon the mechanics that have laid a firm foundation for the analytical methods currently used. Beginning with this premise, this study has begun to build a new predictive methodology based upon composite laminated plate theory.

In chapter 2, a finite element methodology was developed. It was demonstrated that the effect of the number of elements is very significant in the convergence of the solution. The solution convergence time is essentially linear with the increase in the number of elements. Based upon this numerical study, it has been found that for the basic geometry of this composite crush analysis, 4480 elements are sufficient in achieving a steady-state solution irrespective of stacking sequence. This corresponds to a maximum aspect ratio of 4.7 and a maximum element size of 5.89 e^3 inches by 7.85e^2 inches or 4.62e^3 inches^2.

Having a methodology in place, the validity of the underlying mechanics was then evaluated. In chapter 3 a study of the effect of fiber stacking sequence on the specific energy response has shown that for glass/epoxy tubes, there is some correlation between numerical results presented here and experimental results presented elsewhere. Particularly it has been noted in this study that beyond an angle of 45°, a very good correlation is observed between the experimental and numerical results. It has been established that the response of the cylinder is not only dependent on the axial and lateral stiffness, but also on the shear stiffness and strength, which are maximum at an angle of 45°. The
discrepancy between the experimental and numerical results below 45° angle samples can be attributed to the viscoelastic behavior of the epoxy. It is theorized that incorporating the viscoelastic behavior into the finite element model with lead to better correlation. Further work in this area has to be done to better understand and quantify this phenomenon.

It has also been experimentally determined that impact velocity is an important factor influencing the energy absorption. The numerical study in chapter 4 also supports the experimental finding that there is indeed a threshold value which characterized two distinctly different energy absorption regions. The first region, corresponding to low impact energies, features a significant amount of strain energy crushing. The second region experiences crushing failure due to high impact energy. This is analogous to the experimental findings which suggest the existence of a velocity threshold. Furthermore, this work conclusively illustrates that impact energy is a far more reliable driver for energy absorption than impact velocity. Thus, for the tubes used in this study the energy absorption at low velocities is mainly due to the elastic strain energy. This has been confirmed by the elastic spring back study. Finally, for all tubes, the buckling load was higher than the crush load; therefore, it can be concluded that the energy absorption was due to compressive crushing.

In chapter 5, this numerical study has also addressed the question of the effect of the geometry on the specific energy absorption in glass/epoxy composite tubes. The specific energy absorption is a function of the tube radius and the results show that the absorption reaches a peak and subsequently drops off. This phenomenon can be attributed to the combination of crushing failure and the onset of in-plane buckling of the tube wall when the radius becomes large. Number of plies or the wall thickness increase shows a unique peak for various diameters and as the diameter increases the energy absorption increases. It was observed that the specific energy absorption peaked at a D/t value of 14.85. As D/t ratio decreased or increased, the energy absorption values dropped. It is noteworthy to point out that this phenomenon was not witnessed in the experimental data. This could be attributed to the lowering of the critical buckling load as the D/t increases. On the other hand, when the D/t
reduces the elastic deformation dominates and then the energy absorption again reduces. This has been seen to be true from the elastic bounce back study in chapter 4. It is therefore clear that there is some noticeable disagreement between the numerical and experimental data. However, the fundamental mechanics are clear. Critical buckling load should vary as tube geometry specifically the D/t ratio varies. This suggests that, it is plausible to distinctly see up to three crushing regions: strain energy, brittle fracturing and buckling. This did not occur in the experimental study, most likely due to the impact energy values at which the tubes were crushed. Thus it is inconclusive if the numerical and experimental data sets are indeed at odds. Further experimental to numerical correlation is warranted.

Thus this numerical study was able to capture the fundamental behavioral response of glass/epoxy composite tubes subject to changes in fiber stacking sequence, impact energy and tube geometry. However it is notable that an absolute and universal relationship linking the numerical and experimental results was not established. This is primarily due to the absence of additional key principles that need to be incorporated into the numerical model. Thus although a firm foundation has been established, this endeavor is not yet complete.

