INTRODUCTION

This paper discusses the application of a numerical analysis technique to study eddy-current transducers. Probe design and operating parameters are determined by the constraints of excitation current and the material being tested. Because the depth of penetration of the induced eddy-current is related to the excitation frequency as well as the permeability and conductivity of the specimen, the operating frequency may be selected for a certain depth of resolution. However, subsurface field depth is also a function of relative probe size and lift-off. Although there are no known closed form solutions relating probe size and lift-off to penetration, approximate analytical relationships have been developed by Mottl[3].

The probe design process can be facilitated by a numerical model. The impedance signature of a probe that is scanned over a surface flaw will depend upon the relative size of the sensor, the drive frequency, lift-off position and material parameters of the specimen. Numerical models can help determine signal sensitivity to each parameter. For NDT purposes it is desirable to optimize the appropriate values of coil characteristics that would increase the chances of flaw detection within a given material.

The results of this paper were generated using the finite element method developed and applied to specific NDT problems by Palanisamy[1]. Because the flexibility of this approach permits the modeling of an arbitrary 2-D input probe it can be extended to study the effect of misaligned or tilted coils. These cases have important practical implications since scanning probes are often not perfectly aligned with the specimen.

The study of impedance changes in coils due to flaws is extensive and includes analytical, numerical and experimental techniques. Many of the analytical formulations consider the relationship between the radius of a relatively small flaw and the radius of a circular coil suspended above the material. For these cases the volume flaw theory developed by Auld is incorporated[4]. Dodd et al[5] compared these analytic results with experimental for absolute and differential axisymmetric probes. Numerically, axisymmetric probes signals were predicted for an arbitrary 2-D crack opening by L. D. Sabbagh[6].

This finite element code is capable of analyzing axisymmetric and 2-D problems. A generic single frequency 2-D geometry for an absolute air-core transducer is considered as shown in Figure 1. The analyzed specimen is assumed to be a conducting half-space where the boundaries in the x and y directions terminate at infinity.
FORMULATION

The electromagnetic field problem can be formulated in terms of the magnetic vector potential $A$ where, for 2-D geometries, only the $z$ component is non-zero. The governing equation for sinusoidal field quantities takes on the form

$$\frac{1}{\mu} \nabla^2 A_z - j \omega \sigma A_z = -J_z \quad (1)$$

where $\mu$, $\sigma$, $\omega$ are, respectively, material permeability and conductivity and drive frequency. $J_z$ is the impressed current density supplied to the coil. Equation (1) is solved numerically by finite elements using the Galerkin weighted residual method. The steps involved are part of standard procedures which include casting the equation in a weak form by forming an inner product with a polynomial weighting function $\phi_i$

$$< \left( \frac{1}{\mu} \nabla^2 A_z - j \omega \sigma A_z + J_z \right) \phi_i > = 0 \quad (2)$$

where $<>$ indicates integration over the region of interest. The field quantity $A_z$ is approximated by unknown coefficients $A_j$ multiplying basis functions $\phi_j$ which are chosen from the same family as the weighting functions

$$A_z = \sum_j A_j \phi_j \quad (3)$$

Here the indices $j$ denote nodes throughout the solution domain $\Omega$. Therefore the continuous representation (2) is transformed into a discrete system of equations

$$\sum_j \left[ < \left( \frac{1}{\mu} \nabla^2 \phi_j - j \omega \sigma \phi_j \right) \phi_i > A_j \right] = -< J_z \phi_i > \quad (4)$$
where $A_j$ is understood to be the nodal value of $A_z$ and $\phi_j$ is chosen to be a linear basis function over triangular elements. Integrating (4) by parts the final numerical formulation includes only first derivatives and known boundary conditions on $A_z$

$$
\left[ -\frac{1}{\mu} \nabla \phi_i \cdot \nabla \phi_j - j \omega \sigma \phi_i \phi_j \right] A_j = -<J_z \phi_i> + \frac{1}{\mu} \int_j \frac{\partial A_z}{\partial n} \phi_i \, dl
$$

(5)

The circular absolute coil with current density $J_z$ is modeled in 2-D by two sets of elements as shown in Figure 2a. The same coil can be approximated using nodal currents $I_z$ at the center node as in Figure 2b where $I_z = J_z \Delta_c$ and $\Delta_c$ equals the coil cross sectional area.

