EXPERIMENTAL PLATE WAVE DISPERSION ANALYSIS

Steven M. Ziola and Michael R. Gorman
Naval Postgraduate School
Department of Aeronautics and Astronautics
Monterey, CA 93943

INTRODUCTION

In an effort to verify theoretical models which relate matrix cracking to strength reduction, acoustic emission (AE) was used to locate matrix cracking in flat graphite/epoxy cross-ply tensile coupons. Using independent test methods [1] it was discovered that errors of up to 2 inches were common in the AE data (specimen length - 8 inches). Dispersion of the AE event caused the AE analyzer location clocks to trigger on different phase components of the event [2,3].

The first step toward understanding how dispersion could affect location was to experimentally determine the dispersion curves for these materials. While there have been several experimental verifications of dispersion theory for frequencies > 1 MHz [4,5,6], there are few results for the sub-megahertz region associated with AE [7].

Both for the range of frequencies and plate thicknesses that were tested, only the lowest order extensional (symmetric) and flexural (antisymmetric) plate modes were of importance. Aluminum and graphite/epoxy plates were tested. The data for the aluminum was compared against the Rayleigh-Lamb dispersion theory, while for the graphite/epoxy classical plate theory was used.

For the aluminum it was found that when using tone bursts, the data for the lowest extensional mode agreed with neither the phase nor the group velocity dispersion curves. The flexural mode however seemed to follow the group velocity dispersion curve. For the graphite/epoxy plates the extensional mode data agreed with the classical plate theory, while for the flexural mode, the theory was correct only for kh/2π values of ~0.1. At that point there was a large deviation between theory and experiment.
THEORY

In many experimental studies, the specimen dimensions in two directions (x and y) are much greater than the third (the thickness, or z direction), and can be considered plate-like. Lamb [8], considered the propagation of sinusoidal waves in an infinite, perfectly elastic, homogeneous, isotropic plate with stress free boundary conditions at the upper and lower surfaces. By solving the equations of motion for the above assumptions, he found that the resulting frequency relationship was,

\[
\frac{\tan(\beta h/2)}{\tan(\alpha h/2)} + \left[ \frac{4\alpha \beta k^2}{(k^2 - \beta^2)^2} \right]^{\pm1} = 0 \begin{cases} +1 & \text{Extensional modes} \\ -1 & \text{Flexural modes} \end{cases}
\]

where \( \alpha^2 = \omega^2/c_1^2 - \kappa^2 \) and \( \beta^2 = \omega^2/c_2^2 - \kappa^2 \). \( c_1 \) and \( c_2 \) are the bulk longitudinal and shear velocities, \( h \) is the plate thickness and \( \omega \) is the circular frequency. The phase velocity is calculated using the relationship, \( c_{ph} = \omega/k \).

Classical laminated plate theory was used to analyze the composite materials. While classical plate theory is correct only in the limit when the wavelength is much longer than the thickness of the plate, higher order theories [9] were not used owing to the complexity of the numerical solutions. The governing equations for the propagation of extensional and flexural waves in an orthotropic laminated plate can be found in reference [10].

For an extensional wave propagating in a composite, the velocity is,

\[ c_e = \sqrt{A_{11}/\rho h} \]

where \( A_{11} \) is the extensional stiffness in the wave propagation direction.

For a symmetric laminated orthotropic plate the flexural wave velocity is,

\[ c_f = k \sqrt{D_{11}/\rho h} \]

Where \( D_{11} \) is the bending stiffness in the direction of wave propagation. It is seen that for the long wavelength limit the flexural wave mode is no longer dispersionless, but varies linearly with the wavenumber, \( k \).

INSTRUMENTATION AND EXPERIMENTAL PROCEDURE

TEST MATERIALS - Table 1 lists the plates used in the experiment. The aluminum plates were included in the testing since the theory for isotropic materials is well documented and would allow calibration of our test setup. Plates of different sizes and boundary conditions were included for two reasons, one, to see if changes in the boundary conditions affected the dispersion characteristics, and two, in the larger plates, reflections were delayed and did not cause interference in the lowest extensional and flexural modes.
### Table 1. MATERIAL PROPERTIES

#### ALUMINUM

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness (in)</th>
<th>Length (in)</th>
<th>Width (in)</th>
<th>Boundary conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>6064</td>
<td>0.5000</td>
<td>6.875</td>
<td>3.0</td>
<td>free</td>
</tr>
<tr>
<td>5052-H38</td>
<td>0.0800</td>
<td>16.0</td>
<td>16.0</td>
<td>free</td>
</tr>
<tr>
<td>2024-0</td>
<td>0.0625</td>
<td>30.0</td>
<td>19.0</td>
<td>cantilevered</td>
</tr>
<tr>
<td>7178-T6</td>
<td>0.0560</td>
<td>72.0</td>
<td>48.0</td>
<td>free</td>
</tr>
</tbody>
</table>