The implications of this work are many. In its current state, this work can be extremely useful in aiding experimental research. By using these codes in their current incarnation, one can better refine the range of fiber stacking sequence, impact energy and tube geometry to investigate experimentally. Ultimately this work will proved the basis for creating a holistic methodology for numerically predicting energy absorption in composite crush tubes irrespective of fiber stacking sequence, impact energy or tube geometry.
APPENDIX

ANSYS Preprocessor Input Deck

/PREP7
ST, 1, SHELL63
K, 1, 0, 0, 4/0/12
K, 4, 0, 0, 5/12
K, 21, 0, 0, 75/12, 3.9875/12
K, 22, 0, 0, 75/12, 4.0/12
K, 23, 0, 0, 75/12, 3.975/12
K, 24, 0, 0, 75/12, 3.9875/12
K, 25, 0, 0, 75/12, 3.9625/12
K, 26, 0, 0, 75/12, 3.975/12
K, 27, 0, 0, 75/12, 3.95/12
K, 28, 0, 0, 75/12, 3.9625/12
K, 29, 0, 0, 75/12, 3.9375/12
K, 30, 0, 0, 75/12, 3.95/12
K, 31, 0, 0, 75/12, 3.925/12
K, 32, 0, 0, 75/12, 3.9375/12
LSTR, 21, 22
LSTR, 23, 24
LSTR, 25, 26
LSTR, 27, 28
LSTR, 29, 30
LSTR, 31, 32
LSTR, 3, 4
FLST, 2, 1, 4, ORDE, 1
FITEM, 2, 6
FLST, 8, 2, 3
FITEM, 8, 3
FITEM, 8, 4
AROTAT, P51X, , , , , P51X, , 360, ,
FLST, 2, 1, 4, ORDE, 1
FITEM, 2, 5
FLST, 8, 2, 3
FITEM, 8, 3
FITEM, 8, 4
AROTAT, P51X, , , , , P51X, , 360, ,
FLST, 2, 1, 4, ORDE, 1
FITEM, 2, 4
FLST, 8, 2, 3
FITEM, 8, 3
FITEM, 8, 4
AROTAT, P51X, , , , , P51X, , 360, ,
FLST, 2, 1, 4, ORDE, 1
FITEM, 2, 3
FLST, 8, 2, 3
FITEM, 8, 3
FITEM, 8, 4
AROTAT, P51X, , , , , P51X, , 360, ,
FLST, 2, 1, 4, ORDE, 1
FITEM, 2, 2
FLST, 8, 2, 3
FITEM, 8, 3
FITEM, 8, 4
AROTAT, P51X, , , , , P51X, , 360, ,
FLST, 2, 1, 4, ORDE, 1
FITEM, 2, 1
FLST, 8, 2, 3
FITEM, 8, 3
FITEM, 8, 4
AROTAT, P51X, , , , , P51X, , 360, ,
K, 51, 0, 0, 75/12
K, 52, 0, 0, 75/12, 3.925/12
LSTR, 51, 52
FLST, 2, 1, 4, ORDE, 1
FITEM, 2, 74
FLST, 8, 2, 3
FITEM, 8, 3
FITEM, 8, 4
ARCAT, PSIX, , , , PSIX, , 360, ,
LESIZE, ALL, , , 20, , 1, , 1,
FLST, 5, 24, 4, ORDE, 24
FITEM, 5, 1
FITEM, 5, 2
FITEM, 5, 3
FITEM, 5, 4
FITEM, 5, 5
FITEM, 5, 6
FITEM, 5, 8
FITEM, 5, 9
FITEM, 5, 10
FITEM, 5, 19
FITEM, 5, 20
FITEM, 5, 21
FITEM, 5, 30
FITEM, 5, 31
FITEM, 5, 32
FITEM, 5, 41
FITEM, 5, 42
FITEM, 5, 43
FITEM, 5, 52
FITEM, 5, 53
FITEM, 5, 54
FITEM, 5, 63
FITEM, 5, 64
FITEM, 5, 65
CM, _Y, LINE
LSEL, , , PSIX
CM, _Y1, LINE
CMSEL, _Y
LESIZE, _Y1, , 1, , , 1
FLST, 5, 4, 4, ORDE, 4
FITEM, 5, 74
FITEM, 5, 75
FITEM, 5, 76
FITEM, 5, 77
CM, _Y, LINE
LSEL, , , PSIX
CM, _Y1, LINE
CMSEL, _Y
LESIZE, _Y1, , 50, , , 1
LLIST, ALL, ,
AMESH, 21, 24, 1
AMESH, 17, 20, 1
AMESH, 13, 16, 1
AMESH, 9, 12, 1
AMESH, 5, 8, 1
AMESH, 1, 4, 1
AMESH, 25, 28, 1
NLIST, ALL, , , NODE, NODE, NODE
e1ist, all, , 0, 0
LLIST, ALL, ,
NUMMRG, ALL, , , LOW
NUMCMP, ALL
NWRITE, '4480_6ply_n3g',
BWRITE, '4480_6ply_e3g',
SAVE, HL4480_6ply3g, db,
**Typical LS-DYNA Input Deck**