The nodal inputs are represented by delta functions, $\delta(\cdot)$, and (5) becomes

$$
\left[ -\frac{1}{\mu} \nabla \phi_i \cdot \nabla \phi_j - j \omega \sigma \phi_i \phi_j \right] A_j = -I_z \delta(y-h) \delta(x-x_1) \phi_i >
$$

$$
+ I_z \delta(y-h) \delta(x-[x_1+r]) \phi_i > + \frac{1}{\mu} \int_l \frac{\partial A_z}{\partial n} \phi_i \, dl
$$

(6)

where $h$ is the lift-off, $r$ is the mean coil width and $x_1$ is the position of the positive current. The impedance calculations for the single wire loop of 2b requires the flux through the two wires and corresponds to the results for a finite coil. Voltage is defined by [1]

$$
V = j \omega \oint_l A \cdot dl
$$

(7)

where

$$
\oint_l A \cdot dl = \int_s (\nabla \times A) \cdot dS
$$

(8a)

and

$$
\nabla \times A = e_x \frac{\partial A_z}{\partial y} - e_y \frac{\partial A_z}{\partial x} ; \quad dS = e_y \, dx \, dz
$$

(8b)

Fig 2a. Coil modeled with 16 elements each with current density $J_z$.

Fig 2b. Coil modeled with nodal values at the coil center.
with $e_x$ and $e_y$ representing unit vectors. Explicitly evaluating the integral (8a) along discrete nodes yields

$$ \int_S (\nabla \times A) \cdot dS = - \int_0^l \int_{x_1}^{x_n} \frac{\partial A_z}{\partial x} \, dx \, dz $$

with

$$ \int_{x_1}^{x_n} \frac{\partial A_z}{\partial x} \, dx = \int_{x_1}^{x_2} \frac{\partial A_z}{\partial x} \, dx + \int_{x_2}^{x_3} \frac{\partial A_z}{\partial x} \, dx + \ldots + \int_{x_{n-1}}^{x_n} \frac{\partial A_z}{\partial x} \, dx $$

where it is assumed that $l$ is a characteristic length of the coil and $x_n$ is the position of the negative wire (i.e. $x_n = x_1 + r$). By numerically calculating these integrals the impedance per unit length is related to the nodal values of $A_z$ by

$$ Z \frac{l}{I} \propto j \omega [A_n - A_{-1}] $$

Note that $A_n$ and $A_{-1}$ have opposite signs so that the magnitudes of their real and imaginary parts add together as required to model an absolute coil. In contrast, the impedance of a differential coil is proportional to the sum $A_n + A_{-1}$, thus detecting the difference in their magnitudes. This implies that a differential coil is more sensitive to an impedance change than an absolute coil. Eddy-current probes measure a relative change in impedance and are usually calibrated with respect to an unflawed specimen prior to scanning. The normalized impedance is obtained by dividing the signal from the flawed half-space by the signal from the flawless half-space. The magnitude and phase of the impedance is recorded at every position and plotted versus the center point of the probe.

For this analysis the discretized region used to model the half-space has a length of 26 mm and a height of 3.75 mm with a simulated crack width of 1.33 mm. The total height of the discretized region is 15 mm. These dimensions are chosen so that the outer boundary which is Dirichlet and set at $A_z = 0$ will not affect the solution. To study the effects of probe sensitivity the crack width remains constant and the coil width $r$, lift-off $h$, permeability of material $\mu$ and drive frequency are varied one at a time.

**TILTED COIL FORMULATION**

Nodal currents are also used to simulate the tilted coil but the delta functions are shifted from the original positions to account for the tilt angle. The tilted sensor can be completely defined by its center lift-off, probe width and angle of inclination $\phi$ with respect to the material surface as shown in Figure 3. The positive and negative input currents have different lift-off heights $h_1$ and $h_2$ and are separated along the x-axis by a distance $d$ such that

$$ h_1 = h - \frac{r}{2} \sin \phi $$

$$ h_2 = h + \frac{r}{2} \sin \phi $$

$$ d = r \cos \phi $$

$$ 1940 $$
An analytic solution for an inclined probe over a half-space is obtained by superimposing the results of two single wires with appropriate lift-offs and x-axis position. The single wire solution was worked out be Stoll [2] and can be superimposed so that for y ≤ 0 in Figure 3 the result is

\[ A_z(x,y) = \frac{\mu_0 I_z}{\pi} \int_0^{\infty} \frac{e^{-kh_1}}{k \mu + \gamma} e^{\gamma y} \cos(kx) \, dk - \]

\[ \frac{\mu_0 I_z}{\pi} \int_0^{\infty} \frac{e^{-kh_2}}{k \mu + \gamma} e^{\gamma y} \cos(k[x-d]) \, dk \] (12)