#### WAVESPEEDS IN ALUMINUM

Bulk Longitudinal Velocity: \( c_1 = 0.25 \text{ in/\mu s} \)

Bulk Shear Velocity: \( c_2 = 0.12 \text{ in/\mu s} \)

Extensional Plate Velocity: \( c_p = 0.208 \text{ in/\mu s} \)

\[
E = 10 \times 10^6 \text{ lb/in}^2 \\
\rho = 0.000255 \text{ lb-sec}^2/\text{in}^4 \\
\nu = 0.33
\]

#### GRAPHITE/EPOXY

**[0/90]_s PLATE**

IM6/3501-6 (Hercules)

Laminate thickness: \( h = 0.021 \text{ in} \)

Density: \( \rho = 1.49 \times 10^{-4} \text{ lb-sec}^2/\text{in}^4 \)

Extensional Velocity: \( c_e = 0.323 \text{ in/\mu s} \)

Flexural Velocity: \( c_f = 0.323 f, f=\text{frequency in Hz.} \)

**[0/90]_2 PLATE**

AS4/976 (Fiberite, 1376F)

Laminate thickness: \( h = 0.025 \text{ in} \)

Density: \( \rho = 1.46 \times 10^{-4} \text{ lb-sec}^2/\text{in}^4 \)

Extensional Velocity: \( c_e = 0.236 \text{ in/\mu s} \)

Flexural Velocity: \( c_f = 3.07 f, f=\text{frequency in Hz.} \)

#### MATERIAL PROPERTIES

<table>
<thead>
<tr>
<th>( [0/90]_s )</th>
<th>( [0/90]_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_x ) (lb/in²)</td>
<td>( 29.4 )</td>
</tr>
<tr>
<td>( E_y ) (lb/in²)</td>
<td>( 1.63 )</td>
</tr>
<tr>
<td>( \nu_x )</td>
<td>( 0.32 )</td>
</tr>
<tr>
<td>( \nu_y )</td>
<td>( 0.018 )</td>
</tr>
<tr>
<td>( E_s ) (lb/in²)</td>
<td>( 1.22 )</td>
</tr>
<tr>
<td>( A_1 ) (lb/in²)</td>
<td>( 0.329 )</td>
</tr>
<tr>
<td>( D_1 ) (lb-in)</td>
<td>( 20.2 )</td>
</tr>
</tbody>
</table>

**INSTRUMENTATION**

A LeCroy 9100 arbitrary function generator (AFG) was used to generate a 20 V p-p, gated, four cycle sine wave tone burst. The repetition rate of the tone burst was controlled by a Wavetek model 145, 20 MHz pulse generator. The tone burst was then amplified (if needed) using a Krohn-Hite model DCA-50 direct coupled amplifier (frequency range of \( 0-500,000 \text{ Hz} \)), which in turn was input into a Krohn-Hite model MT-55 matching transformer. The amplified signal was then input into a Harisonic model G1004, piezoelectric transducer, resonant at 10 MHz. (For
data points over 500,000 Hz the amplifier and transformer
were disconnected from the system and the tone burst fed
directly into the pulser) The receiving transducer was a
Harisonic model G0504, piezoelectric transducer, resonant at
5 MHz. Both of the transducers had a diameter of 0.25
inches. The signal from the receiving transducer was then
amplified by 60 dB using a Physical Acoustics Corporation
(PAC) preamplifier, model 1220A, with a 20 kHz highpass
filter. The tone burst from the AFG was used to trigger a
LeCroy 9400A digital storage oscilloscope (DSO) which was
then used to capture the waveform detected by the receiver.

PROCEDURE

TONE BURSTS - The transducers were placed a known distance
apart on the plate, and the tone burst of known frequency
was applied to the pulser. The DSO was triggered by the
tone burst from the AFG and the resulting waveform from the
receiver was captured. Both sinusoidal and gaussian tone
bursts were tried with the sinusoidal tone burst being used
since it was easier to locate the four cycles of the tone
burst in the captured waveform. The DSO's cursor was then
placed on a known phase point of the waveform and the time
was read from the DSO's digital readout. Due to noise, the
firmware of the DSO was used to continuously average the
waveform signal, otherwise finding the correct phase point
became difficult. The receiver was then moved a known
distance and the procedure was repeated. By dividing the
distance between the sensors by the Δt from the DSO, the
phase velocity, \(c_{ph}\), of the extensional and flexural waves
could be calculated. Then the wavenumber, \(k = \omega/c_{ph}\), could
be found. \(\omega\) is the circular frequency.