*KEYWORD*

*TITLE*

256 ELEMENTS 0+/-45 3/4 INCH CONSTRAINED TOP

*NODE*

$NODE,X,Y,Z$

9999, 0.0, 0.0, 0.333334

9996, 0.0, 0.0

1. 0.00000E+00, 6.25000E-02, 0.33229167
2. -6.25000E-02, 0.00000E+00, 0.33229167
3. -2.39177E-02, 5.77425E-02, 0.33229167
4. -4.41942E-02, 4.41942E-02, 0.33229167
5. -5.77425E-02, 2.39177E-02, 0.33229167
6. -6.25000E-02, 0.00000E+00, 0.33333333
7. 0.00000E+00, 6.25000E-02, 0.33333333
8. -2.39177E-02, 5.77425E-02, 0.33333333
9. -4.41942E-02, 4.41942E-02, 0.33333333
10. -5.77425E-02, 2.39177E-02, 0.33333333
11. 0.00000E+00, -6.25000E-02, 0.33229167
12. -5.77425E-02, -2.39177E-02, 0.33229167
13. -4.41942E-02, -4.41942E-02, 0.33229167
14. -2.39177E-02, -5.77425E-02, 0.33229167
15. 0.00000E+00, -6.25000E-02, 0.33333333
16. -5.77425E-02, -2.39177E-02, 0.33333333
17. -4.41942E-02, -4.41942E-02, 0.33333333
18. -2.39177E-02, -5.77425E-02, 0.33333333
19. 6.25000E-02, 0.00000E+00, 0.33229167
20. 2.39177E-02, -5.77425E-02, 0.33229167
21. 4.41942E-02, -4.41942E-02, 0.33229167
22. 5.77425E-02, -2.39177E-02, 0.33229167
23. 6.25000E-02, 0.00000E+00, 0.33333333
24. 2.39177E-02, -5.77425E-02, 0.33333333
25. 4.41942E-02, -4.41942E-02, 0.33333333
26. 5.77425E-02, -2.39177E-02, 0.33333333
27. 5.77425E-02, 2.39177E-02, 0.33229167
28. 4.41942E-02, 4.41942E-02, 0.33229167
29. 2.39177E-02, 5.77425E-02, 0.33229167
30. 5.77425E-02, 2.39177E-02, 0.33333333
31. 4.41942E-02, 4.41942E-02, 0.33333333
32. 2.39177E-02, 5.77425E-02, 0.33333333
33. 0.00000E+00, 6.25000E-02, 0.33125000
34. -6.25000E-02, 0.00000E+00, 0.33125000
35. -2.39177E-02, 5.77425E-02, 0.33125000
36. -4.41942E-02, 4.41942E-02, 0.33125000
37. -5.77425E-02, 2.39177E-02, 0.33125000
38. 0.00000E+00, -6.25000E-02, 0.33125000
39. -5.77425E-02, -2.39177E-02, 0.33125000
40. -4.41942E-02, -4.41942E-02, 0.33125000
41. -2.39177E-02, -5.77425E-02, 0.33125000
42. 6.25000E-02, 0.00000E+00, 0.33125000
43. 2.39177E-02, -5.77425E-02, 0.33125000
44. 4.41942E-02, -4.41942E-02, 0.33125000
45. 5.77425E-02, -2.39177E-02, 0.33125000
46. 5.77425E-02, 2.39177E-02, 0.33125000
47. 4.41942E-02, 4.41942E-02, 0.33125000
48. 2.39177E-02, 5.77425E-02, 0.33125000
49. 0.00000E+00, 6.25000E-02, 0.33020833
50. -6.25000E-02, 0.00000E+00, 0.33020833
51. -2.39177E-02, 5.77425E-02, 0.33020833
52. -4.41942E-02, 4.41942E-02, 0.33020833
53. -5.77425E-02, 2.39177E-02, 0.33020833
54. 0.00000E+00, -6.25000E-02, 0.33020833
55. -5.77425E-02, -2.39177E-02, 0.33020833
56. -4.41942E-02, -4.41942E-02, 0.33020833
57. -2.39177E-02, -5.77425E-02, 0.33020833
58. 6.25000E-02, 0.00000E+00, 0.33020833
<table>
<thead>
<tr>
<th>x1</th>
<th>x2</th>
<th>x3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00000E+00</td>
<td>6.25000E-02</td>
<td>0.32708333</td>
</tr>
<tr>
<td>-6.25000E-02</td>
<td>0.00000E+00</td>
<td>0.32916667</td>
</tr>
<tr>
<td>-2.39177E-02</td>
<td>5.77425E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>6.25000E-02</td>
<td>0.00000E+00</td>