Upon making the appropriate substitutions (11a-c) equation (12) takes the form

\[ A_z(x,y) = \frac{\mu_0 I_z}{\pi} \int_0^{\infty} \frac{e^{-k(h-\frac{r}{2}\sin \phi)}}{k \mu + \gamma} e^{\gamma y} \cos(kx) \, dk - \]

\[ \frac{\mu_0 I_z}{\pi} \int_0^{\infty} \frac{e^{-k(h+\frac{r}{2}\sin \phi)}}{k \mu + \gamma} e^{\gamma y} \cos(k[x-r\cos \phi]) \, dk \] (13)

The comparison of the finite element model with the analytical predictions provided by (13) is in excellent agreement for x = 0.

For the numerically tilted probe the input currents are again delta functions but this time their position, in general, will not coincide with a node as illustrated in Figure 4. Rewriting (6) for the shifted currents results in
\[
\left[ < \frac{1}{\mu} \nabla \phi_i \cdot \nabla \phi_j - j \omega \sigma \phi_i \phi_j > A_j \right] = -I_z < \delta(y-h_1) \delta(x-x_1) \phi_i > \\
+ I_z < \delta(y-h_2) \delta(x-[x_1+d]) \phi_i > + \frac{1}{\mu} \int \frac{\partial A_2}{\partial n} \phi_i \, dl \quad (14)
\]

The integration of the source terms on the right hand side leads to nodal currents weighted by the relative area covered by the input node and the other two nodes of the element. Thus the current is redistributed to the three surrounding nodes to simulate an input at any point within an element. This is a consequence of the weighted residual formulation. The impedance of the tilted coil is calculated based on (10) where the values of \( A_n \) and \( A_1 \) are found at the coordinates of the input locations by an interpolation of the basis functions [7]. These impedances are normalized by the signals of the flat coil over a half-space.

SIMULATED RESULTS

Impedance magnitude and phase are plotted for different relative probe sizes in Figure 6. As the coil width increases from half of the crack width to five times the flaw opening, the magnitude of the signal also is amplified as more flux passes through a wider coil. The space between the maxima are closer as the coil width decreases, however, making smaller probes more sensitive to the crack edges. Figure 10a shows this peak separation as a function of normalized probe size thus indicating flaw detection capability.

Figure 7 shows the effect of driving frequency on signal strength. The magnitude increases with frequency because of the smaller skin depth in the material. The higher frequency restricts the fields to the surface thus increasing the signal strength at the measurement location. Permeability also affects the depth of penetration of the so that the
percent change in signal magnitude increases with $\mu_r$ as seen in Figure 8. Overall the relative impedance is inversely proportional to $\mu_r$ because of the boundary conditions enforced at the interface.

The effect of tilt angle on impedance measurements is clarified by Figure 9. Signal strength decreases as the probe is more inclined since fewer normal flux lines pass through the coil. As expected the peaks in Figure 9 are different corresponding to the two lift-offs $h_1$ and $h_2$. Both maxima are discernible up to an angle of 30 degrees. Finally Figure 10 is a quantitative indication of probe sensitivity due to probe size and tilt angle. It illustrates the effects of wider probes and larger tilt angles in terms of flaw imaging capability. Future investigations are required to investigate tilt angles between 10 and 20 degrees, to explain the effects of phase angle and to study issues such as crack depth and varying conductivity.

CONCLUSIONS

This paper discusses in two dimensions how numerical analysis techniques can be used to optimize design parameters of a scanning single frequency eddy-current probe. The influence of drive frequency, coil size and material parameters on impedance signatures is clearly illustrated and can be chosen for improved flaw detection. In addition probe misalignment with respect to the surface of the specimen are investigated by calibrating the numerical model against an analytical solution of a tilted coil over a conducting half-space. Subsequent results of a tilted scanning probe over a surface-breaking crack exemplify how the tilt angle affects the signal response.
Fig 9. Numerical impedance signatures (magnitude) for different coil tilt angles.

Fig 10. Signal strength versus tilt angle.

REFERENCES