RESONANCE METHOD - For this method, the same instrumentation
was used, however the sensors were placed on opposing edges
of the plate and a continuous sine wave was input into the
pulser. The frequency was then varied until a large
increase in amplitude was detected at the receiving
transducer, thereby indicating that a resonant frequency had
been found. Then the receiving transducer was coupled to
the surface of the plate and moved along the direction of
wave propagation until either two successive maxima or
minima in the displacement had been found, giving one-half
the wavelength. By knowing the wavelength, the wavenumber,
\(k = 2\pi/\lambda\), could be calculated, which gave the phase velocity
through the relationship, \(c_{ph} = \omega/k\), where \(\omega\) is the circular
frequency.

RESULTS

ALUMINUM - Figure 1 shows a waveform generated at 500 kHz
(input tone burst shown) in a 72 x 48 in² sheet of 7178-T6
aluminum. The distance between the sensors was 17 inches.
As can be seen there were two pulses propagating in the
plate, corresponding to the lowest extensional and flexural
modes, which are labeled in the figure. These modes were
verified by applying thumb pressure between the two sensors.
Since the flexural mode displacement is out-of-plane, this
causd the flexural mode to be damped out. The extensional
mode however was relatively unaffected since its
displacement is in-plane.
Fig. 1. Plate wave in 7178-T6 aluminum. X-axis: time, Y-axis: volts. Time/Div=10 μs, Pretrigger=68.6 μs, Channel 1=5 V/Div, Channel 2=0.5 V/Div, zero reference at channel markers.

Fig. 2. Extensional theoretical and experimental dispersion curves for aluminum. \( c_p = 0.208 \text{ in/μs} \).

Figure 2 shows the plot of the extensional Rayleigh-Lamb theory and experimental data for the two measurement techniques in aluminum (\( \nu = 0.33 \)). \( c_p = \sqrt{\frac{E}{\rho(1-\nu^2)}} \) and \( k = kh/2\pi \). The tone bursts fell below the theoretical phase velocity dispersion curve. Why the tone bursts did not propagate with the phase velocity is not clear. The reason may be that the tone bursts were actually narrow band pulses, which should have propagated at the group velocity. However, the data do not fit on the theoretical group velocity dispersion curve either. It can be seen that the data from the resonance method agreed quite well with the phase velocity dispersion curve.
Figure 3 shows the curves for the Rayleigh-Lamb theory and experimental data for the flexural mode in aluminum. This data is for tone bursts only, as trying to create a standing wave in the flexural mode proved too difficult. The data for this mode appears to follow the group velocity curve.

GRAPHITE/EPOXY - The data for the graphite/epoxy plates is shown in Figures 4 and 5. The direction of the wave propagation was parallel to the outer fiber (0°) direction. No resonance data was taken because of the difficulty of mounting the transducers on the edges of the plates. As can be seen, agreement is shown between the classical laminated plate theory for the extensional phase velocity and the experimental data.

For the flexural mode the simple classical plate theory models the dispersion correctly up to $kh/2\pi$ values of approximately 0.1. After this the data begins to deviate substantially from the theory.

DISCUSSION

ALUMINUM - At this point the discrepancy between the theory and the experimental data for the aluminum plates has not been resolved. Reviewing the meager literature in this area, we found that Worlton [6] has confirmed the Rayleigh-Lamb theory using a Leaky-Lamb wave method in the mega-hertz region for isotropic plates. However, his data points near the knee of the flexural dispersion curve begin to diverge from the theory, propagating at a faster velocity than predicted. Also, as $fd$ (frequency $\times$ thickness) increases, his data approaches the bulk shear velocity, $(G/\rho)^{\frac{1}{3}}$, before reaching the Rayleigh velocity. This is also observed in our data. His extensional data similarly exhibits the same departure from theory as ours. Stiffler and Henneke [7] have studied the dispersion characteristics.
in an aluminum plate at sub-megahertz frequencies. In their experimental procedures they used the same method as here to excite the plate waves and the same phase point location method for finding the phase velocities. The results from their study for both the extensional and flexural modes were in agreement with the classical plate theory for phase velocities at low frequencies. However, there seems to be an inconsistency between the data values shown in their plot and the values they give for the material properties and geometry.

GRAPHITE/EPOXY - At this time theoretical plots based on
exact theory for these composites are needed for the low frequency range investigated. The data followed the phase velocity dispersion curve calculated from classical plate theory, up to $kh/2\pi \sim 0.1$.

CONCLUSIONS

The goal of this study was to confirm the low frequency dispersion curves for plates. These curves are needed to improve the location resolution of AE measurements. In studying the propagation of waves in aluminum plates, discrepancies were found between theory and experiment. The cause of these differences has not been resolved. For the composite materials, up to the applicability of classical laminated plate theory, theory and experiment are in agreement.

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REFERENCES