<td>0.32708333</td>
</tr>
<tr>
<td>2.39177E-02</td>
<td>-5.77425E-02</td>
<td>0.32812500</td>
</tr>
<tr>
<td>4.41942E-02</td>
<td>-4.41942E-02</td>
<td>0.32812500</td>
</tr>
<tr>
<td>5.77425E-02</td>
<td>2.39177E-02</td>
<td>0.32812500</td>
</tr>
<tr>
<td>4.41942E-02</td>
<td>4.41942E-02</td>
<td>0.32812500</td>
</tr>
<tr>
<td>2.39177E-02</td>
<td>5.77425E-02</td>
<td>0.32812500</td>
</tr>
<tr>
<td>0.00000E+00</td>
<td>6.25000E-02</td>
<td>0.32708333</td>
</tr>
<tr>
<td>-6.25000E-02</td>
<td>0.00000E+00</td>
<td>0.32812500</td>
</tr>
<tr>
<td>-2.39177E-02</td>
<td>5.77425E-02</td>
<td>0.32812500</td>
</tr>
<tr>
<td>6.25000E-02</td>
<td>0.00000E+00</td>
<td>0.32708333</td>
</tr>
<tr>
<td>2.39177E-02</td>
<td>-5.77425E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>4.41942E-02</td>
<td>-4.41942E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>5.77425E-02</td>
<td>2.39177E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>4.41942E-02</td>
<td>4.41942E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>2.39177E-02</td>
<td>5.77425E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>0.00000E+00</td>
<td>6.25000E-02</td>
<td>0.32708333</td>
</tr>
<tr>
<td>-6.25000E-02</td>
<td>0.00000E+00</td>
<td>0.32812500</td>
</tr>
<tr>
<td>-2.39177E-02</td>
<td>5.77425E-02</td>
<td>0.32812500</td>
</tr>
<tr>
<td>6.25000E-02</td>
<td>0.00000E+00</td>
<td>0.32708333</td>
</tr>
<tr>
<td>2.39177E-02</td>
<td>-5.77425E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>4.41942E-02</td>
<td>-4.41942E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>5.77425E-02</td>
<td>2.39177E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>4.41942E-02</td>
<td>4.41942E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>2.39177E-02</td>
<td>5.77425E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>0.00000E+00</td>
<td>6.25000E-02</td>
<td>0.32708333</td>
</tr>
<tr>
<td>-6.25000E-02</td>
<td>0.00000E+00</td>
<td>0.32812500</td>
</tr>
<tr>
<td>-2.39177E-02</td>
<td>5.77425E-02</td>
<td>0.32812500</td>
</tr>
<tr>
<td>6.25000E-02</td>
<td>0.00000E+00</td>
<td>0.32708333</td>
</tr>
<tr>
<td>2.39177E-02</td>
<td>-5.77425E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>4.41942E-02</td>
<td>-4.41942E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>5.77425E-02</td>
<td>2.39177E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>4.41942E-02</td>
<td>4.41942E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>2.39177E-02</td>
<td>5.77425E-02</td>
<td>0.32916667</td>
</tr>
<tr>
<td>0.00000E+00</td>
<td>6.25000E-02</td>
<td>0.32708333</td>
</tr>
</tbody>
</table>
119

266, 2.39177E-02, 5.77425E-02, 0.09812500
267, 2.39177E-02, 5.77425E-02, 0.13083333
268, 2.39177E-02, 5.77425E-02, 0.16354167
269, 2.39177E-02, 5.77425E-02, 0.19625000
270, 2.39177E-02, 5.77425E-02, 0.22895833
271, 2.39177E-02, 5.77425E-02, 0.26166667
272, 2.39177E-02, 5.77425E-02, 0.29437500

* ELEMENT_SHELL
$ ELEM,pid,n1,n2,n3,n4
1, 2, 4, 5, 9, 10
2, 2, 9, 8, 3, 4
3, 2, 8, 7, 1, 3
4, 2, 5, 2, 6, 10
5, 2, 13, 14, 18, 17
6, 2, 17, 16, 12, 13
7, 2, 16, 6, 2, 12
8, 2, 14, 11, 15, 18
9, 2, 21, 22, 26, 25
10, 2, 25, 24, 20, 21
11, 2, 24, 19, 23, 26
12, 2, 28, 29, 32, 31
13, 2, 31, 30, 27, 28
14, 2, 30, 23, 19, 27
15, 2, 29, 17, 7, 32
16, 2, 21, 22, 26, 25
17, 2, 37, 34, 2, 35
18, 3, 5, 4, 36, 37
19, 3, 4, 3, 35, 36
20, 3, 3, 1, 33, 35
21, 3, 41, 38, 11, 14
22, 3, 14, 13, 40, 41
23, 3, 13, 12, 39, 40
24, 3, 12, 2, 34, 39
25, 3, 45, 42, 19, 22
26, 3, 22, 21, 44, 45
27, 3, 21, 20, 43, 44
28, 3, 20, 11, 38, 43
29, 3, 48, 33, 1, 29
30, 3, 29, 28, 47, 48
31, 3, 28, 27, 46, 47
32, 3, 27, 19, 42, 46
33, 4, 35, 33, 49, 51
34, 4, 51, 52, 36, 35
35, 4, 52, 53, 37, 36
36, 4, 53, 50, 34, 37
37, 4, 39, 34, 50, 55
38, 4, 55, 56, 40, 39
39, 4, 56, 57, 41, 40
40, 4, 57, 54, 38, 41
41, 4, 43, 38, 54, 59
42, 4, 59, 60, 44, 43
43, 4, 60, 61, 45, 44
44, 4, 61, 58, 42, 45
45, 4, 46, 43, 58, 62
46, 4, 62, 63, 47, 46
47, 4, 63, 64, 48, 47
48, 4, 64, 49, 33, 48
49, 5, 69, 66, 50, 53
50, 5, 53, 52, 68, 69
51, 5, 52, 51, 67, 68
52, 5, 51, 49, 65, 67
53, 5, 73, 70, 54, 57
54, 5, 57, 56, 72, 73
55, 5, 56, 55, 71, 72
56, 5, 55, 50, 66, 71
57, 5, 77, 74, 58, 61
58, 5, 61, 60, 76, 77
59, 5, 60, 59, 75, 76
60, 5, 59, 54, 70, 75
130, 1, 143, 152, 153, 144
131, 1, 152, 161, 162, 153
132, 1, 161, 125, 126, 162
133, 1, 135, 144, 99, 97
134, 1, 144, 153, 100, 99
135, 1, 153, 162, 101, 100
136, 1, 162, 126, 98, 101
137, 1, 124, 164, 176, 118
138, 1, 164, 165, 185, 176
139, 1, 165, 166, 194, 185
140, 1, 166, 163, 169, 194
141, 1, 118, 176, 177, 119
142, 1, 176, 185, 186, 177
143, 1, 185, 194, 195, 186
144, 1, 194, 167, 168, 195
145, 1, 129, 177, 178, 120
146, 1, 177, 186, 187, 178
147, 1, 186, 195, 196, 187
148, 1, 195, 168, 169, 196
149, 1, 120, 178, 179, 121
150, 1, 178, 187, 188, 179
151, 1, 187, 196, 197, 188
152, 1, 196, 169, 170, 197
153, 1, 121, 179, 180, 122
154, 1, 179, 188, 189, 180
155, 1, 186, 197, 196, 189
156, 1, 197, 170, 171, 198
157, 1, 122, 180, 181, 123
158, 1, 180, 189, 190, 181
159, 1, 189, 198, 199, 190
160, 1, 198, 171, 172, 199
161, 1, 123, 161, 162, 124
162, 1, 181, 190, 191, 182
163, 1, 190, 199, 200, 191
164, 1, 199, 172, 173, 200
165, 1, 124, 182, 183, 125
166, 1, 182, 191, 192, 183
167, 1, 191, 200, 201, 192
168, 1, 200, 173, 174, 201
169, 1, 125, 183, 184, 126
170, 1, 183, 192, 193, 184
171, 1, 192, 201, 202, 193
172, 1, 201, 174, 175, 202
173, 1, 126, 184, 103, 98
174, 1, 184, 193, 104, 103
175, 1, 193, 202, 105, 104
176, 1, 202, 175, 102, 105
177, 1, 163, 204, 216, 167
178, 1, 204, 205, 225, 216
179, 1, 205, 206, 234, 225
180, 1, 206, 203, 207, 234
181, 1, 167, 216, 217, 168
182, 1, 216, 225, 226, 217
183, 1, 225, 234, 235, 226
184, 1, 234, 207, 208, 235
185, 1, 168, 217, 218, 169
186, 1, 217, 226, 227, 218
187, 1, 226, 235, 236, 227
188, 1, 235, 208, 209, 236
189, 1, 169, 218, 219, 170
190, 1, 218, 227, 228, 219
191, 1, 227, 236, 237, 228
192, 1, 236, 209, 210, 237
193, 1, 170, 219, 220, 171
194, 1, 219, 228, 229, 220
195, 1, 228, 237, 238, 229
196, 1, 237, 210, 211, 238
197, 1, 171, 220, 221, 172
198, 1, 220, 229, 230, 221
199, 229, 238, 239, 230
200, 238, 211, 212, 239
201, 232, 221, 222, 213
202, 221, 230, 232, 222
203, 230, 239, 240, 231
204, 239, 212, 213, 240
205, 1, 222, 223, 174
206, 222, 231, 232, 223
207, 231, 240, 241, 232
208, 240, 213, 214, 241
209, 1, 174, 223, 224
210, 223, 232, 233, 224
211, 232, 241, 242, 233
212, 241, 214, 215, 242
213, 1, 195, 224, 107, 102
214, 224, 233, 108, 107
215, 233, 242, 109, 108
216, 242, 215, 106, 109
217, 233, 243, 246, 207
218, 243, 244, 255, 246
219, 244, 245, 264, 255
220, 245, 113, 132, 264
221, 1, 207, 246, 247, 208
222, 246, 295, 256, 247
223, 1, 255, 264, 265, 256
224, 264, 127, 128, 265
225, 208, 247, 248, 209
226, 247, 256, 257, 248
227, 256, 265, 266, 257
228, 1, 265, 128, 129, 266
229, 1, 209, 248, 249, 210
230, 248, 257, 258, 249
231, 257, 132, 267, 258
232, 1, 266, 129, 130, 267
233, 1, 210, 249, 250, 211
234, 1, 249, 258, 259, 250
235, 1, 258, 267, 268, 259
236, 1, 267, 130, 131, 268
237, 1, 211, 250, 251, 212
238, 1, 250, 259, 260, 251
239, 1, 259, 268, 269, 260
240, 1, 268, 131, 132, 269
241, 1, 232, 251, 252, 213
242, 1, 251, 260, 261, 252
243, 1, 260, 269, 270, 261
244, 1, 269, 132, 133, 270
245, 1, 233, 252, 253, 214
246, 1, 252, 261, 262, 253
247, 1, 261, 270, 271, 262
248, 1, 270, 133, 134, 271
249, 1, 214, 253, 254, 215
250, 1, 253, 262, 263, 254
251, 1, 262, 271, 272, 263
252, 1, 271, 134, 135, 272
253, 1, 215, 254, 110, 106
254, 1, 254, 263, 111, 110
255, 1, 263, 272, 112, 111
256, 1, 272, 135, 97, 112

$**CONSTRAINTS AND BOUNDARY CONDITIONS**$
$**BOUNDARY SPC_NODE**$
$nid,cid,dofx,dofy,dofz,dofrx,dofry,dofrz$
$Final decision is to have constraints at top and bottom (fixed) OR just bottom$
$1,0,1,0,0,0,0,
<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$\text{BASE}$

$\text{PART}

$\text{pid sid mid ecosid hgid grav adpopt tmid}$

$\text{1 1 1 0 0 0 0 0 0 0}$

$\text{PART}$

$\text{1 ply}$

$\text{pid sid mid ecosid hgid grav adpopt tmid}$

$\text{2 2 1 0 0 0 0 0 0 0}$

$\text{PART}$

$\text{2 plies}$

$\text{pid sid mid ecosid hgid grav adpopt tmid}$

$\text{3 3 1 0 0 0 0 0 0 0}$

$\text{PART}$

$\text{3 plies}$

$\text{pid sid mid ecosid hgid grav adpopt tmid}$

$\text{4 4 1 0 0 0 0 0 0 0}$

$\text{PART}$

$\text{4 plies}$
$ pid sid mid eesid hgid grav adpopt tmid
5 5 1 0 0 0 0 0
$ *PART
$ % plies
$ pid sid mid eesid hgid grav adpopt tmid
6 6 1 0 0 0 0 0
$ *SECTION_SHELL
$ base mateial
$ 1 SECID EFORM shRF NIP PROPT QR/IRID ICOMP SETYP
$ 2 t1 t2 t3 t4 nloc area
0.0084167 .0084167 .0084167 .0084167
$ b1 b2 b3 b4 b5 b6
0.0 -45.0 45.0 0.0 -45.0 45.0
$ *SECTION_SHELL
$ 1 ply
$ 1 SECID EFORM shRF NIP PROPT QR/IRID ICOMP SETYP
$ 2 t1 t2 t3 t4 nloc area
.0014028 .0014028 .0014028 .0014028
$ b1 b2 b3 b4 b5 b6
0.0
$ *SECTION_SHELL
$ 2 plies
$ 1 SECID EFORM shRF NIP PROPT QR/IRID ICOMP SETYP
$ 2 t1 t2 t3 t4 nloc area
.0028056 .0028056 .0028056 .0028056
$ b1 b2 b3 b4 b5 b6
0.0 -45.0 45.0
$ *SECTION_SHELL
$ 3 plies
$ 1 SECID EFORM shRF NIP PROPT QR/IRID ICOMP SETYP
$ 4 t1 t2 t3 t4 nloc area
.0042083 .0042083 .0042083 .0042083
$ b1 b2 b3 b4 b5 b6
0.0 -45.0 45.0
$ *SECTION_SHELL
$ 4 plies
$ 1 SECID EFORM shRF NIP PROPT QR/IRID ICOMP SETYP
$ 5 t1 t2 t3 t4 nloc area
.0056111 .0056111 .0056111 .0056111
$ b1 b2 b3 b4 b5 b6
0.0 -45.0 45.0 0.0
$ *SECTION_SHELL
$ 5 plies
$ 1 SECID EFORM shRF NIP PROPT QR/IRID ICOMP SETYP
$ 6 t1 t2 t3 t4 nloc area
.0070138 .0070138 .0070138 .0070138
$ b1 b2 b3 b4 b5 b6
0.0 -45.0 45.0 0.0 -45.0
$ *MAT_COMPOSITE_FAILURE SHELL MODEL
$ Material 59
$........1........2........3........4........5........6........7........8
$1 mid ro ea eb ec prba pca prcb 1.117E9 3.697E+8 3.697E+8 0.09201 0.09201 0.400
$2 gab gbc gca kfc aopt macf 1.218E+8 1.320E+8 1.218E+8 3.0
$3 xp yp zp a1 a2 a3
$4 \quad v1 \quad v2 \quad v3 \quad d1 \quad d2 \quad d3 \quad \text{beta}
0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 90.0$
$5 \quad \text{tsize} \quad \text{alp} \quad \text{soft} \quad \text{fbst} \quad \text{yfcfac(st)} \quad \text{sf(??)}
1.0 \cdot 10^{-9} \quad .94 \quad .94 \quad 0.5$
$6 \quad \text{xc} \quad \text{xt} \quad \text{yc} \quad \text{yt}
1.913 \cdot 10^{7} \quad 2.318 \cdot 10^{7} \quad 2.318 \cdot 10^{7} \quad 7.310 \cdot 10^{6} \quad 1.504 \cdot 10^{6}
2.549 \cdot 10^{7} \quad 3.499 \cdot 10^{7} \quad 4.406 \cdot 10^{6} \quad 1.008 \cdot 10^{6} \quad 1.008 \cdot 10^{6}$

*RIGIDWALL_PLANAR_MOVING_FORCES*

$\text{........1........2........3........4........5........6........7........8}$

$1 \quad \text{NSID} \quad \text{NSIDEX} \quad \text{BOXID} \quad \text{OFFSET}
0 \quad 0 \quad 0 \quad 0$

$2 \quad \text{xt} \quad \text{yt} \quad \text{zt} \quad \text{xh} \quad \text{yh} \quad \text{zh} \quad \text{fric} \quad \text{wvel}
0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0.32$

$D \quad \text{mass} \quad \text{vo}
3.0843 \quad 25.0$

$E \quad \text{scft} \quad \text{ssid} \quad n1 \quad n2 \quad n3 \quad n4
0 \quad 0 \quad 9999$

*RIGIDWALL_PLANAR_FORCES*

$\text{........1........2........3........4........5........6........7........8}$

$1 \quad \text{NSID} \quad \text{NSIDEX} \quad \text{BOXID} \quad \text{OFFSET}
0 \quad 0 \quad 0 \quad 0$

$2 \quad \text{xt} \quad \text{yt} \quad \text{zt} \quad \text{xh} \quad \text{yh} \quad \text{zh} \quad \text{fric} \quad \text{wvel}
0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0.4$

$E \quad \text{scft} \quad \text{ssid} \quad n1 \quad n2 \quad n3 \quad n4
0 \quad 0 \quad 9998$

*LOAD_BODY_GENERALIZED*

$\text{........1........2........3........4........5........6........7........8}$

$1 \quad \text{N1} \quad \text{N2} \quad \text{LCID} \quad \text{DRLCID} \quad \text{XC} \quad \text{YC} \quad \text{ZC}
1 \quad 9999 \quad 1$

$2 \quad \text{AX} \quad \text{YX} \quad \text{ZX} \quad \text{OMX} \quad \text{OMY} \quad \text{OMZ}
\text{ft/s2}
32.2$

*DEFINE_CURVE*

$\text{LCID}
1$

$\text{ABSCISSA} \quad \text{ORDINATE}
0.0 \quad 1.0
1.0 \quad 1.0$

*CONTACT_AUTOMATIC_SINGLE_SURFACE*

$\text{........1........2........3........4........5........6........7........8}$

$1 \quad \text{ssid} \quad \text{msid} \quad \text{ssstyp} \quad \text{msstyp} \quad \text{aboxid} \quad \text{mboxid} \quad \text{spr} \quad \text{mpr}
0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

$2 \quad \text{fs} \quad \text{fd} \quad \text{dc} \quad \text{vc} \quad \text{vdc} \quad \text{penchk} \quad \text{bt} \quad \text{dt}
.40 \quad .32$

$E \quad \text{sf} \quad \text{ssf} \quad \text{ssst} \quad \text{msst} \quad \text{sfst} \quad \text{ssfmt} \quad \text{fsf} \quad \text{vsf}$

*CONTROL_ENERGY*

$\text{........1........2........3........4........5........6........7........8}$

$1 \quad \text{bgen} \quad \text{rwen} \quad \text{slnten} \quad \text{rylen}
2 \quad 2 \quad 2 \quad 2$

*CONTROL_SHELL*

$\text{........1........2........3........4........5........6........7........8}$

$1 \quad \text{wrpanel} \quad \text{esort} \quad \text{imxxx} \quad \text{istupd} \quad \text{theory} \quad \text{bwc} \quad \text{miter} \quad \text{proj}
15 \quad 16$

$\text{........1........2........3........4........5........6........7........8}$

$\text{................1........2........3........4........5........6........7........8}$

$1 \quad \text{endim} \quad \text{endcyc} \quad \text{dtmin} \quad \text{endeng} \quad \text{endmas}
1.8E-2$
$ cmpflg ievep beamip doom comp shge stsz n3thdt
  1  2  2  2  2
$........1........2........3........4........5........6........7........8
$ *DATABASE_BINARY_D3PLOT
$ dt/cycl ldc npltc psetid istats tstart iavg
  5E-4
$ *DATABASE_BINARY_D3THDT
$ dt/cycl ldc npltc psetid istats tstart iavg
  5E-4
$ *DATABASE_HISTORY_NODE
$ dt/cycl ldc npltc psetid istats tstart iavg
  9999 9998 1 2 3 4 5 17
$ *DATABASE_RWFORCE
$ dt
  5E-4
$ *DATABASE_NODOUT
$ $ see DATABASE_HISTORY_OPTION (handled above)
$ DT
  5E-4
$ *DATABASE_GLSTAT
$ dt
  5E-4
$ *DATABASE_MATSUM
$ dt
  5E-4
$ *END